Mathematical Methods for International Commerce

Week 12/1: Indefinite Integration (6.1)

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Why It Matters in Economics & Finance

- Integration helps in determining total cost, total revenue, and accumulated savings over time.
- It is crucial in calculating areas under curves, which represent total accumulated quantities.
- Applications include:
  - Finding total cost from marginal cost
  - Determining total revenue from marginal revenue
  - Estimating accumulated savings from marginal propensity to save

Learning Objectives

- Recognize the notation for indefinite integration
- Integrate power functions and exponential functions
- Apply integration to find total cost and total revenue
- Apply integration to determine consumption and savings functions
- Use inspection for integrating complex functions

Understanding Indefinite Integration

- Indefinite integration is the reverse of differentiation.
- The general form:

$$\int f(x)\,dx = F(x) + C$$

  where:

  - $f(x)$ is the integrand
  - $F(x)$ is the antiderivative
  - C is the constant of integration

Note: The antiderivative, also known as the indefinite integral, is a function whose derivative equals the original function $f(x)$. In essence, finding the antiderivative reverses the process of differentiation.

Understanding Indefinite Integration (cont.)

- Example:
  - If $f(x) = 3x^2$, then:

$$\int 3x^2 \, dx = x^3 + C$$

  - Why? Because $\frac{d}{dx}(x^3 + C) = 3x^2$.

The Constant of Integration

- Why do we add $+C$?
  - Represents an **arbitrary constant** since differentiation eliminates constants.
- Example:
  - $\int 3x^2 \, dx = x^3 + C$
  - Both $x^3 + 5$ and $x^3 - 7$ differentiate to $3x^2$.
- From economics perspective, C can represent fixed costs, initial revenue/values, or other baseline values in economic models.
- In practice, we often determine C using initial conditions or boundary values.

Power Rule for Indefinite Integrals

- General formula:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

- Examples:
  - $\int x^5 \, dx = \frac{x^6}{6} + C$
  - $\int x^{-3} \, dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

Integration of Exponential Functions

- Formula:

$$\int e^{mx}\,dx = \frac{1}{m}e^{mx} + C$$

- Example:
  - $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$
  - $\int e^{-0.5x}\,dx = -2e^{-0.5x} + C$

Practical Example: Marginal Revenue to Total Revenue (1)

- Suppose the **marginal revenue function** is given by:

$$MR(x) = 50 - 2x$$

- To find the total revenue function:

$$TR(x) = \int (50 - 2x)\, dx$$

$$= 50x - x^2 + C$$

- **Economic Interpretation:**
  - The area under the MR curve represents the total revenue.

Practical Example: Marginal Revenue to Total Revenue (2)

- If the marginal revenue is given by:

$$MR(x) = 12 - 2x$$

- The total revenue function is:

$$TR(x) = \int (12 - 2x)\, dx = 12x - x^2 + C$$

- Interpretation:
  - The total revenue function represents the total income generated from selling x units of a good.
  - The constant C can be interpreted as fixed costs or initial revenue.
  - The area under the marginal revenue curve gives the total revenue.
  - The total revenue function changes quadratically with respect to x.

Practical Example: Marginal Cost to Total Cost (1)

- If the marginal cost is given by:

$$MC(x) = 3x^2 + 5$$

- The total cost function is found by integrating:

$$TC(x) = \int (3x^2 + 5)\, dx = x^3 + 5x + C$$

Practical Example: Marginal Cost to Total Cost (2)

- Given the marginal cost function:

$$MC(x) = 4x^2 - 3x + 7$$

- Integrate to find the total cost function:

TC(x) = \int (4x^2 - 3x + 7) \, dx = \frac{4x^3}{3} - \frac{3x^2}{2} + 7x + C $$

Economic Example: Consumption & Savings

- Given the marginal propensity to consume (MPC):

$$MPC = 0.8$$

- Integrate to find the consumption function:

$$\int 0.8\, dY = 0.8Y + C$$

- The savings function can be derived similarly by considering the marginal propensity to save (MPS).

Method of Inspection

- For more complex integrals, identify a form that simplifies to a known pattern:

$$\int (5x^2 + 3x + 2)\,dx = \int 5x^2\,dx + \int 3x\,dx + \int 2\,dx$$

$$= \frac{5x^3}{3} + \frac{3x^2}{2} + 2x + C$$

Practice Problems

1. Integrate:
    - (a) $\int 4x^3 \, dx$
    - (b) $\int e^{2x} \, dx$
2. Find the total cost given $MC = 5x^2 - 3x + 4$.
3. Determine the total revenue function given $MR = 15 - 0.5x$.
4. Given the marginal propensity to consume is $MPC = 0.8$, determine the consumption function if initial consumption is 100.

Summary

- Indefinite integration helps in finding total values given marginal functions.
- Applications in economics include calculating total cost, total revenue, and accumulated savings.
- Key formulas:
  - Power Rule
  - Exponential Function
  - Method of Inspection

2. Group Activity

Math Battle: Indefinite Integrals

Objective: Apply indefinite integration concepts to solve economic problems.
Format: 4 groups, 4 students each. Each group receives a distinct problem.
Time: 20 minutes to solve, 5 minutes to present.

Math Battle Rules:

- Each group presents their solutions.
- Other groups can challenge the solution if they find discrepancies.
- Correct challenges earn 2 points; incorrect challenges lose 1 point.
- Winning group gets a prize!

Group 1: Marginal Cost Analysis

A firm's marginal cost function is given by:

$$MC(x) = 15x^2 - 40$$

1. Determine the total cost function by integrating the MC function.

2. Identify the constant of integration given that the fixed cost is $120.

Group 2: Consumption and Savings

The marginal propensity to save is given by:

$$MPS = 0.2x - 0.05$$

1. Integrate the MPS to find the savings function.

2. If the initial savings is $50, determine the complete savings function.

Group 3: Investment Accumulation

A firm invests \$200 annually in a project, and the marginal investment function is given by:

$$MI(t) = 200e^{0.03t}$$

1. Integrate to find the total investment function.

2. Determine the accumulated investment after 5 years.

Group 4: Demand Function Analysis

A product's marginal demand function is given by:

$$MD(Q) = -3Q^2 + 12Q - 5$$

1. Integrate to find the demand function.

2. Determine the constant of integration if the initial demand is $50.

3. Home work #2

Homework #2

- **Due Date:** June 13, 2025, before the start of class.
- **Submission Format:** Submit your solutions as a single PDF file via the Cyber Campus.

- **Instructions:**
  - Clearly show all steps and calculations.
  - Include explanations for your answers where applicable.
  - Ensure your submission is neat and well-organized.
  - Bring any questions to the office hours or email me.

- Work on your Home Assignment #2 (Jacques, 10th edition, Chapters 5-9):
  - Chapter 5.1: Exercise 5.1, Problem 5 (p. 411)
  - Chapter 5.2: Exercise 5.2, Problem 7 (p. 427)
  - Chapter 5.3: Exercise 5.3, Practice Problem inside the chapter (p. 432 only)
  - Chapter 5.4: Exercise 5.4, Problem 5 (p. 457)
  - Chapter 5.5: Exercise 5.5, Problems 6 and 7 (p. 467)
  - Chapter 5.6: Exercise 5.6, Problem 4 (p. 479)
  - Chapter 6.1: Exercise 6.1, Problem 3 (p. 506)
  - Chapter 6.2: Exercise 6.2, Problem 6 (p. 521)
  - Chapter 7.1: Exercise 7.1, Problem 5 (p. 553)
  - Chapter 7.2: Exercise 7.2, Problem 6 (p. 572)
  - Chapter 7.3: Exercise 7.3, Problem 5 (p. 584)
  - Chapter 8.1: Exercise 8.1, Problem 6 (p. 615)
  - Chapter 8.2: Exercise 8.2, Problem 3 (p. 626)
  - Chapter 9.1: Exercise 9.1, Problem 4 (p. 655)
  - Chapter 9.2: Exercise 9.2, Problem 3 (p. 670)

Good luck!

Any QUESTIONS?

Thank you for your attention!

Next Classes

- (May 23) Definite Integration (6.2)

Reminder:

- **Quiz 2**: May 28, 2025