

Mathematical Methods for International Commerce

Week 14/2-15/1: Linear Programming (8.1)

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Agenda

1. Linear Programming (8.1)
2. Group activity:
3. Homework #2

1. Linear Programming (8.1)

Why It Matters in Economics & Business

Linear programming is a powerful tool to:

- Optimize **profits, costs, resource use**
- Solve **production, transportation, investment** problems
- Understand **feasibility** and **trade-offs** in economic models

Real-world applications include:

- Supply chain optimization
- Product mix decisions
- Agricultural planning
- Finance: portfolio risk constraints

| Linear programming helps make **data-driven** decisions in complex environments.

What is Linear Programming?

A linear programming (LP) problem:

- Optimize a linear objective function
- Subject to a set of linear inequalities (constraints)

General form:

$$\text{Maximize or Minimize } Z = c_1x + c_2y$$

Subject to:

$$a_1x + b_1y \leq d_1$$

$$a_2x + b_2y \leq d_2$$

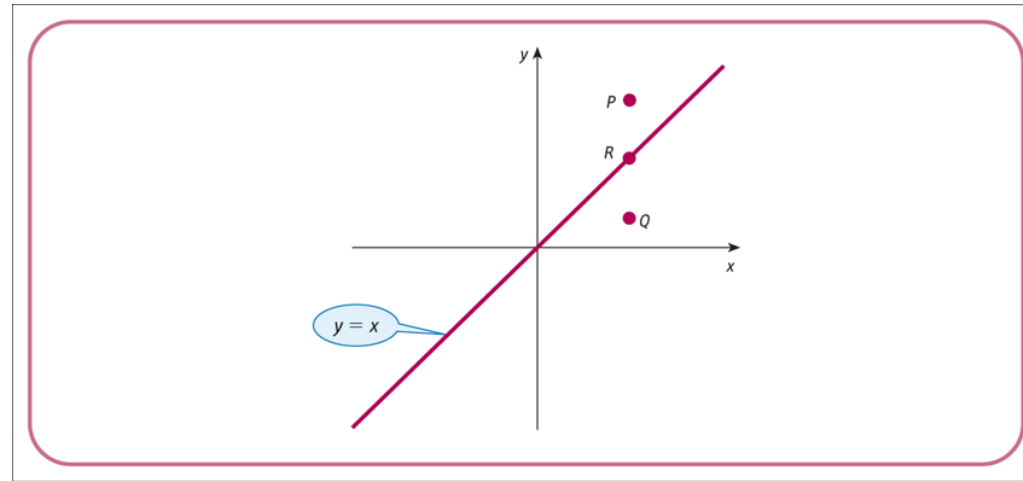
$$x, y \geq 0$$

Interpreting Linear Inequalities Graphically

Linear inequality: $y \geq x$

- Points **above** the line $y = x$: satisfy $y > x$
- Points **on** the line: satisfy $y = x$
- Points **below** the line: satisfy $y < x$

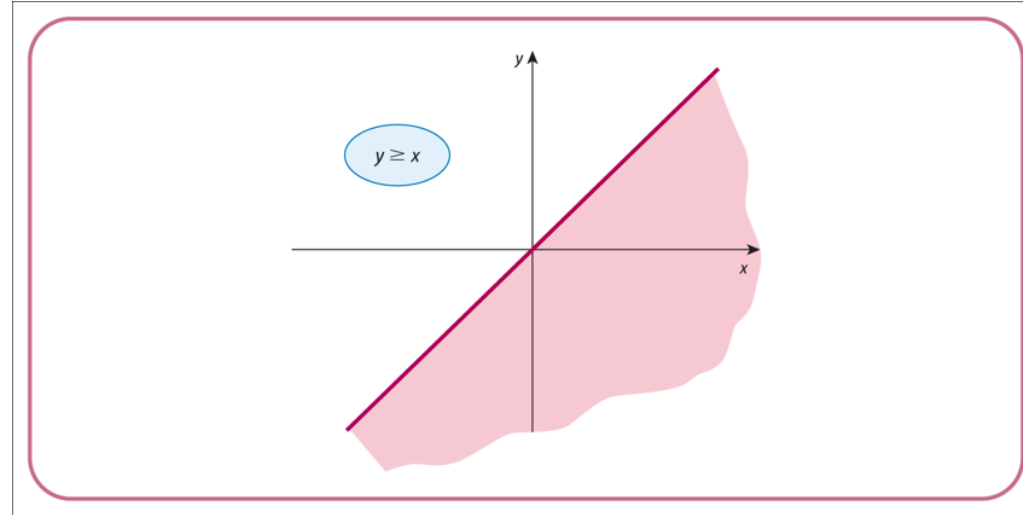
Graph of $y = x$ with Sample Points



- Point **P** lies above $y = x$
- Point **Q** lies below $y = x$
- Point **R** lies on $y = x$

Graph of $y \geq x$: Shading False Region

We shade the region not satisfying the inequality:



This helps highlight the feasible region clearly for optimization.

General Strategy

To sketch any linear inequality:

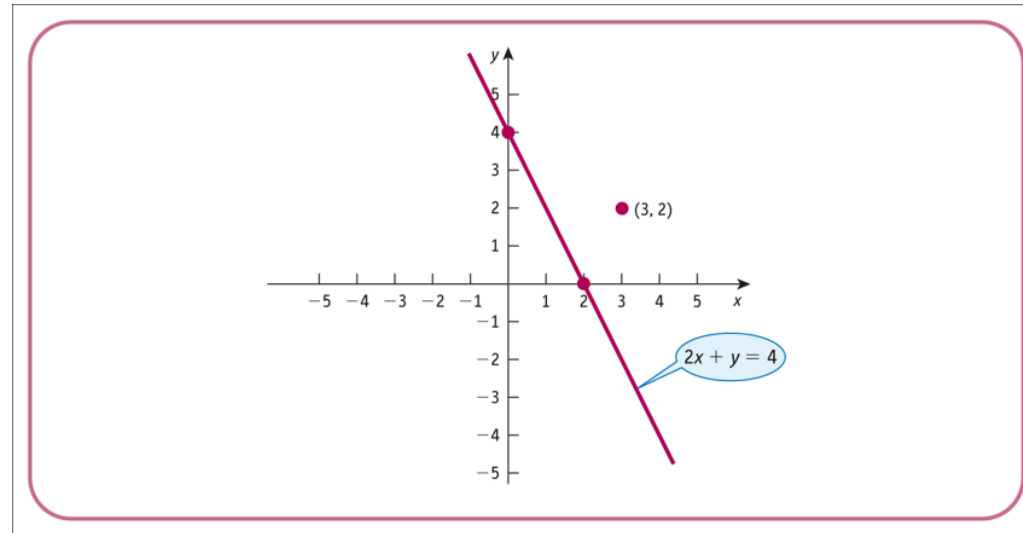
1. Draw the boundary line $dx + ey = f$
2. Use a test point to determine shading:
 - If inequality holds \rightarrow region of interest
 - Otherwise \rightarrow shade opposite side
3. Use **solid line** for \leq, \geq , **dashed** for $<, >$

Example: $2x + y < 4$

Step 1: Plot $2x + y = 4$

- When $x = 0$, $y = 4$
- When $y = 0$, $x = 2$

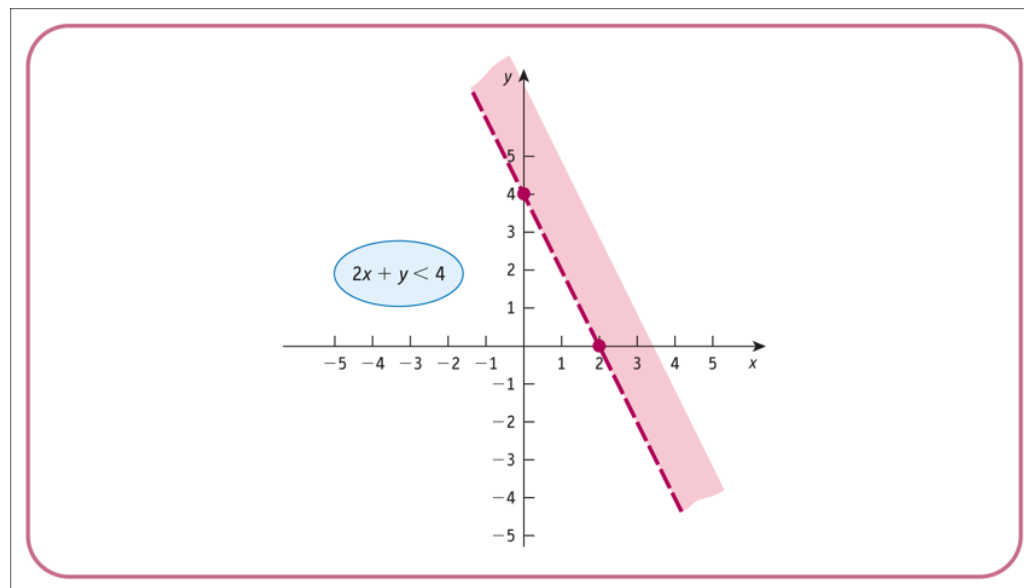
Plot line through $(0, 4)$ and $(2, 0)$



Step 2: Choose a Test Point

Test point: $(3, 2)$

- Evaluate: $2(3) + 2 = 8 \not< 4$
- So: region of interest lies **below** the line
- Use **broken line** for $<$

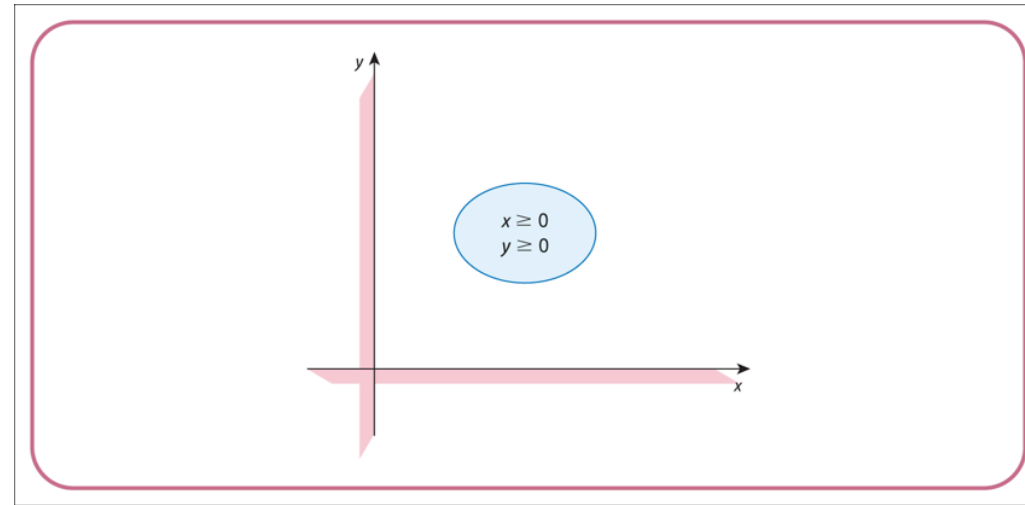


Feasible Region from Multiple Inequalities

To define a feasible region, plot and intersect:

- $x + 2y \leq 12$
- $-x + y \leq 3$
- $x \geq 0, y \geq 0$

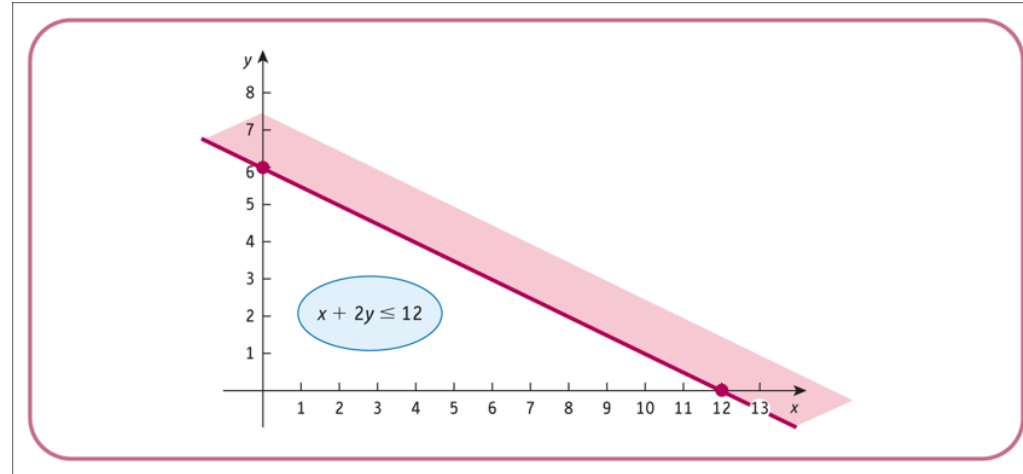
Non-Negativity Constraints



Top-right quadrant: restricts us to economically meaningful values of x and y .

Constraint $x + 2y \leq 12$

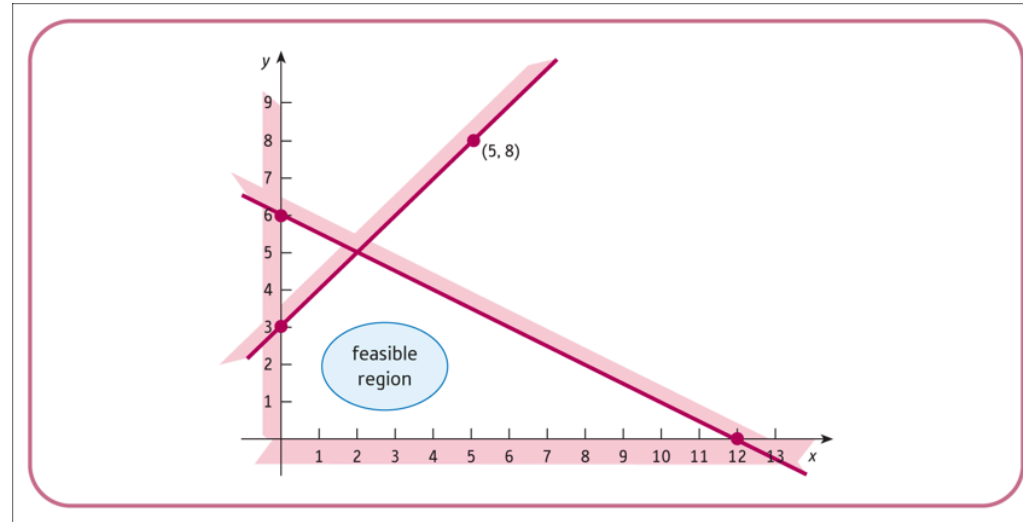
- Line passes through $(0, 6)$ and $(12, 0)$
- Test point $(0, 0)$: inequality holds \rightarrow shade above



Final Constraint $-x + y \leq 3$

- Line passes through $(0, 3)$ and $(5, 8)$
- Test point $(0, 0)$: holds \rightarrow shade above

The **feasible region** is the unshaded area satisfying all inequalities.



Summary: Graphical Approach (2 Variables)

Steps:

1. Express constraints as **equalities** to sketch lines
2. Identify **feasible region** (satisfying all inequalities)
3. Find **corner points** of feasible region
4. Evaluate **objective function** at each corner
5. Choose max or min value

Note: Only works for 2-variable problems. For higher dimensions: use Simplex Method.

Example: Maximize Profit

A firm produces two goods: x and y

Objective: Maximize $Z = 5x + 3y$

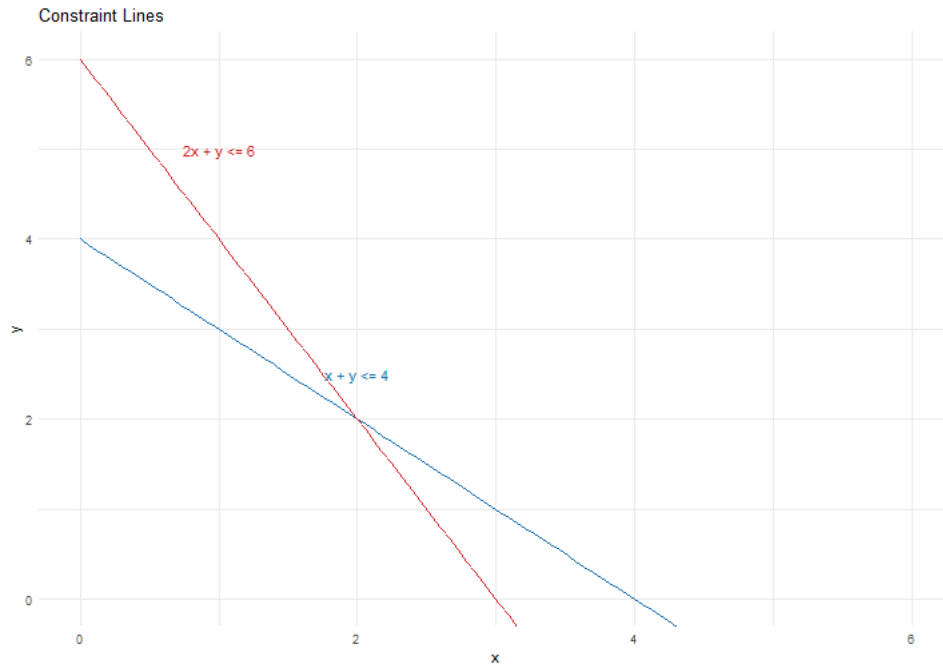
Subject to:

$$x + y \leq 4$$

$$2x + y \leq 6$$

$$x, y \geq 0$$

Step 1: Plot Constraints



Step 2-4: Feasible Region and Objective Evaluation

Feasible points: Intersections and axis cuts:

- (0, 0)
- (0, 4)
- (2, 2)
- (3, 0)

Evaluate $Z = 5x + 3y$ at each:

Point	Z
(0,0)	0
(0,4)	12
(2,2)	16 ← Max
(3,0)	15

Interpretation in Business Context

This tells the firm:

- To **maximize profits**, it should produce 2 units of x and 2 units of y
- Constrained by available **resources**

This helps with:

- **Resource allocation**
- **Product mix decisions**

Special Cases

1. No Solution (Infeasible)

- When constraints **conflict**

2. Infinite Solutions

- Objective function is **parallel** to a constraint edge

3. Unbounded Solution

- No upper limit; occurs when constraints don't bound the feasible region

Add visual plots to illustrate each.

Practice Problem (Group)

A bakery makes bread x and muffins y .

Profit: $Z = 3x + 4y$

Subject to:

$$\begin{aligned}x + 2y &\leq 8 \\5x + 3y &\leq 15 \\x, y &\geq 0\end{aligned}$$

Tasks:

- Graph the constraints
- Identify feasible region
- Compute profit at each corner point
- Find optimal solution

Summary

- Linear programming optimizes linear functions under constraints
- Graphical method works for 2-variable problems
- Corner points of feasible region yield optimal solutions
- Special cases include infeasibility, infinite solutions, and unbounded solutions

2. Group activity: Solving matrixes

3. Home work #2

Homework #2

- **Due Date:** June 13, 2025, before the start of class.
- **Submission Format:** Submit your solutions as a single PDF file via the Cyber Campus.
- **Instructions:**
 - Clearly show all steps and calculations.
 - Include explanations for your answers where applicable.
 - Ensure your submission is neat and well-organized.
 - Bring any questions to the office hours or email me.
- Work on your Home Assignment #2 (Jacques, 10th edition, Chapters 5-9):
 - Chapter 5.1: Exercise 5.1, Problem 5 (p. 411)
 - Chapter 5.2: Exercise 5.2, Problem 7 (p. 427)
 - Chapter 5.3: Exercise 5.3, Practice Problem inside the chapter (p. 432 only)
 - Chapter 5.4: Exercise 5.4, Problem 5 (p. 457)
 - Chapter 5.5: Exercise 5.5, Problems 6 and 7 (p. 467)
 - Chapter 5.6: Exercise 5.6, Problem 4 (p. 479)
 - Chapter 6.1: Exercise 6.1, Problem 3 (p. 506)
 - Chapter 6.2: Exercise 6.2, Problem 6 (p. 521)
 - Chapter 7.1: Exercise 7.1, Problem 5 (p. 553)
 - Chapter 7.2: Exercise 7.2, Problem 6 (p. 572)
 - Chapter 7.3: Exercise 7.3, Problem 5 (p. 584)
 - Chapter 8.1: Exercise 8.1, Problem 6 (p. 615)
 - Chapter 8.2: Exercise 8.2, Problem 3 (p. 626)
 - Chapter 9.1: Exercise 9.1, Problem 4 (p. 655)
 - Chapter 9.2: Exercise 9.2, Problem 3 (p. 670)

Good luck!

Any QUESTIONS?

Thank you for your attention!

Next Classes

- (June 6: Recorded lecture) Linear Programming (8.1)