Mathematical Methods for International Commerce

Week 2/2: Supply and Demand Analysis, Transposition of Formulae, National Income Determination

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Why Are These Concepts Essential in Economics?

1. **Supply and Demand Analysis** → Understanding **market equilibrium**.
2. **Transposition of Formulae** → Rearranging equations to **solve for key economic variables**.
3. **National Income Determination** → Finding **equilibrium GDP** in macroeconomic models.

Learning Objectives

At the end of this section, you should be able to:

1. **Use function notation**, $y = f(x)$.
2. **Identify** endogenous and exogenous variables.
3. **Sketch and interpret** a **linear demand function**.
4. **Sketch and interpret** a **linear supply function**.
5. **Determine** equilibrium price and quantity **graphically** and **algebraically**.
6. **Solve simultaneous equations** for multi-commodity equilibrium.
7. **Transpose formulae** to solve for unknown variables in economic models.
8. **National Income Determination**: Understand the concept of national income and its determinants.

1. Supply and Demand Analysis

Function Notation in Economics

Economic models often use **function notation**:

$$y = f(x)$$

where:

- (x) is the **independent variable** (e.g., price, income).
- (y) is the **dependent variable** (e.g., quantity demanded, total revenue).

Note: **Functions** describe **relationships** between variables.

Example: Demand Function
If demand depends on price:

$$Q_d = f(P) = 100 - 5P$$

where:

- Q_d is **quantity demanded**.
- P is **price**.
- The function shows that as **price increases, demand decreases**.

Endogenous vs. Exogenous Variables

- **Endogenous Variables** → **Determined within the model** (e.g., equilibrium price and quantity).
- **Exogenous Variables** → **Determined outside the model** (e.g., government policies, income levels).

Example:

$$Q_d = 100 - 5P$$

$$Q_s = 20 + 3P$$

Here:

- **(P)** (price) and **(Q)** (quantity) are **endogenous.**
- **Shocks like taxes or subsidies** are **exogenous.**

Demand and Supply Functions

Linear Demand Function

A linear demand function follows:

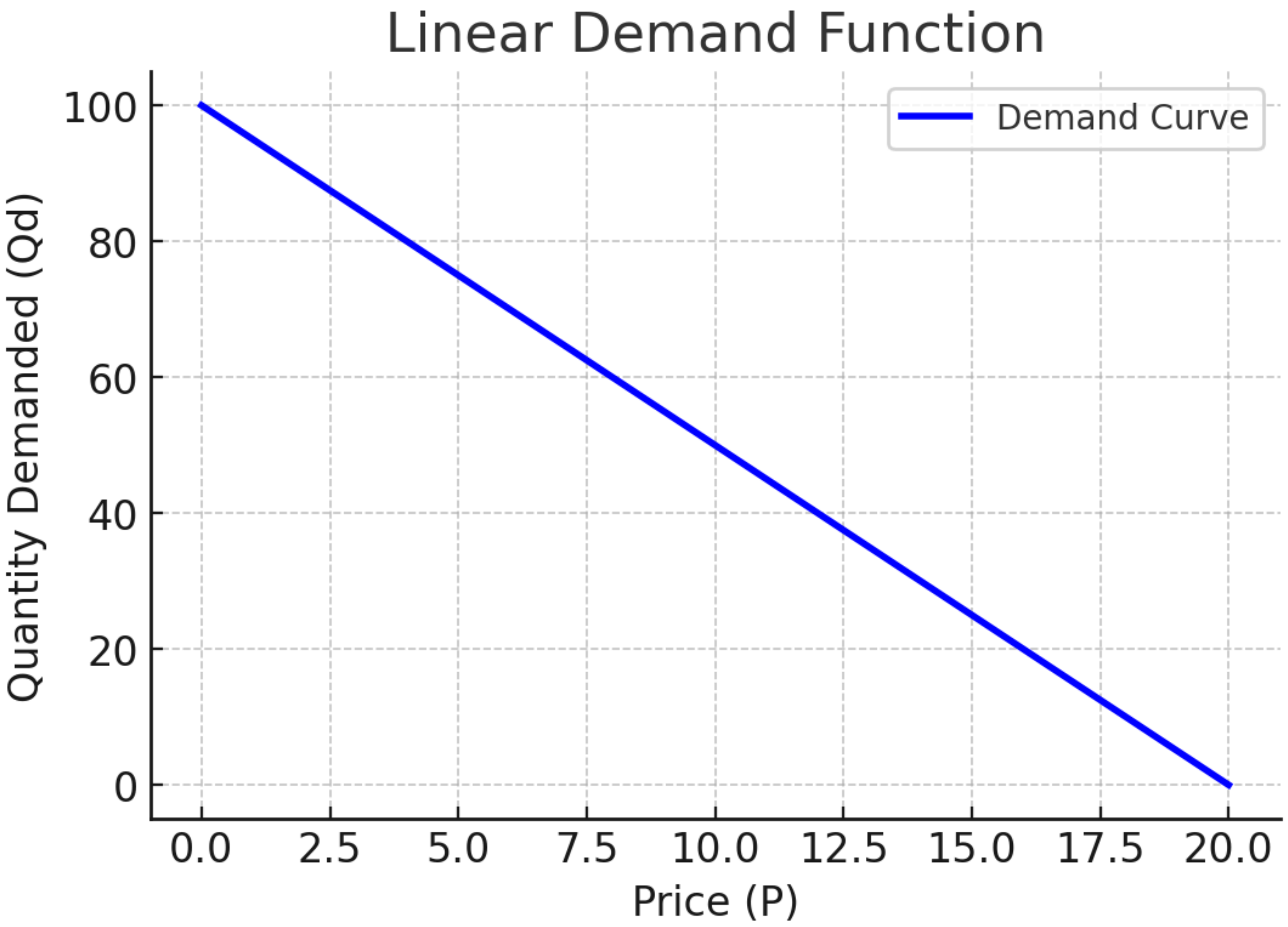
$$Q_d = a - bP$$

where:

- (a) is the **intercept** (max demand at price = 0).
- (b) is the **slope** (rate at which demand falls when price rises).

Demand and Supply Functions (cont)

Demand Graph



Demand and Supply Functions (cont)

Supply Function

A **linear supply function** follows:

$$Q_s = c + dP$$

where:

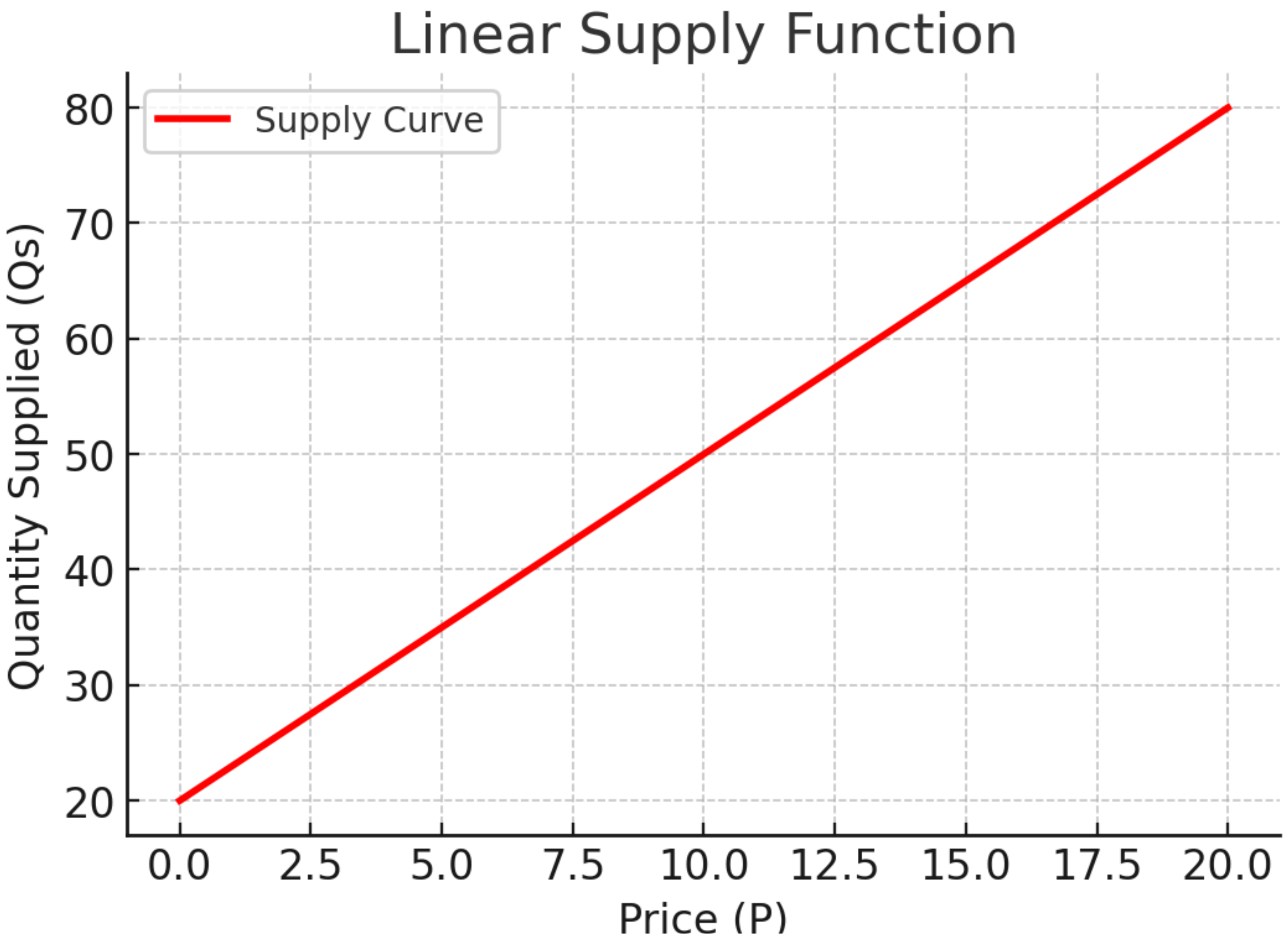
- (c) is the **intercept** (minimum quantity supplied at zero price).
- (d) is the **slope** (rate at which supply increases as price rises).

Example Supply Function

$$Q_s = 20 + 3P$$

Demand and Supply Functions (cont)

Supply Graph



Market Equilibrium Concept

Market equilibrium occurs where **quantity demanded** equals **quantity supplied**:

$$Q_d = Q_s$$

Using the demand function:

$$Q_d = 100 - 5P$$

And the supply function:

$$Q_s = 20 + 3P$$

Setting them equal:

$$100 - 5P = 20 + 3P$$

Solving for Equilibrium Price and Quantity

Solving for P^*:

$$100 - 5P = 20 + 3P$$

$$80 = 8P$$

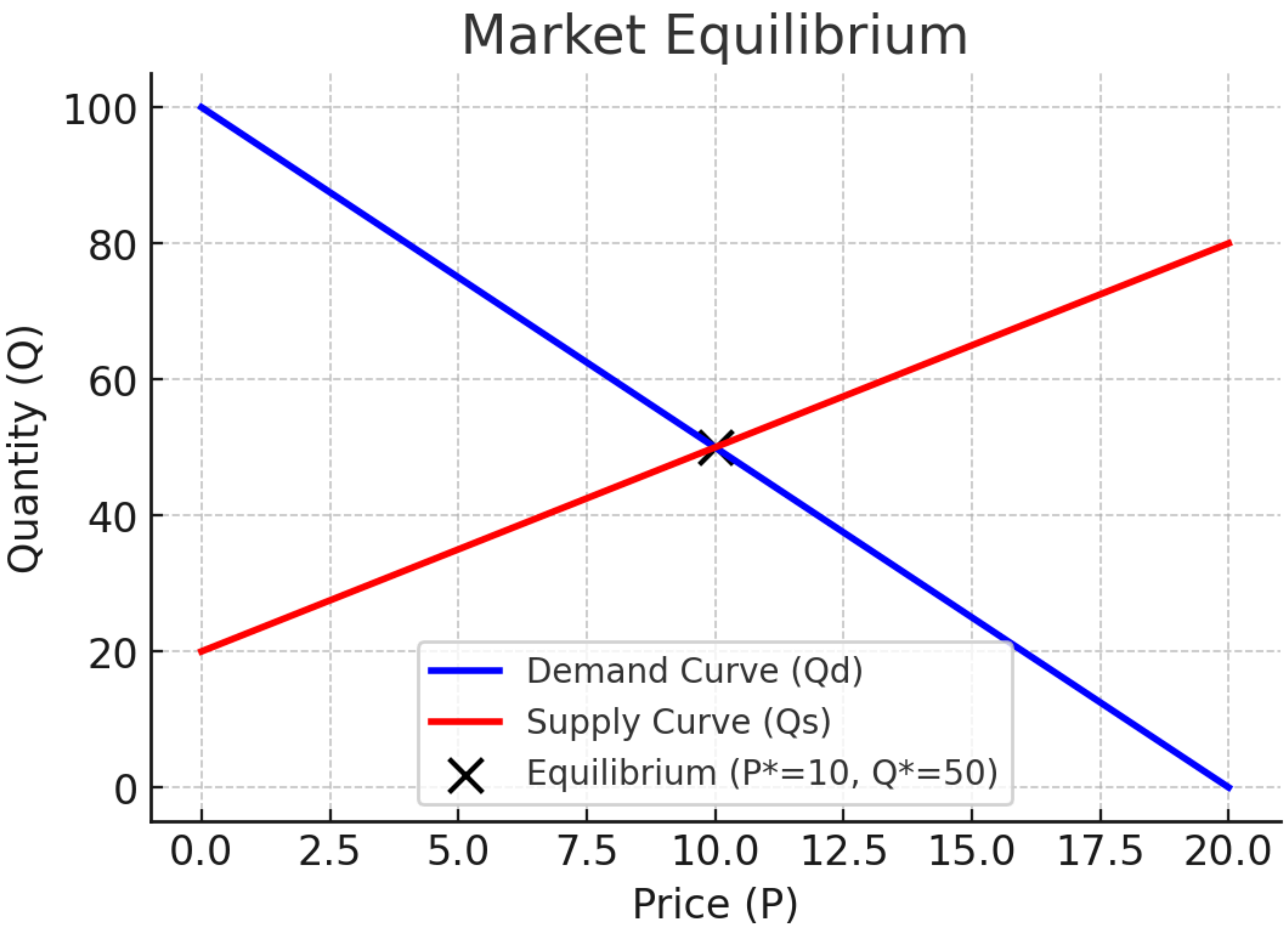
$$P^* = 10$$

Substituting P^* into the demand equation:

$$Q^* = 100 - 5(10) = 50$$

Equilibrium Price: P^* = 10;
Equilibrium Quantity: Q^* = 50

Market Equilibrium Plot



Your Turn: Practice Problem

Given the **demand and supply functions** for one good:

Demand Function:

$$Q_d = 50 - 2P$$

Supply Function:

$$Q_s = -10 + 3P$$

1. **Find the equilibrium price** P^* and **quantity** Q^*.
2. **Plot the demand and supply functions.**
3. **Show the equilibrium point.**

Your Turn: Practice Problem (cont)

Given the **demand and supply functions** for one good:

Demand Function:

$$Q_d = 40 - 7P$$

Supply Function:

$$Q_s = -30 + 4P$$

1. **Find the equilibrium price** P^* and **quantity** Q^*.
2. **Plot the demand and supply functions.**
3. **Show the equilibrium point.**

Multi-Commodity Market Equilibrium

In **multi-commodity markets**, equilibrium is determined by **simultaneously solving multiple demand and supply equations**.

- **Each good has its own demand and supply function.**
- **Equilibrium occurs when** demand equals supply **for all goods**.
- **Requires solving a system of linear equations.**

Two-Commodity Market Equilibrium

Consider an economy with **two goods** where:

Demand Functions:

$$Q_{d1} = 50 - 2P_1 + P_2$$
$$Q_{d2} = 60 - P_1 - 3P_2$$

Supply Functions:

$$Q_{s1} = -10 + 3P_1$$
$$Q_{s2} = 5 + 2P_2$$

Find the equilibrium prices P_1^* and P_2^*.

Step 1: Rearrange into Standard Form

At **equilibrium**, demand = supply:

$$\begin{aligned} -5P_1 + P_2 &= -60 \quad (1) \\ -P_1 - 5P_2 &= -55 \quad (2) \end{aligned}$$

Rearranged equations are now in standard form.

Step 2: Solve for P_2^*

Multiply **Equation (2)** by **5**:

$$-5P_1 + P_2 = -60$$
$$-5P_1 - 25P_2 = -275$$

Now subtract:

$$(-5P_1 - 25P_2) - (-5P_1 + P_2) = -275 + 60$$

$$-26P_2 = -215$$

$$P_2^* = \frac{-215}{-26} = 8.27$$

Equilibrium price for Good 2: $P_2^* = 8.27$.

Step 3: Solve for P_1^*

Substituting $P_2 = 8.27$ into **Equation (1)**:

$$-5P_1 + 8.27 = -60$$

$$-5P_1 = -60 - 8.27$$

$$P_1^* = \frac{-68.27}{-5} = 13.65$$

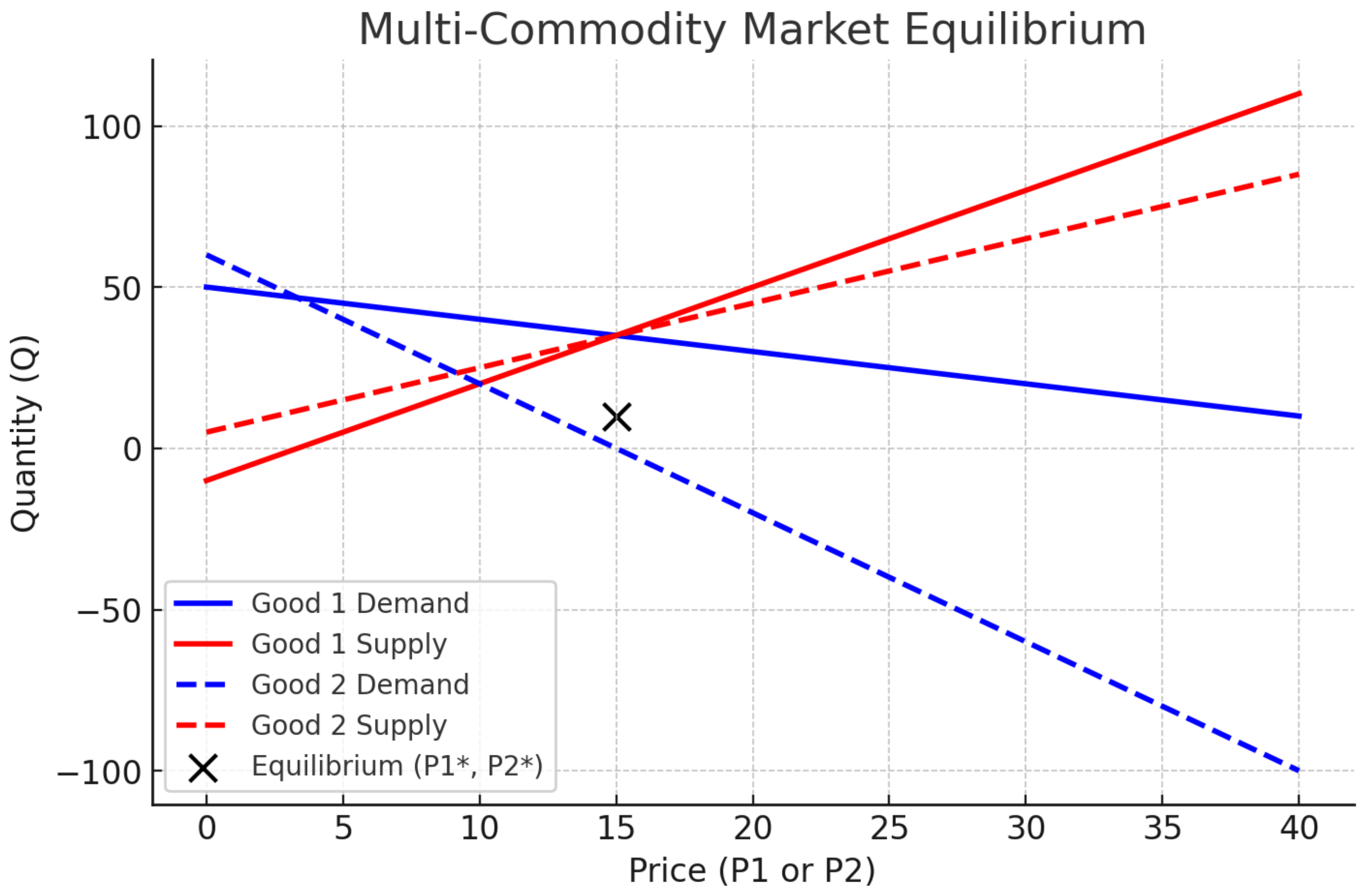
Equilibrium price for Good 1: $P_1^* = 13.65$.

Final Answer

[\boxed{ P_1^ *= 13.65, \quad P_2^* = 8.27 }]

These are the equilibrium prices where demand equals supply.

Graphical Solution



Your Turn: Practice Problem

Solve for equilibrium prices P_1^* and P_2^* in the following two cases.

Example 1: Two-Commodity Market

Consider an economy with **two goods** where:

Demand Functions:

$$\begin{aligned} Q_{d1} &= 80 - 4P_1 + 2P_2 \\ Q_{d2} &= 100 - 3P_1 - P_2 \end{aligned}$$

Supply Functions:

$$\begin{aligned} Q_{s1} &= 20 + 5P_1 \\ Q_{s2} &= 10 + 4P_2 \end{aligned}$$

Find the equilibrium prices P_1^* and P_2^* by solving:

$$\begin{aligned} 80 - 4P_1 + 2P_2 &= 20 + 5P_1 \\ 100 - 3P_1 - P_2 &= 10 + 4P_2 \end{aligned}$$

Example 2: Alternative Market System

A different economy has the following market equations:

Demand Functions:

$$
\begin{aligned}
Q_{d1} &= 60 - 3P_1 + P_2 \\
Q_{d2} &= 90 - 2P_1 - 2P_2
\end{aligned}
$$

Supply Functions:

$$
\begin{aligned}
Q_{s1} &= -5 + 4P_1 \\
Q_{s2} &= 15 + 3P_2
\end{aligned}
$$

Find the equilibrium prices P_1^* and P_2^* by solving:

$$
\begin{aligned}
60 - 3P_1 + P_2 &= -5 + 4P_1 \\
90 - 2P_1 - 2P_2 &= 15 + 3P_2
\end{aligned}
$$

Instructions

- **Rearrange equations** into standard form ($Ax + By = C$).
- **Use elimination or substitution** to solve.
- **Interpret the results**: What do the equilibrium prices mean for the market?

2. Transposition of Formulae

What is Transposition of Formulae?

1. Rearranging an equation to express one variable in terms of others.
2. Useful in economics, finance, and business analysis.
3. Allows solving for unknown variables in different contexts.

Example:

$$A = B + C$$

To express (C) in terms of (A) and (B):

$$C = A - B$$

Example 1: Solving for Price in Demand Function

Consider a **linear demand function**:

$$Q_d = a - bP$$

We want to solve for (P):

$$P = \frac{a - Q_d}{b}$$

If $Q_d = 100 - 5P$, solve for (P):

$$P = \frac{100 - Q_d}{5}$$

Now we can **calculate the price for any given quantity**.

Example 2: Solving for Time in Interest Formula

The compound interest formula is:

$$A = P(1 + r)^t$$

Solve for (t):

1. Divide both sides by (P):

$$\frac{A}{P} = (1 + r)^t$$

1. Take the natural logarithm:

$$\ln\left(\frac{A}{P}\right) = t \ln(1 + r)$$

1. Solve for (t):

$$t = \frac{\ln(A/P)}{\ln(1 + r)}$$

3. National Income Determination

What is National Income?

- **Total value of goods and services produced** in a country over a period.
- **Measures economic activity** and **standard of living**.
- **Components of National Income**:
  - **Consumption (C)**: Spending by households.
  - **Investment (I)**: Spending by firms.
  - **Government Spending (G)**: Public expenditure.
  - **Net Exports (NX)**: Exports minus imports.
- **National Income Formula**:

$$Y = C + I + G + NX$$

The Consumption Function

The **linear consumption function**:

$$C = C_0 + cY$$

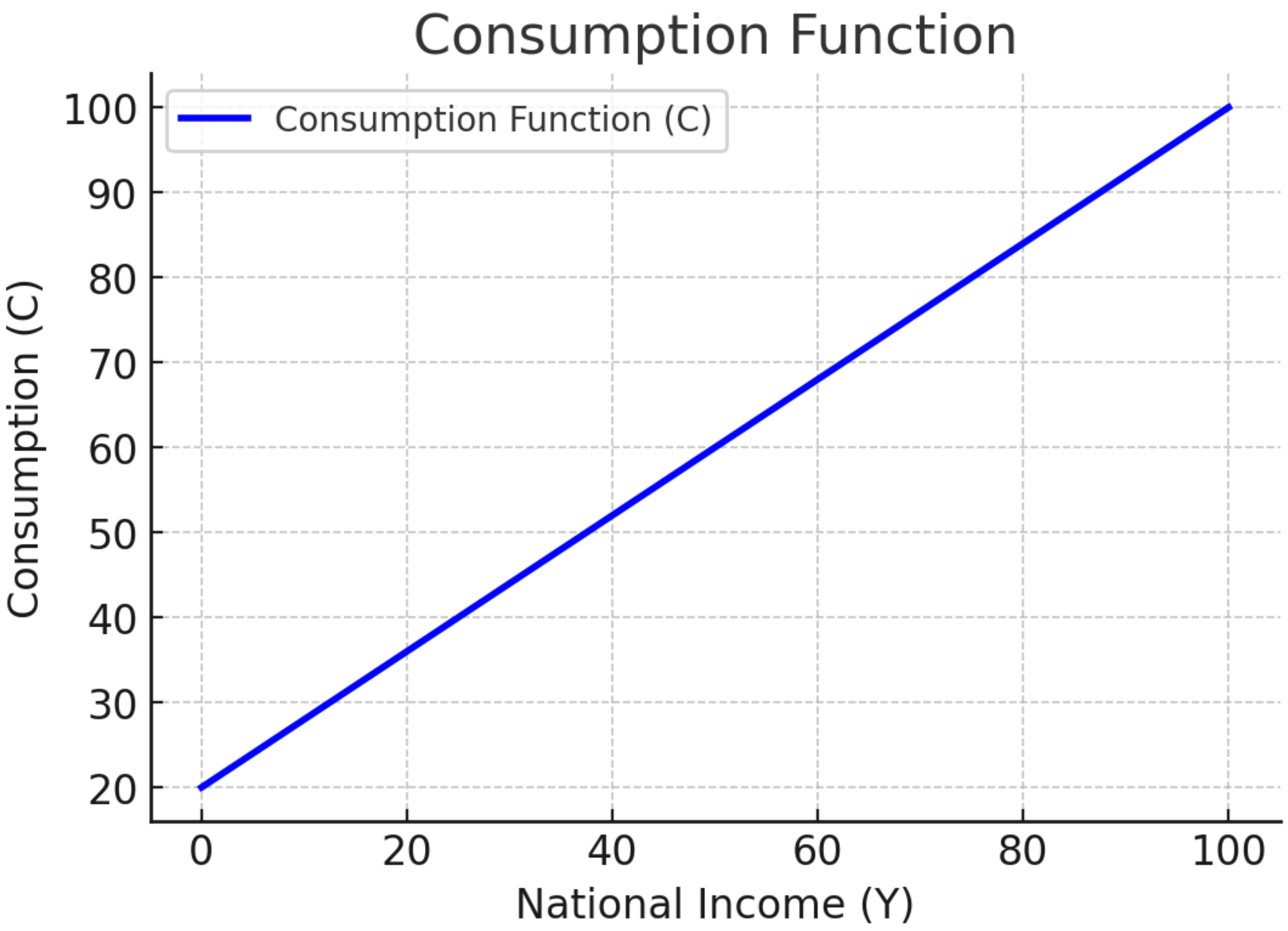
where:

- (C) = Total consumption
- (C_0) = Autonomous consumption (spending even if income is zero)
- (c) = Marginal propensity to consume (MPC) (i.e., how much of each extra unit of income is consumed)
- (Y) = National income

Interpretation:

- If **income increases, consumption increases.**
- The slope (c) tells us **how much of each extra unit of income is consumed.**

Plotting the Consumption Function



The Savings Function

In macroeconomics, **savings** is the portion of income that is **not spent on consumption**.

The **linear savings function** is:

$$S = S_0 + sY$$

where:

- (S) = **Total savings**
- (S_0) = **Autonomous savings** (can be negative if dissaving occurs)
- (s) = **Marginal propensity to save (MPS)** (i.e., how much of each extra unit of income is saved)
- (Y) = **National income**

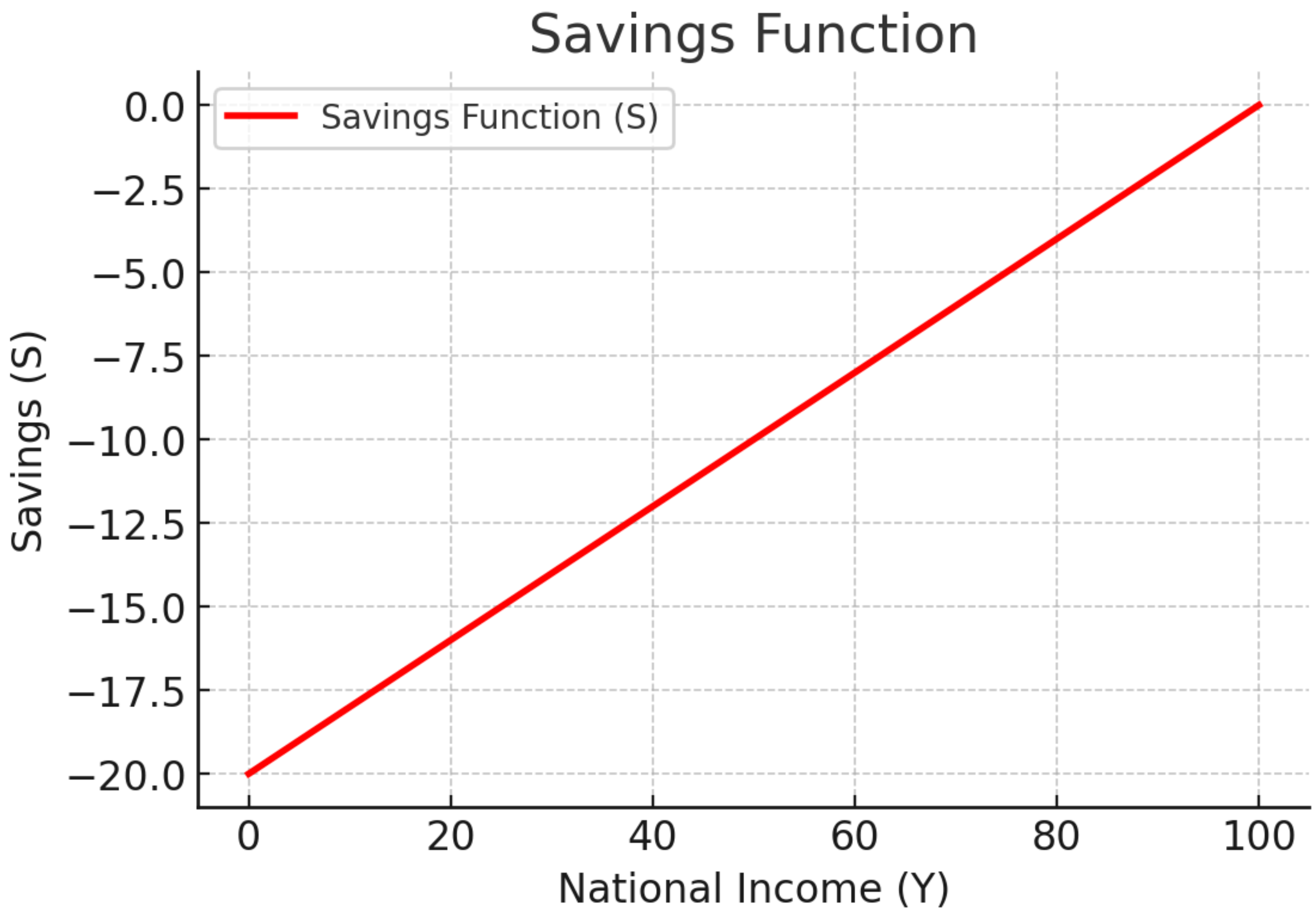
Since **income is either spent or saved**, we know:

$$MPC + MPS = 1$$

Interpretation:

- If **MPC = 0.8**, then **MPS = 0.2**.
- **Higher income leads to higher savings.**

Plotting the Savings Function



The Income-Expenditure Model

The **Income-Expenditure Model** determines **equilibrium national income**.

In a **simple Keynesian model**, total spending is:

$$Y = C + I + G + (X - M)$$

where:

- (C) = **Consumption**
- (I) = **Investment**
- (G) = **Government spending**
- (X - M) = **Net exports** (ignored for a closed economy)

At **equilibrium**, income equals total spending:

$$Y = C_0 + cY + I + G$$

Solving for **equilibrium income**:

$$Y^* = \frac{C_0 + I + G}{1 - c}$$

Problem Statement

Given:

$$G = 40, \quad I = 55, \quad C = 0.8Y_d + 25, \quad T = 0.1Y + 10$$

Find the **equilibrium level of national income** Y^*.

Step 1: Define Disposable Income

Disposable income Y_d is the **income left after taxation**:

$$Y_d = Y - T$$

Using the tax function:

$$T = 0.1Y + 10$$

$$Y_d = Y - (0.1Y + 10) = 0.9Y - 10$$

Disposable income depends on total income Y.

Step 2: Write the Consumption Function

Consumption function is given by:

$$C = 0.8Y_d + 25$$

Substituting $Y_d = 0.9Y - 10$:

$$C = 0.8(0.9Y - 10) + 25$$

$$C = 0.72Y - 8 + 25$$

$$C = 0.72Y + 17$$

Consumption increases with national income (Y).

Step 3: Write the Equilibrium Condition

In equilibrium:

$$Y = C + I + G$$

Substituting known values:

$$Y = (0.72Y + 17) + 55 + 40$$

$$Y = 0.72Y + 112$$

Now solve for (Y).

Step 4: Solve for Equilibrium Income

Rearrange the equation:

$$Y - 0.72Y = 112$$

$$0.28Y = 112$$

$$Y^* = \frac{112}{0.28} = 400$$

Equilibrium national income is $Y^* = 400$.

Your Turn: Practice Problem

Given the following information:

$$G = 50, \quad I = 60, \quad C = 0.7Y_d + 30, \quad T = 0.15Y + 10$$

Find the **equilibrium level of national income** Y^*.

Summary

1. **Supply and Demand Analysis** helps us understand market equilibrium.
2. **Transposition of Formulae** allows us to solve for unknown variables in economic models.
3. **National Income Determination** helps us find equilibrium GDP in macroeconomic models.

Math is powerful—and fun!

Next Steps

1. Practice **problems from the textbook** (Jacques 10ed, Sections 1.5-1.7).

2. Bring any questions to our **next class discussion!**

3. Work on your Home Assignment #1 (due next Friday, 13:30pm).

    - Chapter 1.1: Exercise 1.1, Problems 12, 14-16, 19 (p. 21-22)
    - Chapter 1.2: Exercise 1.2, Problems 9, 11, 14 (p. 39)
    - Chapter 1.3: Exercise 1.3, Problems 10 and (p. 53-54)
    - Chapter 1.4: Exercise 1.4, Problem 4 (p. 65)
    - Chapter 1.5: Exercise 1.5, Problems 3, 5, 8 (p. 81)
    - Chapter 1.6: Exercise 1.6, Problem 5 (p. 92)
    - Chapter 1.7: Exercise 1.7, Problem 6 and 7 (p. 106)

Any QUESTIONS?

Thank you for your attention!

Next Class

- (Mar 19) Quadratic Functions (2.1), Revenue, Cost, and Profit (2.2)