

Financial Institutions and System

Week 4: The Meaning and Behavior of Interest Rates

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Agenda

1. What is an Interest Rate?
2. The Behavior of Interest Rates
3. Class Activity: Understanding Interest Rates in Action

1. What is an Interest Rate?

Interest Rates: The Cost of Money

Definition: The **interest rate** is the **price of borrowing money** or the **return earned from lending or saving money**.

Why it matters: It affects everything—from mortgages and student loans to investment decisions and inflation.

Interest rate = Risk-free rate + Risk premium

where, **Risk-free rate = Real rate + Expected inflation**

Note:

- **Real rate** is the rate of return on an investment after adjusting for inflation.
- Expected inflation is the rate at which prices are expected to rise.
- Risk-free rate is the return on an investment that carries no risk of financial loss.
- Risk premium is the return in excess of the risk-free rate.

Measuring Interest Rates

Present Value (PV): Reflects the idea that **money today is worth more than money in the future.**

- A dollar today earns interest and grows to $1 \times (1 + i)$ tomorrow.
- Ex: Would you prefer \\$20M **today**, or \\$1M every year for 20 years?

The answer depends on the **discount rate** or interest rate used.

Compound Interest Example

Let $i = 0.10$ (10%)

- Year 1:

$$\$100 \times (1 + 0.10) = \$110$$

- Year 2:

$$\$110 \times (1 + 0.10) = \$121 \rightarrow \text{or } \$100 \times (1 + 0.10)^2$$

- Year 3:

$$\$121 \times (1 + 0.10) = \$133 \rightarrow \text{or } \$100 \times (1 + 0.10)^3$$

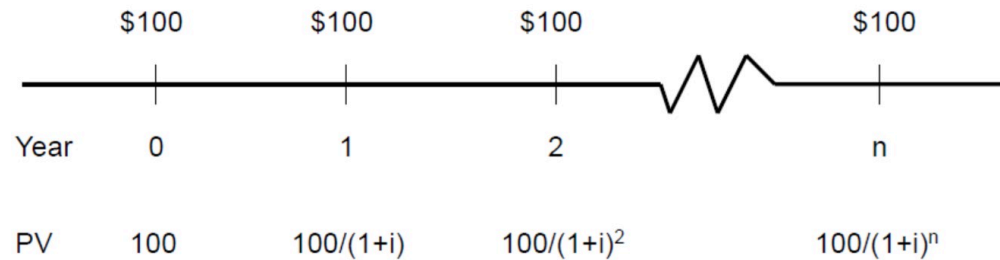
General formula: $FV = PV \times (1 + i)^n$

Simple Present Value

$$PV = \frac{CF}{(1+i)^n}$$

Where:

- **PV** = Present Value
- **CF** = Future Cash Flow
- **i** = Interest Rate
- **n** = Time in years



This formula helps us **compare money received at different times**.

How Much Is That Jackpot Worth?

You win \$20M, paid as \$1M per year for 20 years.

- Is it really worth \$20M today?
- Let's assume an interest rate of 10%.

Calculate Present Value:

$$PV = \frac{1M}{(1+0.10)} + \frac{1M}{(1+0.10)^2} + \dots + \frac{1M}{(1+0.10)^{20}}$$

This will result in a value significantly **less** than \$20M. That's the power of **discounting**.

Four Types of Credit Market Instruments

1. **Simple Loan**: Lump sum borrowed and repaid with interest (e.g., short-term bank loan).
2. **Fixed Payment Loan**: Equal regular payments (e.g., car loans, mortgages).
3. **Coupon Bond**: Pays fixed interest (coupon) annually + face value at maturity.
 - **Coupon rate** = Annual coupon ÷ Face value
 - **Zero-coupon bonds**: No interim payments, only repayment at maturity.
4. **Discount Bond**: Sold below face value, pays no coupon (e.g., Treasury bills).

Yield to Maturity (YTM)

YTM is the interest rate that **equates the bond's current price with the present value of its future payments.**

Why is YTM important?

- It reflects the **true return** on a bond if held to maturity.

Example – Simple Loan:

- $PV = \$100$
- $CF = \$110$ in 1 year

$$100 = \frac{110}{1+i} \Rightarrow i = 10$$

Fixed Payment Loan Formula

$$LV = \sum_{t=1}^n \frac{FP}{(1+i)^t}$$

Where:

- **LV** = Loan Value
- **FP** = Fixed annual payment
- **n** = Number of years

Used in amortized loans like mortgages.

Coupon Bond Formula

$$P = \sum_{t=1}^n \frac{C}{(1+i)^t} + \frac{F}{(1+i)^n}$$

Where:

- **P** = Current bond price
- **C** = Annual coupon payment
- **F** = Face value
- **i** = Interest rate
- **n** = Years to maturity

Bond Pricing Insights

- **YTM = Coupon Rate** → Bond trades at **par** (face value).
- **YTM > Coupon Rate** → Bond trades **below par** (discount).
- **YTM < Coupon Rate** → Bond trades **above par** (premium).

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = \$1,000)

Price of Bond (\$)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Remember: Bond price and Yield to Maturity (YTM) are inversely related — when price increases, yield decreases.

Why? Because paying a higher price for the same fixed future payments results in a lower return.

Consol (Perpetuity)

A **consol bond** or **perpetuity** pays **coupon payments forever** and never matures.

$$P = \frac{C}{i_c}$$

Where:

- **P** = Price
- **C** = Annual coupon
- **i_c** = Current yield

Used as a **simple approximation** of YTM when maturity is distant.

Discount Bond Formula

$$i = \frac{F - P}{P}$$

Where:

- **F** = Face value
- **P** = Current price

Like other bonds, **price** ↑ → **yield** ↓.

Interest Rates vs Returns

$$RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

Where:

- **RET** = Return from holding bond for 1 period
- **C** = Coupon payment
- P_t = Price at time t
- P_{t+1} = Price at time t+1

Two components:

1. **Current yield**
2. **Capital gain/loss**

Return vs Yield

- Return equals YTM **only** if held to maturity.
- Longer maturity → **greater interest-rate risk**.
- If interest rates **rise**, prices fall → **capital loss**.

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price (\$)	(4) Price Next Year* (\$)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1,000	503	-49.7	-39.7
20	10	1,000	516	-48.4	-38.4
10	10	1,000	597	-40.3	-30.3
5	10	1,000	741	-25.9	-15.9
2	10	1,000	917	-8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

Interest-Rate Risk

- **Long-term bonds** are more sensitive to interest rate changes.
- If you sell before maturity → you face price risk.
- But if you hold to maturity, your return = YTM.

Rule: The longer the maturity, the **higher the volatility** of bond price.

Real vs Nominal Interest Rates

- **Nominal rate (i)**: Not adjusted for inflation
- **Real rate (i_r)**: Adjusted for inflation

Types of real rates:

- **Ex ante**: Based on **expected inflation**
- **Ex post**: Based on **actual inflation**

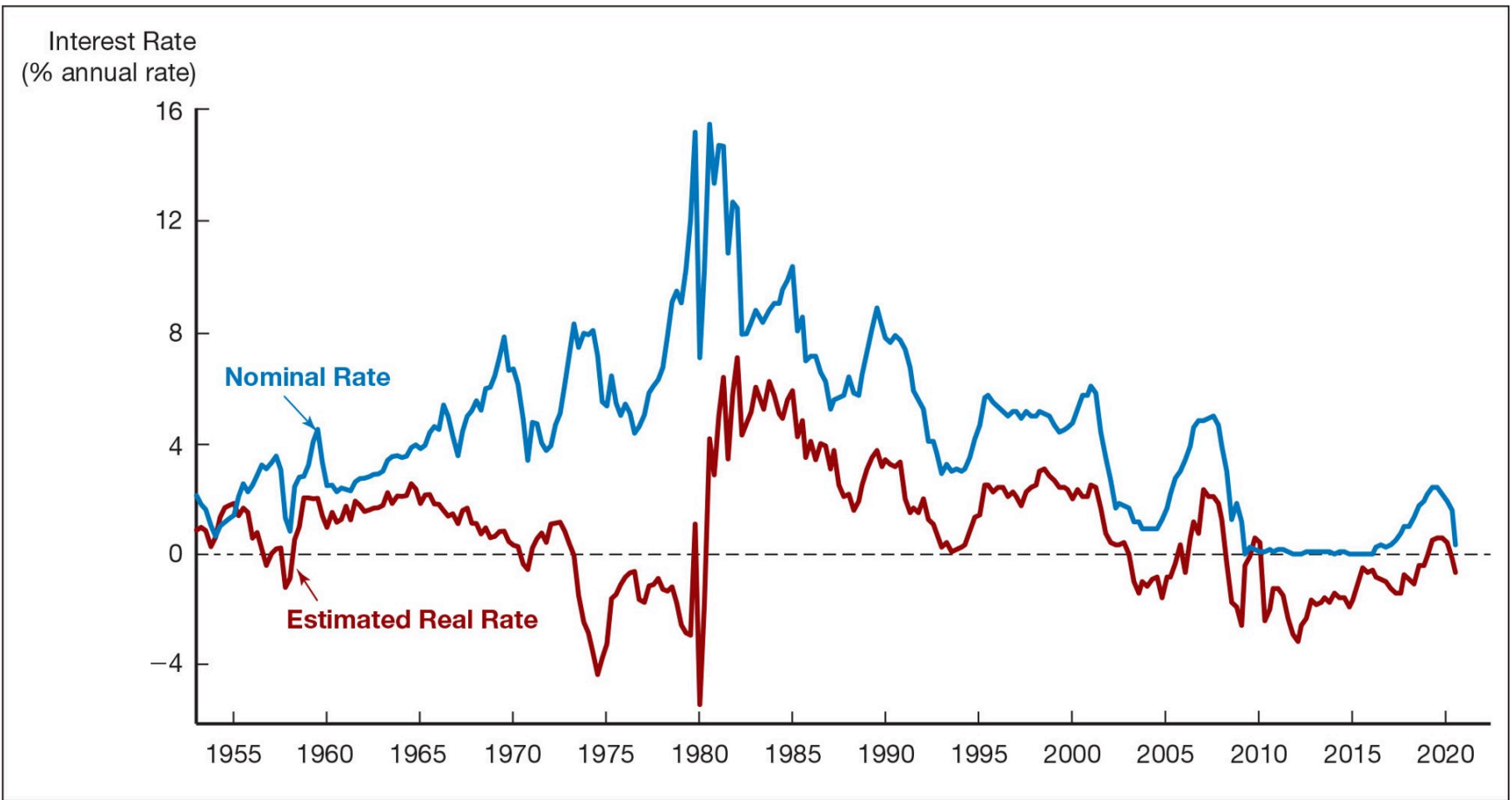
Fisher Equation

$$i = i_r + \pi^e$$

Where:

- i = Nominal rate
- i_r = Real rate
- π^e = Expected inflation

Low real rates → borrowing is cheap → **more demand for credit**



Real-World Example: Fed Rate Hikes & Bank Failures



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U.S. Markets

Fed's Barr says supervisors more aggressive, honing in on interest rate risk

By **Pete Schroeder**

February 16, 2024 11:33 PM GMT+9 · Updated 24 days ago



Federal Reserve Board Vice Chair for Supervision, Michael Barr, testifies before a Senate Banking, Housing, and Urban Affairs Committee hearing in the wake of recent bank failures, on Capitol Hill in Washington, U.S., May 18, 2023. REUTERS/Evelyn Hockstein/File Photo

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Fed Commentary: Financial Stability Risks

- Fed has increased exams after Silicon Valley Bank (SVB) collapse.
- Issue: **Unrealized losses** on long-term assets due to rising rates.
- Banks now face **tighter supervision and capital requirements**.

Shows how **interest rate changes ripple through the financial system.**

2. The Behavior of Interest Rates

Determinants of Asset Demand

An asset is anything that can be owned and has value.

Examples: money, laptops, cellphones, bonds, stocks, art, land, houses, and machinery.

Asset demand is the amount of an asset that people are willing to hold at a given price.

Financial assets are claims on real assets or income.

Factors Affecting Asset Demand

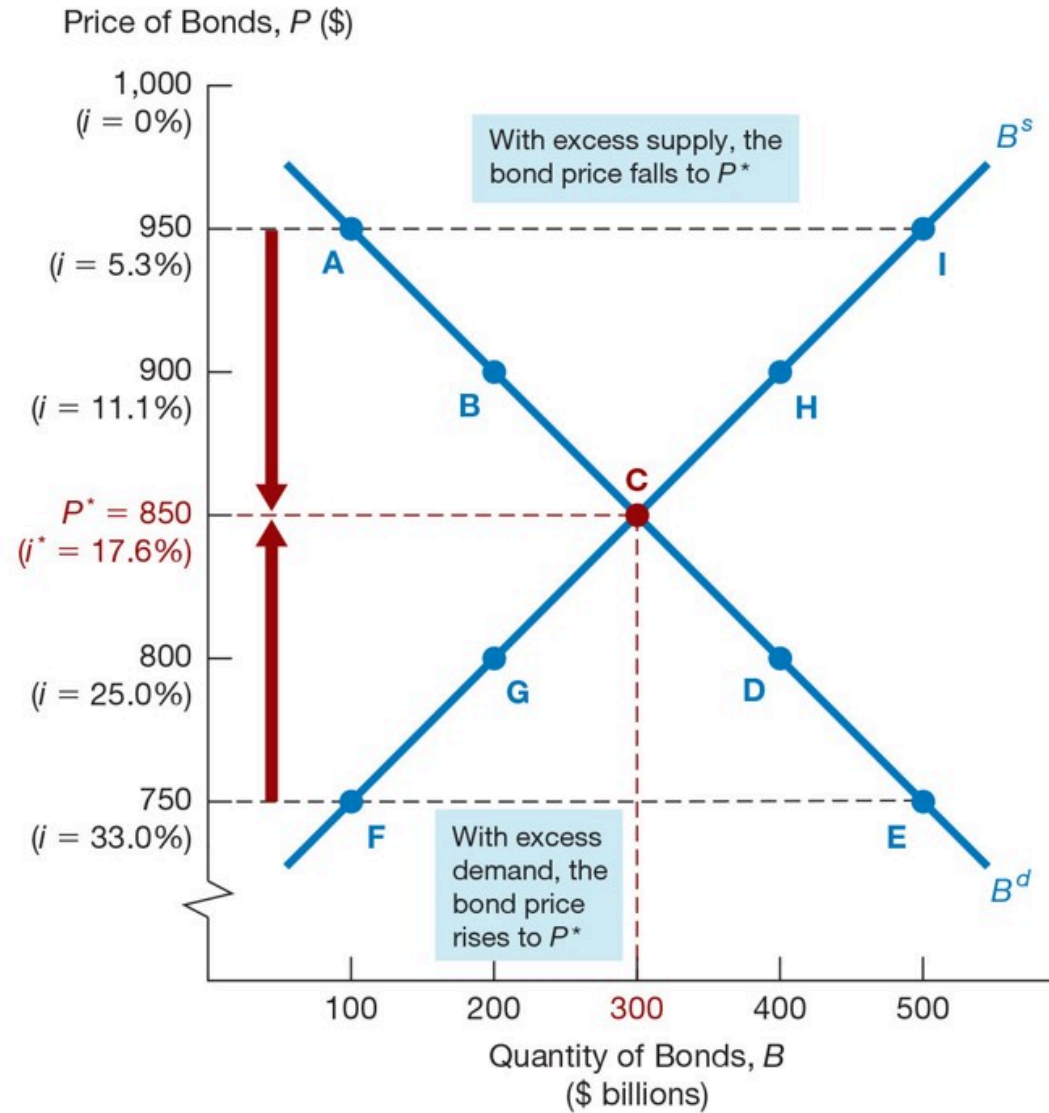
- **Wealth:** Total resources, including all assets
- **Expected Return:** Return relative to other assets
- **Risk:** Uncertainty of returns
- **Liquidity:** Ease of converting to cash

Theory of Portfolio Choice

1. Demand \uparrow with **wealth**
2. Demand \uparrow with **expected return**
3. Demand \downarrow with **risk**
4. Demand \uparrow with **liquidity**

Bond Market: Supply & Demand

- Lower price → Higher interest rate → Higher demand for bonds
- Lower price → Higher interest rate → Lower supply of bonds



Market Equilibrium in Bonds

- Market clears when: $B^d = B^s$
- $B^d > B^s$: Excess demand \rightarrow Price $\uparrow \rightarrow$ Interest rate \downarrow
- $B^d < B^s$: Excess supply \rightarrow Price $\downarrow \rightarrow$ Interest rate \uparrow

Changes in Equilibrium Interest Rates

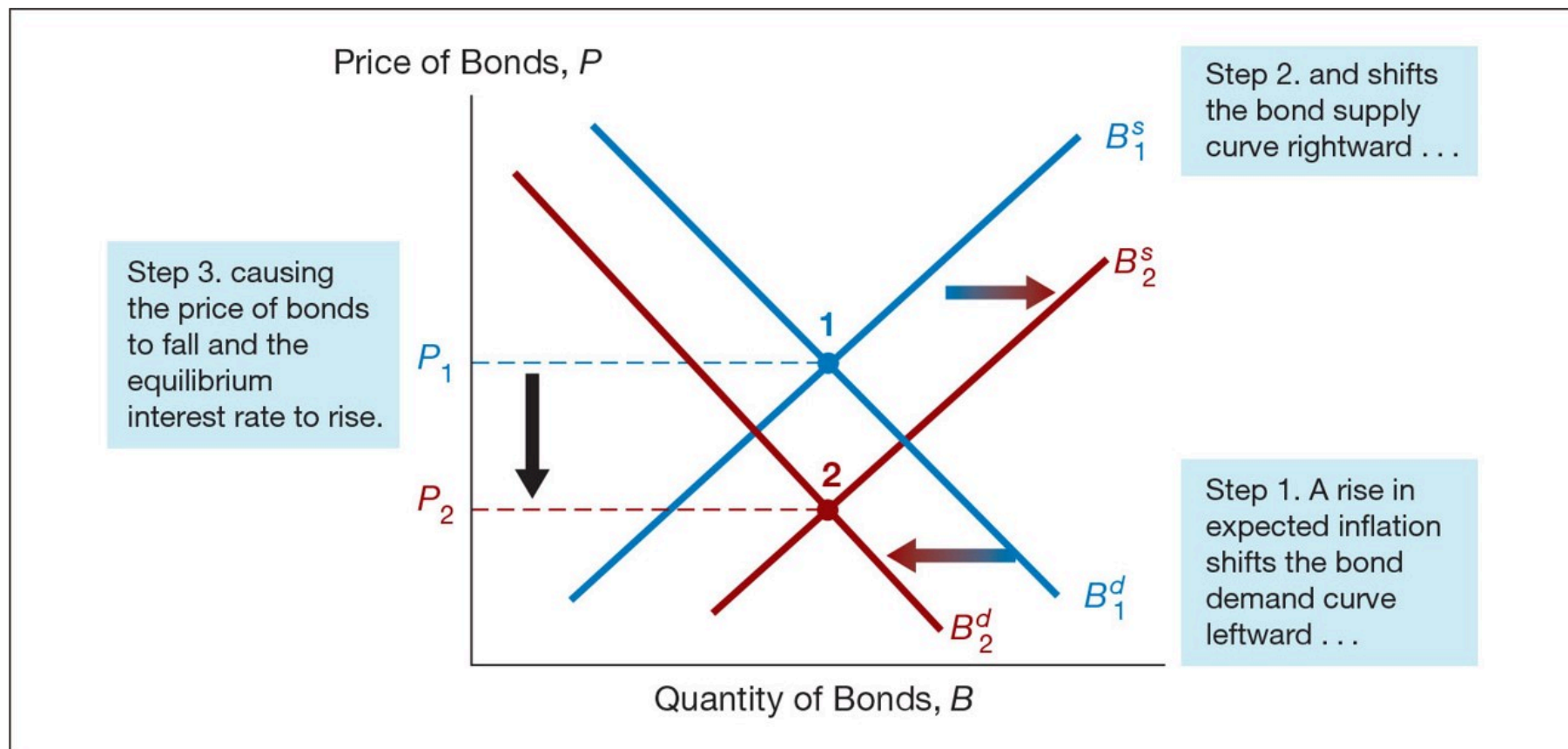
Demand for Bonds Shifts:

- \uparrow Wealth \rightarrow Demand \uparrow
- \uparrow Expected interest rate \rightarrow Demand \downarrow
- \uparrow Expected inflation \rightarrow Demand \downarrow
- \uparrow Risk \rightarrow Demand \downarrow
- \uparrow Liquidity \rightarrow Demand \uparrow

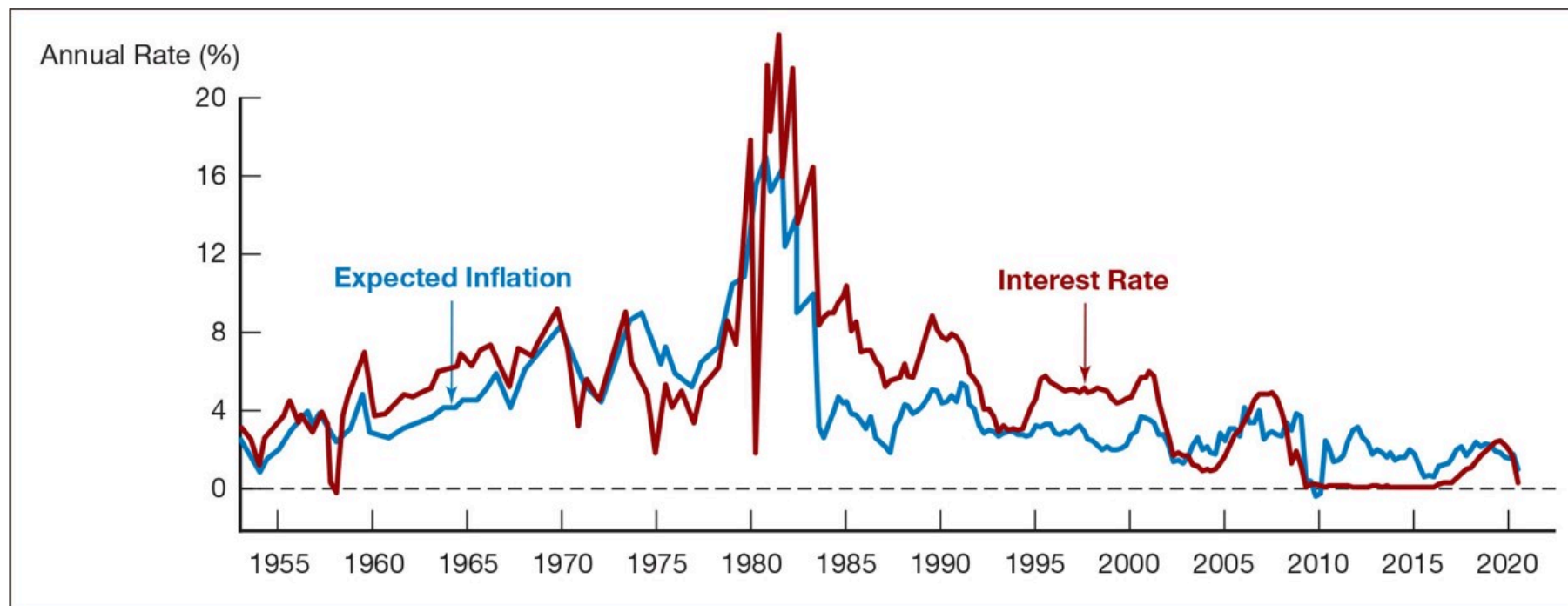
Supply of Bonds Shifts

- \uparrow Profitability \rightarrow Supply \uparrow
- \uparrow Expected inflation \rightarrow Supply \uparrow
- \uparrow Budget deficit \rightarrow Supply \uparrow

Fisher Effect: Response to Expected Inflation



Expected Inflation vs Interest Rates (1953-2020)



Liquidity Preference Framework

- Keynesian model: equilibrium interest rate via money supply & demand
- Wealth held in either **money** or **bonds**

$$B^s + M^s = B^d + M^d \rightarrow \text{Rearranged: } B^s - B^d = M^s - M^d$$

If $M^s = M^d \rightarrow$ then $B^s = B^d$

Demand for Money

As interest rate \uparrow :

- Opportunity cost of holding money \uparrow
- Expected return of money $\downarrow \rightarrow$ Demand for money \downarrow

Assume money supply is **fixed** by central bank.

Meaning:

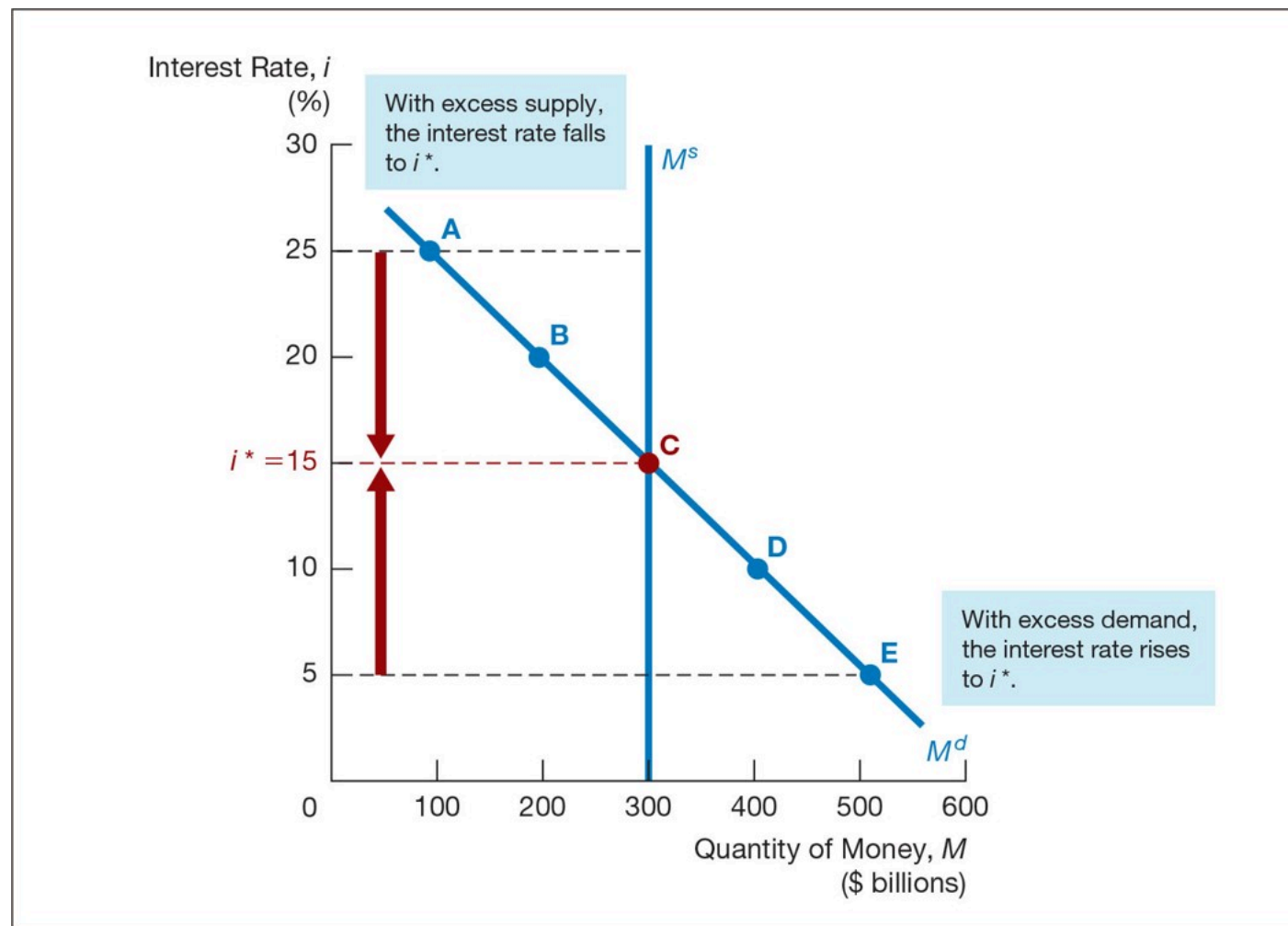
- When interest rates go up:

You're missing out on more potential earnings by just holding onto cash — that's the opportunity cost.

Since cash doesn't earn interest, its return becomes less attractive compared to alternatives like bonds or savings accounts.

- Result: People prefer to hold less money and instead invest it — so the demand for money falls.

Equilibrium in Money Market



Changes in Equilibrium (Liquidity Preference)

Demand for Money Shifts:

- **Income Effect:** \uparrow income \rightarrow \uparrow money demand
- **Price-Level Effect:** \uparrow prices \rightarrow \uparrow money demand

Shifts in Supply of Money

- Central bank controls money supply
- Fed increases $M^s \rightarrow$ Supply curve shifts **right**

Money & Interest Rates

- Liquidity preference says $\uparrow M^s \rightarrow \downarrow$ interest rate
- But...
 - **Price-level effect:** $\uparrow M^s \rightarrow \uparrow$ prices $\rightarrow \uparrow$ interest rate

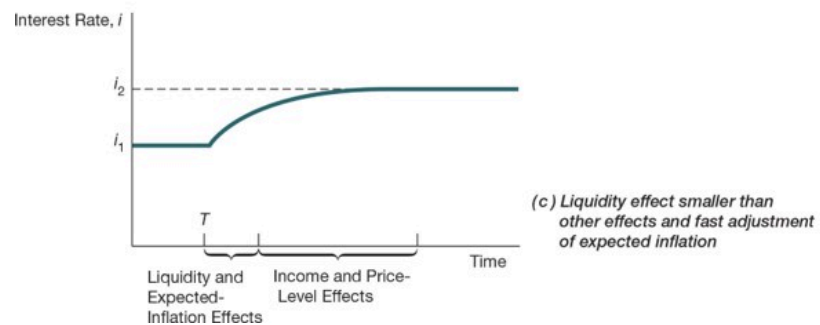
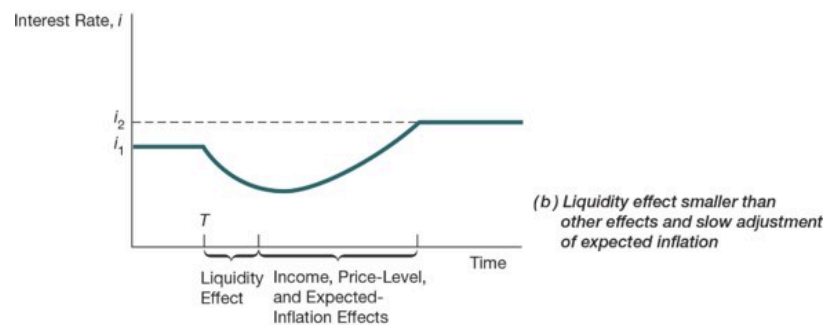
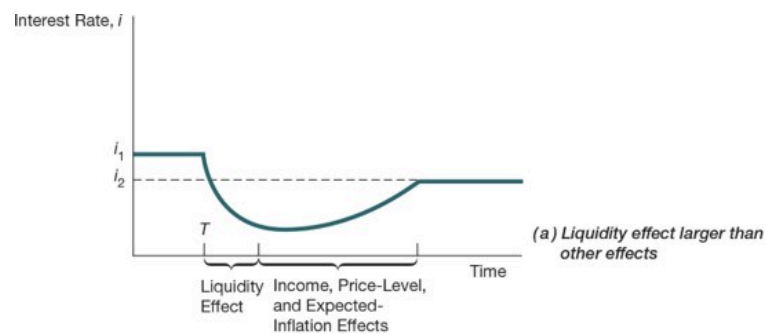
More Effects of Money Growth

- **Expected-inflation effect:** if people expect rising inflation, they demand higher interest
- **Income effect:** $\uparrow M^s \rightarrow \uparrow \text{income} \rightarrow \uparrow \text{interest}$

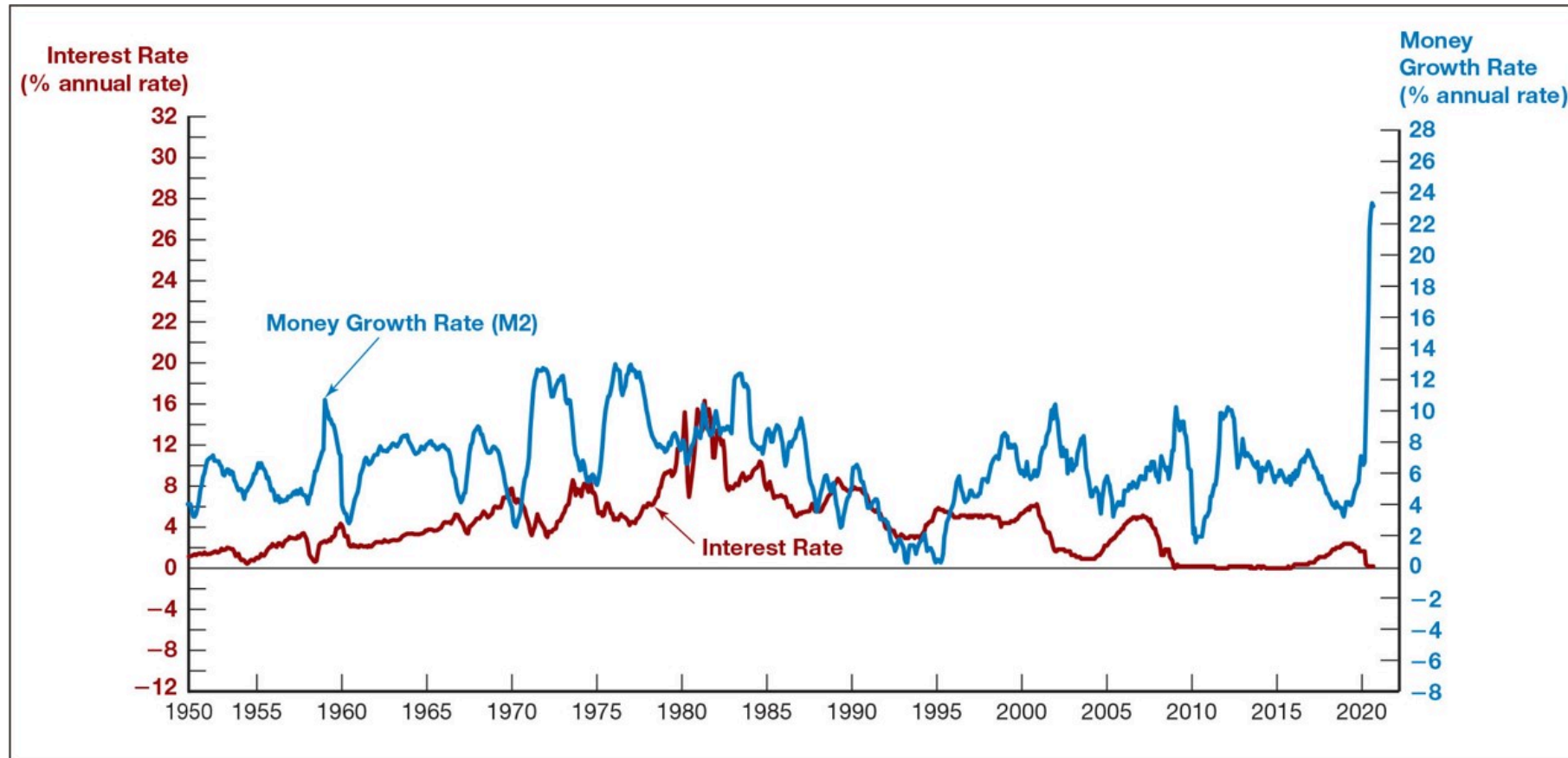
Net Effect? Does \uparrow Money Supply \downarrow Rates?

- Liquidity effect: \downarrow rates
- Income effect: \uparrow rates
- Price-level effect: \uparrow rates
- Expected-inflation effect: \uparrow rates

Response Over Time to Money Growth



Money Growth & Interest Rates (1950-2020)



3. Class Activity: Understanding Interest Rates in Action

Any QUESTIONS?

Thank You!

Next Class

- (Apr 4)
 - **Chap 6.** The Risk and Term Structure of Interest Rates;
 - **Chap 7.** The Stock Market, the Theory of Rational Expectations, and the Efficient Market Hypothesis