Mathematical Methods for International Commerce

Week 1/2: Basic Algebra

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Why Does Math Matter in Economics?

- **Mathematics is everywhere** in economics and finance.
- Helps us **analyze markets, optimize decisions, and manage risk**.
- Today, we'll explore **how banks use algebra** in international finance.

Let's dive in!

What we need to know for today's class?

1. **Basic algebraic operations** (addition, subtraction, multiplication, division).
2. **Negative numbers** and their use in financial calculations.
3. **Algebraic expressions** and their evaluation.
4. **Solving equations** and **inequalities** for financial decision-making.
5. **Fractions** and their application in loan payments.

1. Real-World Example of Algebra in International Finance

Example: International Loan Risk

Scenario: A company in Europe borrows **$1 million** from a U.S. bank.

Problem: The company earns revenue in euros (€), but the loan is in dollars ($).

Key Question: How does exchange rate fluctuation impact the company's debt?

Solution: Use algebra to analyze the exchange rate risk, loan repayment and other questions.

Step 1: Working with Negative Numbers

Currency Depreciation & Loan Impact

- The exchange rate was **€1 = $1.10** but dropped to **€1 = $1.05**
- This means the **euro lost value**, making it **more expensive** for the company to repay the loan.

Using Negative Numbers in Calculations

Change in Exchange Rate

$$\Delta ER = 1.05 - 1.10 = -0.05$$

The negative value indicates a depreciation of the euro.

Step 1: Working with Negative Numbers (cont)

Using Negative Numbers in Calculations (cont)

Change in Debt Due to Depreciation

- Before depreciation:

$$D_{\text{old}} = \frac{1,000,000}{1.10} = 909,091 \text{ EUR}$$

- After depreciation:

$$D_{\text{new}} = \frac{1,000,000}{1.05} = 952,381 \text{ EUR}$$

- **Increase in debt** (using subtraction):

$$\Delta D = 909,091 - 952,381 = -43,290 \text{ EUR}$$

- Since the result is **negative** *(old-new)*, this means the company **owes more** after depreciation.

Step 1: Working with Negative Numbers (cont)

Using Negative Numbers in Calculations (cont)

Multiplying Negative Numbers: Predicting Future Losses

- Suppose the **exchange rate continues to drop** at a rate of **-0.05 per year**.
- If the debt increase last time was **43,290 EUR**, what happens **after 3 years**?

$$\text{Total Debt Increase} = \Delta D \times (-3)$$

$$= 43,290 \times (-3) = -129,870 \text{ EUR}$$

The negative result means the company's debt grows significantly due to continuous depreciation.

Practice: Please calculate the debt increase after 5 years.

Step 1: Working with Negative Numbers (cont)

Using Negative Numbers in Calculations (cont)

Dividing Negative Numbers: Recalculating Exchange Rate Risk

- If the euro depreciation of **-0.05 per year** continues, how many years until the exchange rate drops to **€1 = $0.85**?
- Given the total depreciation needed:

$$\text{Total Change} = 1.05 - 0.85 = 0.20$$

- Solve for time:

$$\text{Years} = \frac{\text{Total Change}}{\text{Annual Change}} = \frac{0.20}{-0.05} = -4$$

Since time cannot be negative, this means in 4 years, the exchange rate could reach €1 = $0.85 if depreciation continues.

Practice: Please calculate the time needed for the exchange rate to drop to €1 = $0.75.

Step 2: Understanding algebraic expressions & evaluating numerically

The Company's Debt Formula

The company's debt can be represented by the following **algebraic expression**:

$$D = \frac{L}{ER}$$

Where:

- (D) = Debt in euros (€)
- (L) = Loan amount in dollars ($)
- (ER) = Exchange rate (€ per $)

Step 2: Understanding algebraic expressions & evaluating numerically (cont)

Evaluating the Expression Numerically

Case 1: Initial Exchange Rate

If:

- L = 1,000,000 USD
- ER = 1.20

Then:

$$D = \frac{1,000,000}{1.20}$$

$$= 833,333 \text{ EUR}$$

Practice: Please calculate the debt amount when the exchange rate is (ER = 0.95).

Step 3: Simplifying Expressions by Collecting Like Terms

Interest Payments on a Loan

The bank considers **interest payments** when determining the total amount to be repaid.

The formula for total repayment is:

$$T = P + rP$$

Where:

- (T) = Total repayment
- (P) = Principal loan amount
- (r) = Interest rate

Step 3: Simplifying Expressions by Collecting Like Terms (cont)

Simplifying the Expression

If the interest rate is 5% (r = 0.05), substitute the value into the equation:

$$T = P + 0.05P$$

Now, factor out (P) (collecting like terms):

$$T = (1 + 0.05)P$$

$$T = 1.05P$$

This simplified formula helps calculate interest payments efficiently in banking.

Practice: Please calculate the total repayment when the principal loan amount is (P = 250,000).

Step 4: Multiplying Out Brackets

- The company signs a **hedging contract** to manage exchange rate risk:

$$C = (D + F)(1 + r)$$

Where:

- (C) = Cost of the contract
- (D) = Debt in euros
- (F) = Fixed fee for hedging. Assumed to be financed by the bank (incl. in loan).
- (r) = Interest rate

Step 4: Multiplying Out Brackets (cont)

Expanding the Brackets

Using the **distributive property** to expand:

$$C = D(1 + r) + F(1 + r)$$

Expanding each term:

$$C = D \times 1 + D \times r + F \times 1 + F \times r$$

$$C = D + Dr + F + Fr$$

Step 4: Multiplying Out Brackets (cont)

Final Expression: Total Cost Breakdown

$$C = (D + F) + (D + F)r$$

- **First part (D + F)** represents the original debt and fee.
- **Second part (D + F)r** represents the additional cost due to interest.

This expanded formula helps in accurately calculating the total cost, including fees and interest.

Note: hedging contract includes forward contract (F) to lock in exchange rate.

Practice: Please calculate the total cost when the debt is (D = 1,000,000), fee is (F = 50,000), and interest rate is (r = 0.05).

Step 5: Factorizing Expressions

- The bank wants to **simplify the total cost formula** for easier calculations:

$$C = D(1+r) + F(1+r)$$

We notice that (1 + r) is a common factor in both terms.

- Since (1 + r) appears in both terms, we can **factor it out**:

$$C = (D+F)(1+r)$$

This simplifies the formula and makes calculations easier when adjusting values.

Practice: Please factorize the expression (T = P + 0.05P).

Step 6: Fractions in Loan Payments & Risk Calculation

Quarterly Loan Payments

If a **$1,000,000 loan** is split into **quarterly payments**, each payment is calculated as:

$$P = \frac{1,000,000}{4}$$

Converting Payments to Euros (€)

If the company **pays in euros**, and the exchange rate is **€1 = $1.10**, the payment in euros is:

$$P_{\text{EUR}} = \frac{250,000}{1.10}$$

Simplifying the fraction:

$$P_{\text{EUR}} = \frac{250,000 \div 10}{1.10 \div 10} = \frac{25,000}{0.11} = 227,273 \text{ EUR}$$

Each quarterly payment costs €227,273 at an exchange rate of 1.10.

Step 6: Fractions in Loan Payments & Risk Calculation (cont)

Impact of Exchange Rate Drop

If the exchange rate **drops to 1.05**:

$$P_{\text{EUR}} = \frac{250,000}{1.05}$$

Simplifying:

$$P_{\text{EUR}} = \frac{250,000 \times 100}{1.05 \times 100} = \frac{25,000,000}{105} = 238,095 \text{ EUR}$$

Now, each quarterly payment costs €238,095.

Step 6: Fractions in Loan Payments & Risk Calculation (cont)

Increase in Payment Due to Exchange Rate Change

$$\Delta P_{\text{EUR}} = 238,095 - 227,273$$
$$= 10,822 \text{ EUR}$$

The company must now pay an additional €10,822 per quarter due to currency depreciation.

Step 6: Fractions in Loan Payments & Risk Calculation (cont)

Multiplying Fractions – Adjusting Loan Payments

If the bank charges a **processing fee of 1/200 (0.5%)** on each quarterly payment, the extra fee per quarter is:

$$F = P_{\mathrm{EUR}} \times \frac{1}{200}$$

$$F = \frac{238,095}{200}$$

- Divide numerator and denominator by 5:

$$\frac{47,619}{40} = 1,190$$

The additional fee per quarter is €1,190.

Practice: Please calculate the processing fee if the quarterly payment is €200,000.

Step 7: Solving Equations for Loan Repayment

Loan Repayment Over 5 Years

The company wants to **repay the loan over 5 years** with equal annual payments.

The total loan amount is **$1,000,000**, and the **annual payment is (P)**.

The equation for equal payments

$$1,000,000 = 5P$$

Solving for (P) (Annual Payment)

To isolate (P), **divide both sides by 5**:

$$P = \frac{1,000,000}{5} = 200,000 \text{ USD}$$

Each annual payment is $200,000.

Practice: Please calculate the annual payment if the total loan amount is $500,000 and the repayment period is 3 years.

Step 8: Linear Inequalities in Risk Management

Recognizing Inequality Symbols

The following symbols are used in **risk management** and **financial decision-making**:

Symbol	Meaning	Example
$<$	Less than	Profit (< 100,000) (profit is below €100K)
$>$	Greater than	Revenue (> 500,000) (revenue exceeds €500K)
$\leq$	Less than or equal to	Risk score ≤ 70 (acceptable risk level)
$\geq$	Greater than or equal to	Capital reserves $\geq 1,000,000$ (must maintain reserves of €1M or more)

Step 8: Linear Inequalities in Risk Management (cont)

Bank's Risk Threshold Example

The bank considers a loan **risky** if the company's **annual revenue falls below €900,000**.

Thus, the company's revenue must satisfy the inequality:

$$R - C \geq 900,000$$

Where:

- (R) = Annual revenue
- (C) = Annual costs
- (900,000) = Minimum revenue required to avoid risk

Step 8: Linear Inequalities in Risk Management (cont)

Solving the Inequality for Minimum Required Revenue

If the company has annual costs of €150,000, we substitute (C = 150,000):

$$R - 150,000 \geq 900,000$$

To isolate (R), add 150,000 to both sides:

$$R \geq 1,050,000$$

The company must earn at least €1.05M per year to remain financially stable.

Practice: Please solve the inequality for a minimum revenue of €1,200,000 if costs are €250,000.

Step 8: Linear Inequalities in Risk Management (cont)

Graphical Representation of the Inequality

The inequality

$$R \geq 1,050,000$$

can be visualized on a number line:

- **Red region**: Revenue below €1.05M (Risky)
- **Green region**: Revenue €1.05M or more (Safe)

$$\bullet \quad \bullet \quad \bullet \quad \bullet \quad | \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \cdots \quad \cdots \quad \mathit{900K} \quad \mathit{950K} \quad \mathit{1M} \quad \mathbf{1.05M} \quad \mathit{1.1M} \quad \mathit{1.2M} \quad \mathit{1.3M}$$

The company must be in the green region to meet the bank's stability requirements.

Conclusion: Why This Math Matters

1. Algebra helps predict financial outcomes.
2. Fractions, equations, and inequalities are tools for banking decisions.
3. Math makes economic decision-making precise and reliable.

Next Steps

- Practice **algebra problems from the textbook** (Jacques, Sections 1.1, 1.2).
- Bring any questions to our **next class discussion!**

Math is powerful—and fun!

Any QUESTIONS?

Next Class

- (Mar 12) Graphs of Equations (1.3), Solving Equations (1.4)