Mathematical Methods for International Commerce

Week 2/1: Graphs of Equations, Solving Equations

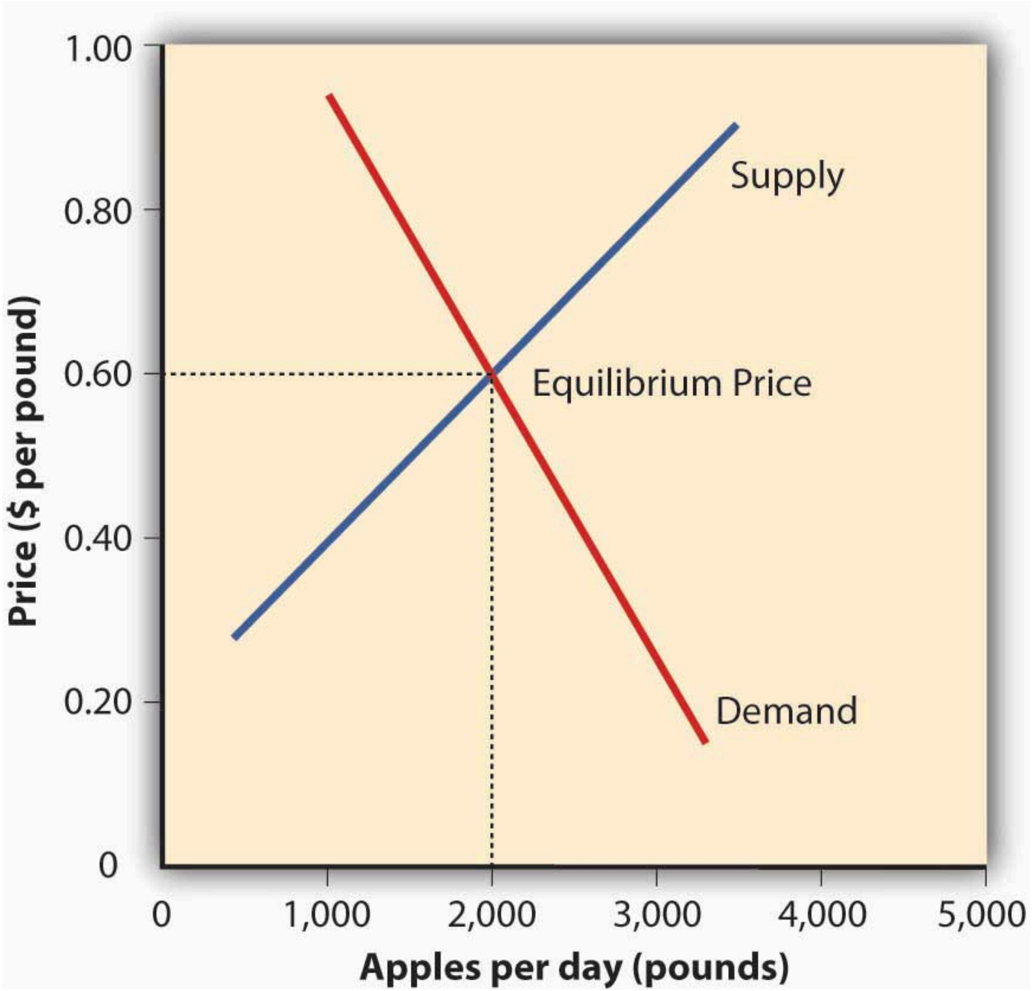
Iegor Vyshnevskyi, Ph.D.

Sogang University

March 12, 2025

Why Are These Concepts Essential in Economics?

Supply and Demand Graph



Why Are These Concepts Essential in Economics? (cont)

Visualizing Economic Relationships

- **Supply and Demand Curves**: Graphs illustrate how prices and quantities interact in markets.
- **Cost and Revenue Functions**: Visual tools to analyze profitability and break-even points.

Solving Equations for Economic Analysis

- **Equilibrium Analysis**: Determining market-clearing prices and quantities.
- **Optimization**: Maximizing profit or minimizing cost functions.

Real-World Applications

- **Policy Modeling**: Predicting outcomes of fiscal and monetary policies.
- **Business Strategy**: Informing pricing, production, and investment decisions.

Let's dive in!

Learning Objectives

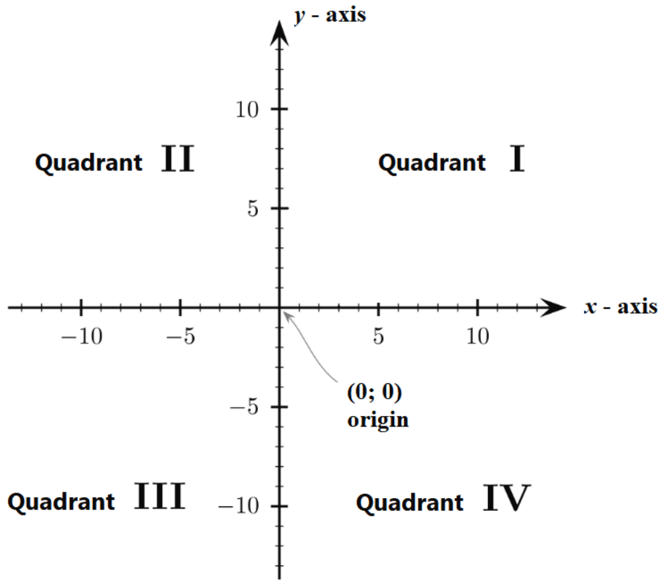
At the end of this class, you should be able to:

- **Graph** linear equations in one/two variables.
- **Solve** a linear equation and a system of two simultaneous linear equations using **elimination**.
- **Detect** when a system of equations **has no solution**.
- **Identify** when a system of equations has **infinitely many solutions**.
- **Solve** a system of **three equations with three unknowns** using **elimination**.

Section 1.3: Graphs of Equations

Understanding the Cartesian Coordinate System

- **X-axis**: Horizontal line
- **Y-axis**: Vertical line
- **Origin**: Intersection of X and Y axes (0,0)
- **Quadrants**: Four sections divided by the axes



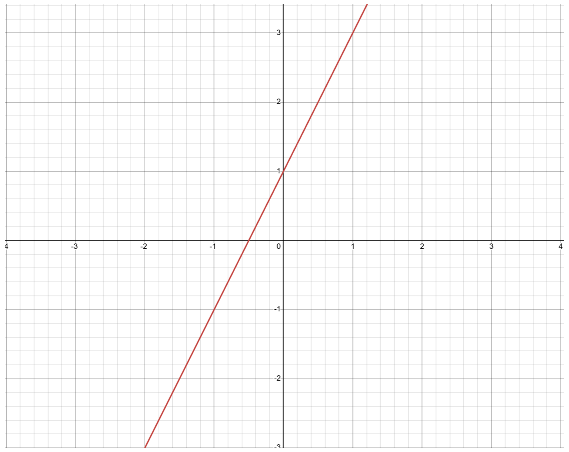
Plotting Linear Equations

Understanding Linear Equations

- **Linear Equation:** $y = mx + c$
- m: **Slope of the line**
- c: **Y-intercept** (point where the line crosses the Y-axis)

Example:

- Plot the equation (y = 2x + 1).



Source: Desmos Graphing Calculator

Your turn

Please plot the following linear equations:

1. (y = 3x - 2)
2. (y = -2x + 3)
3. (y = 0.5x + 1)

Impact of Exchange Rate on Loan Payments

How Exchange Rate Affects Loan Repayments

- A company must pay **$200,000 annually** for a loan.
- The payment in **EUR depends on the exchange rate**.
- If the **exchange rate changes**, the **amount in EUR** fluctuates.
  - Say, exchange rate fluctuates from **€1 = $0.90** to **€1 = $1.20**.
- There is a fixed commission fee of **€50,000** for the transaction.

Debt and Exchange Rate Relationship

Equation for Debt Calculation

A company borrows in **USD**, but its total debt in **EUR** depends on the exchange rate and a **fixed commission**.

$$D = \frac{L}{ER} + C$$

Where:

- D = Total debt in **EUR**
- L = Loan amount in **USD**
- ER = Exchange rate **(EUR/USD)**
- C = Fixed commission **(€50,000 EUR)**

Example Calculation

A company borrows **$1,000,000 USD**, and the exchange rate is **1 EUR = 1.10 USD**, with a **€50,000 commission**.

$$D = \frac{1,000,000}{1.10} + 50,000$$

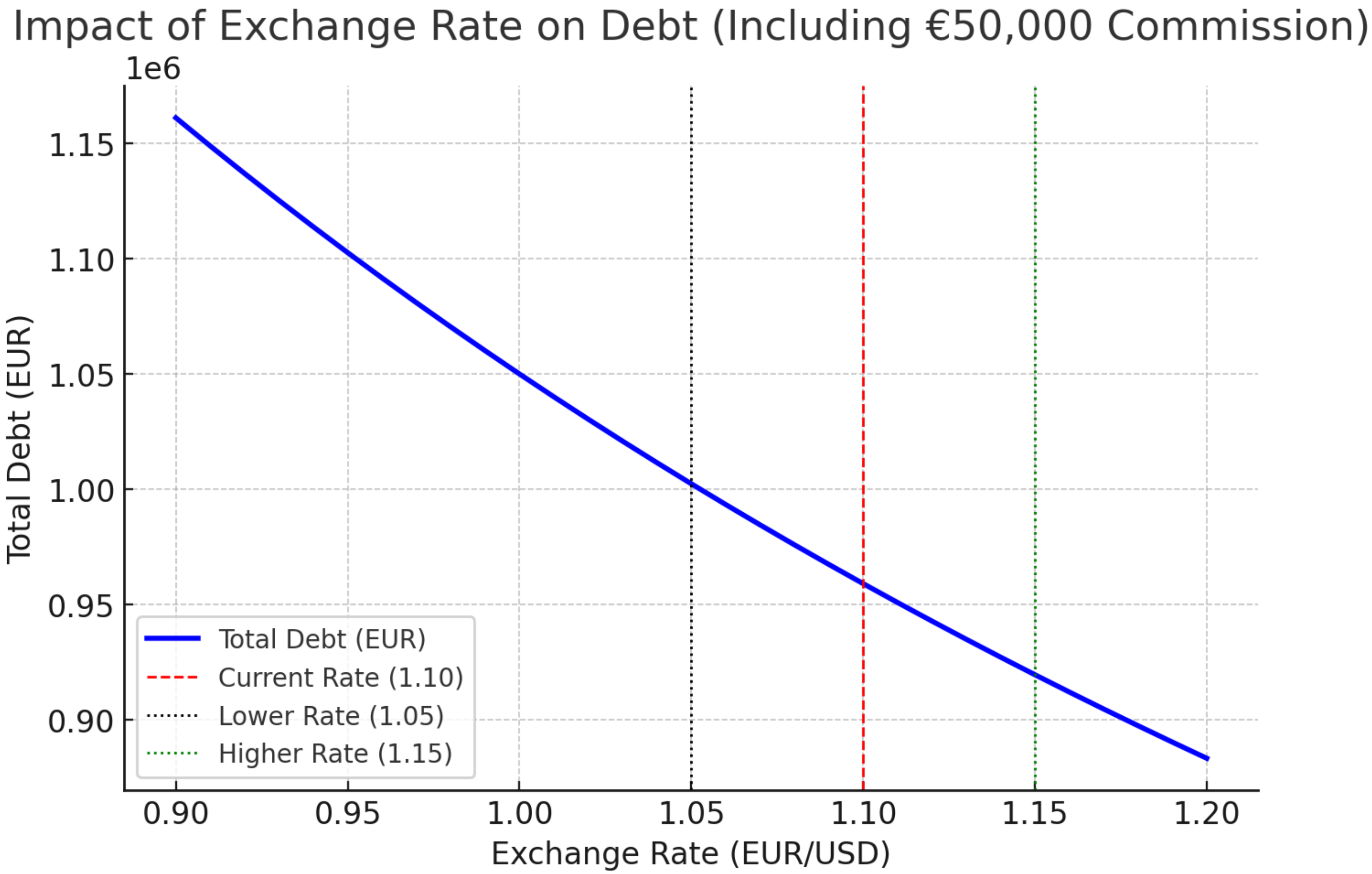
$$D = 909,091 + 50,000 = 959,091 \text{ EUR}$$

If the **exchange rate drops** to **1.05 EUR/USD**, the debt increases:

$$D = \frac{1,000,000}{1.05} + 50,000 = 952,381 + 50,000 = 1,002,381 \text{ EUR}$$

As the EUR weakens, the debt in EUR increases even more!

Visualizing Impact of Exchange Rate on Loan Payments (cont)



What is a Simultaneous Equation?

- A **system of equations** consists of two or more equations with multiple variables.
- The goal is to **find the values of the unknowns** that satisfy all equations simultaneously.

Example of a system with two unknowns:

$$\begin{aligned} 2x + 3y &= 12 \\ 4x - y &= 5 \end{aligned}$$

Method: Solving by Elimination (Two Equations)

Steps to solve:

1. **Multiply or adjust** the equations to align one variable.
2. **Add or subtract** the equations to eliminate one variable.
3. **Solve** for the remaining variable.
4. **Substitute** back to find the second variable.

Example: Solve the system

$$
\begin{aligned}
3x + 2y &= 14 \\
5x - 2y &= 10
\end{aligned}
$$

Step 1: Add the two equations

$$(3x + 2y) + (5x - 2y) = 14 + 10$$

$$8x = 24 \Rightarrow x = 3$$

Example: Solve the system (cont)

Step 2: Substitute (x = 3) into one equation

$$3(3) + 2y = 14$$

$$9 + 2y = 14 \Rightarrow 2y = 5 \Rightarrow y = 2.5$$

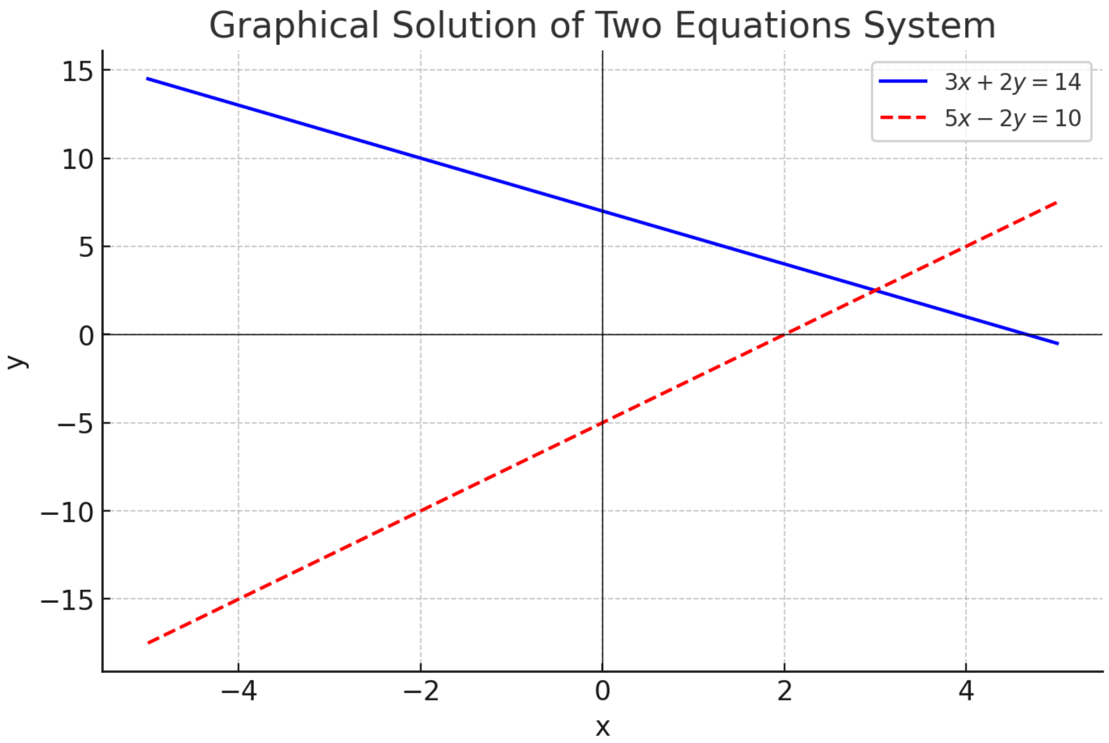
Solution: (x = 3, y = 2.5)

Graphical Interpretation of a System of Equations

Graphical Solution

- **Graph each equation** on the same axes.
- **Intersection point** is the solution to the system.

Example:



No Solution Case (Inconsistent System)

A system has **no solution** if the equations are contradictory.

Example:

$$\begin{aligned} 2x + 4y &= 10 \\ x + 2y &= 6 \end{aligned}$$

Dividing the first equation by 2:

$$x + 2y = 5$$

This contradicts the second equation **(x + 2y = 6)**!

No solution exists → The lines are **parallel**.

Infinitely Many Solutions (Dependent System)

A system has **infinitely many solutions** if the equations are identical.

Example:

$$\begin{aligned} 2x + 3y &= 6 \\ 4x + 6y &= 12 \end{aligned}$$

Divide the second equation by **2**:

$$2x + 3y = 6$$

The equations are identical, meaning **infinitely many solutions** exist.

Graphically, the lines overlap completely.

Your Turn: Practice Problems

Please solve the following systems of equations:

1. ($x + 2y = 5$) and ($2x - y = 3$)
2. ($3x + 2y = 14$) and ($5x - 2y = 10$)
3. ($2x + 4y = 10$) and ($x + 2y = 6$)

Solving a 3×3 System Using Elimination

Problem: Solve the System

$$
\begin{aligned}
x + y + z &= 6 \quad (1) \\
2x - y + 3z &= 14 \quad (2) \\
x + 2y - z &= 4 \quad (3)
\end{aligned}
$$

Step 1: Eliminate (z)

Adding **(1) and (3)**:

$$2x + 3y = 10 \quad (4)$$

Multiplying **(1) by -2** and adding to (2):

$$-3y + z = 2 \quad (5)$$

Step 2: Express (z) in Terms of (y)

From (5):

$$z = 3y + 2$$

Substituting into (1):

$$x + y + (3y + 2) = 6$$

$$x + 4y = 4 \quad (6)$$

Now solve:

$$\begin{aligned} 2x + 3y &= 10 \quad (4) \\ x + 4y &= 4 \quad (6) \end{aligned}$$

Step 3: Solve for (y) and (x)

Multiply (6) by -2 and add:

$$-2x - 8y + 2x + 3y = -8 + 10$$

$$-5y = 2 \Rightarrow y = -\frac{2}{5}$$

Substituting (y) into (6):

$$x = 4 + \frac{8}{5} = \frac{28}{5}$$

Step 4: Solve for (z)

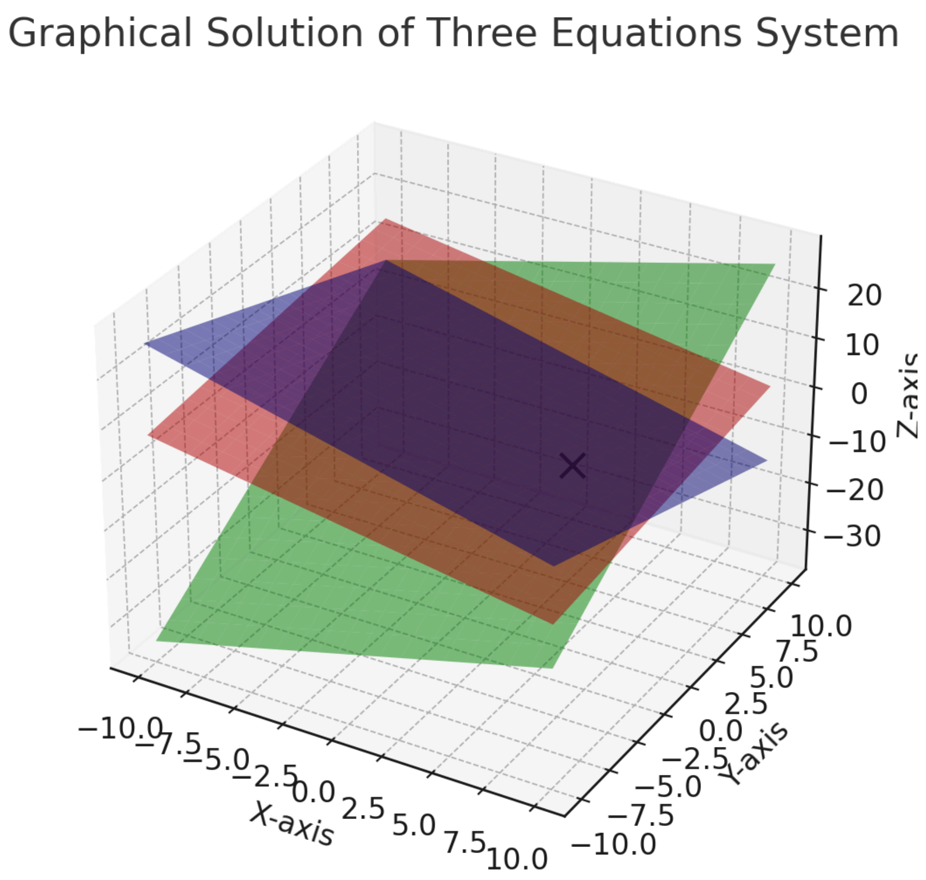
$$z = 3(-\frac{2}{5}) + 2 = \frac{4}{5}$$

Final Answer

$$x = \frac{28}{5}, \quad y = -\frac{2}{5}, \quad z = \frac{4}{5}$$

Graphical Solution of a 3*3 System of Equations

Example:



Your Turn: Practice Problems

Please solve the following 3*3 systems of equations:

1. $(x + y + z = 6)$, $(2x - y + 3z = 14)$, $(x + 2y - z = 4)$
2. $(x + 2y + z = 6)$, $(2x - y + 3z = 14)$, $(x + 2y - z = 4)$

Conclusion: Why This Math Matters

1. **Graphs of Equations**: Visual tools to understand relationships in economics.
2. **Solving Equations**: Essential for equilibrium analysis and optimization in economics.
3. **Systems of Equations**: Used to model complex economic relationships.
4. **Real-World Applications**: Informing policy decisions and business strategies.

Next Steps

1. Practice **algebra problems from the textbook** (Jacques, Sections 1.3, 1.4).

2. Bring any questions to our **next class discussion!**

Math is powerful—and fun!

Any QUESTIONS?

Next Class

- (Mar 14) Supply and Demand Analysis (1.5), Transposition of Formulae (1.6), National Income Determination (1.7)