

Mathematical Methods for International Commerce

Week 6/1: Marginal Functions

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Agenda

1. Marginal Functions
2. Group Activity: Marginal Strategy Showdown

1. Marginal Functions

Why Marginal Functions Matter

Marginal functions help us understand how **economic quantities change** when we adjust inputs or make small decisions.

- In **microeconomics**, marginal revenue and marginal cost are essential to determine **optimal production** and **pricing decisions**.
- In **macroeconomics**, marginal propensity to consume and save help model **aggregate demand**.
- In **production theory**, marginal product helps evaluate **efficiency** of labour and capital.

Learning Objectives

By the end of this lecture, you should be able to:

- Calculate **marginal revenue** and **marginal cost**
- Calculate **marginal product of labour**
- Derive the link between **marginal and average revenue** under different market structures
- State the **law of diminishing marginal productivity** using calculus
- Calculate **marginal propensity to consume** and **save**

Marginal Revenue & Marginal Cost

- **Marginal Revenue (MR)** is the **additional revenue** earned from selling one more unit:

$$MR = \frac{dR}{dx}$$

- **Marginal Cost (MC)** is the **additional cost** from producing one more unit:

$$MC = \frac{dC}{dx}$$

Example:

Let revenue: $R(x) = 40x - 0.5x^2$ and cost: $C(x) = 10x + 50$

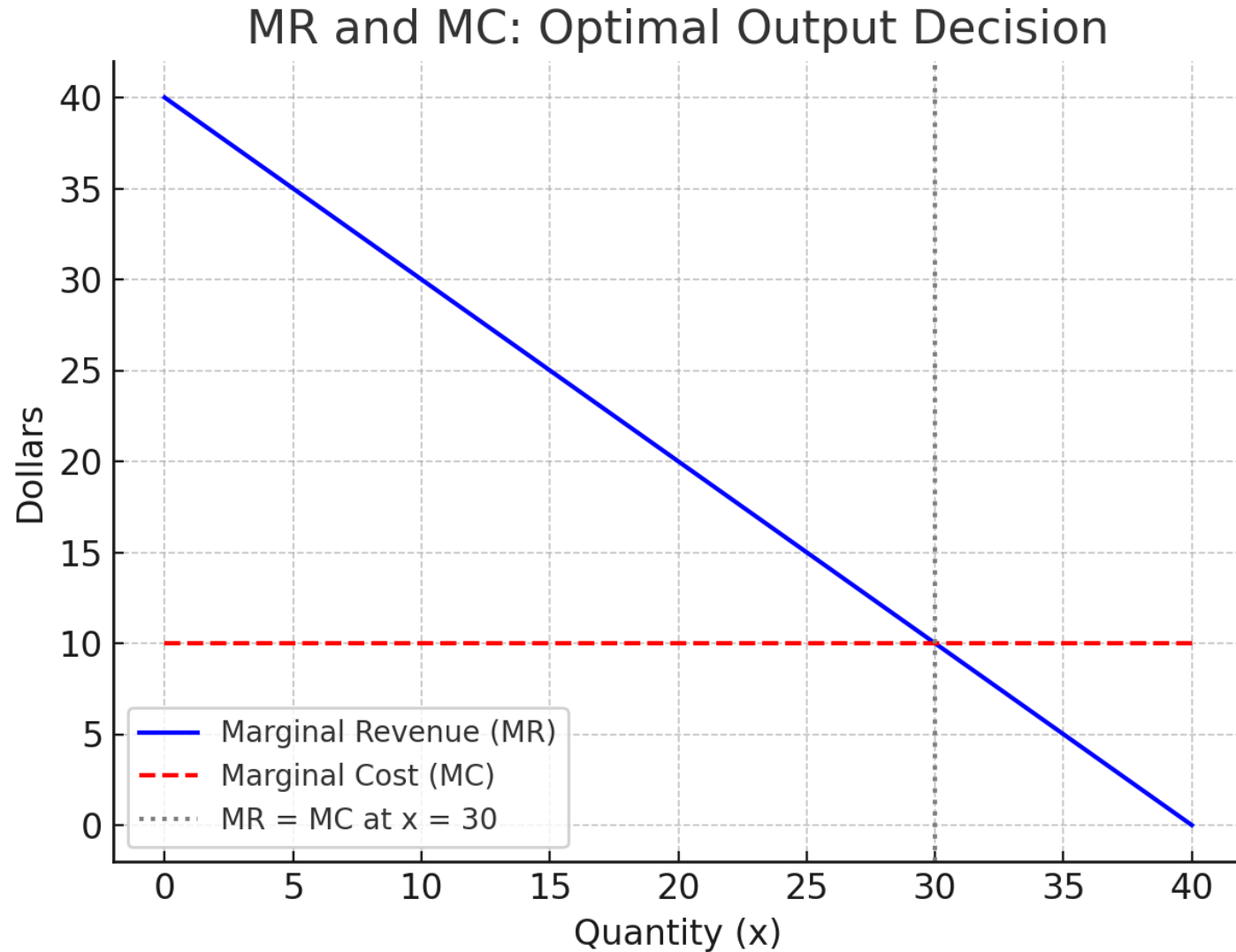
- $MR = \frac{dR}{dx} = 40 - x$
- $MC = \frac{dC}{dx} = 10$

At $x = 20$:

- $MR = 20, MC = 10$

The firm should produce more since $MR > MC$.

Visualizing MR and MC



Marginal Product of Labour

- The **Marginal Product of Labour (MP_L)** is the additional output from hiring one more worker:

$$MP_L = \frac{dQ}{dL}$$

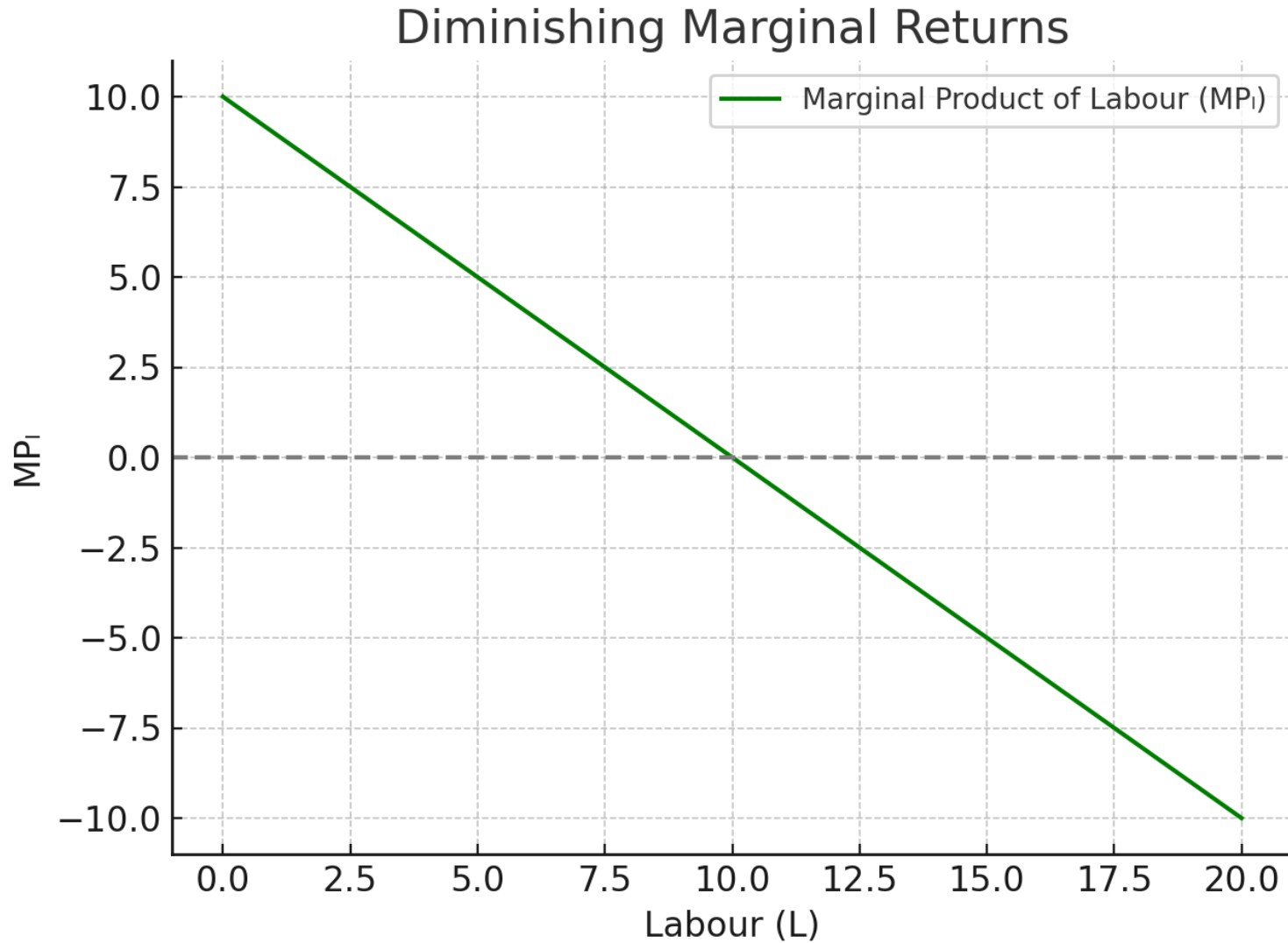
Example:

Let $Q = 10L - 0.5L^2$

- $MP_L = \frac{dQ}{dL} = 10 - L$
- When $L = 4$: $MP_L = 6$

Output increases but at a diminishing rate — see law of diminishing returns.

Visualizing Diminishing Marginal Returns



Law of Diminishing Marginal Productivity

The **law states** that, after a point, adding more of a variable input leads to smaller increases in output:

$$MP_L = \frac{dQ}{dL} \text{ decreases as } L \text{ increases}$$

- Seen in the production function $Q = 10L - 0.5L^2$
- Marginal product falls as L increases

Monopoly vs Perfect Competition

Perfect Competition:

- $MR = AR = P$ (Price is constant)
- Average Revenue and Marginal Revenue are equal

Monopoly:

- Price falls as more units are sold
- $MR < AR$
- Total revenue function $R(x) = P(x) \cdot x$, where $P(x)$ is decreasing

Example:

$$P(x) = 100 - 2x \Rightarrow R(x) = (100 - 2x)x = 100x - 2x^2$$

- $MR = \frac{dR}{dx} = 100 - 4x$
- $AR = R(x)/x = 100 - 2x$

Always: $MR < AR$

Marginal Propensity to Consume (MPC) and Save (MPS)

- **MPC** is the fraction of additional income that is consumed:

$$MPC = \frac{dC}{dY}$$

- **MPS** is the fraction of additional income that is saved:

$$MPS = \frac{dS}{dY}$$

Always: $MPC + MPS = 1$

Example: MPC and MPS

Let consumption function be:

$$C = 30 + 0.8Y \Rightarrow MPC = 0.8$$

$$\text{Since } S = Y - C = Y - (30 + 0.8Y) = -30 + 0.2Y$$

$$\text{Then } MPS = \frac{dS}{dY} = 0.2$$

$$MPC + MPS = 0.8 + 0.2 = 1$$

Your Turn!

Practice Problems:

1. Let $R(x) = 50x - x^2$, $C(x) = 10x + 60$

- Find MR and MC
- At what x is $MR = MC$?

2. If $Q = 20L - L^2$, find the marginal product and show diminishing returns

3. For $C = 40 + 0.75Y$, calculate MPC and MPS

4. A monopolist faces $P(x) = 120 - 3x$

- Derive total revenue, AR, MR
- Compare MR and AR

Summary

- Marginal functions describe how **economic values change at the margin**.
- **MR and MC** determine **optimal output**.
- **Marginal product** explains productivity in firms.
- **MPC and MPS** are foundational for understanding **macroeconomic behaviour**.

| In economics, it's not just the level that matters — it's how fast it changes.

2. Group Activity: Marginal Strategy Showdown

Marginal Strategy Showdown

Objective: Apply marginal concepts (MR, MC, MP_i , MPC) to make business or policy decisions.

Setup:

- 4 groups of 4 students each.
- Each group receives a scenario card (provided below).
- Must answer the question and present their logic in 2 minutes.

Scenario Cards

1. *Group 1: Production Strategy (MR vs. MC)* Your firm produces gadgets. The MR from the last unit sold was \$20, and the MC was \$25.
 - Should you increase, reduce, or maintain current production? Explain using marginal principles.
2. *Group 2: Hiring Decision (Marginal Product)* You added one more worker, and output rose by 3 units. The previous worker added 6 units.
 - Are you experiencing diminishing returns? Should you hire another?
3. *Group 3: Consumer Behaviour (MPC & MPS)* A consumer's income increases by \$100. She spends \$70 and saves \$30.
 - Calculate MPC and MPS. What would a policy-maker learn from this?
4. *Group 4: Monopoly Pricing* A monopolist sees that when they sell 2 more units, total revenue increases by \$10.
 - Is $MR < AR$? Should the firm lower or raise the price to increase profit?

Any QUESTIONS?

Thank you for your attention!

Next Class

- (April 11) Further Rules of Differentiations (4.4), Elasticity (4.5)