Mathematical Methods for International Commerce

Week 10/1: Comparative Statics (5.3)

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Why It Matters in Economics, Business & Finance

Comparative statics is an essential technique used to understand how changes in **policy, shocks, or parameters** affect outcomes in economic models.

- In **macroeconomics**, we can predict how national income responds to a change in government spending or taxation.
- In **finance**, understanding multipliers helps estimate how economic indicators respond to policy adjustments.
- In **microeconomics**, it shows how supply and demand curves shift in response to taxes, subsidies, or preferences.

It allows decision-makers to simulate "what if" scenarios and make **informed, strategic choices**.

Learning Objectives

By the end of this class, you should be able to:

- Derive the **reduced form** of macroeconomic models
- Compute **national income multipliers**
- Interpret **qualitatively** how changes in parameters affect outcomes
- Use **quantitative** multiplier analysis
- Apply **multipliers** in a simple one-good market model

Agenda

1. Comparative Statics (5.3)
2. Class Activity

1. Comparative Statics (5.3)

Structural Form → Reduced Form

Example:

Suppose:

$$Y = C + I + G$$

$$C = a + b(Y - T)$$

This is the **structural form**.

We substitute C into the Y equation:

$$Y = a + b(Y - T) + I + G$$

Solve for $Y \rightarrow$ this is the **reduced form**.

Solving for Y

Start with:

$$Y = a + b(Y - T) + I + G$$

Distribute:

$$Y = a + bY - bT + I + G$$

Group terms:

$$Y - bY = a - bT + I + G$$

Factor:

$$Y(1 - b) = a - bT + I + G$$

Solve:

$$Y = \frac{1}{1 - b}(a - bT + I + G)$$

This is the **reduced form**: it shows Y as a function of **exogenous variables**

Multiplier Concept

From the reduced form:

$$Y = \frac{1}{1-b}(a - bT + I + G)$$

- The **multiplier** is $\frac{1}{1-b}$
- It shows how **sensitive** Y is to a change in **G**, **I**, etc.

If $b = 0.8$, multiplier = $\frac{1}{1-0.8} = 5$

So:

- $\Delta G = 10 \rightarrow \Delta Y = 5 \times 10 = 50$

From economics perspectives it means that Y is **5 times more sensitive** to changes in G than G itself.

Practice Problem

Given:

- $a = 20, b = 0.75$
- $T = 10, I = 30, G = 40$

1. Write reduced form for Y

2. Compute multiplier

3. Calculate Y

4. If G increases to 50, what is the new Y?

Solution

1. Reduced form:

$$Y = \frac{1}{1 - 0.75}(20 - 0.75 \cdot 10 + 30 + 40) = 4(20 - 7.5 + 30 + 40) = 4(82.5)$$

2. Multiplier = $\frac{1}{1-0.75} = 4$

3. $Y = 330$

4. New $G = 50 \rightarrow$

$$Y = 4(20 - 7.5 + 30 + 50) = 4(92.5) = 370$$

Market Model Example

Supply and demand:

$$Q_d = 100 - 5P \quad Q_s = 20 + 3P$$

Equilibrium: $Q_d = Q_s$

Solve: $100 - 5P = 20 + 3P \Rightarrow 80 = 8P \Rightarrow P^* = 10 \Rightarrow Q^* = 100 - 5 \cdot 10 = 50$

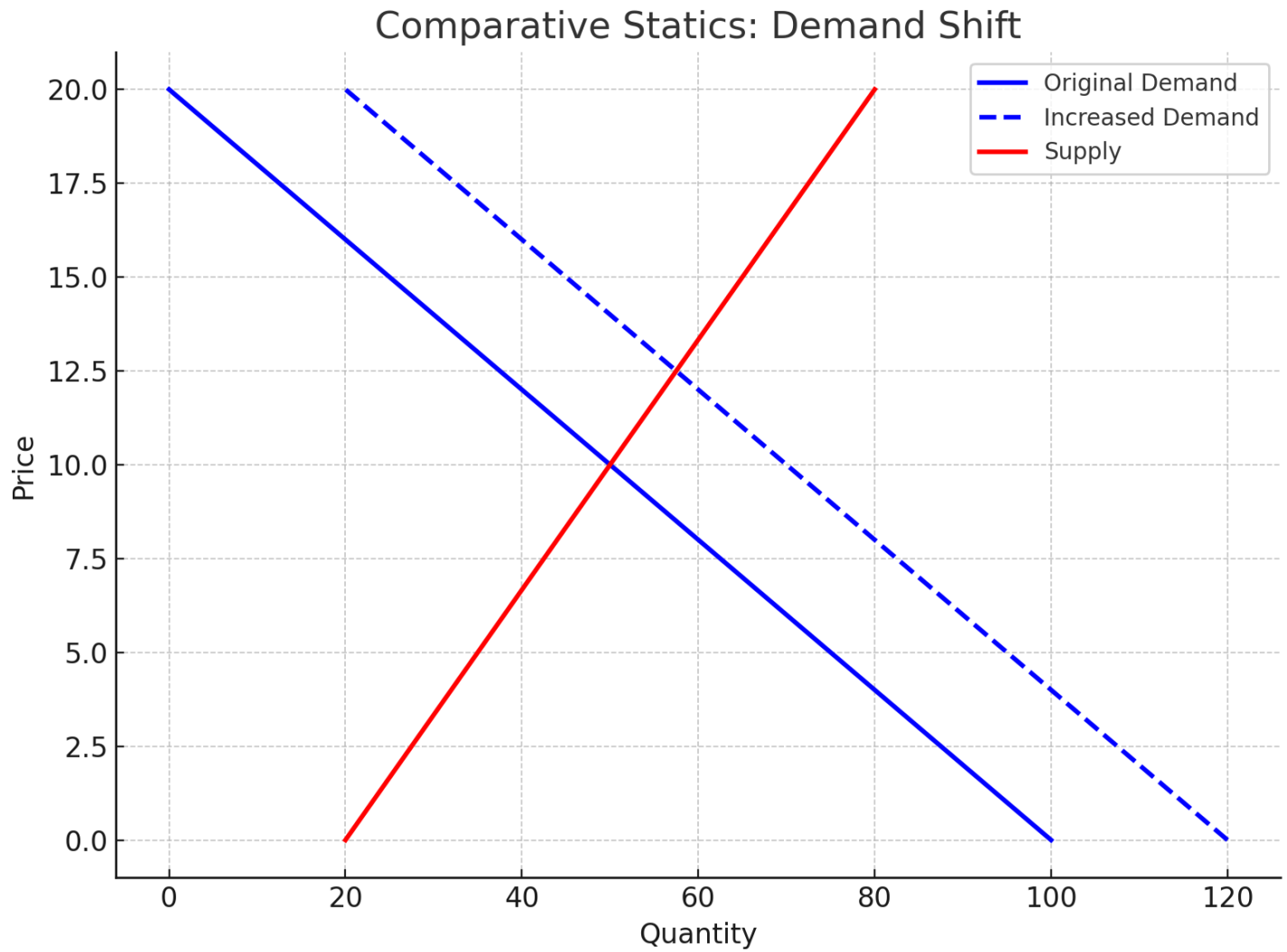
Now increase demand:

$$Q_d = 120 - 5P$$

$\rightarrow$ Solve again:
$120 - 5P = 20 + 3P \Rightarrow 100 = 8P \Rightarrow P^* = 12.5, Q^* = 120 - 5 \cdot 12.5 = 57.5$

Shows how change in **parameters** shifts equilibrium.

Plot: Comparative Statics in Supply & Demand



Summary

- Comparative statics helps **evaluate how equilibrium changes** with a shift in exogenous variables
- Multiplier tells us the **magnitude of change**
- We derived **reduced form** from structure
- We applied the concept to:
  - National income
  - Market models

2. Group Activity

Group Activity: Comparative Statics Battle

- **4 teams of 4 students**.
- Each team competes to solve a set of comparative statics challenges.
- Teams get **2 minutes** per question.
- Points awarded for:
  - Correct answer ✅
  - Economic interpretation 💡
  - Bonus if solved under 1 minute ⏱️

Round 1: Solve the Model!

Structural Form:

$$Y = C + I + G, \quad C = a + b(Y - T)$$

Each team:

1. Derive the **reduced form** of Y.

2. Identify the **multiplier**.

3. Compute Y if $a = 10, b = 0.8, I = 30, G = 40, T = 5$

Round 2: Policy Shock Simulation

Suppose:

$$Y = \frac{1}{1-b}(a - bT + I + G)$$

Government increases spending: $\Delta G = +20$

Each team:

- Calculate the new Y if $b = 0.75$
- Discuss: "Why does a higher b mean a bigger impact?"

Round 3: Market Shock

Market model:

$$Q_d = 120 - 4P, \quad Q_s = 30 + 2P$$

Task:

1. Find equilibrium P^*, Q^*
2. If Q_d shifts to $140 - 4P$, find new P, Q
3. Sketch **before/after curves**

Wrap-Up & Debrief

- Which team explained their **economic reasoning** best?
- Where did **math meet policy**?
- Pints for participation!

Any QUESTIONS?

Thank you for your attention!

Next Classes

- (May 9) Unconstrained Optimization (5.4)