Mathematical Methods for International Commerce

Week 15/1-15/2: Applications of Linear Programming (8.2)

Iegor Vyshnevskyi, Ph.D.

Sogang University

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Agenda

1. Applications of Linear Programming (8.2)
2. Group activity
3. Final Exam Arrangements

1. Applications of Linear Programming (8.2)

Why It Matters in Economics & Business

Linear programming (LP) is a powerful optimization technique used in:

- **Production planning**
- **Cost minimization**
- **Profit maximization**
- **Resource allocation**

Used by:

- Manufacturers
- Service providers
- Financial institutions

> Linear programming helps make **data-driven** decisions in complex environments.

Learning Objectives

By the end of this class, you should be able to:

- Identify decision variables in a word-based LP problem
- Formulate the objective function (maximize or minimize)
- Write down all constraints from the problem context
- Solve LP problems using graphical methods
- Interpret and calculate **shadow prices**

Graphical Method (Recap)

1. Convert inequalities to equalities to find constraint lines
2. Graph the feasible region
3. Identify the corner points (vertices)
4. Evaluate the objective function at each corner
5. Choose the optimal value (max/min)

Example: Glass Studio (Ian Jacques)

A pottery studio makes **bowls** and **plates**:

Let:

- x: number of bowls
- y: number of plates

Objective: Maximize profit

$$\text{Profit} = 150x + 100y$$

Constraints (from studio resources)

1. **Glassblowing time** (70 hrs max):

$$2x + y \leq 70$$

2. **Annealing time** (130 hrs max):

$$4x + y \leq 130$$

3. **Sand available** (45 kg max):

$$x + y \leq 45$$

4. **Non-negativity**:

$$x \geq 0, \quad y \geq 0$$

These form the **feasible region** in the solution space.

The complete example setup

Context: A studio produces bowls (x) and plates (y)

- **Profit**: $150 per bowl, $100 per plate

Objective: Maximize profit

$$Z = 150x + 100y$$

Subject to:

$$\begin{aligned} 2x + y &\leq 70 & \text{(glassblowing)} \\ 4x + y &\leq 130 & \text{(annealing)} \\ x + y &\leq 45 & \text{(silica sand)} \\ x, y &\geq 0 \end{aligned}$$

Step 1: Plot Feasible Region

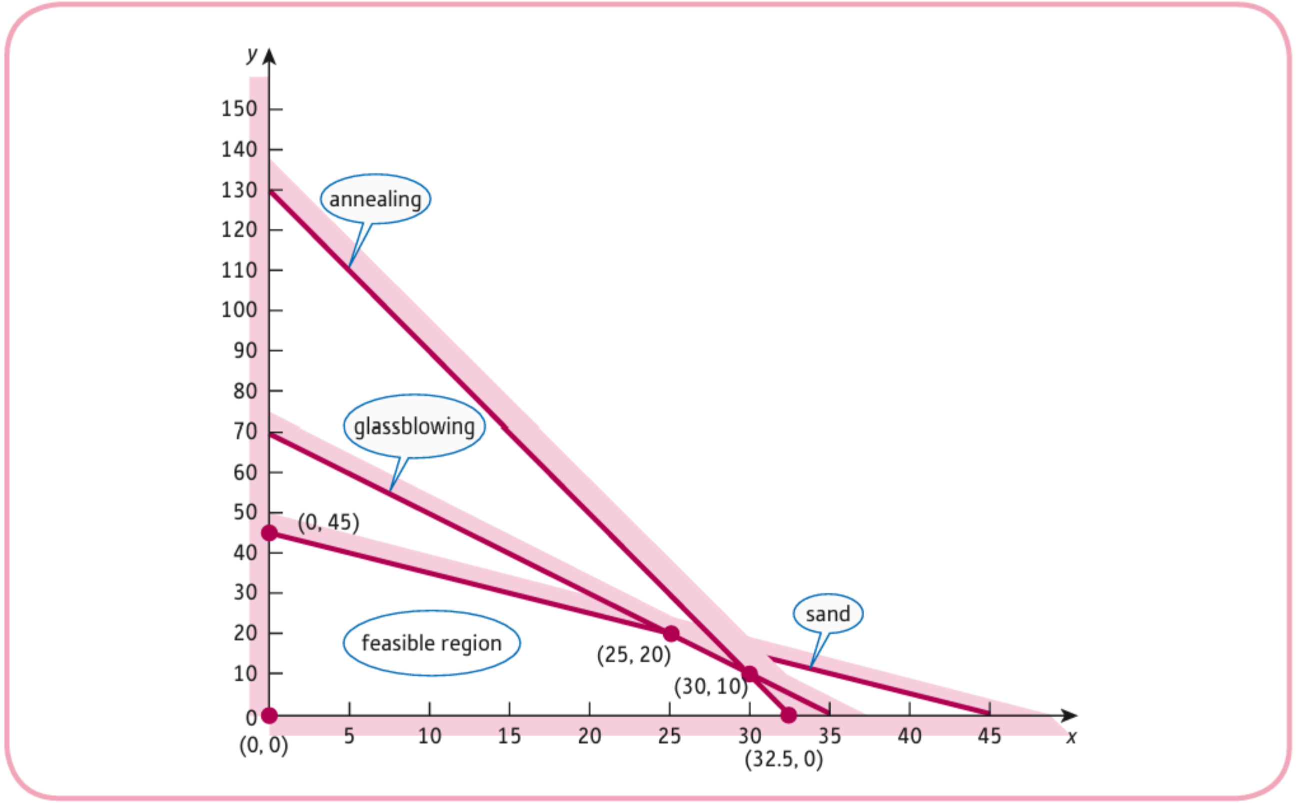

Figure 8.13

Source: the course book

Step 2: Identify Corner Points

- (0, 0)
- (0, 45)
- (25, 20) — Intersection of $2x + y = 70$ and $x + y = 45$
- (30, 10) — Intersection of $2x + y = 70$ and $4x + y = 130$
- (32.5, 0)

Step 3: Evaluate Objective Function

Point	Profit = $150x + 100y$
(0, 0)	0
(0, 45)	4500
(25, 20)	**5750** (Optimal)
(30, 10)	5500
(32.5, 0)	4875

- Optimal solution: **(25, 20)** with profit **$5750**.
- Substituting $x = 25$ and $y = 20$ into $4x + y$ gives *120*.
- As such, there were 130 hours available, so 10 of these hours are unused.

Resource Adjustment Example

One important task when planning business projects is to decide whether it is worthwhile buying in extra resources.

For the glassmaking problem the studio might consider:

- Increasing hours for **glassblowing** or **annealing**
- Buying an extra **kilogram of sand**

To explore this:

- Rework the LP problem by **changing one constraint at a time**
- Software tools provide this info to assess **sensitivity** of the solution

We'll consider next:

> A case when **one of the glassblowers works one extra hour** (glassblowing constraint increases)

Shadow Prices

- If **glassblowing hours increase by 1**:
  - Constraint changes: $2x + y \leq 71$
  - New solution: $x = 26$, $y = 19$
  - New profit: $150(26) + 100(19) = 5800$
  - **Shadow price** of glassblowing = $5800 - 5750 = 50$

Shadow price tells us **how much profit changes** with a marginal increase in a resource.

Example: Labour Cost Minimization

Company needs **900 hours** work:

- Full-time: 40 hrs/week, $800/wk
- Part-time: 20 hrs/week, $320/wk
- Max: part-time $\leq \frac{1}{3}$ full-time

$x = \text{number of full-time workers } y = \text{number of part-time workers}$

Objective: Minimize

$$Z = 800x + 320y$$

Subject to:

$$\begin{aligned} 40x + 20y &\geq 900 \\ y &\leq \frac{x}{3} \\ x, y &\geq 0 \end{aligned}$$

Step 1: Plot Constraints to see Feasible Region

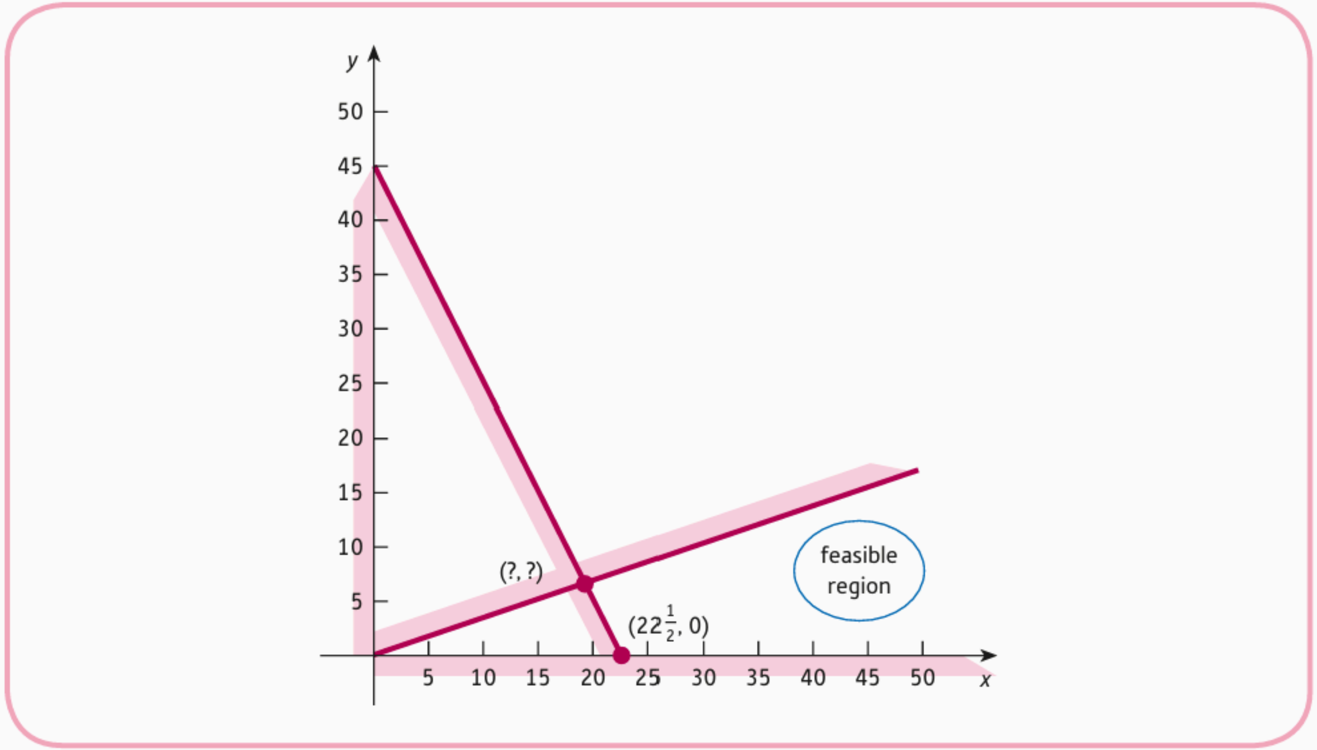

Figure 8.14

Source: the course book

Step 2: Identify Corner Points

Solving Algebraically (from Textbook)

The feasible region has two corners. One is clearly $(22.5, 0)$. The other is the intersection of:

1. $y = \frac{x}{3}$
2. $40x + 20y = 900$

Substitution:

$$\begin{aligned} 40x + 20\left(\frac{x}{3}\right) &= 900 \\ 40x + \frac{20x}{3} &= 900 \\ \frac{140x}{3} &= 900 \\ x &= \frac{2700}{140} = \frac{135}{7} \end{aligned}$$

Now solve for y:

$$y = \frac{135}{7} \cdot \frac{1}{3} = \frac{45}{7}$$

The **feasible region corner points**: $\left(\frac{135}{7}, \frac{45}{7}\right), (22.5, 0)$

Integer Programming Note

Solution: $\left(\frac{135}{7}, \frac{45}{7}\right)$, but we can't hire fractional workers. In practice, we round to the nearest feasible integers.

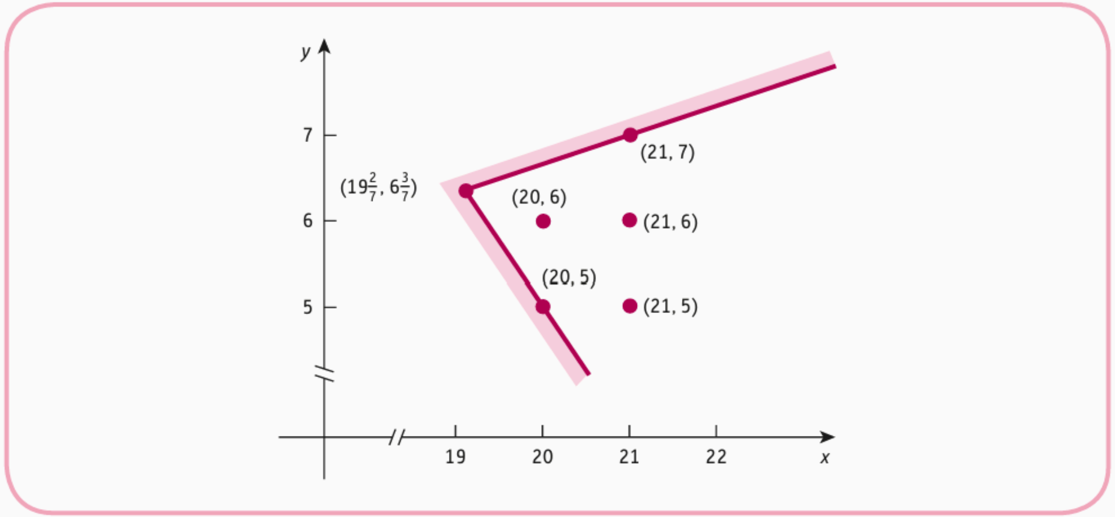

Figure 8.15

Source: the course book

Nearest feasible point with integers:

- (20, 5) with cost = $17,600 (optimal)

Key Takeaways

- **Formulate LP problems** from real-world scenarios
- Use graphical solution for 2-variable cases
- Evaluate at **corner points**
- Understand **shadow prices** and what they tell us
- For integer-only decisions, adjust your answer accordingly

2. Group activity

Practice Problem (Group)

Electronics firm:

- Two models: TAB1 and TAB2
- Costs: \$120 (TAB1), \$160 (TAB2), total budget \$4000
- Max total: 30 tablets
- Profits: \$600 (TAB1), \$700 (TAB2)

Let:

$$x = \text{number of TAB1 tablets}$$
$$y = \text{number of TAB2 tablets}$$

Maximize:

$$Z = 600x + 700y$$

Subject to:

$$120x + 160y \leq 4000$$
$$x + y \leq 30$$
$$x \geq 0, \quad y \geq 0$$

Task

Solve the electronics firm LP problem:

1. Plot constraints
2. Find feasible region
3. Identify corner points
4. Evaluate profit function at each
5. Determine optimal mix of TAB1 and TAB2

3. Final Exam Arrangements

Final Exam Arrangements

- Coverage: all topics after the Midterm

Practice & Prep

Review your:

- Lecture slides and solved examples
- Homework #2
- Quiz #2

Follow exam instructions!

- June 20, 1:30 pm.
- Closed book / Individual work.
- You may use a calculator (no phones or financial calculators).
- Only one A4 double-sided **handwritten** cheat sheet (formulas only) is permitted.
- The time limit (75 minutes) will be strictly enforced.

Time to shine – you've got this!

Good luck!

Any QUESTIONS?

Thank you for your attention!

Next Classes

- (June 20) Final Exam

Good luck with your preparation!