MO824 - Atividade 1

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Formulation

Objective Function

Our objective, as said in the activity, is to minimize the costs of production and shipment of goods from a set of factories to another set of clients. Since we have cost associated with each of those operations, we will utilize two set of variables: $x_{p,l,f}$ to represent the amount (in tons) of product $p \in P$ produced by machine $l \in L$ at factory $f \in F$ and $y_{p,f,j}$ to represent the amount (in tons) of product $p \in P$ transported from factory $f \in F$ to client $j \in J$.

That way, we can describe our objective function as

$$\min z = \sum_{p \in P} \sum_{f \in F} (\sum_{l \in L} x_{p,f,l} p_{p,f,l} + \sum_{j \in J} y_{p,f,j} t_{p,f,j)}).$$

Restrictions

First, we need to ensure every demand is satisfied, which means the amount of each product transported for each client should be at least greater than the client's demand. In another terms, we have

$$D_{p,j} \leq \sum_{f \in F} y_{p,f,j} \ \forall p \in P \ \forall j \in J.$$

Then, we have to ensure the amount of products produced should match the amount transported from each factory.

$$\sum_{l \in L} x_{p,f,l} = \sum_{j \in J} y_{p,f,j} \ \forall p \in P \ \forall f \in F.$$

Lastly, we have to be able to produced said products, which means, we must have the materials required and the capacity.

$$R_{m,f} \geq \sum_{p\in P}\sum_{l\in L} x_{p,f,l} r_{m,p,l} \ \forall f \in F \ \forall m \in M$$
$$C_{f,l} \geq \sum_{p\in P} x_{p,f,l} \ \forall l \in L \ \forall f \in F.$$

To complete our restrictions, since we modeled each variable as the amount produced, they should be positive.

$$x_{p,l,f} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall f \in F$$
$$y_{p,l,j} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall j \in J.$$

Final program

Combining everything, we have our Linear Program as follows.

$$\min z = \sum_{p\in P}\sum_{f\in F}(\sum_{l\in L} x_{p,l,f} p_{p,f,l} + \sum_{j\in J} y_{p,f,j} t_{p,f,j)})$$
$$\text{subject to } \sum_{f\in F} y_{p,f,j} \geq D_{p,j} \ \forall p \in P \ \forall j \in J.$$
$$\sum_{l\in L} x_{p,f,l} - \sum_{j\in J} y_{p,f,j} = 0 \ \forall p \in P \ \forall f \in F.$$
$$\sum_{p\in P}\sum_{l\in L} x_{p,f,l} r_{m,p,l} \leq R_{m,f} \ \forall f \in F \ \forall m \in M$$
$$\sum_{p\in P} x_{p,f,l} \leq C_{f,l} \ \forall l \in L \ \forall f \in F.$$
$$x_{p,l,f} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall f \in F$$
$$y_{p,l,j} \geq 0 \ \forall p \in P \ \forall l \in L \ \forall j \in J.$$

Results

Running our model using Gurobi 9.2 solver in a machine with a Ryzen 1800x octacore at 3.6GHz with 16GB of ram and 16GB of swap (which we saw being used). We obtained the following results.

As of work time, we can see in the Figure 1 that we have an exponential relationship between number of client and work/running time.

Table 1: Result of running 10 random instances.

# Clients	# Variables	# Constrains	Result cost	Execution time	Work
100	146880	4128	11005953	0.8959	0.8408
200	282880	6984	20663983	1.9750	2.0171
300	1378600	12040	22862187	8.7141	10.3167
400	2400830	21023	52976044	18.7725	20.8948
500	4558284	28428	79913943	38.5863	39.2341
600	5070720	27684	67383596	52.1588	43.8014
700	5857758	37248	119142323	64.8838	51.0371
800	9353070	43124	118680540	183.6494	82.7664
900	9546845	37947	74096075	314.5878	143.8355
1000	11612258	44927	97121488	203.0819	95.4361

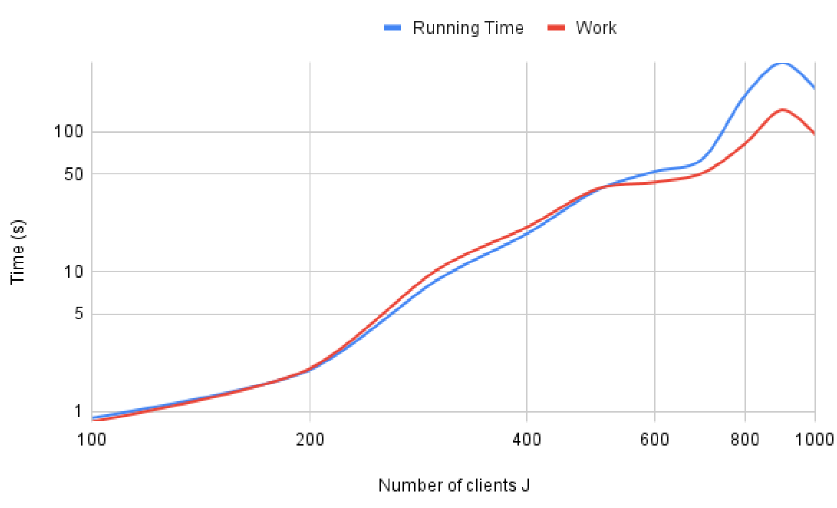

Figure 1: Plotting in logarithm scale of execution time by number of clients.

We do note the number of variables is proportional to the number of clients and we did not put the non-negativity constrains in the total number of constrains.

Also, we can note some weirdness in our data: the randomness of the instances made the instances with 700 an 800 clients have out of proportion result value, compared to its peers; we also note the execution time of the instance with 900 clients, which came out to be a lot slower than others. We do believe that anomaly maybe caused by some external factors involving the machine.