Thermal System Modeling

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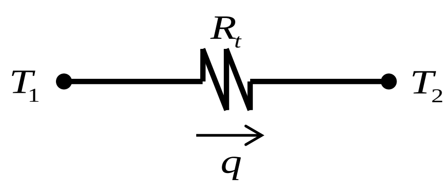
Overview

System	Effort	Position	Flow	Dissipation	Compliance	Inertia
Mech.	Force /Torque	(Angular) Displacement	(Angular) Velocity	Damper	Stiffness	Mass
Elec.	Voltage	Charge	Current	Resistance	Capacitor	Inductance
Fluid	Pressure	Volume, V_{fluid}	Flow Rate, $\dot{V}$	Resistance	Volume, V_{tank}	Inertance
Pneumatic	Pressure	Mass, m_{fluid}	Flow Rate, $\dot{m}$	Resistance	Mass, m_{tank}	Inertance
Thermal	Temp.	Heat	Heat Flux	Resistance	Mass	-

- **Thermal Resistance:** how difficult it is for heat to be conducted

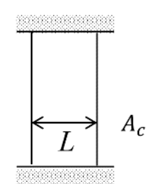


$$effort = resistance \times flow$$

$$\Delta T = T_1 - T_2 = R_t \cdot q$$

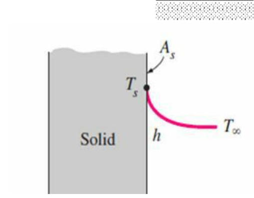
$$temperature\ difference\ [°C] \equiv thermal\ resistance\ [°C/W] \times heat\ transfer\ rate\ [W]$$

<u>Conduction</u>:



$$R_{T,cond} = \frac{L}{k \cdot A_c} \equiv \frac{length\ [m]}{thermal\ conductivity\ [W \cdot °C/m] \times cross\ sectional\ area\ [m^2]}$$

<u>Convection</u>:



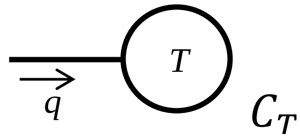
$$R_{T,conv} = \frac{1}{h \cdot A_s} \equiv \frac{1}{convective\ coeff.\ [W \cdot °C/m^2] \times surface\ area\ [m^2]}$$

<u>Radiation</u>:

$$R_{T,rad} = \frac{\Delta T}{\sigma \cdot \epsilon \cdot A_{ij} \cdot F_{ij} \cdot \left(T_i^4 - T_j^4\right)} \approx \frac{1}{h_{rad} \cdot A_s} \equiv \frac{1}{rad.coeff.\ \times surf.area}$$

Where, $R_{T,rad}$ can be linearized about the operating point. $q \propto (T_1^4 - T_2^4)$

- **Thermal Capacitance:** the ability for a material to store and release heat over time



$$flow \equiv capacitance \times \frac{d}{dt}(effort)$$

$$q = C_T \cdot \frac{dT}{dt} = m \cdot c_p \cdot \frac{dT}{dt} = \rho \cdot V \cdot c_p \cdot \frac{dT}{dt}$$

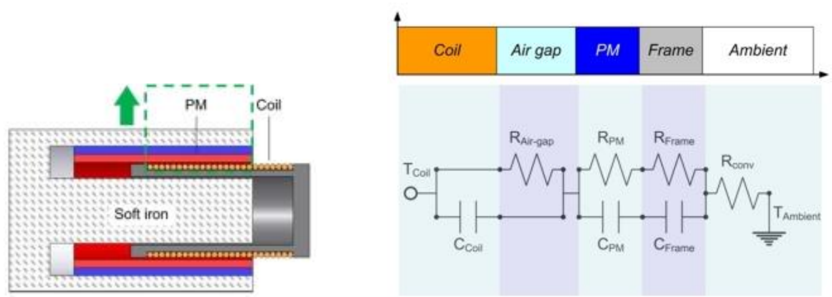
Where,

$C_T \equiv thermal\ capacitance\ [J/°C]$

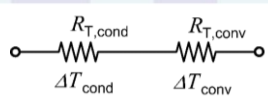
$\rho \equiv density\ [kg/m^3]$, $V \equiv volume\ [m^3]$, $m \equiv mass\ [kg]$

$c_p \equiv specific\ heat\ capacity\ [J \cdot °C/kg]$

- **Lumped Capacitance Approach**: reduces a thermal system to discrete lumps



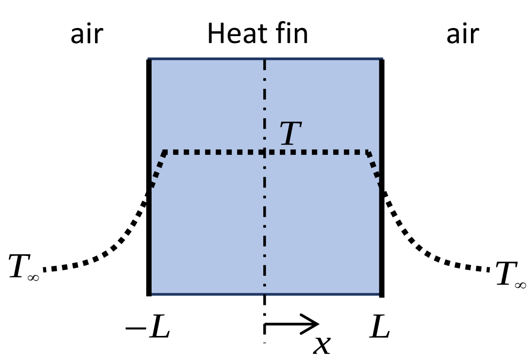
Biot Number, $Bi = \frac{\Delta T_{cond}}{\Delta T_{conv}} = \frac{R_{T,cond}}{R_{T,conv}} = \frac{L_c}{kA}\frac{hA}{1} = \frac{hL_c}{k}$



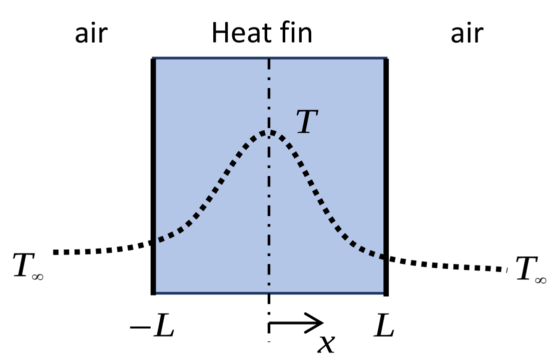
Where, $k \equiv$ thermal conductivity of solid, $L_c = \frac{V}{A_s}$, the largest dimension, to be conservative

If $Bi < 0.1$, the temperature drop across the solid is much smaller than the temperature drop across the fluid, we can neglect $R_{T,cond}$. Heat transfer is limited by convection.

If $Bi < 0.1$, lumped capacitance approximation may be used.



$Bi << 1$



$Bi > 1$

Nusselt Number, $Nu = \frac{\text{heat transfer by convection thru fluid}}{\text{heat transfer by conduction thru fluid}} = \frac{hA\Delta T}{1}\frac{L}{kA\Delta T} = \frac{hL}{k}$

Where, $k \equiv$ thermal conductivity of <u>fluid</u>

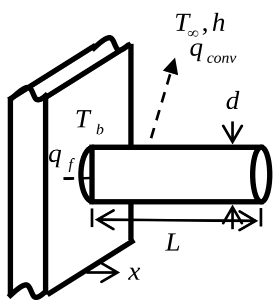
If $Nu = 1$, there is no convection; the heat is transferred purely by conduction.

If $Nu \geq 1$, there is convection.

Forced and free convection coefficients can be determined from correlations for the Nusselt number as a function of the Reynolds and Prandtl numbers:

$\overline{Nu}_x = \frac{\overline{h_x}x}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3}$ for $0.6 \leq Pr \leq 500$

- **Extended Surfaces (Fins)**



Assume uniform cross-section and steady state conditions

$$\frac{d^2T}{dx^2} - \frac{hp}{kA_c}(T - T_\infty) = 0$$

Where, $p \equiv perimeter\ [m]$, and $A_c \equiv cross\ sectional\ area\ [m^2]$

$$\frac{d^2T}{dx^2} - \frac{h \cdot (\pi d)}{k \cdot \left(\frac{\pi}{4} \cdot d^2\right)}(T - T_\infty) = \frac{d^2T}{dx^2} - \frac{4 \cdot h}{k \cdot d}(T - T_\infty) = 0$$

Let $m^2 \equiv \frac{hp}{kA_c}$, and $\theta(x) \equiv T(x) - T_\infty$

$$\frac{d^2\theta}{dx^2} - m^2 \cdot \theta = 0$$

Solve the ODE for θ

$$\theta(x) = C_1 \cdot e^{mx} + C_2 \cdot e^{-mx}$$

For boundary conditions, consider

1. Base temperature $[°C]$: $T_{base} = T_b = \theta_b + T_\infty$
2. Adiabatic tip (no heat transfer): $\frac{d\theta}{dt}|_{x=L} = 0$

Define $\eta_f =$ fin efficiency $= \dfrac{\text{actual fin heat transfer}}{\text{max fin heat transfer (if entire fin were at } \theta_b\text{)}} = \dfrac{q_f}{q_{max}} = \dfrac{q_f}{hA_f\theta_b} = \dfrac{tanh(mL)}{mL}$

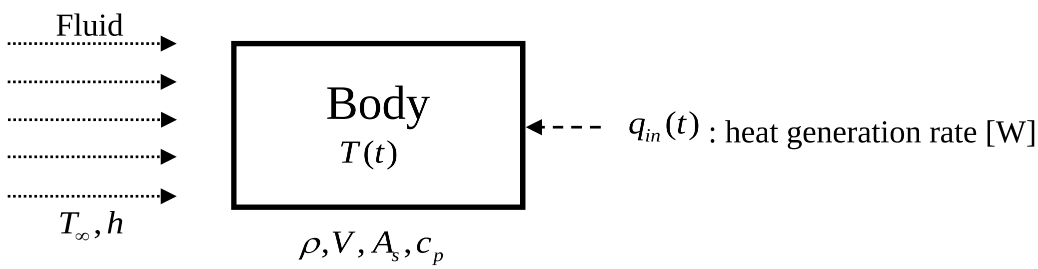
If there is convection from the tip, then use the adiabatic tip condition with effective length:

$$L_c = L + \frac{A_c}{p} = L + \frac{d}{4}$$

Which leads to

$$\eta_f = \frac{\tanh(mL_c)}{mL_c}$$

- **Transient Problem (Heating with a Step Input)**



Assume that

1. There is only convection, and the internal temperature is uniform
2. $T_0 > T_\infty$

$$q_{in} - q_{out} = q_{stored}$$

$$q_{in} - hA_s(T - T_\infty) = q_{in} - hA_s\theta = \rho V c_p \frac{d\theta}{dt}$$

$$\frac{\rho V c_p}{hA_s} \cdot \frac{d\theta}{dt} + \frac{hA_s}{hA_s} \cdot \theta = C_t R_t \cdot \frac{d\theta}{dt} + \theta = \frac{1}{hA_s} \cdot q_{in} = K \cdot q_{in}$$

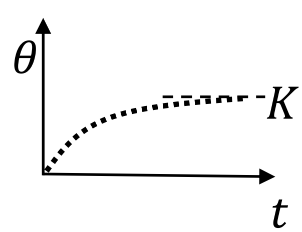
$$\tau \cdot \frac{d\theta}{dt} + \theta = K \cdot q_{in}(t)$$

$$\mathcal{L}\left\{\tau \cdot \frac{d\theta(t)}{dt} + \theta(t)\right\} = \mathcal{L}\{K \cdot q_{in}(t)\}$$

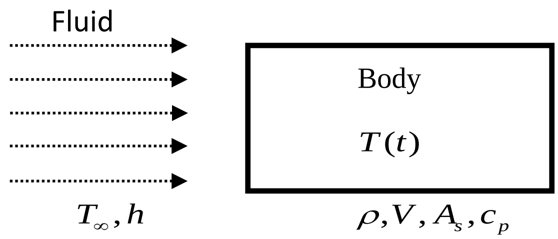
$$\tau \cdot s \cdot \theta(s) + \theta(s) = [\tau \cdot s + 1] \cdot \theta(s) = \frac{K}{s} \cdot q_{in}(s)$$

$$\theta(s) = \frac{K}{s \cdot (\tau \cdot s + 1)}$$

$$\theta(t) = K \cdot \left(1 - e^{-t/\tau}\right)$$



- **Cooling (Initial Value Problem)**



Assume:

1. convection only
2. $T(0) = T_0 > T_\infty$

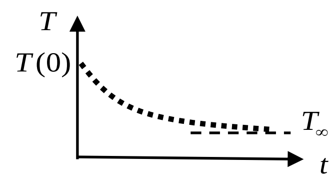
$$\tau \cdot \frac{d\theta(t)}{dt} + \theta(t) = K \cdot q_{in}(t) = 0$$

$$\tau \cdot r + 1 = 0, \;\; r = -\frac{1}{\tau}$$

$$\theta(t) = C \cdot e^{-t/\tau}$$

Initial condition:

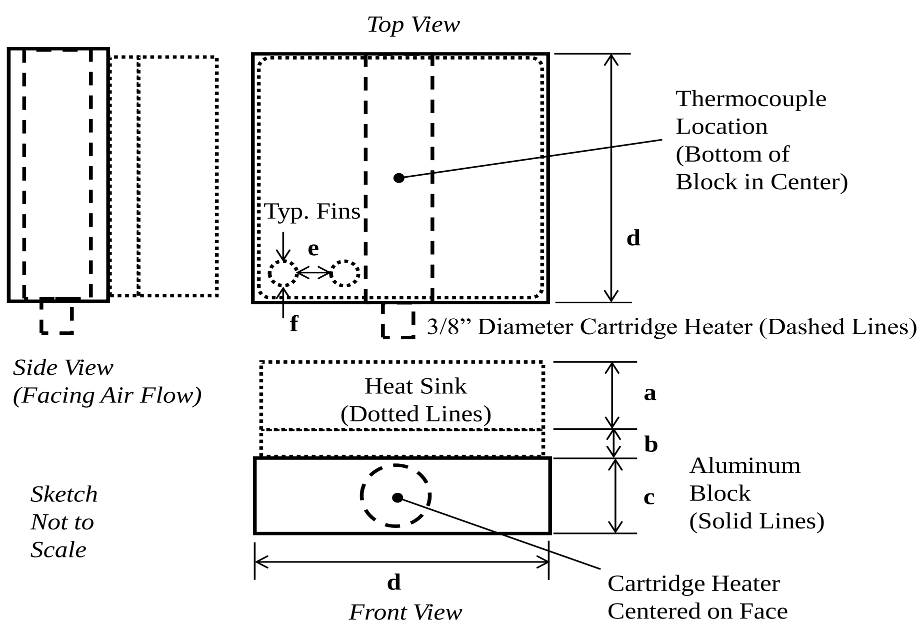
$$\theta_0 = \theta(0) = T(0) - T_\infty = T_0 - T_\infty$$



Solution:

$$\theta(t) = \theta_0 \cdot e^{-t/\tau}$$

- **Setup**



Part 1: Heating-Cooling Curve

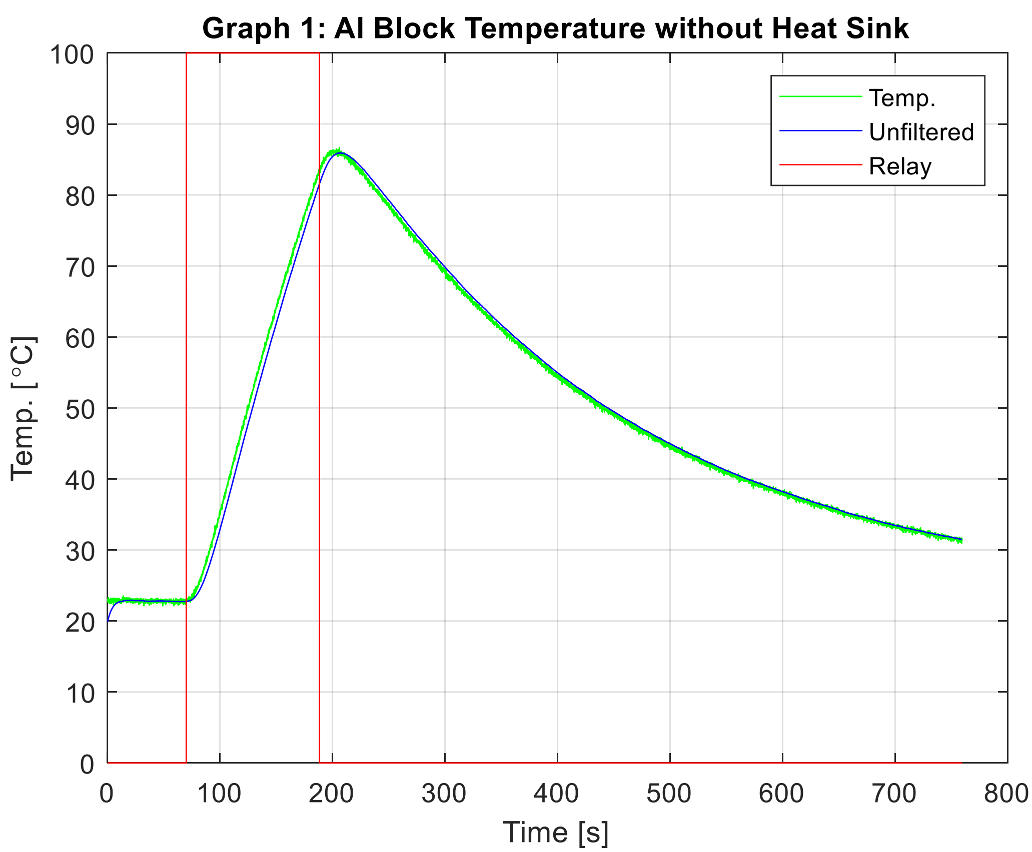
T = measured temperature of aluminum block, unfiltered

T_f = filtered temperature of aluminum block

u = control signal = signal from Arduino to relay = signal to cartridge heater

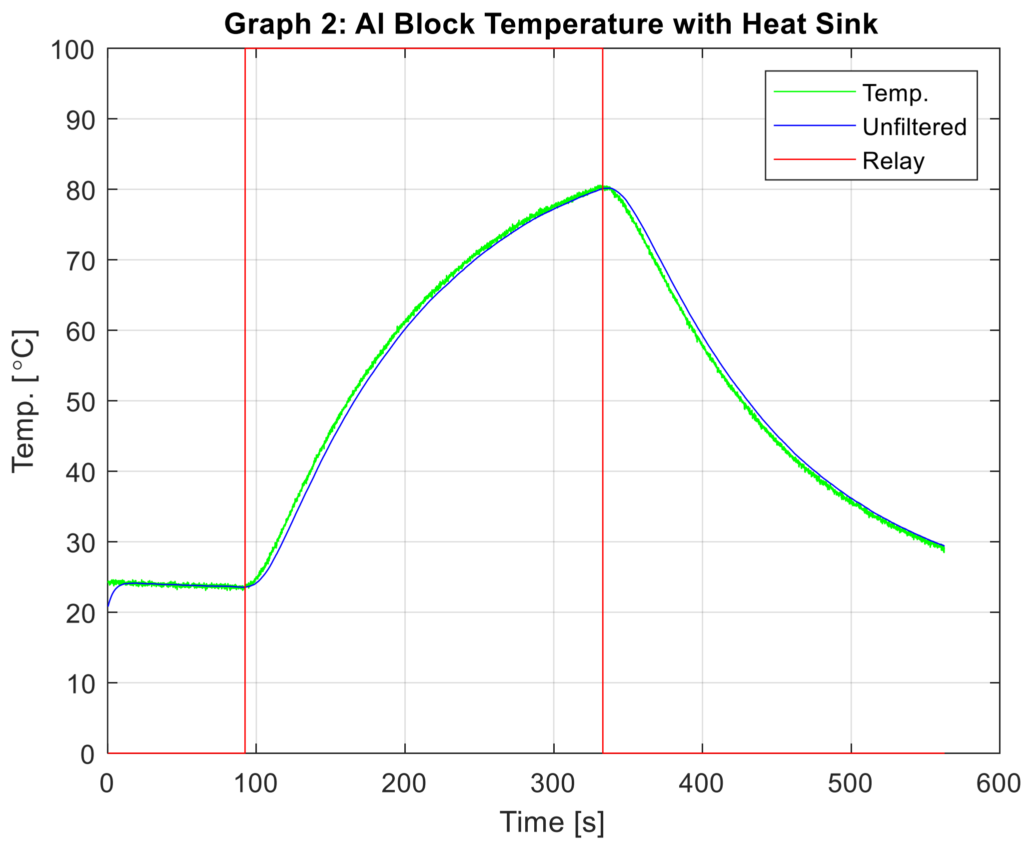
Plotted below are heating-cooling curve without and with heat sink:

Graph 1, without heat sink: (t, T), (t, T_f), and $(t, 100u)$

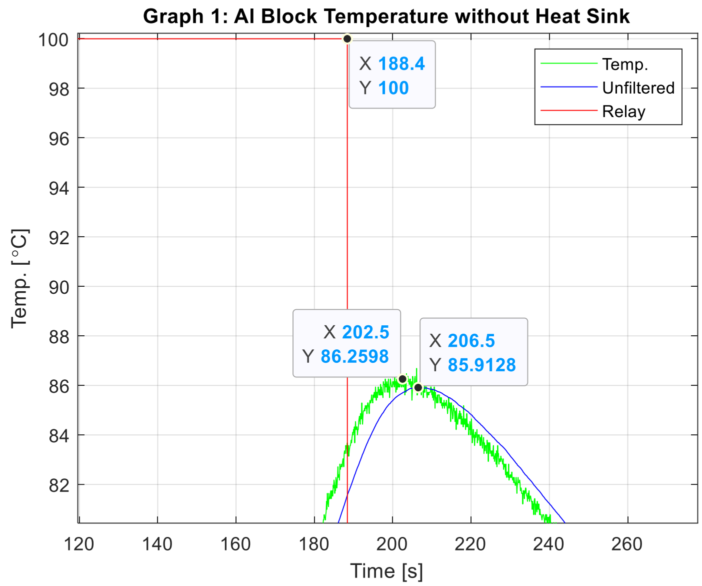


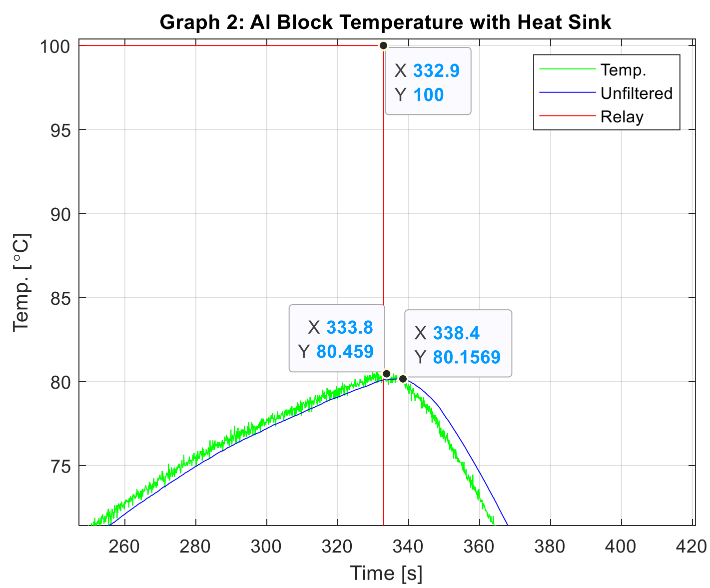

```
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 1: Al Block Temperature with Heat Sink')
```

Graph 2, with heat sink: (t, T), (t, T_f), and $(t, 100u)$




```
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink')
```





According to the plots, both the filtered and unfiltered responses exhibit delays, the delay for the curve without heat sink is around 4.1~8.1 sec, while it is about 0.8~5.5 sec for when it is with the heat sink. When the heat sink is present, the delay is reduced by 2.6~3.3 sec. By comparing the filtered and unfiltered responses, it can be seen that the filtered response has a lower delay, for both with and without heat sink scenarios.

A low pass filter is used to reduce the high frequency noise from the signal.

The delay caused by a first-order low-pass filter in Simulink is an inherent result of its filtering characteristics, and the extent of delay is related to filter's time constant and/or cutoff frequency.

Let $x \equiv unfiltered\ signal$, $y \equiv filtered\ signal$, then the first-order lower pass filter is

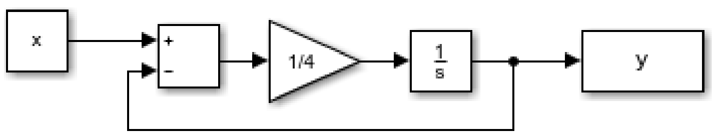
$$\tau \cdot \dot{y} + y = x$$

$$\mathcal{L}[\tau \cdot \dot{y} + y] = \mathcal{L}[x]$$

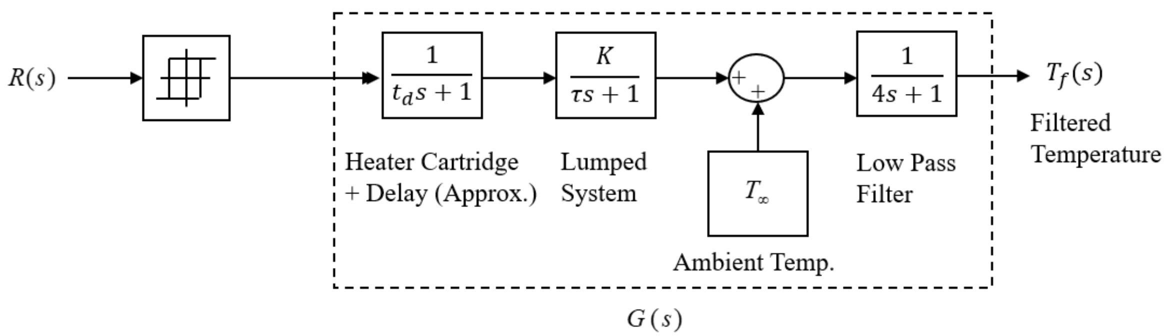
$$\tau \cdot s \cdot Y(s) + Y(s) = (\tau \cdot s + 1) \cdot Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau \cdot s + 1} = \frac{1}{4 \cdot s + 1}$$

The filter can be implemented in Simulink by solving for the highest order derivative,



The block diagram for the entire system is



The sub-system block $G_1(s) = \frac{1}{t_d \cdot s+1}$ in the plant $G(s)$ can be approximated assuming delay t_d,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} \approx 1 + x$$

$$e^{-x} = e^{-t_d s} = \frac{1}{1+x} = \frac{1}{1+t_d s}$$

The low pass filter block $G_3(s) = \frac{1}{4s+1}$ has a time constant of $\tau = 4\ s$, which is equivalent to a cut-off frequency $f_c = 0.25\ rad/s$. This correlates with the results, as the delay is observed to be slightly above 4 seconds, as expected.

Part 2: Parameter Estimation

To estimate parameters time constant τ, process time delay t_d, and gain K, six points are needed.

Define $u \equiv relay\ signal$, $t \equiv time\ arry$, $T \equiv temp.signal$, $Te \equiv experimental\ temp.$, and $N \equiv number\ of\ data\ points$

The six points are:

Pt 1: when relay is on

Pt 2: when $T_e > T_{init} + 1$

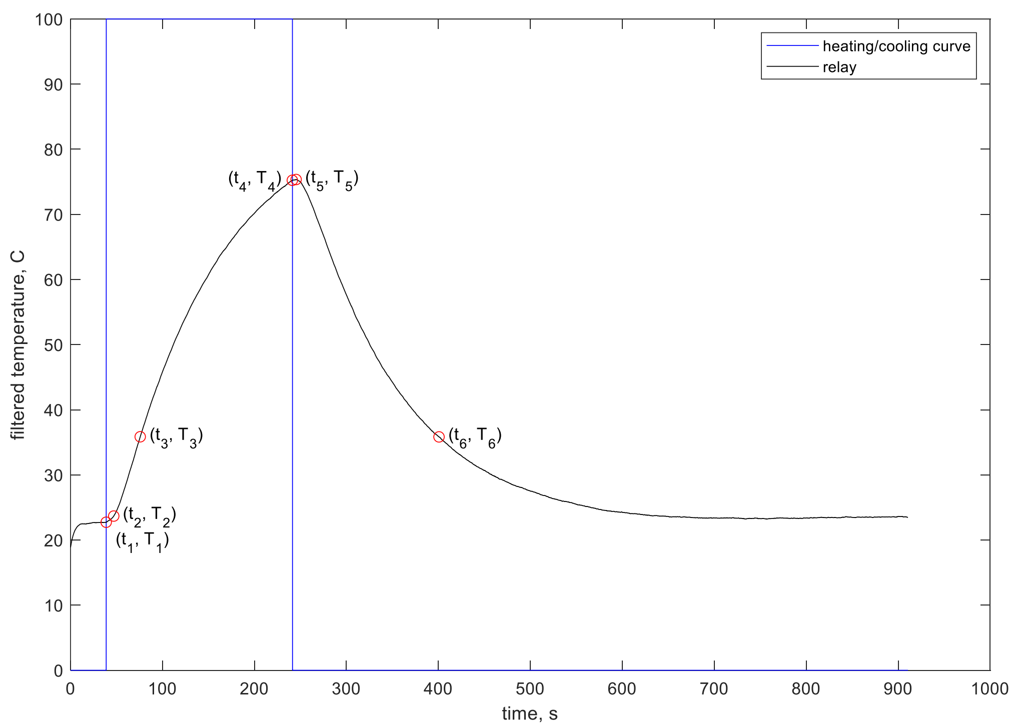
Pt 3: when T_e is x % of the way up (arbitrary)

Pt 4: when relay is off

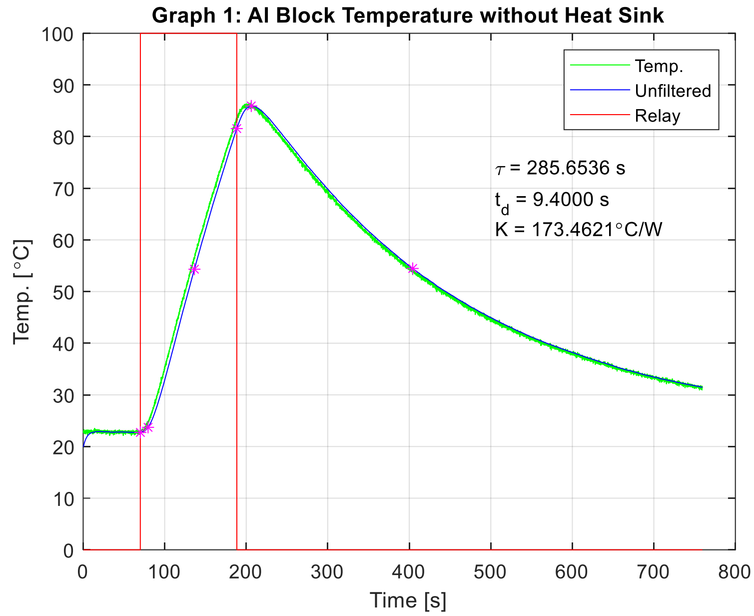
Pt 5: when $T = \max(T_e)$

Pt 6: when T_e is x % of the way down (arbitrary)

Sample:



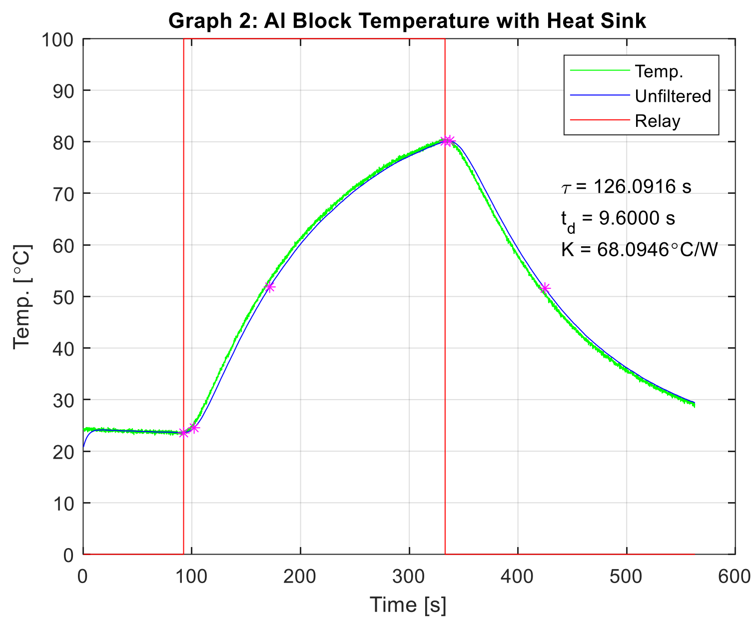
Actual plots and parameters:



Time constant, $\tau \approx 285.653640109794\ s$

Process time delay, $t_d \approx 9.39999847412109\ s$

Gain (Static sensitivity), $K \approx 173.462053571547\ °C/W$



Time constant, $\tau \approx 126.091581795742\ s$

Process time delay, $t_d \approx 9.59999847412109\ s$

Gain (Static sensitivity), $K \approx 68.0946142195335\ °C/W$

Calculation Scheme:

Let $t_5^* = 0$, $\theta(t_5^*) = T_5 - T_\infty$

Let $t_6^* = t_6 - t_5$, $\theta(t_6^*) = T_6 - T_\infty$

$$\theta(t_6^*) = T_6 - T_\infty = T_6 - [T_5 - \theta(t_5^*)] = T_6 - T_5 + \theta(t_5^*)$$

$$\theta(t_6^*) = \theta(t_5^*) \cdot e^{-t_6^*/\tau}$$

$$\ln\left[\frac{\theta(t_6^*)}{\theta(t_5^*)}\right] = \ln\left[e^{-t_6^*/\tau}\right] = -\frac{t_6^*}{\tau}$$

$$time\ constant, \tau = -\frac{t_6^*}{\ln\left[\frac{\theta(t_6^*)}{\theta(t_5^*)}\right]}$$

$$process\ delay, t_d = t_2 - t_1$$

Let $t_3^* = t_3 - t_2$, $\theta(t_3^*) = T_3 - T_\infty$

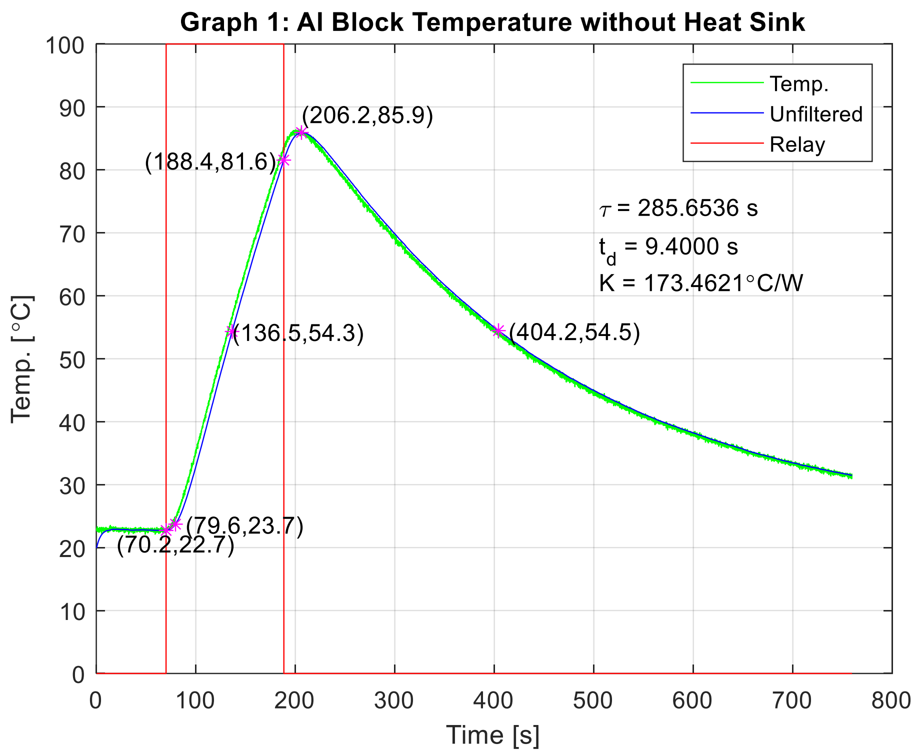
$$\theta(t_3^*) = T_3 - T_\infty = K \cdot \left(1 - e^{-t_3^*/\tau}\right)$$

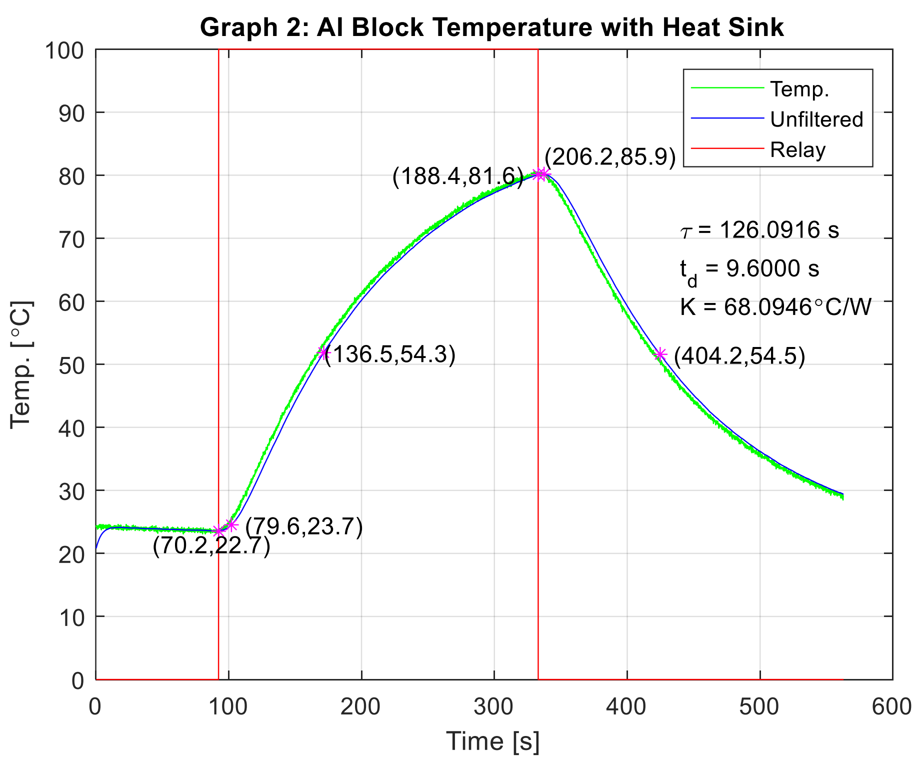
$$gain, K = \frac{\theta(t_3^*)}{1 - e^{-t_3^*/\tau}}$$

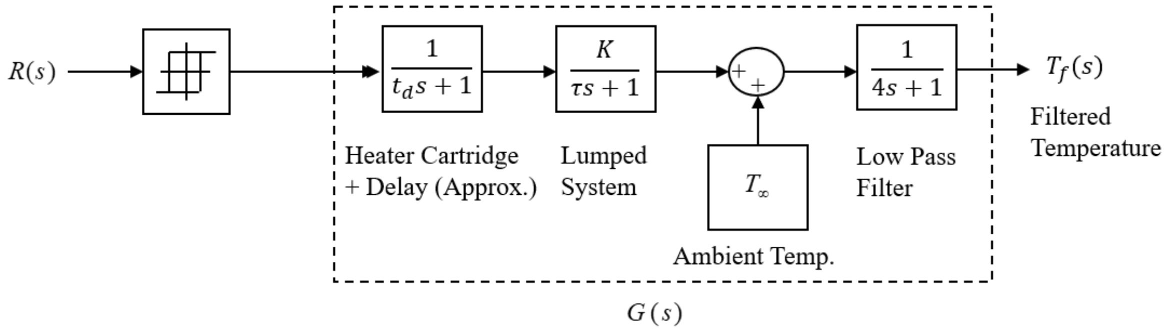
```
%% Part 2: Parameter Estimation (without Heat Sink)
t1=70.2; [~,I]=min(abs(tn-t1)); T1=Tnf(I); [~,I]=min(abs(Tnf(1:2019)-T1-1));
T2=Tnf(I); t2=tn(I); t4=188.4; [~,I]=min(abs(tn-t4)); T4=Tnf(I); Tamb=23;
[T5,I]=max(Tnf); t5=tn(I); T3=T1+0.5*(T5-T1); [~,I]=min(abs(Tnf(1:2019)-T3));
t3=tn(I); T6=T5+0.5*(Tamb-T5); [~,I]=min(abs(Tnf(2019:end)-T6)); t6=tn(I+2019);


t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'),
title('Graph 1: Al Block Temperature without Heat Sink'),
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+100,T6+20,['\tau' ' = 285.6536 s']),
text(t6+100,T6+13,['t_d' ' = 9.4000 s']),
text(t6+100,T6+8,['K' ' = 173.4621' '\circC/W'])
```

```
%% Part 2: Parameter Estimation (with Heat Sink)
t1=92.5; [~,I]=min(abs(th-t1)); T1=Thf(I); [~,I]=min(abs(Thf(1:2019)-T1-1));
T2=Thf(I); t2=th(I); t4=332.9; [~,I]=min(abs(th-t4)); T4=Thf(I); Tamb=23;
[T5,I]=max(Thf); t5=th(I); T3=T1+0.5*(T5-T1); [~,I]=min(abs(Thf(1:2019)-T3));
t3=th(I); T6=T5+0.5*(Tamb-T5); [~,I]=min(abs(Thf(2019:end)-T6)); t6=th(I+2019);


t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink')
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+15,T6+20,['\tau' ' = 126.0916 s']),
text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
text(t6+15,T6+8,['K' ' = 68.0946' '\circC/W'])
```

Plotted below, on each figure, from left to right are $(t_1, T_1), (t_2, T_2), \ldots (t_6, T_6)$







One may use the linear simulation ($lsim$) command in MATLAB to study the linear system response to relay signal input, u

To do so, first recognize that the plant consists of three sub-plants: cartridge heater + approximated delay, metal block, and low pass filter. In such order, define

$$G_1(s) = \frac{1}{t_d s + 1}$$

$$G_2(s) = \frac{K}{\tau s + 1}$$

$$G_3(s) = \frac{1}{4s + 1}$$

$$G(s) = [G_1(s) \cdot G_2(s) \cdot G_3(s)] + T_\infty = \left[\frac{1}{t_d s + 1} \cdot \frac{K}{\tau s + 1} \cdot \frac{1}{4s + 1}\right] + T_\infty$$

The transfer function of the system is

$$G(s) = \frac{T_f(s)}{R(s)} = \left[\frac{1}{t_d s + 1} \cdot \frac{K}{\tau s + 1} \cdot \frac{1}{4s + 1}\right]$$

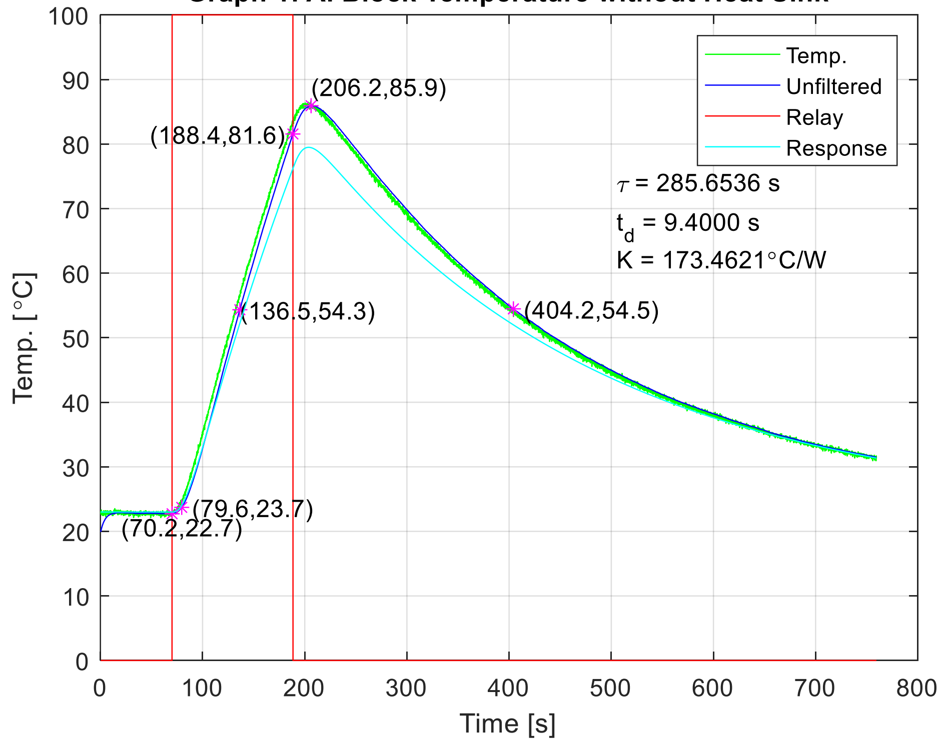
Hence the time domain response of the system is

$$T_f(t) = \mathcal{L}^{-1}[T_f(s)] = \mathcal{L}^{-1}\left[\left(\frac{1}{t_d s + 1} \cdot \frac{K}{\tau s + 1} \cdot \frac{1}{4s + 1}\right) \cdot R(s)\right]$$

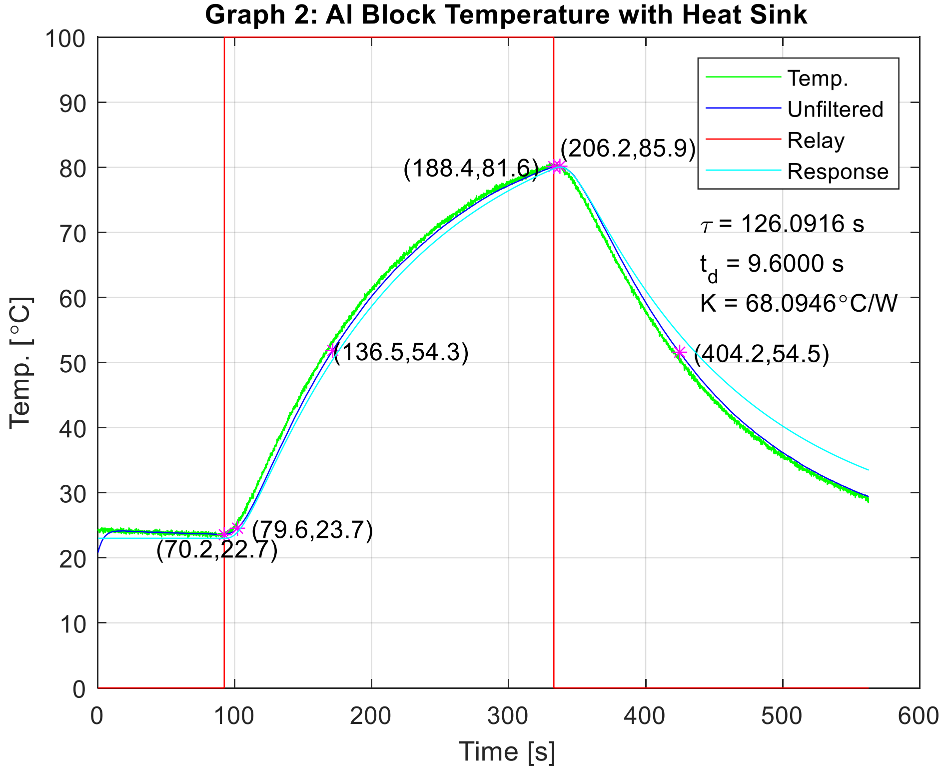
$$T_f(t) = \mathcal{L}^{-1}[T_f(s)] = \mathcal{L}^{-1}\left[\left(\frac{1}{t_d s + 1} \cdot \frac{K}{\tau s + 1} \cdot \frac{1}{4s + 1}\right) \cdot \frac{1}{s}\right] + T_\infty$$

Plotted below are the simulated system response and experimental results for the two scenarios:

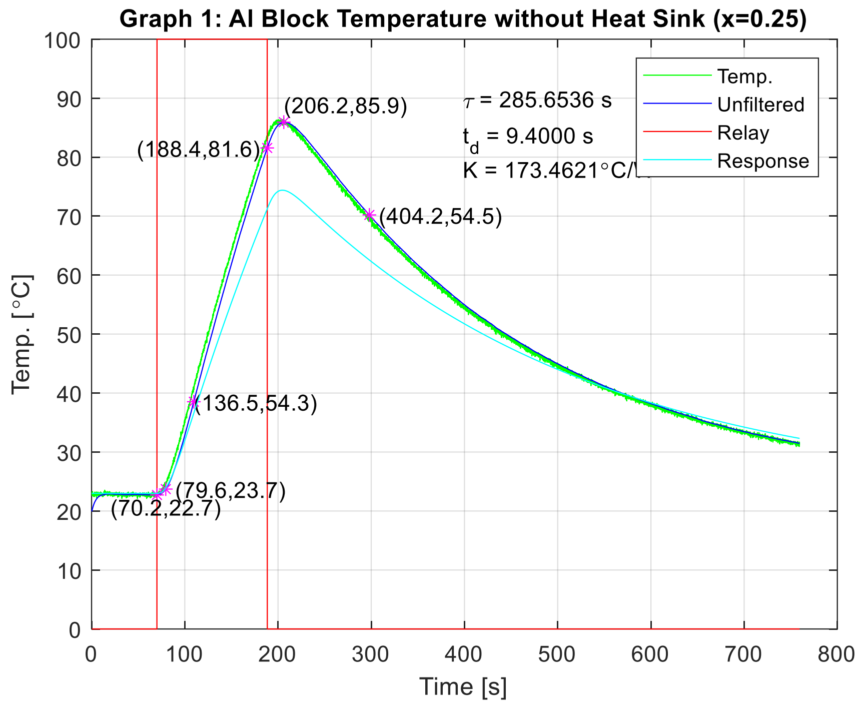
Graph 1: Al Block Temperature without Heat Sink

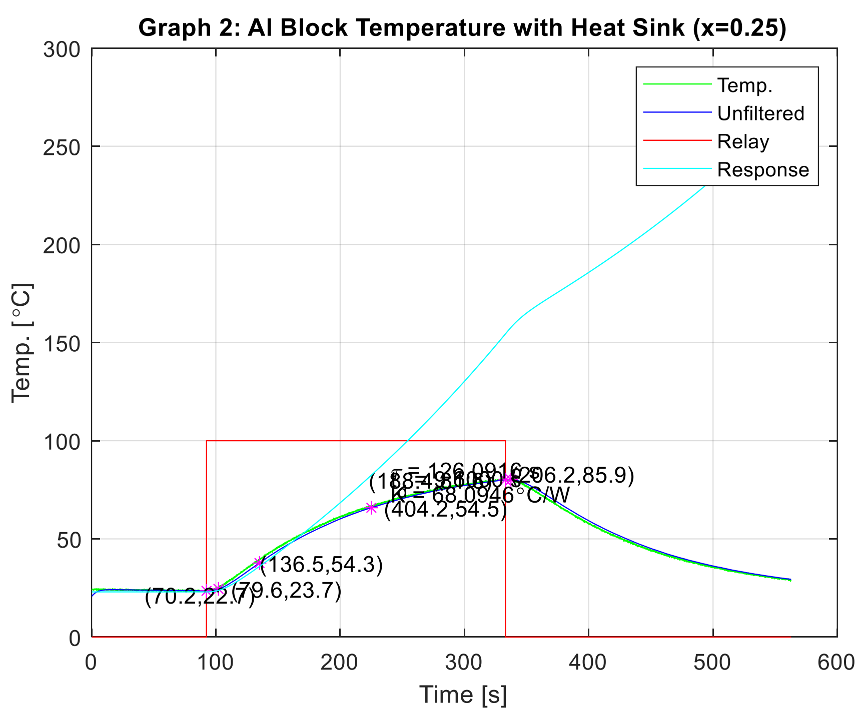


Graph 2: Al Block Temperature with Heat Sink

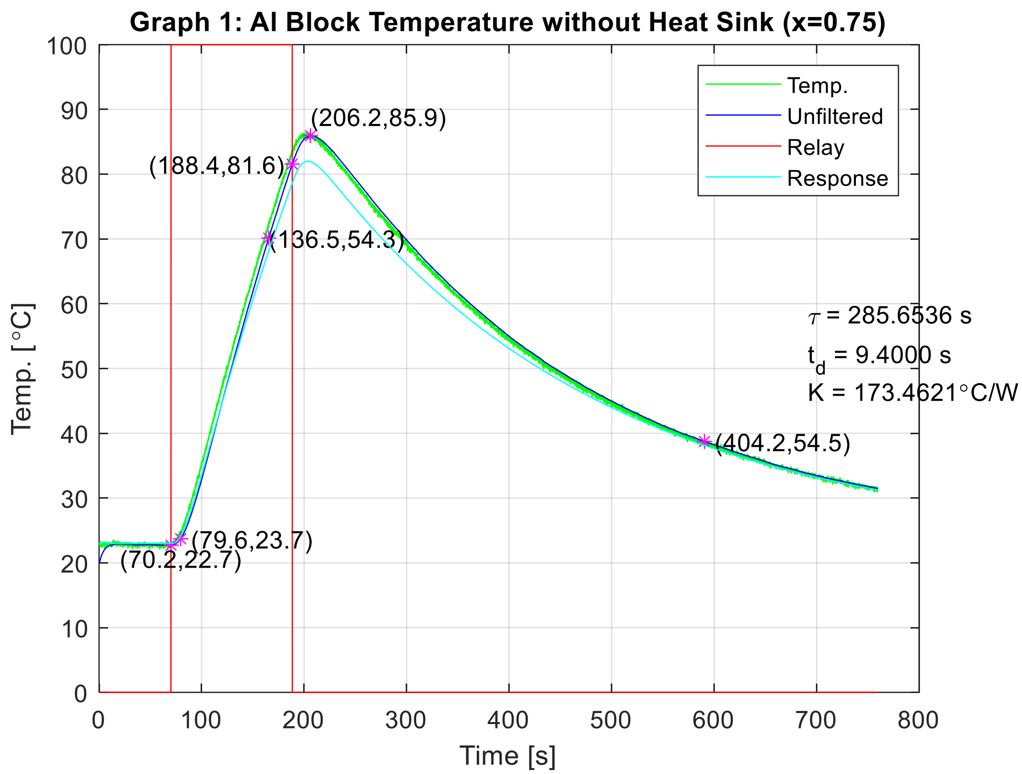


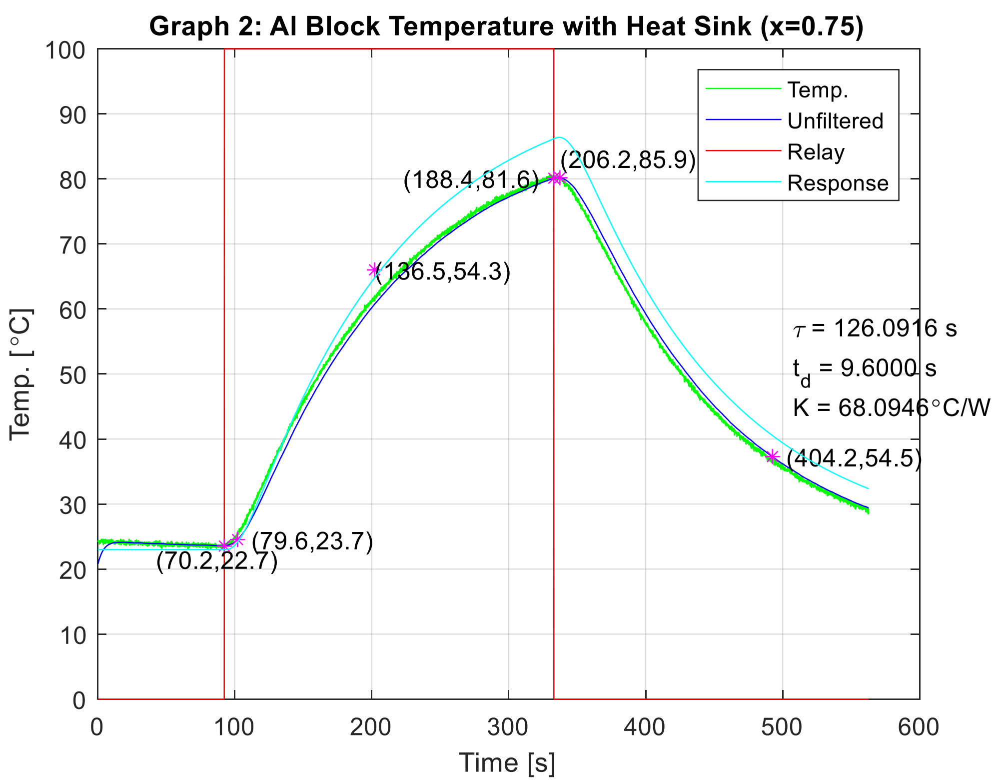
To determine what x value would leave to the best agreement between model and experiment, try the following numbers: $x = 0.25, 0.5, 0.75, 0.63$





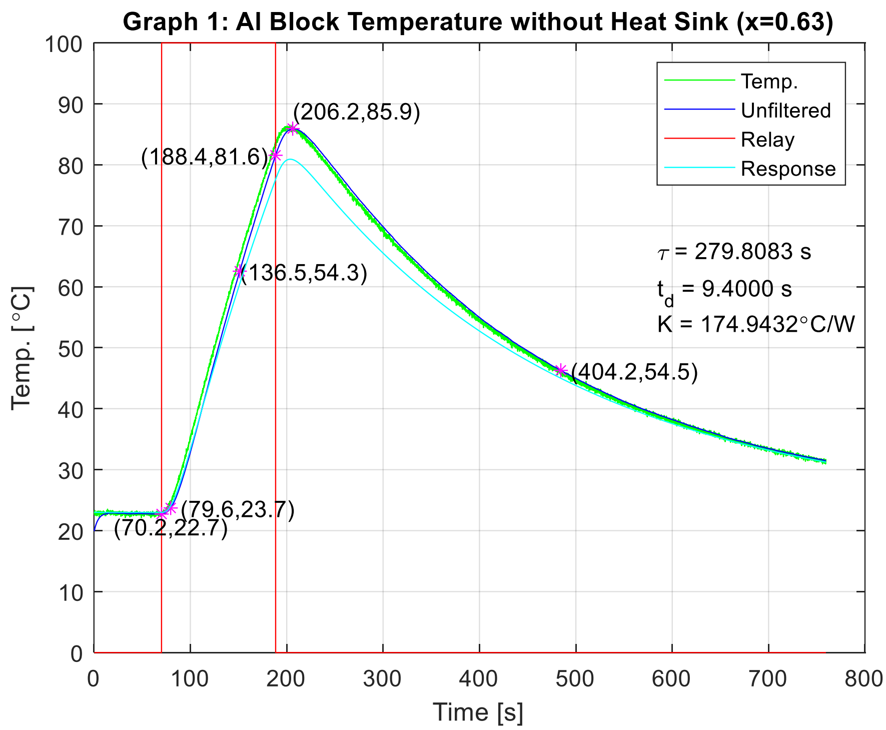
The simulated response does not match experimental result.

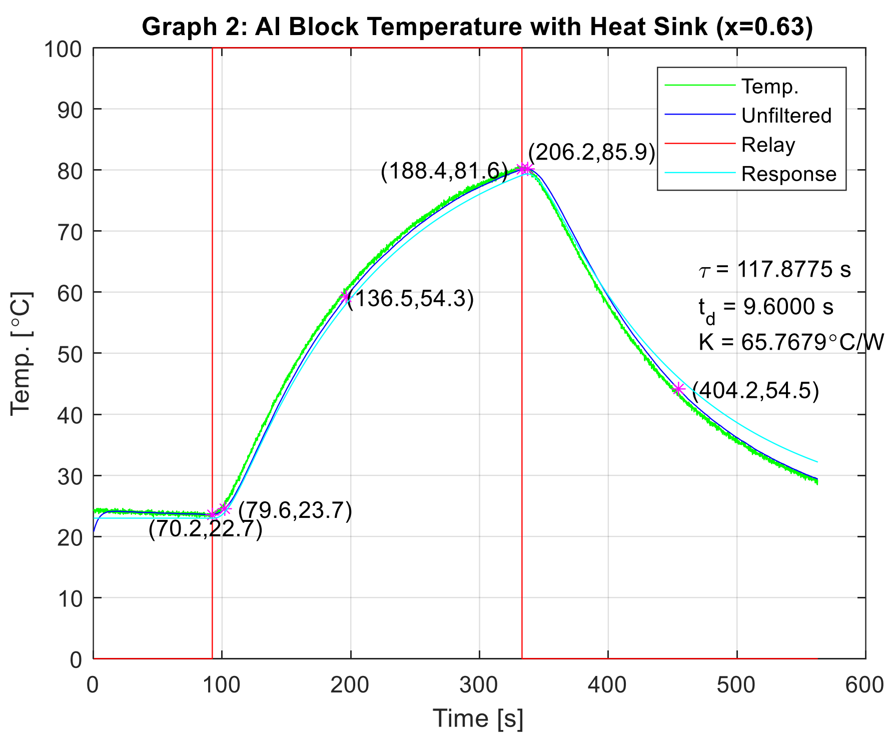




The $x = 0.75$ simulation of heat sink case has a higher overestimated peak than $x = 0.5$ case.

When $x = 0.63$,





When $x = 0.63$, the model and experiment have a higher level of agreement.

For $x = 0.63$ case, the system parameters are as follows:

Without Heat Sink

Time constant, $\tau \approx 279.80825833327\ s$

Process time delay, $t_d \approx 9.39999847412109\ s$

Gain (Static sensitivity), $K \approx 174.943167106269\ °C/W$

With Heat Sink

Time constant, $\tau \approx 117.877540086345\ s$

Process time delay, $t_d \approx 9.59999847412109\ s$

Gain (Static sensitivity), $K \approx 65.7679155117167\ °C/W$

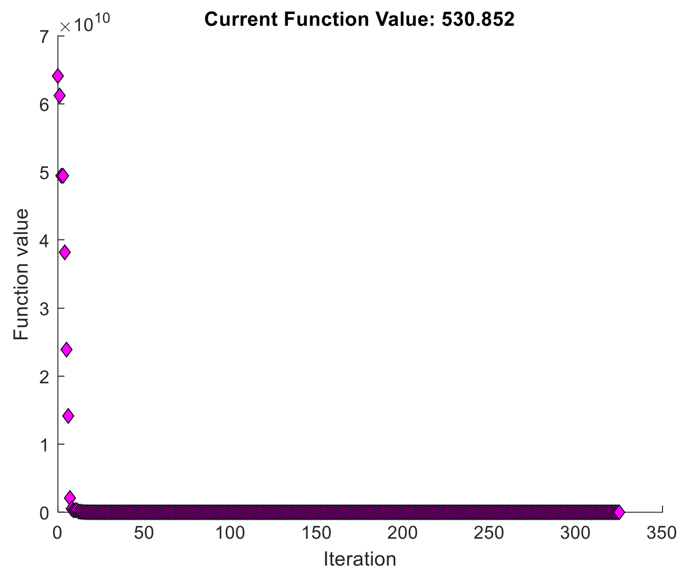
Part 3: Estimation Refinement

For the case without heat sink, the corresponding MATLAB code:

```
load('no_heat_sink.mat'); tn=Temperature.time; Tn=Temperature.signals.values;
Tnf=Temperature_Filtered.signals.values; Rn=Relay_Command.signals.values*100;
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 1: Al Block Temperature without Heat Sink'), x=0.63;

t1=70.2; [~,I]=min(abs(tn-t1)); T1=Tnf(I); [~,I]=min(abs(Tnf(1:2019)-T1-1));
T2=Tnf(I); t2=tn(I); t4=188.4; [~,I]=min(abs(tn-t4)); T4=Tnf(I); Tamb=23;
[T5,I]=max(Tnf); t5=tn(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Tnf(1:2019)-T3));
t3=tn(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Tnf(2019:end)-T6)); t6=tn(I+2019);
t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
te=tn; Te=Tn; ue=Rn/100; Tinit=Tamb;
x0=[td tau K];   % from initial estimate
fun=@(x)Fcurvefit(x,te,Te,ue,Tinit);
OPTIONS=optimset('Display','iter','PlotFcns',@optimplotfval,'TolFun',1e-8,'TolX',1e-8);
x=fminsearch(fun,x0,OPTIONS);
```

Command Window Output:

```
Iteration   Func-count    min f(x)       Procedure

  131         252        530.852        reflect

x =       8.52237915374391         268.0615695286        185.703164146439
```



Without Heat Sink

Time constant, $\tau \approx 268.0615695286\ s$

Process time delay, $t_d \approx 8.52237915374391\ s$

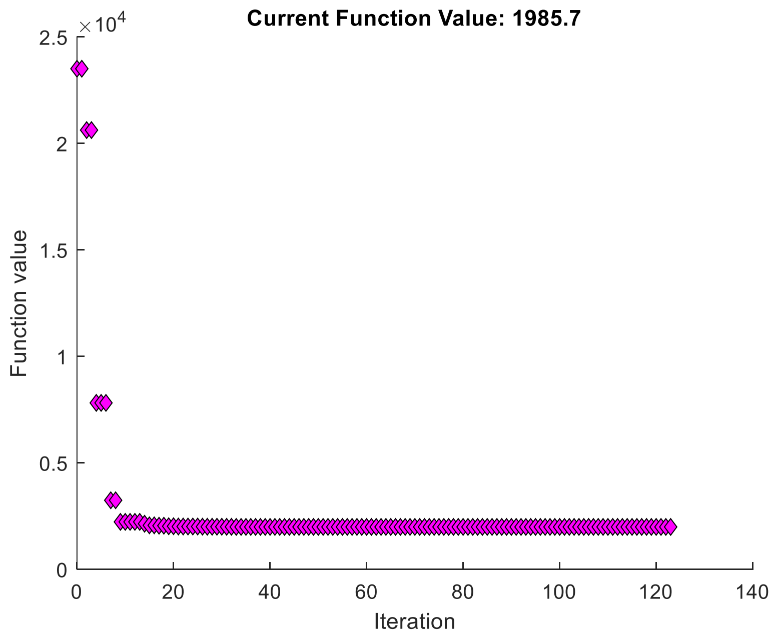
Gain (Static sensitivity), $K \approx 185.703164146439\ ^{\circ}C/W$

For the case <u>with heat sink</u>, the corresponding MATLAB code:

```
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink'), x=0.63;

t1=92.5; [~,I]=min(abs(th-t1)); T1=Thf(I); [~,I]=min(abs(Thf(1:2019)-T1-1));
T2=Thf(I); t2=th(I); t4=332.9; [~,I]=min(abs(th-t4)); T4=Thf(I); Tamb=23;
[T5,I]=max(Thf); t5=th(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Thf(1:2019)-T3));
t3=th(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Thf(2019:end)-T6)); t6=th(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));

te=th; Te=Th; ue=Rh/100; Tinit=Tamb;
x0=[td tau K];    % from initial estimate
fun=@(x)Fcurvefit(x,te,Te,ue,Tinit);
OPTIONS=optimset('Display','iter','PlotFcns',@optimplotfval,'TolFun',1e-8,'TolX',1e-8);
x=fminsearch(fun,x0,OPTIONS);
```

Command Window Output:

```
Iteration   Func-count    min f(x)       Procedure

  123         242          1985.7        shrink

x =       8.71638350875727        102.53800369082        64.0861535210426
```



Without Heat Sink

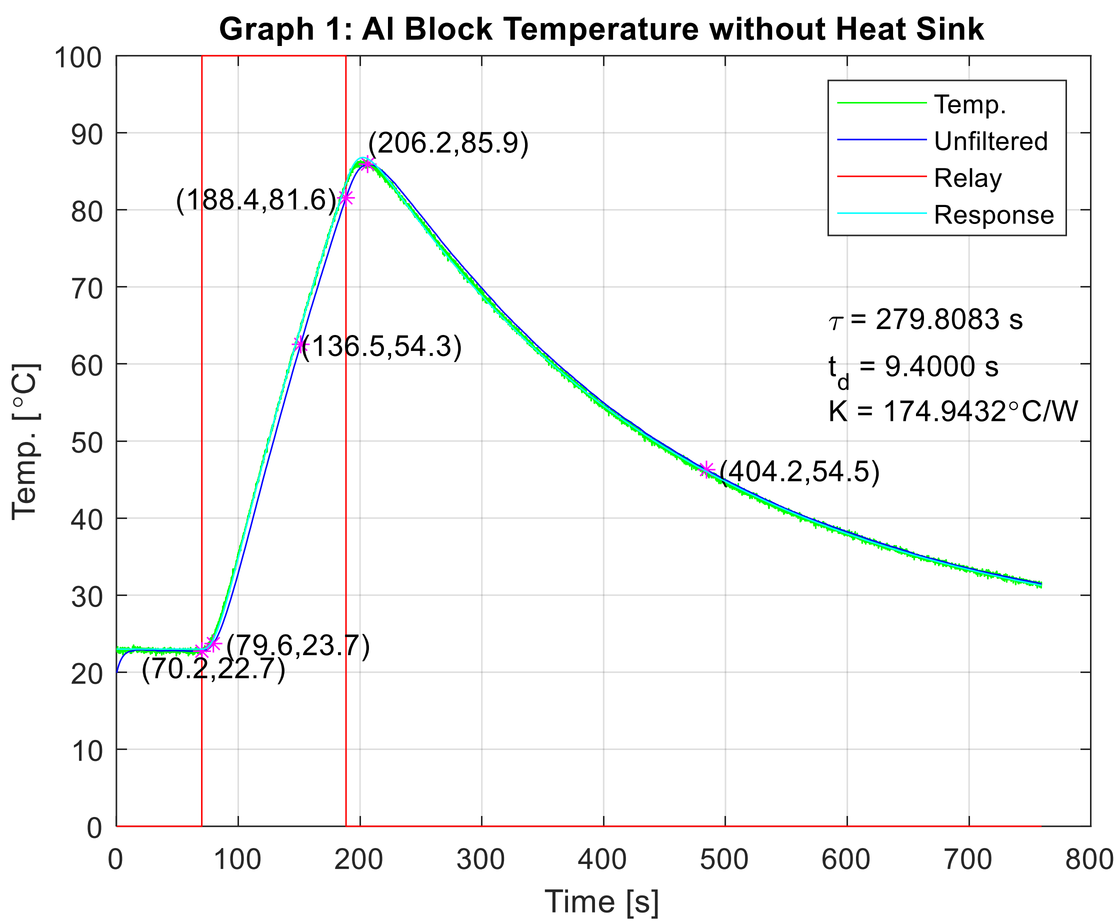
Time constant, $\tau \approx 102.53800369082\ s$

Process time delay, $t_d \approx 8.71638350875727\ s$

Gain (Static sensitivity), $K \approx 64.0861535210426\ °C/W$

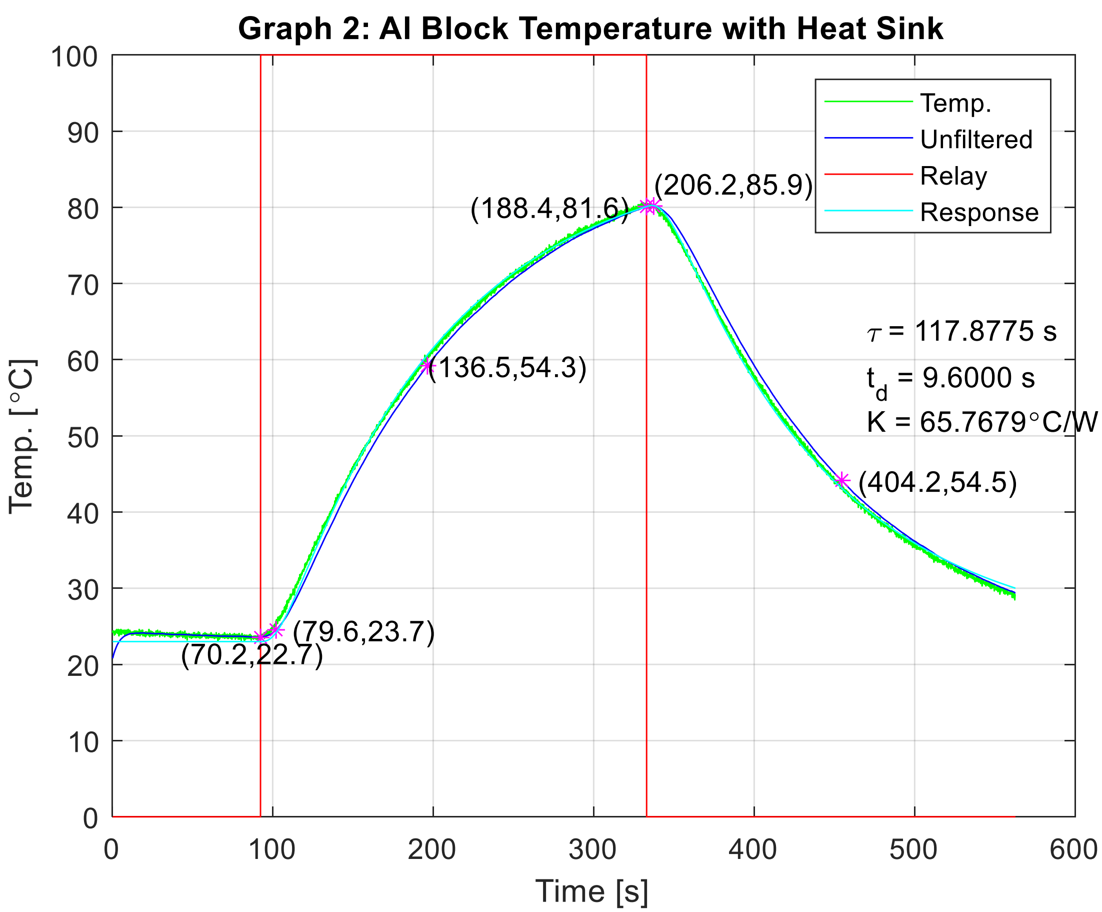
Case 1: Without Heat Sink

```
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'),
title('Graph 1: Al Block Temperature without Heat Sink'),
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'),
text(t6+100,T6+20,['\tau' ' = 279.8083 s']),
text(t6+100,T6+13,['t_d' ' = 9.4000 s']),
text(t6+100,T6+8,['K' ' = 174.9432' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-140,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

td=x(1); tau=x(2); K=x(3);
s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),
```

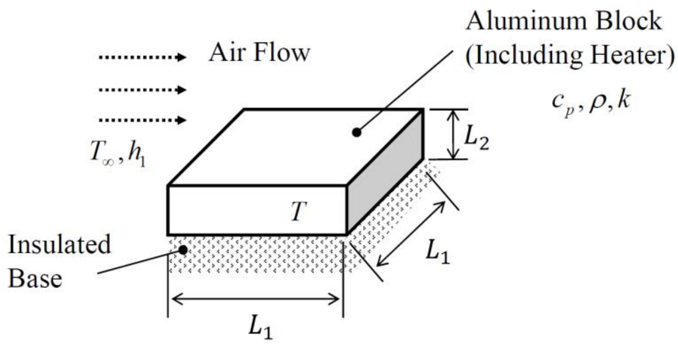


Case 2: With Heat Sink

```
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink (x=0.63)')
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+15,T6+20,['\tau' ' = 117.8775 s']),
text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
text(t6+15,T6+8,['K' ' = 65.7679' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-110,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

td=x(1); tau=x(2); K=x(3);
s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rh/100,th); hold on, plot(th,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),
```



Part 4: Heat Transfer Coefficients



For the case with no heat sink, the volume of the aluminum block is

$$V_1 = L_1^2 L_2$$

Which leads to thermal capacitance

$$C_{t1} = \rho V_1 c_p$$

Assuming the bottom surface is insulated, the exposed surface area is

$$A_{s1} = 4L_1 L_2 + L_1^2$$

The time constant is

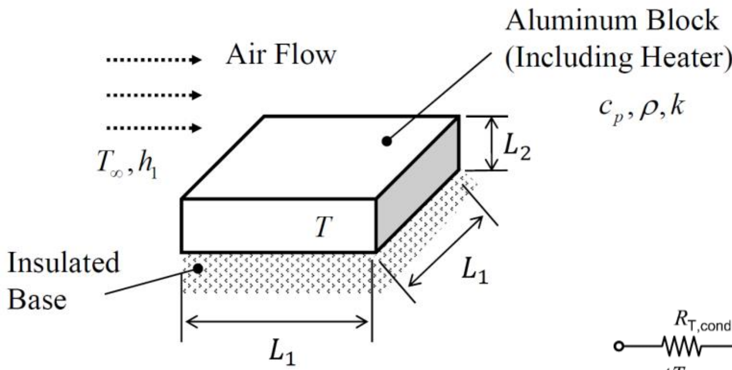
$$\tau_1 = C_{t1} \cdot R_{t,conv1} = C_{t1} \cdot \frac{1}{h_1 \cdot A_{s1}}$$

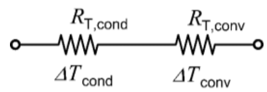
Therefore, the heat transfer coefficient is

$$h_1 = \frac{C_{t1}}{\tau_1 \cdot A_{s1}}$$

```
L1=1.5*0.0254; %[m] length of al block
L2=0.75*.0254; %[m] height of al block
V1=L1^2*L2; %[m^3] volume of al block
rho=2710; %[kg/m^3] density of al
cp=921.096; %[J/(kg*C)] specific heat capacity of al
Ct1=rho*V1*cp; %[J/C] themal capacitance of al
As1=4*L1*L2+L1^2; %[m^2] exposed surface area
h1=Ct1/(tau*As1) %[W/(m^2*K)] conv. h.t. coeff.
```

$$h_1 \approx 56.6483656001345\, W/(m^2 \cdot K)$$



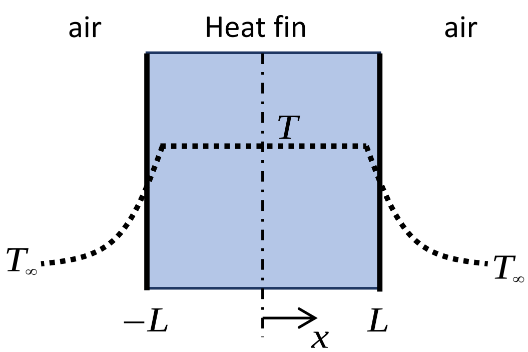


Biot Number, $Bi = \frac{\Delta T_{cond}}{\Delta T_{conv}} = \frac{R_{T,cond}}{R_{T,conv}} = \frac{L_c}{kA}\frac{hA}{1} = \frac{hL_c}{k}$

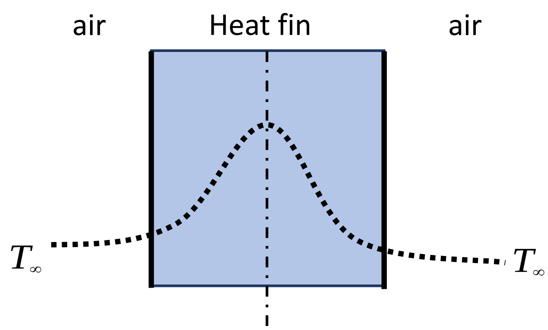
Where, $k \equiv$ thermal conductivity of solid, $L_c = \frac{V}{A_s}$, the largest dimension, to be conservative

If $Bi < 0.1$, the temperature drop across the solid is much smaller than the temperature drop across the fluid, we can neglect $R_{T,cond}$. Heat transfer is limited by convection.

If $Bi < 0.1$, lumped capacitance approximation may be used.



$Bi << 1$



$Bi > 1$

To calculate Biot Number, use

$$Bi = \frac{h_1 L_1}{k}$$

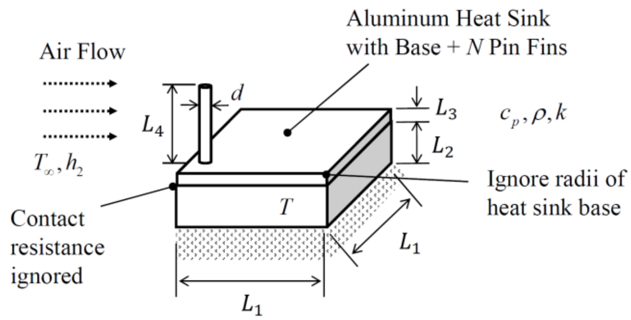
Where $L_1 \equiv largest\ dimension$

```
k=205; %[W/(m*K)]
Bi=h1*L1/k %[dimensionless]
```

$$Bi \approx 0.0105283059969031$$

Since $Bi_1 < 0.1$, then lumped capacitance approximation can be used.

Part 5: Heat Transfer Coefficient Comparison



Area of sides is

$$A_{s2} = 4L_1(L_2 + L_3)$$

The exposed base top area is

$$A_b = L_1^2 - N \cdot \frac{\pi}{4} \cdot d^2$$

Area of the fins (adjusted adiabatic tip)

$$A_f = \pi d L_c$$

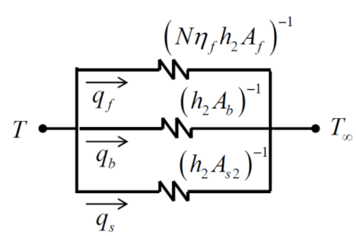
Where, $L_c = L_4 + \frac{d}{4}$

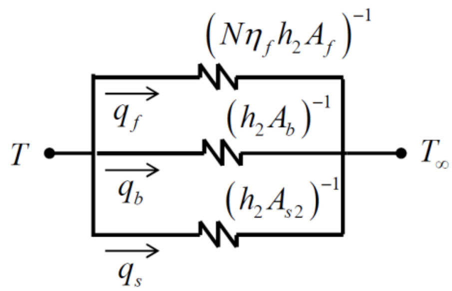
The volume of fin

$$V_f = \frac{\pi}{4} d^2 L_4$$

Hence, the total volume is

$$V_2 = L_1^2(L_2 + L_3) + NV_f$$





Given that

$$C_{t2} = \rho V_2 c_p$$

And,

$$A_t = A_{s2} + A_b + NA_f \qquad\qquad for\ A_t > A_{S1}$$

Then,

$$h_2 = \frac{C_{t2}}{A_t \tau_2}$$

```
% calculation of convective heat tsf. coeff. h with heat sink
d=0.06*.0254; %[m] diameter of fin
L4=0.8*.0254; %[m] length of fin
Vf=pi/4*d^2*L4; %[m^3] volume of fin
L3=0.1*0.0254; %[m] heat sink base height
N=100; % number of fins on heat sink
V2=L1^2*(L2+L3) + N*Vf; %[m^3] total volume of base + sink
Ct2=rho*V2*cp; %[J/C] thermal capacitance of al
Ab=L1^2 - N*pi/4*d^2; %[m^2] surface area of base
Af=pi*d*(L4+d/4); %[m^2] surface area of one fin
As2=4*L1*(L2+L3); %[m^2] surface area of sides
At=As2 + Ab + N*Af; %[m^2] exposed surface area
h2=Ct2/At/tau %[W/(m^2*C)]
```

$$h_2 = \frac{87.4830920069355}{0.014470709598672 \times 117.877540086345} \frac{W}{m^2 \cdot C} \approx 51.2865200721016\ \frac{W}{m^2 \cdot C}$$

To find ***the fin efficiency, η_f***

We first define the efficiency of a fin as

$$\eta_{fin} = \frac{actual\ heat\ transfer\ with\ F_{in}}{heat\ transfer\ if\ \theta = \theta_b\ everywhere\ on\ fin} = \frac{q_{actual}}{q_{ideal}}$$

$$\eta_{fin} = \frac{q_{actual}}{q_{ideal}} = \frac{\int_0^L h\rho \cdot (T - T_\infty)\ dx}{h \cdot (\rho L) \cdot (T_b - T_\infty)}$$

$$\eta_{fin} = \frac{\sqrt{kAh\rho} \cdot \theta_b \cdot \tanh(mL)}{h \cdot (\rho L) \cdot \theta_b}$$

$$\eta_{fin} = \sqrt{\frac{kA}{h\rho}} \cdot \frac{\tanh(mL)}{L}$$

Since $\frac{1}{m} = \sqrt{\frac{kA}{h\rho}}$,

$$\eta_{fin} = \frac{\tanh(mL)}{mL} \times 100\%$$

$$\eta_{fin} = \frac{0.485733461653791}{0.530461077364263} \times 100\%$$

$$\eta_{fin} \approx 91.5681625628946\ \%$$

To determine ***the overall convective thermal resistance, $R_{T,conv}$ of the heat sink surface***

$$R_{T,conv} = \frac{1}{h \cdot A_s} \equiv \frac{1}{convective\ coeff.\ [W \cdot {}^\circ C/m^2] \times surface\ area\ [m^2]}$$

$$R_{t,conv2} = \frac{1}{N\eta_f h_2 A_f + h_2 A_b}$$

```
% convective heat resistance (for heat sink top only)
R_t_conv = 1/(N*nf*h2*Af + h2*Ab) %[C/W]
```

$$R_{t,conv2} = \frac{1}{0.465450923855736 + 0.065092610614793}$$

$$R_{t,conv2} \approx 1.88485946020995\ C/W$$

To find ***the Biot number,***

$$Bi \equiv \frac{heat\ transfer\ coefficient}{heat\ conduction\ to\ interior\ of\ part}$$

$$Bi = \frac{h}{\left(\frac{k}{L}\right)} = \frac{h \cdot L}{k}$$

$$Bi_2 = \frac{h_2 \cdot (L_2 + L_3 + L_4)}{k}$$

Here we use the largest dimension to be conservative

```
% Biot number
Bi2 = h2*(L2+L3+L4)/k %[dimensionless]
```

$$Bi_2 = \frac{51.2865200721016 \times (0.04191)}{205}$$

$$Bi_2 \approx 0.0104849661279111$$

Since $Bi_2 < 0.1$, then lumped capacitance approximation can be used.

From earlier, heat transfer coefficient for the case without heat sink

$$h_1 \approx 56.6483656001345\ W/(m^2 \cdot K)$$

With heat sink,

$$h_2 \approx 51.2865200721016\ W/(m^2 \cdot K)$$

In comparison,

$$h_1 > h_2$$

The effective convective heat transfer coefficient for the heat sink is slightly higher compared to the case without the heat sink. This makes sense as the heat sink hinders the airflow, reducing air velocity and subsequently diminishing the convective heat transfer coefficient.

The thermal resistance of the heat sink surface was found to be

$$R_{t,conv2} \approx 1.88485946020995\ C/W$$

According to the specification sheet,

PART #	HEIGHT (mm)	CHIP SIZE (mm)	NATURAL CONVECTION	FORCED CONVECTION (C/W)		
				200 LFM	400 LFM	600 LFM
908-35-2-12-2-B-0	12	35	10.03 C/W	3.06 C/W	1.97 C/W	1.49 C/W
908-35-2-15-2-B-0	15	35	9.5 C/W	2.85 C/W	1.81 C/W	1.34 C/W
908-35-2-18-2-B-0	18	35	8.98 C/W	2.6 C/W	1.64 C/W	1.19 C/W
908-35-2-21-2-B-0	21	35	8.46 C/W	2.4 C/W	1.5 C/W	1.07 C/W
908-35-2-23-2-B-0	23	35	8.32 C/W	2.19 C/W	1.34 C/W	.97 C/W
908-35-2-28-2-B-0	28	35	7.99 C/W	1.97 C/W	1.19 C/W	.83 C/W
908-35-2-33-2-B-0	33	35	7.65 C/W	1.82 C/W	1.06 C/W	.7 C/W

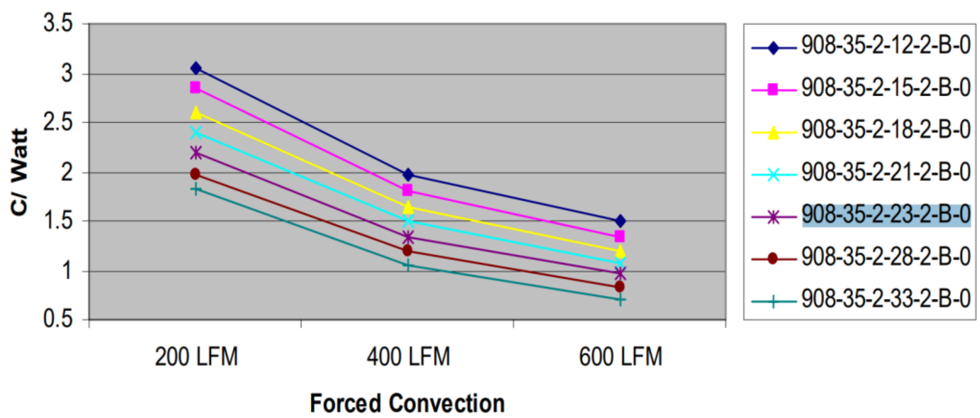
The forced convection coefficient for 908-35-2-23-2-B-0 ranges from 0.97 to 2.19 C/W, based on the Linear Feet per Minute (LFM).

The average flow in the experiment is

```
% flow in LFM
front_face = 4.8/.0254*60/12; %[ft/min]
rear_face = 1.4/.0254*60/12; %[ft/min]
avg = (front_face+rear_face)/2
```

avg = 610.236220472441

$$flow_{avg} \approx 610.236220472441\ LFM$$



By interpolating the sample plot, the thermal resistance can be estimated to be around 0.95 C/W.

The calculated value is about twice as large as the nominal value from the data sheet.

The thermal resistance calculated through experimental data matches the values associated with the air flow velocities that were measured. Even though the table's lower limit has thermal resistances for flows with an LFM of 200 (which is smaller than the lowest measured flow of 1.4 m/s or 275.6 LFM), and the upper limit has thermal resistances at 600 LFM (which is substantially smaller than the highest measured flow of 4.8 m/s or 944.9 LFM), the experimental thermal resistance falls within the tabulated values. Specifically, the calculated value of $1.88485946020995\ C/W$ corresponds to a flow rate of 325.111646142317 LFM.

The lumped capacitance approximation can be reasonably applied, as suggested by the Biot number analysis. The calculated Biot number of 0.0104849661279111 is significantly lower than the 0.1 cutoff, indicating its validity.

The conclusion can be drawn that the fin's actual heat transfer closely matches what it would have been if the entire fin had the same temperature as the base, as indicated by the fin efficiency of 91.5681625628946%. This further strengthens the assumption of uniform temperature across the block/heat sink.

Part 6: Theoretical Results

To start with, several assumptions are proposed:

1. The physical properties of air are assessed at room temperature.
2. The physical properties of air remain unchanged throughout the process.
3. The average velocity of the air responsible for convecting heat from the block is 5 m/s.
4. The calculation for convective heat transfer was determined at length L_1.
5. The lumped capacitance approximation is applied, assuming a small Biot number.
6. The flow of air is assumed to be laminar, which can be verified by the Reynolds number.
7. The air and the block are flowing in parallel.
8. The block is an isothermal, flat plate.
9. The system is assumed to be in a steady state for the purpose of calculation.

Nusselt Number

$$Nu = \frac{heat\ transfer\ by\ convection\ thru\ fluid}{heat\ transfer\ by\ conduction\ thru\ fluid} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k}$$

Where, $k \equiv thermal\ conductivity\ of\ fluid$

Heat transfer coefficients for forced and natural convection can be determined from correlations for the Nusselt number as a function of the Reynolds and Prandtl numbers:

$$\overline{Nu_x} = \frac{\overline{h_x}x}{k} = 0.664 \cdot Re_x^{1/2} \cdot Pr^{1/3} \qquad for\ 0.6 \le Pr \le 500$$

Where,

$$Re = \frac{\rho u L}{\mu}$$

And, for air in room temperature,

$$Pr = 0.71$$

$$\overline{Nu_x} = \frac{\overline{h_x}x}{k} = 0.664 \cdot Re_x^{1/2} \cdot Pr^{1/3}$$

$$\overline{Nu_x} = 0.664 \cdot \left(\frac{\rho u L}{\mu}\right)^{1/2} \cdot (0.71)^{1/3}$$

And,

$$\because\ Nu = \frac{hL}{k}$$

$$\therefore h = \frac{Nu \cdot k}{L}$$

```
% calculation of the heat transfer coefficient with forced convection
rho_air = 1.225; %[kg/m^3] density of air @ room temp
V_air = 5; %[m/s] avg velocity of air over al block
mu_air = 1.789*10^-5; %[N*s/m^2] dynamic viscosity of air at room temp
Re=rho_air*V_air*L1/mu_air %[dimensionless]
Pr=0.71 %[dimensionless] value of Pr for ait at room temp
Nu=0.664*Re^.5*Pr^(1/3);
k_air=0.028; %[W/(m*C)]
h=Nu*k_air/L1
```

```
Re =        13044.298490777

Pr =                   0.71

h =       49.7199803899994
```

Based on Nusselt number, the convective coefficient is determined to be 49.7199803899994 $\mathrm{W/(m^2 \cdot {}^\circ C)}$.

In part 5, heat transfer coefficient for the case without heat sink is

$$h_1 \approx 56.6483656001345\, W/(m^2 \cdot K)$$

With heat sink,

$$h_2 \approx 51.2865200721016\, W/(m^2 \cdot K)$$

They are slightly greater than the value just found.

This can be ascribed to the different assumptions made when conducting the calculation.

Appendix A: MATLAB

```
%%
close all, clear, clc

%% Part 1: Heating-Cooling Curve (without Heat Sink)
load('no_heat_sink.mat'); tn=Temperature.time; Tn=Temperature.signals.values;
Tnf=Temperature_Filtered.signals.values; Rn=Relay_Command.signals.values*100;
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 1: Al Block Temperature without Heat Sink')

%% Part 2: Parameter Estimation (without Heat Sink)
t1=70.2; [~,I]=min(abs(tn-t1)); T1=Tnf(I); [~,I]=min(abs(Tnf(1:2019)-T1-1));
T2=Tnf(I); t2=tn(I); t4=188.4; [~,I]=min(abs(tn-t4)); T4=Tnf(I); Tamb=23;
[T5,I]=max(Tnf); t5=tn(I); T3=T1+0.5*(T5-T1); [~,I]=min(abs(Tnf(1:2019)-T3));
t3=tn(I); T6=T5+0.5*(Tamb-T5); [~,I]=min(abs(Tnf(2019:end)-T6)); t6=tn(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'),
title('Graph 1: Al Block Temperature without Heat Sink'),
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'),
text(t6+100,T6+20,['\tau' ' = 285.6536 s']),
text(t6+100,T6+13,['t_d' ' = 9.4000 s']),
text(t6+100,T6+8,['K' ' = 173.4621' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-140,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

% s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2+Tamb)*G3;
% Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta,'c'), grid on
% legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%% Part 1: Heating-Cooling Curve (with Heat Sink)
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink')

%% Part 2: Parameter Estimation (with Heat Sink)
t1=92.5; [~,I]=min(abs(th-t1)); T1=Thf(I); [~,I]=min(abs(Thf(1:2019)-T1-1));
T2=Thf(I); t2=th(I); t4=332.9; [~,I]=min(abs(th-t4)); T4=Thf(I); Tamb=23;
[T5,I]=max(Thf); t5=th(I); T3=T1+0.5*(T5-T1); [~,I]=min(abs(Thf(1:2019)-T3));
t3=th(I); T6=T5+0.5*(Tamb-T5); [~,I]=min(abs(Thf(2019:end)-T6)); t6=th(I+2019);
```

```
t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink')
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+15,T6+20,['\tau' ' = 126.0916 s']),
text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
text(t6+15,T6+8,['K' ' = 68.0946' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-110,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

% s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2+Tamb)*G3;
% Theta=lsim(G,Rh/100,th); plot(th,Rh,'r',th,Theta,'b'), grid on
% xlabel('Time [s]'), ylabel('Temp. [\circC]'), legend('Relay','Response')
% title('Filtered Temperature as a System Response over Time (with Heat Sink)')

s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rh/100,th); hold on, plot(th,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%%







%% Part 1: Heating-Cooling Curve (without Heat Sink)
load('no_heat_sink.mat'); tn=Temperature.time; Tn=Temperature.signals.values;
Tnf=Temperature_Filtered.signals.values; Rn=Relay_Command.signals.values*100;
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 1: Al Block Temperature without Heat Sink'), x=0.63;

%% Part 2: Parameter Estimation (without Heat Sink)
t1=70.2; [~,I]=min(abs(tn-t1)); T1=Tnf(I); [~,I]=min(abs(Tnf(1:2019)-T1-1));
T2=Tnf(I); t2=tn(I); t4=188.4; [~,I]=min(abs(tn-t4)); T4=Tnf(I); Tamb=23;
[T5,I]=max(Tnf); t5=tn(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Tnf(1:2019)-T3));
t3=tn(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Tnf(2019:end)-T6)); t6=tn(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'),
title('Graph 1: Al Block Temperature without Heat Sink (x=0.63)'),
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'),
text(t6+100,T6+20,['\tau' ' = 279.8083 s']),
```

```
text(t6+100,T6+13,['t_d' ' = 9.4000 s']),
text(t6+100,T6+8,['K' ' = 174.9432' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-140,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

% s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2+Tamb)*G3;
% Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta,'c'), grid on
% legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%% Part 1: Heating-Cooling Curve (with Heat Sink)
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink'), x=0.63;

%% Part 2: Parameter Estimation (with Heat Sink)
t1=92.5; [~,I]=min(abs(th-t1)); T1=Thf(I); [~,I]=min(abs(Thf(1:2019)-T1-1));
T2=Thf(I); t2=th(I); t4=332.9; [~,I]=min(abs(th-t4)); T4=Thf(I); Tamb=23;
[T5,I]=max(Thf); t5=th(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Thf(1:2019)-T3));
t3=th(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Thf(2019:end)-T6)); t6=th(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink (x=0.63)')
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+15,T6+20,['\tau' ' = 117.8775 s']),
text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
text(t6+15,T6+8,['K' ' = 65.7679' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-110,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

% s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2+Tamb)*G3;
% Theta=lsim(G,Rh/100,th); plot(th,Rh,'r',th,Theta,'b'), grid on
% xlabel('Time [s]'), ylabel('Temp. [\circC]'), legend('Relay','Response')
% title('Filtered Temperature as a System Response over Time (with Heat Sink)')

s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rh/100,th); hold on, plot(th,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%%
```

```
%% Part 3: w/o
load('no_heat_sink.mat'); tn=Temperature.time; Tn=Temperature.signals.values;
Tnf=Temperature_Filtered.signals.values; Rn=Relay_Command.signals.values*100;
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 1: Al Block Temperature without Heat Sink'), x=0.63;

t1=70.2; [~,I]=min(abs(tn-t1)); T1=Tnf(I); [~,I]=min(abs(Tnf(1:2019)-T1-1));
T2=Tnf(I); t2=tn(I); t4=188.4; [~,I]=min(abs(tn-t4)); T4=Tnf(I); Tamb=23;
[T5,I]=max(Tnf); t5=tn(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Tnf(1:2019)-T3));
t3=tn(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Tnf(2019:end)-T6)); t6=tn(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));

te=tn; Te=Tn; ue=Rn/100; Tinit=Tamb;
x0=[td tau K];   % from initial estimate
fun=@(x)Fcurvefit(x,te,Te,ue,Tinit);
OPTIONS=optimset('Display','iter','PlotFcns',@optimplotfval,'TolFun',1e-8,'TolX',1e-
8);
x=fminsearch(fun,x0,OPTIONS);
%%
figure, plot(tn,Tn,'g',tn,Tnf,'b',tn,Rn,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'),
title('Graph 1: Al Block Temperature without Heat Sink'),
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'),
text(t6+100,T6+20,['\tau' ' = 279.8083 s']),
text(t6+100,T6+13,['t_d' ' = 9.4000 s']),
text(t6+100,T6+8,['K' ' = 174.9432' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-140,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

td=x(1); tau=x(2); K=x(3);
s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rn/100,tn); hold on, plot(tn,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%%
load('heat_sink.mat'); th=Temperature.time; Th=Temperature.signals.values;
Thf=Temperature_Filtered.signals.values; Rh=Relay_Command.signals.values*100;
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink'), x=0.63;

t1=92.5; [~,I]=min(abs(th-t1)); T1=Thf(I); [~,I]=min(abs(Thf(1:2019)-T1-1));
T2=Thf(I); t2=th(I); t4=332.9; [~,I]=min(abs(th-t4)); T4=Thf(I); Tamb=23;
```

```
[T5,I]=max(Thf); t5=th(I); T3=T1+x*(T5-T1); [~,I]=min(abs(Thf(1:2019)-T3));
t3=th(I); T6=T5+x*(Tamb-T5); [~,I]=min(abs(Thf(2019:end)-T6)); t6=th(I+2019);

t5s=0; t6s=t6-t5; theta5=T5-Tamb; theta6=T6-Tamb; tau=-t6s/log(theta6/theta5);
td=t2-t1; t3s=t3-t2; theta3=T3-Tamb; K=theta3/(1-exp(-t3s/tau));
% figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
% ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
% title('Graph 2: Al Block Temperature with Heat Sink (x=0.63)')
% hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
% plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
% text(t6+15,T6+20,['\tau' ' = 117.8775 s']),
% text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
% text(t6+15,T6+8,['K' ' = 65.7679' '\circC/W'])
%
% text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
% text(t3,T3,'(136.5,54.3)'), text(t4-110,T4,'(188.4,81.6)')
% text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')
%
% % s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2+Tamb)*G3;
% % Theta=lsim(G,Rh/100,th); plot(th,Rh,'r',th,Theta,'b'), grid on
% % xlabel('Time [s]'), ylabel('Temp. [\circC]'), legend('Relay','Response')
% % title('Filtered Temperature as a System Response over Time (with Heat Sink)')

% s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
% Theta=lsim(G,Rh/100,th); hold on, plot(th,Theta+Tamb,'c'), grid on
% legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

te=th; Te=Th; ue=Rh/100; Tinit=Tamb;
x0=[td tau K];   % from initial estimate
fun=@(x)Fcurvefit(x,te,Te,ue,Tinit);
OPTIONS=optimset('Display','iter','PlotFcns',@optimplotfval,'TolFun',1e-8,'TolX',1e-
8);
x=fminsearch(fun,x0,OPTIONS);

%%
figure, plot(th,Th,'g',th,Thf,'b',th,Rh,'r'), xlabel('Time [s]'), grid on,
ylabel('Temp. [\circC]'), legend('Temp.','Unfiltered','Relay'),
title('Graph 2: Al Block Temperature with Heat Sink')
hold on, plot(t1,T1,'m*'), plot(t2,T2,'m*'), plot(t3,T3,'m*'), plot(t4,T4,'m*'),
plot(t5,T5,'m*'), plot(t6,T6,'m*'), legend('Temp.','Unfiltered','Relay'),
text(t6+15,T6+20,['\tau' ' = 117.8775 s']),
text(t6+15,T6+13,['t_d' ' = 9.6000 s']),
text(t6+15,T6+8,['K' ' = 65.7679' '\circC/W'])

text(t1-50,T1-2,'(70.2,22.7)'), text(t2+10,T2,'(79.6,23.7)')
text(t3,T3,'(136.5,54.3)'), text(t4-110,T4,'(188.4,81.6)')
text(t5,T5+3,'(206.2,85.9)'), text(t6+10,T6,'(404.2,54.5)')

td=x(1); tau=x(2); K=x(3);
s=tf('s'); G1=1/(td*s+1); G2=K/(tau*s+1); G3=1/(4*s+1); G=(G1*G2)*G3;
Theta=lsim(G,Rh/100,th); hold on, plot(th,Theta+Tamb,'c'), grid on
legend('Temp.','Unfiltered','Relay','','','','','','','Response'),

%% Prob 4
L1=1.5*0.0254; %[m] length of al block
```

```
L2=0.75*.0254; %[m] height of al block
V1=L1^2*L2; %[m^3] volume of al block
rho=2710; %[kg/m^3] density of al
cp=921.096; %[J/(kg*C)] specific heat capacity of al
Ct1=rho*V1*cp; %[J/C] themal capacitance of al
As1=4*L1*L2+L1^2; %[m^2] exposed surface area
h1=Ct1/(tau*As1) %[W/(m^2*K)] conv. h.t. coeff.

k=205; %[W/(m*K)]
Bi=h1*L1/k %[dimensionless]

%% Prob 5
% calculation of convective heat tsf. coeff. h with heat sink
d=0.06*.0254; %[m] diameter of fin
L4=0.8*.0254; %[m] length of fin
Vf=pi/4*d^2*L4; %[m^3] volume of fin
L3=0.1*0.0254; %[m] heat sink base height
N=100; % number of fins on heat sink
V2=L1^2*(L2+L3) + N*Vf; %[m^3] total volume of base + sink
Ct2=rho*V2*cp; %[J/C] thermal capacitance of al
Ab=L1^2 - N*pi/4*d^2; %[m^2] surface area of base
Af=pi*d*(L4+d/4); %[m^2] surface area of one fin
As2=4*L1*(L2+L3); %[m^2] surface area of sides
At=As2 + Ab + N*Af; %[m^2] exposed surface area
h2=Ct2/At/tau %[W/(m^2*C)]

% fin efficiency
Lc = L4 + d/4; %[m]
m = sqrt(4*h2/k/d); %[m^-1]
nf = tanh(m*Lc)/(m*Lc) %[dimensionless]

% convective heat resistance (for heat sink top only)
R_t_conv = 1/(N*nf*h2*Af + h2*Ab) %[C/W]

% Biot number
Bi2 = h2*(L2+L3+L4)/k %[dimensionless]

% flow in LFM
front_face = 4.8/.0254*60/12; %[ft/min]
rear_face = 1.4/.0254*60/12; %[ft/min]
avg = (front_face+rear_face)/2

%%
% calculation of the heat transfer coefficient with forced convection
rho_air = 1.225; %[kg/m^3] density of air @ room temp
V_air = 5; %[m/s] avg velocity of air over al block
mu_air = 1.789*10^-5; %[N*s/m^2] dynamic viscosity of air at room temp
Re=rho_air*V_air*L1/mu_air %[dimensionless]
Pr=0.71 %[dimensionless] value of Pr for ait at room temp
Nu=0.664*Re^.5*Pr^(1/3);
k_air=0.028; %[W/(m*C)]
h=Nu*k_air/L1
```

Appendix B: Parameter Definitions

Parameter	Symbol	Units
Specific Heat Capacity	c_p	J/kg/˚C
Density	ρ	kg/m^3
Thermal Conductivity	k	W/m/˚C
Convection Heat Transfer Coefficient	h	W/m^2/˚C
Thermal Resistance	R_T	˚C/W
Thermal Capacitance	C_T	J/˚C
Time Constant	τ	s
Temperature	T	˚C
Relative Temperature	θ	˚C
Ambient Temperature	T_∞	˚C
Volume	V	m^3
Area	A	m^2
Length	l	m
Diameter	d	m