EC413 Applied Econometrics Assignment 3:

Forecasting Canadian GDP Growth

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Introduction

GDP Growth is a measure of how much the economy has grown since the previous period. It is a key macroeconomic performance indicator of interest to many stakeholders from government, central banks investors and businesses as it informs decision makers about the trajectory the economy is on, as well as its point in the business cycle, allowing them to plan. By forecasting this value greater insight into the future is given thus this report aims to build and evaluate econometric models forecasting Canada's GDP growth, comparing the models' performances over the evaluation horizon using several forecast error measures.

Data Discussion

The data provided was a stationary set of the following variables: the explanatory variables being US Inflation (*US Infl*), US GDP Growth (*US GDPG*), Canadian Inflation (*CAN Infl*) and finally Canadian GDP Growth (*CAN GDPG*), the dependent variable of interest. The data spans quarterly from Q2 1973 to Q4 2019, with no missing values.

Forecasting Approach

An iterated forecasting method was chosen as literature often finds that iterated approaches tend to outperform direct methods, especially over longer horizons.

Due to the quarterly nature of the data a forecast horizon of 4 was selected as this would estimate GDP growth up to 1 year in advance. A popular time horizon among the stakeholders for which these models will be relevant to. Longer horizons may introduce larger forecast errors as model uncertainty grows.

Over the entire sample period the US and Canadian economies underwent numerous economic shocks e.g. Dot-Com bubble of 2000 and Global Financial Crash of 2008. As opposed to cutting these periods from the sample I opted to roll the estimation window. This is where the window is kept at a fixed size of 146 as the start point and end points are incremented as forecasts are made. In doing this old data that is less relevant to the current and future state of the economy is removed from the estimation window. In contrast to an expanding window where the start is fixed, and the estimation window grows as new forecasts are made. The estimation period was from Q2 1973 to Q4 2010 using the rest of the data set for model evaluation.

A simple AR(4) model seen below was selected as the benchmark using a max lag order of 4 on the dependent variable as explanatory variables for the estimation. The lag specification was chosen based on AIC. The rationale being it strikes a balance between overfitting with too many lags and not giving the model enough impactful parameters. As a secondary check the autocorrelation functions were plotted in Figures 3.1,2,4,5. The output shows 2 lags are an appropriate lower bound for lag length. This philosophy behind the lower bound was extended to the competing models to test if the lower bound, 2lag models would outperform the 4lag.

$$CAN\widehat{GDP}G_{t+h} = \alpha + \sum_{j=0}^{4} \beta_j CANGDPG_{t-j} + \varepsilon_{t+h}, \qquad \varepsilon \sim N(0, \sigma^2)$$

Competing Models follow a similar line of reasoning and take a multivariate form of a VAR(p,q) (shown below) for the 6 bivariate models. These test the predictive ability of each of the explanatory variables for 2 and 4 max lag lengths.

$$CAN\widehat{GDP}G_{t+h} = \alpha + \sum_{j=0}^{p} \beta_{1,j} CANGDPG_{t-j} \sum_{j=0}^{q} \beta_{2,j} x_{t-j} + \varepsilon_{t+h},$$

After Identifying the strongest predictor variables they were included in a model together in models *Combine2&4* specified below, resulting in a VAR(p,q,s). In theory building the best possible model. This was again done for 2 and 4 lag length.

$$CAN\widehat{GDP}G_{t+h} = \alpha + \sum_{j=0}^{p} \beta_{1,j} CANGDPG_{t-j} \sum_{j=0}^{q} \beta_{2,j} CANInfl_{t-j} \sum_{j=0}^{s} \beta_j USInfl_{t-j} + \varepsilon_{t+h},$$

Finally the *ALL* models were estimated using all 4 variables to predict Canadian GDP growth. This was for the purposes of comparing the strategically formulated model to one build from using all the available data.

Several forecast evaluation techniques were employed in this report. Firstly, the Root Mean Squared Forecast Error (RMSFE) was used as the measure of forecast error as it combines attractive qualities of both the Mean Absolute Forecast Error (MAFE) and Mean Squared Forecast Error (MSFE). MAFE maintains the units of the dependent variable of GDP growth but fails to penalise large estimation errors while MSFE achieves a desirable penalty but reports in an unintuitive scale. The Sum of the RMSFE across the forecast horizon was also computed to aid in analysis.

Then for robustness the Cumulative Sum of Squared Forecast Errors Differences (CSSED) was then used to check for any forecast inconsistencies and better discuss the comparative performance of the models over the evaluation window. With this measure a positive CSSED means the model had lower errors in the period than that of the benchmark and *vice versa* with negative figures. Particular attention is paid to step climbs and falls in slope as they signify a significant over or underperformance where a stable line denotes consistency.

Using these tools I investigate which of these is the best model for estimating Canadian GDP growth, discussing their strengths, weaknesses and providing recommendations.

Results Summary & Discussion

The first row of Figure 1 reports the Root Mean Squared Forecast Error (RMSFE) of the benchmark AR(4) model while all others are reported as ratios of the RMSFE of the model using that of the benchmark as the denominator. A value below one means that the model has a lower RMSFE than the benchmark (and thus outperforms it), and *vice versa* for values above 1. The bold figures denote the best value for the given estimation horizon while the italics denotes each model's best performing horizon. For when these are the same value, a bold and underline is given.

Shown by the output of the bivariate models in Figure 1lines 2-7, variables predictive ability tends to worsen relative to the benchmark as estimation horizon increases, in line with intuition as uncertainty compounds the further out the estimation horizon (Svetunkov, 2024). US GDP Growth models weakly see the opposite trend shown by the increases of up to 6% on line 5. Though in the 4lag model the score does increase for steps 1-3 before falling to its best performance in h=4, the only horizon at which it outperformed the benchmark. This suggests that there is a year lag before shocks to US GDP growth will reflect in Canada. Interestingly, US GDPG models are the only models to see RMSFE improvement on the 4^{th} step estimation, the poor performance of non-US GDPG augmented models suggests that they lose reliability after 3quarters. When US GDPG is reintroduced to the horse race in the ALL models we again see the characteristic 4^{th} step improvement.

US inflation was a very strong predictive indicator performing on par with the *Combined* VAR models and holding the best comparative performance for the 3 step ahead forecast. With the two nations sharing a boarder and a large proportion of each other's Trade Balances (The Observatory of Economic Complexity, 2022), a high degree of covariance in their macroeconomic indicators is expected. But not to the degree of outperforming Canda's own inflation on every horizon irrespective of lag length.

The *Combined* models include the autoregressive element being lags of Canadian GDP growth in addition to *CAN Infl* and *US Infl* based on the process of elimination and summing the RMSFE ratios to find the lowest values. These models prove to be some of thes best in this report with the best 1-2step performances in the case of the 4 lag model. Though due to its high CSSED volatility the 4 lag model cannot be seen as stable and reliable. Combine 2 achieves a strong performance RMSFE, though not the best in any one horizon, and is proved to be more stable than the 4 lag model with its persistently lower CSSED standard error shown in Figure 4.

The only difference between the Combined and ALL models is the inclusion of US GDP growth data which appears to have hampered the performance of the model severely, only outperforming the benchmark on the 1step ahead forecast.

Conclusions & Recommendations

Counterintuitively Canada's own inflation rate was not the best predictor of Canadian GDP growth rates but rather US inflation. Particularly when a maximum of 4 lags are considered. Though, with 2 lags it significantly outperforms all other models on the 3rd horizon estimates shown by the stable and persistent rise in Figure 2.3. With the added benefit for being bivariate, it is the simplest yet one of the most powerful predictors.

The *Combine4* model is the best for estimation between 1-2steps ahead. But falls off with estimation horizon. *Combine2* on the other hand is more consistent across all horizons and points in the evaluation window shown by the consistently lower CSSED standard error. Though it did not perform exceedingly well in any one horizon due to its reliability, I conclude that *Combine2* is the best predictive model of Canadian GDP Growth over this sample period.

Though due to the similarity to the benchmark across the models it cannot be concluded that these models are significantly better than the benchmark. A Diebold-Mariano test of forecast accuracy could have been run to obtain clearer a conclusion on the varying performances of the models and to test if any of them have significantly different predictive power than the benchmark.

Appendix

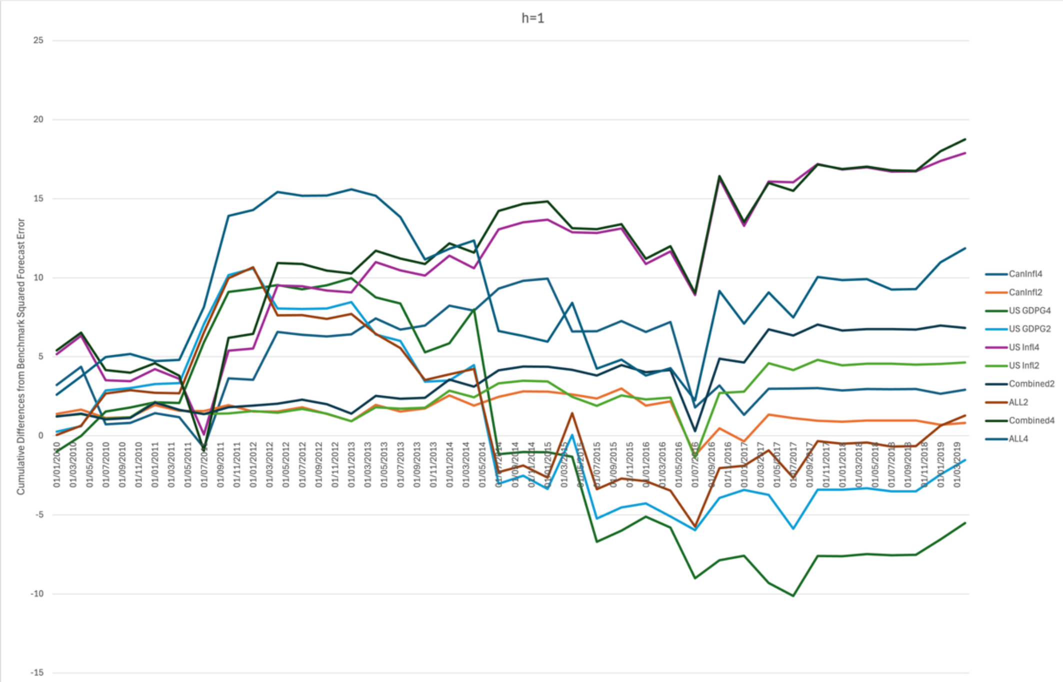
Figure 1: RMSE Ratio

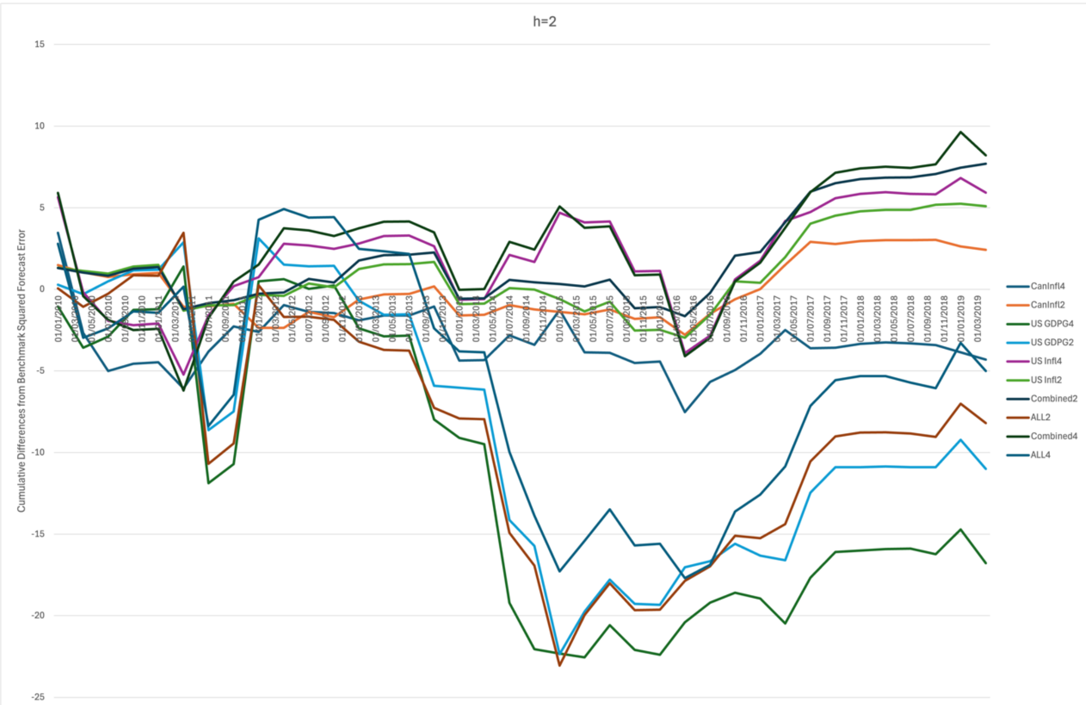
		h =1	h =2	h =3	h =4	∑
1	Benchmark AR(4)r	1.91759	2.05134	1.96032	1.99656	
2	CAN Infl2	0.99705	0.99241	*0.99181*	1.00853	3.98980
3	CAN Infl4	*0.98949*	1.01337	1.00875	1.03438	4.04598
4	US GDPG2	1.00549	1.03385	1.08219	*0.99300*	4.11454
5	US GDPG4	1.01958	1.05117	1.08476	***0.99160***	4.14710
6	US Infl2	0.98326	0.98396	***0.96256***	0.99791	3.92768
7	US Infl4	*0.93379*	0.98131	0.97836	1.04554	3.93901
8	Combined2	0.97526	0.97563	*0.96373*	0.99970	3.91433
9	Combined4	***0.93047***	**0.97398**	0.98128	1.05009	3.93581
10	ALL2	*0.99539*	1.02532	1.08426	0.99545	4.10042
11	ALL4	*0.95660*	1.01552	1.07520	1.01477	4.06209

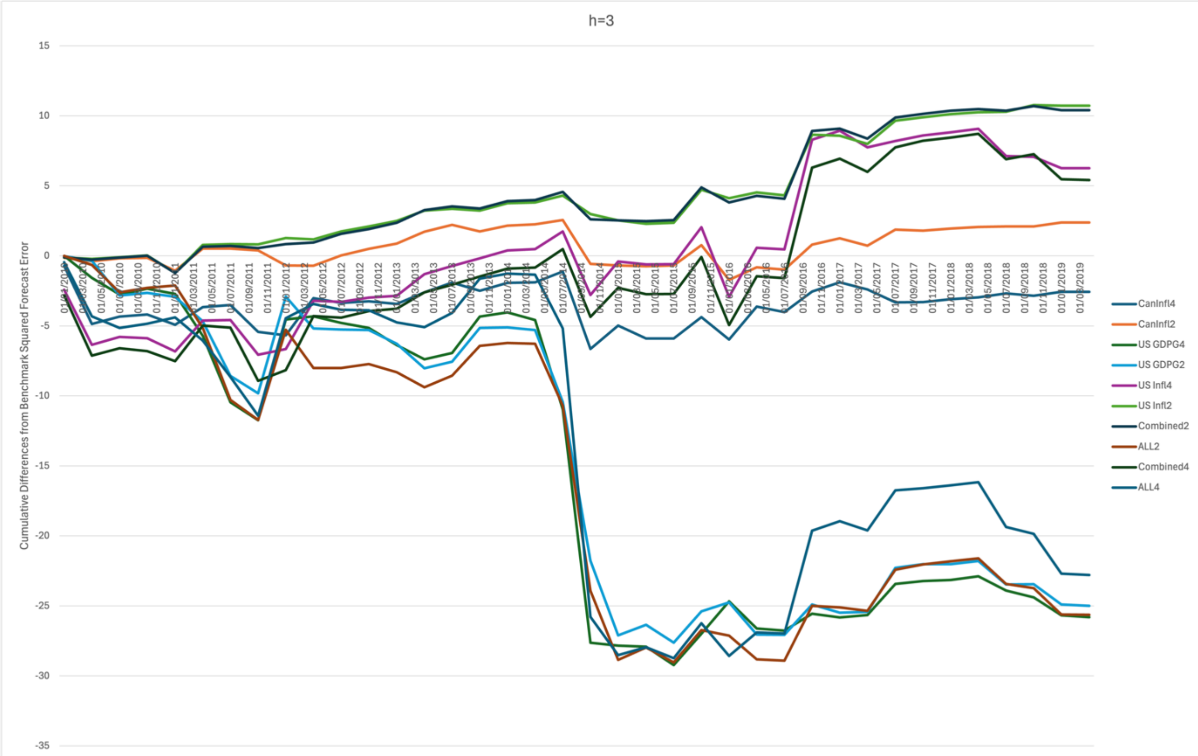
Figure 4: CSSED Means & Standard Errors

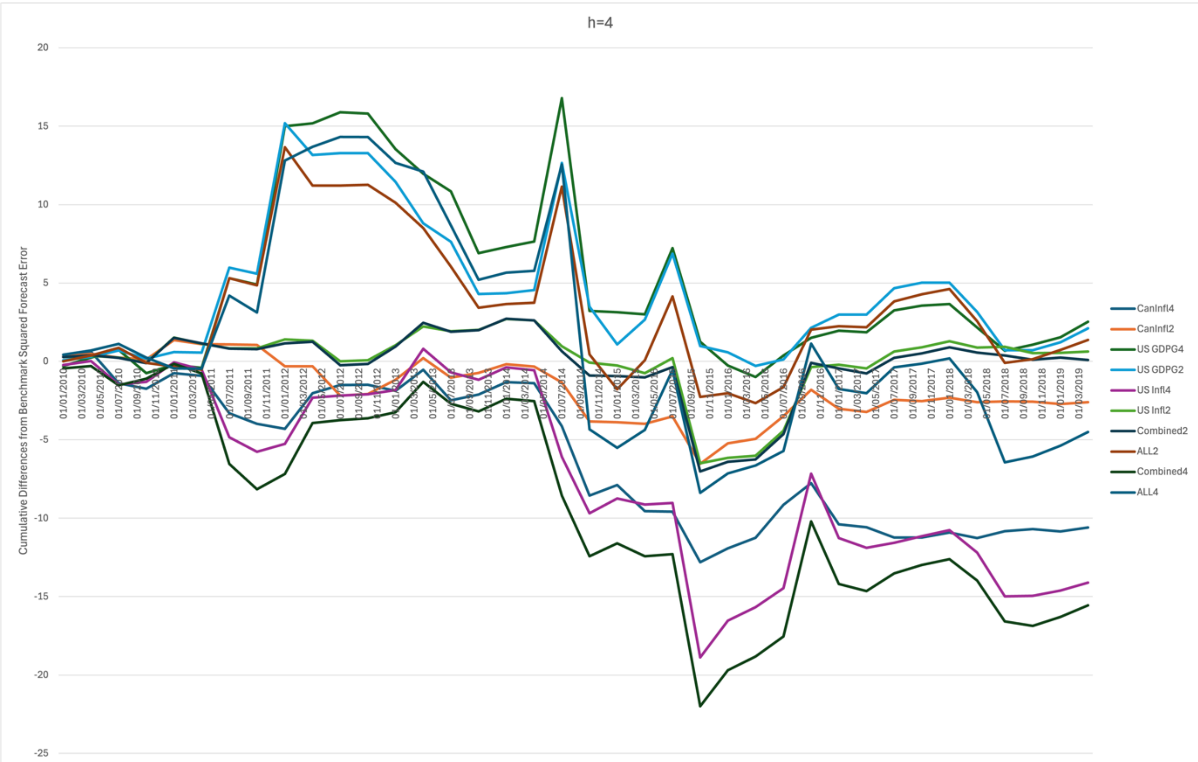
	h =1	h =2	h =3	h =4
CAN Infl2	1.50223	-0.008154	0.6890759	-1.775994
	(0.84819)	(1.81797)	(1.25194)	(1.93885)
CAN Infl4	4.65822	-3.2657583	-3.5244016	-6.0830249
	(2.79392)	(1.84268)	(1.42858)	(4.54147)
US GDPG2	0.67248	-8.4868002	-14.899741	4.4440079
	(5.12677)	(7.78242)	(10.14926)	(4.47267)
US GDPG4	-0.43559	-11.647311	-15.470968	4.923742
	(6.87161)	(8.48801)	(10.61156)	(5.50398)
US Infl2	2.52489	0.9533384	4.3885336	0.071355
	(1.41167)	(2.35926)	(3.80270)	(2.20398)
US Infl4	**11.05465**	2.0366115	0.5272737	-7.1759845
	(4.77379)	(3.11460)	(5.30577)	(5.95301)
Combined2	3.67175	1.904361	4.3887986	-0.1459558
	(2.09754)	**(2.87227)**	**(3.88369)**	**(2.28195)**
Combined4	**11.54332**	2.5915724	-0.6486221	-9.0953588
	(4.77173)	(3.76158)	(5.44193)	(6.57730)
ALL2	1.60922	-8.868255	-15.86595	3.2264258
	(4.17589)	(7.14412)	(10.14022)	(4.39184)
ALL4	8.91761	-5.8264045	-13.493684	1.4910703
	(4.06958)	(6.94099)	(9.90673)	(6.86599)

Figures 2.1-2.4:

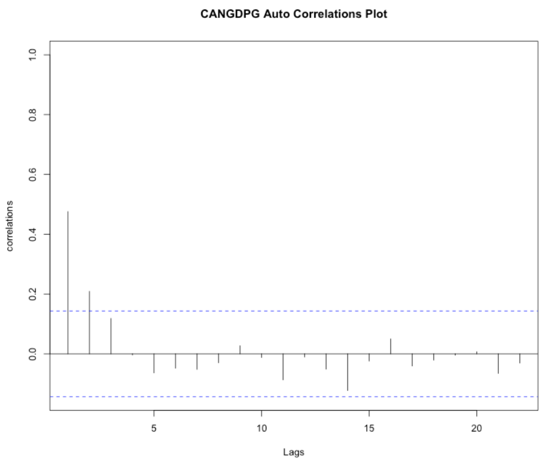


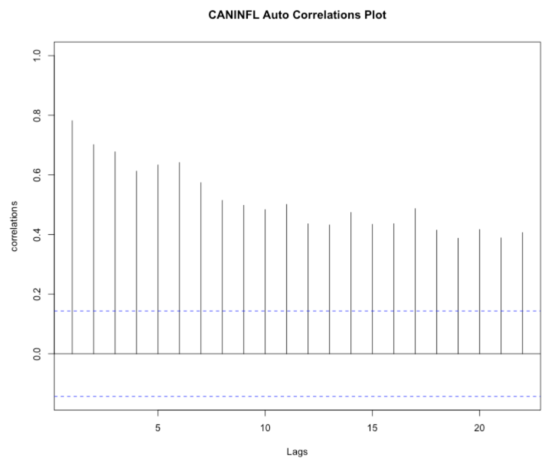


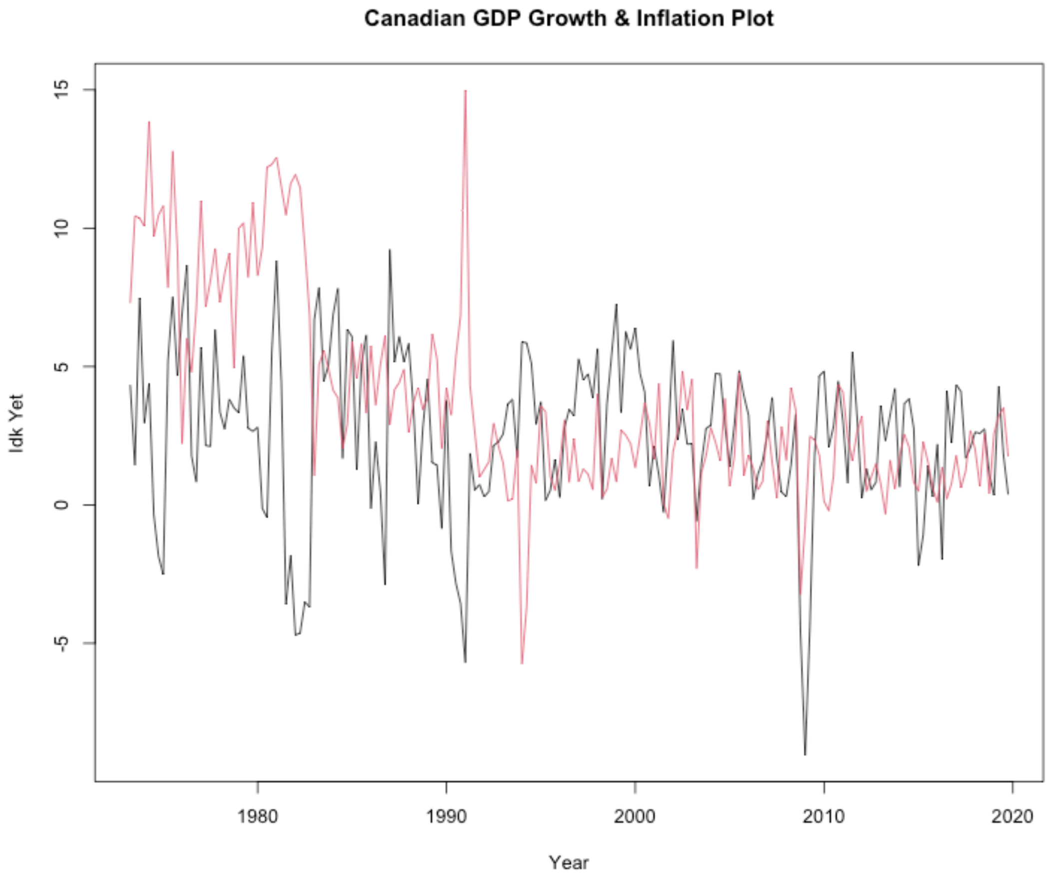

h=3
Cumulative Differences from Benchmark Squared Forecast Error
CanInfl4
CanInfl2
US GDPG4
US GDPG2
US Infl4
US Infl2
Combined2
ALL2
Combined4
ALL4

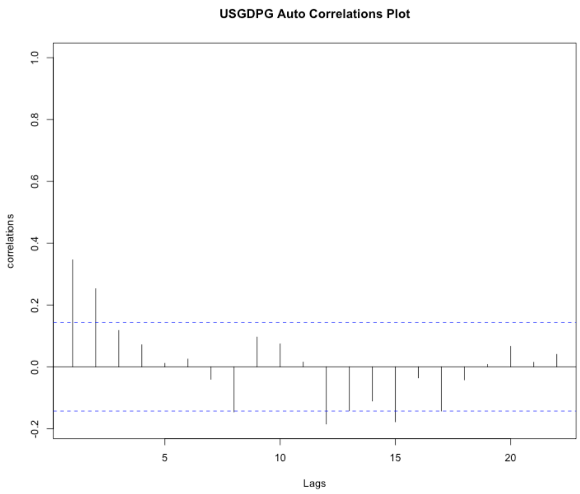

h=4
Cumulative Differences from Benchmark Squared Forecast Error
CanInfl4
CanInfl2
US GDPG4
US GDPG2
US Infl4
US Infl2
Combined2
ALL2
Combined4
ALL4

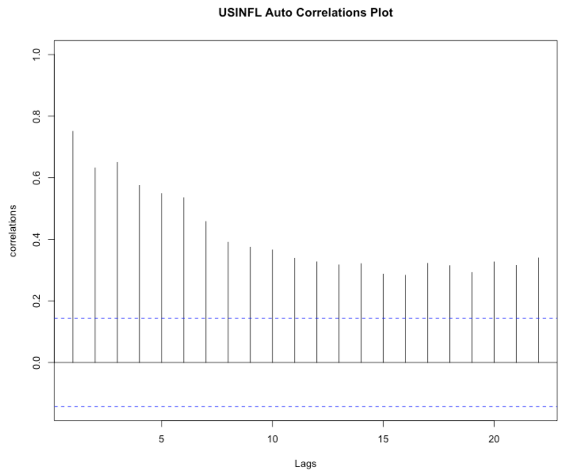
Figures 3.1-3.6:








USGDPG Auto Correlations Plot
correlations
Lags
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
5
10
15
20


USINFL Auto Correlations Plot
correlations
Lags
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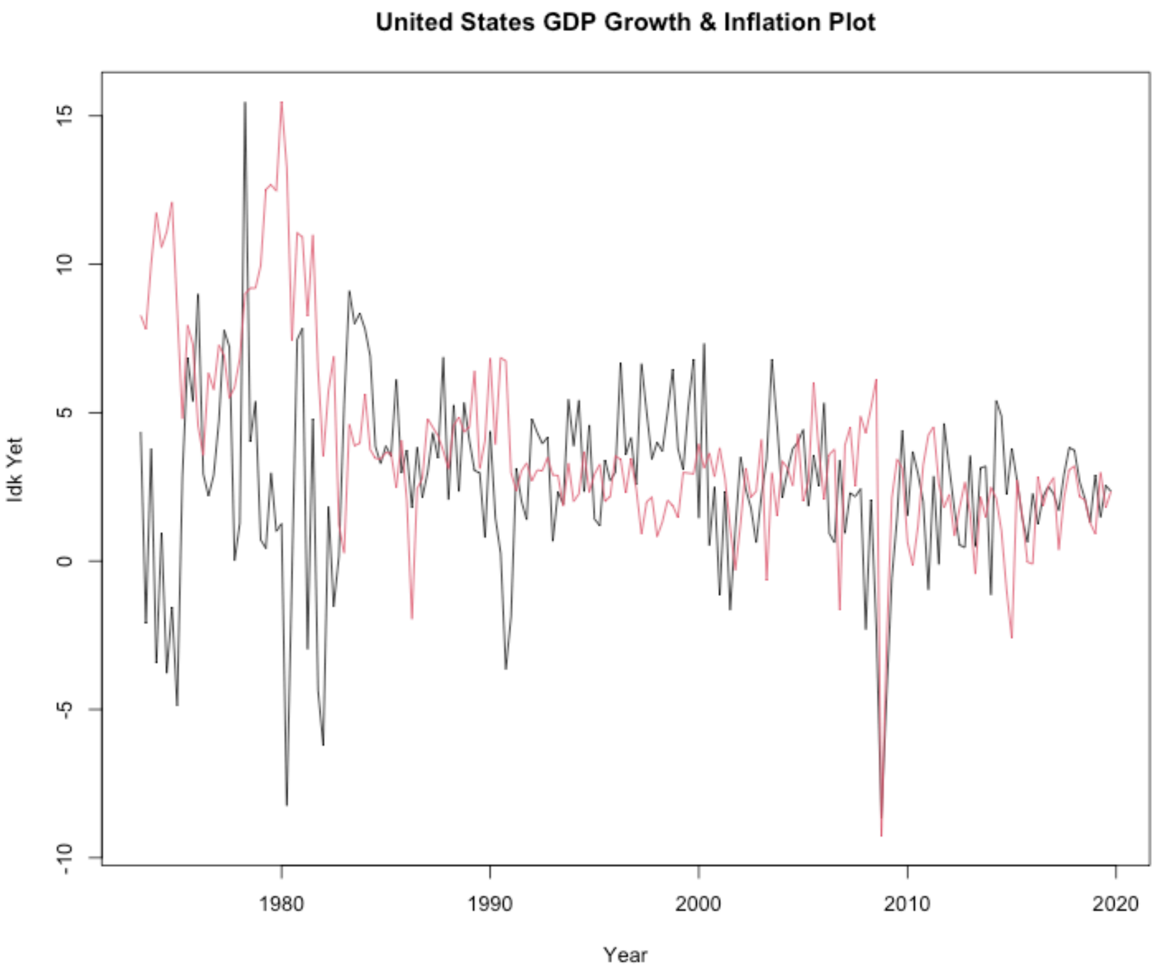
0.2
0.4
0.6
0.8
1.0
5
10
15
20


United States GDP Growth & Inflation Plot
Idk Yet
Year
-10
-5
0
5
10
15
1980
1990
2000
2010
2020

References

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