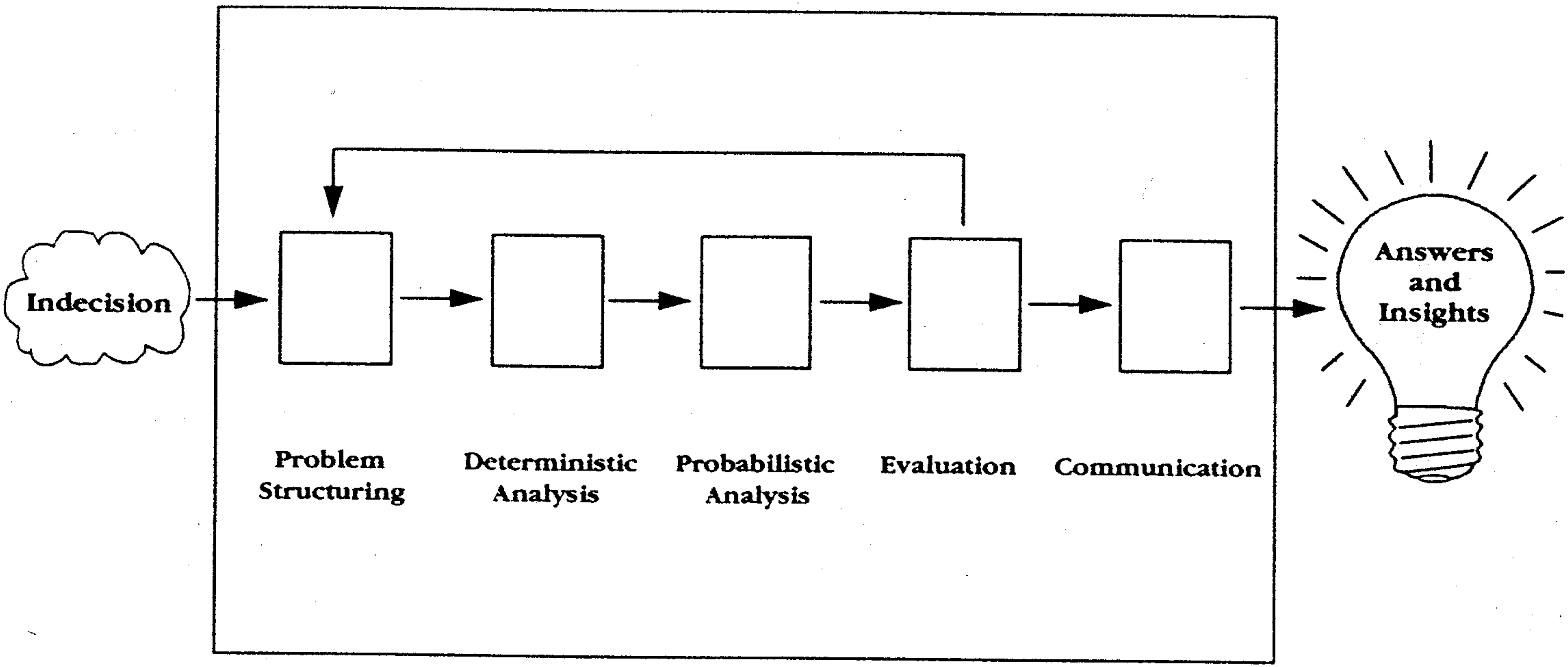
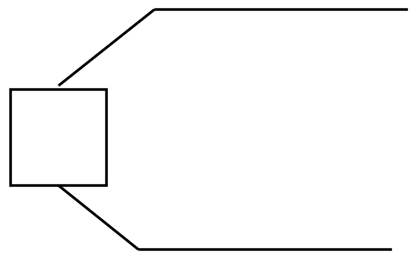
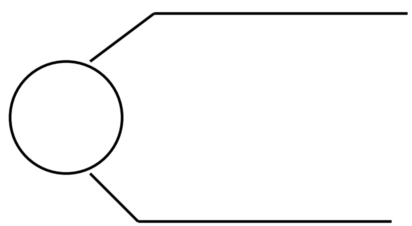
Decision Analysis Methodology



Decision Tree Notation



decision node

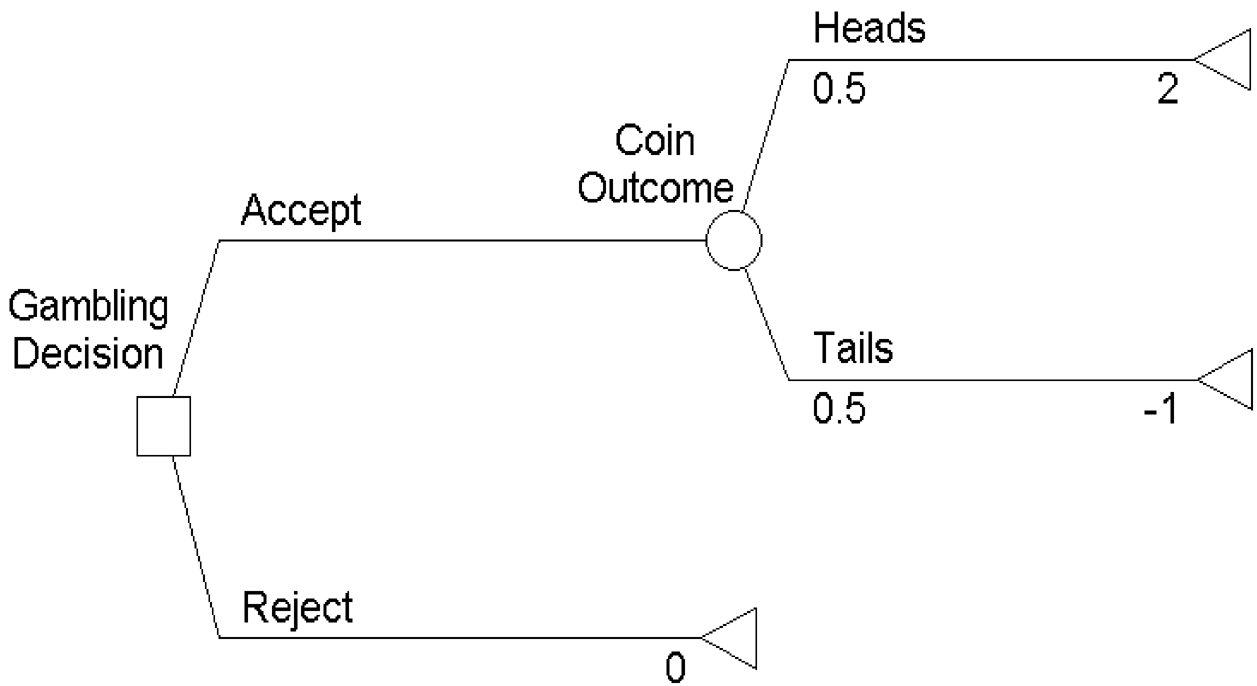


chance node

logical sequence between nodes chronological from left to right

Simple Decision Tree Example

- Coin is to … Would you accept the



- Expected profit = 0.5×2+0.5×(-1)=£0.50
So accept.

Oil Drilling Problem

- Decision regarding whether or not to drill.

- Cost of drilling the site = \$70m

  If "dry", all lost.

  If "wet", profit will be \$150m.

  If "soaking", profit will be \$600m.

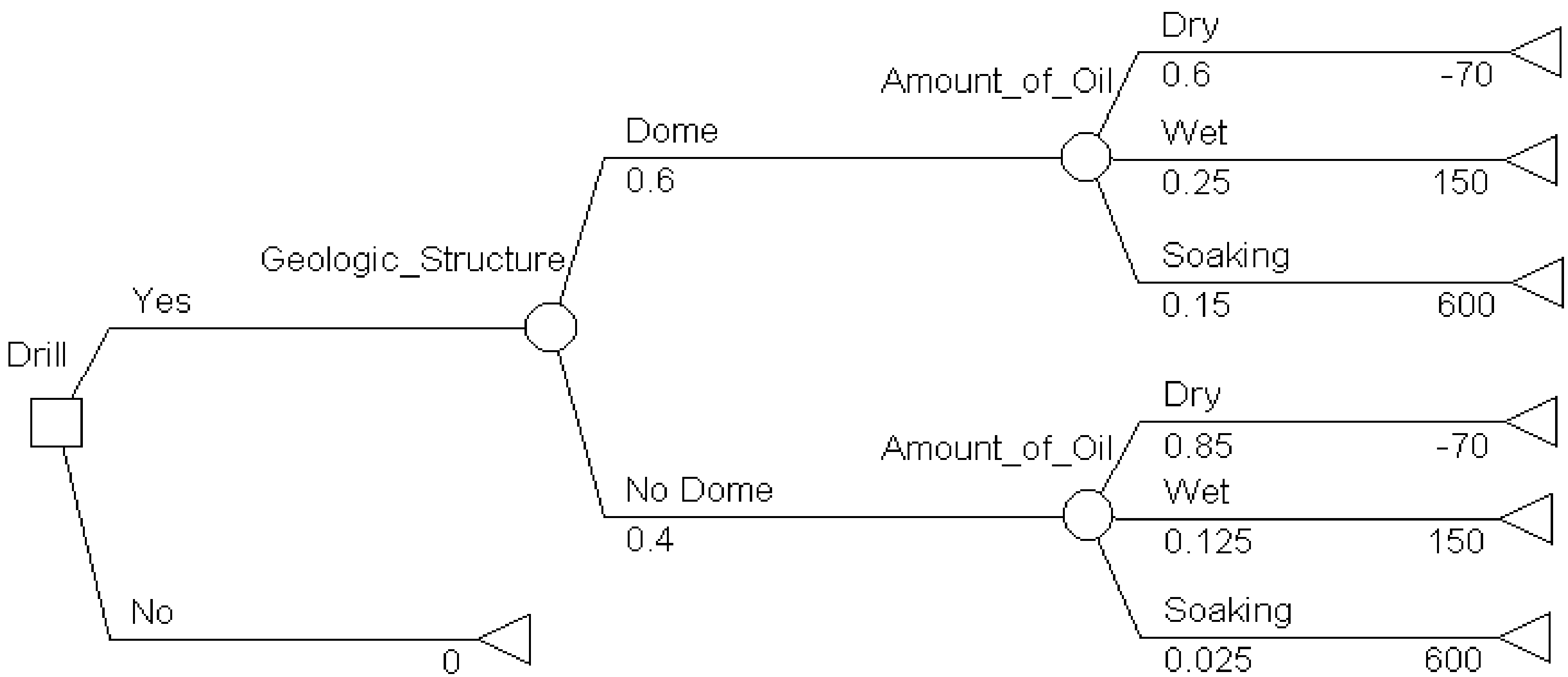
- If underlying lime-shale rises into flat dome shape, better chances of finding oil. We don't know yet if there is a dome. Estimate is 6 chances in 10 of finding a dome on current site.

- Conditional probabilities of oil outcomes:

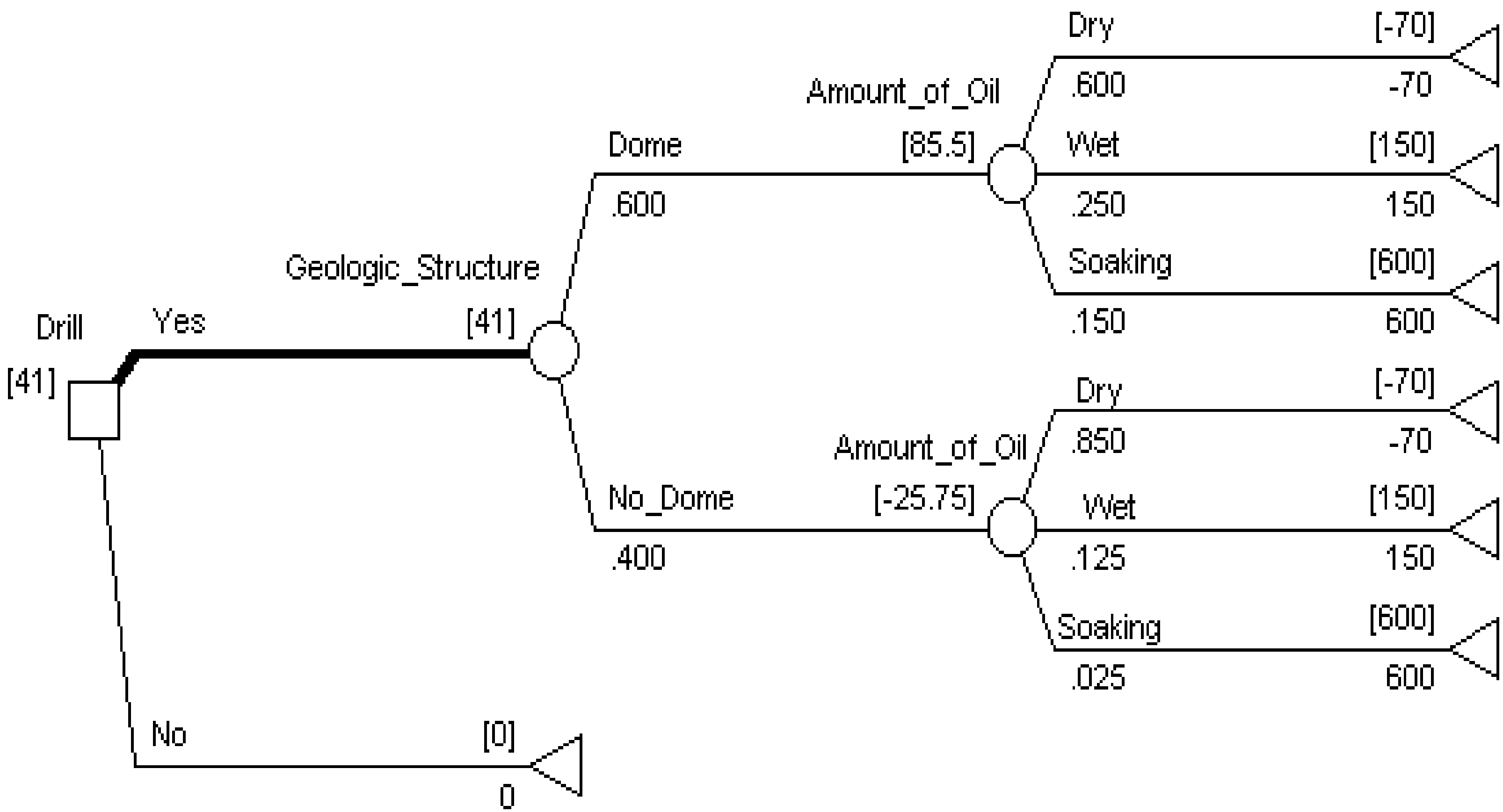
  e.g. $P(Dry \mid No\ Dome) = 0.85$

	If dome	If no dome
Dry	0.60	0.85
Wet	0.25	0.125
Soaking	0.15	0.025
	1.00	1.00

Oil Drilling Decision Tree



Expected Value and "Roll-Back"



Expected Value and Risk Attitude

- *Expected value* is what you expect to get on average - masks possibility of catastrophic payoff.
- *Risk aversion* - accept lower expected value in return for less risk, e.g. house fire insurance.
- *Risk seeking* - accept lower expected value in return for possibility of higher payoff, e.g. betting at bookies.
- *Risk neutral* - decision maker who looks only at expected value.

Alternative Criterion

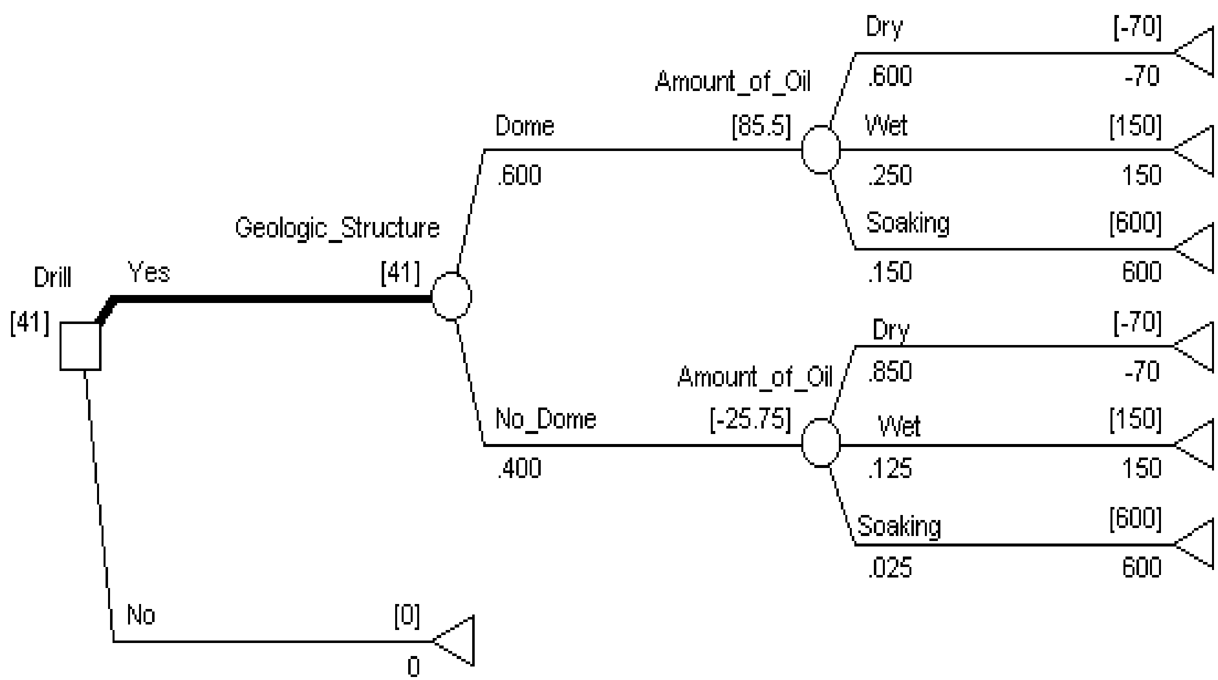
- Alternatives to expected monetary value criterion:
  - *Maximin* – Choose decision whose worst payoff is better than any other decision's worst.
  - *Maximax* – Choose decision whose best payoff is better than any other decision's best.
  - *Utility functions* – Monetary values replaced by values expressing relative preference.

Oil Drilling Probabilities

- Consideration of risk analysis motivates us to consider probabilities of outcomes, rather than simply expected values.

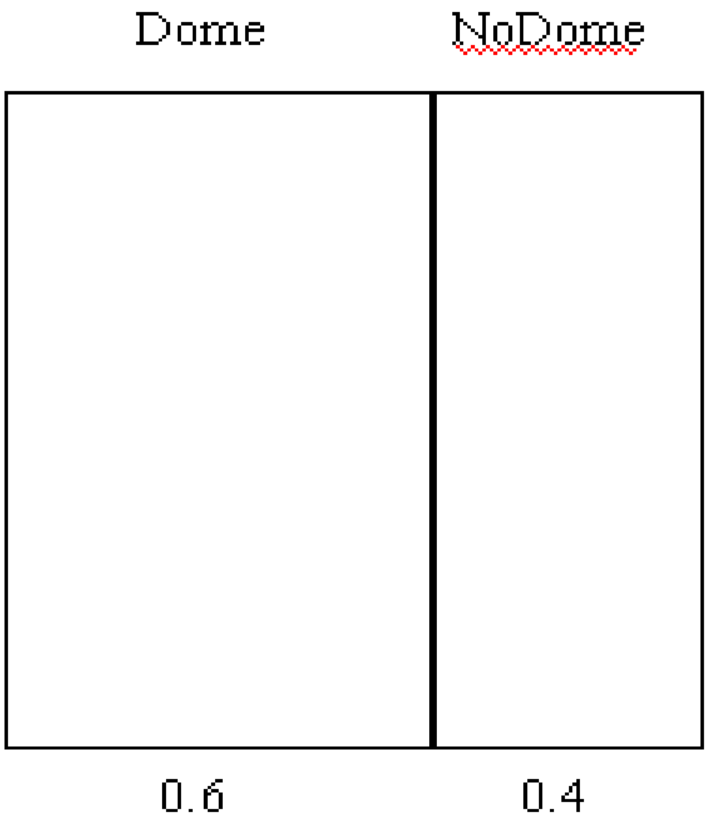
- Interesting to calculate *marginal* probabilities: P(*Dry*), P(*Wet*) and P(*Soak*).

- Might also be useful to calculate *joint* probabilities, e.g. P($Dry \cap NoDome$).

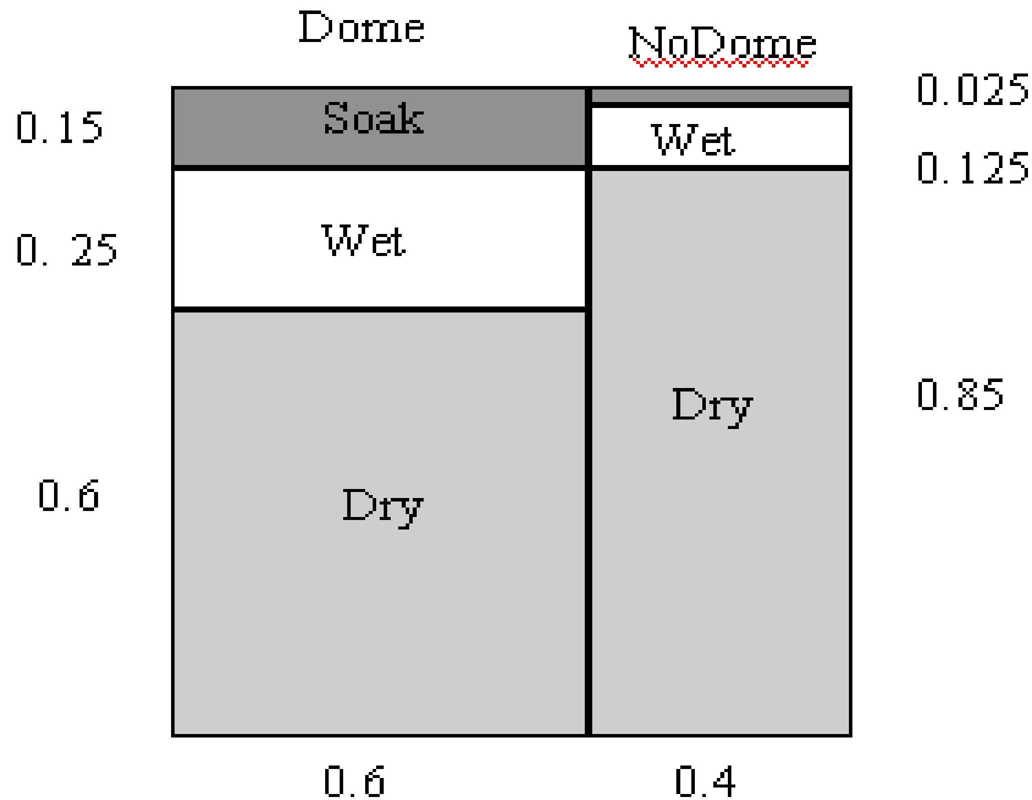


Venn Diagram

Step 1:



Step 2:

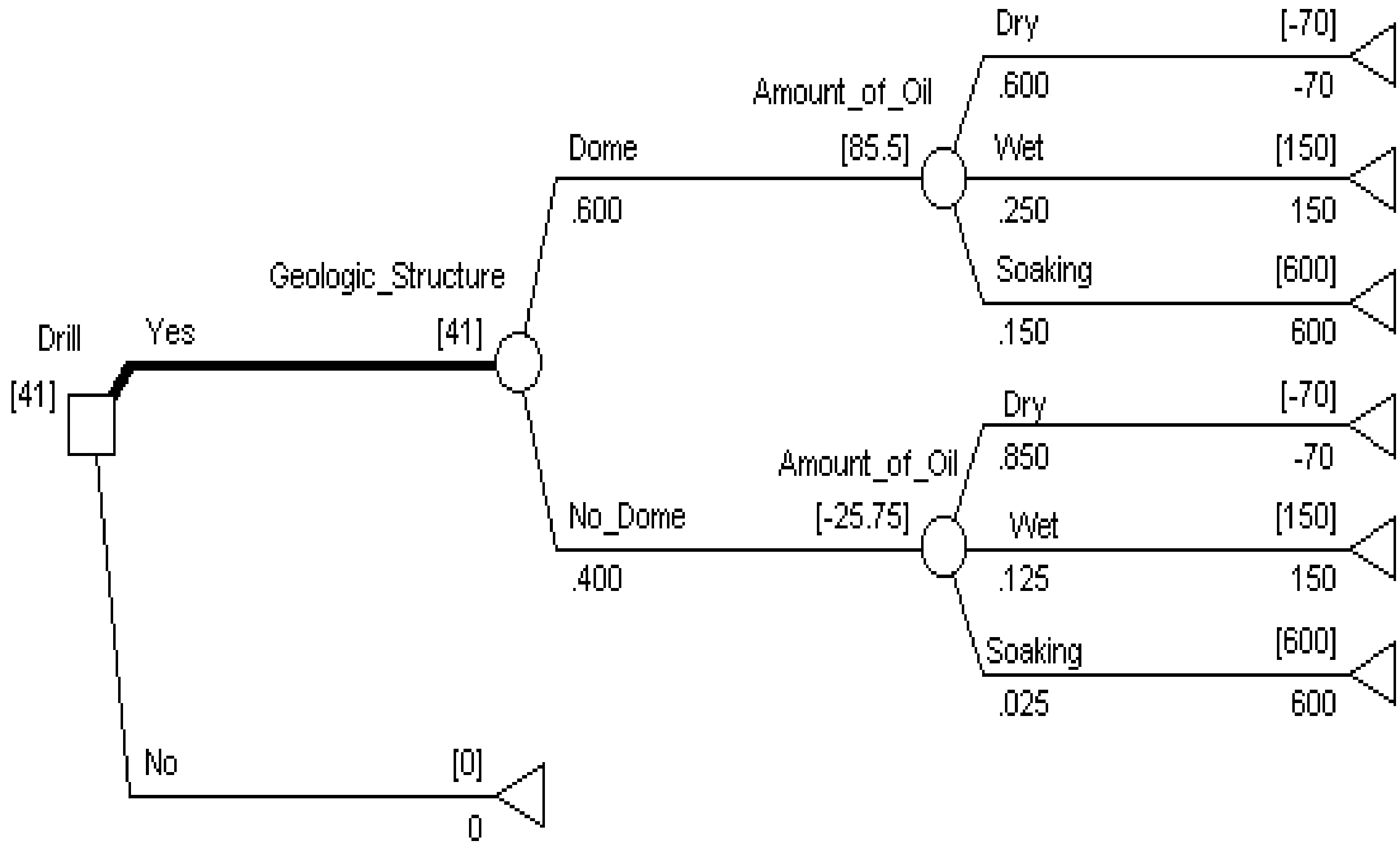


Step 3: Interpret areas as joint probabilities

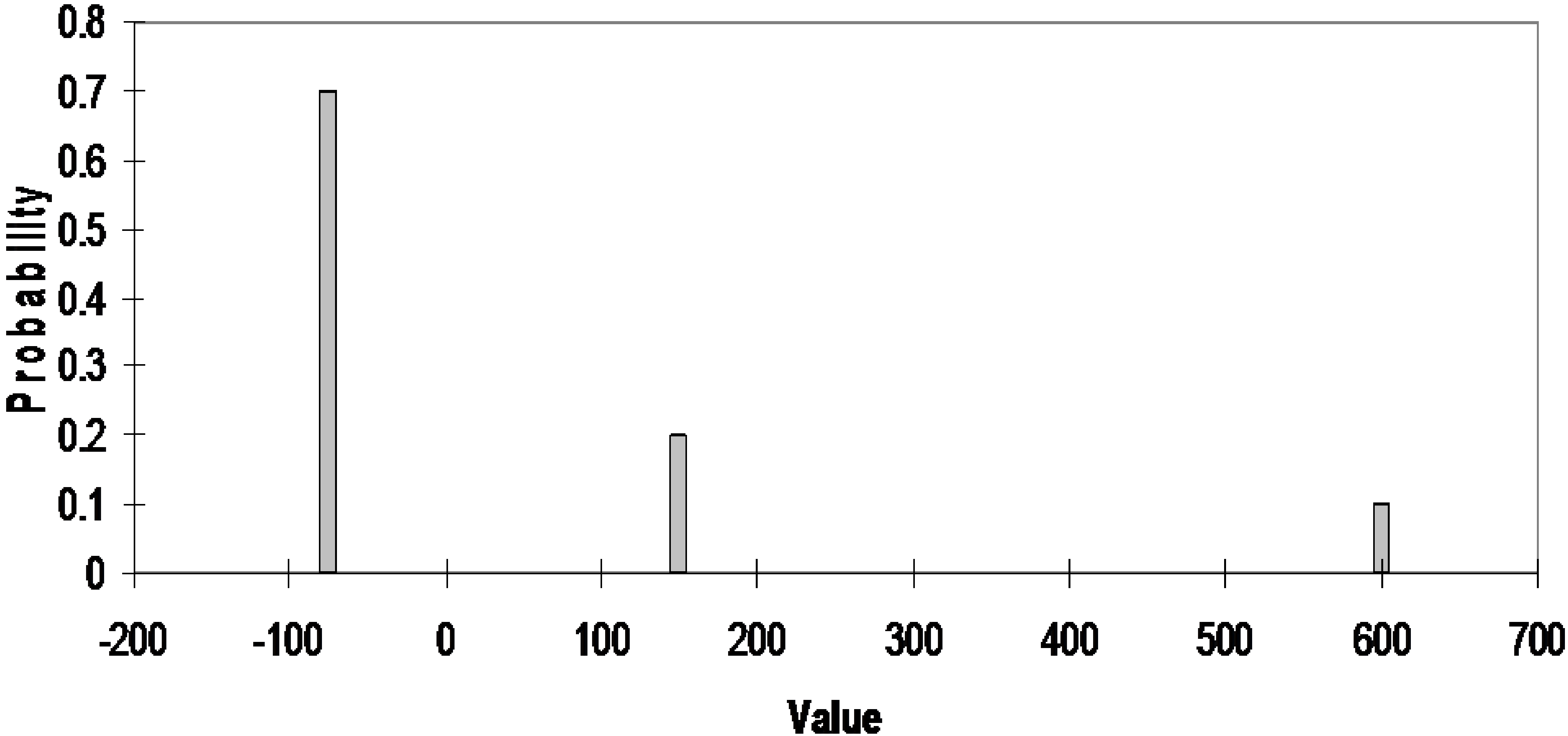
e.g. $P(Dry \cap NoDome) = 0.85 \times 0.4 = 0.34$

$P(Dry) = 0.6 \times 0.6 + 0.85 \times 0.4 = 0.7$

Oil Drilling Decision Tree



Risk Profile



Expected Value of Perfect Information

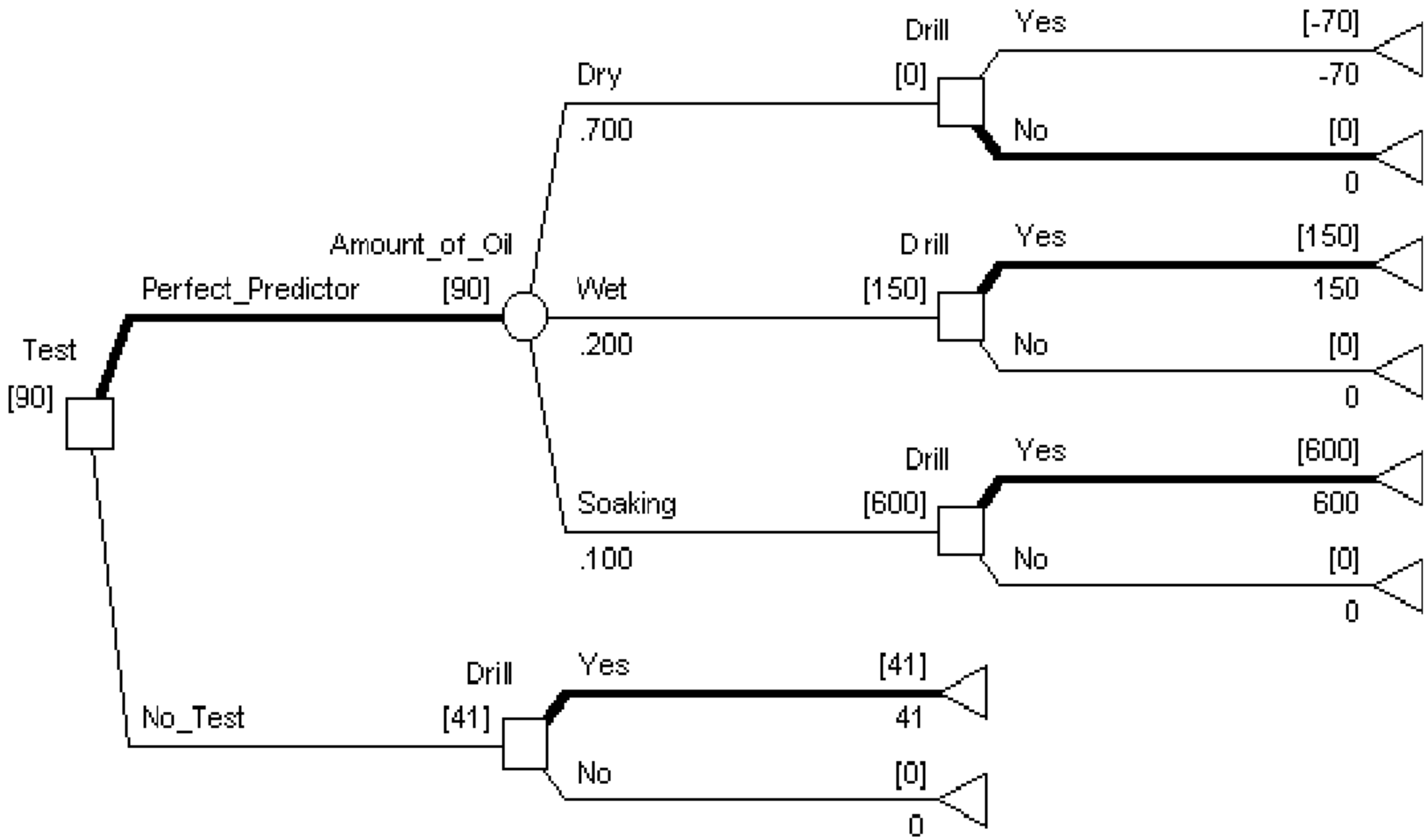
- Value of perfect information sets upper bound on value of new information.

- Concept of value of information provides basis for measuring benefit of additional information.

  e.g. Used in market research and product tests.

- Expected value of perfect information is the change in expected value if an uncertain event could be observed before decision is made.

Perfect Predictor of Dry, Wet or Soaking



- Expected value of perfect information = 90 – 41 = 49.

  Pay no more than this for information regarding dry, wet or soaking.

Seismic Test for Dome

- Seismic test is available for \$10m.

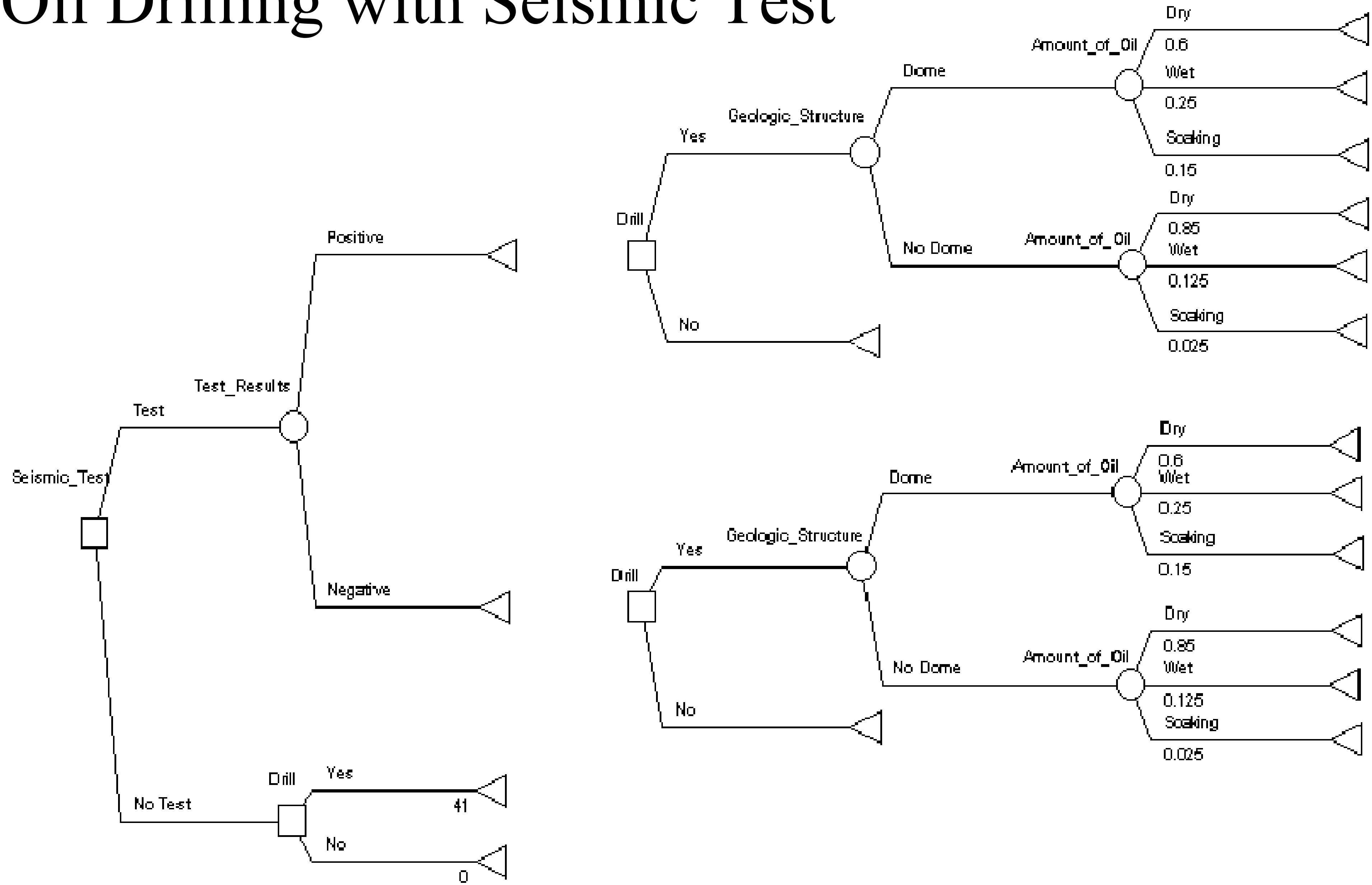
  Test gives indication of lime-shale depth, and hence existence of dome.

- Test is not perfect.

  Estimates of test reliability:

	Reading from Seismic Test	
Actual State	Positive	Negative
Dome Exists	0.90	0.10
No Dome Exists	0.20	0.80

Oil Drilling with Seismic Test



After drawing tree, we need probabilities and payoffs.

Bayes Revision of Probabilities

Prior probabilities

Test accuracy

Bayes theorem $\longrightarrow$ Posterior probabilities

e.g.

P(Dome)

P(Positive|Dome)

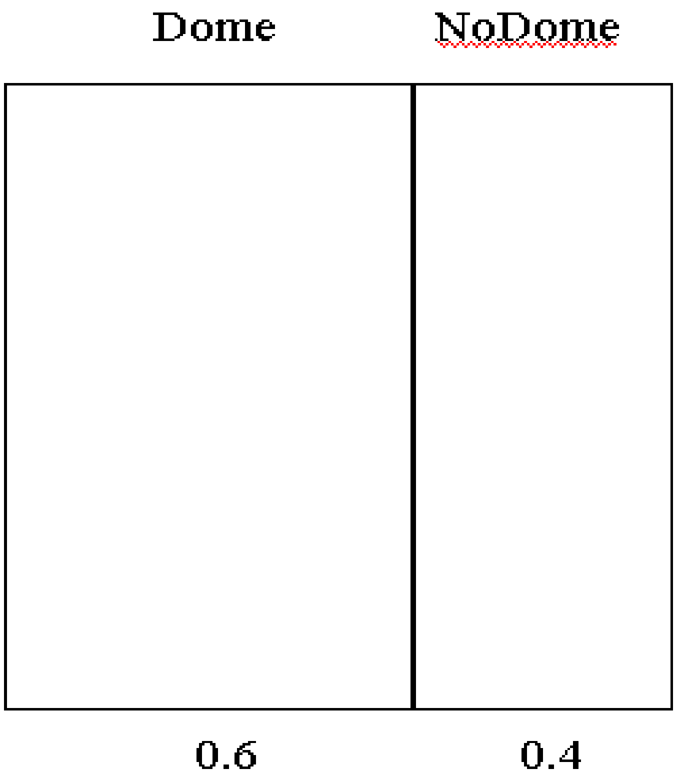
Bayes theorem $\longrightarrow$ *P(Dome|Positive)*

Bayes theorem:

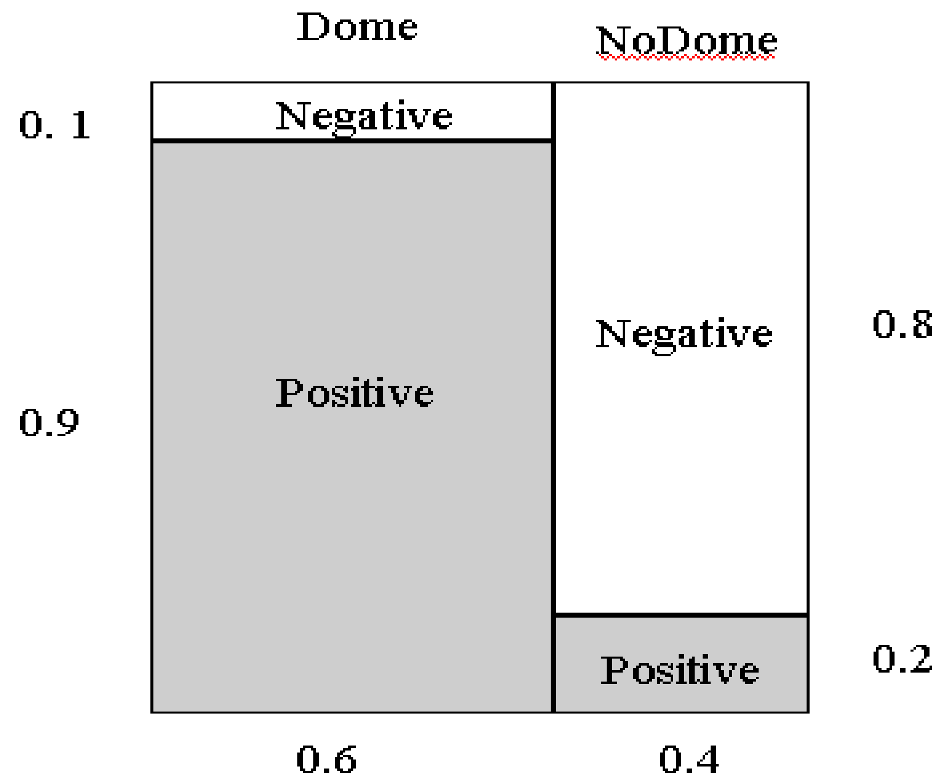
$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Venn Diagram for Bayes Theorem

Step 1:



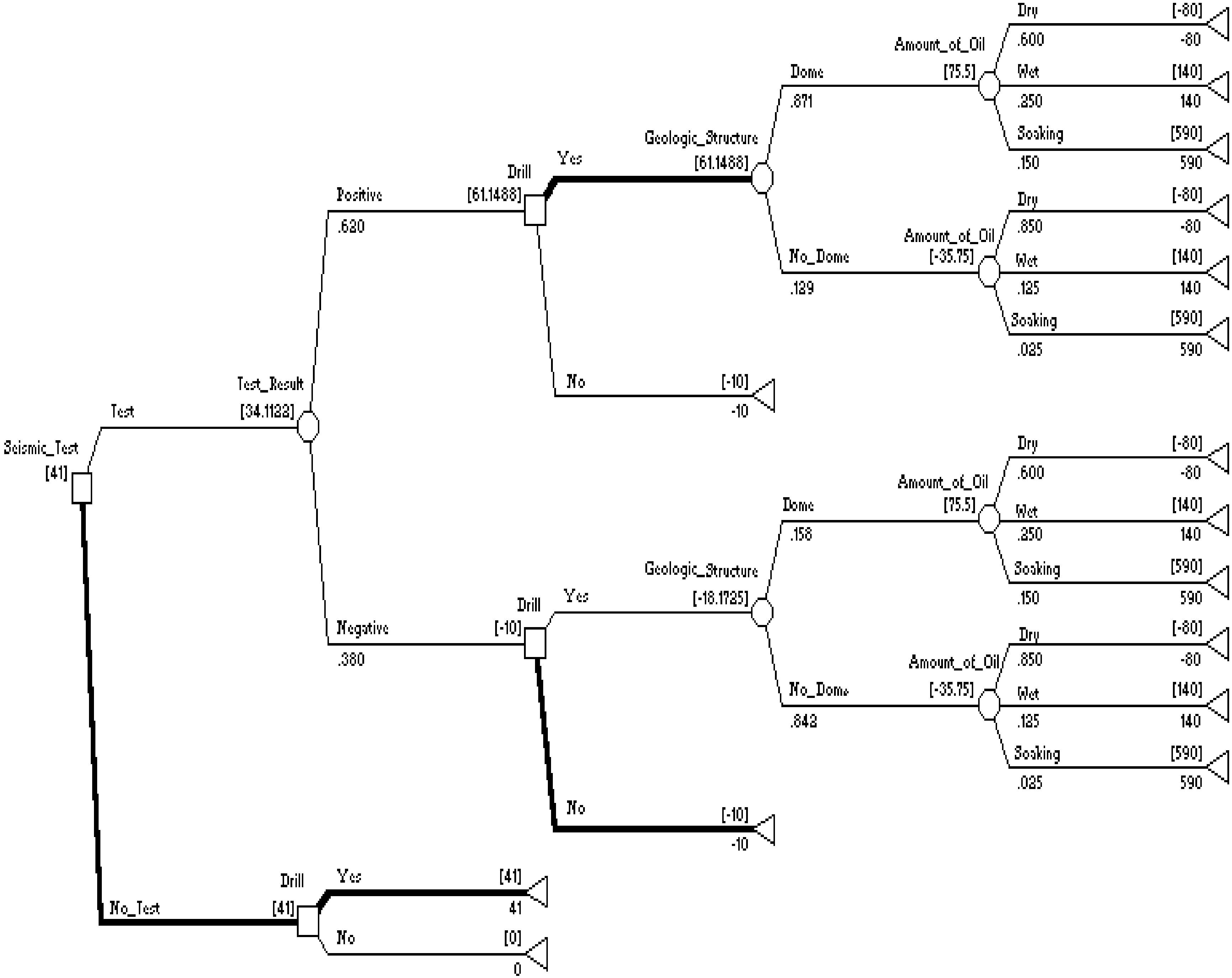
Step 2:



Step 3:

$$P(Dome|Positive) = \frac{0.9 \times 0.6}{0.9 \times 0.6 + 0.2 \times 0.4} = 0.871$$

$$P(Dome|Positive) = \frac{P(Positive|Dome)P(Dome)}{P(Positive)}$$


Seismic_Test
[41]
Test
Test_Result
[34.1122]
Positive
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Drill
[61.1488]
Yes
Geologic_Structure
[61.1488]
Dome
.871
Amount_of_Oil
[75.5]
Dry
[-80]
.600
-80
Wet
[140]
.250
140
Soaking
[590]
.150
590
No_Dome
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Amount_of_Oil
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.850
-80
Wet
[140]
.125
140
Soaking
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.025
590
No
[-10]
-10
Negative
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Drill
[-10]
Yes
Geologic_Structure
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Dome
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-80
Wet
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140
Soaking
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590
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Dry
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Wet
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140
Soaking
[590]
.025
590
No
[-10]
-10
No_Test
Drill
[41]
Yes
[41]
41
No
[0]
0

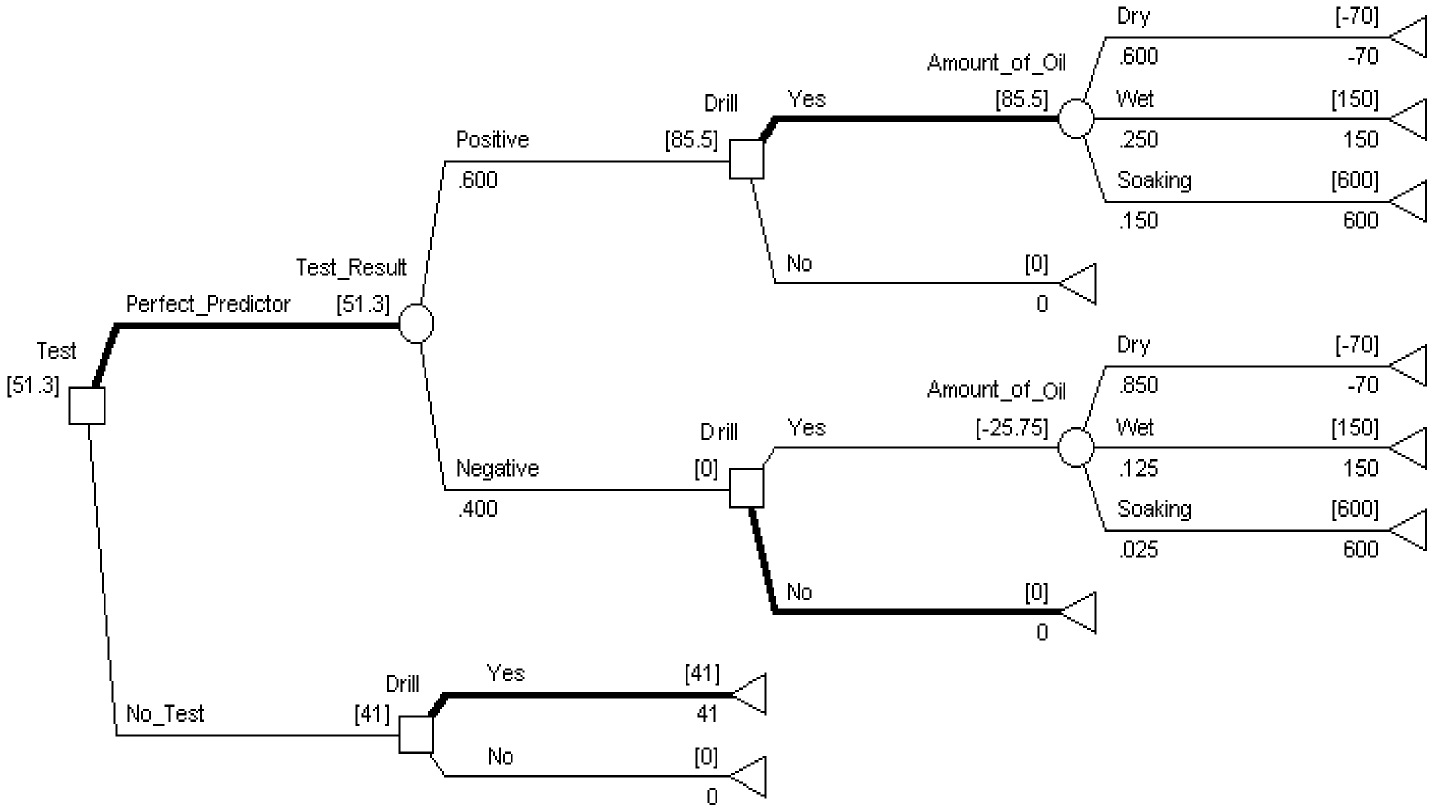
Value of Information

- Paying \$10m for test gives EV = \$34.1m

- Paying nothing for test would give EV = 34.1 + 10 = \$44.1m
  Compare with 'no test' benchmark of \$41m.
  So, for this test, we should pay no more than: 44.1 - 41 = \$3.1m
  This is value of information provided by this test.

- If offered a 'better' test, what is the most worth paying?

Perfect Seismic Test for the Presence of a Dome



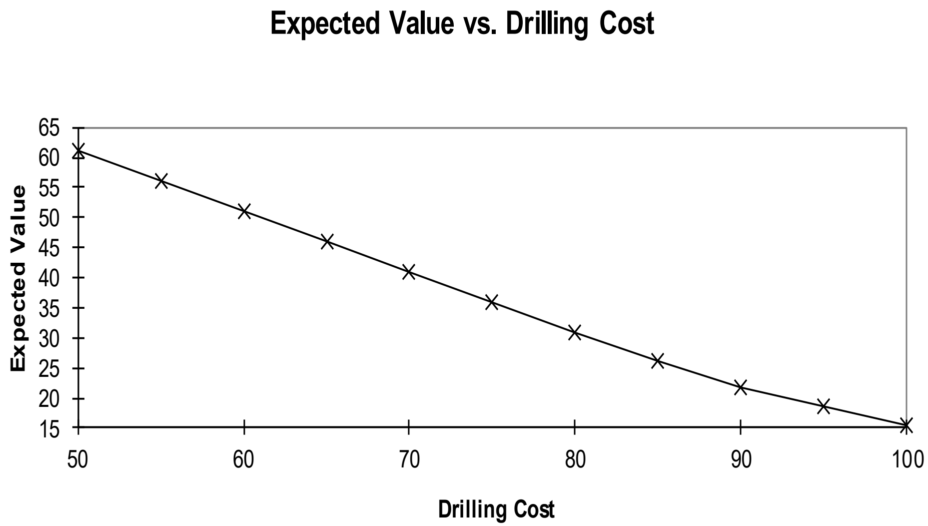
- With a perfect seismic test for presence of a dome:

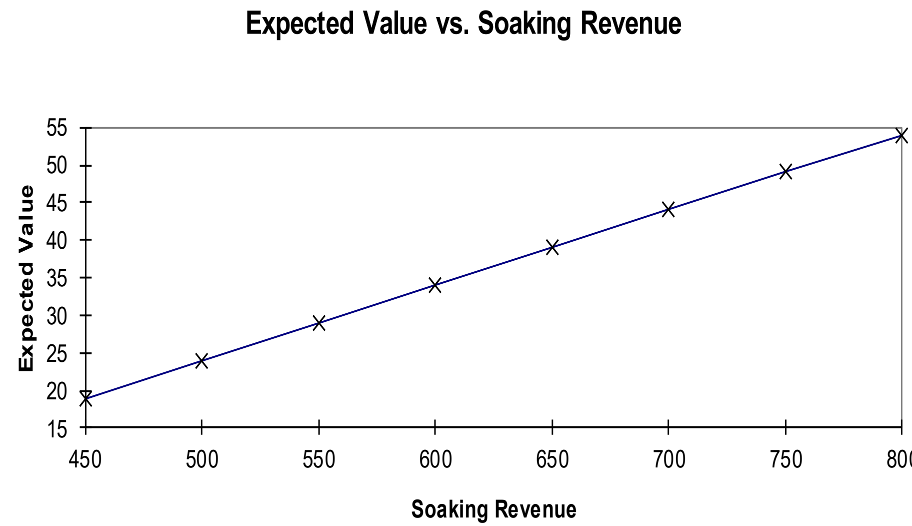
  P(Positive) = P(Dome)

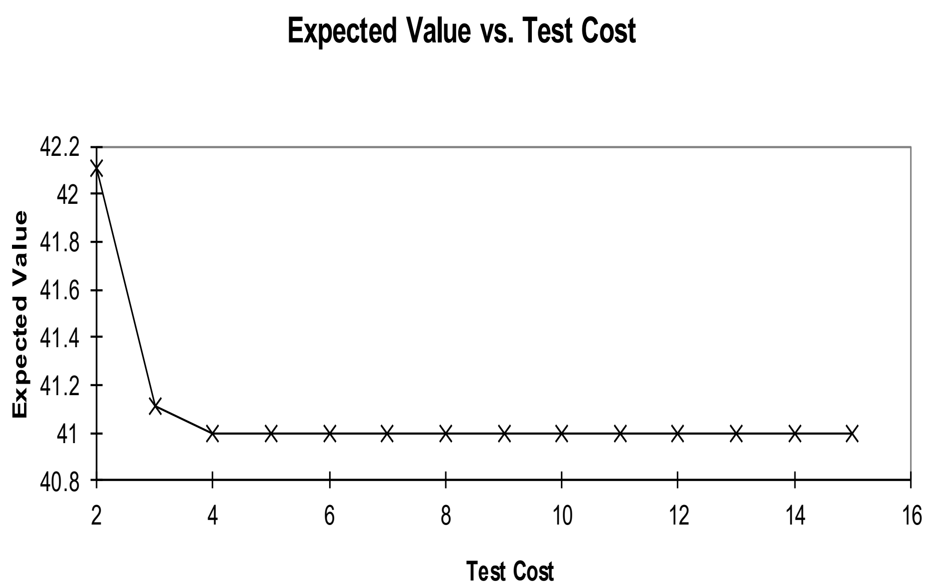
  P(Negative) = P(NoDome)

- Expected value is 51.3, which is 10.3 more than “no test” option. So expected value of perfect information is 10.3. Pay up to 10.3 for more info.

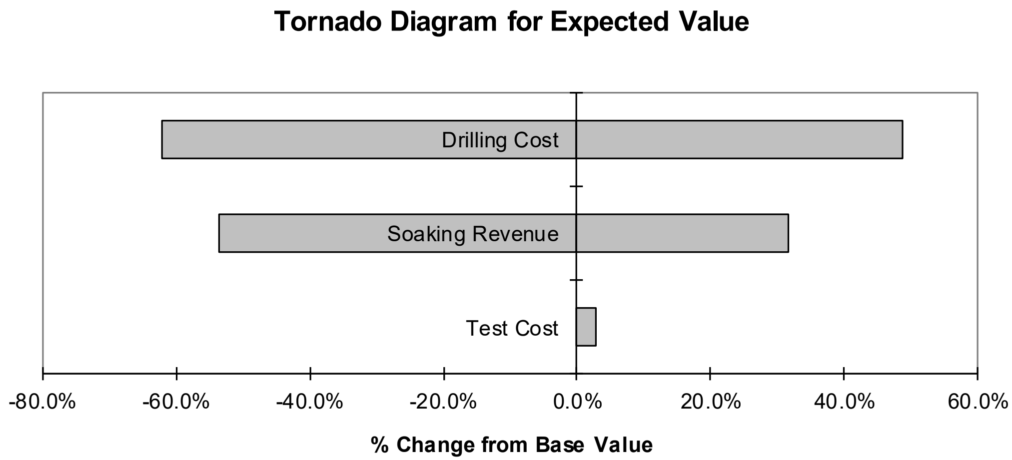
One-Way Sensitivity Analysis

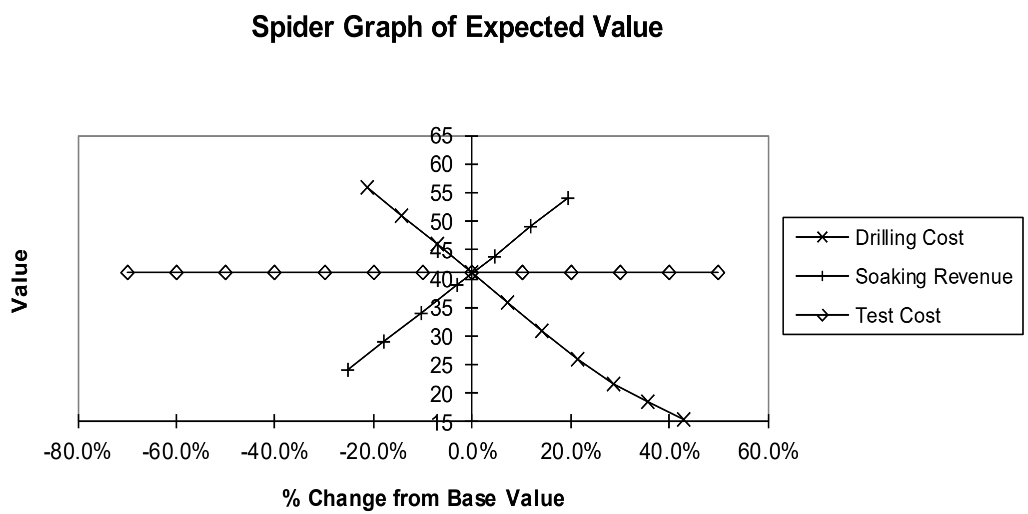






Sensitivity Analysis Charts





Decision Analysis Seeks to Improve

- Insight through the process of modelling and sensitivity analysis
- Communication through visual structure
- Defensibility with explicit modelling
- Consensus and commitment through shared involvement
- Awareness of important strategic drivers

Summary

- Decision trees enable a convenient evaluation of decision problems.
- Expected value criterion is standard approach to evaluating decision trees.
- Should also consider risk profiles and sensitivity analysis.
- Expected value of perfect information provides an upper bound on value of new information.
- Bayes theorem reverses conditional probabilities: $\mathrm{P}(A|B) \rightarrow \mathrm{P}(B|A)$
  Venn diagram avoids confusion with formulae.

Appendix - Utility Functions - Health Care Example

- Utility functions are also useful for quantifying qualitative outcomes.