# PROBLEM 2 Sudoku Puzzle:

A Sudoku board consists of 81 squares (see textbook section 6.2.6), some of which are initially filled with digits from 1 to 9. The puzzle is to fill in all the remaining squares such that no digit appears twice in any row, column, or 3×3 box. A row, column, or box is called a **unit**.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | | | 3 | | 2 | | 6 | | |
| B | 9 | | | 3 | | 5 | | | 1 |
| C | | | 1 | 8 | | 6 | 4 | | |
| D | | | 8 | 1 | | 2 | 9 | | |
| E | 7 | | | | | | | | 8 |
| F | | | 6 | 7 | | 8 | 2 | | |
| G | | | 2 | 6 | | 9 | 5 | | |
| H | 8 | | | 2 | | 3 | | | 9 |
| I | | | 5 | | 1 | | 3 | | |

(a)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| A | 4 | 8 | 3 | 9 | 2 | 1 | 6 | 5 | 7 |
| B | 9 | 6 | 7 | 3 | 4 | 5 | 8 | 2 | 1 |
| C | 2 | 5 | 1 | 8 | 7 | 6 | 4 | 9 | 3 |
| D | 5 | 4 | 8 | 1 | 3 | 2 | 9 | 7 | 6 |
| E | 7 | 2 | 9 | 5 | 6 | 4 | 1 | 3 | 8 |
| F | 1 | 3 | 6 | 7 | 9 | 8 | 2 | 4 | 5 |
| G | 3 | 7 | 2 | 6 | 8 | 9 | 5 | 1 | 4 |
| H | 8 | 1 | 4 | 2 | 5 | 3 | 7 | 6 | 9 |
| I | 6 | 9 | 5 | 4 | 1 | 7 | 3 | 8 | 2 |

(b)

Figure 6.4 (a) A Sudoku puzzle and (b) its solution.

# CSP Formulation:

Whenever we formulate a problem into a CSP, we need to identify the following: set of variables *Xi*, set of domains *Di*, and set of constraints *C*:

X is a set of variables, $\{X_1, \ldots, X \ \}$.

D is a set of domains, $\{D_1, \ldots, D \ \}$, one for each variable.

C is a set of constraints that specify allowable combinations of values.

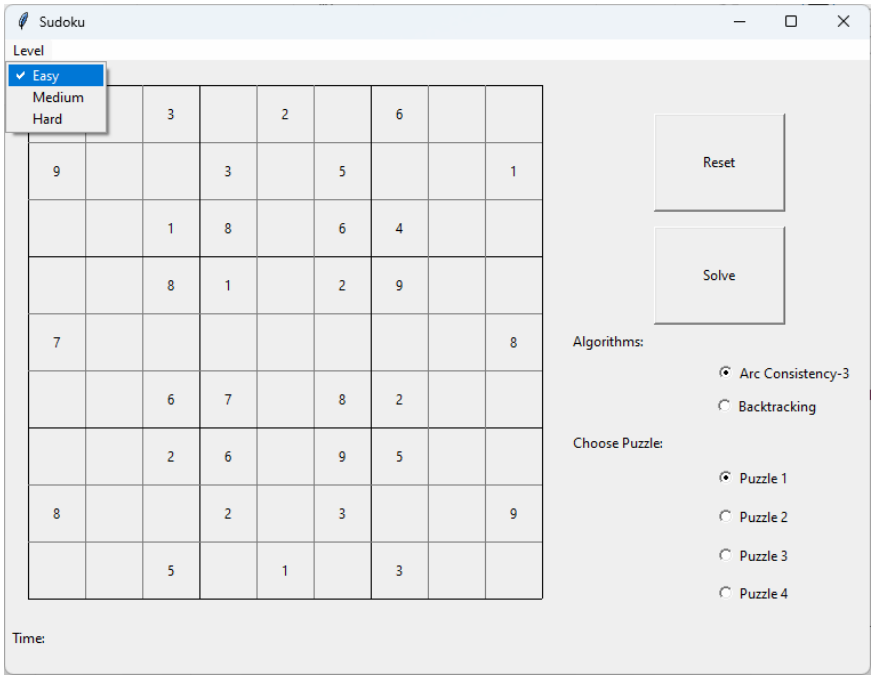
All sudoku puzzles can be formulated as CSP by considering each **cell** as a variable. The initial domain of all cells is {1, 2, 3, 4, 5, 6, 7, 8, 9}. The constraints are formulated by the fact that in the solution of a sudoku puzzle, no two cells in a row, column or block can have identical numbers.

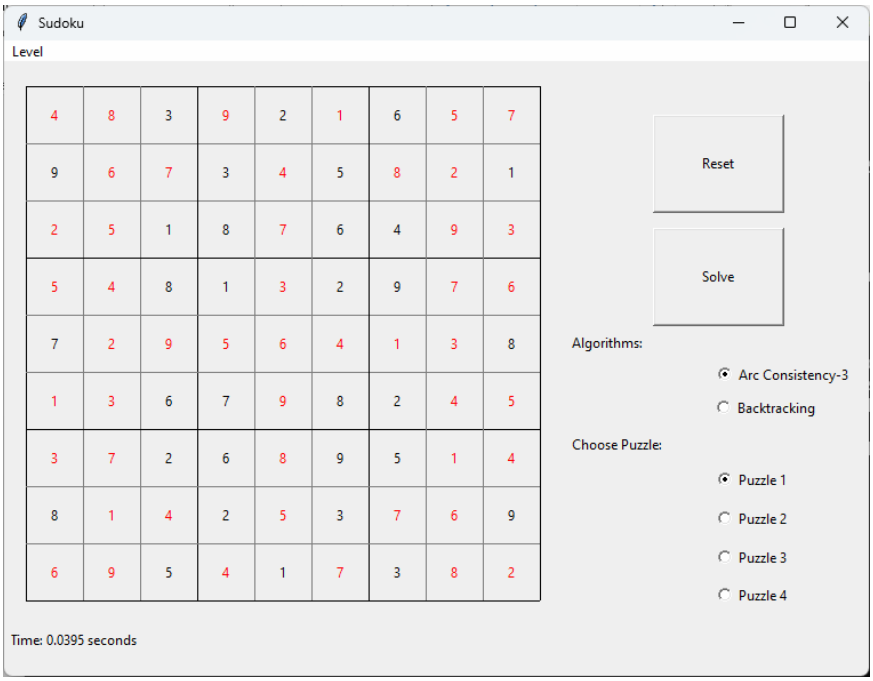
# Implementation Activities:

The Sudoku puzzle is a classic constraint satisfaction problem (CSP). Implement the Arc-Consistency 3 (AC-3) Algorithm and the Backtracking Algorithm to solve the Sudoku puzzle in Python, and compare their time complexities. The Sudoku puzzle board should feature a graphical user interface (GUI) using Tkinter. The dataset for the Sudoku puzzles is provided.

```
function AC-3(csp) returns false if an inconsistency is found and true otherwise
  inputs: csp, a binary CSP with components (X, D, C)
  local variables: queue, a queue of arcs, initially all the arcs in csp

  while queue is not empty do
      (Xi, Xj) ← POP(queue)
      if REVISE(csp, Xi, Xj) then
          if size of Di = 0 then return false
          for each Xk in Xi.NEIGHBORS − {Xj} do
              add(Xk, Xi) to queue
  return true

function REVISE(csp, Xi, Xj) returns true iff we revise the domain of Xi
  revised ← false
  for each x in Di do
      if no value y in Dj allows (x, y) to satisfy the constraint between Xi and Xj then
          delete x from Di
          revised ← true
  return revised

function BACKTRACKING-SEARCH(csp) returns a solution, or failure
  return BACKTRACK({}, csp)

function BACKTRACK(assignment, csp) returns a solution, or failure
  if assignment is complete then return assignment
  var ← SELECT-UNASSIGNED-VARIABLE(csp)
  for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
      if value is consistent with assignment then
          add {var = value} to assignment
          inferences ← INFERENCE(csp, var, value)
          if inferences ≠ failure then
              add inferences to assignment
              result ← BACKTRACK(assignment, csp)
              if result ≠ failure then
                  return result
          remove {var = value} and inferences from assignment
  return failure
```

1. Stuart Russell and Peter Norvig. Artificial Intelligence: A Modern Approach, 4th edition. Pearson, 2022.
2. Code for the book "Artificial Intelligence: A Modern Approach" https://github.com/aimacode/aima-python (accessed April 15, 2024).
3. Wei-Meng Lee, "Programming Sudoku" www.apress.com/9781590596623 (accessed April 15, 2024).
4. Graphical User Interfaces, http://newcoder.io/gui/ (accessed April 15, 2024).



The project will be evaluated based on the following criteria:

1. Both algorithms should be correctly implemented and fully functional.
2. The Sudoku Board should offer the option to select either the Arc-3 algorithm or backtracking to solve the puzzle.
3. The Sudoku Board should allow users to select a difficulty level—easy, medium, or hard—and choose from one of four available puzzles, provide in dataset.
4. The time taken to solve the puzzle, also known as time complexity, should be displayed.
5. Quality of Code.

## Dataset:

### Easy Sudoku - 1

Puzzle #1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 7 | | 3 | 5 | | 8 | | |
| | 3 | 8 | 7 | | 4 | | 6 | 9 |
| 6 | 4 | 5 | | | | 7 | | 3 |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

Puzzle #2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 7 | | | 8 | | | 3 | |
| 3 | | 9 | 6 | 5 | 7 | | | |
| | | 2 | | | 9 | 6 | 8 | |
| | 6 | | 2 | 3 | 5 | | | |
| | 2 | 3 | | 9 | | | | |
| 9 | | | 4 | 3 | 5 | | | |
| | | 8 | | | 6 | 3 | 9 | |
| | | | | | | | | |
| | | | | | | | | |

Puzzle #3

Puzzle #4

### Medium Sudoku - 2

Puzzle #1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 7 | 3 | | | | | 8 | 5 |
| | | | 8 | | | | 7 | |
| 5 | | | | | | | | |
| | | | | 8 | 9 | | 4 | |
| | | 8 | | 6 | 5 | | 3 | 7 |
| 4 | | 7 | | | 2 | 8 | 5 | |
| 3 | 5 | | | 7 | | | 2 | 4 |
| | | | | | | 7 | | |
| 7 | | | 9 | | 3 | | 6 | 8 |

Puzzle #2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 6 | | 8 | | 9 | | |
| 2 | | | | | 3 | 7 | 5 | 6 |
| 9 | | 7 | | | | | | 8 |
| 4 | 8 | | 6 | | | 3 | | |
| | | | 8 | 3 | | | | |
| | 3 | | | | 2 | 6 | 8 | 5 |
| 5 | 6 | 4 | | | 9 | 8 | | |
| | | | | | | | | |
| | | | 4 | | | | | 7 |

Puzzle #3

Puzzle #4

# High Sudoku

Puzzle #1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 9 | 6 | 8 | 2 | 9 | 6 | 4 |
| | 7 | | 3 | 7 | 8 | 4 | | |
| 8 | | | | | 6 | | | 7 |
| | | | | | | | | |
| | | | | | | | | |
| | | 7 | | | 5 | | 8 | 2 |
| | | | 9 | | | | 4 | |
| 4 | | | | | | | | 5 |
| | 7 | 5 | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | 7 | 8 |
| | | | | | | 8 | | |
| | 3 | 7 | | | | | 9 | |
| | | | | | | | 6 | |
| 9 | 8 | | 3 | 2 | 6 | | | |
| | | 3 | | | | | 9 | |
| 3 | | | | 6 | 4 | | 5 | 9 |
| | 7 | 9 | 2 | 5 | | 3 | 4 | |
| 6 | 1 | 4 | 9 | 3 | | | 2 | |

– 3

1

Puzzle #2

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | | | | | 5 | | | |
| | | | 1 | | 4 | 6 | 5 | |
| | | | | 6 | | 3 | | 1 |
| 8 | | 6 | 4 | | | | | 3 |
| | | 4 | 8 | | 4 | 7 | | 9 |
| 9 | | 2 | | 7 | | 4 | 6 | |
| | | 1 | 5 | 8 | 3 | 9 | 7 | 2 |
| 5 | | | | | | 9 | 5 | 1 |
| | | 8 | | 4 | | 5 | | 4 |
| 9 | | | | | 7 | | | |
| | | | 4 | | 6 | 2 | | |
| | | | 8 | | | 6 | 4 | 9 |

Puzzle #3

Puzzle #4

Dataset

1

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 9 | | | | 3 | | |
| | | 1 | | | 4 | 9 | 6 | |
| | 6 | | | 8 | | | | 4 |
| 8 | 7 | 9 | 4 | 5 | 2 | 8 | 9 | |
| 2 | | | 8 | | | 7 | 8 | 9 |
| 5 | | 8 | | 6 | | 2 | 3 | |
| 6 | | 2 | | 4 | | | | 6 |
| | 5 | 4 | 5 | 9 | 6 | | | |
| | 1 | | | 2 | | | | |
| | 6 | | 7 | 2 | | 4 | | |
| | | 3 | 6 | | | | | 7 |
| | 2 | | 4 | | | 6 | | |