5.

$$T(n) = \begin{cases} O(n^{\log_b a}) & \text{if } b^k < a \\ O(n^k \log n) & \text{if } b^k = a \\ O(n^k) & \text{if } b^k > a \end{cases}$$

1. $T(n) = T(n/2) + n - 1, \quad T(1) = 0$

$a = 1, \; b = 2, \; c = 0, \; k = 1$

$\therefore \; 2^1 > a$

$2^1 > 1$

Worst case time complexity, $O(n^k)$

$= O(n^1)$

3. $T(n) = T(n/3) + 2T(n/3) + n$

$a = 3, \; b = 3, \; c = 1, \; k = 1$

$3^1 = 3$

Worst case time complexity $O(n^k \log n)$

$= O(n^1 \log n)$

$= O(n \log n)$

4. $T(n) = 2T(n/2) + n^2$

$a=2,\ b=2,\ c=1,\ k=2$

$2^2 > 2$

The worst case time complexity, $O(n^k)$

$O(n^2)$ [shown]

2. $T(n) = T(n-1) + n - 1,\ T(1) = 0$ ——— ①

$T(n-1) = T(n-2) + (n-1) - 1$

$= T(n-2) + n - 2$ ——— ②

$T(n-2) = T(n-3) + (n-2) - 1$

$= T(n-3) + n - 3$ ——— ③

$T(n) = T(n-2) + n - 2 + n - 1$

$= T(n-2) + 2n - 3$

$T(n) = T(n-3) + n - 3 + 2n - 3$

$= T(n-3) + 3n - 6$ ——— ④

Now,

$T(n-k) + kn - C$

$n - k = 0,\quad n = k$

$T(n-n) + n(n*n) - C$

$T(0) + n^2 - C$ [$T(0) = 1$]

$\therefore O(n^2)$