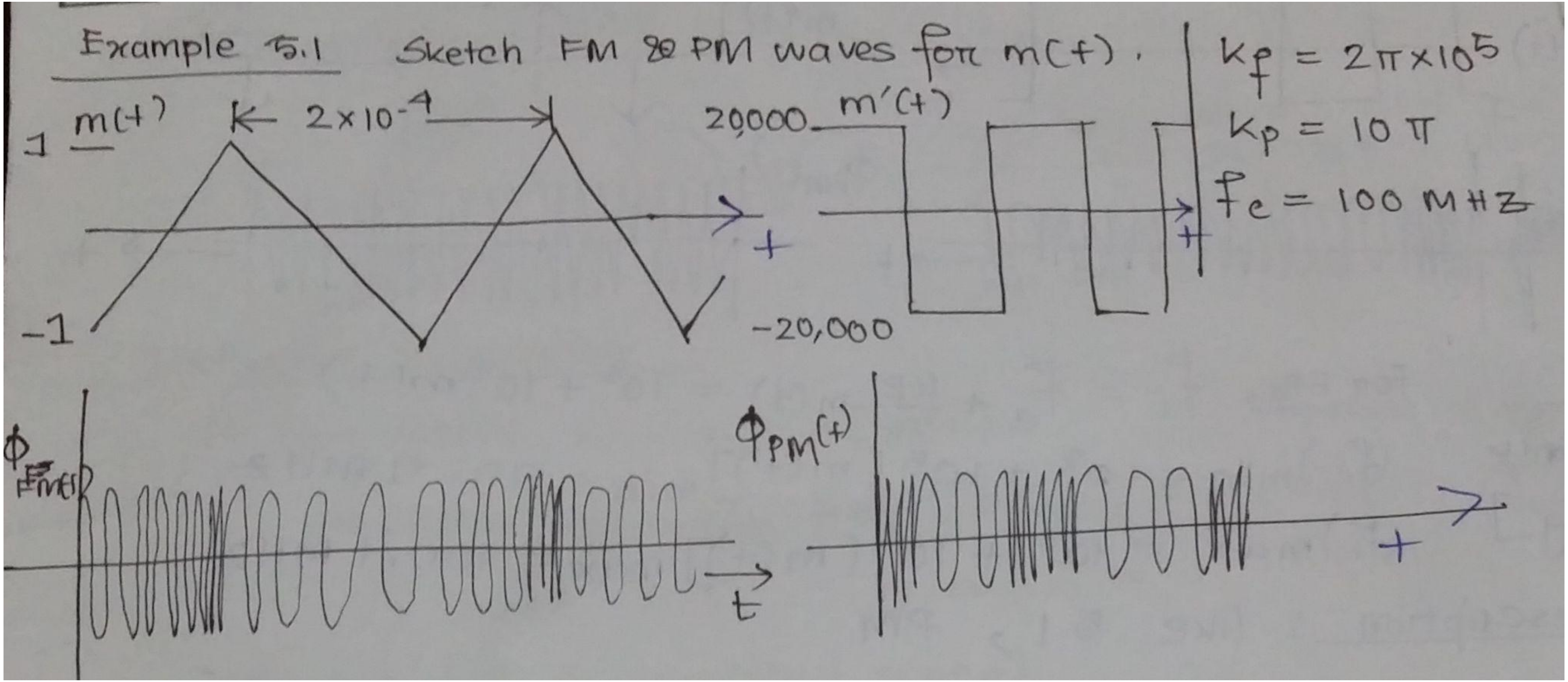
Lecture 10 & 11

Topics

- Graphical Representation of PM and FM
- Bandwidth Calculation of FM
- Types of FM
- NBFM
- WBFM

Graphical Representation of PM and FM



Graphical Representation of PM and FM

* FM $\qquad \omega_i = \omega_c + k_f m(t)$

$$\Rightarrow 2\pi f_i = 2\pi f_c + k_f m(t)$$

$$\Rightarrow f_i = f_c + \frac{k_f m(t)}{2\pi} = 10^8 + 10^5 m(t)$$

$$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min} = 99.9 \text{ MHz}$$

$$(f_i)_{max} = 10^8 + 10^5 [m(t)]_{max} = 100.1 \text{ MHz}$$

* f_i increases linearly from 99.9 to 100.1 MHz over a half cycle and then decreases to 99.9 MHz over the remaining half cycle.

Graphical Representation of PM and FM

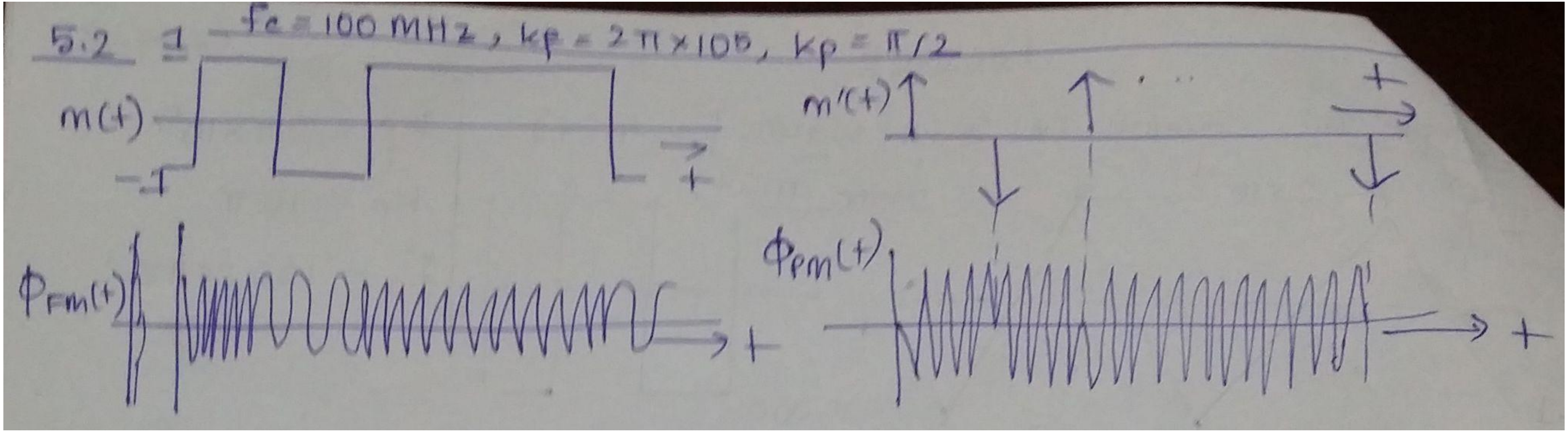
* PM $\quad f_i = F_c + \frac{k_p}{2\pi} m'(t) = 10^8 + 5\, m'(t)$

$$(f_i)_{min} = 10^8 + 5\,[m'(t)]_{min} = 10^8 - 10^5 = 99.9\ \text{MHz}$$

$$(f_i)_{max} = 10^8 + 5\,[m'(t)]_{max} = 100.1\ \text{MHz}$$

* the carrier frequency remains 99.9 MHz for a half cycle, and then stays at 100.1 MHz for the remaining half, switching back and forth.

Graphical Representation of PM and FM



Graphical Representation of PM and FM

[FSK - frequency shift keying]

For FM, $f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

$(f_i)_{min} = 10^8 + 10^5 [m(t)]_{min} = 99.9$ MHz

$(f_i)_{max} = 10^8 + 10^5 [m(t)]_{max} = 100.1$ MHz

Graphical Representation of PM and FM

Description : like চ.1 , PM

PM : $f_i = f_c + \frac{k_p}{2\pi} m'(t) = 10^8 + \frac{1}{4} m'(t)$

How can we change the frequency by an infinite amount and then come back to the original frequency in zero time????

Another approach, $\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$

$$= A\cos\left[\omega_c t + \frac{\pi}{2} m(t)\right]$$

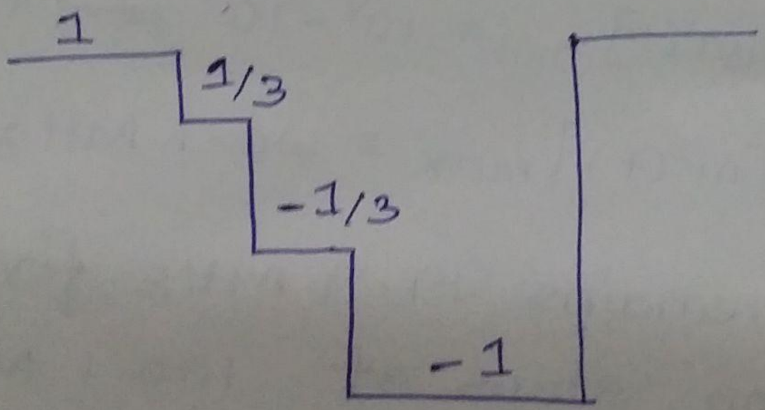
$$= \begin{cases} A\sin\omega_c t & \text{for } m(t) = -1 \\ -A\sin\omega_c t & \text{when } m(t) = 1 \end{cases}$$

[এটাকে PSK বলে, Phase shift keying]

Graphical Representation of PM and FM

* The amount of phaseshift is controlled by k_p. but k_p cannot be arbitrary. The val of k_p should be such that

phase diff $\longrightarrow [-\pi, +\pi]$



1 and $-1/3$

Graphical Representation of PM and FM

Let $k_p = \frac{3\pi}{2}$, $A\cos(\omega_c t + \frac{3\pi}{2}\cdot 1)$

$= \cancel{A\cos}\ A\sin\omega_c t$

Then for $-1/3$,

$A\cos(\omega_c t + \frac{3\pi}{2}(-\frac{1}{3}))$

$= A\sin\omega_c t$

The same signal is transmitted in case of 1 and -⅓ !!! -> can't be distinguished

Graphical Representation of PM and FM

Let $k_p = \frac{3\pi}{2}$, $A\cos(\omega_c t + \frac{3\pi}{2}\cdot 1)$

$= \cancel{A\cos}\ A\sin\omega_c t$

Then for $-1/3$,

$A\cos\left(\omega_c t + \frac{3\pi}{2}\left(-\frac{1}{3}\right)\right)$

$= A\sin\omega_c t$

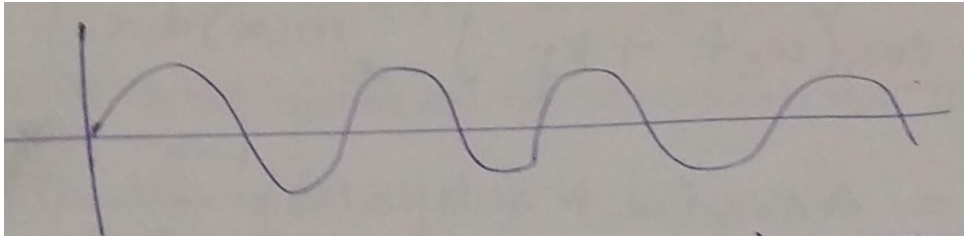
The same signal is transmitted in case of 1 and -⅓ !!! -> can't be distinguished

Graphical Representation of PM and FM

cosθ = cos(θ+2nπ), where n is an integer

maximum phase diff $\longrightarrow [k_p \times (m_{p+} - m_{p-})]$

Graphical Representation of PM and FM



In case of such signals, phase shift occurs every moment. So, there is no constraint on the value of k_p

Bandwidth Calculation of FM

Bandwidth Calculation

For freq mod, $\omega_i = \omega_c + k_f m(t)$

* highest possible freq, $\omega_{max} = \omega_c + k_f m_p$

lowest " " , $\omega_{min} = \omega_c - k_f m_p$

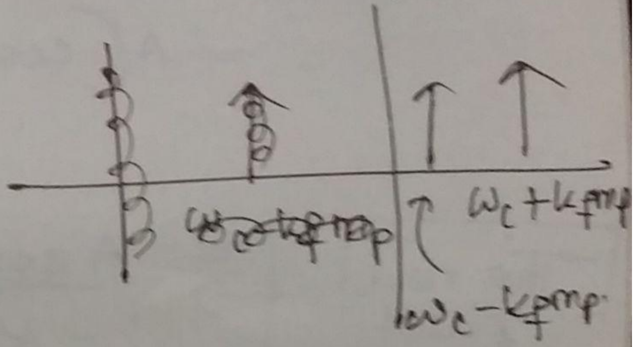
m_p = ~~am max~~ amplitude of $m(t)$

So, B/ω of transmitted signal

∴ $\omega_{max} - \omega_{min} = 2 k_f m_p$

if $k_f \to 0$
then B/ω $\to 0$

No noise, No B/W !!

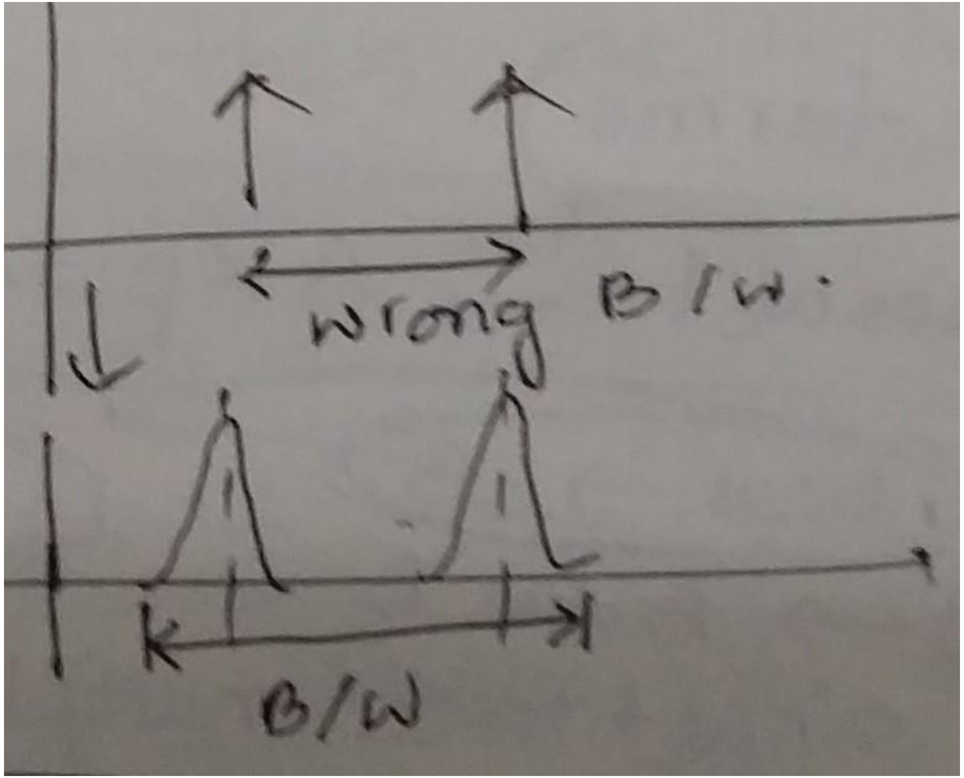


Bandwidth Calculation of FM

However, this assumption is not true!!!

- Bandwidth = highest frequency - lowest frequency, this concept only works in cases where a sinusoid continues for an **infinite amount of time**
- If the sinusoid continues for a **finite amount of time**, its spectrum will not be like that of an infinite sinusoid -> a **spreading effect** will be added

Bandwidth Calculation of FM



- This additional part will contribute in b/w calculation!

Types of FM

- Narrow Band FM (NBFM)
    - The interval 2kfmp is small
- Wide Band FM (WBFM)
    - The interval 2kfmp is large

NBFM

Narrow Band FM :

* instantaneous freq : $\omega_i = \omega_c + k_f\, m(t)$

The FM wave, $A \cos\left(\omega_c t + k_f \underbrace{\int_{-\infty}^{t} m(\alpha)\, d\alpha}_{\to\, a(t)}\right)$

$$FM = A \cos(\omega_c t + k_f a(t)) \text{ ——— (i)}$$

Now, (i) is the real component of the complex quantity, $\hat{Q}_{FM} = A e^{j(\omega_c t + k_f a(t))}$

$$= A \cos(\omega_c t + k_f a(t)) + jA \sin(\omega_c t + k_f a(t))$$

NBFM

$$\hat{Q}_{FM} = A e^{j(\omega_c t + k_f a(t))} = A e^{j\omega_c t} \cdot e^{j k_f a(t)}$$

$$= A(\cos\omega_c t + j\sin\omega_c t)\left[1 + j k_f a(t) + \frac{j^2 k_f^2 a^2(t)}{2!} + \frac{j^3 k_f^3 a^3(t)}{3!} + \cdots\right]$$

$\boxed{\text{The real part}} \rightarrow$

$$= \underbrace{A\cos\omega_c t}_{\text{carrier in DSB/SSB/VSB}} - A k_f a(t)\sin\omega_c t - \frac{k_f^2 a^2(t)}{2!} A\cos\omega_c t$$

* If $m(t) \xrightarrow{B/W} B$

then $\int m(\alpha)\,d\alpha \xrightarrow{B/W} B$

NBFM

If we take convolution, then $B/\omega = 2B$

$$a^2(t) = a(t) * a(t)$$

Similarly, for $a^3(t) \longrightarrow 3B$

If we consider all the terms of the infinite series $\longrightarrow$ B/ω will be infinite

When $k_f a(t) << 1$ then $\longrightarrow$ NBFM (neglecting higher order terms)

NBFM

For NBFM, $A\cos\omega_c t - Ak_f\, a(t)\sin\omega_c t$ [B/W $\rightarrow$ 2B]

this looks a lot like DSB-WC

Intuition: $m(t) \rightarrow$ B/W = B

So, $a(t) \rightarrow$ B/W = B

Then, if we multiply with $\sin\omega_c t$, double sideband is formed $\longrightarrow$ B/W = 2B

$$A\cos\omega_c t + Ak_f\, a(t)\cos\left(\omega_c t + \frac{\pi}{2}\right)$$

This can be constructed using a DSB modulator

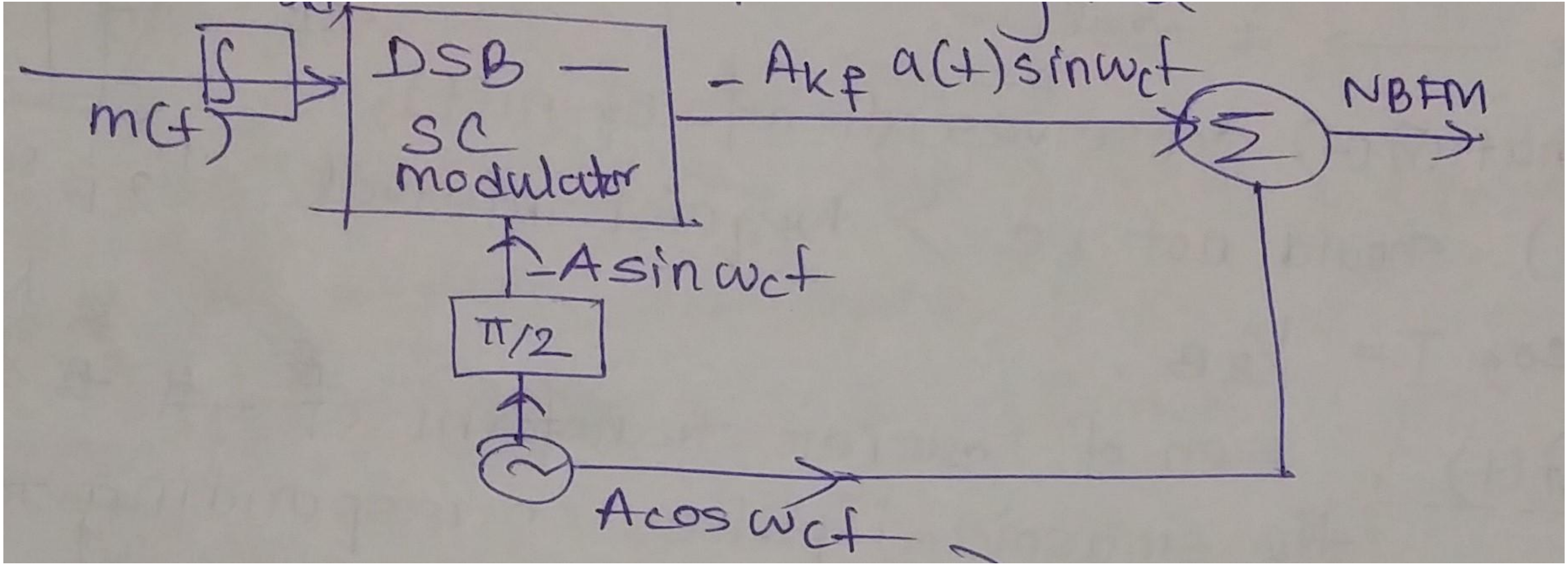
So, if $k_f\, a(t) >> 1 \longrightarrow$ Wide band

NBFM and WBFM

$$\varphi_{NBPM}(t) \simeq A\cos\omega_c t - Ak_p m(t)\sin\omega_c t$$

$$\varphi_{NBFM}(t) \simeq A\cos\omega_c t - Ak_f a(t)\sin\omega_c t.$$

NBFM: Block Diagram



- WBFM: From textbook

NBFM: Demodulation

Demodulation for NBFM

$$\phi_{NBFM}(t) = A\cos\omega_c t - A k_f m(t)\sin\omega_c t$$

* DSB $\longrightarrow$ Multiply by cos$\omega_c t$

Here, freq modulated wave, $A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)$

$\downarrow$ Differentiate

$$-A\sin\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)\left[\omega_c + k_f m(t)\right]$$

If we now use envelope detection, then we'll get $\longrightarrow$

$$A + \omega_c + k_f m(t))$$

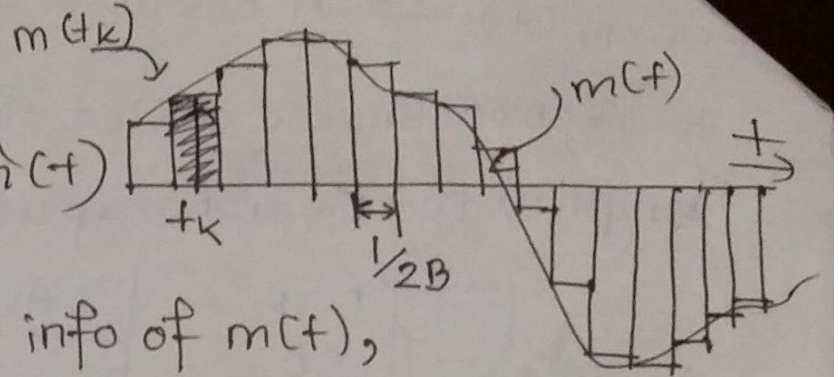
WBFM

WBFM : [the previous approach will be too complicated]

$m(t) \longrightarrow$ band-limited to B Hz

it can be approximated by staircase signal $\hat{m}(t)$

* each pulse $\longrightarrow$ a cell



To ensure that $\hat{m}(t)$ conserves all info of $m(t)$,

cell width of $\hat{m}(t)$ should not be > Nyquist interval $1/2B$ sec.

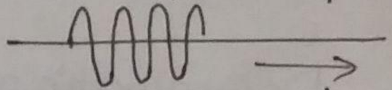
so, $T = 1/2B$

WBFM

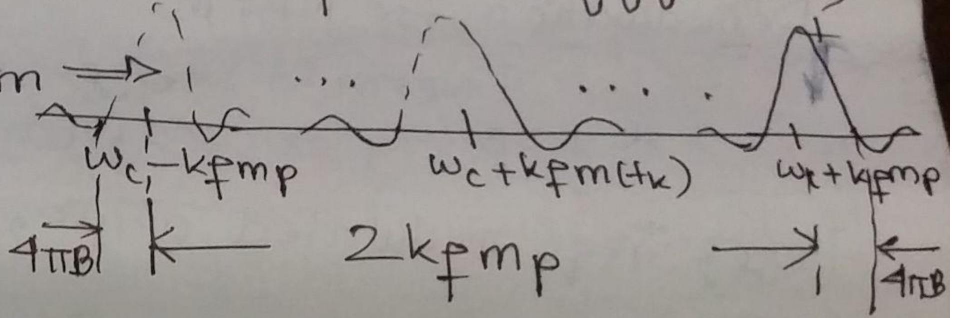
* FM for $\hat{m}(t)$: sum of Fourier transform of ~~all~~ ~~B~~ ~~of~~ the sinusoidal pulses corresponding to all the cells.

$\omega_i = \omega_c + k_f m(t_k)$

* Fourier transform of shaded pulse:



* Now, the spectrum $\Longrightarrow$ of this pulse is spread out on either side of freq $\omega_c + k_f m(t_k)$ by ~~that~~ $2\pi/T = 4\pi B$



WBFM

min value of $m(t) = -m_p$ & $\max = m_p$,

then minimum freq $= \omega_c - k_f m_p$

max " $= \omega_c + k_f m_p$

So, max significant freq − min significant freq

$$= (\cancel{\omega_c} + \cancel{k_f m_p} + 4\pi B) - (\omega_c \cancel{+} - k_f m_p - 4\pi B)$$

$$= 2k_f m_p + 8\pi B$$

WBFM

Let, $\Delta\omega = k_f m_p$

So, $\Delta f = k_f m_p / 2\pi$

$$B_{FM} = \frac{1}{2\pi}(2k_f m_p + 8\pi B)$$

$$= 2(\Delta f + 2B)$$

WBFM

So, $B_{FM} = 2(\Delta f + 2B)$

But actual b/w will be a bit smaller since this approximation is for $\hat{m}(t) \longrightarrow m(t)$ is smoother

Bandwidth of FM

π NB case : $k_f \longrightarrow$ very small, so Δf is very small.

Then we can ignore Δf term

$$B_{FM} = 2\Delta f + 4B$$

ignore

$$B_{FM} \approx 4B$$

But we know, for NBFM, $B/W = 2B$,

A better estimate β is,

$$B_{FM} = 2(\Delta f + B)$$

$$= 2\left(\frac{k_f m_p}{2\pi} + B\right)$$

Carson's rule

Bandwidth of FM

Note that, for a very wide band scenario,

$$\Delta f \gg B$$

So, $B_{FM} \approx 2\Delta f$

But this won't work for NBFM, since for NBFM, $\Delta f \ll B$, so, for NBFM, $B_{FM} \neq 2k_f m_p$