Name - Iftekhar Hakim Kaowsar
ID - 1705045
Course - CSE 311 (Assignment 1)

$a = 14$

2.7-4

For each of the periodic ~~table~~ signals shown in Figure-2.7-4, find the exponential Fourier Series and sketch the amplitude and phase spectra. Note any symmetric property.

a)

$$g(t) = \begin{cases} 1 ; & -1 < t < 1 \\ -1 ; & -3 < t < -1 \\ g(t+4) ; & \text{otherwise} \end{cases}$$

$T_0 = 4$, so, $\omega_0 = \frac{\pi}{2}$.

Now,

$$D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn2\pi f_0 t}$$

$$= \frac{1}{4}\left[- \int_{-3}^{-1} e^{-jn\frac{\pi}{2}t} dt + \int_{-1}^{1} e^{-jn\frac{\pi}{2}t} dt \right]$$

$$= \frac{1}{4} \times \frac{2}{jn\pi} \times \left[e^{-jn\frac{\pi}{2}t} \right]_{-3}^{-1} - \frac{1}{4} \times \frac{2}{jn\pi} \times \left[e^{-jn\frac{\pi}{2}t} \right]_{-1}^{1}$$

$$= \frac{1}{4} \times \frac{2}{jn\pi} \left[e^{jn\frac{\pi}{2}} - e^{jn\frac{3\pi}{2}} - e^{-jn\frac{\pi}{2}} + e^{jn\frac{\pi}{2}} \right]$$

$$= \frac{1}{2jn\pi} \left[\cos\left(\frac{n\pi}{2}\right) + j\sin\left(\frac{n\pi}{2}\right) - \cos\left(\frac{3n\pi}{2}\right) - j\sin\left(\frac{3n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right) + j\sin\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{2}\right) + j\sin\left(\frac{n\pi}{2}\right) \right]$$

$$= \frac{1}{2jn\pi} \times \left(4j \sin\frac{n\pi}{2}\right)$$

$$= \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

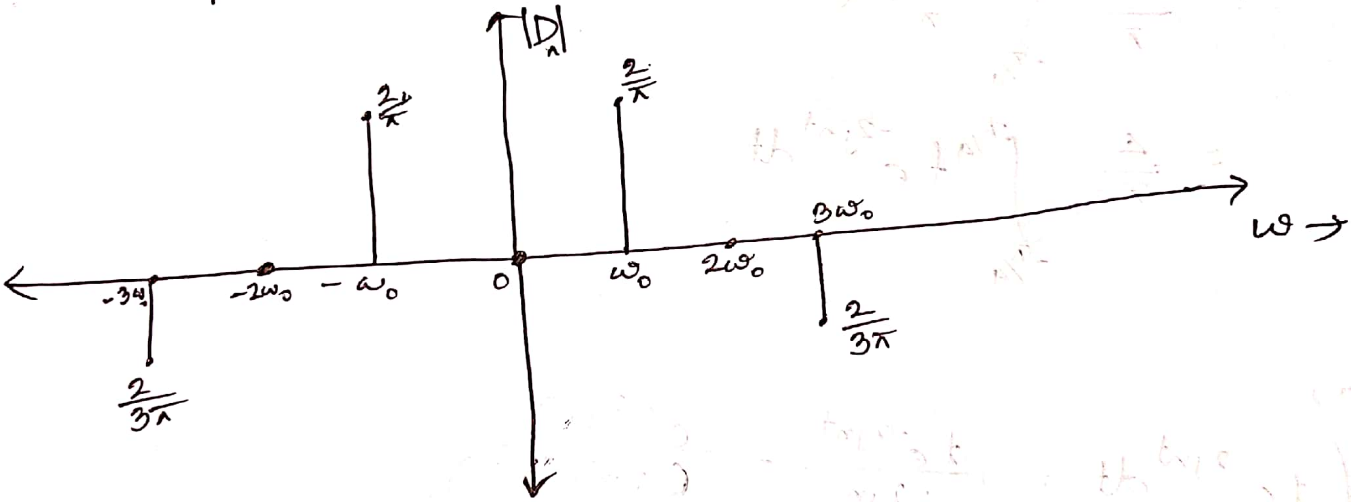
Again,

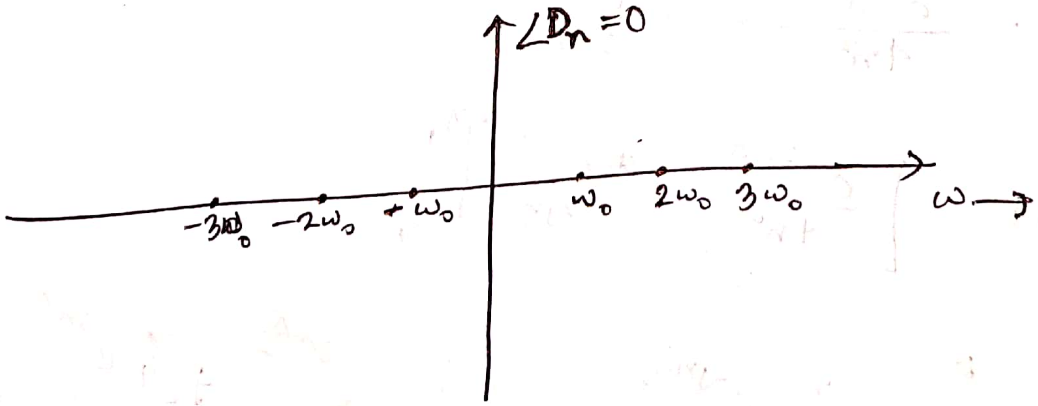
$$D_0 = \frac{1}{4} \times \int_{-3}^{-1} (-1)\,dt + \frac{1}{4}\int_{-1}^{1} dt = 0$$

$$\therefore g(t) = \sum_{-\infty}^{\infty} D_n e^{jn\frac{\pi}{2}t}$$

Here, $g(t)$ is even function. That's why, we got $\angle D_n = 0^\circ$

Moreover, $|D_n| = |D_{-n}|$. It has even symmetry.





(b)
$$g(t) = \begin{cases} 1 & ; -\pi < t < \pi \\ 0 & ; \pi < t < 9\pi \\ g(t+10\pi), & \text{otherwise} \end{cases}$$

$T_0 = 10\pi,\ \omega_0 = \frac{1}{5}$

$$\therefore D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn\frac{2\pi}{10\pi}t}\, dt$$

$$= \frac{1}{10\pi} \int_{-\pi}^{\pi} e^{-\frac{jnt}{5}}\, dt$$

$$= \frac{1}{10\pi} \times \frac{-5}{jn} \times \left[e^{-\frac{jnt}{5}} \right]_{-\pi}^{\pi}$$

$$= \frac{-1}{2jn\pi} \times \left(e^{-\frac{jn\pi}{5}} - e^{\frac{jn\pi}{5}} \right)$$

$$= \frac{1}{2jn\pi} \times \left(e^{\frac{jn\pi}{5}} - e^{-\frac{jn\pi}{5}} \right)$$

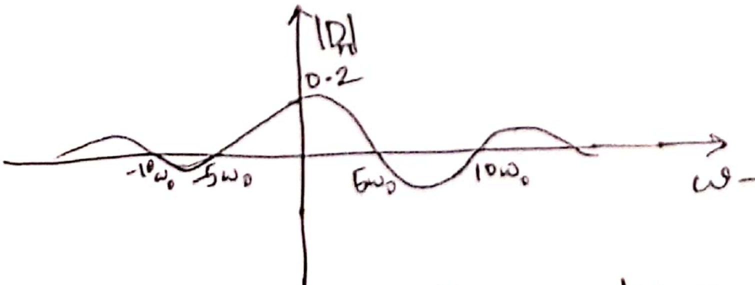
$$= \frac{1}{n\pi} \sin\left(\frac{n\pi}{5}\right)$$

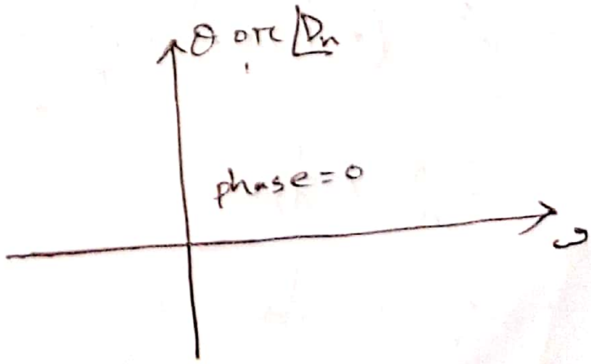
Again, $D_0 = \frac{1}{10\pi} \int_{-\pi}^{\pi} dt = \frac{1}{10\pi} \times 2\pi = \frac{1}{5}$

$$\therefore g(t) = \frac{1}{5} + 2 \sum_{n=1}^{\infty} \left(\frac{1}{n\pi} \sin\left(\frac{n\pi}{5}\right) \times e^{\frac{jnt}{5}} \right)$$

Amplitude $= |D_n|$, phase $= 0$

Amplitude





$g(t)$ is even, hence, phase $= 0$. As always, $|D_n| = |D_{-n}|$.
It has even symmetry.

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Pg-4

(c)

$$g(t) = \begin{cases} \frac{t}{2\pi} & ; 0 < t < 2\pi \\ g(t+2\pi) & ; \text{otherwise} \end{cases}$$

So, $T_0 = 2\pi$, $\omega_0 = \frac{2\pi}{2\pi} = 1$

$$\therefore D_n = \frac{1}{T_0} \int_{T_0} g(t)\, e^{-jn\frac{2\pi}{T_0}t}\, dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{t}{2\pi} e^{-jnt}\, dt$$

$$= \frac{1}{4\pi^2} \int_0^{2\pi} t e^{-jnt}\, dt$$

Now, $\int t e^{-jnt}\, dt = t \times \frac{e^{-jnt}}{-jn} - \int \frac{e^{-jnt}}{-jn}\, dt$

$$= \frac{t e^{-jnt}}{-jn} - \frac{1}{(-jn)(-jn)} e^{-jnt}$$

$$\therefore \cancel{g(t)}\, D_n = \left[\frac{t e^{-jnt}}{-jn} + \frac{e^{-jnt}}{n^2}\right]_0^{2\pi} \times \frac{1}{4\pi^2}$$

$$= \left[\frac{2\pi e^{-2\pi jn}}{-jn} + \frac{e^{-2\pi jn}}{n^2} - \frac{1}{n^2}\right] \times \frac{1}{4\pi^2}$$

$$= \left[\frac{2\pi}{-jn}\left(\cos(2n\pi) - j\sin(2n\pi)\right) + \frac{1}{n^2}\left(\cos 2n\pi - j\sin 2n\pi\right) - \frac{1}{n^2}\right] \times \frac{1}{4\pi^2}$$

$$= \left[\frac{2\pi}{-jn}(1) + \frac{1}{n^2} - \frac{1}{n^2}\right] \times \frac{1}{4\pi^2}$$

$$= \frac{2\pi}{-jn} \times \frac{1}{4\pi^2}$$

$$= \frac{2\pi}{n} j \times \frac{1}{4\pi^2}$$

$$\therefore D_n = \frac{j}{2n\pi}$$

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Pg-5

$$D_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{t}{2\pi}\,dt$$

$$= \frac{1}{4\pi^2}\int_0^{2\pi} t\,dt$$

$$= \frac{1}{4\pi^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

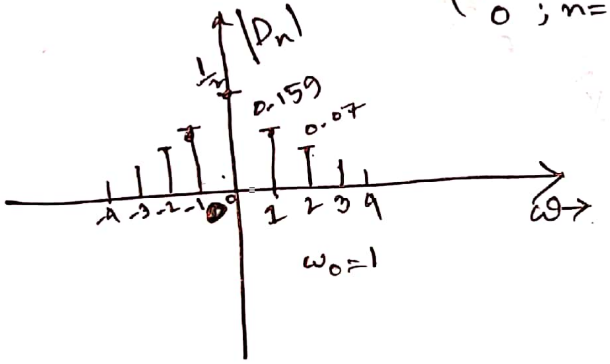
$$= \frac{1}{4\pi^2}\left(\frac{4\pi^2}{2} - 0\right)$$

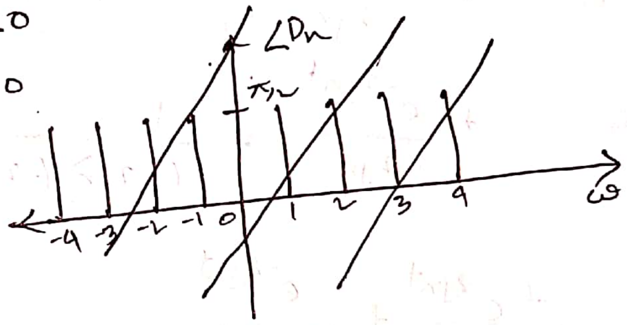
$$= \frac{1}{2}$$

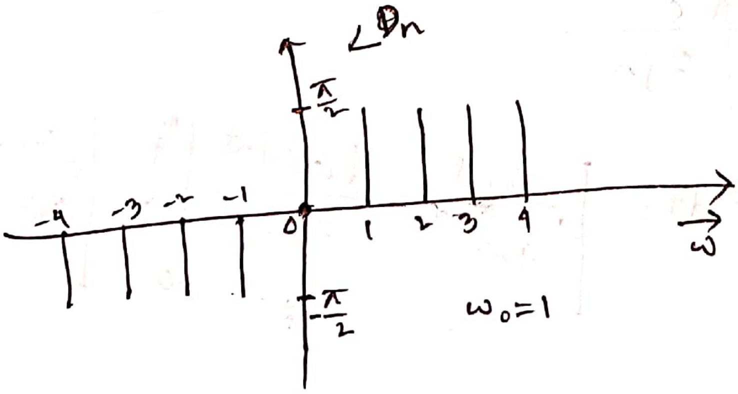
$$\therefore g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jnt}$$

Amplitude, $|D_n| = \frac{1}{2n\pi}$; $|D_0| = \frac{1}{2}$.

Phase, $\theta_n = \angle D_n = \begin{cases} \frac{\pi}{2} ; n>0 \\ -\frac{\pi}{2} ; n<0 \\ 0 ; n=0 \end{cases}$







It has odd symmetry.

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~~(b)~~
(d)

$$g(t) = \begin{cases} \frac{4t}{\pi} ; & -\frac{\pi}{4} < t < \frac{\pi}{4} \\ 0 ; & \frac{\pi}{4} < t < \frac{3\pi}{4} \\ g(t+\pi) ; & \text{otherwise} \end{cases}$$

$T_0 = \pi, \quad \omega_0 = 2$

$$D_n = \frac{1}{T_0} \int_{T_0} g(t)\, e^{-jn2\pi f_0 t} \cdot dt$$

$$= \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-jn2\pi f_0 t} dt$$

$$= \frac{4}{\pi^2} \int_{-\pi/4}^{\pi/4} t e^{-2jnt} dt$$

Now,

$$\int t e^{-2jnt} dt = \frac{t e^{-2jnt}}{-2jn} - \frac{e^{-2jnt}}{(-2jn)(-2jn)}$$

$$= \frac{e^{-2jnt}}{4n^2} - \frac{t e^{-2jnt}}{2jn}$$

$$\therefore \int_{-\pi/4}^{\pi/4} t e^{-2jnt} dt = \left[\frac{e^{-2jnt}}{4n^2} - \frac{t e^{-2jnt}}{2jn} \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{4n^2} e^{-jn\frac{\pi}{2}} - \frac{\frac{\pi}{4} e^{-jn\frac{\pi}{2}}}{2jn} - \left(\frac{1}{4n^2} e^{jn\frac{\pi}{2}} + \frac{\frac{\pi}{4} e^{jn\frac{\pi}{2}}}{2jn} \right)$$

$$= \frac{1}{4n^2} \left(e^{-jn\frac{\pi}{2}} - e^{jn\frac{\pi}{2}} \right) - \frac{\pi}{8jn} \left(e^{-jn\frac{\pi}{2}} + e^{jn\frac{\pi}{2}} \right)$$

$= \frac{1}{4n^2} \times \cos$

$= \frac{-1}{4n^2} \times \sin\left(\frac{n\pi}{2}\right) \times 2j - \frac{\pi}{4jn} \cos\left(\frac{n\pi}{2}\right)$

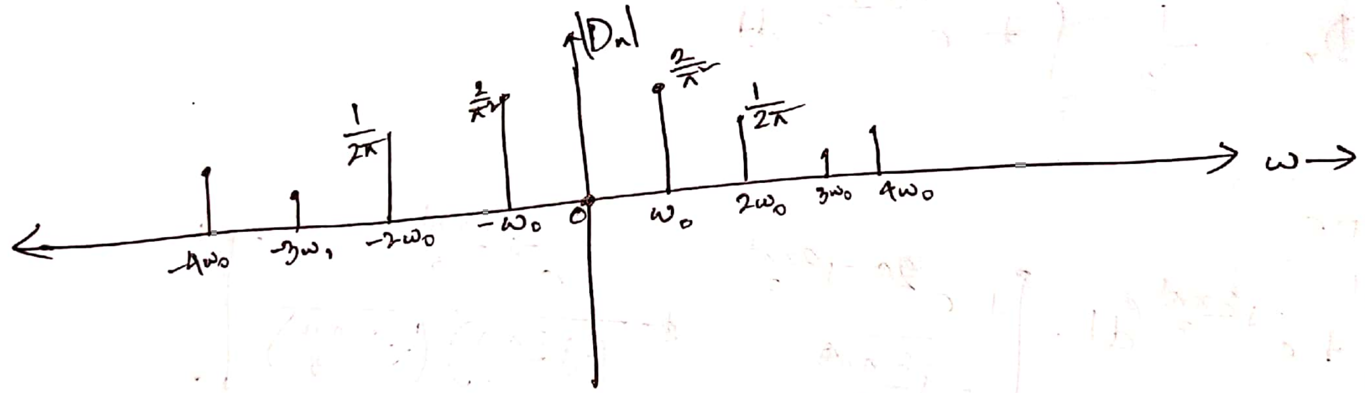
$= \frac{-j}{2n^2} \sin\left(\frac{n\pi}{2}\right) + \frac{\pi j}{4n} \cos\left(\frac{n\pi}{2}\right)$

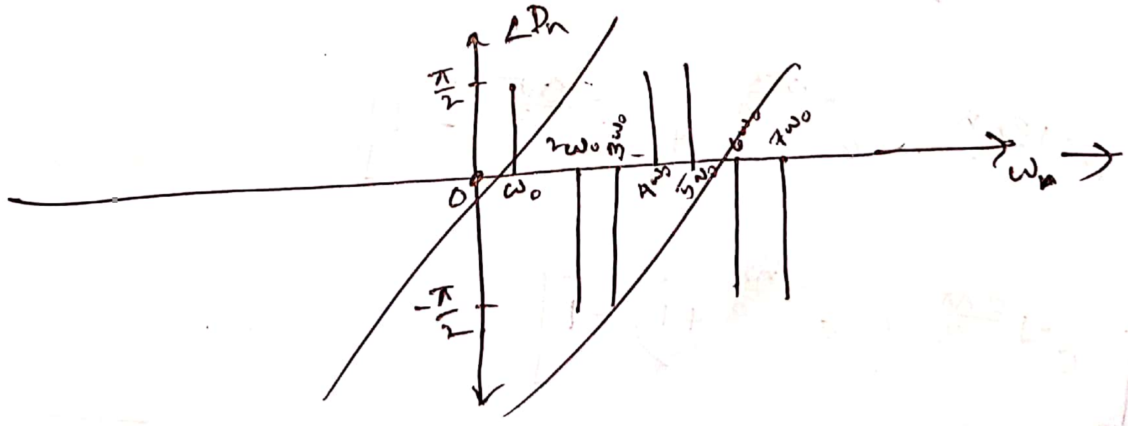
$\therefore D_n = \frac{4}{\pi^2} \times \left[\frac{-j}{2n^2} \sin\left(\frac{n\pi}{2}\right) + \frac{\pi j}{4n} \cos\left(\frac{n\pi}{2}\right)\right]$

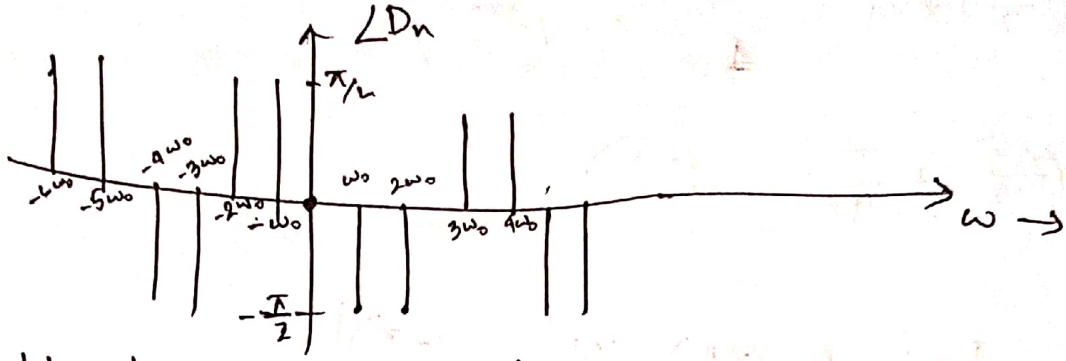
$= \frac{-j}{n\pi} \left(\frac{2}{n\pi} \sin\frac{n\pi}{2} - \cos\frac{n\pi}{2}\right)$

Again,

$D_0 = \frac{1}{\pi} \int_{-\pi/4}^{\pi/4} \frac{4t}{\pi}\, dt = \frac{4}{\pi^2} \left[\frac{t^2}{2}\right]_{-\pi/4}^{\pi/4} = \frac{4}{\pi} \times 0 = 0$







It has odd symmetry.

(e)

$$g(t) = \begin{cases} t ; & 0 < t < 1 \\ 0 ; & 1 < t < 3 \end{cases}$$

$T_0 = 3$, $\omega_0 = \frac{2\pi}{3}$

$$D_n = \frac{1}{3} \int_0^1 t\, e^{-j\frac{2\pi n t}{3}} dt$$

Here,

$$\int_0^1 t\, e^{-j\frac{2\pi n t}{3}} dt = \left[\frac{t\, e^{-j\frac{2\pi n t}{3}}}{-j\frac{2\pi n}{3}} - \frac{e^{-j\frac{2\pi n t}{3}}}{\left(-j\frac{2\pi n}{3}\right)\left(-j\frac{2\pi n}{3}\right)} \right]_0^1$$

$$= \frac{3}{4\pi^2 n^2} \left[e^{-j\frac{2\pi n}{3}} \times e^{-jn\frac{\pi}{3}} + e^{-j\frac{2n\pi}{3}} - 1 \right]$$

$$= \frac{3}{4n^2\pi^2} \left[e^{-j\frac{2n\pi}{3}} \left(e^{-j\frac{n\pi}{3}} + 1 \right) - 1 \right]$$

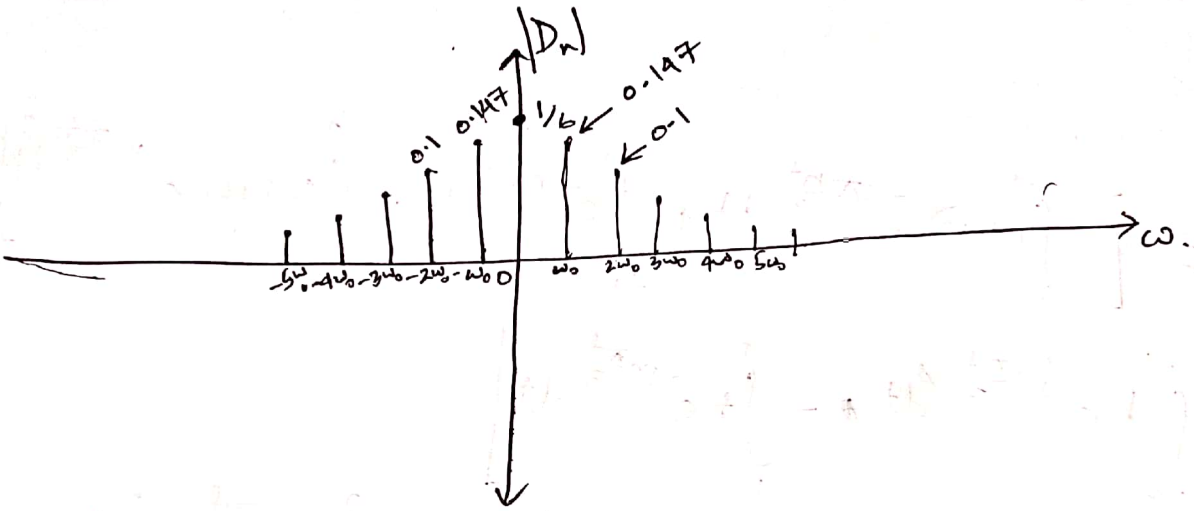
$$\therefore |D_n| = \frac{3}{4n^2\pi^2} \sqrt{2 + \frac{4n^2\pi^2}{9} - 2\cos\frac{2n\pi}{3} - \frac{4n\pi}{3} \sin\frac{2n\pi}{3}}$$

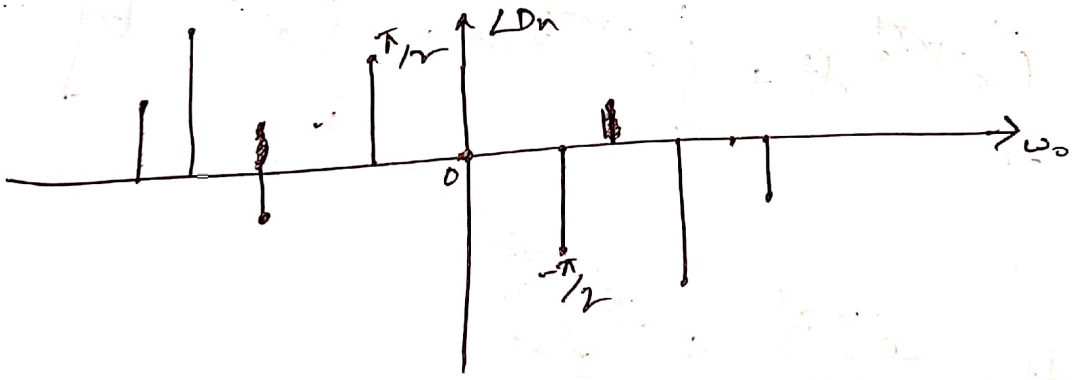
$$\angle D_n = \tan^{-1}\left(\frac{\frac{2\pi n}{3}\cos\frac{2\pi n}{3} - \sin\frac{2\pi n}{3}}{\cos\frac{2\pi n}{3} + \frac{2\pi n}{3}\sin\frac{2\pi n}{3} - 1}\right)$$

$$D_0 = \frac{1}{3}\int_0^1 t\,dt$$

$$= \frac{1}{3}\left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{1}{6}$$





~~From phase sketch, it has~~

(f)

$$g(t) = \begin{cases} \cancel{(t)}\ t+2 & ; -2 < t < -1 \\ 1 & ; -1 < t < 1 \\ (-t+2) & ; 1 < t < 2 \\ 0 & ; 2 < t < 4 \\ g(t+6) & ; \text{otherwise} \end{cases}$$

$T_0 = 6.$

$\omega_0 = \frac{\pi}{3}.$

$$D_n = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-j\pi\frac{nt}{3}}dt + \int_{-1}^{1}e^{-j\pi\frac{nt}{3}}dt + \int_{1}^{2}(-t+2)e^{-jn\frac{\pi t}{3}}dt\right]$$

$$= \frac{1}{6}\left[\int_{-2}^{-1}t e^{-jn\frac{\pi t}{3}}dt - \int_{1}^{2}t e^{-jn\frac{\pi t}{3}}dt\right]$$

$$- \frac{1}{6}\left[2\int_{-2}^{-1}e^{-jn\frac{\pi t}{3}}dt + \int_{-1}^{1}e^{-jn\frac{\pi t}{3}}dt + 2\int_{1}^{2}e^{-jn\frac{\pi t}{3}}dt\right]$$

We know,

$$\int t e^{-\frac{jn\pi t}{3}}dt = \frac{e^{-jn\frac{\pi t}{3}}}{-jn\frac{\pi}{3}} + \frac{e^{-jn\frac{\pi t}{3}}}{\frac{n^2\pi^2}{9}}$$

$$\int e^{-\frac{jn\pi t}{3}}dt = \frac{e^{-\frac{jn\pi t}{3}}}{-jn\frac{\pi t}{3}}$$

From this, we get,

$$D_n = \frac{3}{n^2\pi^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3}\right)$$

D_n has no term with j.

So,

$\angle D_n = 0$

Again,

$$D_0 = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)\,dt + \int_{-1}^{1}dt + \int_{1}^{2}(-t+2)\,dt\right]$$

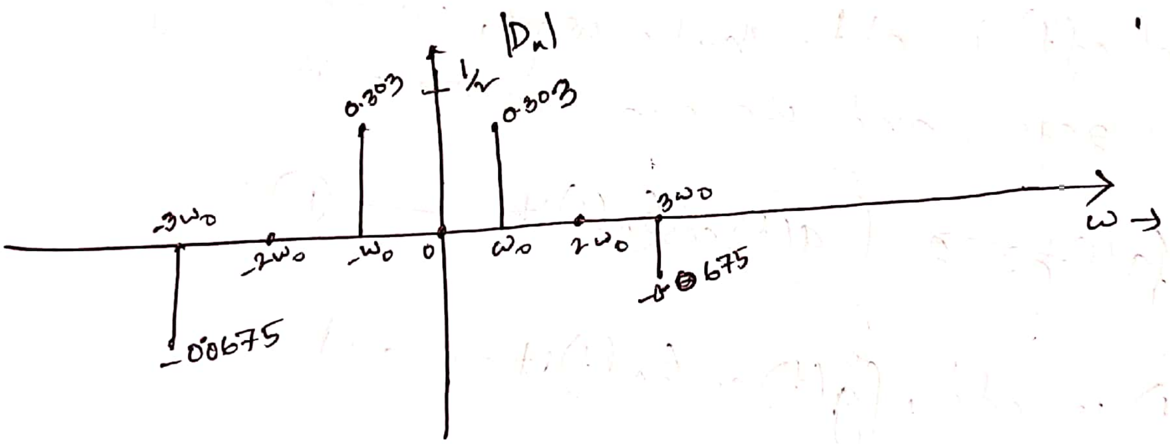
$$= \frac{1}{6}\left[\frac{(t+2)^2}{2}\right]_{-2}^{-1} + \frac{1}{6}\times 2 + \frac{1}{6}\left[\frac{(-t+2)^2}{2}\right]_{1}^{2}$$

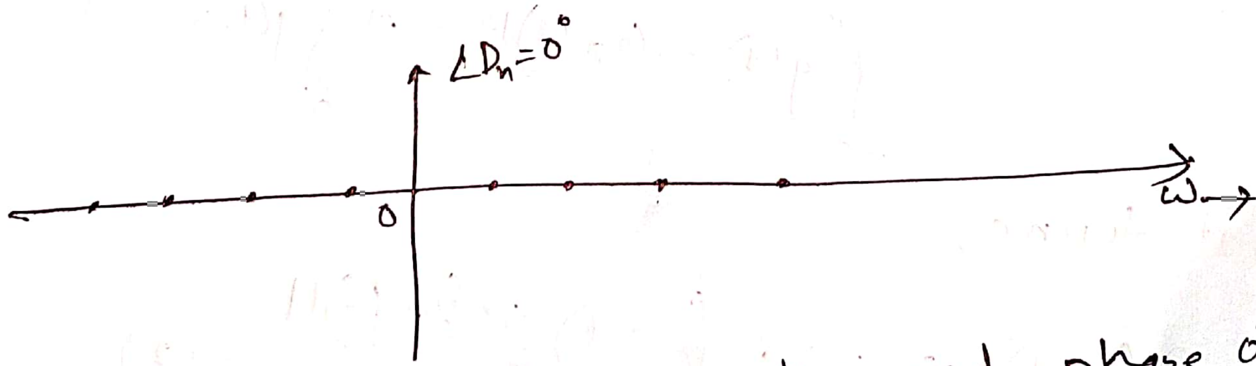
$$= \frac{1}{6}\times\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\times\frac{1}{2}$$

$$= \frac{1}{2} = |D_0|$$

If $n \neq 0$,

$$|D_n| = \frac{3}{n^2\pi^2}\left(\cos\frac{n\pi}{3} - \cos\frac{2n\pi}{3}\right)$$





g(t) Even function. It has even symmetry, and phase 0°.

b= 17

3.1-1

We know,

$$\mathcal{F}\{g(t)\} = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} g(t) (\cos \omega t - j \sin \omega t) dt$$

$$= \int_{-\infty}^{\infty} g(t) \cos(2\pi f t) dt - j \int_{-\infty}^{\infty} g(t) \sin(2\pi f t) dt$$

If $g(t)$ is even, $g(t)\cos(2\pi f t)$ is even too, and $g(t)\sin(2\pi f t)$ is odd. That's why, second integral turns to zero, and we can rewrite it

$$\mathcal{F}\{g(t)\} = 2 \int_{0}^{\infty} g(t) \cos(2\pi f t) dt \quad \text{--- (1)}$$

when $g(t)$ is odd, $\int_{-\infty}^{\infty} g(t) \cos(2\pi f t) dt = 0$ and

$$\int_{-\infty}^{\infty} g(t) \sin(2\pi f t) dt = 2 \int_{0}^{\infty} g(t) \sin(2\pi f t) dt$$

As it turns,

$$\mathcal{F}\{g(t)\} = -2j \int_{0}^{\infty} g(t) \sin(2\pi f t) dt \quad \text{--- (2)}$$

$= $

(1) When $g(t)$ is real and even —

By eqn (1),

$\mathcal{F}\{g(t)\} = G(f)$ is real, as coefficient of j is zero

Again,

$$G(-f) = 2\int_0^\infty g(t)\cos(2\pi(-f)t)\,dt$$
$$= 2\int_0^\infty g(t)\cos(2\pi f t)\,dt$$
$$= G(f)$$

So, it is even too.

(2) When $g(t)$ is real and odd —

by eqn (2),

$\mathcal{F}\{g(t)\} = G(f)$ is imaginary, as there is only one term which a real coefficient of j

Again,

$$G(-f) = -2j\int_0^\infty g(t)\sin(2\pi(-f)t)\,dt$$
$$= 2j\int_0^\infty g(t)\sin(2\pi f t)\,dt$$
$$= -G(f)$$

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③ when $g(t)$ is imaginary and even,
by eqn ①,
$G(f) = \mathcal{F}\{g(t)\}$ is imaginary, because $\int g(t) \cos 2\pi ft\,dt$ is imaginary, here $g(t)$ is imaginary and $\cos 2\pi ft$ is real.

Again,

$$G(-f) = 2\int_0^\infty g(t) \cos(2\pi(-f)t)\,dt$$

$$= 2\int_0^\infty g(t) \cos(2\pi ft)\,dt$$

$$= G(f)$$

So, $G(f)$ is even too.

④ when $g(t)$ is complex and even,
by eqn ①,
$G(f)$ is complex too. Because $g(t)$ is complex, and that's why, $g(t)\cos 2\pi ft$ is complex and that's why, $\int g(t)\, 2\pi ft\,dt$ is complex.

Again,

$$G(-f) = 2\int_0^\infty g(t) \cos(2\pi(-f)t)\,dt$$

$$= 2\int_0^\infty g(t) \cos(2\pi ft)\,dt$$

$$= G(f).$$

Hence it is even too.

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5 When $g(t)$ is complex and odd,
by eqn 2,
As $g(t)$ is ~~of~~ complex, $g(t)\sin 2\pi ft$ is complex.
That's why $\int_0^\infty g(t)\,2\pi ft\,dt$ is complex.

Again,

$$G(-f) = -2j\int_0^\infty g(t)\sin(2\pi(-f)t)\,dt$$
$$= 2j\int_0^\infty g(t)\sin(2\pi ft)\,dt$$
$$= -G(f)$$

~~Ans~~ That's why, $G(f)$ is odd.