

Solution: To construct the truth table for a compound proposition, we work from the inside out. In each case, we will show the intermediate steps. In part (d), for example, we first construct the truth table for $p \vee q$, then the truth table for $p \wedge q$, and finally combine them to get the truth table for $(p \vee q) \rightarrow (p \wedge q)$. For parts (a) and (b) we have the following table (column three for part (a), column four for part (b)).

| P | $\sim P$ | $p \wedge \sim p$ | $p \vee \sim p$ |
|---|----------|-------------------|-----------------|
| T | F | F | T |
| F | T | F | T |

For part (c) we have the following table.

| p | q | $\sim q$ | $p \vee \sim q$ | $(p \vee \sim q) \rightarrow q$ |
|---|---|----------|-----------------|---------------------------------|
| T | T | F | T | T |
| T | F | T | T | F |
| F | T | F | F | T |
| F | F | T | T | F |

For part (d) we have the following table.

| P | q | $p \vee q$ | $p \wedge q$ | $(p \vee q) \rightarrow (p \wedge q)$ |
|---|---|------------|--------------|---------------------------------------|
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | F | F | T |

For part (e) we have the following table. This time we have omitted the column explicitly showing the negations of p and q. Note that this true proposition is telling us that a conditional statement and its contrapositive always have the same truth value.

| p | q | $p \rightarrow q$ | $\sim q \rightarrow \sim p$ | $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ |
|---|---|-------------------|-----------------------------|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

For part (f) we have the following table. The fact that this proposition is not always true tells us that knowing a conditional statement in one direction does not tell us that the conditional statement is true in the other direction.

| p | q | $p \rightarrow q$ | $q \rightarrow p$ | $(p \rightarrow q) \rightarrow (q \rightarrow p)$ |
|-----|-----|-------------------|-------------------|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |