

1. Fill in the probability value rows for the given observation and hidden path sequence, where Fair coin (F) has equal probability for Head (H) or Tail (T) emission and Bias coin (B) has 65% chance of coming up with Head (H). Finally, compute the **10 based log value** of the joint probability of the observation and hidden path. (10)

i	1	2	3	4	5	6	7	8	9	10	11
x	T	T	T	H	H	H	T	H	H	T	T
π	F	F	F	B	B	F	B	B	F	F	F
$\Pr(\pi_i \rightarrow \pi_{i+1})$	$\frac{1}{2}$	0.9	0.9	0.1	0.9	0.1	0.1	0.9	0.1	0.9	0.9
$\Pr(x_i \pi_i)$	0.5	0.5	0.5	0.65	0.65	0.5	0.35	0.65	0.5	0.5	0.5

Joint Probability Calculations:

$$\begin{aligned}
 &= \log 0.5 \times 0.9 \times 0.9 \times 0.1 \times 0.9 \times 0.1 \times 0.1 \times 0.9 \times 0.1 \times 0.9 \times 0.5 \\
 &\quad \times 0.5 \times 0.5 \times 0.5 \times 0.65 \times 0.65 \times 0.5 \times 0.35 \times 0.65 \times 0.5 \times 0.5 \times 0.5 \\
 &= \log [0.5^8 \times 0.9^6 \times 0.1^4 \times 0.65^3 \times 0.35^1] \\
 &= 8 \log 0.5 + 6 \log 0.9 + 4 \log 0.1 + 3 \log 0.65 + \log 0.35 \\
 &= \cancel{5.5918} - 2.4082 - 0.2745 - 4 - 0.5613 - 0.4559 = \boxed{-7.6999}
 \end{aligned}$$

2. The figure below shows an HMM with two states α and β . Given that the HMM emitted the string GC, draw the Viterbi graph. Use log-scores (base 10), rather than straight probability scores. (10)

