CSE-103
DISCRETE MATHEMATICS

ASSIGNMENT NO : ***03***
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DATE OF ASSIGNED *:*
04-11-2018
DATE OF SUBMISSION *:*
06-11-2018

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SECTION *: B*
LEVEL-1 TERM-2

Chapte-1 Section-1
Problem No : 40

Problem Statement :

Explain , without using a truth table , why $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p,q and r have the same truth value and it is false otherwise.

Answer :

Method 1:

#Case 1 :

Propositions having same truth value "true"

So for every pair

$(p \vee \neg q)$, $(q \vee \neg r)$, $(r \vee \neg p)$ has the truth value "true" .

So,

$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) = \text{true}$ [$\text{true} \wedge \text{true} = \text{true}$]

#Case 2 :

Propositions having same truth value "false"

So for every pair

$(p \vee \neg q)$, $(q \vee \neg r)$, $(r \vee \neg p)$ has the truth value "true" .

[$\neg\text{false} = \text{true}$ && $\text{false} \vee \text{true} = \text{true}$]

So,

$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) = \text{true}$ [$\text{true} \wedge \text{true} = \text{true}$]

#Case 3 :

The given proposition holds for only the same value of each proposition as if at least one proposition becoming different truth value than others , leads at least one case of 'logical or' having the truth value false , which leads the whole proposition 'false'.

Let the truth value of ' p ' is true , ' q ' is false and ' r ' also true.
So for $(q \vee \neg r)$ has the truth value 'false', so the proposition becomes false .

Method 2:

The equivalent proposition

$$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \equiv ((p\wedge q\wedge r) \vee (\neg p\wedge \neg q\wedge \neg r))$$

So this proposition will be true **if and only if** the truth value of each propositions are same . Because for first case $(p\wedge q\wedge r)$ will be true if and only if the all are true/false and so on for second case . Otherwise the proposition becomes false.