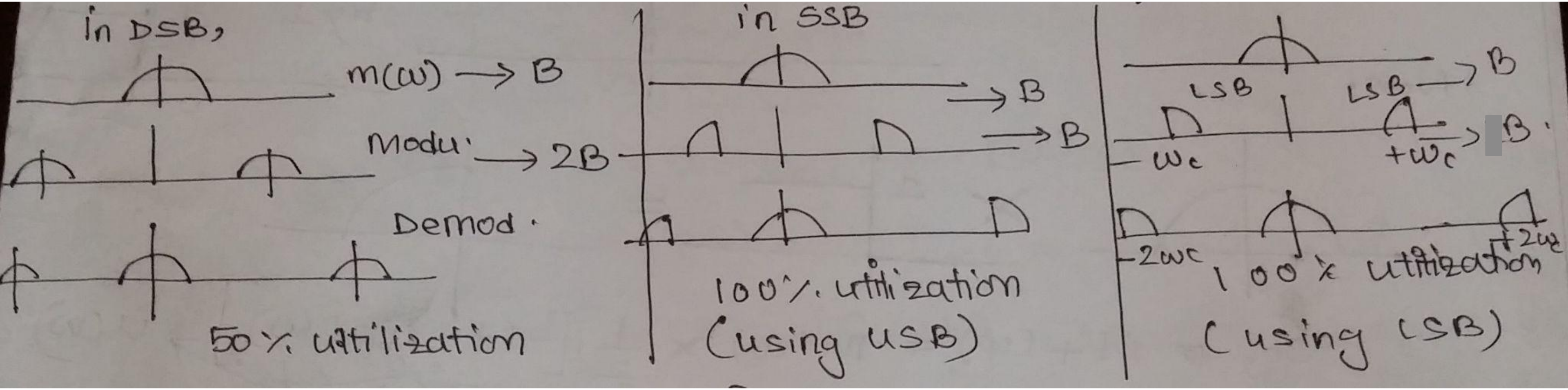
Lectures 6 & 7

Topics

- SSB: Modulation
- SSB: Coherent and Envelope Detection
- Problems of SSB
- Drawing Spectrum of SSB
- Concept of VSB

DSB vs SSB

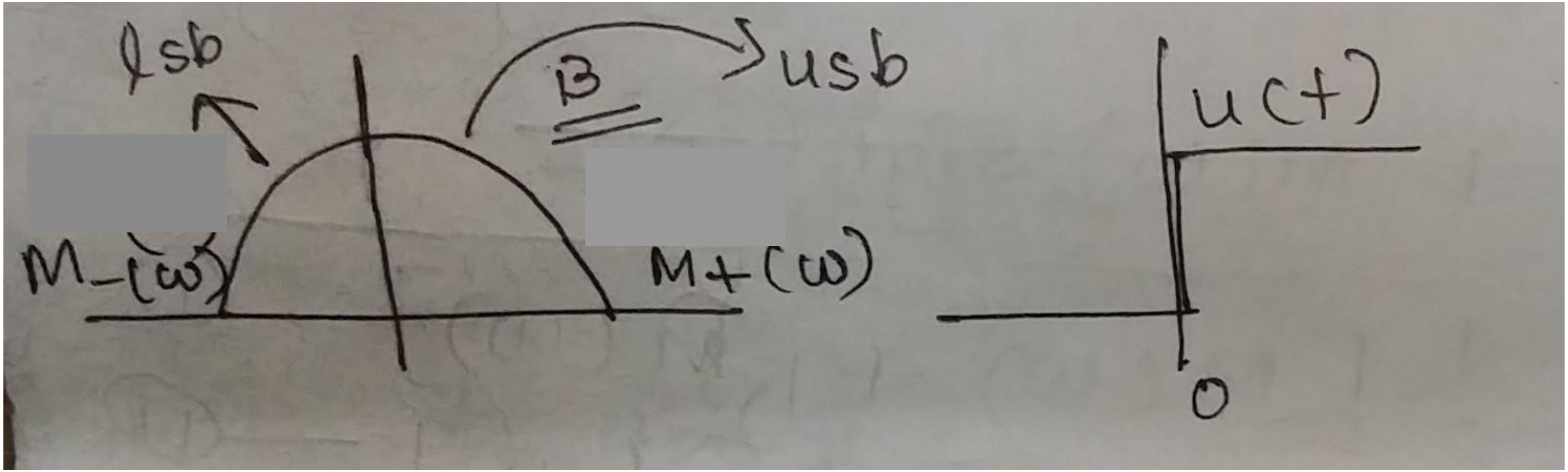
- In **DSB,** bandwidth utilization is **50%**
- In **SSB,** bandwidth utilization is **100%**



SSB

- In **SSB,** bandwidth utilization is **100%**
- **Full signal recovery** is possible from SSB
- **Cutoff** is done using a **filter**

SSB: Modulation



SSB: Modulation

$$M_+(\omega) = M(\omega)\,u(\omega)$$

$$M_-(\omega) = M(\omega)\,u(-\omega)$$

$m_+(t), m_-(t) \longrightarrow$ inverse fourier transform

SSB: Modulation

*** Since amplitude spectra $|M_+(\omega)|$ and $|M_-(\omega)|$ are not even functions of ω, the signals $m_+(t)$ & $m_-(t)$ cannot be real $\longrightarrow$ they are complex.

& $M_+(\omega)$ & $M_-(\omega)$ are two halves of $M(\omega)$

So, $M_+(\omega)$ & $M_-(\omega)$ $\longrightarrow$ conjugates &

$m_+(t)$ & $m_-(t)$ $\longrightarrow$ conjugates.

$M(\omega) = M_+(\omega) + M_-(\omega)$

Also, $m_+(t) + m_-(t) = m(t)$

SSB: Modulation

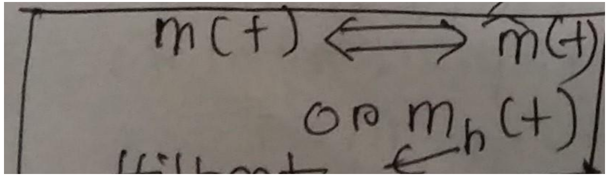
Now, $\Phi_{USB}(\omega) = M_+(\omega-\omega_c) + M_-(\omega+\omega_c)$

Taking Inverse Fourier Transform,

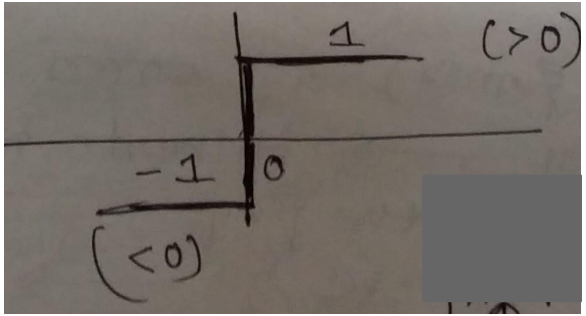
$$\phi_{USB}(t) = m_+(t)\,e^{j\omega_c t} + m_-(t)\,e^{-j\omega_c t}$$

$$\left.\begin{aligned} m_+(t) &= \frac{1}{2}\left[m(t) + j\hat{m}(t)\right] \\ m_-(t) &= \frac{1}{2}\left[m(t) - j\hat{m}(t)\right] \end{aligned}\right\} \text{conjugate form}$$

SSB: Modulation

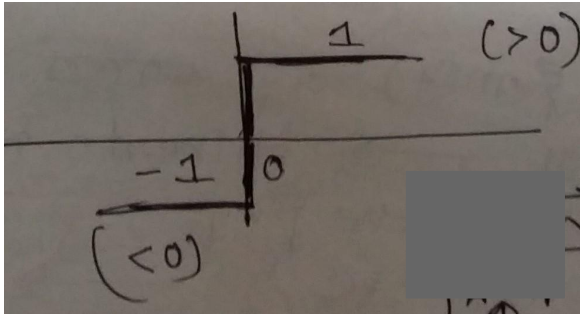


Hilbert Transform



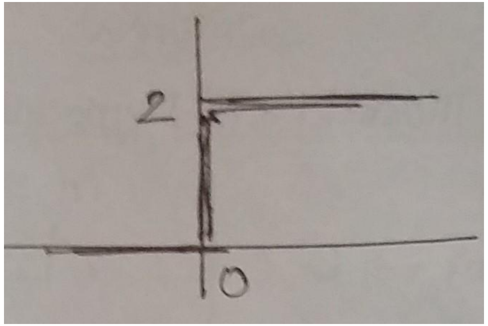
Signum Function

SSB: Modulation

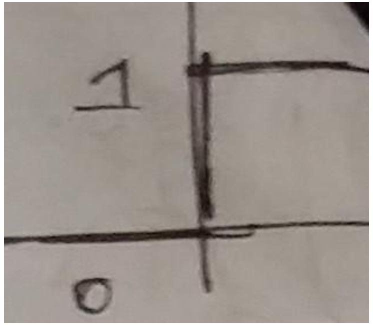


Signum Function

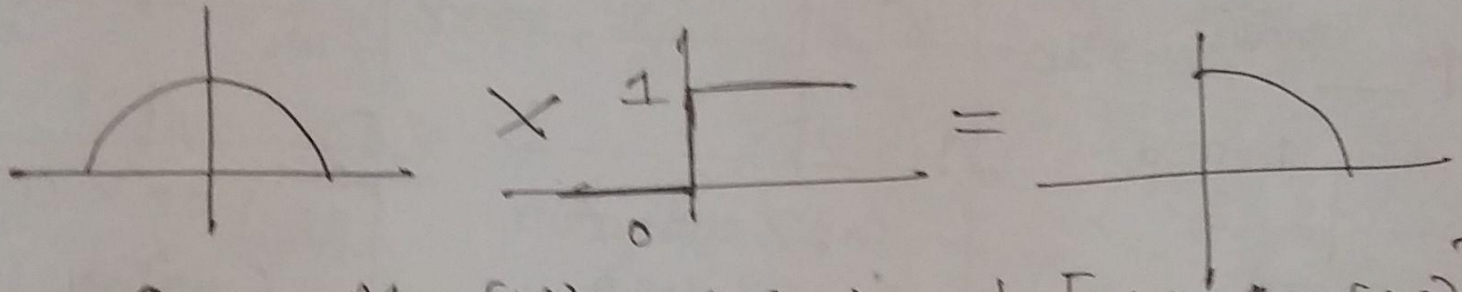
+1 ->



Divided by 2 ->



SSB: Modulation



So, $M_+(\omega) = M(\omega) \times \underbrace{\frac{1}{2}\left[1 + \text{sign}(\omega)\right]}_{U(\omega)}$

SSB: Modulation

Now, $m_+(t) = \frac{1}{2} m(t) + j\frac{1}{2}\hat{m}(t)$

$$M_+(\omega) = \frac{1}{2}M(\omega) + \frac{1}{2}M(\omega)\,\mathrm{sign}(\omega) \quad \text{— (*)}$$

Now, $m_+(\omega) = \frac{1}{2}\left[M(\omega) + j\hat{M}(\omega)\right]$ — (i)

$$M_-(\omega) = \frac{1}{2}\left[M(\omega) - j\hat{M}(\omega)\right] \quad \text{— (ii)}$$

From (i), $j\hat{M}(\omega) = 2M_+(\omega) - M(\omega)$

$$= \cancel{M(\omega)} + M(\omega)\,\mathrm{sign}(\omega) - \cancel{M(\omega)} \quad \text{(From *)}$$

$$= M(\omega)\,\mathrm{sign}(\omega)$$

SSB: Modulation

So, $j\hat{M}(\omega) = M(\omega)\,\mathrm{sgn}(\omega)$

$$\Rightarrow \hat{M}(\omega) = -j\,M(\omega)\,\mathrm{sgn}(\omega) \quad \text{——— (iii)}$$

Inverse Fourier transform of $M(\omega)$ will yield $m(t)$

Inverse fourier transform of $\hat{M}(\omega)$ will yield $\hat{m}(t)$

To invert the two together, we will use convolution operation

$$\Downarrow$$

$$\hat{m}(t) = m(t) * \frac{2}{\pi}$$

convolution

from $\hat{M}(\omega)$ from $M(\omega)$ time domain representation of $-j\,\mathrm{sgn}(\omega)$

SSB: Modulation

*** Hilbert Transform : convolution of a signal $m(t)$ & $\frac{2}{\pi}$

+ve frequencies of the signal $\longrightarrow -\pi/2$ phase shift

−ve frequencies of the signal $\longrightarrow +\pi/2$ phase shift

SSB: Modulation

$$m(t) \longrightarrow \boxed{\begin{array}{l} +\text{ve freq} \to -\pi/2 \text{ phase shift} \\ -\text{ve freq} \to +\pi/2 \end{array}} \longrightarrow \hat{m}(t)$$

So, $M_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega - \omega_c)$

$$= m_+(t)\, e^{j\omega_c t} + m_-(t)\, e^{-j\omega_c t}$$

$$2M_{USB}(\omega) = [m(t) + j\hat{m}(t)](\cos\omega_c t + j\sin\omega_c t)$$

$$+ [m(t) - j\hat{m}(t)](\cos\omega_c t - j\sin\omega_c t)$$

$$= 2m(t)\cos\omega_c t \pm 2\hat{m}(t)\sin\omega_c t$$

So, $M_{USB}(\omega) = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t$

SSB: Modulation

Hilbert

+ve freq component ($\omega > 0$) $\longrightarrow$ $-\pi/2$ phase shift

$\theta_h(\omega)$

−ve " " ($\omega < 0$) $\longrightarrow$ $+\pi/2$ " "

Actually, $m(t) \xleftrightarrow[\text{Hilbert transform}]{} \hat{m}(t) \,/\, m_h(t)$

SSB: Modulation

Hilbert Transform for cosine signal ->

Shifting cosine signal by $+\pi/2$ (for negative frequencies) or $-\pi/2$ (for positive frequencies) ->

Transmitting cosine signal with a phase delay of $-\pi/2$

SSB: Modulation

$-\pi/2$ phase shift : $\cos\left(\omega_c - \frac{\pi}{2}\right)$ $\boxed{\text{+ve freq component}}$

$$\cos\left(-\omega_c - \frac{\pi}{2}\right) \quad \boxed{\text{-ve freq comp}}$$

$$= \cos\left(\omega_c + \pi/2\right) \longrightarrow \text{so}$$

$+\pi/2$ phase shift

SSB: Modulation

Example: $m(t) = \cos 100t$

$\hat{m}_h(t) = \cos(100t - \pi/2) = \sin 100t$

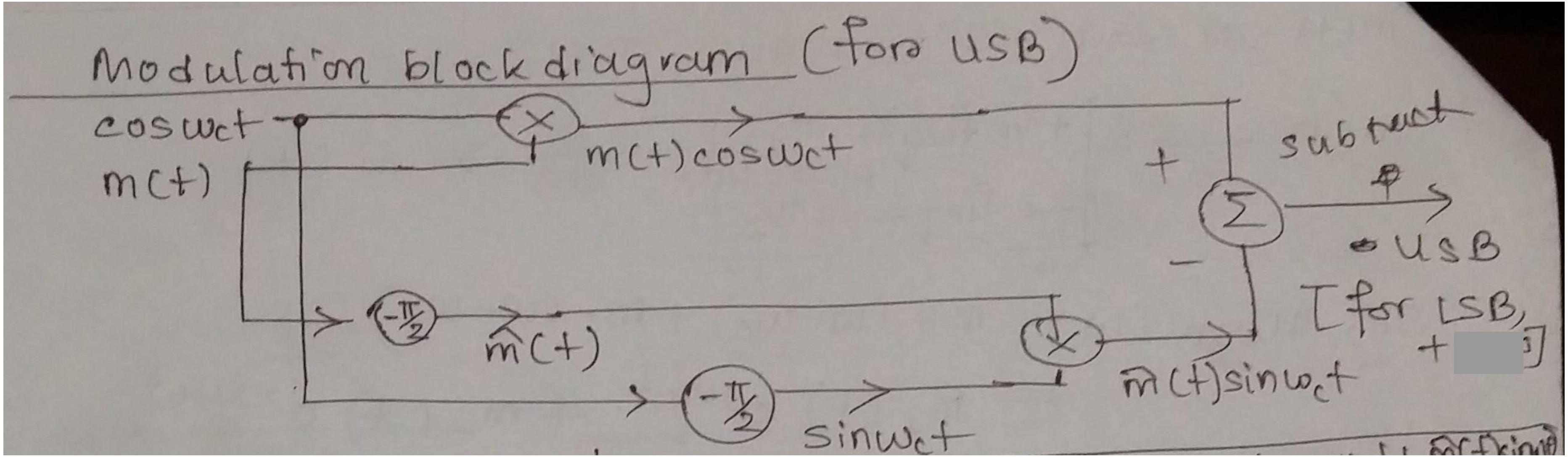
SSB: Modulation

* $M_{LSB}(\omega) = m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t$

so, $M_{SSB}(\omega) = m(t)\cos\omega_c t \pm \hat{m}(t)\sin\omega_c t$

[$+ \rightarrow$ LSB, $- \rightarrow$ USB]

SSB: Modulation



SSB: Coherent Demodulation

Demodulation (coherent) $= [m(t)\cos\omega_c t + \hat{m}(t)\sin\omega_c t]$

$$\times \cos\omega_c t = \frac{1}{2}\left[m(t)(1+\cos 2\omega_c t) - \hat{m}(t)\sin 2\omega_c t\right]$$

$$= \frac{1}{2} m(t) + \underbrace{\text{— — —}}_{\text{filter out.}}$$

SSB-WC: Envelope Detection

to transmit this signal $\longrightarrow m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t + A \cos \omega_c t$

$$\boxed{A \cos \omega_c t + B \sin \omega_c t = \sqrt{A^2 + B^2} \cos\left(\omega_c t + \tan^{-1} B/A\right)}$$

$$= [m(t) + A] \cos \omega_c t - \hat{m}(t) \sin \omega_c t$$

$$= \sqrt{(m(t) + A)^2 + (\hat{m}(t))^2} \; \cos\Big(\omega_c t + \underbrace{\tan^{-1} \frac{\hat{m}(t)}{m(t) + A}}_{\theta}\Big)$$

After expanding this part using binomial expansion and discarding higher order terms,

$$\approx A + m(t)$$

So, ultimately $\approx [A + m(t)] \cos(\omega_c t + \theta)$

SSB-WC: Condition for Envelope Detection

So, ultimately $\approx [A + m(t)] \cos(\omega_c t + \theta)$

In AM, condition: $A \geq |m(t)|$

SSB+C: $A >> |m(t)|$ $\longrightarrow$ the required amplitude is much larger than in AM

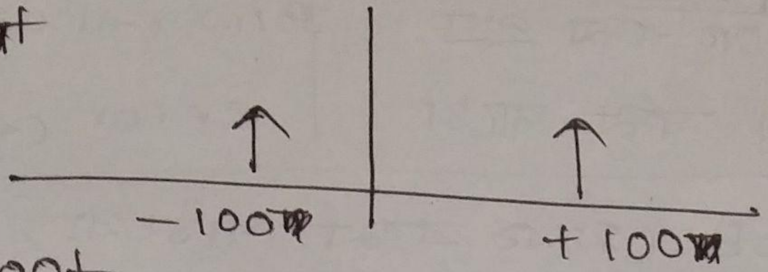
$\longrightarrow$ so efficiency of SSB+C is very low.

SSB: Problems

- In practice, it is often not feasible to sharply cut off double sideband to produce single sideband using filters
- It is not always possible to multiply a signal and its Hilbert transform -> since real signals have many components, it is not possible to transform each component -> approximation methods are used

Drawing Spectrum of SSB

$m(t) = \cos 100t$



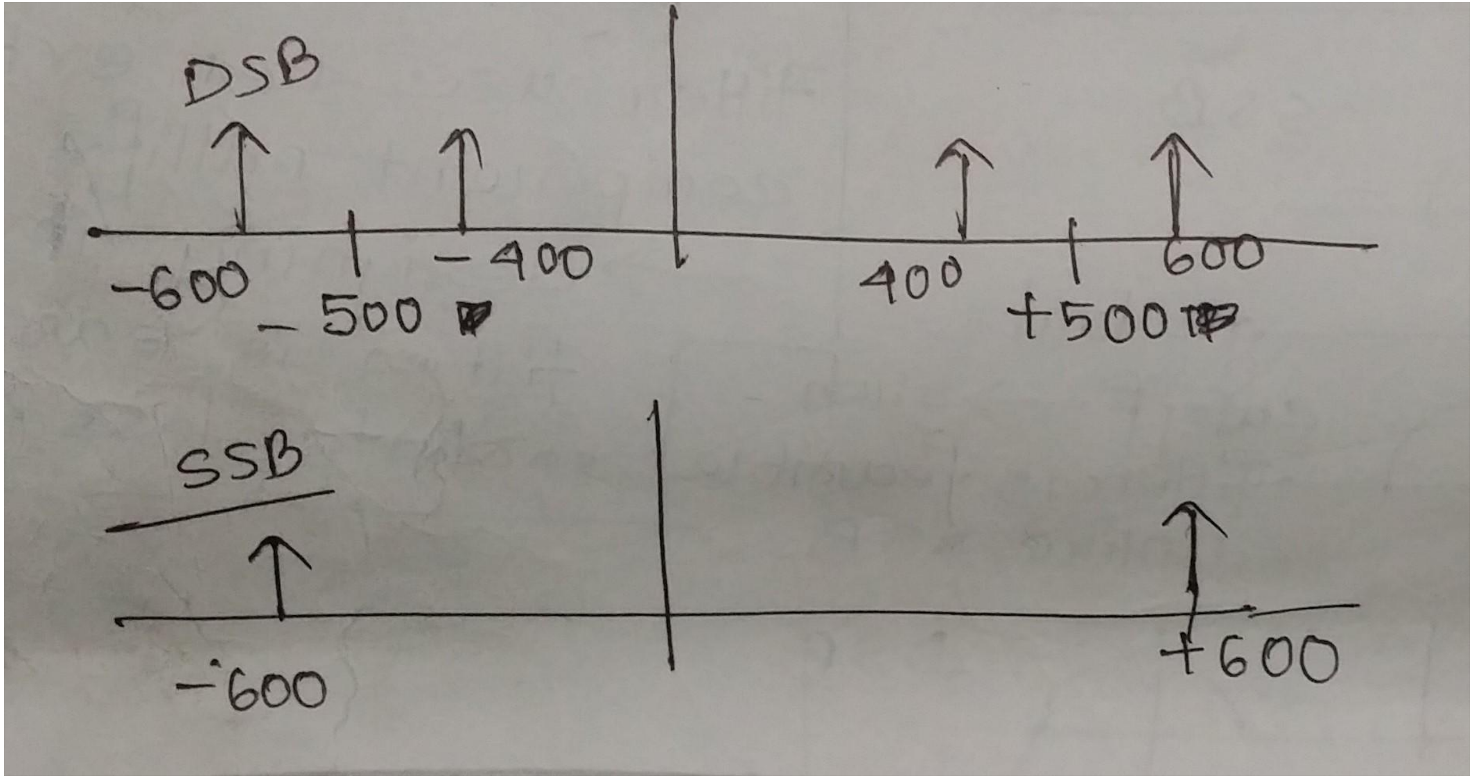
* carrier $= \cos 500t$

* So, transmitted signal $= m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t$

$$= \cos 100t \cos 500t - \sin 100t \sin 500t$$

$$= \cos(500t + 100t) = \cos 600t$$

Drawing Spectrum of SSB



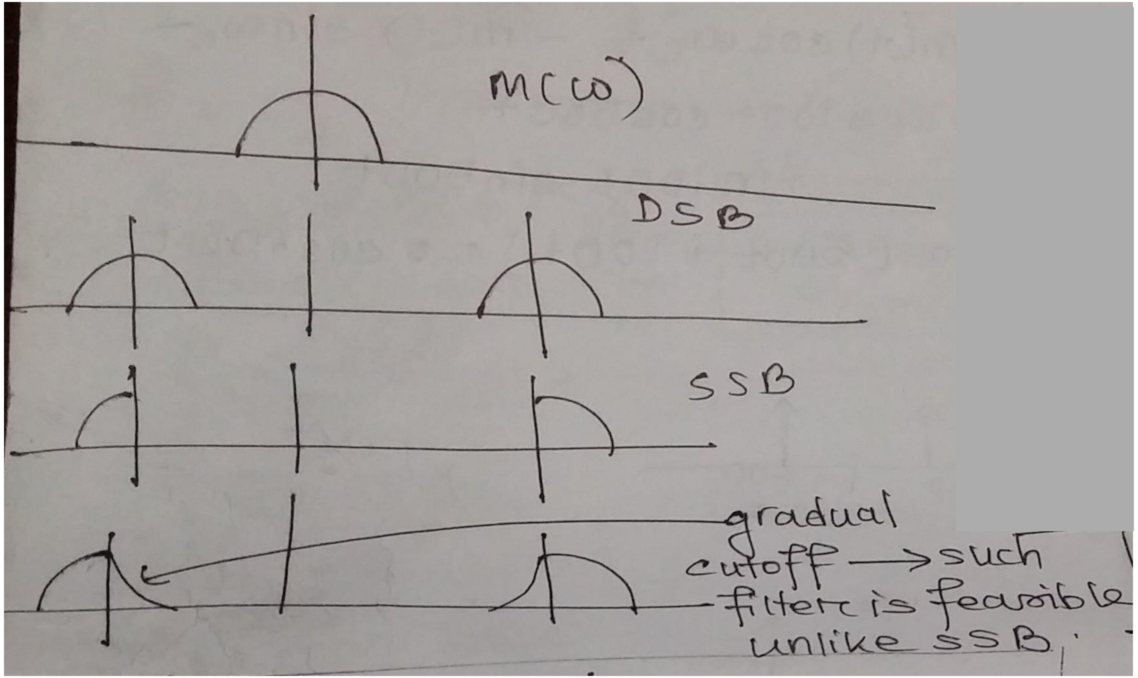
Pros and Cons of DSB and SSB

DSB: Less error-prone, but requires more bandwidth!

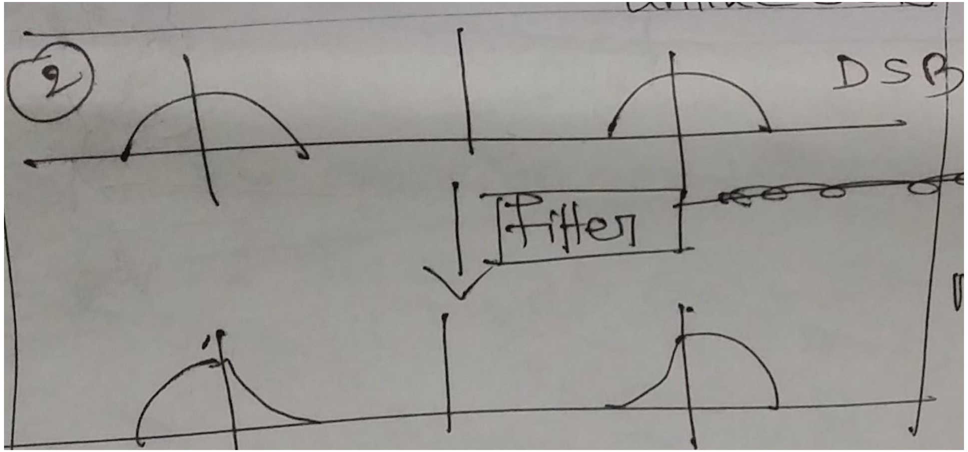
SSB: Requires less bandwidth, but more error-prone!

Concept of VSB (Vestigial Sideband): Sending more than SSB, but less than DSB!!

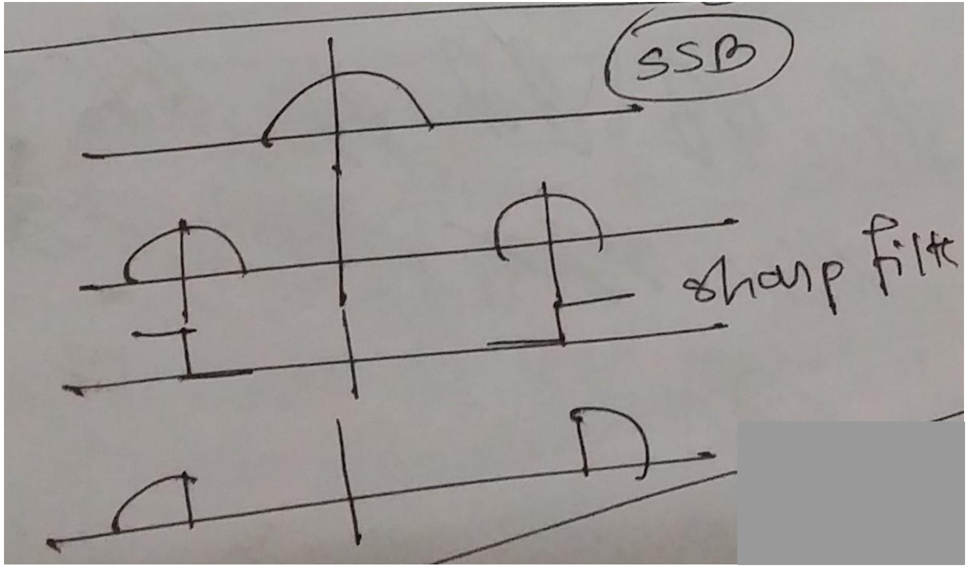
Concept of VSB

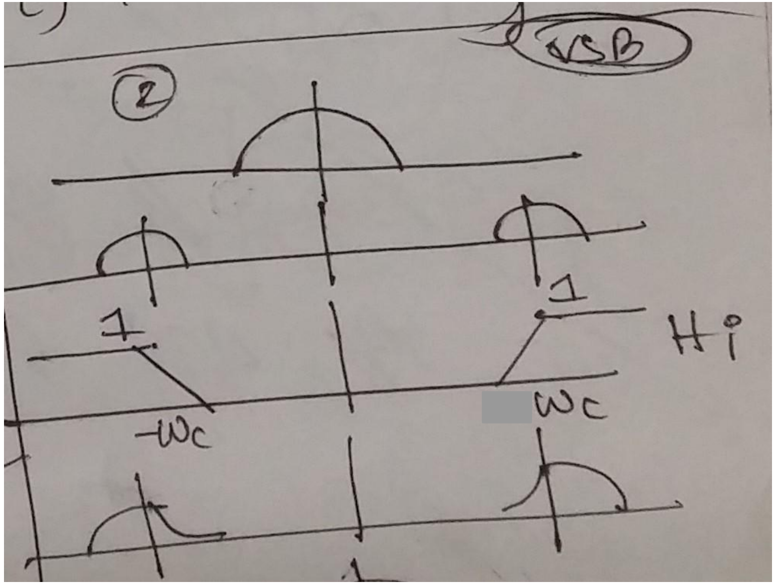


Concept of VSB



Concept of VSB





Concept of VSB: Problem with Demodulation

- During demodulation, the extra portion will overlap in the middle and become distorted ->
  has to be nullified using an inverse filter

- Since designing an inverse filter is quite easy, data loss will not occur!