17050045
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(1) $f(x) = \lambda e^{-\lambda x}$

$$F(x) = \int_0^x \lambda e^{-\lambda x}\,dx = \lambda\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^x$$

$$= -(e^{-\lambda x} - 1) = 1 - e^{-\lambda x}$$

$\therefore U = F(x)$

$\Rightarrow U = 1 - e^{-\lambda x}$

$\Rightarrow e^{-\lambda x} = 1 - U$

$\Rightarrow -\lambda x = \ln(1-U)$ $\therefore x = -\frac{1}{\lambda}\ln(1-U)$

Simplified form by replacing $1-U$ with U.

So,

$$X = -\frac{1}{\lambda}\ln U$$

So, column 2, 3, 4 are correct.

(2) $f(x) = \frac{6}{\pi(x^2+6^2)}$

$$F(x) = \frac{6}{\pi}\int_{-\infty}^{x} \frac{1}{x^2+6^2}\,dx$$

$$= \frac{6}{\pi}\left[\arctan\left(\frac{x}{6}\right) \times \frac{1}{6}\right]_{-\infty}^{x}$$

$$= \arctan\left(\frac{x}{6}\right)\frac{1}{\pi} + \frac{\pi}{2\pi}$$

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$$= \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x}{\gamma}\right)$$

So,

$$U = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x}{\gamma}\right)$$

$$\Rightarrow \left(U - \frac{1}{2}\right)\pi = \arctan\left(\frac{x}{\gamma}\right)$$

$$\Rightarrow \tan\left(\left(U - \frac{1}{2}\right)\pi\right) = \frac{x}{\gamma}$$

$$\Rightarrow x = \gamma \tan\left(\left(U - \frac{1}{2}\right)\pi\right)$$

We can simplify replacing $U - \frac{1}{2}$ by U.

~~4th column is not correct~~

So, $\cancel{x = \gamma \tan\left(\left(U - \frac{1}{2}\right)\pi\right)}$

$\cancel{\gamma \tan(\pi U)}$

So, ~~c~~ all columns are correct.

3

$$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

$$F(x) = \int_0^x \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \, dx$$

Now,

$$\cancel{\int \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx}$$

$$= \cancel{x\, x}$$

~~Let~~ $\int \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx$ | let $x^2 = z$, $2x\,dx = dz$

$$= \int \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} dz$$

$$= \frac{1}{2\sigma^2} \frac{e^{-z/2\sigma^2}}{-\frac{1}{2\sigma^2}}$$

$$= e^{-z/2\sigma^2}$$

$$= e^{-\frac{x^2}{2\sigma^2}}$$

So, $F(x) = \left[e^{-x^2/2\sigma^2} \right]_0^x$

$$= 1 - e^{-x^2/2\sigma^2}$$

So, $U = 1 - e^{-x^2/2\sigma^2}$

$$\Rightarrow e^{-x^2/2\sigma^2} = 1 - U$$

$$\Rightarrow \frac{x^2}{2\sigma^2} = -\ln(1-U)$$

$$\Rightarrow x^2 = 2\sigma^2(-\ln(1-U))$$

$$\Rightarrow x = \sigma\sqrt{-2\log(1-U)}$$

So, 3rd column is not correct.

4th column is not correct too. Simplified version is $= \sigma\sqrt{-2\log U}$

(4)

$$f(x) = \frac{2}{a}\left(1 - \frac{x}{a}\right)$$

$$F(x) = \frac{2}{a}\int_0^x \left(1 - \frac{x}{a}\right) dx$$

$$= \frac{2}{a}\left[x - \frac{x^2}{2a}\right]_0^x$$

$$= \frac{2}{a}\left[x - \frac{x^2}{2a}\right]$$

So, $U = \frac{2}{a}\left[x - \frac{x^2}{2a}\right]$

$$\Rightarrow U = 2\frac{x}{a} - \frac{x^2}{a^2}$$

$$\Rightarrow \left(\frac{x}{a}\right)^2 - 2\cdot\frac{x}{a}\cdot 1 + 1^2 + U - 1 = 0$$

$$\Rightarrow \left(\frac{x}{a} - 1\right)^2 = 1 - U$$

$$\Rightarrow \frac{x}{a} - 1 = \pm\sqrt{1 - U}$$

$$\Rightarrow \frac{x}{a} = 1 \pm \sqrt{1 - U}$$

~~$\Rightarrow x = a^2 +$~~

$$\Rightarrow x = a\left(1 \pm \sqrt{1-U}\right)$$

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But $x \in [0, a]$

So, $x = a\left(1 - \sqrt{1-U}\right)$

We can replace $1-U$ by U.

So, simplified form is $a\left(1 - \sqrt{U}\right)$

So, all columns are correct.

(5) $f(x) = x e^{\frac{a^2 - x^2}{2}}$

$$F(x) = \int_a^x x e^{\frac{a^2-x^2}{2}} dx$$

$$= \frac{1}{2}\int_a e^{\frac{a^2-z}{2}} dz$$

$$= \frac{1}{2} \frac{e^{\frac{a^2-z}{2}}}{-1/2}$$

$$= -e^{\frac{a^2-z}{2}}$$

$$= \left[-e^{\frac{a^2-x^2}{2}}\right]_a^x$$

$$= 1 - e^{\frac{a^2-x^2}{2}}$$

$$\therefore U = 1 - e^{\frac{a^2-x^2}{2}}$$

$$\Rightarrow e^{\frac{a^2-x^2}{2}} = 1-U$$

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$\therefore x = \sqrt{a^2 - 2\log(1-U)}$

We can replace $1-U$ by U.

So,

$$x = \sqrt{a^2 - 2\log U}$$

⑥ Pareto

$$f(x) = \frac{ab^a}{x^{a+1}}$$

$$F(x) = \int_b^x \frac{ab^a}{x^{a+1}}\,dx$$

$$= ab^a \left[\frac{x^{-a}}{-a}\right]_b^x$$

$$= -\cancel{a}b^a \left[\frac{1}{x^a} - \frac{1}{b^a}\right]$$

$$= 1 - \left(\frac{b}{x}\right)^a$$

So, $U = 1 - \left(\frac{b}{x}\right)^a$

$$\Rightarrow \left(\frac{b}{x}\right)^a = 1-U$$

$$\Rightarrow \frac{b^a}{1-U} = x^a$$

$$\Rightarrow x = \frac{b}{(1-U)^{1/a}}$$

We can replace $1-U$ by U.

So,
Simplified form is

$$x = \frac{b}{U^{1/a}}$$

So, all columns are correct.