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Assignment - 2

6.1

By central limit theorem, it is normally distributed as n is large.

6.9 Average number of failure is $\bar{X}(n)$. So, we expect a success every $(\bar{X}(n)+1)$ th round.

So, $\hat{p} = \frac{1}{\bar{X}(n)+1}$

$\boxed{6.10c}$

Here,

$$L(a,b) = \left(\frac{1}{b-a}\right)^n$$

We need b-a to be smallest, but [a,b] must contain all X_i.

So, $a = \min(X_i)$

$b = \max(X_i)$

$\boxed{6.10d}$

$$N(\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

So, $L(\mu,\sigma^2) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n \exp\left(-\sum \frac{(X_i-\mu)^2}{2\sigma^2}\right)$

$\therefore\ l(\mu,\sigma^2) = -n\ln(\sqrt{2\pi}) - n\ln\sigma - \sum \frac{(x_i-\mu)^2}{2\sigma^2}$

$$\frac{dl}{d\mu} = \sum \frac{(x_i - \mu)}{\sigma^2} = 0$$

$$\Rightarrow \sum x_i = \mu n$$

$$\Rightarrow \boxed{\hat{\mu} = \frac{\sum x_i}{n}}$$

$$\frac{dl}{d\sigma} = -\frac{n}{\sigma} + \sum \frac{(x_i - \mu)^2}{\sigma^3} = 0$$

So, $\boxed{\hat{\sigma}^2 = \dfrac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$

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(2) If observations are $x_1, x_2, \ldots, x_n$

$$L(\lambda) = \frac{e^{-\lambda}\lambda^{x_1}}{x_1!} \times \frac{e^{-\lambda}\lambda^{x_2}}{x_2!} \times \cdots \times \frac{e^{-\lambda}\lambda^{x_n}}{x_n!}$$

$$= \frac{e^{-\lambda n} \times \lambda^{x_1+x_2+\cdots+x_n}}{\text{constant}}$$

$$l(\lambda) = -\lambda n + (x_1+x_2+\cdots+x_n)\ln\lambda - \ln(\text{constant})$$

$$l'(\lambda) = 0$$

$$\Rightarrow -n + \frac{(x_1+x_2+\cdots+x_n)}{\lambda} = 0$$

$$\Rightarrow \lambda = \frac{x_1+x_2+\cdots+x_n}{n}$$