Assignment

P-1
Roll - 1705045
Name - Iftekharr Hakim

① For insert operation, we can prove that at most one single/double rotation is needed. Trivially, when no violation happens after inserting, no rotation is needed.

Case of single rotation:-

The violated subtree part can be pictured as below. Assumed y to be right child without loss of generality.

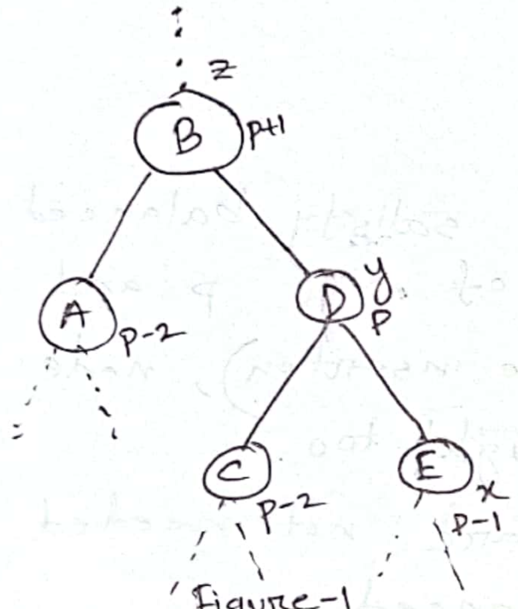

Figure-1

Here, if the height of node z is p+1, height of y is p, height of x is p-1. Height of A has to be p-2. Also, before this insertion, height of node z (or B) has to p. and height of y (or D) has to be p-1.

Note that subtree rooted at A or C were untouched during insertion and height of y were p-1 before insertion, so hight of node C is p-2 in figure-1.

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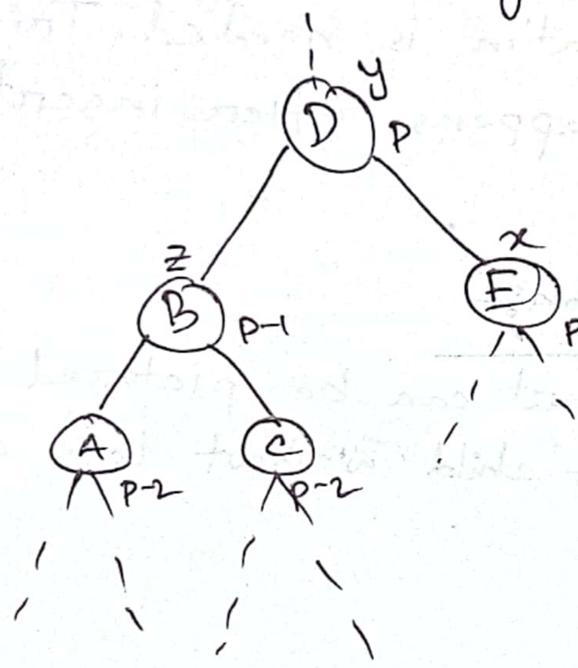
Now, if we do a single rotation,



Here, node A, B, C, E satisfy balanced height.
Again, as new height of D is p and height of z was p (before insertion), node D satisfy balanced height too.
So, extra rotations are not needed anymore.
It is completely balanced.

Case of double rotation:-

Assuming y to be right child without loss of generality, the violated part can be—

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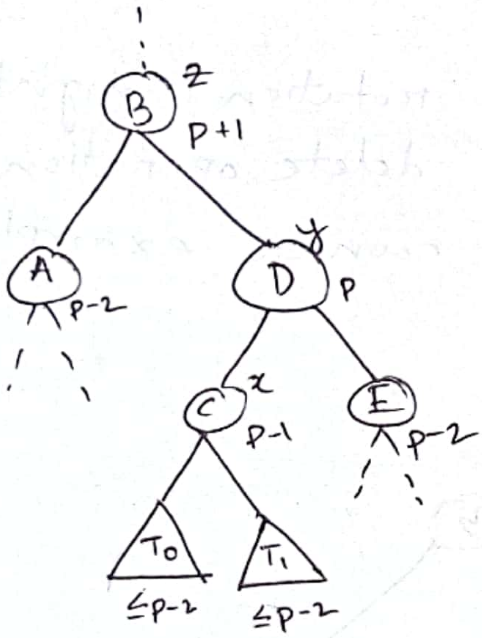
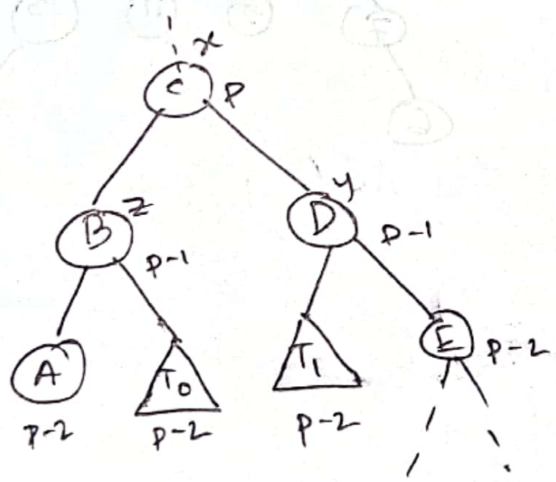


At least one of T0 and T1 has height p-2. Other heights are written with same reason as before.

Now, if we do rotation(double),



Here, nodes A,B,D,E ~~se~~ are seen to have balanced height. Again, new height of C is p and height (before insertion) of z was p, so node C has balanced ~~weight~~ height too. So, it balanced now.

That's why, more than one rotation is not needed.

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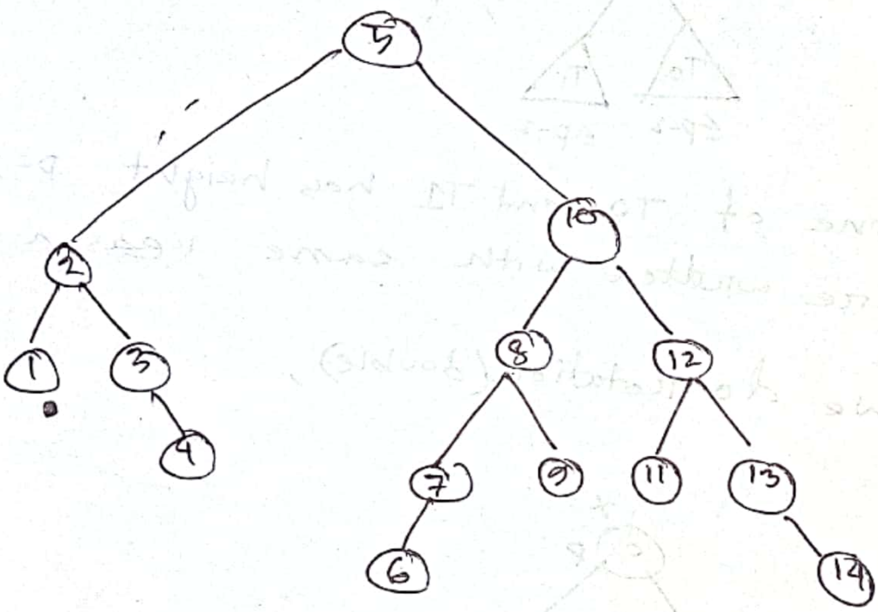
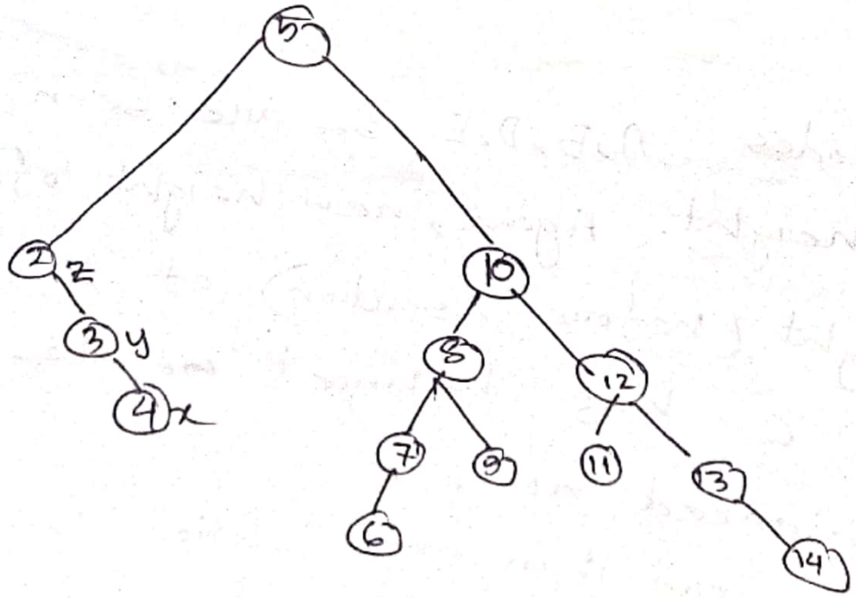
(2) More than one rotation might be necessary after delete operation. We can prove it with counter example.

Initial tree:-



Deleting 1,



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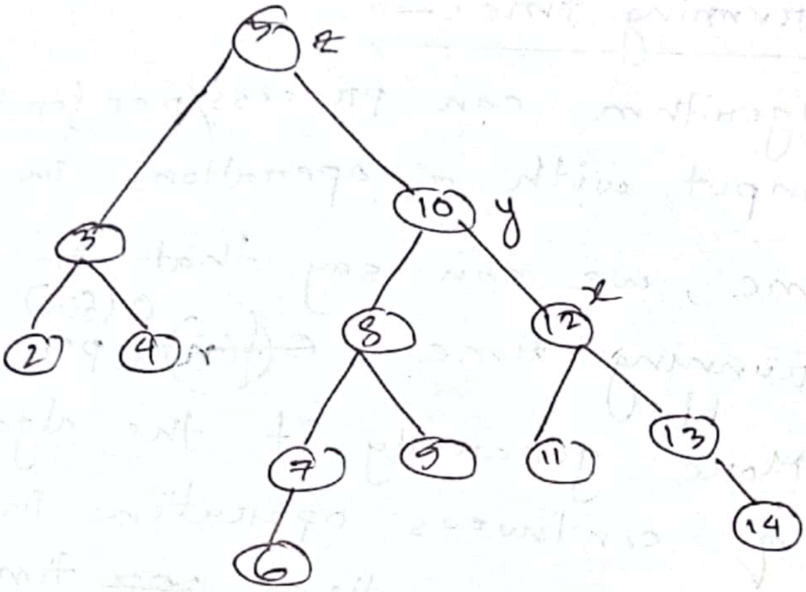
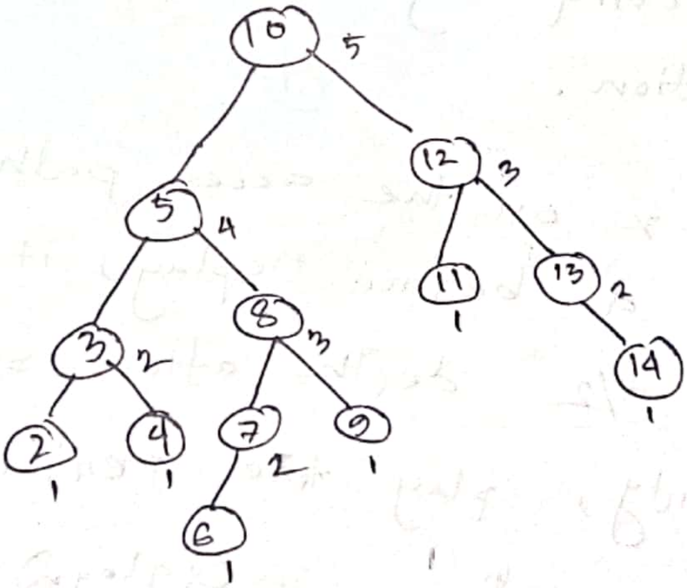
Doing single rotate,



Still not balanced,
Doing single rotate,



It is now balanced.
By this example, we can say that more than one rotation can be necessary for delete operation

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(3)

Amortized running time:-

If an algorithm can process/perform any valid input with q operations in $O(qf(n))$ time, we can say that it has amortized running time ~~$\Theta(f(n))$~~ $O(f(n))$ per operation. More generally, if the algorithm performs q continuous operation in a time complexity, amortized ~~opr~~ time complexity is said to be the average time per operation. But it does not bound the time complexity for a single and lone operation.

For splay tree,

If a node x on the access path is at depth d before splay, it is at about $d/2$ depth after splay. With this property, splay tree perform q continuous operation in $O(q\log n)$ time. That's why it has $O(\log n)$ amortized time complexity per operation.

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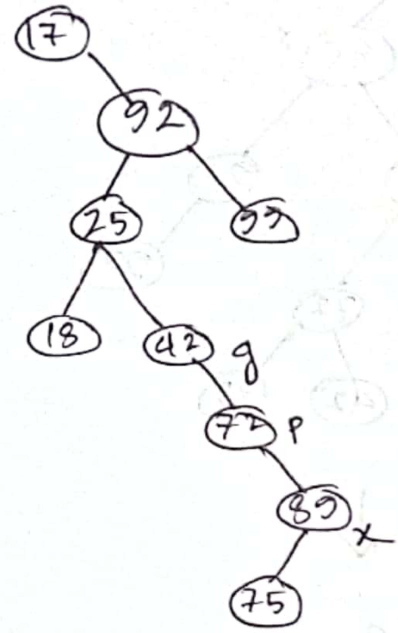
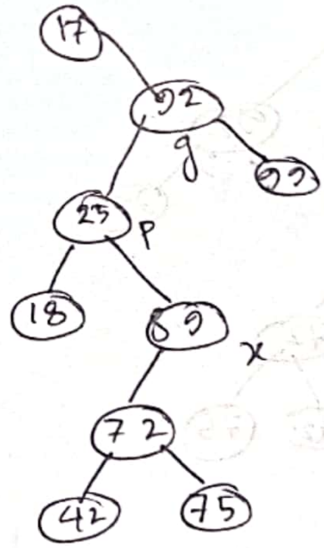
~~1st Rotation:~~



After rotation-1 :-



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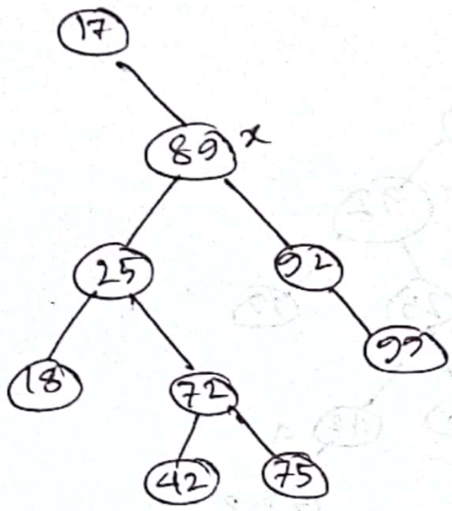
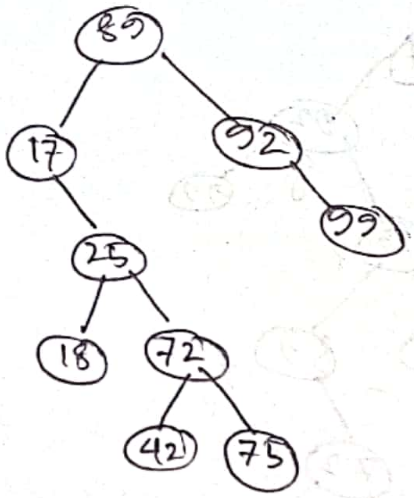
After rotation-2,



After rotation 3,



Now, 89 is at the root.

So, 3 rotations are needed.