Assignment-1

For the following six distributions, how can you generate random variates using the inverse transform method? Assume, you are given the first column or the density function, verify the entries of the other three columns. Are there any errors in the third or fourth columns?

Density $f(x)$	$F(x)$	$X=F^{-1}(U)$	Simplified form
Exponential(λ) $\lambda e^{-\lambda x}, x \geq 0$	$1-e^{-\lambda x}$	$-\frac{1}{\lambda}\log(1-U)$	$-\frac{1}{\lambda}\log(U)$
Cauchy(σ) $\frac{\sigma}{\pi(x^2+\sigma^2)}$	$\frac{1}{2}+\frac{1}{\pi}\arctan(\frac{x}{\sigma})$	$\sigma\tan(\pi(U-\frac{1}{2}))$	$\sigma\tan(\pi U)$
Rayleigh(σ) $\frac{x}{\sigma}e^{-\frac{x^2}{2\sigma^2}}, x \geq 0$	$1-e^{-\frac{x^2}{2\sigma^2}}$	$\sigma\sqrt{-\log(1-U)}$	$\sigma\sqrt{-\log(U)}$
Triangular on(0,a) $\frac{2}{a}(1-\frac{x}{a}), 0 \leq x \leq a$	$\frac{2}{a}(x-\frac{x^2}{2a})$	$a(1-\sqrt{1-U})$	$a(1-\sqrt{U})$
Tail of Rayleigh $xe^{\frac{a^2-x^2}{2}}, x \geq a > 0$	$1-e^{\frac{a^2-x^2}{2}}$	$\sqrt{a^2-2\log(1-U)}$	$\sqrt{a^2-2\log U}$
Pareto(a, b) $\frac{ab^a}{x^{a+1}}, x \geq b > 0$	$1-(\frac{b}{x})^a$	$\frac{b}{(1-U)^{1/a}}$	$\frac{b}{U^{1/a}}$

Assignment-2

1. Exercises 6.1, 6.9, 6.10 (c), 6.10 (d)

2. Show that for a Poisson distribution, the MLE for lambda is the sample mean.

Assignment-3

Study two sections with examples from the following webpage

Section-1: Strictly increasing functions of a discrete random variable

Section-2: Strictly increasing functions of a continuous random variable

https://www.statlect.com/fundamentals-of-probability/functions-of-random-variables-and-their-distribution

Now find the expression of density function of Lognormal distribution.

[Hint: If Y is distributed as Normal, then e^Y (which is a strictly increasing function of Y) is distributed as Lognormal.]

Assignment-4

1. Excercises 4.30, 4.31, 4.32
2. Assume that a point is chosen in uniformly random manner inside a m X n (m by n) rectangle. What will be the joint probability distribution f (x, y) [x,y is the cartesian coordinate of the point]?
3. What will be the joint probability distribution f (x, y) and f (r, θ), assuming the point is chosen in uniformly random manner inside a circle? [(x,y) is cartesian and (r, θ) is polar coordinate of the point]