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Q3

$$S = \sum_{j,k} [1 \le j \le k \le n]$$

$$= \sum_{1 \le k \le n} \sum_{1 \le j \le k} (1)$$

$$= \sum_{1 \le k \le n} k$$

$$= \frac{n(n+1)}{2}$$

Q2

$$S = \sum_{0 \le k \le n} (-1)^{n-k}$$

$$= \sum_{0 \le n-k \le n} (-1)^{k}$$

$$= \sum_{0 \le k \le n} (-1)^{k}$$

Finding range,

$n-k \ge 0$	$n-k \le n$
$\Rightarrow k \le n$	$\Rightarrow k \ge 0$

$$\therefore S = \sum_{\substack{0 \le k \le n \\ k \text{ even}}} 1 - \sum_{\substack{0 \le k \le n \\ k \text{ odd}}} 1$$

So, if n if odd, there will be equal number of even and odd number in $[0,n]$. And, $S=0$
Otherwise, there will one more even than odd numbers. And, $S=1$

Scanned with CamScanner

so, $S = \begin{cases} 1 ; n\text{-even} \\ 0 ; n \text{ odd}. \end{cases}$

① we Know,

$\sum U \Delta V \delta x = UV - \sum EV \Delta U \delta x$

~~Setting, $x^2 = \Delta V$ and $U = H_x$,~~

$\cancel{\sum x^2 H_x = H_x \sum x^2 \delta x - \sum}$

First find,

$\sum x H_x \delta x$

let $A = \sum x H_x \delta x = H_x \frac{x^{\underline{2}}}{2} - \sum \frac{(x+1)^{\underline{2}}}{2} \times \frac{1}{x} \delta x$

$= H_x \frac{x(x-1)}{2} - \frac{1}{2} \sum (x+1) \delta x$

$= \frac{1}{2} H_x x(x-1) - \frac{1}{2} \times \frac{(x+1) \times x}{2}$

$\therefore A = \frac{1}{2} x(x-1) H_x - \frac{1}{4} x(x+1)$

$\sum x^2 \delta x$

$= \sum (x^{\underline{2}} + x^{\underline{1}}) \delta x$

$= \frac{x^{\underline{3}}}{3} + \frac{x^{\underline{2}}}{2}$

$= \frac{x(x-1)(x-2)}{3} + \frac{x(x-1)}{2}$

$= x(x-1) \frac{2(x-2)+3}{6}$

$= \frac{x(x-1)(2x-1)}{6}$

Now,

$S = \sum (x)(x H_x) \delta x$

$= x\left(\frac{1}{2} x(x-1) H_x - \frac{1}{4} x(x+1)\right) - \sum \left(\frac{1}{2}(x+1) x H_x - \frac{1}{4}(x+1)(x+2)\right) \cdot \delta x$

$= \frac{1}{2} x^2 (x-1) H_x - \frac{1}{4} x^2 (x+1) - \frac{1}{2} \sum x^2 H_x - \frac{1}{2} \sum x H_x \delta x + \frac{1}{4} \sum (x+2)^{\underline{2}} \delta x$

$S = \frac{1}{2} x^2 (x-1) H_x - \frac{1}{4} x^2 (x+1) - \frac{1}{2} S - \frac{1}{2} A + \frac{1}{4} \frac{(x+2)^{\underline{3}}}{3}$

$$\therefore \frac{3}{2}S = \frac{1}{2}x^2(x-1)H_x - \frac{1}{4}x^2(x+1) - \frac{1}{2}A + \frac{(x+2)^3}{12}$$

where,

$$A = \frac{1}{2}x(x-1)H_x - \frac{1}{4}x(x+1)$$

$$S = \frac{1}{3}x^2(x-1)H_x - \frac{1}{6}x^2(x+1) - \frac{1}{3}A + \frac{(x+2)^3}{18}$$