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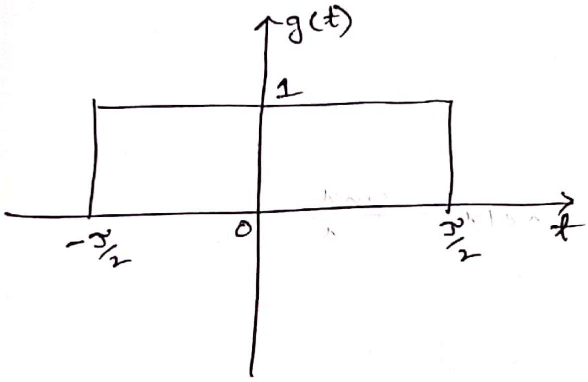
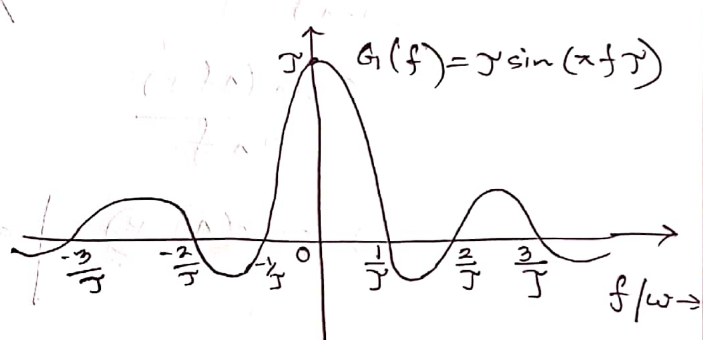
(1) By definition,

$$g(t) = \Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 \;; & |t| \leq \frac{\tau}{2} \\ 0.5 \;; & |t| = \frac{\tau}{2} \\ 0 \;; & \text{otherwise} \end{cases}$$



In Time Domain



In Frequency Domain

We know, fourier transform is for aperiodic signal.

That is,

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \Pi\left(\frac{t}{\tau}\right) e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{+\tau/2} e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{-j2\pi f} \left(e^{-j\pi f \tau} - e^{j\pi f \tau} \right)$$

$$= \frac{1}{-j2\pi f} \left\{ \cos(\pi f \tau) - j\sin(\pi f \tau) - \cos(\pi f \tau) - j\sin(\pi f \tau) \right\}$$

$$= \frac{1}{-j2\pi f} \left(-2j\sin(\pi f \tau) \right)$$

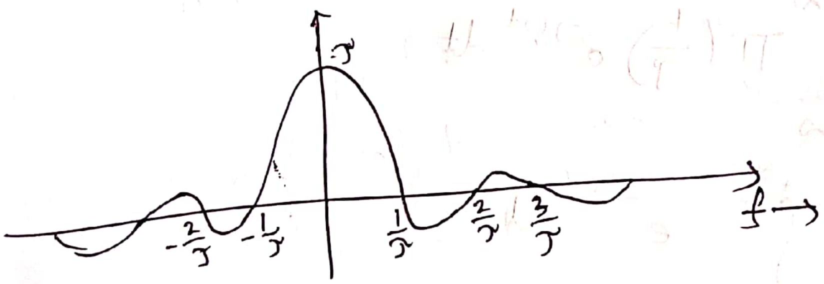
$$= \frac{2\sin(\pi f \tau)}{2\pi f}$$

$$= \tau \ \mathrm{sinc}(\pi f \tau) \quad \Big| \text{ as } \mathrm{sinc}(x) = \frac{\sin x}{x}$$

So,

$$\Pi\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau \ \mathrm{sinc}(\pi f \tau)$$

(2) In frequency domain,



Now, we see, if we take $\tau \to \infty$, in time domain graph, it only have value when $f=0$, and

$$\lim_{\tau \to \infty} G(0) = \infty$$

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This is the definition of an impulse function too.

Because,

$$\lim_{T\to\infty} G(f) = \begin{cases} \infty, & \text{when } f=0 \\ 0, & \text{otherwise} \end{cases}$$

That's why, as $T\to\infty$, the spectrum converges to an impulse function.

Mathematically, $G(f)$ becomes,

$$G(f) = \int_{-\infty}^{\infty} 1 \times e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-\infty}^{\infty}$$

$$\cancel{= \frac{1}{-j\omega}\left(e^{-j\omega t} - \right.}$$

$$= \frac{1}{-j\omega}\left[\frac{1}{e^{j\omega\infty}} - e^{j\omega\infty} \right]$$

$$= \frac{1}{-j\omega 2\pi f}\left[\frac{1}{e^{j\infty \times 2\pi f \times \infty}} - e^{j2\pi f\infty} \right] = \frac{1}{\cancel{j2\pi f}} \times e^{2\pi f}$$

We see, $G(f)$ has value when, $f=0$; And the value is ∞.

Otherwise, $G(f) = 0$

Alternatively, if $T\to\infty$, $g(t) = \{1 ; -\infty < t < \infty$.

Now, we know, $1 \Leftrightarrow \delta(f)$.

So, by this way, $G(f)$ converges to impulse too

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Dirichlet condition is not necessary.
It is sufficient only.

Let $G(f) = \delta(f)$

So, taking inverse fourier

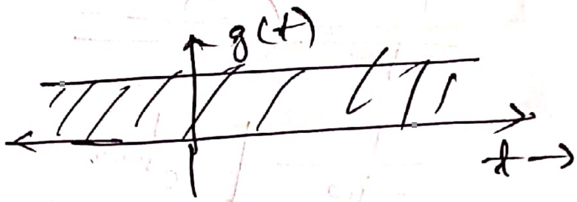
$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j\omega t}\, df$$

$$= \int_{-\infty}^{\infty} \delta(f)\, e^{j\omega t}\, df$$

$$= e^{2\pi \times 0 \times t}$$

$$= 1$$

So, $1 \Leftrightarrow \delta(f)$



we see g(t) does not
have a finite area ~~of~~ under its curve. But
~~it~~ it has corresponding G(f). that's why
it is justified that dirichlet condition is
not necessary for the existence of
fourier transform of a time domain signal.