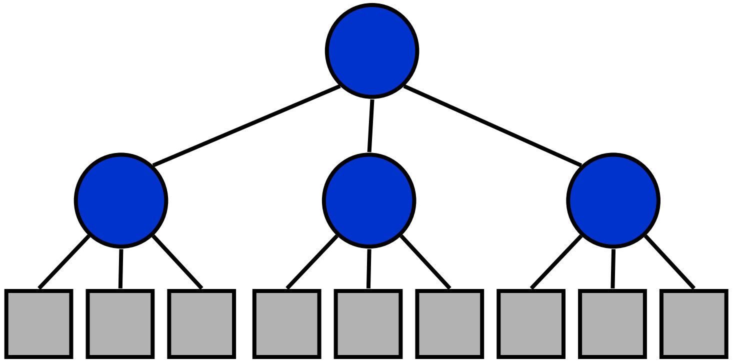
Divide-and-Conquer Technique:

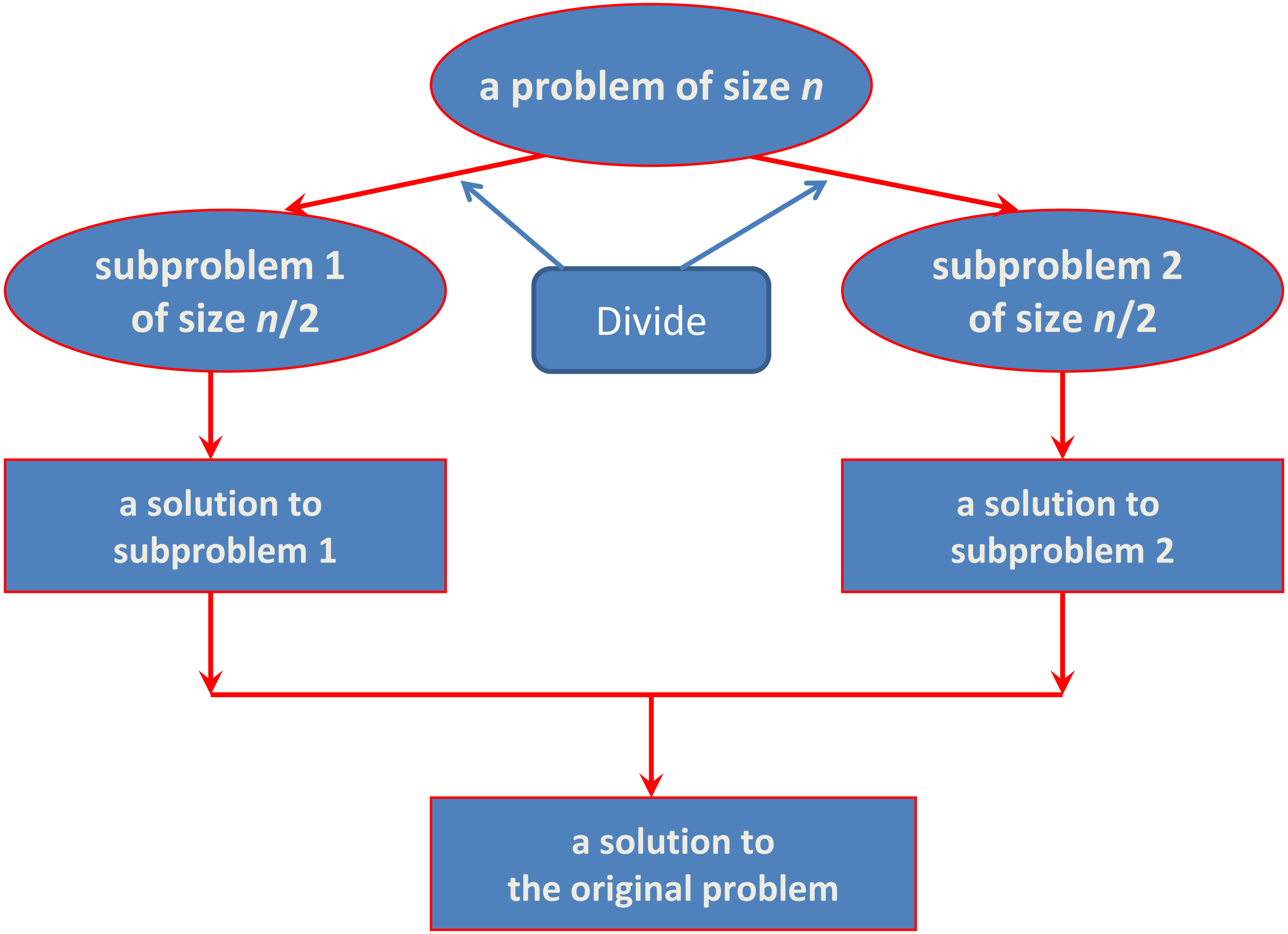
Finding Maximum & Minimum

Divide-and-Conquer

- Divide-and-Conquer is a general algorithm design paradigm:
  - Divide the problem into a number of subproblems that are smaller instances of the same problem
  - Conquer the subproblems by solving them recursively
  - Combine the solutions to the subproblems into the solution for the original problem
- The base case for the recursion are subproblems of constant size
- Analysis can be done using **recurrence equations**



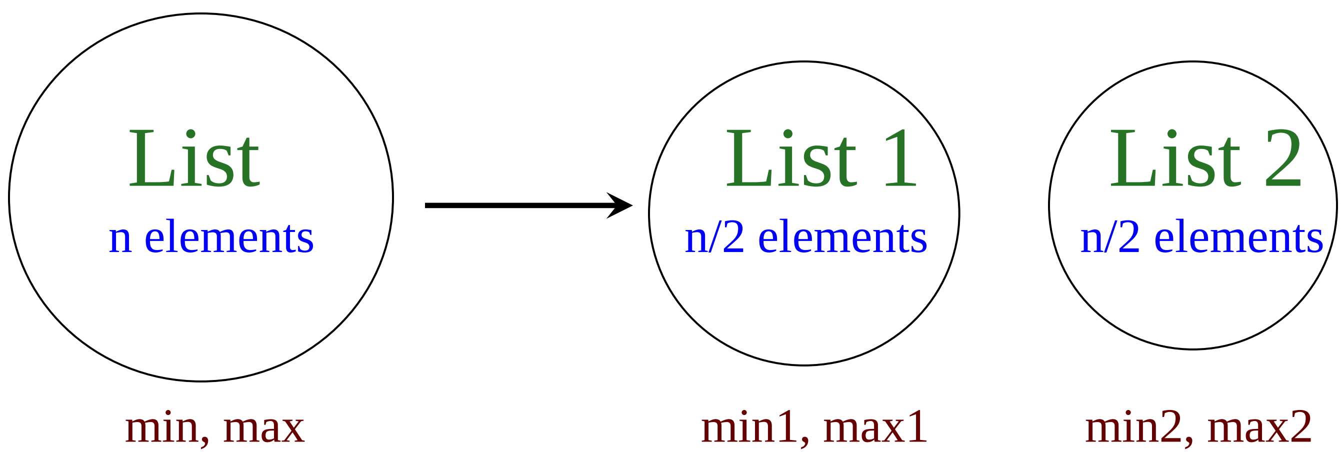
Divide-and-Conquer



Finding Maximum and Minimum

- *Input:* an array $A[1..n]$ of n numbers
- *Output:* the maximum and minimum value

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	13	-13	-25	20	-3	-16	-23	18	20	-7	12	-5	-22	15	-4	7



min = MIN (min1, min2)
max = MAX (max1, max2)

Finding Maximum and Minimum

The straightforward algorithm:

max $\leftarrow$ min $\leftarrow$ A [1];
for $i \leftarrow 2$ to n do
    if (A [i] > max) then max $\leftarrow$ A [i];
    if (A [i] < min) then min $\leftarrow$ A [i];

No. of comparisons: $2(n-1)$

Finding Maximum and Minimum

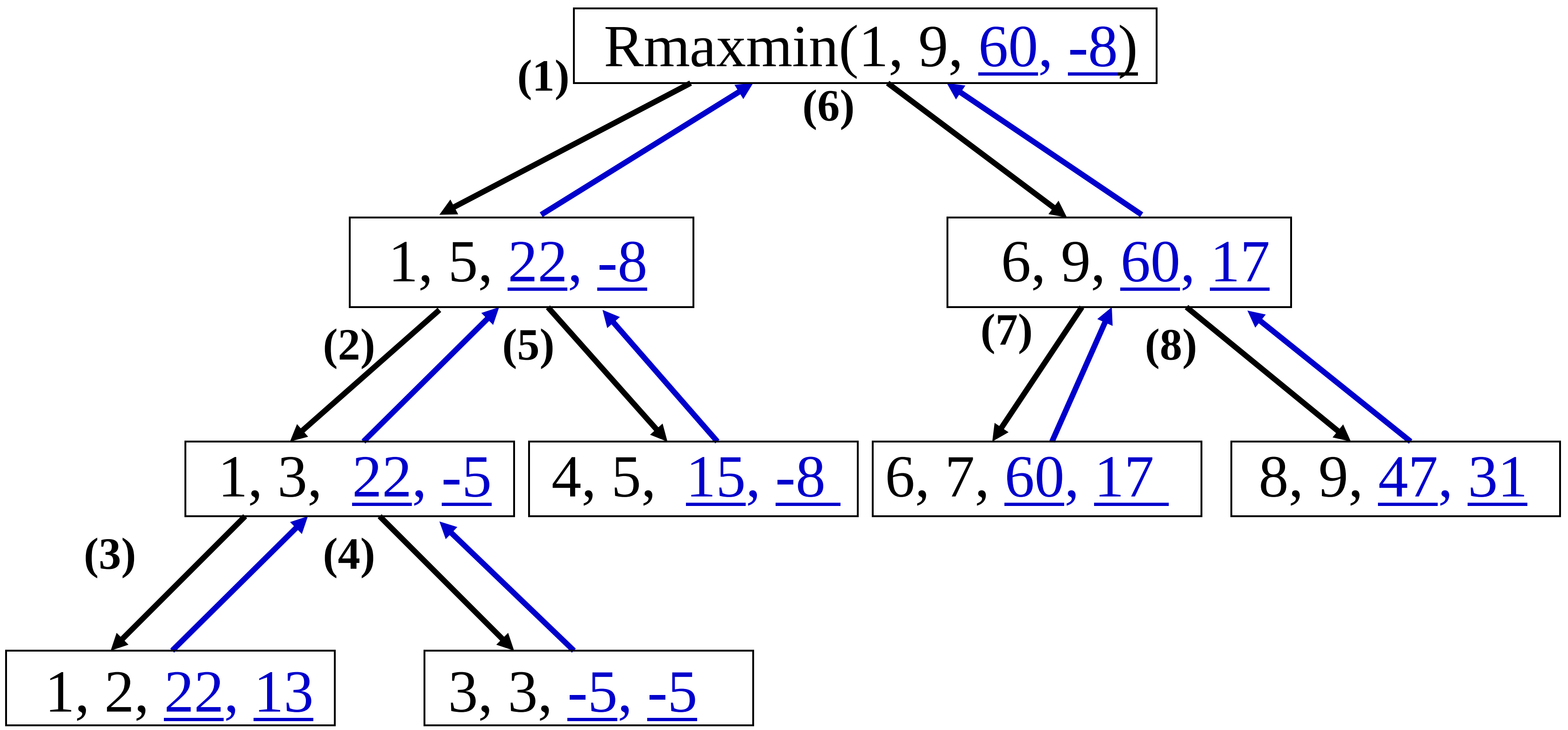
The Divide-and-Conquer algorithm:

```
procedure Rmaxmin (i, j, fmax, fmin);        // i, j are index #, fmax,
    begin                                    // fmin are output parameters
      case:
          i = j:        fmax ← fmin ← A[i];
          i = j –1:     if A[i] < A[j] then   fmax ← A[j];
                                              fmin ← A[i];
                        else   fmax ← A[i];
                               fmin ← A[j];
          else:         mid ← (i + j)/2;

                        call Rmaxmin (i, mid, gmax, gmin);
                        call Rmaxmin (mid+1, j, hmax, hmin);
                        fmax ← MAX (gmax, hmax);
                        fmin ← MIN (gmin, hmin);
      end
    end;
```

Finding Maximum and Minimum

Index:	1	2	3	4	5	6	7	8	9
Array:	22	13	-5	-8	15	60	17	31	47



Finding Maximum and Minimum

The recurrence for the worst-case running time $T(n)$ is

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \text{ or } 2 \\ 2\mathrm{T}(n/2) + \Theta(1) & \text{if } n > 2 \end{cases}$$

equivalently

$$T(n) = \begin{cases} b & \text{if } n = 1 \text{ or } 2 \\ 2T(n/2) + b & \text{if } n > 2 \end{cases}$$

By solving the recurrence, we get

$T(n)$ is $O(n)$