40. Explain, without using a truth table, why $(p \vee \neg q) \wedge (q \vee \neg p) \wedge (r \vee \neg p)$ is true when p, q and r have the same truth value and it is false otherwise.

Answer:
~~Step 1~~:

The objective is to explain $(p \vee \neg q) \wedge (q \vee \neg p) \wedge (r \vee \neg p)$ is true when p, q and r have the same truth values and is false otherwise.

step 1:

If p, q and r are all true, then clearly each of the three disjunctions is true and their conjunction is true.

If p, q and r are all false, then clearly each of the three disjunctions is true and the conjunction is true.

step 2:

If p, q are true and r is false, then
(r ∨ ¬p) is false and other two disjunctions
are true which makes the conjunction false.

If p, q are ~~true~~ false and r is true, then
(q ∨ ¬p) is false and other two disjunctions
are true which makes the conjunction false.

step 3:

If p, r are true and q is false,
then (q ∨ ¬p) is false and other two
disjunctions are true which makes the
conjunction false.

If p, r are false and q is true,
then all disjunctions are true
except (p ∨ ¬q) and so the conjunction
becomes false.

step 4:

If q, r are true and p is false, then $(p \vee \neg q)$ becomes false and other two disjunctions are true and so the conjunction is false.

If q, r are false and p is true, then $(r \vee \neg p)$ is false and other two disjunctions are true which makes the conjunction false.

So, we can see that the conjunction is true when p, q, r ~~are~~ all ~~be~~ have the same truth value.