Digital Modulation

Topics

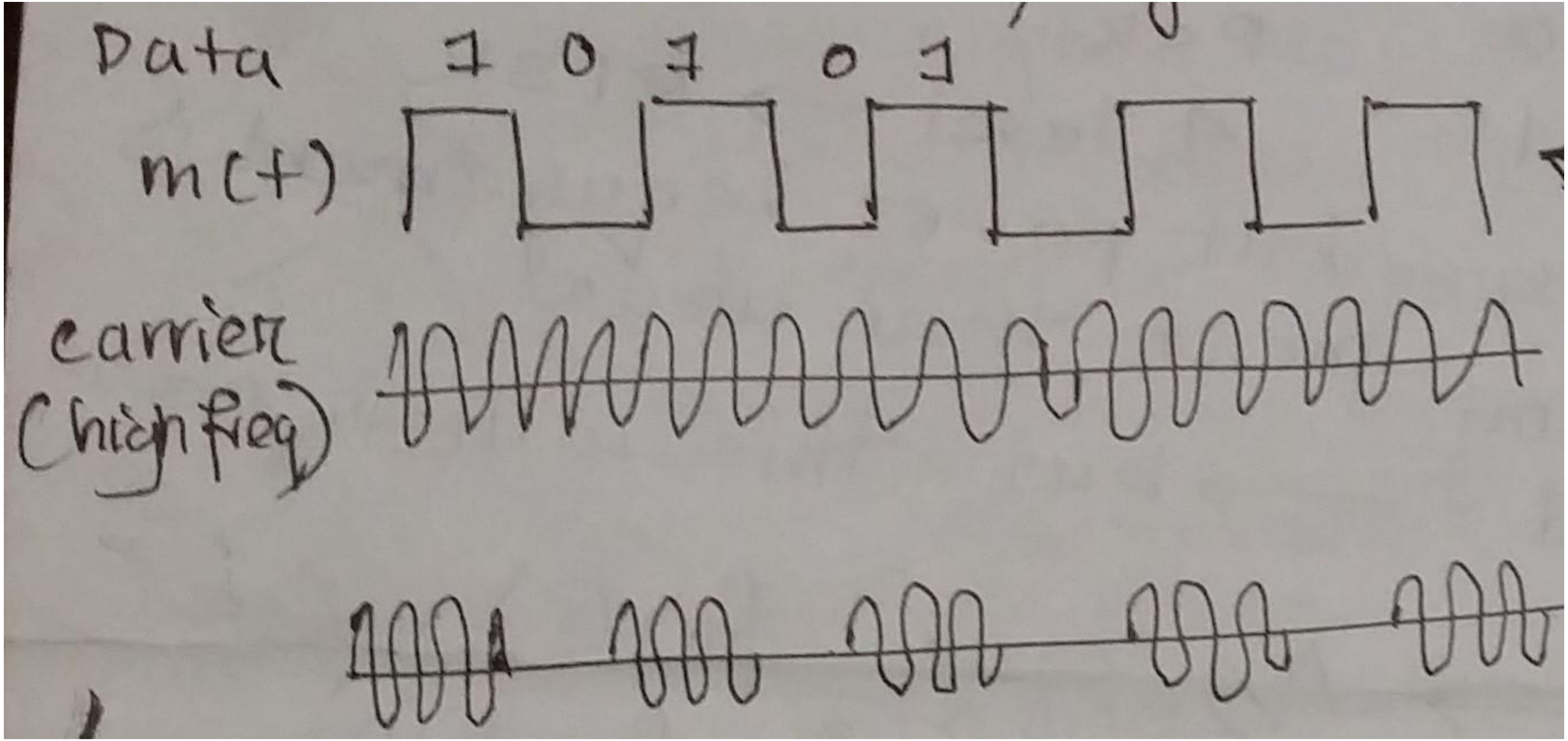
- Amplitude Shift Keying (ASK)
- Phase Shift Keying (PSK)
- Frequency Shift Keying (FSK)
- Differential Phase Shift Keying (DPSK)
- M-ary Modulation
- M-ary ASK
- M-ary FSK
- M-ary PSK
- M-QAM, QPSK, OQPSK
- Minimum Shift Keying (MSK)

Digital Modulation

Advantages over Analog Modulation:

- A Digital signal has a finite number of states while an analog signal has an infinite number of states
- Leads to easier modulation and demodulation

Amplitude Shift Keying (ASK) / On-Off Keying (OOK)



Amplitude Shift Keying (ASK) / On-Off Keying (OOK)

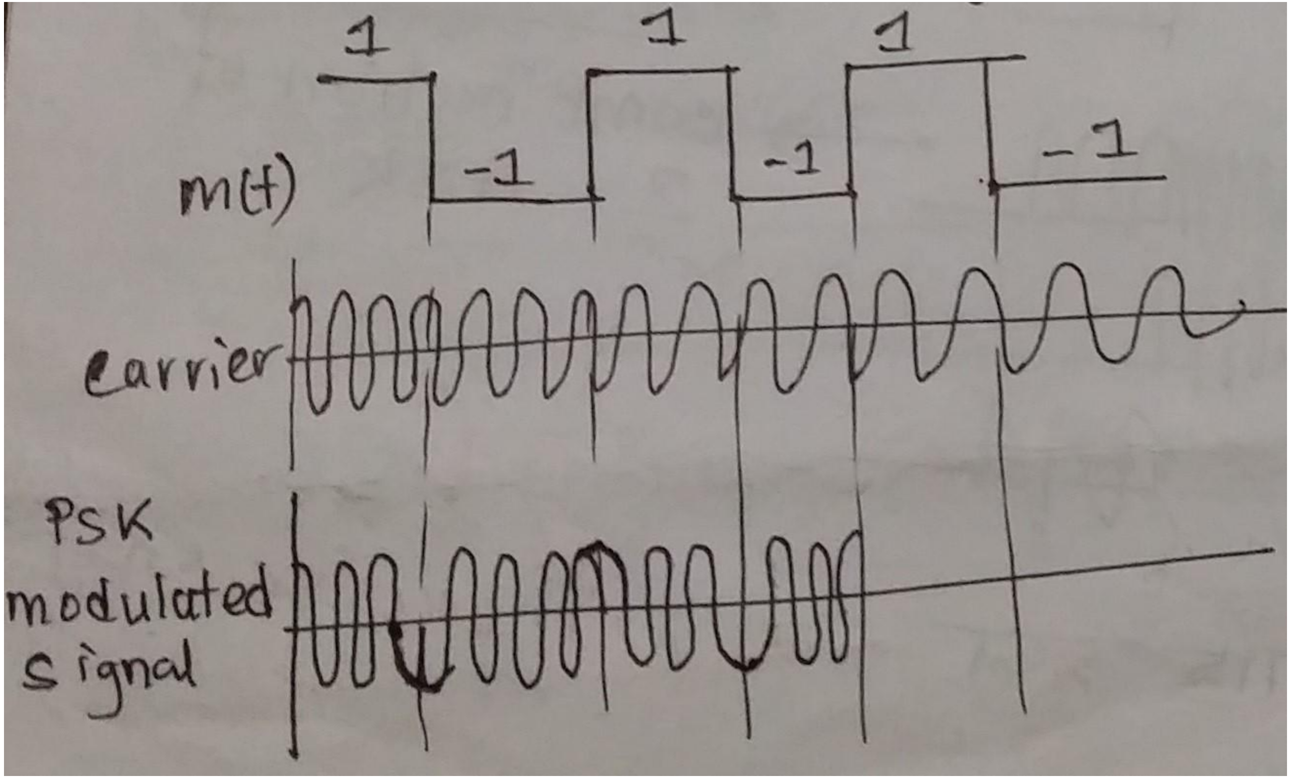
Modulation:

- When m(t)=1, send carrier as it is
- When m(t)=0, send nothing

Demodulation:

- Coherent Detection: Multiply the modulated signal with cosω_ct and then apply a filter -> synchronization is harder!
- Envelope Detection: If nothing is received, then m(t)=0 (Otherwise m(t)=1)
    - Accuracy will be similar, but this is better than coherent detection since it is less costly

Phase Shift Keying (PSK)



Phase Shift Keying (PSK)

Modulation: $A\cos(\omega_c t + \theta_k)$

- When m(t)=1, send carrier with a phase difference of 0 ($\theta_k=0$)
- When m(t)=-1, send carrier with a phase difference of π ($\theta_k=\pi$)

Demodulation:

- ***Coherent Detection: Multiply the modulated signal with $\cos\omega_c t$ and then apply a filter
- Envelope Detection: This will not work here, since the amplitude of the resultant signal is fixed (envelope will be a straight line)

Phase Shift Keying (PSK)

Demodulation:

- Multiply the signal by Acosωct

For m(t)=1 $\frac{A^2}{2} \cos \omega_c^2 t$

For m(t)=-1 $-\frac{A^2}{2} \cos \omega_c^2 t$

Then check whether the value >0 or <0

if (-1), $-A^2 \cos^2 \omega_c t$

$= -\frac{A^2}{2}(1 + \cos 2\omega_c t)$ filter out

$= -\frac{A^2}{2}$

Phase Shift Keying (PSK)

Demodulation:

- Multiply the signal by Acosωct

  For m(t)=1 $\frac{A^2}{2} \cos^2\omega_c t$

  For m(t)=-1 $-\frac{A^2}{2} \cos^2\omega_c t$

Then check whether the value >0 or <0

if (1), $A^2\cos^2\omega_c t$

$= \frac{A^2}{2}(1+\underbrace{\cos 2\omega_c t}_{\text{filter out}})$ then

we have,

$\frac{A^2}{2}$

Phase Shift Keying (PSK)

Now, $A\cos(\omega_c t + \theta_k) = A\cos\omega_c t\cos\theta_k - A\sin\omega_c t\sin\theta_k$

This is similar to the equation of QAM, but here a and b are related (In QAM, $m_1(t)$ and $m_2(t)$ were unrelated)

Let $a_k = A\cos\theta_k$, $b_k = -A\sin\theta_k$

Then we have, $A\cos(\omega_c t + \theta_k) = A\cos\omega_c t\cos\theta_k - A\sin\omega_c t\sin\theta_k$

$= a_k\cos\omega_c t + b_k\sin\omega_c t$

[QAM: $m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$]

Phase Shift Keying (PSK)

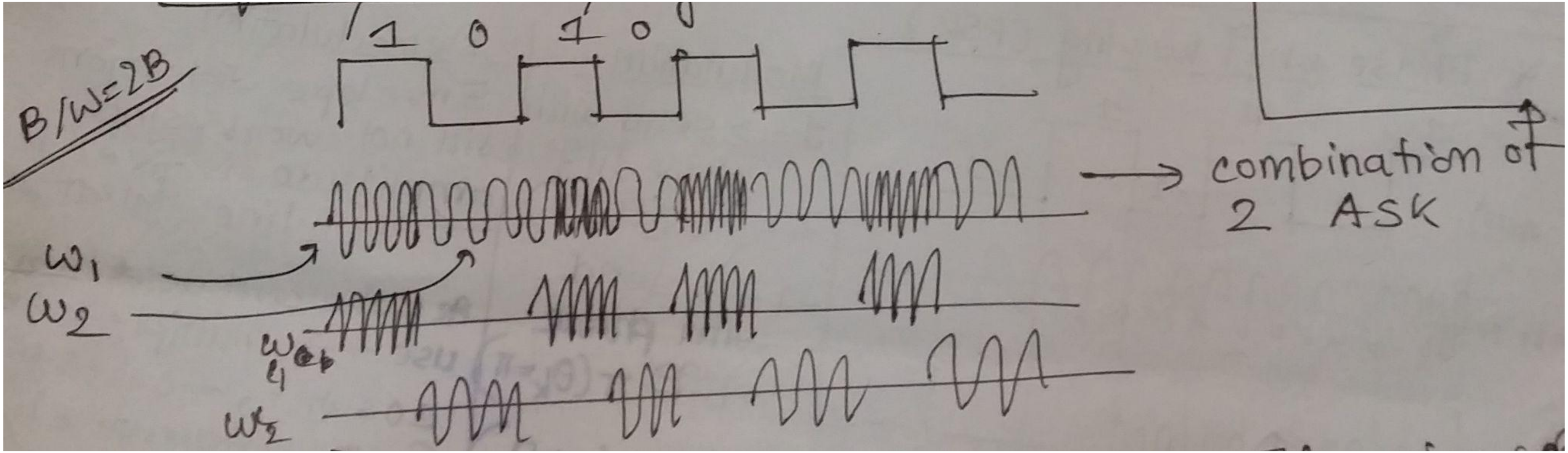
How can we increase bit rate?

- Increasing levels
  - 1-> 00   2-> 01   3-> 10   4-> 11
  - Send using PSK (4 levels -> QPSK)
  - But power requirement will be higher than usual
- Decreasing time period
  - But this will increase bandwidth

Bandwidths of ASK, PSK, FSK

- FSK -> 2B
- ASK -> 2B
- PSK -> 2B

Frequency Shift Keying (FSK)



Frequency Shift Keying (FSK)

Coherent Demodulation:

- Multiply with $\cos\omega_{c1}t$ and then use an LPF
- Multiply with $\cos\omega_{c2}t$ and then use an LPF
- Then compare the values and find out the branch which yields the higher value
- The one producing the higher value is the desired component

Frequency Shift Keying (FSK)

Envelope Detection:

- Pass through a tuned filter with frequency = ω_{c1}
- Pass through another tuned filter with frequency = ω_{c2}
- Then compare the values and find out the branch which yields the higher value
    - If the incoming signal is a component of ω_{c1} , then the first branch will produce a significant value. Similarly, if the incoming signal is a component of ω_{c2} , then the second branch will produce a significant value
    - The one producing the higher value is the desired component

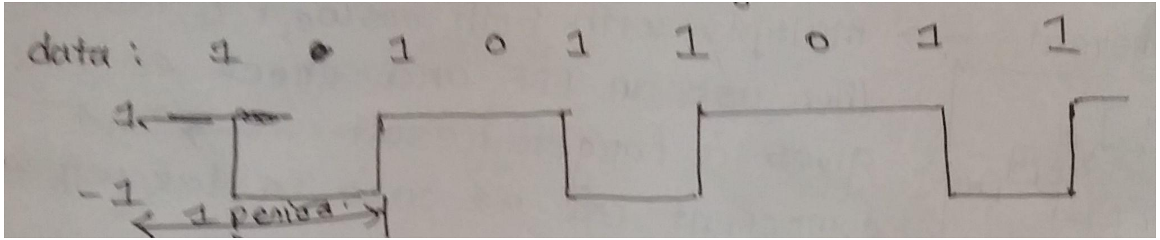
Differential Phase Shift Keying (DPSK)

PSK -> Envelope Detection is not possible

DPSK -> Envelope Detection is possible!

Differential encoder:

- If m(t) = 1, changed
- If m(t) = 0, unchanged



Signal -> Encoded Signal -> Modulation -> Demodulation

Differential Phase Shift Keying (DPSK)

Now, the encoded signal will be phase modulated

Modulation:

- When $m(t)=1$, send $A\cos\omega_c t$ (phase difference = 0)
- When $m(t)=-1$, send $-A\cos\omega_c t$ (phase difference = π)

Demodulation:

- Value at receiver end during time T = $A\cos\omega_c t$
- After 1 time period delay, check whether the value is $A\cos\omega_c t$ or $-A\cos\omega_c t$
- phase change -> 1, phase unchanged -> 0

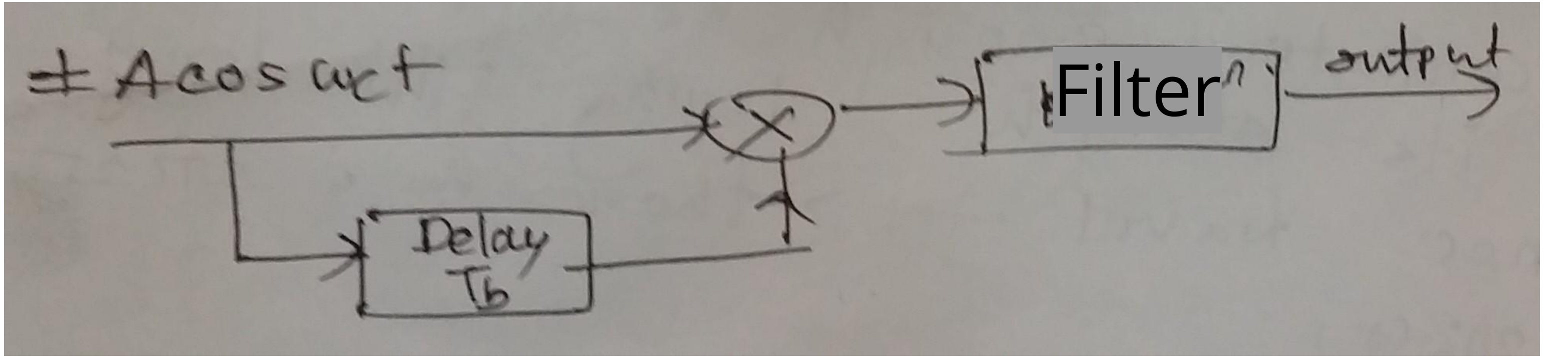
Differential Phase Shift Keying (DPSK)

Benefit:

- Carrier is not needed at the receiver end

Problem:

- Making the delay perfectly timed



M-ary Modulation

Using multiple levels to send more data at a time!

- M-ary ASK
- M-ary PSK
- M-ary FSK

M-ary ASK

- **Modulation:** Transmit $\log_2 m$ bits at a time

level	data
0	0
1	$A \cos \omega_c t$
2	$2A \cos \omega_c t$
$\vdots$	
$m-1$	$(m-1) A \cos \omega_c t$

M-ary ASK

Transmission power:

- Increase in transmission power

$$(m-1)A\cos\omega_c t$$

Power $\propto$

$$(m-1)^2 A^2$$

M-ary ASK

Demodulation:

- Envelope detection:
  - Possible (since there are m different levels)
- Coherent detection:
  - Multiply with $A\cos\omega_c t$, $2A\cos\omega_c t$, … , individually
  - Check which one produces the largest value

Problems with M-ary PSK/FSK

- How to demodulate?
- Will we need any constraints on frequency gap? -> due to noise/error
- What will happen if the number of levels in increased?
    - The gap between the levels will decrease, so error will increase

M-ary FSK

- We will need M different frequency levels
- Large frequency gap:
  - Let, frequency gap = 1 MHz
  - Problem: bandwidth increases !!!

Level	Frequency
0	100 MHz
1	101 MHz
2	102 MHz
…	…

M-ary FSK

- Small frequency gap:
  - Let, frequency gap = 0.01 MHz
  - Problem: frequency gap is too low -> more chances of error during demodulation

Level	Frequency
0	99.90 MHz
1	99.91 MHz
2	99.92 MHz
…	…

M-ary FSK

Orthogonal signals

- We can minimize error even with a smaller frequency gap
- If two orthogonal signals and multiplied and then integrated, the result will be 0
- Example: cos2πfx and cos4πfx are mutually orthogonal, but cos(2π/5)fx and cos4πfx are not mutually orthogonal

M-ary FSK

Orthogonal signals

- cosxcosx = $\cos^2 x$ -> will yield non-zero result when integrated
- If frequencies are same, then multiplication and subsequent integration will lead to a non-zero value
- If frequencies are different, then multiplication and subsequent integration will yield 0
- This will lead to a reduction in error

M-ary FSK

$\boxed{\text{Minimum Frequency Gap}}$ let the two freq be f_m & f_n.

1 period

$$* \int_0^{T_b} A\cos(2\pi f_m x) \times A\cos(2\pi f_n x)\,dx = 0$$

$$\Rightarrow \int_0^{T_b} \frac{A^2}{2} \cdot 2\cos(2\pi f_m x)\cos(2\pi f_n x)\,dx = 0$$

$$\Rightarrow \frac{A^2}{2}\left[\int_0^{T_b}\left(\cos 2\pi(f_m+f_n)x + \cos 2\pi(f_m-f_n)x\right)dx\right] = 0$$

$$\Rightarrow \frac{A^2}{2}\left[\int_0^{T_b}\cos 2\pi(f_m+f_n)x\,dx + \int_0^{T_b}\cos 2\pi(f_m-f_n)x\,dx\right] = 0$$

$$\Rightarrow \frac{A^2}{2}\left[\left[\frac{\sin 2\pi(f_m+f_n)x}{2\pi(f_m+f_n)}\right]_0^{T_b} + \left[\frac{\sin 2\pi(f_m-f_n)x}{2\pi(f_m-f_n)}\right]_0^{T_b}\right] = 0$$

M-ary FSK

$$\Rightarrow \frac{A^2}{2}\left[\frac{\sin 2\pi(f_m+f_n)T_b}{\underbrace{2\pi(f_m+f_n)}} + \frac{\sin 2\pi(f_m-f_n)T_b}{2\pi(f_m-f_n)}\right] = 0$$

Now, Here, f_m+f_n = very high (for high freq)

& so, $\sin 2\pi(f_m+f_n)T_b \approx 0-1$ so,

$$\frac{\overbrace{\sin 2\pi(f_m+f_n)T_b}^{(0-1)}}{2\pi(f_m+f_n)} \approx 0$$

$2\pi(f_m+f_n) \longrightarrow$ high value

So, $\dfrac{\sin 2\pi(f_m-f_n)T_b}{2\pi(f_m-f_n)} \approx 0$

i,e, $\sin 2\pi(f_m-f_n)T_b = 0$

$\Rightarrow \sin 2\pi\Delta f T_b = \sin n\pi$

$\rightarrow 2\cancel{\pi}\Delta f T_b = n\cancel{\pi}$

$\Rightarrow \Delta f = \dfrac{n}{2T_b}$

This is the min frequency gap, since a gap lower than this won't allow the signals to be orthogonal

M-ary PSK

PSK : $A\cos(\omega_c t + \theta)$

if $\theta \rightarrow 0$, data = 1

if $\theta \rightarrow \pi$, data = 0/-1

$= A\cos\omega_c t \cos\theta - A\sin\omega_c t \sin\theta$

Similar to QAM : $m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$

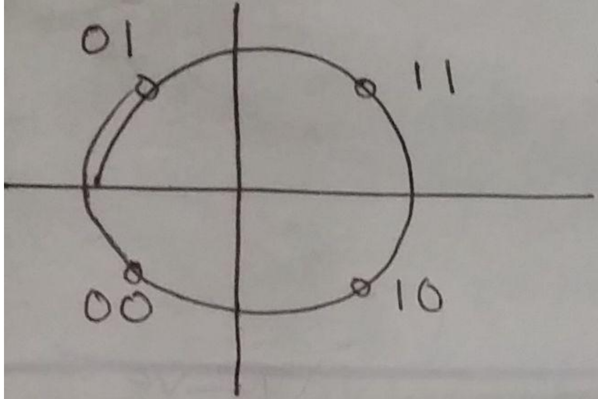
* 90° phase shift in carrier

M-PSK → if M = 4, then it is called QPSK [level = 4 / phases required = 4 & bits transmitted = 2]

* 2 bit info → 4 possible scenarios

M-ary PSK

2 bit info → divide the range from 0 to π into 4 divisions and send the 4 combinations.

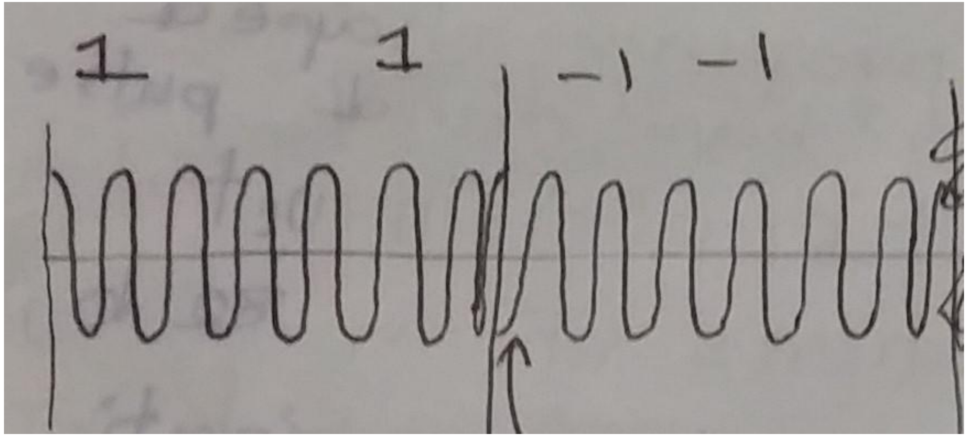


1	1
1	-1
-1	1
-1	-1

1 bit change → 90° phase shift

2 bit change → 180° phase shift

M-ary PSK



180 degree phase shift: sudden change in amplitude

M-ary PSK

→ during transmission, sudden change is not expected, we expect constant amplitude

→ How to fix this? 2 bits are completely separated. can we combine them somehow?

instead of

$(1, 1), (-1, -1) \longrightarrow (1, 1) \quad (1, -1), (-1, -1)$

1 bit change, 90° phase

This is OQPSK (Offset shift QPSK)

M-QAM

$\boxed{\text{M-QAM}}$: In QAM, we have 2 separate signals $m_1(t)$ & $m_2(t)$: $m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$

but, if we consider only one signal $\longrightarrow$

(1) 1st bit $\longrightarrow m_1(t)$

(2) 2nd bit $\longrightarrow m_2(t)$

alternatively

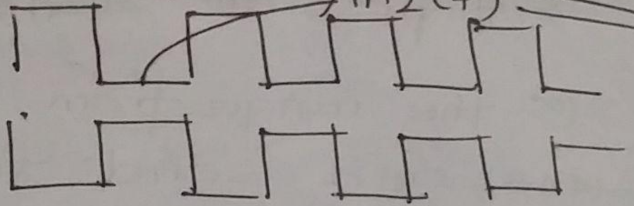
in QAM

$m_1(t)$

$m_2(t)$

$\rightarrow m_1(t)$

$\rightarrow m_2(t)$



in MSK

M-QAM

		encoding	
0	0	-1	-1
0	1	-1	1
1	0	1	-1
1	1	1	1

→ control pulse: +ve / −ve.

$m_1(t) \longrightarrow a_1 p(t)$

→ ~~normal: square pulse~~ properly shaped bare band pulse

Then, we have: $a_1 p(t) \cos \omega_c t + b_1 p(t) \sin \omega_c t$

There can be 4 possible combinations of a_1 & b_1

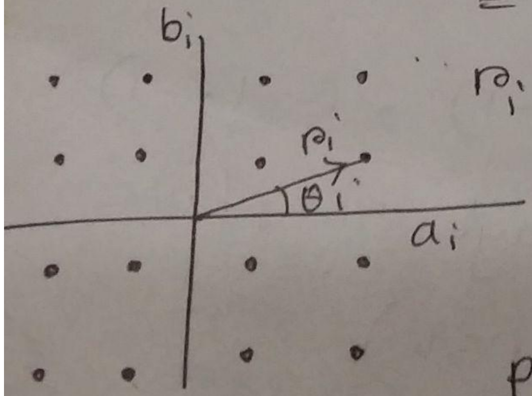
* but we can consider even more combinations.

M-QAM

Example : $M = 16$ / 16-QAM, so info $= \log_2 16 =$ 4 bits at a time.

$$P_i(t) = a_i\, p(t) \cos \omega_c t + b_i\, p(t) \sin \omega_c t \qquad i = 1 \text{ to } 16$$

$$= r_i\, p(t) \cos(\omega_c t - \theta_i)$$



$$r_i = \sqrt{a_i^2 + b_i^2} \quad \text{and} \quad \theta_i = \tan^{-1} \frac{b_i}{a_i}$$

* There are 16 possible sequences of 4 binary digits ~~ant~~, which are indicated by the 16 dots in the figure. So, every possible 4 bit seq is transmitted by a particular value of (a_i, b_i) or (r_i, θ_i)

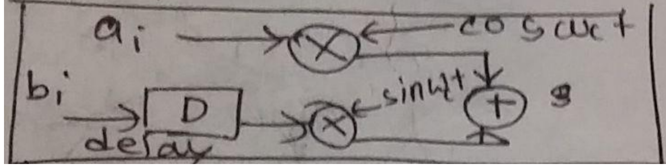
M-QAM

- The bit rate is quadrapled without increasing B/W. Similarly, the transmission rate can be increased further by increasing the value of M.

QPSK and Offset-QPSK

* QPSK → total envelope drop (π phase shift)

* Offset QPSK → some envelope drop at phase shift point



Minimum Shift Keying (MSK)

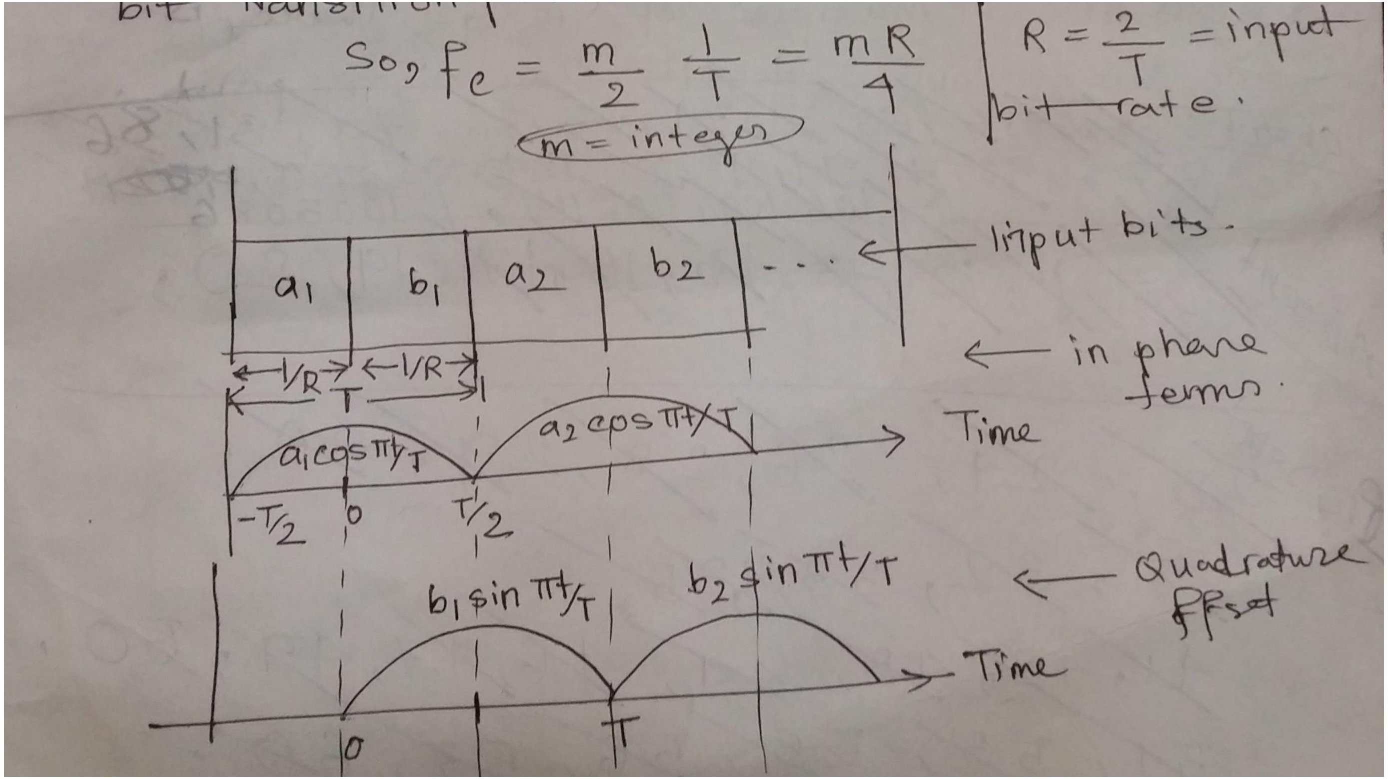
MSK → multiplies the terms ~~ai~~ by a smoothing sine wave.

takes it one step further.

$$s_i(t) = a_i \cos\left(\frac{\pi t}{T}\right) \cos w_c t + b_i \sin\left(\frac{\pi t}{T}\right) \sin w_c t \quad (1) \qquad a_i, b_i = \pm 1$$

* Due to this smoothing sine wave, each carrier is multiplied by a term going to zero at the bit transition point. If the carrier has an ~~even~~ integer # of half cycle within interval T, then there is no phase discontinuity at the bit transition points.

Minimum Shift Keying (MSK)



Minimum Shift Keying (MSK)

So, when $a_i = 1$ and $b_i = \pm 1$, (1) becomes,

$$s_i(t) = \cos\left(\omega_c t \mp \frac{\pi t}{T}\right) \quad \text{——— (2)}$$

when $a_i = -1$ and $b_i = \pm 1$, (1) becomes

$$s_i(t) = \cos\left(\omega_c t \pm \frac{\pi t}{T} + \pi\right)$$

So, $$s_i(t) = \cos\left(\omega_c t - \frac{a_i b_i \pi t}{T} + \theta\right) \quad \text{——— (3)}$$

with $\theta = 0$ if $a_i = 1$

and $\theta = \pi$ if $a_i = -1$

Minimum Shift Keying (MSK)

So, we can now deduce the following facts from eq (2) & (3),

(i) $s_i(t)$ has a constant envelope, as desired

(2) since a_i or b_i only changes every $1/R = T/2$ seconds, the maximum phase change $= \pi/2$

(3) If carrier frequency f_c is chosen to be a multiple of $R/4$, there is no discontinuity ~~at bit transition point~~.