Lecture 9

Topics

- Concept of Angle Modulation
- Phase Modulation
- Frequency Modulation
- Graphical Representation of PM and FM
- PM <-> FM Conversion

Amplitude Modulation

- **Carrier:** $A\cos(\omega_c t+\theta)$

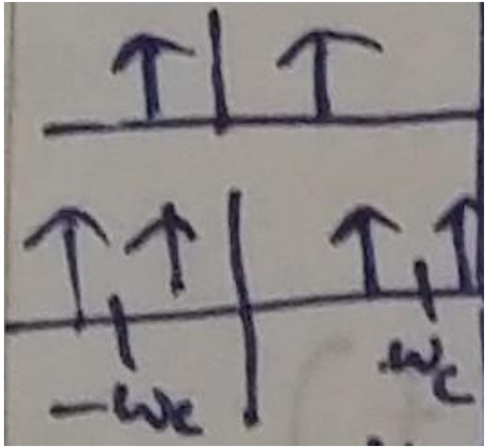
  If we consider $\theta=0$, then the carrier becomes

  $A\cos\omega_c t$

- In frequency modulation, we will change the frequency of the carrier with respect to m(t) -> $\omega_c+km(t)$

Angle Modulation

In frequency modulation, we will change the frequency of the carrier with respect to m(t) -> $\omega_c + km(t)$



Upper limit = $\omega_c + km(t)$
Lower limit = $\omega_c - km(t)$
Let, x=km(t)
If k is very small, then $\omega_c + x$ and $\omega_c - x$ will be very close -> bandwidth will be almost negligible!!
Difference (bandwidth) = 2km(t)
So, if k->0, b/w->0

Unfortunately, this is not true!!! (In reality, the bandwidth requirement is a lot more than AM, at best equal to AM!!!)

Angle Modulation

<u>Angle modulation</u> : $A\cos(\omega_c t + k m(t))$

Instantaneous Frequency: ωc changes constantly with m(t), but frequency is a periodic behaviour (at least over a full/half/quarter cycle)

<u>Example</u> → ~~Instantaneous~~ Instantaneous velocity

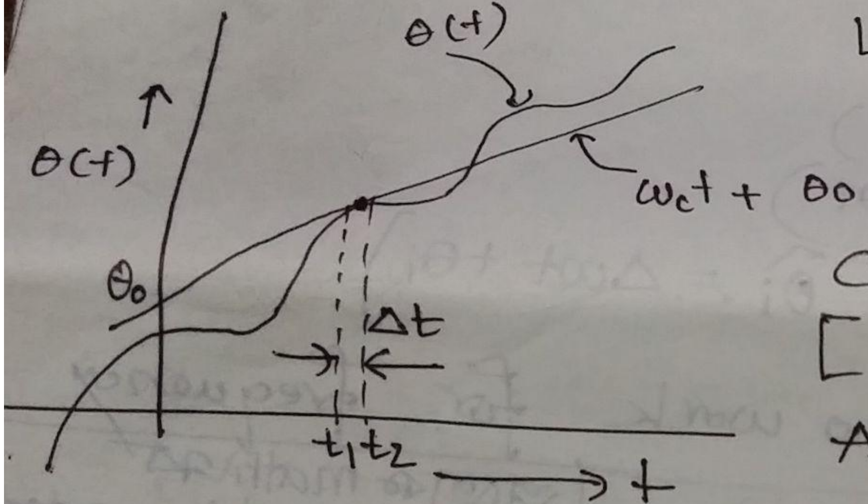
displacement, $s(t) = t^3 + t^2 + 2$

$$\Rightarrow \frac{ds(t)}{dt} = v(t) = 3t^2 + 2t$$ ← changes every second, so we use instantaneous velocity.

Angle Modulation

Instantaneous Frequency: ω_c changes constantly with m(t), but frequency is a periodic behaviour (at least over a full/half/quarter cycle)

* 1st derivative → tangent/rate of change.



Let us consider a generalized sinusoidal signal, $\varphi(t) = A\cos\theta(t)$ → generalized angle

~~Regular sinu~~

Conventional sinusoid, $A\cos(\omega_c t + \theta_0)$

[slope ω_c & intercept θ_0]

At some instant t, $\theta(t)$ & $\omega_c t + \theta_0$ coincide

Angle Modulation

over a small interval Δt, $\phi = A\cos\theta(t)$ and $A\cos(\omega_c t + \cancel{\phi}\theta_0)$

are ~~equal~~ identical $\Rightarrow$ $\phi(t) = A\cos(\omega_c t + \theta_0)$: $t_1 < t < t_2$

So, $\omega_i(t) = \dfrac{d\theta}{dt}$ [instantaneous freq]

Angle Modulation

Since $\omega_c t + \theta_0$ is tangential wrt $\theta(t)$,

at ~~the~~ that point, the slope of $\omega_c t + \theta_0$ = slope of $\theta(t)$

= instantaneous frequency

$$\theta = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

Phase Modulation

* Phase modulation ; $\theta(t) = \omega_c t + \cancel{\phi}\,\theta + k_p m(t)$

Let, θ=0

$\theta(t) = \omega_c t + k_p m(t)$

k_p → controls max/min freq

So PM wave ⇒ $\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p m'(t)$$

Phase Modulation

In PM, ω_i varies linearly with the derivative of modulated signal. $\phi_{PM}(t) = A\cos(\omega_c t + k_p m(t))$

Frequency Modulation

· FM: ω_i varies linearly with the modulated signal

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)$$

so,

$$\omega_i(t) = \omega_c + k_f m(t)$$

Frequency Modulation

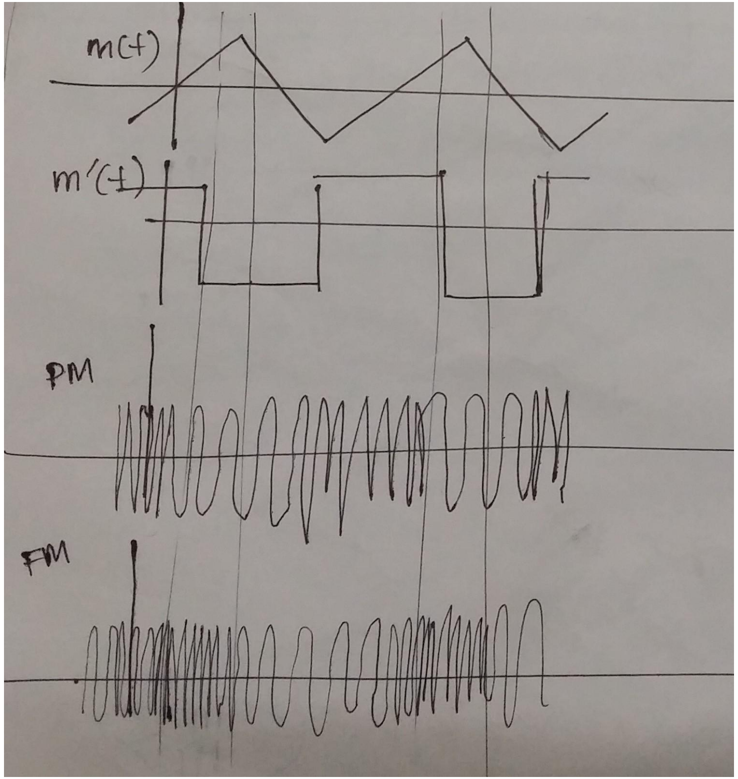
So,

$$\omega_i(t) = \omega_c + k_f m(t)$$

$$\Rightarrow \theta(t) = \int_{-\infty}^{t} (\omega_c + k_f m(\alpha))\, d\alpha$$

$$= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

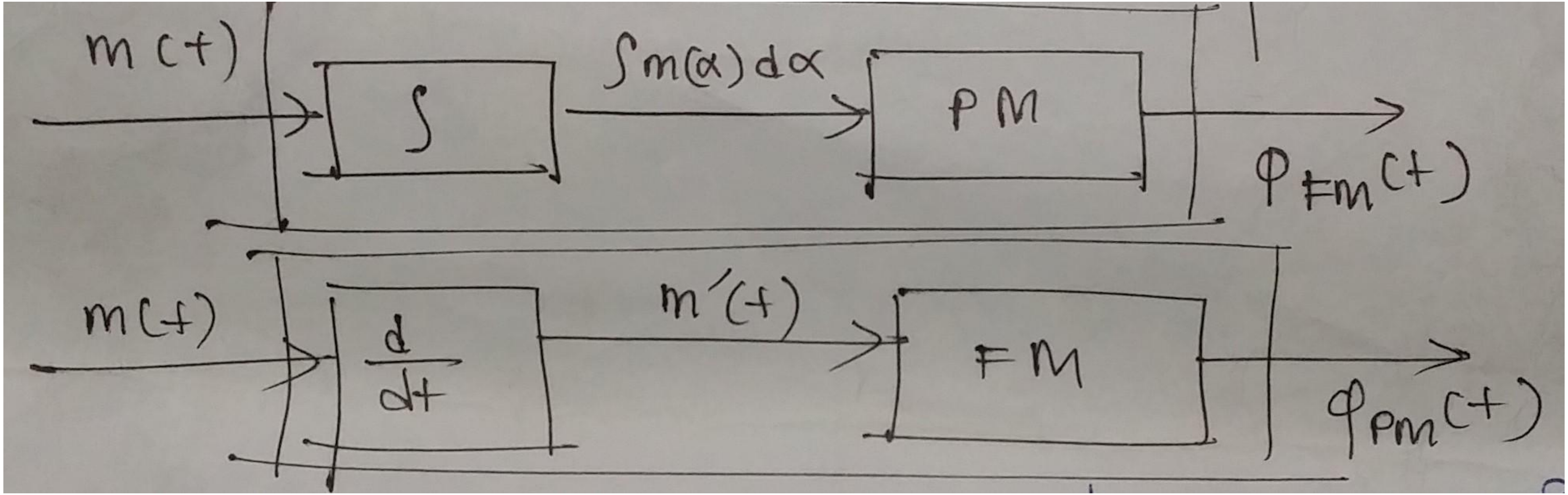
Graphical Representation of PM and FM



PM <-> FM Conversion

- Converting PM to FM -> Replace $m'(t)$ by $m(t)$
- Converting FM to PM -> Replace $m(t)$ by $m'(t)$

PM <-> FM Conversion



Angle Modulation

- Which one is better in practice?
  - Phase modulation is better than frequency modulation
  - Even better: Using an in between transfer function (in place of derivative)