

Quicksort Benchmarking README

Code Breakdown

Quicksort Implementation

```
left = [x for x in arr if x < pivot]
middle = [x for x in arr if x == pivot]
right = [x for x in arr if x > pivot]
```

python syntax

- go through the array
- check the if condition
- if true, then add that x to the array called "left"

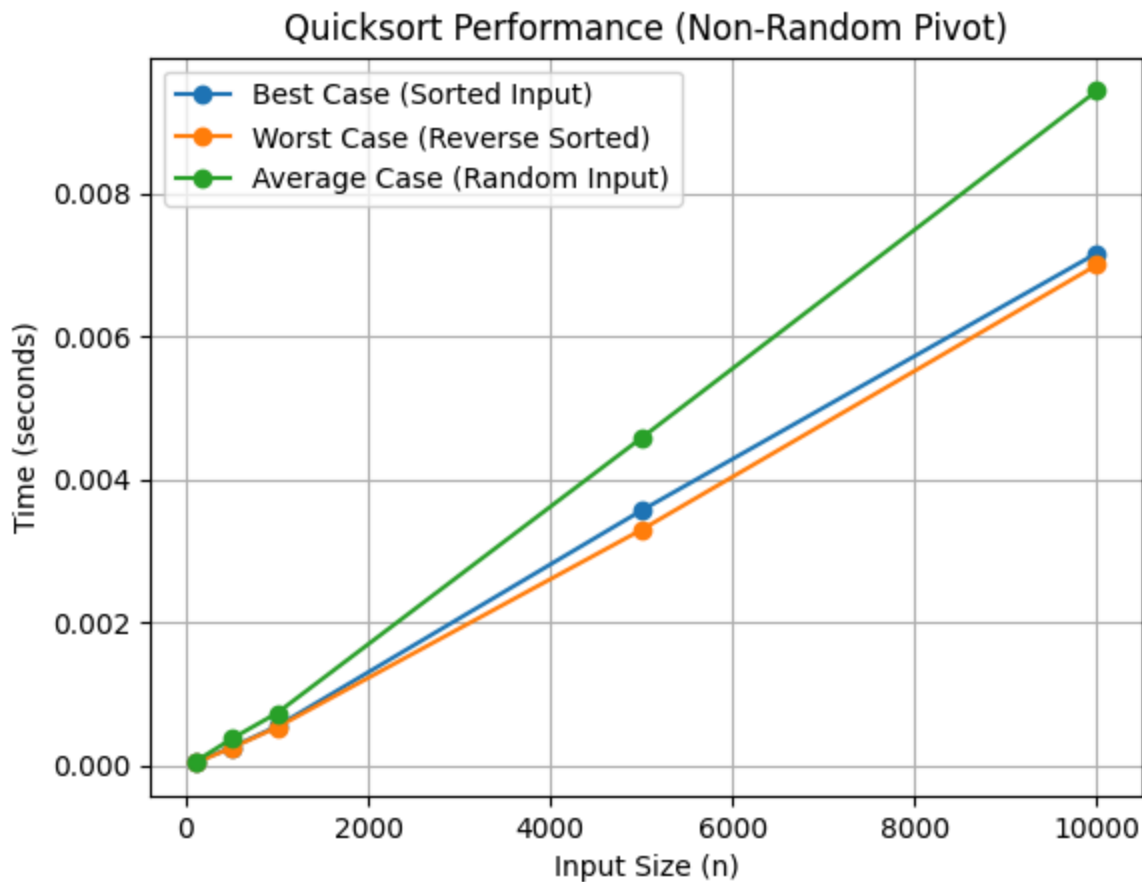
technically this states that we're making a subarray with all elements that meet the condition we're looking for.

Generating Input Cases

```
best_case = list(range(n)) # Already sorted input
worst_case = list(range(n, 0, -1)) # Reverse sorted input
average_case = [random.randint(0, n) for _ in range(n)] # Random input
```

- **Best case:** Sorted list $[0, 1, 2, \dots, n-1]$
- **Worst case:** Reverse sorted list $[n-1, n-2, \dots, 0]$.
 - as I did in a previous assignment, `range(start, stop, increment)`
 - (start at n, stop at 0, decrement by 1)
- **Average case:** Randomly generated list.

Performance Graph



3 Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

we're taking a midpoint ($/2$)

left and right subarray (2)

take the midpoint of those arrays ($1/2$)

repeat

Classic example we've gone over before

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

2t from the fact that we're doing it on left/right array and executing through another $n/2$ times. + the original array n . Specifically comparing each element to the pivot

Expanding by recurrence

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)$$

substitute back in

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + c\left(\frac{n}{2}\right)\right) + c\left(\frac{n}{2}\right)$$

$$T(n) = 4T\left(\frac{n}{4}\right) + 2cn$$

we can already see it's going roughly

$$T(n) = 2^k T(n/2^k) + kcn$$

don't care.

in our base case with 1 element, the time is 1. $T(1) = 1$

so when is it 1?

$$\frac{n}{k^2} = 1$$

$$n = k^2$$

$$k = \log_2 n$$

plug it into the equation

$$T(n) = 2^{\log_2 n} T(1) + (\log_2 n)cn$$

$$T(n) = O(n) + O(n \log n)$$

we take the biggest value

$$O(n \log n)$$