UNIVERSITY OF DHAKA

Department of Applied Mathematics
Program: B.S. (Honours) in Applied Mathematics
Year: 4th , Academic Session: 2020-2021
Course No: AMTH 450, Course Title: MATH LAB IV
Assignment-02: Solving Problems on Ordinary Differential Equations, and Partial Differential Equations with its Applications

Instruction: Write programming code using Mathematica/MATLAB to get the outputs and visualize the results of the following problems.

Name: **Roll:** **Group:**

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1. Use **DSolve** to solve the following first-order differential equations, $(i)\frac{dy}{dt}=-\frac{1}{2}y+5$, and $(ii)\frac{dy}{dt}=-y\sin t$, and then make a list of particular solutions where the values of constant $C[1]$ vary from –6 to 6 in increments of $\frac{1}{2}$. Also, Plot these particular solutions on the intervals $0<t<20$ for (i), and $0<t<6\pi$ for (ii).

2. (a) Use **NDSolve** to find the numerical solutions of the following initial and boundary value problems, $(i)\frac{dy}{dt}=\frac{1}{20}y-\frac{1}{10000}y^2, y(0)=10$, $(ii)\frac{d^2y}{dx^2}+x^2y=\cos x, y(0)=1, y'(1)=-1$; and then plot these solutions on the intervals $0<t<200$ for (i), and $0<x<20$ for (ii). Also, create a table of values for these solutions ranging from 0 to 200 in increments of 20 for (i), and from 0 to 20 in increments of 0.5 for (ii), and hence show these tabular values in graphical representations.

(b) Suppose an oscillator has an equation of motion as $mx''(t)+kx'(t)+ax^3(t)=0$. Plot the solutions in the same figure with different colors on the interval $0<t<20$, where $m=1$, $k=0.3, a=0.04, x(0)=0, and\ x'(0)=1,2,3,4,5$.

(c) Use **NDSolve** to solve the Blasius equation $f'''(\eta)+\frac{1}{2}f(\eta)f''(\eta)=0$ with $f(0)=0=f'(0)$, $f''(1)=1$, and then draw the graph of f, f', f'' in the same figure using different colours on the interval $0<\eta<5$.

3. (a) Solve the following differential equations using **ParametricNDSolve**,

$$(i)\, y''(x)+2y(x)=\alpha\sin(2x)+\cos(2y(x)) \text{ with } y(0)=\alpha, y'(0)=1,$$

$$(ii)\, y''(x)=-\beta^2y(x) \text{ with } y(0)=0, y'(1)=\beta.$$

Find the parametric function of these equations and hence sketch the graph of these functions on the intervals $0<x<5$ for (i), and $0<x<1$ for (ii) where the values of the parameter vary from $-\frac{3}{5}$ to $\frac{3}{5}$ in the increments of $\frac{1}{10}$ for (i), and from $\frac{3}{2}$ to $\frac{33}{2}$ in the increments of 3 for (ii).

(b) The Falker-Skan equation describing a laminar boundary layer with a pressure gradient is given by,

$$\frac{d^3G}{d\eta^3}+G\frac{d^2G}{d\eta^2}+\beta\left[1-\left(\frac{dG}{d\eta}\right)^2\right]=0,$$

with boundary conditions,

$$G(0)=0, \frac{dG}{d\eta}(0)=0, \frac{dG}{d\eta}(1)=\frac{1}{3}$$

Compute the parametric function of this equation using **ParametricNDSolve** and hence sketch the graph of these functions on the interval $0 < \eta < 4$ for various values of wedge angle parameter β ranging from $-\frac{1}{2}$ to 2 in the increments of $\frac{1}{2}$.

4. The Lorenz-like system at the convection threshold is given by,

$$\begin{aligned}
\frac{dX}{d\tau} &= \sigma\{Y-(1+C_1)X\} \\
\frac{dY}{d\tau} &= \rho X - Y - XZ \qquad , \\
\frac{dZ}{d\tau} &= XY - \beta Z
\end{aligned}$$

with initial conditions,

$$X(0)=1, Y(0)=5, Z(0)=10$$

Use **NDSolve and ParametricPlot3D** to draw the phase trajectories and transient behaviors of the system on the interval $0 < \tau < 60$ for different values of the couple-stress parameter C_1, i.e., for $(i) C_1 = \frac{1}{10}, (ii) C_1 = \frac{1}{5}, (iii) C_1 = \frac{1}{2}$ with $\sigma = 10, \rho = 28, \beta = \frac{8}{3}$.

5. Consider the following one-dimensional heat equation,

$$\begin{aligned}
&k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < a, t > 0, \\
&u(0,t) = T_0, u(1,t) = T_a, t > 0, \\
&u(x,0) = f(x),
\end{aligned}$$

and the solution obtained through separation of variable technique for the problem is given as below,

$$u(x,t) = v(x) + \sum_{n=1}^{\infty} b_n \sin(\lambda_n x) e^{-\lambda_n^2 kt}$$

where, $\lambda_n = \frac{n\pi}{a}, v(x) = T_0 + \frac{1}{a}(T_a - T_0)x,$ and $b_n = \frac{2}{a}\int_0^a (f(x) - v(x)) \sin\frac{n\pi x}{a} dx$.

(a) Solve the problem using the above technique,

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, 0 < x < 1, t > 0, \\
&u(0,t) = 10, u(1,t) = 10, t > 0, \\
&u(x,0) = 10 + 20\sin^2 \pi x,
\end{aligned}$$

(b) Create a table that contains the solution for $n = 1, 3, 5, 7, 10$;

(c) Animate the temperature distribution.

6. Solve the PDE:

$$\begin{aligned}
&u_{tt} = u_{xx}, 0 < x < 1, t > 0, \\
&u(0,t) = 0, u(1,t) = 0, \\
&u(x,0) = \sin \pi x, u_t(x,0) = 3x + 1,
\end{aligned}$$

(a) By using Laplace transform;

(b) Verify the solution by using **DSolve** command.

Good Luck!