UNIVERSITY OF DHAKA

Department of Applied Mathematics
Program: B.S. (Honours) in Applied Mathematics
Year: 4th , Academic Session: 2019-2020
Course No: AMTH 450, Course Title: MATH LAB IV
Assignment Topic: Solving Problems on Mathematical Modeling in Biology and Physiology with its Applications

Instruction: Write programming code using Mathematica/MATLAB to get the outputs and visualize the results of the following problems.

Name: **Roll:** **Group:**

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1. Population growth can be modeled by the equation $\frac{dP}{dt} = rP(1-P)$, $P(0) = P_0$. Solve the model equation and animate the family of curves of the solution for $0.01 \le P_0 \le 1$(thousand), and $-2 \le r \le 2$ on the time interval $0 \le t \le 20$. Also, determine the limiting value of the population when $t \to \infty$.

2. Solve the logistic equation $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$ and hence sketch the solution curves with initial population $N(0) = N_0; N_0 = 5, 50, 125, 250, 300$, and $r = 0.025, K = 125$ on the time interval $0 \le t \le 60$. Also, draw a direction field for the logistic equation.

3. The ecological predator-prey model is given by $\begin{aligned}\frac{dx(t)}{dt} &= x(\alpha - \beta y)\\ \frac{dy(t)}{dt} &= y(\delta x - \gamma)\end{aligned}$, where x is the number of prey and y is the number of predator. Solve the nonlinear system with $\alpha = \frac{2}{3}, \beta = \frac{4}{3}, \gamma = 1, \delta = 1$. Plot the solutions with $x(0) = y(0) = 0.9$ to1.5 while taking step size 0.1. Place dots on the initial points of each solution curve.

4. The Lotka-Volterra model is governed by $\begin{aligned}x'(t) &= ax(t) - bx(t)y(t)\\ y'(t) &= -cy(t) + dx(t)y(t)\end{aligned}$, where a, b, c, and d are all positive constants. Solve the nonlinear system with $x(0) = x_0$, $y(0) = y_0$, and draw the graphs of these periodic solutions for the prey and the predator of the Lotka-Volterra system with $x_0 = y_0 = 0$ to 20 in the interval $0 < t < 60$ when the values of the model parameters vary from -4 to 4.

5. Display the cobweb plots of 10 iterations of the following maps:

$$(i)\ x_{n+1} = rx_n(1 - x_n),\ x_0 = \frac{2}{10}, 0 \le r \le 4$$

$$(ii)\ x_{n+1} = x_n + rx_n(1 - x_n),\ x_0 = \frac{1}{10}, 0 \le r \le 1$$

$$(iii)\ x_{n+1} = \frac{rx_n^2}{x_n^2 + A},\ 0 \le x_0 \le 0.6, 0 \le r \le 4, A = 2$$

6. Solve the SIR model $\begin{aligned} \frac{dS}{d\tau} &= -\beta SI \\ \frac{dI}{d\tau} &= \beta SI - \gamma I \\ \frac{dR}{d\tau} &= \gamma I(\tau) \end{aligned}$ with $S(0) = 0.9999, I(0) = 0.0001, R(0) = 0, \beta = 0.5,$ $\gamma = 0.714$, and plot the results with different colour indicated the susceptible, infected, and recovered population over time ($0 \le t \le 100$). Also, stack these plots to highlight that their sum is always one.

7. Solve the SIS model $\begin{aligned} S' &= -\beta SI + \gamma I \\ I' &= \beta SI - \gamma I \end{aligned}$ with $S(0) = 0.9999, I(0) = 0.0001, \beta = 0.25, \gamma = 0.0357$, and plot the results with different colour indicated the susceptible and infected, and recovered population over time ($0 \le t \le 100$). Also, stack these plots to highlight that their sum is always one.

8. Draw the vector field, nullclines, and equilibrium points for the following autonomous systems.

(i) $\begin{aligned} x' &= 2x - y + 3(x^2 - y^2) + 2xy \\ y' &= x - 3y - 3(x^2 - y^2) + 3xy \end{aligned}$, (ii) $\begin{aligned} x' &= x(1-y) - xy \\ y' &= 2y\left(1 - \frac{y^2}{2}\right) + 3x^2 y \end{aligned}$, (iii) $\begin{aligned} x' &= x(1-x) - xy \\ y' &= 2y\left(1 - \frac{y^2}{2}\right) - 3x^2 y \end{aligned}$,

(iv) $\begin{aligned} x' &= 2x\left(1 - \frac{x}{2}\right) - xy \\ y' &= y\left(\frac{9}{4} - y^2\right) - x^2 y \end{aligned}$, (v) $\begin{aligned} x' &= x(1-x) - xy \\ y' &= 2y\left(1 - \frac{y^2}{2}\right) - 3xy \end{aligned}$, (vi) $\begin{aligned} x' &= 5\left(y + x - \frac{x^3}{3}\right) \\ y' &= 0.2\left(x + 0.7 - 0.5y\right) \end{aligned}$.

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