Distance-based Interpolation and Extrapolation Methods for RSS-based Localization with Indoor Wireless Signals

Jukka Talvitie, Markku Renfors, *Fellow, IEEE*, and Elena Simona Lohan, *Member, IEEE*

Abstract—**Wireless Local Area Network (WLAN) based fingerprinting using Received Signal Strengths (RSS) has been considered as one solution for indoor positioning. However, one widely recognized problem in fingerprinting is the collection and maintenance of a proper fingerprint database. In this paper we consider having an incomplete fingerprint database with realistic coverage gaps and we study the performance of several interpolation and extrapolation methods for recovering the missing fingerprint data. For this purpose, we have collected an extensive set of data at 2.4GHz and 5GHz frequency bands from one university building with four floors. The accuracy of the interpolation and extrapolation methods is studied by artificially removing fingerprints from the database using a randomized procedure, and by comparing the estimated fingerprints with the original ones. The average RSS estimation error of different interpolation and extrapolation methods is shown for various percentages of missing fingerprints. In addition, a cumulative RSS error distribution is studied in order to reveal the dispersion of the error statistics, which affect the user positioning accuracy. Here, the user positioning accuracy is defined in terms of horizontal positioning error and floor detection probability. The user positioning accuracy is also compared in four cases, namely when using the original fingerprints, the partial fingerprints, the interpolated fingerprints, and the interpolated and extrapolated fingerprints. It is shown that both the horizontal positioning accuracy and the floor detection probability can be improved with proper interpolation and extrapolation methods. However, it is also illustrated that the best positioning performance is not necessarily achieved with the best average interpolation and extrapolation accuracy, but it is important to avoid certain type of errors in the interpolation and extrapolation process.**

Index Terms—**Coverage area, Extrapolation, Fingerprinting, Indoor positioning, Interpolation, Received Signal Strength (RSS), Wireless Local Area Network (WLAN)**

I. INTRODUCTION

LOW-COST indoor positioning has become one of the most important research topics in recent years and the emerging market of indoor positioning applications is continuously seeking for novel and efficient positioning solutions. One widely recognized approach for indoor positioning is to exploit IEEE 802.11 standard [1] for Wireless Local Area Networks (WLAN). The market of WLANs has continued to increase rapidly and it has made the WLANs widely available in urban areas, especially indoors. Besides the network availability, the growth of WLAN-equipped mobile devices has made WLAN-based positioning even more appealing, since it has become more likely that the users are already carrying the required positioning technology with them. Moreover, WLAN signals are nowadays transmitted on multiple frequency bands (e.g. 2.4GHz and 5GHz), which may ensure also a certain form of diversity in positioning the wireless users.

Exploiting the Received Signal Strength (RSS) measurements from WLAN Access Points (AP) is one of the most common WLAN based positioning method [2]-[21]. In fact, RSS-based positioning is not limited to WLANs, but it can be applied also with cellular [22]-[23], RFID [24]-[25], Bluetooth [26], or ZigBee [27] signals. From this point of view, our work in here is not limited to WLAN signals, but it could find its usefulness as well for any other wireless signals used for localization via RSS. One option for RSS-based positioning is to exploit the WLAN Access Point (AP) locations and the radio propagation path loss models or more advanced ray-tracing algorithms with building floor plans. However, since the AP locations and the building floor plans are not usually accessible, it makes these approaches less attractive from the global availability point of view. The other option is to collect enough learning data and to use it to estimate the AP positions and the path loss models, or to create fingerprints, which can be used in positioning by comparing the signals measured by the user with the pre-collected data. In this paper, we consider traditional fingerprinting, where the learning data is comprised of fingerprints as done in [10]-[18], because the fingerprinting methods have been traditionally shown to give more accurate positioning results than the probabilistic methods based on path loss modeling [12]-[15],[28]. Nevertheless, fingerprinting requires usually significant efforts to construct and maintain an adequate fingerprint database, but on the other hand, the learning data can be collected and used without any extra knowledge about the physical environment.

In many cases, the collected data does not necessarily cover

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the whole target area, leaving some of the areas in total blackness from positioning point of view. For example, this might occur, if some areas, such as office rooms and apartments, are restricted or inaccessible for the persons (or devices) collecting the data. In addition, parts of the learning data might get outdated or lose their integrity, which can lead to exclusion of the data. In all the cases, some gaps may appear in the fingerprint coverage area. One of the main focuses of this paper is on studying the applicability of different floor-wise interpolation and extrapolation methods for recovering the information of missing fingerprints. Interpolation and similar type of processing of RSS based fingerprints has been earlier studied in [2]-[6] and [8], but none of these have tested the algorithms with large and comprehensive networks as those provided in this paper. For example, instead of using specific network arrangements with only 6 or 10 APs as in [5] and [4], our results are based on extensive measurement campaign with 422 observed APs including more than half a million RSS measurements. In addition, extrapolation methods have usually not been addressed, or addressed in a very basic form (e.g., padding with constant values), and yet the extrapolation step is a crucial step into filling the gaps from the measurement phase.

In order to study the performance of different interpolation and extrapolation methods, a specific randomized process for artificial fingerprint removal, introduced by the authors in [29], is used for generating realistic gaps in the fingerprint coverage area. The results in [29] have demonstrated that the fingerprint removal method has a significant impact on the positioning accuracy, thus we take it into account in our current studies. Since the RSS values of removed fingerprints are known, they can be used as reference points when comparing the accuracy and error statistics of different interpolation and extrapolation methods. Results are evaluated by means of average RSS estimation error and cumulative error distribution for all considered interpolation and extrapolation methods. Moreover, the additional objectives of this paper are: to study the effect of the given fingerprint recovery techniques on user positioning accuracy, to provide an applicable approach for fingerprint removal scheme, to study and evaluate a gradient-based extrapolation in context of WLAN signals, delivering results from real-life large scale WLAN networks (not only a few APs), and to provide comparative analysis in two different frequency bands (2.4GHz and 5GHz). It is shown here that the positioning error can be noticeably decreased with proper interpolation and extrapolation methods, especially in case where most of the fingerprints have been removed from the database. This is a new and important finding, not necessarily intuitive, and it can help the location service provider to choose how to collect the fingerprint data with minimum effort, but still ensuring a desirable target performance.

The remainder of this paper is organized as follows: Section II introduces the measurement set-up and the structure of the used fingerprint database. In addition, the used fingerprint removal scheme for interpolation and extrapolation performance analysis is provided. The considered interpolation and extrapolation techniques are presented in detail in Section III and their estimation accuracy is further analyzed in Section IV. After this, in Section V, the user positioning accuracy with the considered interpolation and extrapolation methods are compared with each other. Finally, the conclusions are drawn in Section VI.

II. MEASUREMENT SET-UP AND FINGERPRINT DATABASE

WLAN measurements were taken with a Nexus 7 tablet by ASUSTeK Computer Inc. (ASUS) in a four-floor University building in Tampere University of Technology campus, in Tampere, Finland, between December 2013 and March 2014. The collected data consists of a set of fingerprints, each including coordinates and RSS values for the heard APs. To the best of our knowledge most of the APs inside the building support at least the 802.11-2012 standard [1] including, for example, amendments 802.11a/b/g and 802.11n. All the measurements considered in this paper are taken at 2.4GHz and 5GHz carrier frequencies and the fingerprints are constructed separately for the both bands. The fingerprint coordinates were manually defined by the user with the help of a building floor plan and dedicated measurement software. The maximum error between fingerprint coordinates and true measurement coordinates is roughly evaluated to be around 1 meter which is adequate for the expected positioning accuracy. Moreover, deviation of RSS values due to body loss and other environment variations introduces considerably larger errors for the learning data and positioning algorithms compared to relatively small reference coordinate errors. The floor plan and the coordinates of collected fingerprints for all 4 floors are illustrated in Fig. 1.

A. Fingerprint database structure

Fingerprints are defined as (x_i, y_i, z_i, $\mathbf{P}_i$), where i=1… N_{FP} is the fingerprint index, x_i, y_i and z_i are the x, y, and z coordinates of the fingerprint, $\mathbf{P}_i$ is a 1xN_i vector of RSS measurements for a set of N_i heard APs in point i and N_{FP} is the number of fingerprints in the database. The RSS measurement vector at i^{th} fingerprint can be further described as

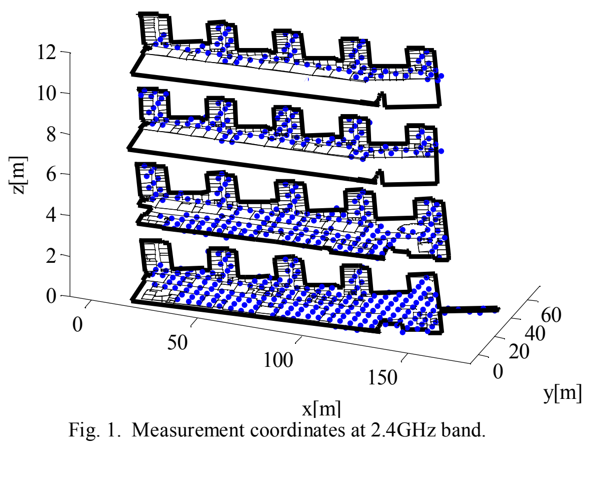

Fig. 1. Measurement coordinates at 2.4GHz band.

$\mathbf{P}_i = \begin{bmatrix} P_{ap_{i,1}} & P_{ap_{i,2}} & \cdots & P_{ap_{i,N_i}} \end{bmatrix}$, where $P_{ap_{i,j}}$ is the RSS value of the j^{th} heard AP and at i^{th} fingerprint. The index $ap_{i,j} \in [1, N_{AP}]$ corresponding to each fingerprint and each heard AP in that particular fingerprint refers to a unique AP identity based on the MAC (Media Access Control) address of the AP and N_{AP} is the total number of APs in the database ($N_i \leq N_{AP}, \forall i$ and the set of $\{ap_{i,j}\}$ has elements with values between 1 and N_{AP}). The number of found individual MAC addresses in the measurement campaign was 316 addresses at 2.4GHz band and 106 addresses at 5GHz band. In this paper, the fingerprint database is constructed separately for both of the bands. Furthermore, fingerprints from separate frequency bands are not interacted with each other, and thus, the frequency band indices are left out from the database structure definitions for the sake of simplicity.

To avoid unnecessary overlapping between fingerprints very close to each other, it is common to merge information of these fingerprints by constructing a uniform rectangular grid with certain grid interval g_{int} as done in [10] and [11]. The same principle is also exploited in [12]-[15] , but without the assumption of a rectangular and uniform grid. Here, each measured fingerprint is mapped into the closest available grid point. In the case when multiple RSS measurements from the same AP are mapped into the same grid point, an arithmetic mean over the RSS values is taken. For the sake of clarity, these grid points with merged fingerprint data are still referred as fingerprints, since their structure and purpose do not differ from the original definition.

B. Fingerprint Removal Process

To be able to study the possibilities of interpolation and extrapolation methods, some of the measured fingerprints are desired to be removed from the database. The fundamental goal is to simulate a situation where learning data collection does not cover the whole operating area. Now, it is essential that fingerprints are not removed randomly using a simple uniform probability distribution, since in average this would result into a case where there would still be some learning data available in every part of the original coverage area. In practical cases though, some data is usually missing from larger spaces, such as whole rooms or corridors, at a time. Hence, the idea is to remove adjacent fingerprints in larger blocks and to introduce much larger and more realistic

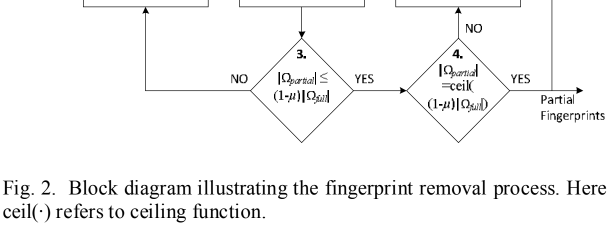

Fig. 2. Block diagram illustrating the fingerprint removal process. Here ceil(·) refers to ceiling function.

coverage gaps in the fingerprint database. In this paper, the interpolation and extrapolation are considered only horizontally in floor-wise manner (2D), since the radio propagation properties are drastically changed in the vertical direction. However, the methods presented here are also applicable in vertical direction with minor modifications.

Separate floors are considered as individual entities from the data collection point of view. Thus, the given fingerprint removal process is defined in a floor-wise manner, similar to the interpolation and extrapolation methods presented further in Section III. The proposed fingerprint removal process is completely defined by the two following parameters: removal percentage μ and removal block radius d_{block}. Here, μ defines the percentage of removed fingerprints with respect to the initial number of fingerprints, and d_{block} defines the radius (in meters) of a circular area, referred as a block, where fingerprints are removed. Here, d_{block} outlines roughly the average size of coverage gaps introduced by the removal process.

Defining the set of original fingerprints as Ω_{full}, the target is to determine the set of fingerprints after the removal process denoted as $\Omega_{partial}$, so that $|\Omega_{partial}|=(1\text{-}\mu)|\Omega_{full}|$, where the operator $|\cdot|$ defines the size of the set, i.e., the number of fingerprints. By initializing the partial database as $\Omega_{partial}=\Omega_{full}$, the removal procedure to fulfil the above condition and to include property of removing fingerprints in larger blocks can be described as follows:

1) Select randomly one fingerprint (x_i, y_i, z_i, $\mathbf{P}_i$) from $\Omega_{partial}$ using uniform probability distribution
2) Remove all fingerprints whose Euclidian distance in horizontal plane (the xy-plane) to the randomly selected fingerprint (x_i, y_i, z_i, $\mathbf{P}_i$) is smaller or equal to the block radius parameter d_{block}. The preserved fingerprints are now defining the new partial database $\Omega_{partial}$.
3) Check if the desired removal percentage is satisfied by $|\Omega_{partial}| \leq (1\text{-}\mu)|\Omega_{full}|$. If yes, continue to the part 4. Otherwise, go back to the part 1.
4) In case $|\Omega_{partial}|=(1\text{-}\mu)|\Omega_{full}|$ is not satisfied with one fingerprint accuracy, retrieve a required number of the fingerprints from the last removed block starting from the fingerprints with largest distance to (x_i, y_i, z_i, $\mathbf{P}_i$).

After the above-mentioned procedure, the value of μ is given with accuracy of one fingerprint. The process is further illustrated in a block diagram shown in Fig. 2. Furthermore, as an example, the fingerprint database with all measurements regarding the first floor is given in Fig. 3 with grid interval of g_{int}=2m. The corresponding partial database after the removal process is then shown in Fig. 4, where 30% of the fingerprints are removed using the block radius of d_{block}=10m. It is worth of noticing that this is just one possible outcome of the random removal process.

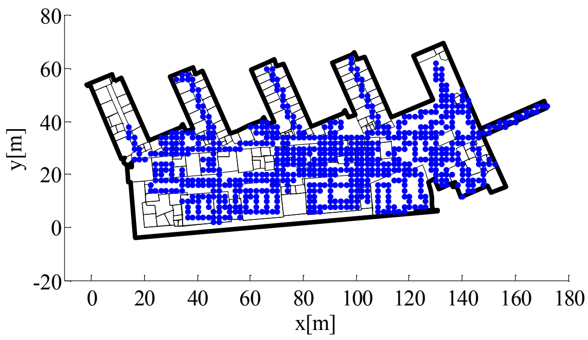

Fig. 3. Original full fingerprint database (Ω_{full}) with g_{int}=2m for the first floor at 2.4GHz band.

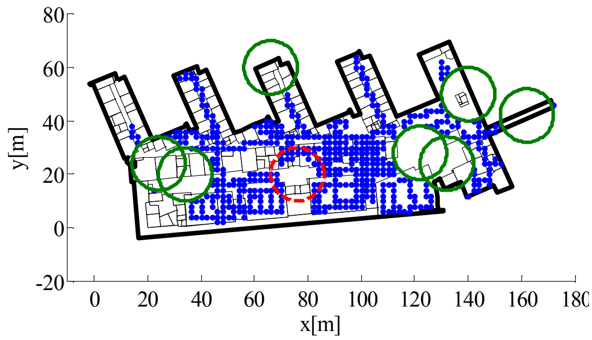

Fig. 4. Example of partial fingerprint database ($\Omega_{partial}$) with, μ=30%, d_{block}=10m, and g_{int}=2m for the first floor (shown in Fig. 3). Here the circles indicate the removed areas of the removal process. The red and dashed circle is the last removed area, where part of the fingerprints has been retrieved in order to satisfy the desired μ.

III. Interpolation and Extrapolation Methods

RSS measurements of an individual AP in one floor create a surface, whose shape is dependent on the radio wave propagation environment. Besides the distance from the transmitter, RSS values are affected by walls and obstacles along with reflections and diffractions in the radio path. For this reason, fingerprints are able to provide more accurate information about the location compared to simple path loss models, especially indoors. In this paper both interpolation and extrapolation are performed only in floor-wise manner because of the differences in radio propagation properties between horizontal and vertical directions. To the best of our knowledge, full 3D interpolation and extrapolation for fingerprint recovery have not been addressed in the literature.

The fingerprint recovery process consists of interpolation and extrapolation of fingerprints for each AP and floor separately. In other words, for each AP and each floor a set of fingerprints denoted as (x_j, y_j, $P_{ap_{j,n}}$) are extracted from the database, where x_j, y_j, and $P_{ap_{j,n}}$ are x coordinate, y coordinate, and the RSS of j^{th} fingerprint for the n-th considered AP and for the considered floor (the floor index has been dropped here for clarity). Notice that now the index j collects all fingerprints from one floor for one AP and the z coordinate can be neglected, since we focus at one floor at the time. As the coordinates x_j and y_j are now located in a rectangular grid, the target of the interpolation is to estimate the RSS values in the grid points between the known fingerprints, and more precisely, inside the convex hull defined by the fingerprints. The convex hull describing the interpolation area along with the definition of extrapolation area is illustrated in Fig. 5. As shown in Fig. 5, the extrapolation area is defined by the area given by the rectangular fitted in the outer limits of original fingerprint coordinates. It should be noticed that in practice, the extrapolation area must be defined without the knowledge of original fingerprints, for example, using reasonable approximation of the maximum coverage of the AP.

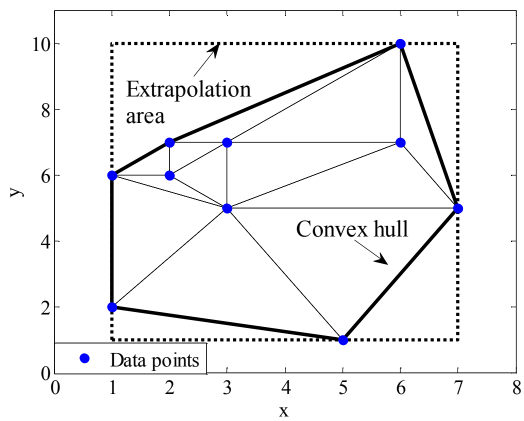

Fig. 5. Illustration of interpolation and extrapolation areas and the considered Delaunay triangulation for data points (fingerprints) originated from one imaginary AP in one floor.

The considered interpolation and extrapolation approaches are divided into two separate categories. In the first category, the interpolation is performed linearly as described further in Part A.1), and the extrapolation is followed by either minimum method, mean method or gradient method as described in Parts A.2), A.3) and A.4), respectively. In the second category the interpolation and extrapolation are performed jointly using approaches based on nearest neighbor method and Inverse Distance Weighting (IDW) method, described in Part B.

A. Disjoint interpolation and extrapolation

Whereas the interpolation refers to filling the gaps between known data points, extrapolation tries to estimate the data outside the known data points. Therefore, extrapolation is, of course, much more challenging and it has to be carried out with great caution to avoid generating extensively misleading data. As the interpolation task is less challenging, a straightforward linear interpolation method is used in all disjointed interpolation and extrapolation approaches, whereas the extrapolation method varies between three considered approaches: minimum method, mean method and gradient method.

1) Linear interpolation

We denote Ω_{AP} as a set of fingerprints $\left\{x_j, y_j, P_{ap_{j,AP}}\right\}_j$ for one specific AP and floor defining the available data for each interpolation and extrapolation procedure. The used linear interpolation method is based on Delaunay triangulation [30], also referred as Delaunay tessellation. The Delaunay triangulation is illustrated in Fig. 5., where the dots represent the fingerprint coordinates (x_j, y_j). Compared to other known data independent triangulation methods, Delaunay triangulation maximizes the observed minimum angle of the tessellation, and thus, results in smoother surfaces. The actual interpolation method exploiting the Delaunay triangulation can be performed in several ways. However, in this paper only the linear interpolation is considered due to uncertainty of the radio propagation environment and relatively large gaps in the data covered by the interpolation. For example, parameters of an indoor radio propagation model can vary considerably between different locations due to walls and other obstacles. Therefore, defining the curvature of the surface model for more advanced interpolation methods can be very challenging, which makes the linear interpolation a reasonable choice.

The first step in linearly interpolating the RSS value for an unknown coordinate point (x,y) is to discover in which tessellation triangle the point belongs to. After this, denoting the triangle vertices as (x_1, y_1, P_1), (x_2, y_2, P_2) and (x_3, y_3, P_3), the interpolated RSS value can be calculated as

$$P_{interp} = \lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 \tag{1}$$

where λ_1, λ_2, and λ_3 are barycentric coordinates of (x,y) on the triangle and $\lambda_1+\lambda_2+\lambda_3=1$. Consequently, the barycentric coordinates can be considered as weights for the known RSS values in the triangle vertices, and they can be calculated as

$$\begin{aligned} \lambda_1 &= \frac{(y_2-y_3)(x-x_3)+(x_3-x_2)(y-y_3)}{(y_2-y_3)(x_1-x_3)+(x_3-x_2)(y_1-y_3)}, \\ \lambda_2 &= \frac{(y_3-y_1)(x-x_3)+(x_1-x_3)(y-y_3)}{(y_2-y_3)(x_1-x_3)+(x_3-x_2)(y_1-y_3)}, \text{ and} \\ \lambda_3 &= 1-\lambda_1-\lambda_2. \end{aligned} \tag{2}$$

The accuracy of the linear interpolation method is greatly affected by the shape of triangles in the tessellation, and therefore, by the geometry of known data point locations.

2) Minimum Method Extrapolation

In the minimum method extrapolation all the extrapolated RSS values are estimated using a single constant defined as the minimum of the observed RSS values of the considered AP and floor as

$$\begin{aligned} P_{extrap,AP} &= K_{AP}, \text{ where} \\ K_{AP} &= \min_i \left\{ P_{ap_{i,AP}} \right\} \end{aligned} \tag{3}$$

for each AP. Since the minimum method extrapolation is performed using a constant value, it is not able to reach the average accuracy of the gradient based method, presented in Part 4), with moderate fingerprint removal percentage, as seen later in Section IV. However, as stated further in Section V, from the positioning accuracy point of view, the minimum method is able to provide better performance than the gradient method, since the errors originated from these are not as critical as the errors originated from the gradient method.

3) Mean Method Extrapolation

Similar to the minimum method extrapolation, the mean extrapolation method estimates the missing RSS values with a single constant, but in this case using a mean of the observed RSS values as

$$\begin{aligned} P_{extrap,AP} &= K_{AP}, \text{ where} \\ K_{AP} &= \frac{1}{N}\sum_i P_{ap_{i,AP}}, \end{aligned} \tag{4}$$

where N is number of RSS values $P_{ap_{i,AP}}$. Whereas the mean value extrapolation can generate better estimates near the true AP location, the minimum value extrapolation works generally better when the extrapolated area is far from the true AP location. As seen later in Section IV, the mean extrapolation method provides smaller average error for the RSS estimates than the minimum method, but the error distribution is different. With the mean method extrapolation the error distribution is concentrated in smaller range resulting in smaller RSS error dynamics.

4) Gradient Method Extrapolation

In [31] the triangulated surface is extended to semi-infinite rectangles on the convex hull exterior, which makes the linear approximation discussed earlier in Part 1) applicable. Similar approach was earlier used in [32] as part of the interpolation solution for a smooth surface fitting. Nevertheless, to create a globally defined extension of the surface outside the convex hull, it is intuitive to concentrate on gradients at the surface edges. When considering indoor RSS measurements, the surface is not expected to be smooth, but it is expected to vary strongly because of walls and other obstacles. This means that purely relying on the convex hull exterior is not practically reasonable. Also, since the process is done separately for each AP found in the fingerprint data set, computational complexity is desired to be maintained at reasonable level. Therefore, the following simplified gradient based method inspired by [31] and [32] is derived for the gradient based extrapolation.

The gradient at point (x_g, y_g, P_g) on the surface edge is defined as $\nabla P = (\Delta x, \Delta y)$, where Δx and Δy are the gradient component values in direction of x and y coordinate axis. Now, based on both x-component and y-component gradients, we can calculate component-wise RSS value estimates at point (x,y) outside the convex hull as

$$\begin{aligned} P_x &= \Delta x\left(x-x_g\right)+P_g \\ P_y &= \Delta y\left(y-y_g\right)+P_g \end{aligned} \tag{5}$$

where P_x and P_y are the estimates of the extrapolated RSS value based on x-component and y-component gradients, respectively. Finally, the extrapolated RSS estimate is approximated by taking an average of the two component estimates as $P_{extrap}=(P_x+P_y)/2$. Because of high deviation of RSS values in fingerprinting due to harsh radio propagation environment, it is beneficial to include gradient information from multiple data points. By this way it is possible to avoid errors in the extrapolation process caused by high RSS deviation in the convex hull exterior. In this paper, based on performed coarse algorithm optimization, the extrapolated RSS values are calculated by taking an average of separate gradient based estimates from three closest data points with known gradients.

B. Joint interpolation and extrapolation

Apart from the two-fold process using separate interpolation and extrapolation algorithms, some algorithms are not making any difference whether the estimated point is inside or outside of the convex hull. This type of joint interpolation and extrapolation approach is naturally included, for example, in nearest neighbor method and IDW method, which are presented next.

1) Nearest neighbor method

With nearest neighbor method, further referred solely as nearest method, the RSS value of a missing fingerprint is defined using the RSS value of the nearest known fingerprint measured in Euclidian distance of the coordinates. Again, denoting the set of known fingerprints for AP-th access point as $\left\{x_j, y_j, P_{ap_{j,AP}}\right\}_j$, the extrapolated RSS value at point (x, y) is obtained as

$$P_{extrap,AP} = P_{ap_{\hat{j},AP}} \tag{6}$$

where the fingerprint index $\hat{j}$ refers to the fingerprint index with minimum distance as

$$\hat{j} = \arg\min_j \sqrt{(x-x_j)^2 + (y-y_j)^2} \tag{7}$$

The resulted RSS values for the whole interpolated and extrapolated area can be presented with Voronoi diagram [30], where each Voronoi cell describes an area estimated based on one known fingerprint. The algorithm is very simple to implement, but the discontinuities in the Voronoi cell edges reduce the estimation accuracy.

2) Inverse Distance Weighting (IDW)

Another approach which performs the interpolation and extrapolation tasks jointly is the IDW, sometimes referred as Shepard's algorithm [33]-[34]. Unlike in the other methods presented in this paper, each estimated data point in IDW is affected by all known data points. The basic idea is to provide weights for the data points based on their distance to the estimated point. The closer is the known data point to the estimated point the more weight is given to it. Hence, the IDW estimate for the unknown RSS value P_{extrap} at coordinate point (x, y) can be given as

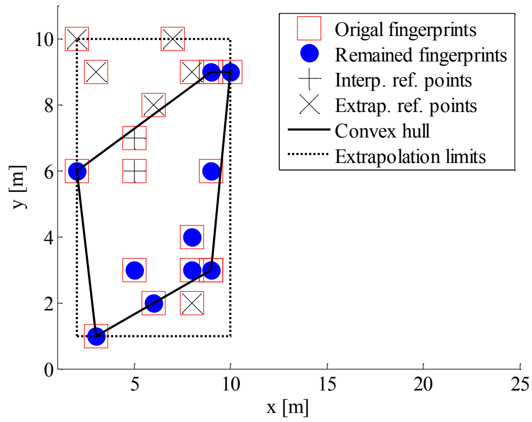

Fig. 6. Illustration of reference points used to calculate interpolation and extrapolation error statistics.

$$\begin{aligned} P_{extrap} &= \sum_{j\in\Omega_{AP}} w_j P_{ap_{j,AP}}, \text{ where} \\ w_j &= \frac{d_j^{-u}}{\sum_{j\in\Omega_{AP}} d_j^{-u}}, \text{ and} \\ d_j &= \sqrt{\left(x_j - x\right)^2 + \left(y_j - y\right)^2}. \end{aligned} \tag{8}$$

Here u is the exponent parameter, which associates weight w_j with the distance d_j. The higher is u the relatively less weight is given for data points further away. In this paper, based on a coarse algorithm optimization, the used value for the exponent is u=4.

IV. Interpolation and Extrapolation Error Statistics

Interpolation and extrapolation methods with high average estimation accuracy enable improved positioning accuracy. However, the higher average accuracy does not directly imply higher positioning accuracy. In some cases it is more important to allow a higher average error but to avoid making critical errors. For example, it is widely recognized that higher RSS measurements in user positioning are more important for accurate positioning than lower RSS values. This is mainly because high RSS values point at specific small area around the measured AP, whereas lower RSS values can be observed further around the AP in all directions. Hence, doing the fingerprint interpolation and extrapolation in RSS based positioning systems, it is not necessarily about choosing the best surface fit for the fingerprint data, but about minimizing the user positioning error. Nonetheless, the recovery of missing data via interpolation and extrapolation methods can improve the positioning performance as it will be shown in Section V.

The accuracy of interpolation and extrapolation methods can be evaluated only in locations where original measurements are available, that is, in the fingerprints which were artificially removed from the original fingerprint database. These specific fingerprints are further called as reference points, since they incorporate information on the

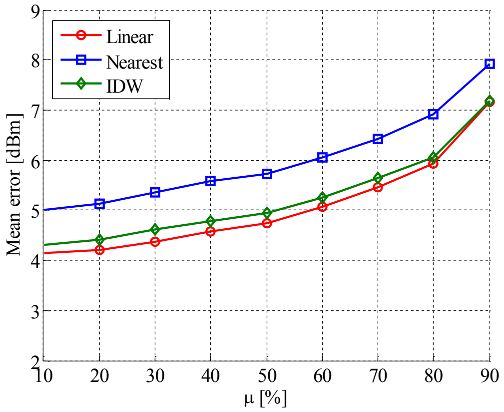

Fig. 7. Mean error of ***interpolated RSS*** values as a function of fingerprint removal percentage μ at 2.4GHz band.

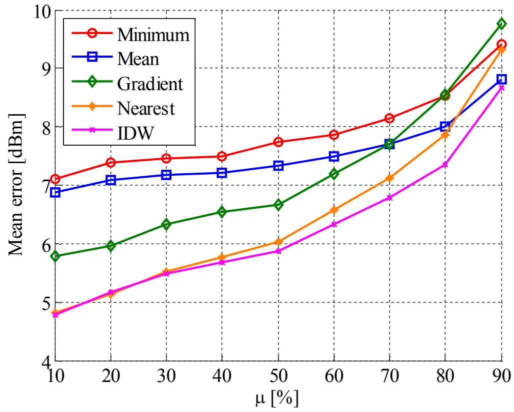

Fig. 8. Mean error of ***extrapolated RSS*** values as a function of fingerprint removal percentage μ at 2.4GHz band.

true RSS measurements besides the estimated ones. Based on the definition of interpolation and extrapolation areas, discussed earlier in Section III, the estimation error statistics are analyzed separately for the interpolated and extrapolated fingerprints. As an example, locations of the reference points used to determine estimate error statistics for interpolation and extrapolation methods are illustrated in Fig. 6.

Since the fingerprint removal process requires random procedures regarding the selection of the removed fingerprints, all the results are given as an average of 100 separate fingerprint removal processes initialized with different random number generator seeds. When comparing different interpolation and extrapolation methods with each other, the same set of fingerprints is always used. The sufficiency of 100 averaging loops was confirmed by observing the standard deviation of the results between different trials. For example, the standard deviation of the presented average RSS error result between different random number seeds was less than 0.5dBm for all the studied methods.

First, the average RSS estimation error was examined with different fingerprint removal block radii d_{block}. The purpose of this was to find a reasonable level for the d_{block} parameter with respect to the dimensions of the operated building. For example, the block area with d_{block}=10m and d_{block}=20m refer to approximately 3-5% and 10-20% of one floor area. Here, the latter one completely removes many of the heard APs from the database, and therefore, it does not serve the fundamental purpose of the study. However, the first case with d_{block}=10m corresponds to scenarios where measurements are removed from certain parts of corridors or from one or multiple rooms at a time. Hence, d_{block}=10m is used in the following results unless otherwise mentioned.

All the results shown in this Section are for the 2.4GHz band. However, similar results were also obtained for the 5GHz band, but because of very small differences in interpolation and extrapolation error statistics between the 2.4GHz and 5GHz bands, the 5GHz band results were excluded from this Section. Nevertheless, based on these results, it appears that there is no significant difference in interpolation and extrapolation error statistics between the 2.4GHz and 5GHz bands.

The average RSS interpolation error as a function of fingerprint removal percent μ is shown in Fig. 7. As expected, the interpolation error increases proportional to μ. Also, the linear interpolation with Delaunay triangulation and the IDW method provide higher accuracy compared to the nearest neighbor method. The same type of comparison is shown for the average RSS extrapolation error in Fig. 8. Here, it is possible to observe larger differences between different methods. In addition, the studied methods are behaving differently as μ increases compared to the curves in the interpolation case. For example, the minimum method and the mean method are clearly less accurate than the IDW and nearest neighbor methods when μ is small, but the gap is clearly reduced when μ is large. Moreover, the gradient method overcomes the minimum and mean methods with small values of μ, but when μ gets larger the accuracy of gradient method collapses down to the lowest accuracy method. The problem of gradient based methods in the RSS measurement extrapolation is its assumption of continuation of the edge of the interpolated surface. However, because of shadowing effects, especially in indoor scenarios, there might be rapid RSS value changes in the edge. As a result, the slope of the gradient can be even positive and the extrapolated RSS values get larger when moving away from the true location of the AP. This substantially decreases the average extrapolation error and the positioning error as seen later in Section V. In terms of average RSS interpolation error, the linear interpolation provides better accuracy than the nearest neighbor and IDW methods. However, the corresponding RSS extrapolation error of the methods used with the linear interpolation are less accurate than the nearest neighbor and IDW methods. As a result, the IDW method seems to provide the best performance when both interpolation and extrapolation accuracy is taken into account.

Cumulative error distributions of interpolated RSS values for different interpolation methods and for two separate values of μ are given in Fig. 9. From here it is possible to see that high error values are dominating the average curves shown

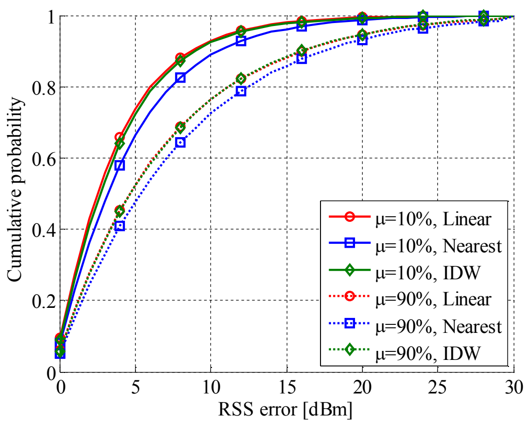

Fig. 9. Cumulative error distribution of ***interpolated RSS*** values at 2.4GHz band.

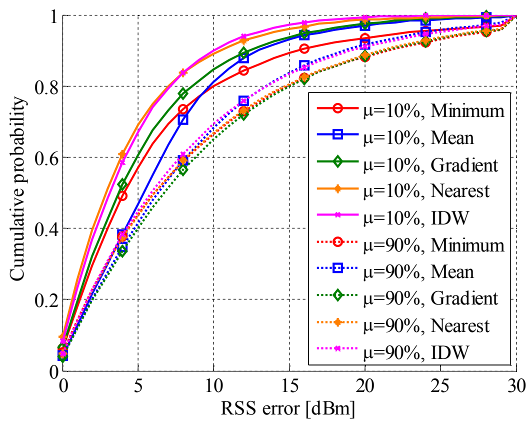

Fig. 10. Cumulative error distribution of ***extrapolated RSS*** values at 2.4GHz band.

earlier in Fig. 7. For example, the linear method achieved 4dBm average error at μ =10%, but from the cumulative error curve it can be seen that approximately 67% of the estimates have lower error than the average error. It is also noticeable that the linear method and the IDW method have almost identical cumulative error distributions. The corresponding curves for different extrapolation methods are shown in Fig. 10. Here, one noteworthy observation is the difference between the curves of minimum and mean methods with μ =10%. Whereas their mean errors are approximately on the same level, as already seen in Fig. 8, the minimum method provides more likely smaller error than the mean method, but at the same time the minimum method has a higher probability for larger errors. As seen later in Section V, the lowest average RSS estimation error does not necessarily provide the lowest positioning error. Hence, the cumulative distributions give important insight into the details of RSS estimation error statistics regarding the considered interpolation and extrapolation methods.

Another aspect in the fingerprint interpolation and extrapolation error analysis is the relation between the estimation accuracy and the distance to the closest known fingerprint. This dependency is studied in Fig. 11, where the average RSS interpolation error is shown as a function of distance to the closest known fingerprint. Here it is noteworthy that all the shown curves are saturated on some specific levels as the distance is increased. Moreover, all the considered interpolation methods are always defining the estimates inside the value limits of the known data points. For example, with linear interpolation the estimated RSS values are evenly distributed between the RSS values of closest known data points. Assuming that the interpolation distance is large enough, the saturated interpolation error is roughly at the same level as shadowing standard deviation. In Fig. 11 the saturated level for the linear method is around 6dBm, which is a common value for indoor shadowing standard deviation. One conclusion here is that if the data is missing in several small areas, the deterioration is less than if the data is missing in a larger area, even if the total area of the missing surface is the same.

In Fig. 12 the average extrapolation error of RSS values as a function of distance to the closest known fingerprint is studied for the extrapolation methods. Unlike in the interpolation case, similar types of saturation levels are not visible, since the estimated surface is not limited by a known data point in the edge of the extrapolated area. Nevertheless, since RSS values

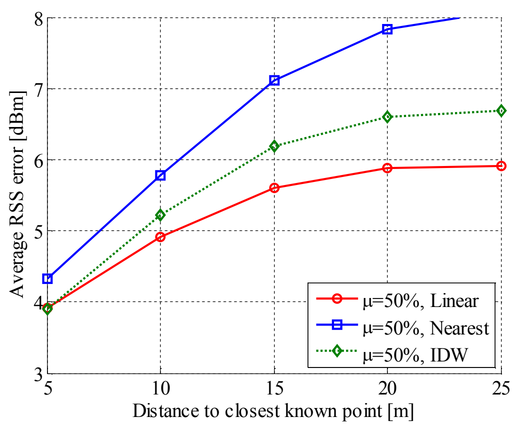

Fig. 11. Average **interpolation RSS** error as a function of distance to the closest known fingerprint at 2.4GHz band.

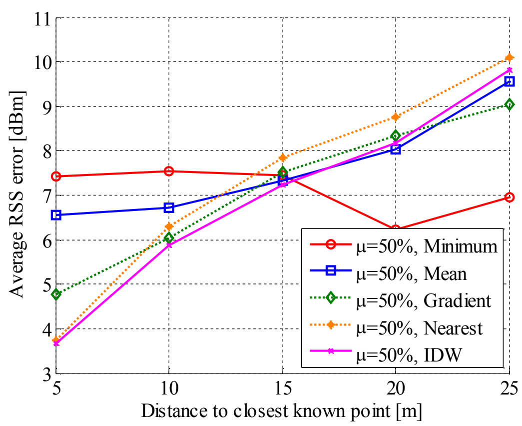

Fig. 12. Average ***extrapolation RSS*** error as a function of distance to the closest known fingerprint at 2.4GHz band.

are physically restricted to some dynamic range, for example due to receiver sensitivity, the maximum extrapolation error is also limited to this range assuming that the estimates are not exceeding the limits. Furthermore, the behavior of the minimum method makes an exception compared to other methods. It appears that the average RSS error is decreasing as the distance increases. This is actually reasonable in case of RSS extrapolation, since low RSS levels are expected at far distances from the AP, where the extrapolated minimum value has most likely been used. In addition, there are usually more low RSS values in the database, for which reason their estimation has larger emphasize in the average error results. On the other hand, the extrapolation accuracy of the minimum method near the AP can get very poor, if there is at least one very low RSS measurement available elsewhere in the fingerprints. On the contrary, the mean method does not follow the behavior of the minimum method curve, although it is also estimating the fingerprints with a constant value. The mean method acquires better estimates at close proximity of AP, but fails to have proper RSS estimates with long distances.

V. POSITIONING RESULTS

Positioning accuracy is studied by using 12 different tracks inside the building measured with the Nexus 7 tablet. For each track the measurements were simultaneously taken for both the 2.4GHz band and the 5GHz band, but the positioning results are considered separately for each of the band. Each user track is tested with each realization of the fingerprint database after the fingerprint removal process described in Section II. Moreover, the positioning accuracy is averaged over 100 different database realizations and the databases are the exact ones that were earlier studied in Section IV for interpolation and extrapolation error statistics. The results are given in forms of positioning accuracy, which refers to floor-wise (2D) Euclidian distance based estimation error, and floor detection probability, which refers to probability of estimating the correct user floor. The floor is determined to be detected correctly, if the true floor of the user is closest to the estimated z coordinate.

The positioning results are obtained using a probabilistic positioning approach discussed in [13]. For each user measurement location, the set of RSS measurements from the heard APs is denoted as $\mathbf{P}_{user} = \left[P_{ap_1} \quad P_{ap_2} \quad \cdots \quad P_{ap_{N_{heard}}} \right]$, where N_{heard} is the number of heard APs. Now, assuming that the measurements from different APs are independent, we can write the probability of the user measurement $\mathbf{P}_{user}$ at the i^{th} fingerprint (i.e., the likelihood of the user being located at the i^{th} fingerprint) as

$$p(\mathbf{P}_{user} \mid i) = \prod_{j=1}^{N_{heard}} p_{RSS}(P_{ap_j} - P_{ap_{i,j}}) \tag{9}$$

where $P_{ap_{i,j}}$ is the RSS value stored in the i^{th} fingerprint for the j^{th} AP, and $p_{RSS}(v)$ describes the probability density function for the measured RSS values (i.e., how does the RSS value vary in one fingerprint when multiple measurement are taken from the same AP). Thus, here we assume that $p_{RSS}(v)$ is the same for all fingerprints and APs, although it is not exactly true in practice. However, since we consider that the fingerprint database includes only one RSS value per each fingerprint and AP, information on the individual probability density functions for each fingerprint and AP is inaccessible.

Using the fact that the fingerprint grid is uniform and assuming that there is no a priori information on the true user position $\mathbf{x}_u = \left[x_u \quad y_u \quad z_u \right]$ (i.e., $p(\mathbf{x})$ is uniformly distributed), we can use the Bayes' rule and write the posterior function for the user position as

$$\begin{aligned} &p(\mathbf{x}_u \mid \mathbf{P}_{user}) \propto \sum_{i=1}^{N_{FP}} p(\mathbf{P}_{user} \mid i) \chi_i(\mathbf{x}_u), \text{ where} \\ &\chi_i(\mathbf{x}_u) = \begin{cases} 1, \text{ if } \mathbf{x}_u \text{ is mapped to the } i^{\text{th}} \text{ fingerprint} \\ 0, \text{ otherwise} \end{cases} \end{aligned} \tag{10}$$

is an indicator function. Here we have also assumed that the posterior probability is constant inside the area of one fingerprint. The maximum a posteriori (MAP) estimate of the user position given the measurement vector $\mathbf{P}_{user}$ is now defined as $\hat{\mathbf{x}}_{u,MAP} = \max_{\mathbf{x}_u} p(\mathbf{x}_u \mid \mathbf{P}_{user})$, which gives the coordinates of the fingerprint in which the posterior function has its maximum value. Since in this case the a priori distribution is uniform, the MAP estimate is also the maximum likelihood (ML) estimate. Another commonly used approach is to use the mean of the posterior function, in which case the estimate becomes as

$$\hat{\mathbf{x}}_{u,MEAN} = \sum_{i=1}^{N_{FP}} \frac{p(\mathbf{P}_{user} \mid i)}{\sum_{j=1}^{N_{FP}} p(\mathbf{P}_{user} \mid j)} \mathbf{x}_i \tag{11}$$

where $\mathbf{x}_i$ is the coordinate vector (x_i, y_i, z_i) of the i^{th} fingerprint.

In this paper the user position estimates are not filtered in anyway and the results are given using the static position estimates. However, the derived posterior probabilities can be directly used in many state-of-the-art filtering methods, such as Bayesian filtering, for further studies. To justify the used positioning approach, we compared the performances of the above described probabilistic approach with the commonly known Nearest Neighbor (NN) and K-Nearest Neighbor (KNN) methods [12],[13], in which the observed RSS values are compared with each fingerprint using the Euclidian distance. In addition, we have considered two different options for the RSS measurement distribution $p_{RSS}(v)$ in the posterior function: Gaussian distribution, defined as $p_{RSS}(v) = 1/\sqrt{2\pi\sigma^2} \exp\left(-v^2/2\sigma^2\right)$, and Exponential

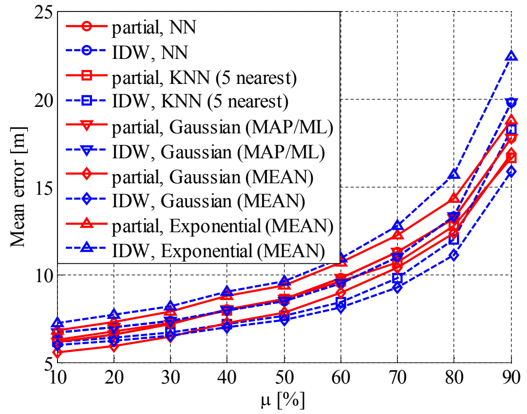

Fig. 13. Mean positioning error as a function of μ considering all user tracks for partial fingerprints and interpolated and extrapolated fingerprints (IDW) for different positioning algorithms at 2.4GHz band.

distribution, defined as $p_{RSS}(v) = 1/2\exp(-|v|)$. Furthermore, the standard deviation σ of the Gaussian distribution was coarsely optimized to be σ=15dB and it was therefore used in all the results further in the paper.

In Fig. 13 the mean positioning error as a function of fingerprint removal percent μ is shown for each considered positioning algorithm. The results are given separately for partial fingerprints (i.e., the remained fingerprints after the removal process without interpolation and extrapolation) and the interpolated and extrapolated fingerprints using the IDW method. From these the probabilistic approach using Gaussian distributed $p_{RSS}(v)$ and the mean of the posterior distribution $\hat{\mathbf{x}}_{u,MEAN}$ provides the smallest average position error. Thus, we choose to use this approach in the following results in this Section.

A. 2.4GHz Band Results

First, the effect of different interpolation methods on the positioning accuracy is studied at the 2.4GHz band. The mean positioning error as a function of fingerprint removal percent μ is given in Fig. 14, where the results are shown separately for original fingerprints, partial fingerprints, and different interpolation methods. Since here only the interpolation is considered, the fingerprints are recovered only for the areas inside the convex hull of the remained fingerprints. The results show that the interpolation alone does not improve the performance compared to using the partial fingerprint database

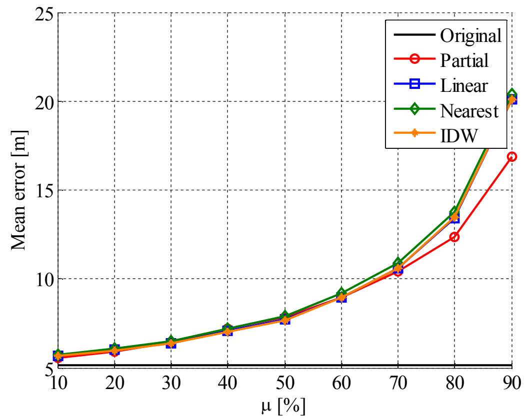

Fig. 14. Mean positioning error as a function of μ considering all user tracks for original fingerprints, partial fingerprints and for different ***interpolation methods at 2.4GHz band***.

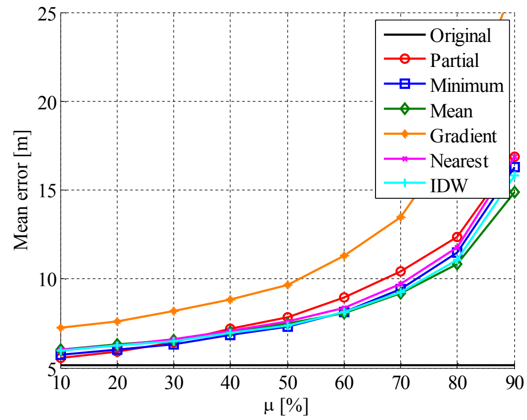

Fig. 16. Mean positioning error as a function of μ considering all user tracks for original fingerprints, partial fingerprints and for different **extrapolation methods** after interpolation ***at 2.4GHz band***.

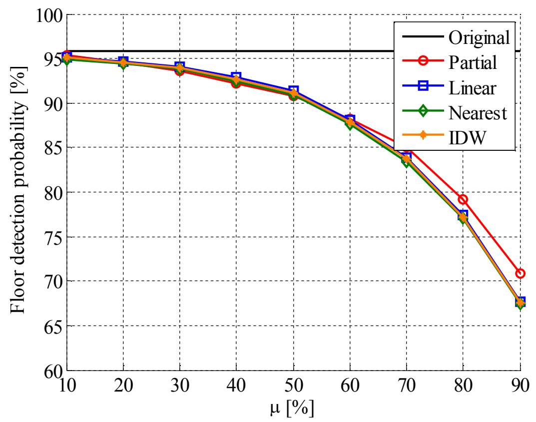

Fig. 15. Mean floor detection probability as a function of μ considering all user tracks for original fingerprints, partial fingerprints and for different ***interpolation methods at 2.4GHz band***.

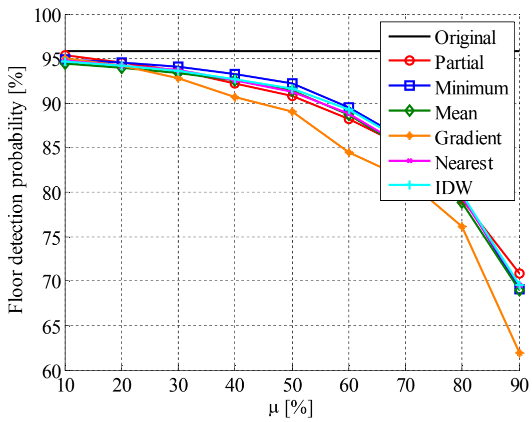

Fig. 17. Mean floor detection probability as a function of μ considering all user tracks for original fingerprints, partial fingerprints and for different ***extrapolation methods*** after interpolation ***at 2.4GHz band***.

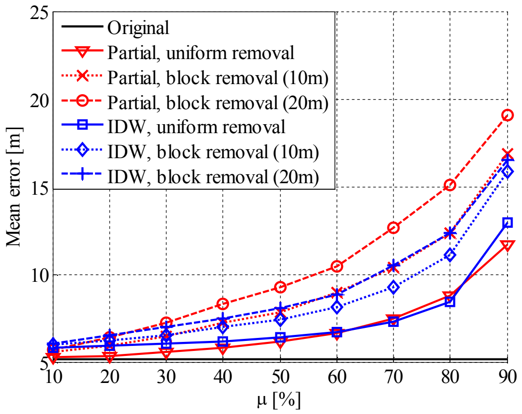

Fig. 18. Mean positioning error as function of μ considering all user tracks for original fingerprints, partial fingerprints and IDW interpolated and extrapolated fingerprints for uniform fingerprint removal and block removal with d_{block}=10m and d_{block}=20m at 2.4GHz band.

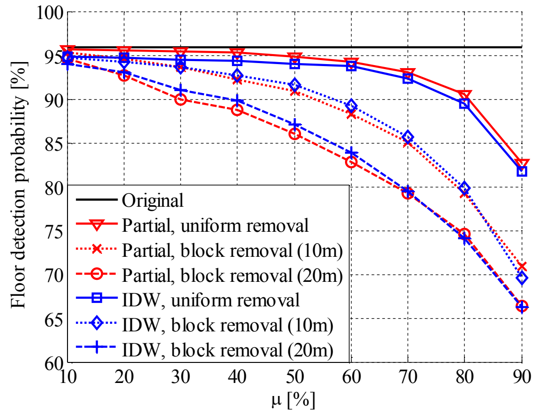

Fig. 19. Mean floor detection probability as function of μ considering all user tracks for original fingerprints, partial fingerprints and IDW interpolated and extrapolated fingerprints for uniform fingerprint removal and block removal with d_{block}=10m and d_{block}=20m at 2.4GHz band.

alone. Similar type of observation can be made for the floor detection probabilities shown in Fig. 15.

In case also the extrapolated areas in the fingerprint recovery process are taken into account, the results begin to change. The corresponding mean error for different extrapolation methods (including the interpolation) is shown in Fig. 16. Here, when μ is increased, compared to the partial fingerprints, the mean error is clearly reduced for all other extrapolation methods except for the gradient method. For the cases with μ>50%, the mean error is roughly 5-12% smaller with the best extrapolation methods compared to the partial fingerprint set. As discussed earlier in Section IV, here it can be clearly seen that better average extrapolation error does not necessarily mean better positioning accuracy. For example, comparing the performance of the minimum method and gradient method in Fig. 16 and in Fig. 8, it can be noticed that the minimum method is clearly lacking the performance in average extrapolation error, but overcomes the gradient method in the positioning error accuracy. As a result, it is very important to recognize that in practice the position solution is originated from a fairly complex network structure, and certain type of errors in interpolation and extrapolation algorithms, such as the ones created with gradient method, might have a huge impact on the positioning solution. However, with the extrapolation methods, the floor detection probability is only 2-3% better than with the partial fingerprints as shown in Fig. 17. Again, the gradient based extrapolation is not able to compete with the other extrapolation methods.

In the end, the impact of the presented fingerprint removal scheme with respect to positioning accuracy and floor detection probability are illustrated in Fig. 18 and Fig. 19. In both of the figures, the uniform removal refers to fingerprint removal with uniform probability distribution. For simplicity, results are only given for the IDW method with extrapolation because of its consistent performance in all of the studied scenarios. Here, it should be emphasized that all the fingerprint removal methods remove exactly the same number of fingerprints based on the value of μ. Nevertheless, the performances of partial fingerprint sets and the IDW method differ considerably between different removal approaches. From fingerprint removal point of view it is more important where the fingerprints are removed, not how much is removed. For example, with uniform fingerprint removal approach the mean horizontal positioning error can be up to 30% in lower level than with block removal approach using d_{block}=20m. Moreover, the benefits of the fingerprint interpolation and extrapolation become visible only if the coverage gaps in the database are considerable. For example, at μ=70% the IDW based interpolation and extrapolation reduces the mean error by 12% with d_{block}=10m, whereas with d_{block}=20m the same mean error is reduced by 17%. On the other hand, the IDW method is not able to considerably improve the floor detection probability compared to the partial fingerprints, but the effect of the block radius of the removal method is also clearly visible here.

B. 5GHz Band Results

The average horizontal positioning accuracies at the 5GHz band for different extrapolation methods are compared in Fig. 20. Except for the gradient method, all the considered extrapolation methods result in approximately the same average positioning error. However, none of extrapolation methods are able to provide better performance compared to using only the partial fingerprint database. On the other hand, in terms of floor detection probability, the performance can be slightly improved by the interpolation when μ<50%, as shown in Fig. 21. Unlike with the corresponding results in the 2.4GHz band, the effect of the removal percentage μ on the floor detection accuracy is very small.

The presented interpolation methods at 5GHz band were also studied, but they were left outside of the given results. When comparing the positioning accuracy differences between the interpolation methods and the extrapolation methods, the 5GHz band results were analogous with the 2.4GHz band results.

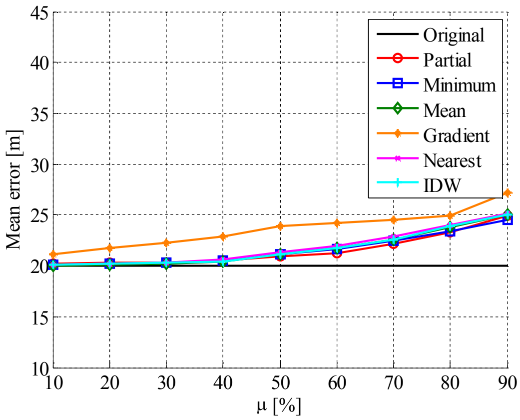

Fig. 20. Mean positioning error as a function of μ considering all user tracks for original fingerprints, partial fingerprints (after removal process) and for different ***extrapolation methods*** after interpolation ***at 5GHz band***.

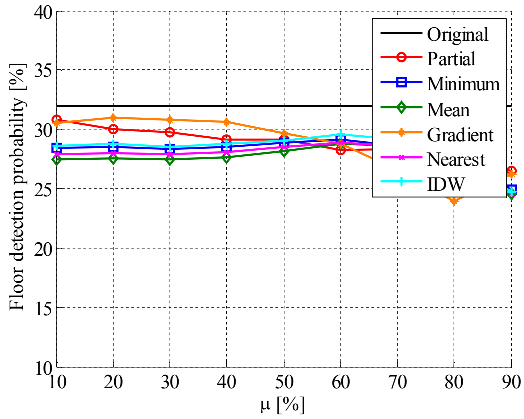

Fig. 21. Mean floor detection probability as a function of μ considering all user tracks for original fingerprints, partial fingerprints (after removal process) and for different ***extrapolation methods*** after interpolation ***at 5GHz band***.

C. Comparative Notes between 2.4GHz and 5GHz bands and Summarization

It can be clearly seen that the positioning accuracy and the floor detection probability at 5GHz band is generally on much lower level than in the 2.4GHz band. This is mainly because of the fact that the availability of APs at the 5GHz band is yet very poor. Unlike at the 2.4GHz band, the results at the 5GHz band indicate that the interpolation and extrapolation methods without enough original fingerprint data are not able to improve the positioning performance compared to the partial fingerprints. However, the comparison between different interpolation and extrapolation methods seems to be consistent for both of the frequency bands. Thus, it is assumed that if the number fingerprints at 5GHz band would be at the same level with the 2.4GHz band, the positioning accuracies would be relatively close to each other. Combining measurements from the two bands is left as an open issue for future research.

The presented results of horizontal positioning accuracy and floor detection probability for the considered interpolation and extrapolation methods have shown that being able to estimate the RSS in missing areas of a building with high average accuracy is not a warrantee for a high positioning performance. In fact, some interpolation and extrapolation approaches which behave well in terms of RSS estimation, have rather poor performance in positioning the mobile. This observation can be very important for positioning system designers, since contrary to the intuition, reducing the average RSS estimate error in the missing areas does not necessarily reduce the positioning error.

When areas of a building are not accessible for fingerprint collection, it is desirable to use proper interpolation and extrapolation methods to estimate the missing fingerprints. The best interpolation and extrapolation methods from the point of view of the positioning accuracy and floor detection probability proved to be the IDW and the linear interpolation with minimum or mean extrapolation. From these two the IDW method is a bit more flexible, since it can be tuned for different type of environments by modifying the exponent parameter u in (8). Also, the interpolation and the extrapolation in IDW are performed jointly without the need of constructing a triangulation and defining the areas for interpolation and extrapolation.

VI. Conclusion

In this paper we have studied recovery of fingerprints in database with considerable coverage gaps by means of different interpolation and extrapolation methods. The results have been validated by using a large set of data collected from one multi-floor university building. Furthermore, it has been shown that simulating the coverage gaps cannot be done by removing a certain percentage of fingerprints according to uniform distribution, but fingerprints should be removed in larger blocks, whose radii are scaled according to the dimensions and properties of the studied area.

The average RSS estimation error for the considered interpolation and extrapolation methods has been compared with each other for different loss rates of missing fingerprints. Besides to the average RSS error, also the cumulative error distribution was studied. Especially, regarding the extrapolation methods, the cumulative distribution helps to explain the differences in the positioning accuracy between the different interpolation and extrapolation methods. Moreover, it was shown that the highest average interpolation and extrapolation accuracy does not necessarily imply the highest positioning accuracy, but more important is the error distribution. For example, whereas the gradient based extrapolation provides better average RSS estimation accuracy than the mean based extrapolation, it does not reach the same user positioning accuracy.

The positioning accuracy was studied in terms of average horizontal accuracy and average floor detection probability. In the case that the fingerprint database was to include considerable coverage gaps, it was shown that the positioning accuracy can be improved by proper interpolation and extrapolation methods, namely IDW, or minimum or mean extrapolation with linear interpolation. However, when using only interpolation, there was no performance gain unlike when using both the interpolation and the extrapolation. On the other hand, this was expected with the considered probabilistic positioning algorithm, since it estimates the user position based on the average of the posterior probability. Because of this, if the likelihood of the user position is distributed around a coverage gap, the user position estimate is able to be located inside the gap. For this reason the extrapolation becomes essential, since the positioning algorithm itself is not able to create estimates outside the area covered by the known fingerprints. Also, as expected, the size of the coverage gaps reduces the interpolation and extrapolation accuracy as well as the positioning accuracy.

Studying other interpolation and extrapolation methods, besides the ones used in this paper, are also under interest, but the ever increasing number of WLAN APs can make too complex methods inapplicable for practical systems. Nevertheless, the design of RSS based interpolation and extrapolation methods should not be based on minimizing the average error of the estimated RSS values, but on maximizing the user positioning accuracy.

ACKNOWLEDGMENT

The authors express their warm thanks to the Academy of Finland (project 250266) for its financial support for this research work. The authors would also like to express their thanks to the team from HERE, Tampere, for providing the set-up for the WLAN measurements.

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