Chapter 2 Problem 6 Faraway

Faraway, Julian J.. Linear Models with R, Second Edition (Chapman & Hall/CRC Texts in Statistical Science). CRC Press.

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Thirty samples of cheddar cheese were analyzed for their content of acetic acid, hydrogen sulfide and lactic acid. Each sample was tasted and scored by a panel of judges and the average taste score produced. Use the cheddar data to answer the following:

- (a) Fit a regression model with taste as the response and the three chemical contents as predictors. Report the values of the regression coefficients.
- (b) Compute the correlation between the fitted values and the response. Square it. Identify where this value appears in the regression output.
- (c) Fit the same regression model but without an intercept term. What is the value of R 2 reported in the output? Compute a more reasonable measure of the goodness of fit for this example.

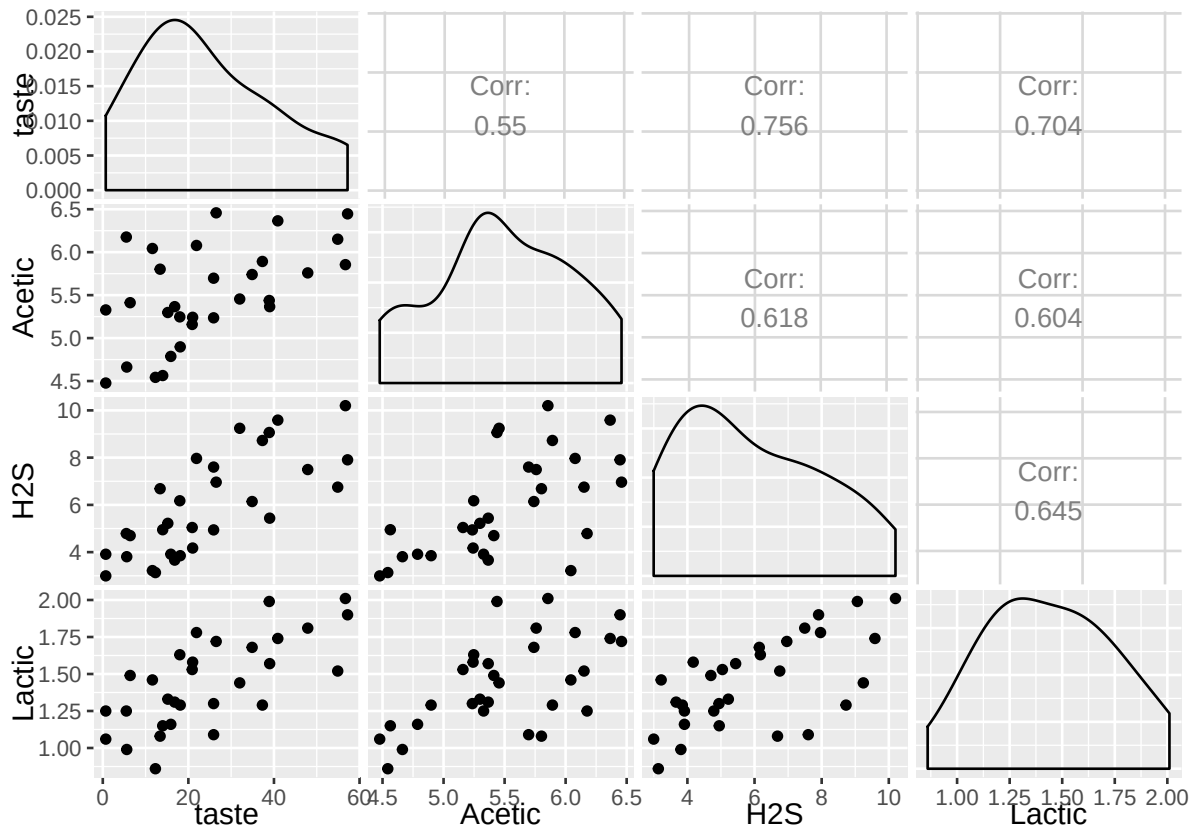
Fit the linear model $taste \sim Acetic + H2S + Lactic$

First we load and inspect the data

```
data(cheddar, package = "faraway")
head(cheddar)
```

```
##   taste Acetic   H2S Lactic
## 1  12.3  4.543 3.135   0.86
## 2  20.9  5.159 5.043   1.53
## 3  39.0  5.366 5.438   1.57
## 4  47.9  5.759 7.496   1.81
## 5   5.6  4.663 3.807   0.99
## 6  25.9  5.697 7.601   1.09
```

```
ggpairs(data = cheddar)
```



Fit the model and display the regression coefficients

```
lm.fit <- lm(taste ~ Acetic + H2S + Lactic, data = cheddar)

summary(lm.fit)
```

```
##
## Call:
## lm(formula = taste ~ Acetic + H2S + Lactic, data = cheddar)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -17.390  -6.612  -1.009   4.908  25.449
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -28.8768    19.7354  -1.463  0.15540
## Acetic        0.3277     4.4598   0.073  0.94198
## H2S           3.9118     1.2484   3.133  0.00425 **
## Lactic       19.6705     8.6291   2.280  0.03108 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.13 on 26 degrees of freedom
## Multiple R-squared:  0.6518, Adjusted R-squared:  0.6116
## F-statistic: 16.22 on 3 and 26 DF,  p-value: 3.81e-06
```

Compute the correlation between the fitted values and the response. Square it. Identify where this value appears in the regression output.

```
yhat <- lm.fit$fitted.values

y <- cheddar$taste

corr.fitted_response <- cor(yhat, y)

pander(data.frame(r_sq = corr.fitted_response^2), caption = "Squared Correlation between actual and pred
```

Table 1: Squared Correlation between actual and predicted

r_sq
0.6518

The value we calculated is the multiple R-squared in the regression output. Note the adjusted R-squared is a modified version of R-squared that has been adjusted for the number of predictors in the model. We use the adjusted R-squared when comparing models - it increases only if the new terms improve the model more than would be expected by chance.

Diagnotic plots for this model - uncomment for display.

```
# plot(lm.fit)
```

Fit the model without an intercept term.

In R a formula has an implied intercept term. To remove this use either $y \sim x - 1$ or $y \sim 0 + x$ in the formula. For our case we will fit $taste \sim Acetic + H2S + Lactic - 1$. We'll need to account for the lack of intercept term when evaluating the quality of the fit. The default R^2 calculation in R assumes a null model with an intercept.

```
lm.fit.nointercept <- lm(taste ~ Acetic + H2S + Lactic - 1, data = cheddar)

summary(lm.fit.nointercept)
```

```
##
## Call:
## lm(formula = taste ~ Acetic + H2S + Lactic - 1, data = cheddar)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.4521  -6.5262  -0.6388   4.6811  28.4744
##
## Coefficients:
##        Estimate Std. Error t value Pr(>|t|)
## Acetic   -5.454      2.111  -2.583  0.01553 *
## H2S       4.576      1.187   3.854  0.00065 ***
## Lactic   19.127      8.801   2.173  0.03871 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.34 on 27 degrees of freedom
## Multiple R-squared:  0.8877, Adjusted R-squared:  0.8752
```

```
## F-statistic: 71.15 on 3 and 27 DF,  p-value: 6.099e-13
```

The reported R^2 in this case is 0.8877 We see the jump in calculated value that the textbook mentions. Now we'll calculate $R^2 = corr^2(y, \hat{y})$

```
yhat <- lm.fit.nointercept$fitted.values

y <- cheddar$taste

corr.fitted_response.nointercept <- cor(yhat, y)


pander(data.frame(r_sq = corr.fitted_response.nointercept^2), caption = "Squared Correlation between ac
```

Table 2: Squared Correlation between actual and predicted

r_sq
0.6244

We see this value is commensurate with the value we obtained when there was an intercept.

Diagnotic plots for this model - uncomment for display.

```
# plot(lm.fit.nointercept)
```