Data Structures HW2 By: Igor Rodrigues Goncalves

1)

a) $f(n) = 5n^5 - 4n^4 + 3n^3 + \sqrt{2}\,n^2 + n - 1$

- $5n^5$ is the dominant term because it grows faster than any other term, therefore the function is $\underline{O(n^5)}$.

b) $f(n) = (n^3 - (\log(n))^3)(n^2 - \log(n)) + 11n^3$

- $= (n^3 \cdot n^2) + (n^3 \cdot -\log(n)) + (-(\log(n))^3 \cdot n^2 + (-(\log(n))^3 \cdot -\log(n)) + 11n^3$
- $= n^5 - n^3\log(n) - n^2(\log(n))^3 + (\log(n))^4 + 11n^3$
- The dominant term is n^5 because it grows the fastest. Therefore the function is $\underline{O(n^5)}$.

c) $f(n) = \dfrac{5n^4 + 10n^3 - 100n^2 - n - 1}{6n^2}$

- $= \dfrac{5n^4}{6n^2} + \dfrac{10n^3}{6n^2} + \dfrac{100n^2}{6n^2} + \dfrac{-n}{6n^2} + \dfrac{-1}{6n^2}$
- $= \dfrac{5}{6}n^2 + \dfrac{10}{6}n + \dfrac{-100}{6} + \dfrac{-1}{6n} + \dfrac{-1}{6n^2}$
- The dominant term is $\frac{5}{6}n^2$, therefore the function is $\underline{O(n^2)}$.

d) $f(n) = \log(n^3 + n + 10) + n^2\log(n+4)$

- $= \log(n^3) + \log(n)$
- $= O(\log(n)) + O(n^2\log(n))$ // BigO of both terms.
- Since the dominant term is $\underline{O(n^2\log(n))}$ that is the BigO of this function.

e) $f(n) = (n\log(n) + 1)^2 + (\log(n) + 1)(n^2 + 1)$

- $= [(n\log(n))^2 + 2(n\log(n))(1) + 1^2] + [(\log(n))(n^2) + (\log(n))(1) + (1)(n^2) + (1)(1)]$
- $= n^2\log^2(n) + 2n\log(n) + 1 + n^2\log(n) + \log(n) + n^2 + 1$
- The dominant term is $n^2\log^2(n)$, therefore the function is $\underline{O(n^2\log^2(n))}$.

f) $f(n) = \log[(5n^5 + 7n^3 + 10)^2 (3n^3 + 4n + 10)]$

- $= \log(5n^5 + 7n^3 + 10)^2 + \log(3n^3 + 4n + 10)$
- $5n^5 + 7n^3 + 10 \approx n^5$; $\log((n^5)^2) = \log(n^{10}) = 10\log(n)$
- $3n^3 + 4n + 10 \approx n^3$; $\log(n^3) = 3\log(n)$
- In this case once we add the two sides, we are left with $13\log(n)$, but since constant factors are ignored, the function is $O(\log(n))$.

2)

a) $f(n) = n^2 + 10 \in O(n^3)$

- $n^2 + 10 \leq cn^3$
- $n^2/n^3 + 10/n^3 \leq c$
- $1/n + 10/n^3 \leq c$
- For $n \geq 2$, $1/n \leq 1/2$
- Thus $n_0 = 2$, $1/2 + 10/8 = 1/2 + 5/4 = 7/4 \leq 2$
- $c = 2$, $n_0 = 2$; Thus $O(n^3)$ ✓

b) $n! \in O(n^n)$

- $n! \leq n^n$ for all $n \geq 1$.
- $c = 1$, $n_0 = 1$; Thus $O(n^n)$ ✓

c) $f(n) = 3n^3 + 7n + 10 \in O(n^3)$

- $3n^3 + 7n + 10 \leq cn^3$
- $3 + 7/n^2 + 10/n^3 \leq c$
- For $n \geq 1$, $7/n^2 \leq 7$, $10/n^3 \leq 10$
- Thus $n \geq 2$; $3 + 7/n^2 + 10/n^3 \leq 3 + 1 = 4$
- $c = 4$, $n_0 = 2$; Thus $O(n^3)$ ✓

→ d) $1^3 + 2^3 + 3^3 + \ldots + n^3 \in O(n^4)$ // Sum of cubes

- $S(n) = \left(n(n+1)/2\right)^2$
- $S(n) = \left(n^2(n+1)^2/4\right)$
- $S(n) = n^2(n^2+2n+1)/4$
- $S(n) = n^4 + 2n^3 + n^2/4$
- $n^4 + 2n^3 + n^2/4 \leq cn^4$
- $n^4/4n^4 + 2n^3/4n^4 + n^2/4n^4 \leq c$
- $1/4 + 2/4n + 1/4n^2 \leq c$
- For $n \geq 2$, $1/4 + 2/8 + 1/16 = 13/16$
- Thus $c = 1$, $1/4 + 2/4n + 1/4n^2 \leq n$
- $c = 1$, $n_0 = 2$; Thus $O(n^4)$ ✓

3)

→ - First Function worst case: Either the first if or the second else if will trigger a loop that runs n^2 times $(n * n)$. The else block (loop runs 10 times) is a constant.

- $O(n) \cdot O(n^2) = O(n^3)$

→ Second function worst case: Both loops run n times.

- $\sum_{i=1}^{N} i = N(N+1)/2$
- $O(n^2)$

→ Third function worst case: Outer loop runs $\log(n)$ times and i is halved each time. Inner loop j doubles each time and runs $\log(n)$ times.

- $O(\log(n)) \cdot O(\log(n)) = O(\log^2(n))$

→ Forth function worst case: Outer loop, i is halved each time and runs $\log(n)$ times. Middle loop doubles each time so it also runs $\log(n)$ times. Inner loop increments by 2, so it will run $n/2$ times, which is $O(n)$.

- $O(\log(n)) \cdot O(\log(n)) \cdot O(n) = O(n\log^2(n))$

4)

→ a) Give the n^{th} term of the sequence.

- $a_n = a_1 + (n-1) \cdot d$; $a_n = 28 + (n-1) \cdot 3$
- $a_n = 28 + 3n - 3$
- $\underline{a_n = 25 + 3n}$

→ b) What term is 112 in this sequence?

- $112 = 25 + 3n$
- $87 = 3n$
- $\underline{n = 29}$ ∴ 112 is the 29th term in the sequence.

→ c) What are the first 10 numbers in the sequence?

- For $n=1$: $a(1) = 3(1) + 25 = \underline{28}$
- For $n=2$: $a(2) = 3(2) + 25 = \underline{31}$
- For $n=3$: $a(3) = 3(3) + 25 = \underline{34}$
- For $n=4$: $a(4) = 3(4) + 25 = \underline{37}$
- For $n=5$: $a(5) = 3(5) + 25 = \underline{40}$
- For $n=6$: $a(6) = 3(6) + 25 = \underline{43}$
- For $n=7$: $a(7) = 3(7) + 25 = \underline{46}$
- For $n=8$: $a(8) = 3(8) + 25 = \underline{49}$
- For $n=9$: $a(9) = 3(9) + 25 = \underline{52}$
- For $n=10$: $a(10) = 3(10) + 25 = \underline{55}$

→ d) Find the sum of first 10 numbers in the sequence.

- $S_n = n/2 \cdot (a_1 + a_n)$
- $S_n = 10/2 \cdot (28 + 55) = 5 \cdot 83 = \underline{415}$