Data Mining and Business Intelligence

Lecture 8: Exponential Smoothing Models

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Agenda

- Midterm Survey
- Online class due to coronavirus
- Review of Lecture 7
- Forecasting
- Exponential Smoothing Models
- Applications

Midterm Feedback

- Relate math to real-life cases and explain business insights
- Outline some key points that we should master
- Slow down a bit and spend more time on practices
- Explain concepts more thoroughly and slowly
- Less dig into math
- More concepts than tools. Dive deeper into specific topics
- Provide non-SAS material (e.g., Python)
- More assignments

Stationarity

- What are the three properties of stationary time series?

- Can a stationary time series exhibit trend or seasonality?

Stationarity of AR(1) Model

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t$$

- Is the model stationary if $|\phi_1| > 1$?

- Is the model stationary if $\phi_1 = 1$?

Multiple AR.R

- Is the model stationary if $\phi_1 = -1$?

- What if $|\phi_1| < 1$ but $\phi_0 > 1$?

Stationarity of MA(1) Model

$$Y_t = \theta_0 + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

- Is the model stationary if $|\theta_1| > 1$?

Multiple MA.R

Which of the following statements are true?

- A random walk model has constant mean
- A random walk model has constant variance
- A random walk with drift model has constant mean
- A random walk with drift model has constant variance

multiple AR.R

Forecasting

Statistical Forecasting — The Math

- Time Series = Signal + Noise

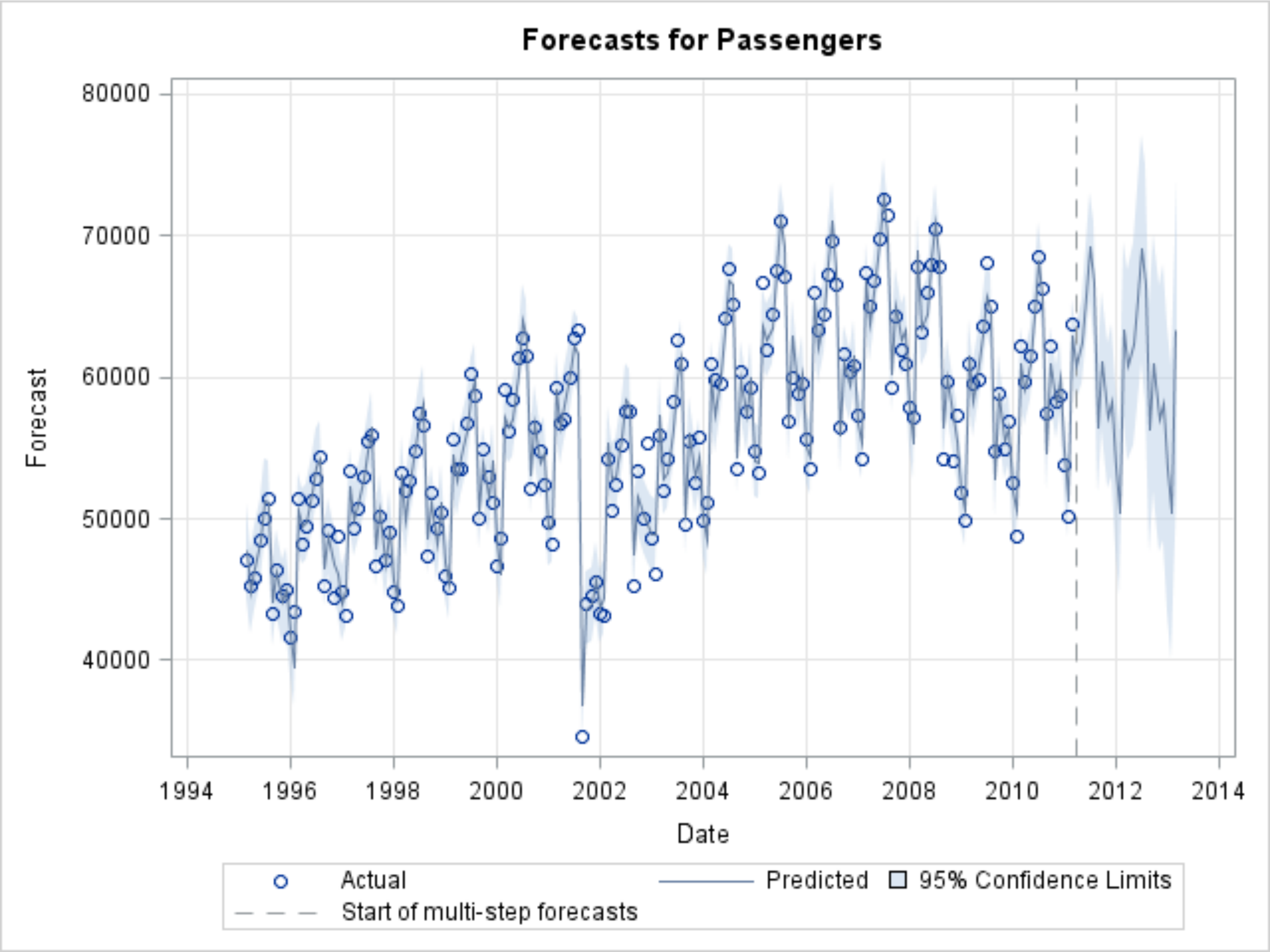
  Systematic Variation   Uncertainty

- Forecast = Extrapolated Signal

- Confidence Interval = Extrapolated Signal +/- Uncertainty

- Example: $Y_t = \phi_0 + \phi_1 Y_{t-1} + \varepsilon_t$

Forecast = Extrapolated Signal



Model Evaluation Metrics

- Root Mean Squared Error

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Y_t - \widehat{Y_t}\right)^2}$$

- Mean Absolute Percent Error:

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n} | Y_t - \hat{Y}_t | / Y_t$$

- Mean Absolute Error:

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} | Y_t - \hat{Y}_t |$$

Evaluation Metrics

- Akaike Information Criteria (AIC):

$$\mathrm{AIC} = -2\log(L) + 2k$$

- Schwarz's Bayesian Information Criteria (SBC or BIC):

$$\mathrm{SBC} = -2\log(L) + k\log(n)$$

- L represents the likelihood of the model
- k represents the number of parameters in the model
- n represents number of observations in the data

Evaluation Process

Fit Sample (Fit)

- **Used to estimate model parameters for accuracy evaluation**

Holdout Sample (Accuracy)

- **Used to evaluate model accuracy**

⚠ **Full = Fit + Holdout** data is used to fit a **final** deployment model

Choosing the Holdout Sample

- Rules of thumb
  - Cover at least a complete seasonal period in holdout sample
  - Length of fit sample is 4 times more than number of parameters
  - Holdout sample often contains no more than 25% of the series
  - The holdout sample is always at the end of the series

- No holdout sample and base accuracy on the entire series
  - If unique behavior occurs within the holdout sample
  - If there is insufficient data to fit a model without the holdout sample

Exponential Smoothing Models

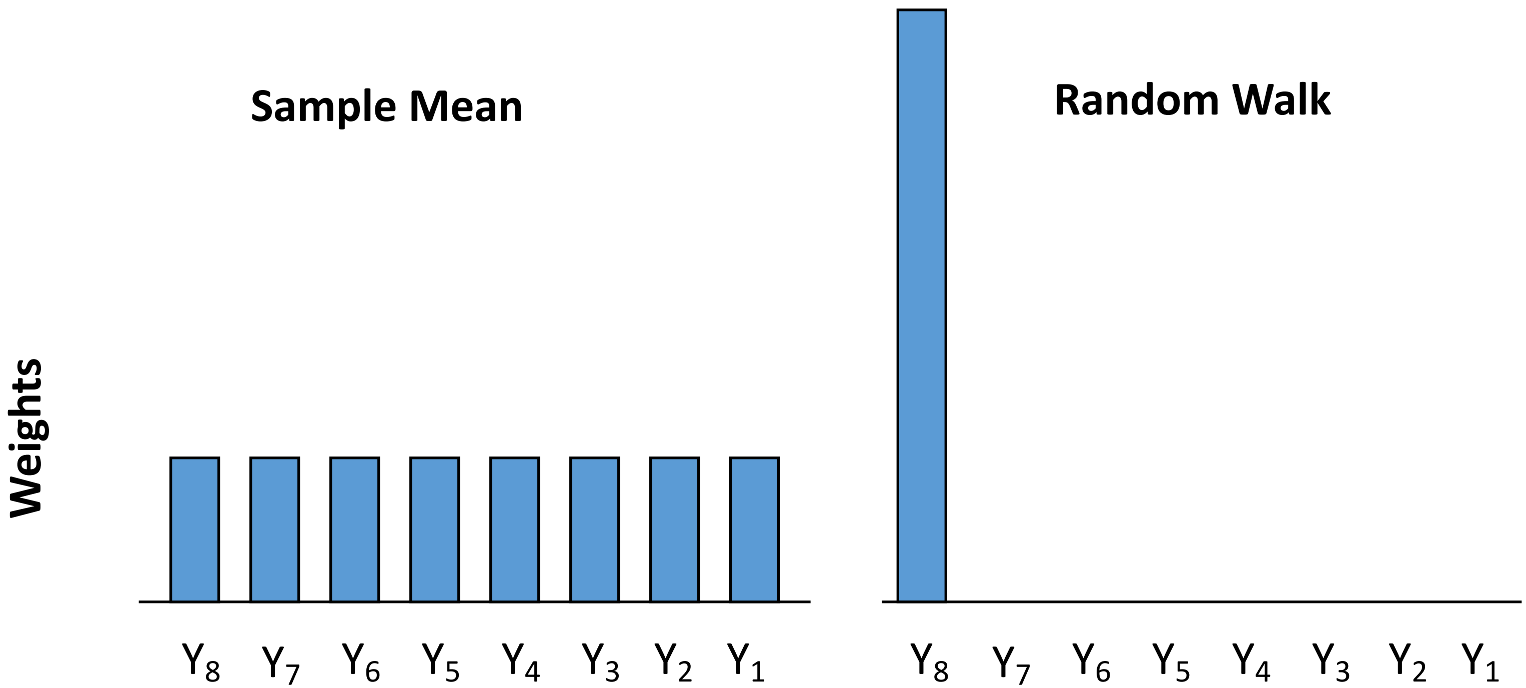
Exponential Smoothing

- Proposed in the late 1950s (Brown, 1959; Holt, 1957; Winters, 1960)

- Produce forecasts using weighted averages of past observations, with the weights decaying exponentially as the observations get older.

- Support data with trend and/or seasonality

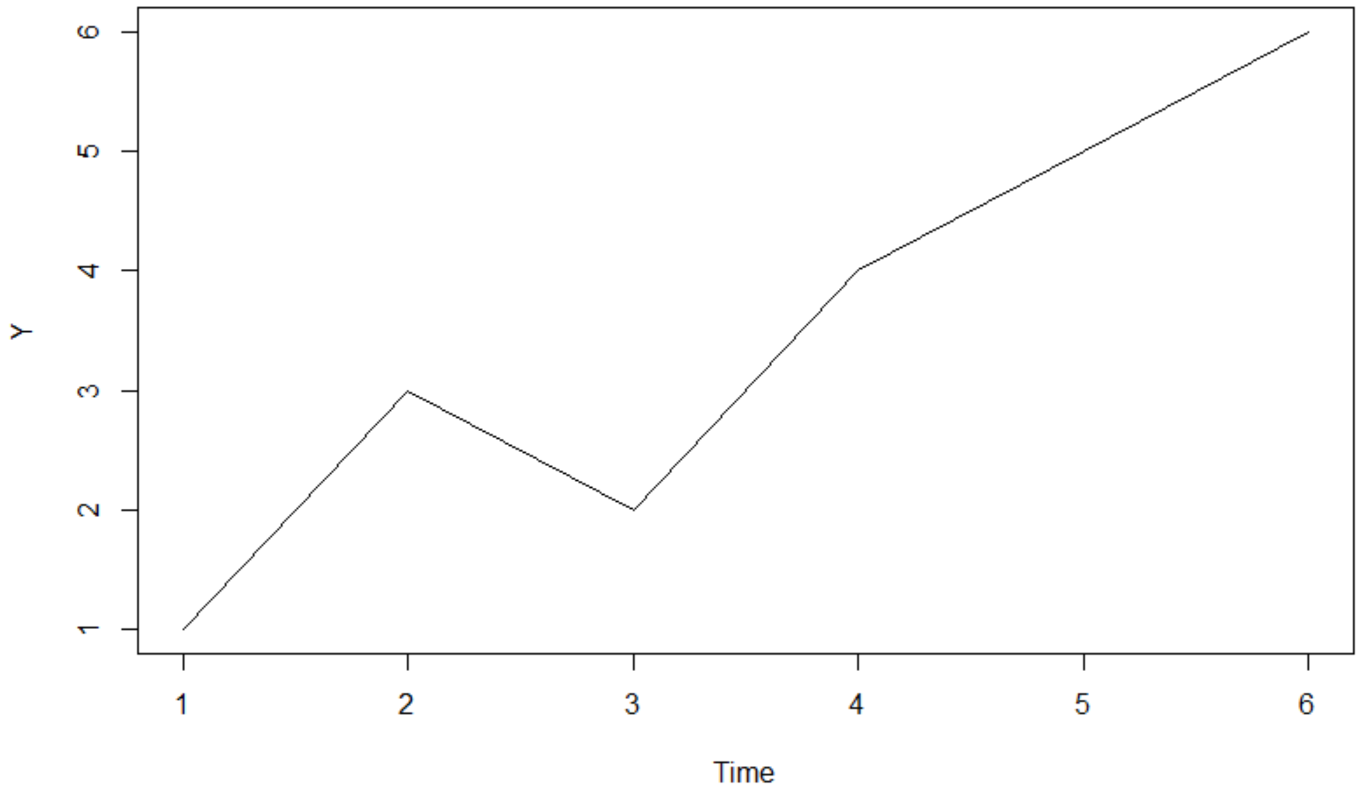
Two Special ESM Models



Weights applied to past values to predict Y_9

- Sample mean: equal weights
- Random walk: all weight to the most recent observation

Example: Predicting Y_7

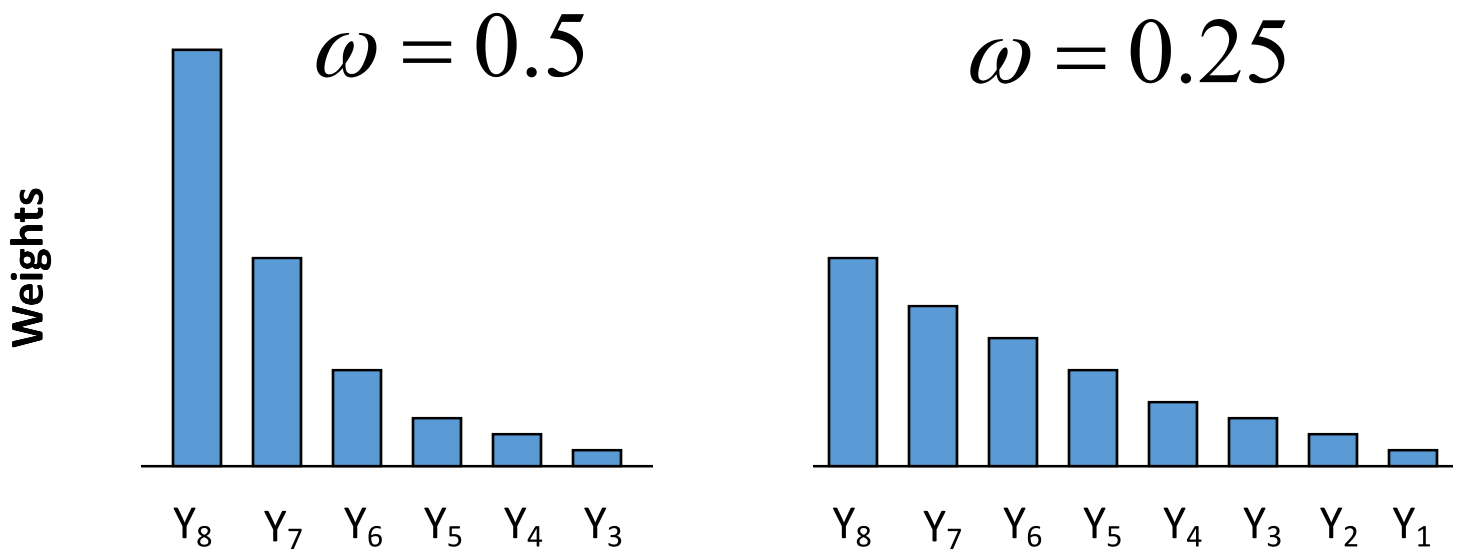


Prediction by sample mean:
Prediction by random walk:

Simple Exponential Smoothing

$$
\begin{aligned}
\hat{Y}_{t+1} &= \omega Y_t + (1-\omega)\hat{Y}_t \\
&= \omega Y_t + (1-\omega)[\omega Y_{t-1} + (1-\omega)\hat{Y}_{t-1}] \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + (1-\omega)^2 \hat{Y}_{t-1} \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + (1-\omega)^2[\omega Y_{t-2} + (1-\omega)\hat{Y}_{t-2}] \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + \omega(1-\omega)^2 Y_{t-2} + \omega(1-\omega)^3 Y_{t-3} + \cdots
\end{aligned}
$$

Simple Exponential Smoothing



Weights applied to past values to predict Y_9

The larger the parameter, the more that
the most recent values are emphasized.

What Happens if ω Approaches Zero

$$
\begin{aligned}
\hat{Y}_{t+1} &= \omega Y_t + (1-\omega)\hat{Y}_t \\
&= \omega Y_t + (1-\omega)[\omega Y_{t-1} + (1-\omega)\hat{Y}_{t-1}] \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + (1-\omega)^2 \hat{Y}_{t-1} \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + (1-\omega)^2[\omega Y_{t-2} + (1-\omega)\hat{Y}_{t-2}] \\
&= \omega Y_t + \omega(1-\omega)Y_{t-1} + \omega(1-\omega)^2 Y_{t-2} + \omega(1-\omega)^3 Y_{t-3} + \cdots
\end{aligned}
$$

Since $1-\omega \approx 1$, the ESM model degenerates to the sample mean model

ESM Parameters

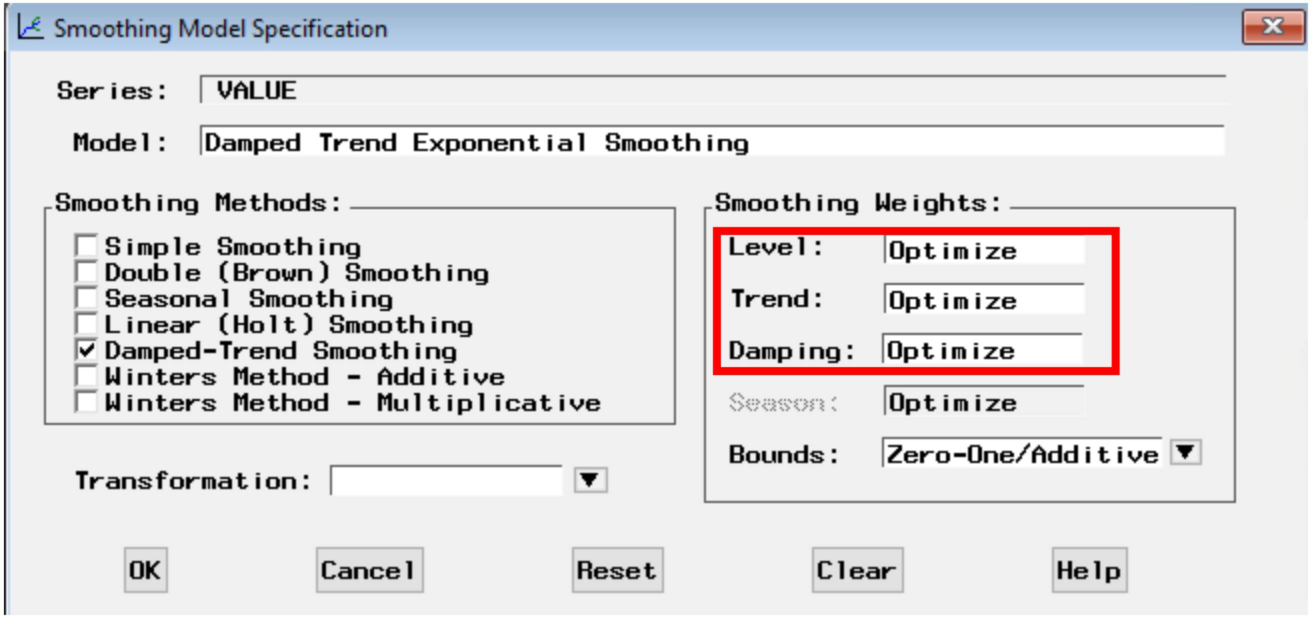
ESM	Parameters	Component
Simple	ω	Level
Double	ω	Level/Trend
Linear (Holt)	ω, γ	Level,Trend
Damped-Trend	ω, γ, ϕ	Level,Trend,Damping

Smoothing Weights

ω Level smoothing weight

γ Trend smoothing weight

ϕ Trend damping weight



Simple Exponential Smoothing Predictions

$\widehat{Y}_1 = Y_0$ (starting value)

$\widehat{Y}_2 = \omega Y_1 + (1-\omega)\widehat{Y}_1$

…

$\widehat{Y}_{t+1} = \omega Y_t + (1-\omega)\widehat{Y}_t$

- The starting value Y_0 is often taken to be the mean of the first *n* observations. SAS TSFS uses *n*=6.
- Does not work well when there is a trend

Double Exponential (Holt) Smoothing

$$L_t = \omega Y_t + (1-\omega)(L_{t-1} + T_{t-1}) \;\; 0 \le \omega \le 1$$

Level equation

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \;\; 0 \le \gamma \le 1$$

Trend equation

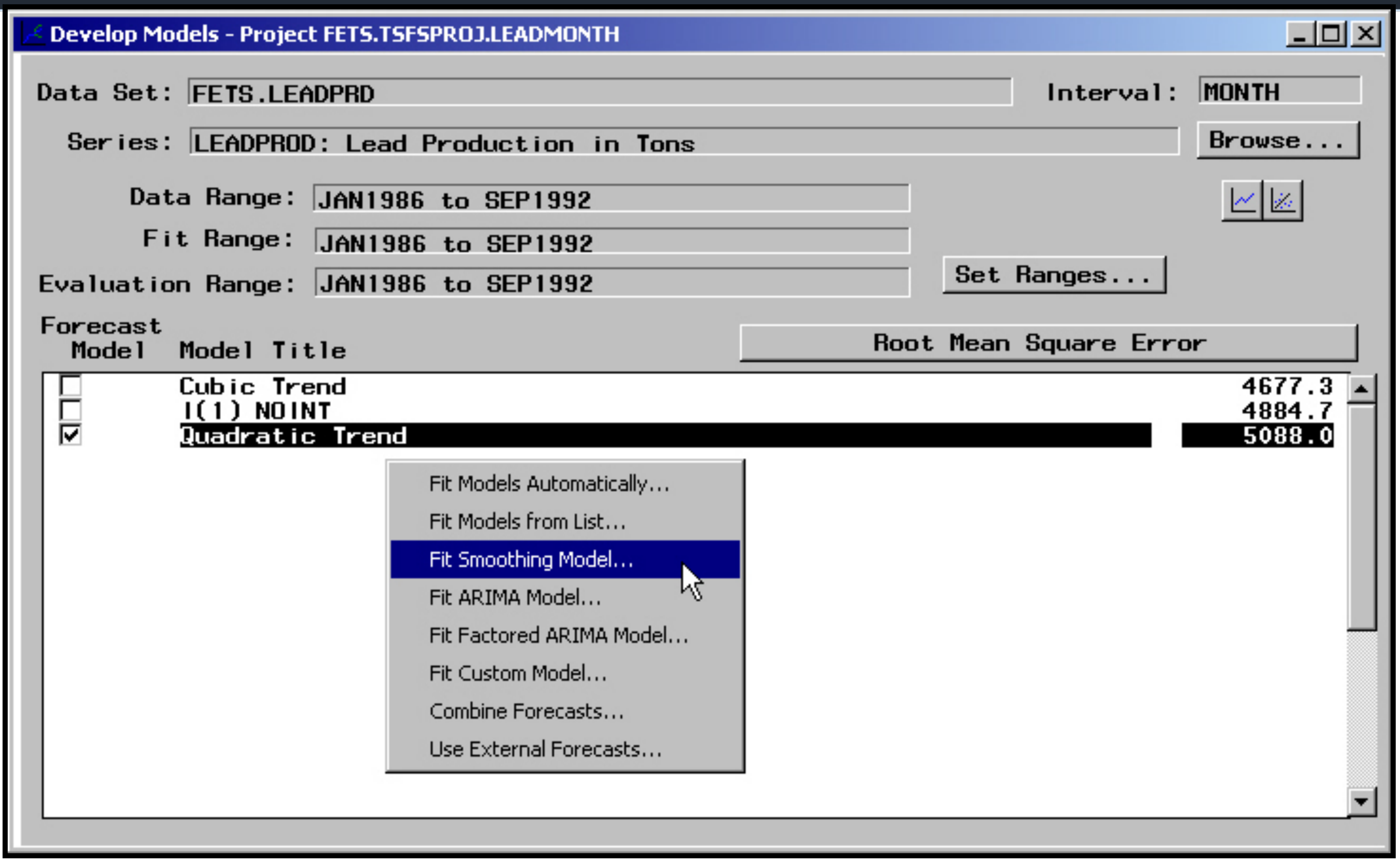
$\hat{Y}_{t+m} = L_t + mT_t$ (m-period-ahead forecast)

Prediction equation

- Smoothing for both level and trend
- TSFS uses Double Exponential Smoothing to refer to a simpler model with only one smoothing parameter (see slide 22)
- Damped-trend smoothing: a third weight on T_{t-1}

Fore more details
https://onlinecourses.science.psu.edu/stat501/node/363
http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4.htm

Exponential Smoothing Models in the TSFS



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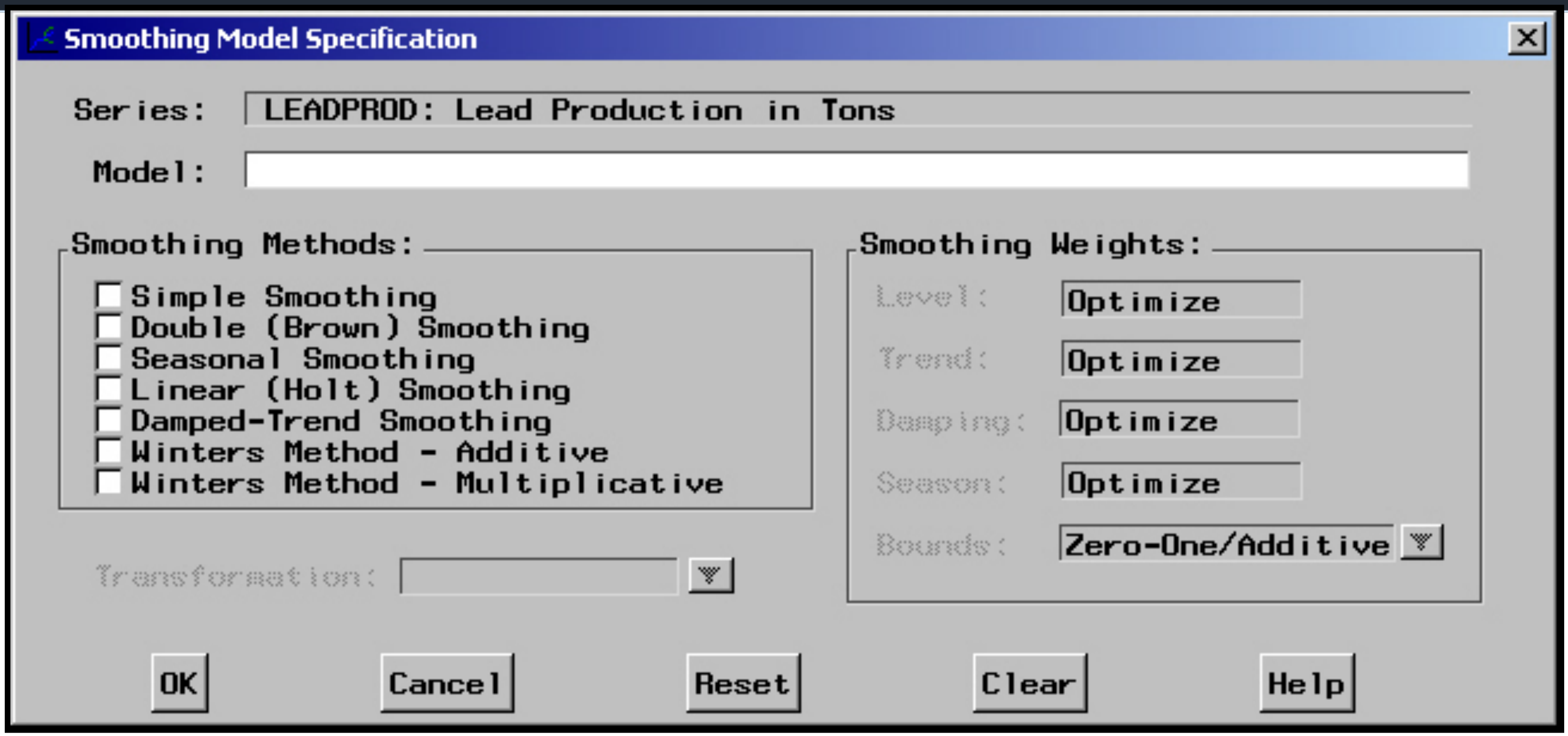
Exponential Smoothing Models in the TSFS



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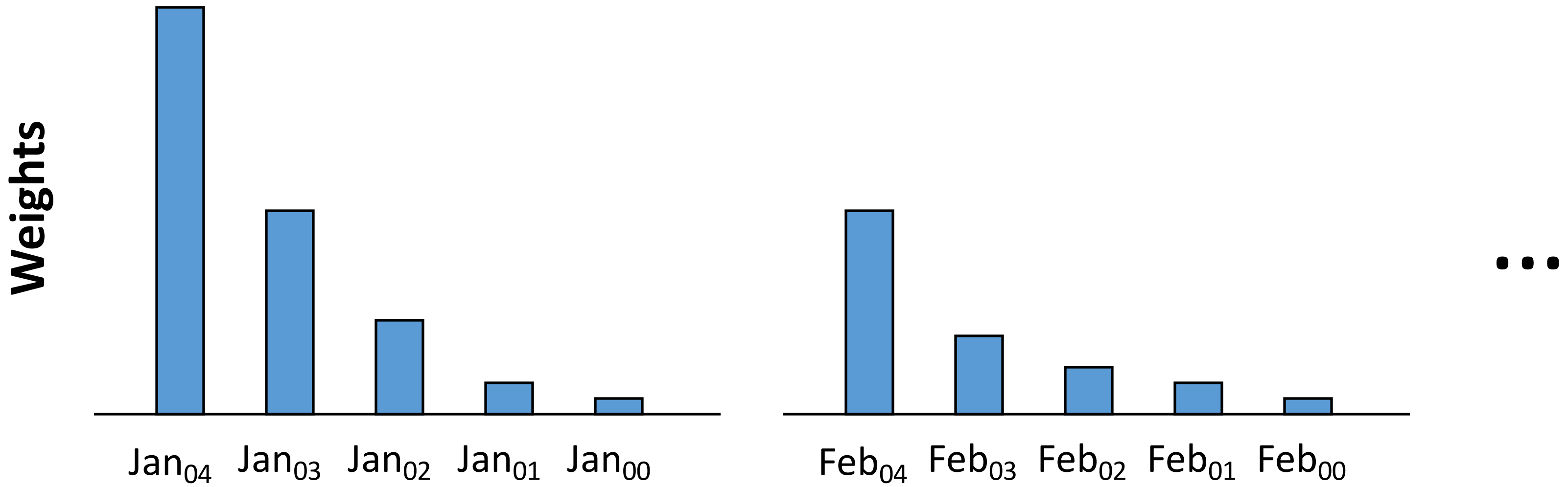
Seasonal Exponential Smoothing

ESM for Seasonal Data



Weights decay with respect to the seasonal factor.

ESM Parameters (Full)

ESM	Parameters
Simple	ω
Double	ω
Linear (Holt)	ω, γ
Damped-Trend	ω, γ, ϕ
Seasonal	ω, δ
Additive Winters	ω, γ, δ
Multiplicative Winters	ω, γ, δ

Smoothing Weights

ω	Level smoothing weight
γ	Trend smoothing weight
δ	Seasonal smoothing weight
ϕ	Trend damping weight

The choice of Greek letter is arbitrary. The software uses names rather than Greek symbols.

Winters Method — Additive

$$L_t = \omega\left(Y_t - S_{t-p}\right) + (1-\omega)(L_{t-1} + T_{t-1})$$

Level equation

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$

Trend equation

$$S_t = \delta(Y_t - L_t) + (1-\delta)S_{t-p}$$

Seasonality equation

$Y_{t+m} = L_t + mT_t + S_t$ (m-period-ahead forecast)

Prediction equation

p is the period of seasonality

Winters Method — Multiplicative

$$L_t = \omega\left(Y_t/S_{t-p}\right) + (1-\omega)(L_{t-1} + T_{t-1})$$

Level equation

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1}$$

Trend equation

$$S_t = \delta(Y_t/L_t) + (1-\delta)S_{t-p}$$

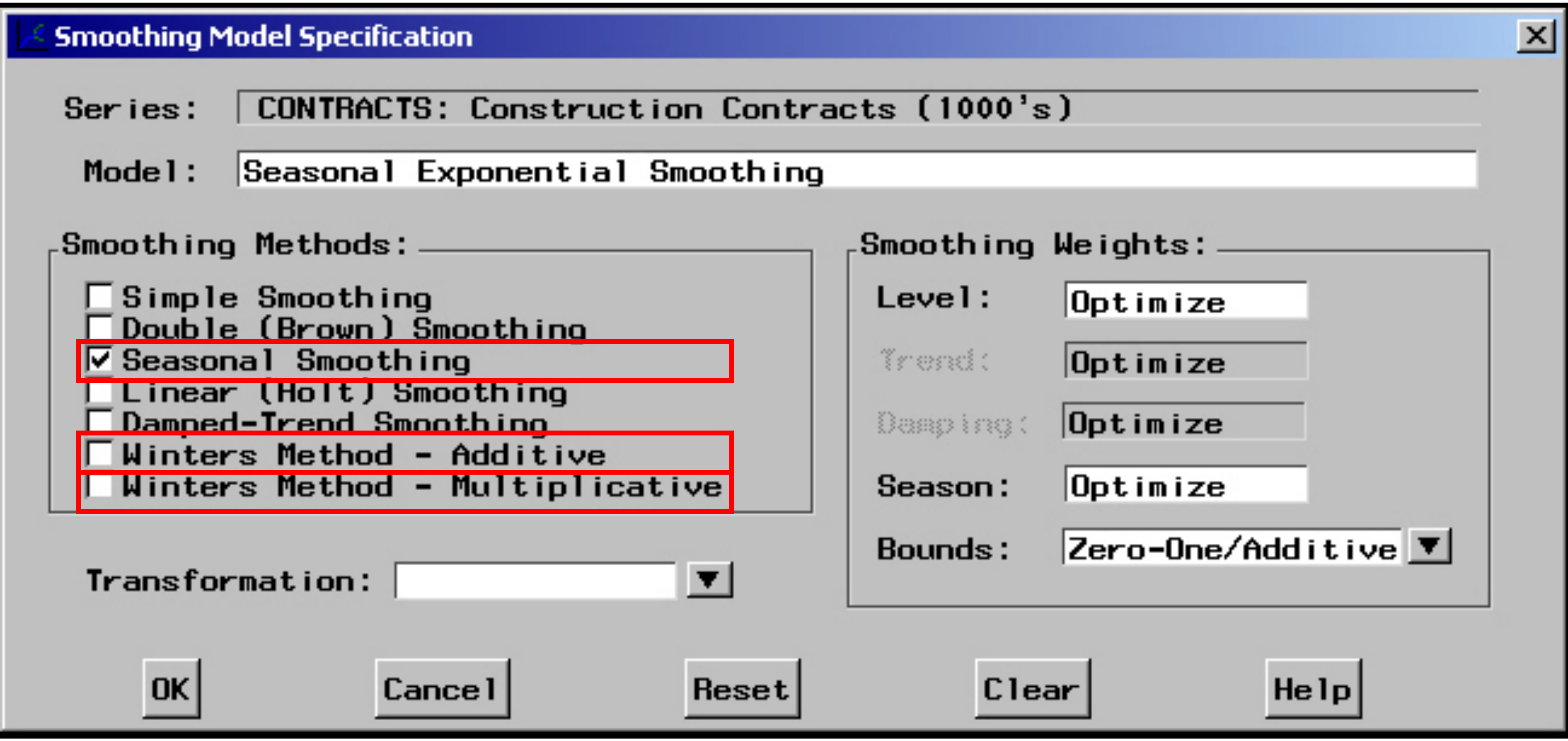
Seasonality equation

$Y_{t+m} = (L_t + mT_t)S_t$ (m-period-ahead forecast)

Prediction equation

p is the period of seasonality

Seasonal Exponential Smoothing Models in the TSFS



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Applications

Fitting ESM for LEADPRD Data

- Models
  - Fit models automatically (use “Diagnose Series” to force SAS to fit all 42 predefined models)
  - Try all non-seasonal ESM models
  - Fit AR(1) and MA(1) models

- Time Series Diagnostics

- Model Residuals Diagnostics

Fitting Seasonal ESM for CONTRACTS Data

- Fit models automatically
- Fit seasonal ESM models
- Setup holdout sample and prediction horizon
- Change evaluation metrics and compare models
- Save predictions

When to Use?

- The prediction of ESM models quickly converges to the mean of time series (plus trend adjustment), not ideal for long-term prediction
  - Simple ESM: short-term prediction
  - Complicated ESM: moderate-term prediction

Readings

- Forecasting Chapter 2
- http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc4.htm
- https://onlinecourses.science.psu.edu/stat501/node/363
- https://people.duke.edu/~rnau/411avg.htm