6

Final Exam due May 20, 2020 05:29 IST *Completed*

Problem 6 (a)

3/3 points (graded)

Starting at time 0, a red bulb flashes according to a Poisson process with rate $\lambda = 1$. Similarly, starting at time 0, a blue bulb flashes according to a Poisson process with rate $\lambda = 2$, but only until a nonnegative random time X, at which point the blue bulb "dies." We assume that the two Poisson processes and the random variable X are (mutually) independent.

1. Suppose that X is deterministically equal to 1. What is the expected total number of flashes (of either color) during the interval $[0, 2]$?

   Expected total number of flashes: 4

   Answer: 4

2. Suppose that $X = \infty$ (i.e., the blue bulb never dies). What is the expected value of the time of the first flash (of either color)?

   Expected value of the time of the first flash:

   1/3

   Answer: 1/3

3. In the time interval $[0, X]$, there are exactly 5 flashes. What is the probability that exactly 2 of them were red?

   Probability that exactly 2 of the 5 flashes were red:

   80/243

   Answer: 80/243

Scroll Down: There are more problems below.

Solution:

1. During the time interval $[0, 1]$, we have a merged Poisson process of total rate $2 + 1 = 3$. In time interval $[1, 2]$, we have a Poisson process of rate 1, corresponding to the red bulb.
   Thus the total expected number of flashes is $3 + 1 = 4$.

2. With both bulbs flashing forever, we have a merged Poisson process of rate $2+1=3$. The expected time until the first flash is thus $\frac{1}{3}$.

3. Looking at the bulb colors, we have a Bernoulli process. The probability that a flash is red is $\frac{1}{2+1}=\frac{1}{3}$. We are looking at the probability that a binomial random variable with $n=5, p=\frac{1}{3}$ takes on the value 2. This is

$$\begin{aligned}\binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 &= 10\frac{1}{9}\frac{8}{27} \\ &= \frac{80}{243}.\end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Problem 6 (b)

2.0/2.0 points (graded)

Suppose that X is equal to either 1 or 2, with equal probability. Write down an expression for the probability that there were exactly 3 arrivals during the time interval $[0,2]$.

(Enter **e** for the constant e. You may use standard notation for this numerical entry even though there will be no parser below the answer box. Enter an exact answer or a numerical answer accurate to at least 3 decimal places.)

Probability that there were exactly 3 arrivals during the time interval $[0,2]$:

0.1423

Answer: 18*e^(-6)+16/3*e^(-4)

STANDARD NOTATION

Scroll Down: There is one more problem below.

Solution:

Conditioned on $\mathbf{X=2}$, we have a merged Poisson process of rate 3, and we want the probability that there were three arrivals of this merged process in $[0,2]$. This is simply

$$\begin{aligned}\frac{e^{-3\cdot 2}(3\cdot 2)^3}{3!} &= \frac{e^{-6}216}{6} \\ &= 36e^{-6}.\end{aligned}$$

Conditioned on $\mathbf{X{=}1}$, we are looking at

$$\begin{aligned} P(3 \text{ total flashes in } [0,2] \,|\, X=1) &= P(3 \text{ red in } [0,2])\, P(0 \text{ blue in } [0,1]) \\ &+ P(2 \text{ red in } [0,2])\, P(1 \text{ blue in } [0,1]) \\ &+ P(1 \text{ red in } [0,2])\, P(2 \text{ blue in } [0,1]) \\ &+ P(0 \text{ red in } [0,2])\, P(3 \text{ blue in } [0,1]) \\ &= \frac{e^{-1\cdot 2}(1\cdot 2)^3}{3!}\frac{e^{-2\cdot 1}(2\cdot 1)^0}{0!} + \frac{e^{-1\cdot 2}(1\cdot 2)^2}{2!}\frac{e^{-2\cdot 1}(2\cdot 1)^1}{1!} \\ &+ \frac{e^{-1\cdot 2}(1\cdot 2)^1}{1!}\frac{e^{-2\cdot 1}(2\cdot 1)^2}{2!} + \frac{e^{-1\cdot 2}(1\cdot 2)^0}{0!}\frac{e^{-2\cdot 1}(2\cdot 1)^3}{3!} \\ &= e^{-4}\left(\frac{8}{6}\frac{1}{1} + \frac{4}{2}\frac{2}{1} + \frac{2}{1}\frac{4}{2} + \frac{1}{1}\frac{8}{6}\right) \\ &= \frac{32}{3}e^{-4}. \end{aligned}$$

Putting them together:
As $X = 1, X = 2$ are equally likely, the final answer is

$$\frac{1}{2}36e^{-6} + \frac{1}{2}\frac{32}{3}e^{-4} = 18e^{-6} + \frac{16}{3}e^{-4}.$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Problem 6 (c)

2.0/2.0 points (graded)
Suppose that X is an exponential random variable with parameter (and mean) equal to 1. Find the MAP estimate of X, given that there were exactly 5 blue flashes.

MAP estimate of X: 5/3 ✔ **Answer:** 5/3

Solution:

Let B denote the number of blue flashes. For the MAP estimate, we wish to find the x which maximizes

$$f_X(x)\,P(B=5|X=x)=e^{-x}\frac{e^{-2x}(2x)^5}{5!}.$$

Taking logarithms, equivalently we wish to maximize $5\log x-3x$.
Differentiating with respect to x and setting equal to 0, we get

$$\frac{5}{x}-3=0,$$

so $x=\frac{5}{3}$.
We note that this value is actually a global maximum; this may be verified by a second derivative test for instance.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

Hide Discussion

Topic: Final Exam:Final Exam / 6

Show all posts by recent activity

Problem 6 c 4
Find the MAP estimate of X , given that there were exactly 5 blue flashes. ---------- Does not give a clear context for t...

6b - My approach 3

[STAFF] Answer typo 4
I got the answer to 6b right in my calculations, however I missed to write in a factor of e in one of the two terms in the ...

Stupid answer 2
will it look like that my answer stupid. in question 6, part a, the answer is 80/243 and I answered 0.3242 and I got wron...

Problem 6c, why don't we use Erlang distribution of order 5? 9
Although my final answer was correct, but I do not totally understand the problem (or maybe the lecture as well). If we ...

(Staff) 6b “Arrival“ term not appropriate. 2
>”... there were exactly 3 arrivals during the time interval...” The use of the term **arrivals** here, instead of **flashes*...

I had the right answer but it is marked as wrong!! 3
Question 6(c), I answered with 1.6 since precision was not specified and the answer was 1.66666...6. Why my answer w...

Access to the course content

Will we have access to the course content indefinitely? Do I need to download everything?

3

? Final Exam vs Capstone Exam

I'd like to ask the question of how similar would be this final exam to the capstone exam ?

7

[STAFF] Alternative interpretation of 6(c)

I am able to understand that intended interpretation was “there were 5 blue flashes and any number of red flashes”, b...

1

Sincere thanks to everyone!

The exam will time out in like 5 minutes. I would like to take this opportunity to thank Prof Tsitsiklis and all the instruct...

5

(Staff) End My Exam Button is damn (Scary)

Dear Staff, It is a humble request to **remove** this **End My Exam** button on the top of the screen while taking th...

11

[STAFF] Q6 part a-2 Point not given

The answer is 1/3. My answer is 0.333 but it is marked as wrong!!

1