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## 6. Upper and lower bounds on the probability of intersection

Problem Set due Feb 5, 2020 05:29 IST Completed

Problem 6. Upper and lower bounds on the probability of intersection

2/2 points (graded)

Given two events A,B with  ${\bf P}(A)=3/4$  and  ${\bf P}(B)=1/3$ , what is the smallest possible value of  ${\bf P}(A\cap B)$ ? The largest? That is, find a and b such that,

$$a \leq \mathbf{P}(A \cap B) \leq b$$
,

holds and any value in the closed interval [a,b] is possible.

$$a = \begin{bmatrix} 1/12 \end{bmatrix}$$
  $\checkmark$  Answer: 1/12

## **Solution:**

First, we begin by recalling that,  $\mathbf{P}\left(A\cap B
ight)$  obeys,

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cup B).$$

Now, for the lower bound, note that,  $\mathbf{P}(A \cup B) \leq 1$ , since the probability of any event can be at most one. Hence,

$$\mathbf{P}\left(A\cap B
ight)\geq\mathbf{P}\left(A
ight)+\mathbf{P}\left(B
ight)-1=rac{1}{12}.$$

Next, for the upper bound, we use the fact that,  $A\cap B\subset A$  and  $A\cap B\subset B$ . Hence,

$$\mathbf{P}(A \cap B) \leq \mathbf{P}(A)$$
 and  $\mathbf{P}(A \cap B) \leq \mathbf{P}(B)$ .

In particular,  $\mathbf{P}(A \cap B)$  must be less than or equal to the smallest of  $\mathbf{P}(A)$  and  $\mathbf{P}(B)$ , which means,

$$\mathbf{P}\left(A\cap B\right)\leq\frac{1}{3}.$$

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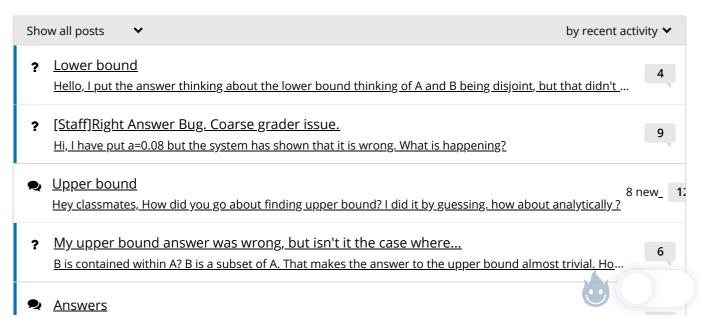
You have used 2 of 3 attempts

**1** Answers are displayed within the problem

## Discussion

**Hide Discussion** 

**Topic:** Unit 1: Probability models and axioms:Problem Set 1 / 6. Upper and lower bounds on the probability of intersection



	Does anyone know when the answers will be provided?	2
<b>∀</b>	Video  Does anyone know what video this relates to?	2
€	lower bound [a] shouldn't the lower bound [a] represent the case where the event A and B are disjoint?	17
2	Recommendation  Ideally, I think this is too soon for set 1. I just started this saturday and had to skip a lot of the lecture to c	5
?	Is there is difference between "P" in normal font and "P" as displayed in this problem?  Hi, everyone! Probably a silly question: but is there a difference between "P" in normal font (for example:	3
2	Answers at some point for revision  Will will get the solutions to these problems to help us revise. I found the answers to the after lecture qui	2
€	Answer number input format  To what decimal precision should the answer be inputted with?	5
4		<b>•</b>

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