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Mid Term due Apr 22, 2020 05:29 IST *Completed*

Problem 5

2.0025/3.0 points (graded)

Let X and Y be independent positive random variables. Let $Z = X/Y$. In what follows, all occurrences of x, y, z are assumed to be positive numbers.

1. Suppose that X and Y are discrete, with known PMFs, p_X and p_Y. Then,

$$p_{Z|Y}(z\,|\,y) = p_X(?)\,.$$

What is the argument in the place of the question mark?

z*y

✔ **Answer:** y*z

$z \cdot y$

2. Suppose that X and Y are continuous, with known PDFs, f_X and f_Y. Provide a formula, analogous to the one in part (a), for $f_{Z|Y}(z\,|\,y)$ in terms of f_X. That is, find A and B in the formula below.

$$f_{Z|Y}(z|y) = A\,f_X(B)\,.$$

$A =$ 1

✖ **Answer:** y

1

$B =$ z*y ✔ **Answer:** y*z

$z \cdot y$

3. Which of the following is a formula for $f_Z(z)$?

$f_Z(z) = \ldots$
(Choose all that apply.)

- [x] $f_Z(z) = \int_0^\infty f_{Y,Z}(y,z)\, dy$ ✱
- [ ] $f_Z(z) = \int_0^\infty f_{Y,Z}(y,z)\, dz$
- [ ] $f_Z(z) = \int_0^\infty f_Y(y)\, f_{Z,Y}(z,y)\, dy$
- [x] $f_Z(z) = \int_0^\infty f_Y(y)\, f_{Z|Y}(z|y)\, dy$ ✱
- [x] $f_Z(z) = \int_0^\infty f_Y(y)\, f_X(yz)\, dy$
- [ ] $f_Z(z) = \int_0^\infty y f_Y(y)\, f_X(yz)\, dy$ ✔

✱

Solution:

Let X and Y be independent positive random variables. Let $Z = X/Y$. In the sequel, all occurrences of x, y, z are assumed to be positive numbers.

1. Since $Z = X/Y$ implies $X = YZ$, the argument is simply yz.

2.

We follow the standard approach of computing the (conditional) CDF and then differentiating to get the conditional PDF.
First, let us obtain the CDF by using the information of the previous part.

$$\begin{aligned}F_{Z|Y}(z|y) &= P(Z \leq z|Y=y)\\ &= P(X/Y \leq z|Y=y)\\ &= P(X/y \leq z|Y=y)\\ &= P(X/y \leq z)\\ &= P(X \leq yz)\\ &= F_X(yz).\end{aligned}$$

Now we simply differentiate with respect to z on both sides to obtain:

$$f_{Z|Y}(z|y) = y f_X(yz).$$

In the above, we applied the chain rule for differentiation.

3. We have

$$\begin{aligned}f_Z(z) &= \int_0^\infty f_{Y,Z}(y,z)\,dy\\ &= \int_0^\infty f_Y(y)\,f_{Z|Y}(z|y)\,dy\\ &= \int_0^\infty y f_Y(y)\,f_X(yz)\,dy.\end{aligned}$$

Alternative ranges for the integration (such as $-\infty, \infty$) are also valid, as long as they contain the interval $[0, \infty)$. This is because the PDFs of X, Y are zero outside the interval $[0, \infty)$. $(0, \infty)$ thus represents the “minimal" acceptable bounds in the generic case.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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