14. Exercise: Theoretical properties

Exercises due Apr 15, 2020 05:29 IST *Completed*

Exercise: Theoretical properties

2/2 points (graded)

Let $\widehat{\Theta}$ be an estimator of a random variable Θ, and let $\widetilde{\Theta} = \widehat{\Theta} - \Theta$ be the estimation error.

a) In this part of the problem, let $\widehat{\Theta}$ be specifically the LMS estimator of Θ. We have seen that for the case of the LMS estimator, $\mathbf{E}\left[\widetilde{\Theta} \mid X = x\right] = 0$ for every x. Is it also true that $\mathbf{E}\left[\widetilde{\Theta} \mid \Theta = \theta\right] = 0$ for all θ? Equivalently, is it true that $\mathbf{E}\left[\widehat{\Theta} \mid \Theta = \theta\right] = \theta$ for all θ?

No ✔ **Answer:** No

b) In this part of the problem, $\widehat{\Theta}$ is no longer necessarily the LMS estimator of Θ. Is the property $\mathsf{Var}\left(\Theta\right) = \mathsf{Var}\left(\widehat{\Theta}\right) + \mathsf{Var}\left(\widetilde{\Theta}\right)$ true for every estimator $\widehat{\Theta}$?

No ✔ **Answer:** No

Solution:

a) There is no reason for this relation to be true. For an example, suppose that Θ is a Bernoulli random variable. With a noisy measurement, $\widehat{\Theta}$ will be somewhere in between 0 and 1, and therefore will never be equal to the true value of θ, which is either 0 or 1 exactly.

b) There is no reason for this to be the case. In fact, the variance of $\widehat{\Theta}$, for a poorly chosen estimator, can be larger than the variance of Θ. For an example, consider the usual model of an observation $X = \Theta + W$ and the estimator $\widehat{\Theta} = 100X$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 7: Bayesian inference:Lec. 16: Least mean squares (LMS) estimation / 14. Exercise: Theoretical properties

Show all posts | by recent activity

?	Part a) why my reasoning is incorrect	2
?	Isn't it the question b) asking if the very definition of the property ok? Am I Reading wrong or the question asks whether the definition of the property is correct in every case? ...	1
?	Point (a) As seen on Unit 6 Lec 13, E[T|T=t] = t since T no longer is a random variable. In here, in a conditional uni...	7
?	Part (a) posterior distribution	1
?	Regarding Part b	1
☑	Covariance of error and estimator	2
?	Calculate $\mathbf{E}[\Theta\hat{}\|\Theta=\theta]$ Hello! I want to find a more formal solution to the exercise 14.a). So, I tried to calculate $\mathbf{E}[\Theta\hat{}\|\Theta=\theta]$ explici...	5