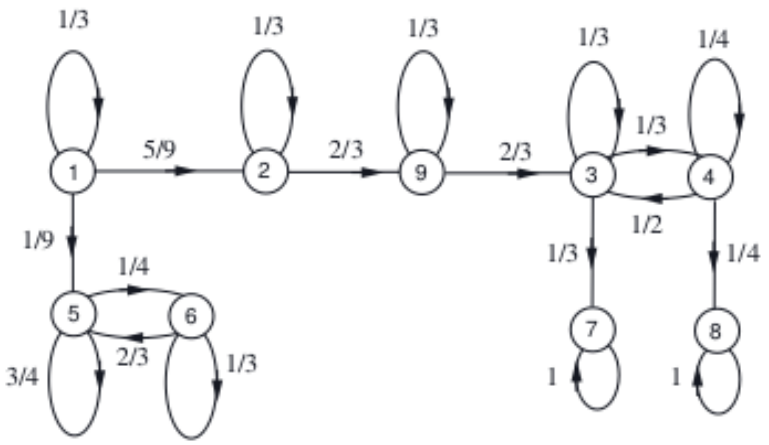
5. Exercise: Steady-state approximation

None due May 29, 2020 05:29 IST

Exercise: Steady-state approximation

0.0/3.0 points (ungraded)

Consider a Markov chain with the following transition probability graph:



Find an approximation to $\mathbf{P}\left(X_{10000} = 5 \mid X_0 = 1\right) = r_{15}\left(10000\right)$.
Hint: First find an (exact) equation relating $r_{15}\left(10000\right)$, $r_{15}\left(9999\right)$ and $r_{55}\left(9999\right)$.

$r_{15}\left(10000\right) \approx$ [ ] **Answer:** 0.12121

Solution:

Following the hint, we look one transition ahead: going from state 1 to state 5 after 10000 transitions can be achieved by either (i) staying at state 1 after the first transition and then going from state 1 to state 5 in 9999 transitions, or (ii) going from state 1 to state 5 after the first transition and then ending back in state 5 after 9999 transitions. The first transition cannot go to state 2 because then there would be no way to end up in state 5. Hence,

$$r_{15}(10000) = p_{11} r_{15}(9999) + p_{15} r_{55}(9999) = \frac{1}{3} r_{15}(9999) + \frac{1}{9} r_{55}(9999).$$

Since 9999 and 10000 are both large numbers of transitions, we use two approximations: (i) $r_{15}(9999) \approx r_{15}(10000)$ and (ii) $r_{55}(9999) \approx \pi_5$, the steady-state probability of being in state 5 when we consider the aperiodic recurrent class $\{5, 6\}$. With these approximations, we have

$$r_{15}(10000) \approx \frac{1}{3} r_{15}(10000) + \frac{1}{9}\pi_5 \Rightarrow r_{15}(10000) \approx \frac{1}{6}\pi_5.$$

The steady-state probabilities π_5 and π_6 are obtained by solving the system of equations

$$\begin{aligned} \frac{1}{4}\pi_5 &= \frac{2}{3}\pi_6 \\ \pi_5 + \pi_6 &= 1, \end{aligned}$$

which leads to $\pi_5 = 8/11$ and $\pi_6 = 3/11$.

Therefore, $r_{15}(10000) \approx \frac{4}{33}$.

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Answers are displayed within the problem

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Ignoring state 7 and 8? 4

The steady state equation pi_5 + pi_6 = 1 totally ignores recurrent states 7 and 8. Is it allowed or did I mis...

? Better approximation?

path ignored?

why the option r15(999)*p55 is not considered?

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