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Final Exam due May 20, 2020 05:29 IST *Completed*

Problem 4

3.0/3.0 points (graded)

Let X and Y be independent random variables, with X uniformly distributed on $[0,1]$ and Y uniformly distributed on $[0,2]$. Find the PDF $f_Z(z)$ of $Z = \max\{X,Y\}$.

For $z < 0$ or $z > 2$:

$f_Z(z) =$ 0

0

✔ **Answer:** 0

For $0 \leq z \leq 1$:

$f_Z(z) =$ z

z

✔ **Answer:** z

For $1 \leq z \leq 2$:

$f_Z(z) =$ 1/2

$\frac{1}{2}$

✔ **Answer:** 1/2

Solution:

We follow the standard method of obtaining the CDF of $Z = \max\{X,Y\}$ and then differentiating.

We have by the independence of X, Y:

$$\begin{aligned} F_Z(z) &= P(Z \leq z) \\ &= P(\max\{X, Y\} \leq z) \\ &= P(X \leq z, Y \leq z) \\ &= P(X \leq z)\, P(Y \leq z) \\ &= F_X(z)\, F_Y(z). \end{aligned}$$

As X, Y are uniform random variables, we have the CDF's of X, Y:

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \leq x \leq 1, \\ 1, & x > 1. \end{cases}$$

$$F_Y(y) = \begin{cases} 0, & y < 0, \\ \frac{y}{2}, & 0 \leq y \leq 2, \\ 1, & y > 2. \end{cases}$$

We may then multiply these CDF's to get:

$$F_Z(z) = \begin{cases} 0, & z < 0, \\ \frac{z^2}{2}, & 0 \leq z \leq 1, \\ \frac{z}{2}, & 1 \leq z \leq 2, \\ 1, & z > 2. \end{cases}$$

Differentiating the CDF of Z, we finally get the PDF of $Z = \max\{X, Y\}$:

$$f_Z(z) = \begin{cases} 0, & z < 0, \\ z, & 0 \leq z \leq 1, \\ \frac{1}{2}, & 1 \leq z \leq 2, \\ 0, & z > 2. \end{cases}$$

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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- [Staff] 2 and 3 question.
  Community TA — 7
- [STAFF] Technical issues - Cannot submit the answer/Question disappeared after 1st attempt of submission — 3
- What does "max" mean in this setting?
  I'm drawing a blank and haven't found any lectures that used/manipulated max except for unit 23 in the ... — 3
- My answers for Q4 of the Final
  a. 0 b. z c. 1/2 — 7
- [STAFF] Are the inequalities stated in pt2 and pt3 definitely correct?
  To me, it seems as though one of the inequalities for part 3 is not strictly correct. Could staff please dou... — 5
- Misspelling
  The question says "Let X and Y be independently random variables". I guess that is copy-paste from som... — 3
- STAFF: are the answers inspected in terms of X,Y or numbers?
  E.g,. g_G(g) = J = c (a pdf that is constant), should we enter J or c? — 2
- f(z=1)?
  Second and third questions include z = 1. Is it correct? I think that in third question should be strict inequ... — 2