4. Exercise: Estimator properties

Exercises due May 1, 2020 05:29 IST *Completed*

Exercise: Estimator properties

3/4 points (graded)

We estimate the unknown mean θ of a random variable X (where X has a finite and positive variance) by forming the sample mean $M_n = (X_1 + \cdots + X_n)/n$ of n i.i.d. samples X_i and then forming the estimator

$$\widehat{\Theta} = M_n + \frac{1}{n}.$$

Is this estimator unbiased?

No ✔ **Answer:** No

Is this estimator consistent?

Yes ✔ **Answer:** Yes

Consider now a different estimator, $\widehat{\Theta}_n = X_1$, which ignores all but the first measurement.

Is this estimator unbiased?

Yes ✔ **Answer:** Yes

Is this estimator consistent?

Yes ✘ **Answer:** No

Solution:

We have $\mathbf{E}\left[\widehat{\Theta}_n\right] = \theta + (1/n) \neq \theta$, so it is not unbiased. On the other hand, M_n converges (in probability) to θ, and $1/n$ converges to zero. So, their sum, $\widehat{\Theta}_n = M_n + (1/n)$ also converges (in probability) to θ, and the estimator is consistent.

The second estimator is unbiased, because $\mathbf{E}\left[\widehat{\Theta}_n\right] = \mathbf{E}\left[X_1\right] = \theta$. But it is not consistent. Its value stays the same (equal to X_1) for all n and therefore cannot converge to θ, unless X_1 is guaranteed to be equal to θ. But this is impossible since X has positive variance.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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Bias & Estimators
What is the difference between estimators Θ^n (with subscript n) & Θ^(without subscript). In the next vl... 7

Unbiased as n approaches infinity
HI. I thought that 1/n becomes infinitely small, one could assume that 1/n added nothing to the distribut... 3 new_

Is bias and consistency only limited to classical statistics?
Is bias and consistency only limited to classical statistics? 2

consistency vs bias (possibly useful info)
I didn't have a clue, but I found this. https://stats.stackexchange.com/questions/31036/what-is-the-differ... 3