13. Exercise: LLMS with multiple observations

Exercises due Apr 15, 2020 05:29 IST *Completed*

Exercise: LLMS with multiple observations

3/3 points (graded)

Suppose that Θ, X_1, and X_2 have zero means. Furthermore,

$$\text{Var}(X_1) = \text{Var}(X_2) = \text{Var}(\Theta) = 4,$$

and

$$\text{Cov}(\Theta, X_1) = \text{Cov}(\Theta, X_2) = \text{Cov}(X_1, X_2) = 1.$$

The LLMS estimator of Θ based on X_1 and X_2 is of the form $\widehat{\Theta} = a_1 X_1 + a_2 X_2 + b$. Find the coefficients a_1, a_2, and b. *Hint:* To find b, recall the argument we used for the case of a single observation.

$a_1 =$ 0.2 ✔ **Answer:** 0.2

$a_2 =$ 0.2 ✔ **Answer:** 0.2

$b =$ 0 ✔ **Answer:** 0

Solution:

By the same argument as in the case of a single observation, we will have $b = \mathbf{E}\left[\Theta - a_1 X_1 - a_2 X_2\right] = 0$. Using the variance and covariance information we are given, the expression we want to minimize is

$$\mathbf{E}\left[\left(a_1 X_1 + a_2 X_2 - \Theta\right)^2\right] = 4a_1^2 + 4a_2^2 + 4 + 2a_1 a_2 - 2a_1 - 2a_2 .$$

Because of symmetry, we see that the optimal solution will satisfy $a_1 = a_2 = a$, so the expression is of the form $8a^2 + 4 + 2a^2 - 4a$. By setting the derivative to zero, we find that $20a = 4$, or $a = 1/5$.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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- Symmetry alone is not enough — 1
- messed up with the signs and wasted 2 hours! — pay attention to the signs of the terms that come out of expanding the square (and the eventual derivati... — 2
- A clear logic or method is way more important than calculation — As stated in the title, I find this the paramount lesson learned in this course, especially after reading give... — 7
- ? Guidance on the expansion and substitution within the solution — Could someone please explain to me why the last three terms of the expanded Expectation (which has t... — 3
- Best value of b question — Can someone help me understand why the best value of b is `E(Theta - a_1 * X_1 - a_2 * X_2)` but not th... — 3
- ? Any hints or videos regarding this question? — I tried minimizing the expression by setting the derivative to 0. But I got 1 equation with 2 unknowns whi... — 7
- ? How to tell the symmetry property in this question? — I did the question right through a method different from the solution and I can't tell how to find that it's

I did the question right through a method different from the solution and I can't tell how to find that it's ...