



6. Correlation coefficients

Problem Set due Apr 1, 2020 05:29 IST Completed

Problem 6. Correlation coefficients

3/3 points (graded)

Consider random variables X , Y and Z , which are assumed to be pairwise uncorrelated (i.e., X and Y are uncorrelated, X and Z are uncorrelated, and Y and Z are uncorrelated). Suppose that

- $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 0$,
- $\mathbf{E}[X^2] = \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 1$,

Find the correlation coefficients $\rho(X - Y, X + Y)$, $\rho(X + Y, Y + Z)$, and $\rho(X, Y + Z)$.

1.

$$\rho(X - Y, X + Y) = \boxed{0} \quad \checkmark \text{ Answer: } 0$$

2.

$$\rho(X + Y, Y + Z) = \boxed{1/2} \quad \checkmark \text{ Answer: } 0.5$$

3.

$$\rho(X, Y + Z) = \boxed{0} \quad \checkmark \text{ Answer: } 0$$

Solution:

1. We have

$$\text{cov}(X - Y, X + Y) = \mathbf{E}[(X - Y)(X + Y)] - \mathbf{E}[X - Y]\mathbf{E}[X + Y]$$



$$\begin{aligned}
&= \mathbf{E}[X^2 - Y^2] - 0 \\
&= \mathbf{E}[X^2] - \mathbf{E}[Y^2] \\
&= 0.
\end{aligned}$$

Hence, $\rho(X - Y, X + Y) = 0$.

2. Since X and Y are uncorrelated, with zero means, we have

$\mathbf{E}[XY] = \text{cov}(X, Y) = 0$. Similarly, we have $\mathbf{E}[XZ] = 0$ and $\mathbf{E}[YZ] = 0$.
Hence,

$$\begin{aligned}
\text{cov}(X + Y, Y + Z) &= \mathbf{E}[(X + Y)(Y + Z)] - \mathbf{E}[X + Y]\mathbf{E}[Y + Z] \\
&= \mathbf{E}[XY + XZ + Y^2 + YZ] \\
&= \mathbf{E}[Y^2] \\
&= 1.
\end{aligned}$$

Also,

$$\begin{aligned}
\text{Var}(X + Y) &= \mathbf{E}[(X + Y)^2] - (\mathbf{E}[X + Y])^2 \\
&= \mathbf{E}[X^2 + 2XY + Y^2] - 0 \\
&= 2.
\end{aligned}$$

Similarly, $\text{Var}(Y + Z) = 2$.

$$\text{Therefore, } \rho(X + Y, Y + Z) = \frac{\text{cov}(X + Y, Y + Z)}{\sqrt{\text{Var}(X + Y)\text{Var}(Y + Z)}} = \frac{1}{2}.$$

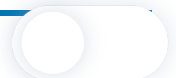
3.

$$\begin{aligned}
\text{cov}(X, Y + Z) &= \mathbf{E}[X(Y + Z)] - \mathbf{E}[X]\mathbf{E}[Y + Z] \\
&= \mathbf{E}[XY + YZ] - 0 \\
&= 0.
\end{aligned}$$

Hence, $\rho(X, Y + Z) = 0$.

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You have used 1 of 3 attempts











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Topic: Unit 6: Further topics on random variables: Problem Set 6 / 6.
Correlation coefficients

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-  Intuition for part 1 4
Is there an intuition for this part? The answer seems like it could have an intuitive explanation. P/S: it loo...
-  Hint on 2? 1 new_
Not sure what I'm doing wrong but I got incorrect answer for point 2). I use formula with expectations to...
-  0 denominator in correlation coeff. 2
How it works the definition in the cases where the denominator is zero (0 variance) ?
-  How can this be solved without knowing if the X, Y and Z are independent? 4
Given that uncorrelated does not necessarily mean the variables are independent, how can we compute...
-  Hint 1
-  Hint: Bilinearity of Covariance 2
It really helps if you take in consideration that the covariance has the distributive property. In that way,y...
-  How to find the expected value of a product? 3
As in a previous problem, I'm stuck on trying to find the expected value of a product, and not a complica...
-  Did the lectures cover expected value of a sum? 2
As above.

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