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5. Hats in a box

Problem Set due Feb 19, 2020 05:29 IST **Completed**

Problem 5. Hats in a box

5/5 points (graded)

Each one of n persons, indexed by $1, 2, \dots, n$, has a clean hat and throws it into a box. The persons then pick hats from the box, at random. Every assignment of the hats to the persons is equally likely. In an equivalent model, each person picks a hat, one at a time, in the order of their index, with each one of the remaining hats being equally likely to be picked. Find the probability of the following events.

(You need to answer all 5 questions before you can submit.)

1. Every person gets his or her own hat back.

☒ $\frac{1}{n!}$

☐ $\frac{1}{(n+1)!}$

☐ $\frac{1}{n}$

☐ $\frac{1}{n+1}$



2. Each one of persons $1, \dots, m$ gets his or her own hat back, where $1 \leq m \leq n$.

☐ $\frac{(n+m)!}{n!}$

☒ $\frac{(n-m)!}{n!}$

☐ $\frac{n!}{(n+m)!}$

☐ $\frac{m!}{n!}$



3. Each one of persons $1, \dots, m$ gets back a hat belonging to one of the last m persons (persons $n-m+1, \dots, n$), where $1 \leq m \leq n$.

☒ $\frac{1}{\binom{n}{m}}$

☐ $\frac{m}{\binom{n}{m}}$

☐ $\frac{n-m}{\binom{n}{m}}$



☐ $\frac{n}{\binom{n}{m}}$



Now assume, in addition, that every hat thrown into the box has probability p of getting dirty (independently of what happens to the other hats or who has dropped or picked it up). Find the probability that:

4. Persons $1, \dots, m$ will pick up clean hats.

☐ $(1 - p)^{n-m}$

☐ $m(1 - p)^m$

☒ $(1 - p)^m$

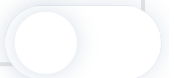
☐ $m(1 - p)^{n-m}$



5. Exactly m persons will pick up clean hats.

☐ $\frac{\binom{n}{m}}{n!} (1 - p)^m p^{n-m}$

☐ $(1 - p)^m p^{n-m}$



☐ $\binom{n}{m} (1-p)^{n-m} p^m$

☒ $\binom{n}{m} (1-p)^m p^{n-m}$



Solution:

1. Consider the sample space of all possible hat assignments. It has $n!$ elements (n hat selections for the first person, after that $n - 1$ for the second, etc.), with every assignment equally likely; hence each assignment has probability $1/n!$. The event that everyone gets his or her own hat back corresponds to exactly one of these $n!$ assignments. Therefore, the answer is $1/n!$.
2. Consider the same sample space and probabilities as in the solution of part 1. The event of interest assigns the first m people to their own hats and allows for an arbitrary assignment of hats to the remaining $n - m$ persons, so that there are $(n - m)!$ possible assignments. The probability of an event with $(n - m)!$ elements is $(n - m)!/n!$.
3. Consider the m hats belonging to the last m persons. There are $m!$ ways to distribute these m hats among the first m persons. Then, there are $(n - m)!$ ways to distribute the remaining $n - m$ hats to everyone else. The probability of an event with $m! (n - m)!$ elements is $m! (n - m)!/n!$, which is equal to $1/\binom{n}{m}$.
4. The probability of a given person picking up a clean hat is $1 - p$. By the independence assumption, the probability of m specific persons picking up clean hats is $(1 - p)^m$.
5. Think of picking a clean hat as an independent Bernoulli trial with success probability $1 - p$. The probability of m successes out of n trials is $\binom{n}{m} (1 - p)^m p^{n-m}$.

Submit

You have used 2 of 2 attempts













i Answers are displayed within the problem



Discussion

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 Am I understanding Pt 3 correctly? Say $n = 5$ and $m = 4$ In layman's terms, the question is asking what is the probability that each of persons...	11
 Problem missing When I checked out my learning progress, I see there should be 6 problems in this problem set. But I ca...	3
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 Grading Issue - Missing Problem? Completed the five problems and earned the full 2, 1, 2, 4, & 5 points on them. Under progress it lists my...	5
 Hint Solved it counting cases using $n=2$ and $n=3$, For eg a is the cap for person 1, b for person 2, c for person ...	2
 Language / Specifications on Q2 On Q2, when saying "1-m persons gets their own hats back", does it automatically imply that $m+1$ to n p...	2
 Interpretation of 3 I'm at loss interpreting the statement given in 3) (even after reading through the posts) Each one of per...	6
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