1. Marie gives away children toys

Problem Set due May 13, 2020 05:29 IST *Completed*

Problem 1. Marie gives away children toys

7/7 points (graded)

Marie distributes toys for toddlers. She makes visits to households and gives away one toy only on visits for which the door is answered **and** a toddler is in residence. On any visit, the probability of the door being answered is $3/4$, and the probability that there is a toddler in residence is $1/3$. Assume that the events "Door answered" and "Toddler in residence" are independent and also that events related to different households are independent.

1. What is the probability that she has not distributed any toys by the end of her second visit?

   9/16

   ✔ **Answer:** 9/16

2. What is the probability that she gives away the first toy on her fourth visit?

   27/256

   ✔ **Answer:** 27/256

3. Given that she has given away her second toy on her fifth visit, what is the conditional probability that she will give away her third toy on her eighth visit?

   9/64

   ✔ **Answer:** 9/64

4. What is the probability that she will give away the second toy on her fourth visit?

   27/256

   ✔ **Answer:** 27/256

5.

Given that she has not given away her second toy by her third visit, what is the conditional probability that she will give away her second toy on her fifth visit?

1/8

✔ **Answer:** 1/8

6. We will say that Marie “needs a new supply"" immediately **after** the visit on which she gives away her last toy. If she starts out with three toys, what is the probability that she completes at least five visits before she needs a new supply?

997/1024

✔ **Answer:** 243/256

7. If she starts out with exactly six toys, what is the expected value of the number of houses with toddlers that Marie visits without leaving any toys (because the door was not answered) before she needs a new supply?

24/12

✔ **Answer:** 2

Solution:

A successful (i.e., the door is answered, and a toddler is present in the residence) visit (“trial//) occurs with probability $p = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$.

1. This is the probability that the first two trials were failures, which happens with probability

$$(1-p)(1-p) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}.$$

2. She gives away her first toy on her fourth visit if and only if the first three trials are failures, and the last (fourth) trial is a success. This happens with probability

$$(1-p)(1-p)(1-p)p = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{27}{256}.$$

3.

The given event of interest happens if and only if the sixth and seventh trials are failures, and the eighth trial is a success. Using the fresh start property of Bernoulli process, this happens with probability

$$(1-p)\,(1-p)\,p = \frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{9}{64}.$$

4. We are interested in the probability that the second success time, Y_2 is equal to 4, namely, $\mathbf{P}\left(Y_2 = 4\right)$. Here, Y_2 has a Pascal PMF and

$$\mathbf{P}\left(Y_2 = 4\right) = \binom{4-1}{2-1} p^2 (1-p)^{4-2} = 3\cdot\left(\frac{1}{4}\right)^2\cdot\left(\frac{3}{4}\right)^2 = \frac{27}{256}.$$

5. We are given the event that $\{Y_2 > 3\}$ and are asked to find the conditional probability of the event $\{Y_2 = 5\}$. Note that the possible values of Y_2 are $2, 3, \ldots$ and therefore,

$$\mathbf{P}\left(Y_2 > 3\right) = 1 - \mathbf{P}\left(Y_2 = 2\right) - \mathbf{P}\left(Y_2 = 3\right).$$

We then have,

$$\begin{aligned}
\mathbf{P}\left(Y_2 = 5 \mid Y_2 > 3\right) &= \frac{\mathbf{P}\left(Y_2 = 5 \cap Y_2 > 3\right)}{\mathbf{P}\left(Y_2 > 3\right)} \\
&= \frac{\mathbf{P}\left(Y_2 = 5\right)}{\mathbf{P}\left(Y_2 > 3\right)} \\
&= \frac{\binom{5-1}{2-1}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{5-2}}{1 - p_{Y_2}(2) - p_{Y_2}(3)} \\
&= \frac{\frac{27}{256}}{1 - \frac{1}{4}\cdot\frac{1}{4} - 2\cdot\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}} \\
&= \frac{\frac{27}{256}}{\frac{27}{32}} = \frac{1}{8},
\end{aligned}$$

6.

This is the probability that the time Y_3 of the third success is greater than or equal to 5:

$$\mathbf{P}(Y_3 \geq 5) = 1 - \mathbf{P}(Y_3 \leq 4) = 1 - \sum_{\ell=3}^{4} \binom{\ell-1}{3-1} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{\ell-3}$$
$$= 1 - \binom{3}{2}\frac{1}{64}\cdot\frac{3}{4} - \binom{2}{2}\frac{1}{64} = \frac{243}{256}.$$

7. In this part, we are only considering the visits to houses with toddlers. At each such visit, either (i) the door is answered ("success"), which happens with probability $3/4$, and a toy is given, or (ii) the door is not answered ("failure") which happens with probability $1/4$. We wish to determine the expected number of failures until the 6th success. The expected number of trials up to and including the 6th success is $6/(3/4) = 8$. The number of failures is the number of such visits minus the number of successes, namely, 6. Therefore, the expected number of failures is $8 - 6 = 2$.

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

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#5 not convinced by the solution
I read "not 2 toys by visit 3" as: in 3V a total of 0 or 1 toys we given out. p(0T in 3V) = 3^3/4^3 actually i... 2

[Staff] Extension of Final Exam Date
Could you please extend the date of Final Exam till May 24? Since most of the students are working pr... 1 new_

Still stuck on 5
I am missing the conditional. But since there are more than one scenarios under which "Mary would ... 4

Question 5 3

(Staff) Revisiting Part 6 Solution
I had solved it by considering the following probability: 1-**P**(3 toys were distributed in 4 visits) (c...

Hint
I'm done with everything else but I'm completely lost on this question, not sure how to even begin. Ca... 1

hardest set of problems of the course
I feel really frustrated because I feel this has been the most tricky and intuitive set of problems of the ... 6

[STAFF] Question 5. "by her third visit" interpretation
My question is about the conditioning event "she has not given away her second toy by her third visit... 1 new_

? Interpretation on #6?
I was thinking of this question as adding the probabilities that she delivers 0, 1, 2, or 3 toys on those fi... 7

? Still confused on Part 7
I've gotten all parts except part 7. Any hints would be appreciated. 6

#5 eludes me 5

Your favorite character?
I'm glad to see another memorable character added to this course's rogues' gallery! Now that we loo... 4

[Staff] PSet 9 and PSet 10 scores not updated in progress
I have completed a few problems in PSet 9 and PSet 10, but the scores are not reflected in the progre... 4