2. Functions of the standard normal

Problem Set due Apr 1, 2020 05:29 IST *Completed*

Problem 2. Functions of the standard normal

2/2 points (graded)

The random variable X has a standard normal distribution. Find the PDF of the random variable Y, where:

1. $Y = 5X - 7$.

- $f_Y(y) = 5 f_X\left(\frac{y-7}{5}\right)$
- **(selected, correct)** $f_Y(y) = \frac{1}{5} f_X\left(\frac{y+7}{5}\right)$
- $f_Y(y) = 5 f_X\left(\frac{y+7}{5}\right)$
- $f_Y(y) = \frac{1}{5} f_X\left(\frac{y-7}{5}\right)$

✔

2. $Y = X^2 - 2X$. For $y \geq -1$,

- $\frac{f_X(1+\sqrt{y+1}) - f_X(1-\sqrt{y+1})}{2\sqrt{y-1}}$

- ○ $\dfrac{f_X\left(1+\sqrt{y+1}\right)-f_X\left(1-\sqrt{y+1}\right)}{2\sqrt{y+1}}$
- ◉ $\dfrac{f_X\left(1+\sqrt{y+1}\right)+f_X\left(1-\sqrt{y+1}\right)}{2\sqrt{y+1}}$
- ○ $\dfrac{f_X\left(1+\sqrt{y+1}\right)+f_X\left(1-\sqrt{y+1}\right)}{2\sqrt{y+1}-2\sqrt{y-1}}$

✔

Solution:

1. We know that when $Y=aX+b$, with $a\neq 0$, we have

$$f_Y(y)=\frac{1}{|a|}f_X\left(\frac{y-b}{a}\right).$$

When $Y=5X-7$, we have $a=5$ and $b=-7$. Therefore,

$$f_Y(y)=\frac{1}{5}f_X\left(\frac{y+7}{5}\right),$$

for all y.

2. $Y=X^2-2X$. We will find the CDF of Y and differentiate to find the PDF. For $y\geq -1$, we have,

$$\begin{aligned}
F_Y(y) &= \mathbb{P}(Y\leq y)\\
&= \mathbb{P}\left((X-1)^2\leq y+1\right)\\
&= \mathbb{P}\left(-\sqrt{y+1}\leq X-1\leq \sqrt{y+1}\right)\\
&= \mathbb{P}\left(1-\sqrt{y+1}\leq X\leq 1+\sqrt{y+1}\right)\\
&= F_X\left(1+\sqrt{y+1}\right)-F_X\left(1-\sqrt{y+1}\right).
\end{aligned}$$

Differentiating and using the chain rule, we obtain

$$f_Y(y) = \frac{f_X(1+\sqrt{y+1}) + f_X(1-\sqrt{y+1})}{2\sqrt{y+1}}.$$

Submit You have used 2 of 2 attempts

Answers are displayed within the problem

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?	Question 2 HELP I don't understand how can you leave the X alone on one side of the equation since you have X^2-2X	3
?	Not sure how to incorporate both X^2 and 2X into Y There were ways we discussed to deal with Y=X^2, and ways with Y=2X, but I wasn't sure what to do with...	4 new
?	Is it useful in any way to know that X has a standard normal distribution? Is it useful in any way to know that X has a standard normal distribution? I solved the problem without a...	2
	Denominator? Quadratic equation Since Y is a quadratic function i was able to find the numerator . but i am struggling to understand wher...	2
	Question 2 confusion	3
	Tip for getting help on Q2. I found part 12 of lecture 11 useful for this problem.	1
	Question about second point Is it possible for y = -1 as stated in the question? If that is true then in the denominator we have either 0 ...	3
?	Question 1 limits Still struggling with Q1. Does anyone have suggestions for the limits?	3