

Course > Unit 7: ... > Proble... > 3. LLM...

3. LLMS estimation

Problem Set due Apr 15, 2020 05:29 IST Completed

Problem 3. LLMS estimation

3/3 points (graded)

Let X=U+W with $\mathbf{E}\left[U\right]=m$, $\mathrm{Var}\left(U\right)=v$, $\mathbf{E}\left[W\right]=0$, and $\mathrm{Var}\left(W\right)=h$. Assume that U and W are independent.

1. The LLMS estimator of U based on X is of the form $\hat{U}=aX+b$. Find a and b. Express your answers in terms of m, v, and h using standard notation.

2. We now further assume that U and W are normal random variables and then construct \hat{U}_{LMS} , the LMS estimator of U based on X, under this additional assumption. Would \hat{U}_{LMS} be identical to \hat{U} , the LLMS estimator developed without the additional normality assumption in Part 1?

Answer: Yes

STANDARD NOTATION

Solution:

1. In order to write the LLMS estimator we need to find $\mathbf{E}\left[X\right]$, $\mathsf{Var}\left(X\right)$, and $\mathsf{cov}\left(U,X\right)$. We have

$$\mathbf{E}\left[X\right] \ = \mathbf{E}\left[U+W\right] = \mathbf{E}\left[U\right] + \mathbf{E}\left[W\right] = \mathbf{E}\left[U\right] = m,$$



$$\begin{aligned} \mathsf{Var}\left(X\right) &= \mathsf{Var}\left(U + W\right) \\ &= \mathsf{Var}\left(U\right) + \mathsf{Var}\left(W\right) & \text{since } U \text{ and } W \text{ are independent} \\ &= v + h, \\ \mathsf{cov}\left(U, X\right) &= \mathbf{E}\left[UX\right] - \mathbf{E}\left[U\right] \mathbf{E}\left[X\right] \\ &= \mathbf{E}\left[U\left(U + W\right)\right] - m^2 \\ &= \mathbf{E}\left[U^2\right] + \mathbf{E}\left[U\right] \mathbf{E}\left[W\right] - m^2 & \text{since } U \text{ and } W \text{ are independent} \\ &= \mathbf{E}\left[U^2\right] - m^2 \\ &= \mathbf{E}\left[U^2\right] - (\mathbf{E}\left[U\right])^2 \\ &= \mathsf{Var}\left(U\right) = v. \end{aligned}$$

Substituting these results into the formula for the LLMS estimator yields

$$\hat{U}=m+rac{v}{v+h}(X-m)\,.$$

2. We know that the LMS estimator of U based on X, under the normality assumption we have introduced, is linear in X. Therefore, it coincides with the LLMS estimator.

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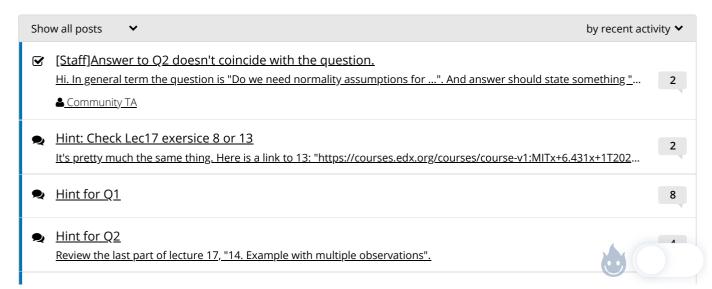
You have used 2 of 3 attempts

1 Answers are displayed within the problem

Discussion

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Topic: Unit 7: Bayesian inference:Problem Set 7b / 3. LLMS estimation



Use the formula given in Lecture 17. "LLMS Example": E(theta) + Cov(theta, X)/ Var(X) *(X - E(X)) Apply this equ...

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