# 5. Maximum likelihood estimation

Problem Set due May 1, 2020 05:29 IST *Completed*

## Problem 5. Maximum likelihood estimation

1/1 point (graded)

The random variables $X_1, X_2, \ldots, X_n$ are continuous, independent, and distributed according to the Erlang PDF

$$f_X(x) = \frac{\lambda^3 x^2 e^{-\lambda x}}{2}, \text{ for } x \geq 0,$$

where $\lambda$ is an **unknown** parameter. Find the maximum likelihood estimate of $\lambda$, based on observed values $x_1, x_2, \ldots, x_n$. Express your answer as a function of $n$ and $s$ where $s = x_1 + x_2 + \ldots x_n$.

$\hat{\lambda}_{\text{ML}} =$ 3*n/s ✔ **Answer:** 3*n/s

$\frac{3 \cdot n}{s}$

**Solution:**

We need to maximize the function,

$$f_X(x; \lambda) = \frac{\lambda^3 x_1^2 e^{-\lambda x_1}}{2} \cdots \frac{\lambda^3 x_n^2 e^{-\lambda x_n}}{2},$$

with respect to $\lambda$. Equivalently, we can maximize its logarithm, which is of the form

$$c + 3n \ln \lambda - \lambda \left( \sum_{i=1}^{n} x_i \right),$$

where $c$ is a term that does not involve $\lambda$ (but can depend on $x_1, x_2, \ldots, x_n$). By taking the derivative with respect to $\lambda$ and setting it to zero, we obtain,

$$\frac{3n}{\lambda} - \sum_{i=1}^{n} x_i = 0,$$

or equivalently,

$$\lambda = \frac{3n}{\sum_{i=1}^{n} x_i} = \frac{3n}{s}.$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

## Discussion

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? Where does the "n" before Ln(Lambda) come from? 3
Can anyone help me understand why there is a "n" before Ln(Lambda) when doing the logarithm? I got t...

I reached the right answer but wonder if this is a right approach? 3
Suppose S = X1 + X2 + ⋯ + Xn, then calculate pS(s;λ), which is approximately N~(3n/λ, 3n/λ^2). Now we ca...

What are the chances for anyone to solve this problem?
The concepts introduced in the sample problem of Lecture 20, video 19 are by no means trivial. By addi

Hint 6