# 4. Convolution calculations

Problem Set due Apr 1, 2020 05:29 IST *Completed*

## Problem 4. Convolution calculations

9/9 points (graded)

1. Let the discrete random variable $X$ be uniform on $\{0, 1, 2\}$ and let the discrete random variable $Y$ be uniform on $\{3, 4\}$. Assume that $X$ and $Y$ are independent. Find the PMF of $X + Y$ using convolution. Determine the values of the constants $a$, $b$, $c$, and $d$ that appear in the following specification of the PMF.

$$p_{X+Y}(z) = \begin{cases} a, & z = 3, \\ b, & z = 4, \\ c, & z = 5, \\ d, & z = 6, \\ 0, & \text{otherwise.} \end{cases}$$

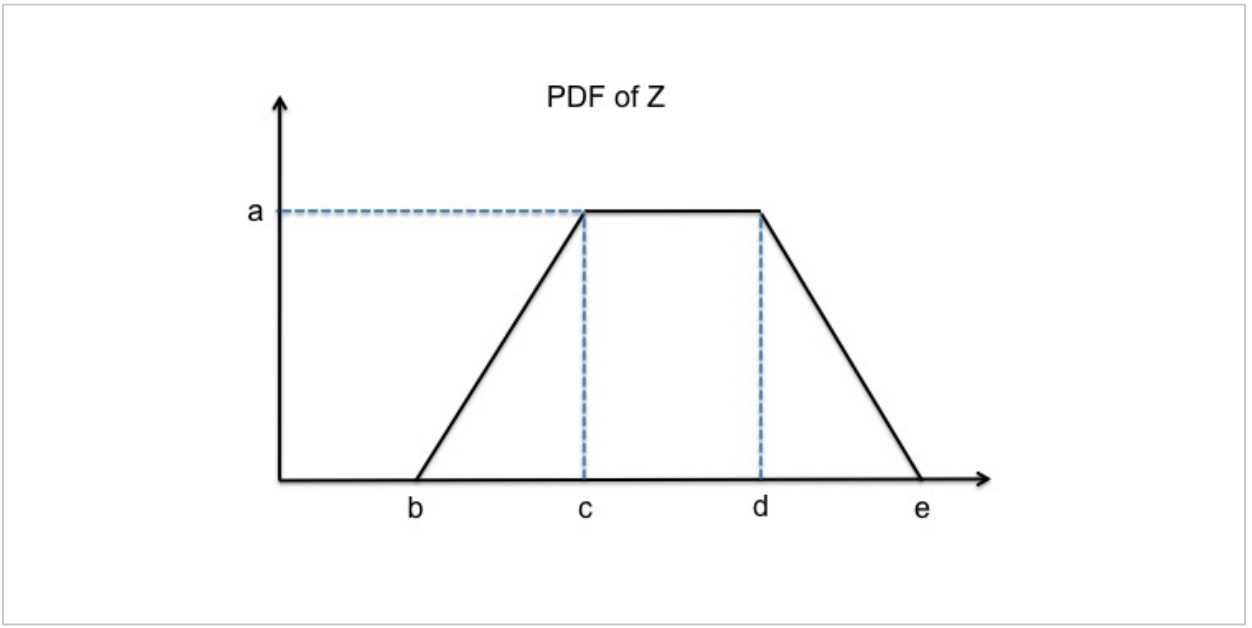
$a =$ 1/6 ✔ **Answer:** 1/6

$b =$ 2/6 ✔ **Answer:** 1/3

$c =$ 2/6 ✔ **Answer:** 1/3

$d =$ 1/6 ✔ **Answer:** 1/6

2. Let the random variable $X$ be uniform on $[0, 2]$ and the random variable $Y$ be uniform on $[3, 4]$. (Note that in this case, $X$ and $Y$ are continuous random variables.) Assume that $X$ and $Y$ are independent. Let $Z = X + Y$. Find the PDF of $Z$ using convolution. The following figure shows a plot of this PDF. Determine the values of $a$, $b$, $c$, $d$, and $e$.




$a =$ 1/2 ✔ **Answer:** 0.5

$b =$ 3 ✔ **Answer:** 3

$c =$ 4 ✔ **Answer:** 4

$d =$ 5 ✔ **Answer:** 5

$e =$ 6 ✔ **Answer:** 6

**Solution:**

1. $$p_{X+Y}(z) = \begin{cases} 1/6, & z \in \{3, 6\} \\ 1/3, & z \in \{4, 5\} \\ 0, & \text{otherwise.} \end{cases}$$

2. The answer is easiest to find graphically, by sliding a rectangle of width 1 along a rectangle of width 2, and is:

$$f_{X+Y}(z) = \begin{cases} \frac{z-3}{2}, & 3 \le z < 4, \\ \frac{1}{2}, & 4 \le z < 5, \\ \frac{6-z}{2}, & 5 \le z \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

A more formal approach involves the convolution formula, but requires careful thought in order to identify the appropriate limits of integration. In particular, if $3 \le z \le 6$, we have

$$\begin{aligned} f_{X+Y}(z) &= \int_{-\infty}^{\infty} f_X(x)\, f_Y(z-x)\, dx \\ &= \int_{\max(0,z-4)}^{\min(2,z-3)} \frac{1}{2}\, dx \\ &= (\min(2, z-3) - \max(0, z-4))/2 \end{aligned}$$

which actually agrees with the answer obtained through the graphical method.

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Answers are displayed within the problem

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- Hint for 2 — 5