10. Exercise: Using the formula for the monotonic case

Exercises due Mar 25, 2020 05:29 IST *Completed*

Exercise: Using the formula for the monotonic case

5/6 points (graded)

The random variable X is exponential with parameter $\lambda = 1$. The random variable Y is defined by $Y = g(X) = 1/(1+X)$.

a) The inverse function h, for which $h(g(x)) = x$, is of the form $ay^b + c$. Find a, b, and c.

$a =$ 1 ✔ **Answer:** 1

$b =$ -1 ✔ **Answer:** -1

$c =$ -1 ✔ **Answer:** -1

b) For $y \in (0, 1]$, the PDF of Y is of the form $f_Y(y) = y^a e^{(b/y)+c}$. Find a, b, and c.

$a =$ 2 ✖ **Answer:** -2

$b =$ -1 ✔ **Answer:** -1

$c =$ 1 ✔ **Answer:** 1

Solution:

a) If x and y obey the relation $y = g(x) = 1/(1+x)$, then $y + yx = 1$, so that

$$x = h(y) = \frac{1-y}{y} = \frac{1}{y} - 1.$$

Note that we are interested in $x \geq 0$ which restricts y to the range $(0, 1]$. Notice also that the functions g and h are monotonically decreasing on the relevant ranges of values.

b) Note that

$$\frac{dh}{dy}(y) = -\frac{1}{y^2}.$$

Therefore,

$$f_Y(y) = f_X\big(h(y)\big)\left|\frac{dh}{dy}(y)\right| = e^{-(1/y)+1} \cdot \frac{1}{y^2}.$$

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Answers are displayed within the problem

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missing minus sign?
what happened to the negative sign in the derivative?
1 new 7

my math has failed me! (hints on what went wrong)
Assuming lambda = 1, f(x) = e^-x, and therefore, y = g(x) = 1 / (1 + e^-x). I proceeded to solve for x in term...
3

What happens to lambda in fx(h(y))?
If fx(x) = lambda * e^(-lambda *x), do we not just substitute x = h(y)? Please can someone explain step by
2

1st question

hello, could somebody clarify. does the first expression should include a logarithmic expression?

5