



5. Hypothesis test between two normals

Problem Set due Apr 8, 2020 05:29 IST **Completed**

Problem 5. Hypothesis test between two normals

2/2 points (graded)

Conditioned on the result of an unbiased coin flip, the random variables T_1, T_2, \dots, T_n are independent and identically distributed, each drawn from a common normal distribution with mean zero. If the result of the coin flip is Heads, this normal distribution has variance 1; otherwise, it has variance 4. Based on the observed values t_1, t_2, \dots, t_n , we use the MAP rule to decide whether the normal distribution from which they were drawn has variance 1 or variance 4. The MAP rule decides that the underlying normal distribution has variance 1 if and only if

$$\left| c_1 \sum_{i=1}^n t_i^2 + c_2 \sum_{i=1}^n t_i \right| < 1.$$

Find the values of $c_1 \geq 0$ and $c_2 \geq 0$ such that this is true. Express your answer in terms of n , and use "ln" to denote the natural logarithm function, as in "ln(3)".

$c_1 =$ **✓ Answer: 3/(8*n*ln(2))**

$c_2 =$ **✓ Answer: 0**

[STANDARD NOTATION](#)

Solution:

Let $\Theta = 0$ denote that the observations t_1, t_2, \dots, t_n were generated from a normal distribution with variance 1, and let $\Theta = 1$ denote that they were generated from a normal distribution with variance 4. For simplicity, let us use the notation $N(t_1, \dots, t_n; 0, \sigma^2)$ to denote the joint PDF.

i.i.d. normal random variables with mean 0 and variance σ^2 , evaluated at t_1, \dots, t_n .

Therefore, given the observations t_1, \dots, t_n , the posterior probability that the samples are generated from a normal distribution with variance 1 is

$$\mathbf{P}(\Theta = 0 \mid T_1 = t_1, \dots, T_n = t_n) = \frac{(1/2) \cdot N(t_1, \dots, t_n; 0, 1)}{(1/2) \cdot N(t_1, \dots, t_n; 0, 1) + (1/2) \cdot N(t_1, \dots, t_n; 0, 4)}.$$

Similarly, the probability that the samples are generated from a normal distribution with variance 4 is given by

$$\mathbf{P}(\Theta = 1 \mid T_1 = t_1, \dots, T_n = t_n) = \frac{(1/2) \cdot N(t_1, \dots, t_n; 0, 4)}{(1/2) \cdot N(t_1, \dots, t_n; 0, 1) + (1/2) \cdot N(t_1, \dots, t_n; 0, 4)}.$$

The MAP rule favors $\Theta = 0$ if the following inequality holds:

$$\mathbf{P}(\Theta = 0 \mid T_1 = t_1, \dots, T_n = t_n) > \mathbf{P}(\Theta = 1 \mid T_1 = t_1, \dots, T_n = t_n)$$

We notice that the denominators in the expressions for $\mathbf{P}(\Theta = 0 \mid \dots)$ and $\mathbf{P}(\Theta = 1 \mid \dots)$ are the same, so it suffices to compare the numerators. Therefore, the MAP rule favors $\Theta = 0$ if the following inequality holds:

$$N(t_1, \dots, t_n; 0, 1) > N(t_1, \dots, t_n; 0, 4)$$
$$\prod_{i=1}^n \frac{1}{\sqrt{2\pi \cdot 1}} e^{-\frac{t_i^2}{2 \cdot 1}} > \prod_{i=1}^n \frac{1}{\sqrt{2\pi \cdot 4}} e^{-\frac{t_i^2}{2 \cdot 4}}.$$

With a little bit of algebra, we obtain

$$\left| \frac{3}{8} \sum_{i=1}^n t_i^2 \right| < n \cdot \ln(2).$$

Note: If the means under the two hypotheses were different, a similar answer would be obtained but with a nonzero coefficient c_2 .

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You have used 1 of 3 attempts



i Answers are displayed within the problem

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<p>? [Staff] <u>I'm not understanding the question correctly I think</u> <u>Usually there is only one "C" as the threshold to pick H1 over H0? Why are there two thresholds here - c1 and c2...</u></p>	2
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<p>? <u>Already failed this after many meticulous hours, only for being misled by "c2" and the lectures.</u> <u>Explanation please?</u> <u>To those who still have tries left on this problem, don't make my same mistake. With that said, I would really like t...</u></p>	2
<p>💬 <u>5 Hours solving time.</u> <u>I solved it. It took me about five hours. Four of them figuring out, how to find c2.:) It is not funny.</u></p>	13
<p>💬 <u>Hint</u> <u>If you assume H1 beeing hypothesis that result is Heads (and hence T is $N(0,1)$), and H2 - hypothesis that result is ...</u></p>	11
<p>💬 <u>Eureka!</u> <u>This short post is my simple way to thank Prof. Tsitsiklis and the team who meticulously put this course together ...</u></p>	1
<p>? <u>There seems to be some problem with the total marks earned for PS7.</u> <u>I only lost one mark in question 4, but my total marks are 16/18 instead of 17/18. I guess there are some problem...</u></p>	1



