# 5. Exercise: Iterated expectations

Exercises due Mar 25, 2020 05:29 IST   *Completed*

## Exercise: Iterated expectations

5/8 points (graded)

In this exercise, do not attempt formal mathematical derivations, which would actually involve some subtle issues when we go beyond discrete random variables. Rather, use your understanding of the concepts involved. For each one of the statements below, indicate whether it is true or false.

(a) The law of iterated expectations tells us that $\mathbf{E}\big[\mathbf{E}\left[X\,|\,Y\right]\big] = \mathbf{E}\left[X\right]$. Suppose that we want apply this law in a conditional universe, given another random variable $Z$, in order to evaluate $\mathbf{E}\left[X\,|\,Z\right]$. Then:

$$\mathbf{E}\big[\mathbf{E}\left[X\,|\,Y,Z\right]\,|\,Z\big] = \mathbf{E}\left[X\,|\,Z\right]$$

| True ⌄ | ✔ **Answer: True** |

$$\mathbf{E}\big[\mathbf{E}\left[X\,|\,Y\right]\,|\,Z\big] = \mathbf{E}\left[X\,|\,Z\right]$$

| False ⌄ | ✔ **Answer: False** |

$$\mathbf{E}\big[\mathbf{E}\left[X\,|\,Y,Z\right]\big] = \mathbf{E}\left[X\,|\,Z\right]$$

| True ⌄ | ✘ **Answer: False** |

(b) Determine whether each of the following statements about the quantity $\mathbf{E}\big[g\left(X,Y\right)\,|\,Y,Z\big]$ is true or false.

The quantity $\mathbf{E}\big[g\left(X,Y\right)\,|\,Y,Z\big]$ is:

- a random variable

| True | ✔ **Answer: True** |

- a number

| False | ✔ **Answer: False** |

- a function of $(X, Y)$

| True | ✘ **Answer: False** |

- a function of $(Y, Z)$

| False | ✘ **Answer: True** |

- a function of $Z$ only

| False | ✔ **Answer: False** |

**Solution:**

(a) The first statement is correct: it is just the law of iterated expectations where all the expectations now involve the additional conditioning on $Z$.

The second statement is incorrect because the inner conditional expectation should be evaluated in a conditional universe where $Z$ is given. For a concrete counterexample, suppose that $X$ and $Y$ are independent and zero mean, and that $X = Z$. Because of independence, $\mathbf{E}[X \mid Y] = \mathbf{E}[X] = 0$, and the left-jand side evaluates to zero. On the other hand, the right-hand side is equal to $Z$.

For the third statement, note that the left-hand side is a number (the ordinary expectation of the random variable $\mathbf{E}[X \mid Y, Z]$ ), whereas the right-hand side is a random variable (a function of $Z$). Hence the statement is incorrect.

(b) A conditional expectation is generally a random variable, a function of the random variables on which we are conditioning, and so a function of $(Y, Z)$ in this case.

| Submit | You have used 1 of 1 attempt |

**ⓘ**   Answers are displayed within the problem

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# Discussion

**Topic:** Unit 6: Further topics on random variables:Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s / 5. Exercise: Iterated expectations

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💬 **(Staff/ Gurus) Real struggle**                                                    `8`

☑ **Iterated Expectation**
Hello Staff, This exercise happens to be the most difficult one I have seen until now in this course. Th...    `3`

**?** **Difference between (a-1) and (a-2).**
Hi, Could anyone please explain the subtle difference between those two cases ? Why does one need ...    `1`

**?** **Geometric Interpretation of Iterated expectaion**
Hi All From the last unit, I had mentally pictured the conditional as a slice of the joint ie X|Y: is the slic...    `3`

💬 **Unfair questions and grading**
I think this is a very bizarre and confusing exercise, and the main problems are: First, it is only 1 atte...    `5 new`

**?** **Higher-order functions?**                                                     `2`

**?** **Part (b) does not include a statement that defines g(X,Y) as a conditioned Expectation function**
Doesn't this matter in answering some of the questions?    `5`

💬 **solution wording**                                                               `2`

💬 **Resources**
I think that the problem of this unit is not that the topic is complicated or hard to grasp. It is rather th...    `1`

💬 **Solution to second statement part (a).**
In the solution, you are assuming that X=Z, and you say that the right-hand side is equal to E[X|Z]=E[X...    `2`

💬 **Confusion over law of iterated expectations**                                     `5`

💬 **Learning from this exercise set**                                                 `5`

**?** **Still not clear**

If E[X|Y]=E[X], then why it should be otherwise as per answers? I am not sure how to formulate it with... 5