# Variance of Bernoulli Random Variable with a Random Variable as parameter

Asked 2 years, 11 months ago Active 2 years, 8 months ago Viewed 835 times

1

Let X be a Bernoulli Random Variable whose parameter is a Uniform Random Variable Q which takes values in the domain [0 , 0.1].

We want to find var(X). My reasoning is the following:

Using the Law of Total Variance we have:

var(X) = E[var(X|Q)] + var(E[X|Q]) = E[Q * (1 - Q)] + var(Q) = E[Q] - E[Q^2] + var(Q) = E[Q] - (var(Q) + (E[Q])^2) + var(Q) = E[Q] - (E[Q])^2 = 0.05 - 0.05^2 = 0.0475.

Do you agree with this line of reasoning?

variance random-variable bernoulli-distribution

edited Apr 19 '17 at 21:53

asked Apr 19 '17 at 19:36

rf7

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2 Please note that the interval [0, 0.1] is a subset of the interval [0, 1] and therefore it is a legitimate domain for the parameter. In the particular problem the parameter is modeled --as I stated-- as a Random Variable uniformly distributed over the interval [0, 0.1]. E.g. assume a biased coin. You know that its bias is between 0 --never Heads-- and 0.1 --10% of the times Heads. Then you model the parameter as a Random Variable uniformly distributed in the interval [0, 0.1]. In dealing with a parameter that is a Random Variable you have to use the Law of Total Variance. – rf7 Apr 19 '17 at 21:12

You are applying a distribution to an unknown fixed parameter. If you were doing Bayesian analysis this could make sense as a prior distribution. Then you would update when you observe a Bernoulli random outcome. – Michael R. Chernick Apr 19 '17 at 21:25

If you are just want to know the variance of a uniform random variable on [0,0,1] there is no need to bring the Bernoulli variable into the picture. You can just integrate 10 (the density) time (x-0.05)$^2 dx to get the variance. – Michael R. Chernick Apr 19 '17 at 21:31

Sorry, this was a typo. I want to find var(X) not var(Q) – rf7 Apr 19 '17 at 21:53

## 1 Answer

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1

You are using the correct approach with the total variance law.

$$\mathbb{V}ar(X) = \mathbb{V}ar(\mathbb{E}(X|Q)) + \mathbb{E}(\mathbb{V}ar(X|Q)).$$

Here you will need the expectations and variances of the Bernoulli and the continuous uniform. These are:

$$\mathbb{E}(X|Q) = Q,$$

$$\mathbb{V}ar(X|Q) = Q(1-Q),$$

$$\mathbb{E}(Q) = \frac{a+b}{2},$$

$$\mathbb{V}ar(Q) = \frac{(a-b)^2}{12},$$

where here $a = 0.1$ and $b = 0.0$, the limits of the continuous uniform. The only other thing I used to solve this was the variance relation

$$\mathbb{V}ar(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2.$$

This is rearranged to get a relationship between the mean and the variance to get the second moment if you know the mean and the variance. So from the total variance law

$$\mathbb{V}ar(X) = \mathbb{V}ar(\mathbb{E}(X|Q)) + \mathbb{E}(\mathbb{V}ar(X|Q)) = \mathbb{V}ar(Q) + \mathbb{E}(Q(1-Q)) = \mathbb{V}ar(Q) + \mathbb{E}(Q) - \mathbb{E}(Q^2) = \frac{0.1^2}{12} + 0.05 - (\frac{.01^2}{12} + 0.05^2) = 0.0008333333 + 0.05 - (0.000833333 + 0.0025) = 0.05 - 0.0025 = 0.0475$$

which matches what you have in the posting.