# 4. Exercise: Checkout counter

None due May 29, 2020 05:29 IST

## Exercise: Checkout counter

2 points possible (ungraded)

Consider our checkout counter example. Assume that there are two types of customers who arrive according to independent Bernoulli processes with rates $p_1 \in (0,1)$ and $p_2 \in (0,1)$, respectively. The overall arrival process of all customers follows a merged Bernoulli process of the two separate Bernoulli processes. All customers who arrive join a single queue, which has a capacity of 10 customers. We are interested in making predictions about the length of the queue at any point in time.

For each of the following parts, choose the correct statement.

1. Assume that service times are not type-dependent and are modelled as independent geometric random variables with parameter $q \in (0,1)$ for all customers in the queue.

- One can model this queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and $q$.
- One can model this queue using the same transition probability graph as in the previous video with $p = 1 - (1 - p_1)(1 - p_2)$ and $q$. ✔
- One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of $p$ and $q$.
- There are no values of $p$ and $q$ for which one can model the queue using the same transition probability graph as in the previous video.

2. Assume now that service times are type-dependent and are modelled as independent geometric random variables with parameters $q_1 \in (0,1)$ and $q_2 \in (0,1)$, respectively, for the two types of customers.

- One can model the queue using the same transition probability graph as in the previous video with $p = (p_1 + p_2)/2$ and $q = (q_1 + q_2)/2$.
- One can model the queue using the same transition probability graph as in the previous video with $p = 1 - (1 - p_1)(1 - p_2)$ and $q = (p_1 q_1 + p_2 q_2)/(p_1 + p_2)$.
- One can model this queue using the same transition probability graph as in the previous video with some other appropriate choice of $p$ and $q$.
- There are no values of $p$ and $q$ for which one can model the queue using the same transition probability graph as in the previous video. ✔

**Solution:**

1. Option 2 is correct. The value of $p$ corresponds to the arrival probability of the merged Bernoulli process.

2. Option 4 is correct. To see this, note that for all of the first three options, the process is a Markov chain. Thus, it suffices to argue that the process with two types of arriving customers is not, in general, a Markov chain. To see this, consider an extreme case where $p_1 = p_2 = 1/2$, $q_1 = 1$, and $q_2$ is very small. Suppose that the previous state was 0 and the current state is 1. This means that we just had an arrival; by symmetry it is equally likely to have been of either type, and the expected time until the next departure is $(1 + (1/q_2))/2$. If we now observe the next state to be again 1, we are pretty certain that it was an arrival of type 2, and the expected time until the next departure is approximately $1/q_2$. Thus, the statistics of the future of the process are not fully determined by the current state — the past history also plays a role, which violates the Markov property.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

## Discussion

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**Topic:** Unit 10: Markov chains:Lec. 24: Finite-state Markov chains / 4. Exercise: Checkout counter

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- Can't understand this part of the solution
  "If we now observe the next state to be again 1, we are pretty certain that it was an arrival of type 2"...wh... — 2
- Why does the final exam have a due date before Unit 10 questions?
  Why does the final exam have a due before Unit 10 questions? — 3
- Submit does not work on edX mobile app for iOS.
  As per above. Maybe an issue with ungraded questions :) — 1
- "Same" transition probability graph
  Hi, I have a query. By the term "**Same** transition probability graph" in the question, does it mean wit... — 3
- #2 Why is it not a Markov chain?
  It is not clear from the solution to me whether # 1 is a Markov chain. Is the fact that in #2 "the service ti... — 2
- If there are two arrivals.
  The question says merge the two process, so when there are two arrivals, should we consider they as on... — 7
- Great website for more clear intuition
  https://setosa.io/ev/markov-chains/ — 1