11. Exercise: Conditional variance definition

Exercises due Mar 25, 2020 05:29 IST *Completed*

Exercise: Conditional variance definition

3/5 points (graded)

For each one of the following statements, indicate whether it is true or false.

(a) If $X = Y$ (i.e., the two random variables always take the same values), then $\mathsf{Var}(X \mid Y) = 0$.

False ✖ **Answer:** True

(b) If $X = Y$ (the two random variables always take the same values), then $\mathsf{Var}(X \mid Y) = \mathsf{Var}(X)$.

True ✖ **Answer:** False

(c) If Y takes on the value y, then the random variable $\mathsf{Var}(X \mid Y)$ takes the value

$$\mathbf{E}\left[(X - \mathbf{E}[X \mid Y = y])^2 \mid Y = y\right].$$

True ✔ **Answer:** True

(d) If Y takes on the value y, then the random variable $\mathsf{Var}(X \mid Y)$ takes the value

$$\mathbf{E}\left[(X - \mathbf{E}[X \mid Y])^2 \mid Y = y.\right]$$

True ✔ **Answer:** True

(e) If Y takes on the value y, then the random variable $\mathsf{Var}(X \mid Y)$ takes the value

$$\mathbf{E}\left[(X - \mathbf{E}[X])^2 \mid Y = y.\right]$$

False ✔ **Answer:** False

Solution:

(a) Conditioned on Y, X is deterministic, and $\mathsf{Var}(X \mid Y = y) = 0$. This implies that the random variable $\mathsf{Var}(X \mid Y)$ is identically equal to zero. Thus, the statement is true.

(b) False, because the previous statement is true.

(c) This statement is just the definition of the numerical value of the conditional variance. We are in a universe where the event $Y = y$ is known to have occurred, and every expectation is replaced by the corresponding conditional expectation.

(d) The outer expectation places us in a universe where $Y = y$. Given this information, the value of the random variable $\mathbf{E}[X \mid Y]$ becomes a known quantity, equal to $\mathbf{E}[X \mid Y = y]$. Thus, this statement is equivalent to the preceding one and is true.

(e) This is false, because all expectations should be conditional on the universe ($Y = y$) within which we are working. For a concrete counterexample, suppose that X is zero-mean and that $Y = X$. Then, as in part (a), $\mathsf{Var}(X \mid Y = y) = 0$. On the other hand, since $\mathbf{E}[X] = 0$, we have

$$\mathbf{E}\left[(X - \mathbf{E}[X])^2 \mid Y = y\right] = \mathbf{E}[X^2 \mid Y = y] = \mathbf{E}[Y^2 \mid Y = y] = y^2.$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Lec. 13: Conditional expectation and variance revisited; Sum of a random number of

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? Is Var(X|Y) not a R.V? 9
This is regarding exercise (a). Even though X=Y, Var(X|Y) is still a R.V. as Y can take any value from its sam...

Abnormally difficult lectures 4 new
Lectures 11 and 13 I found particularly difficult, I did not see how the videos followed into the questions....

[Staff] About the solution of d 5
The solution implies that E[X|Y] can be treated as a number in a specific universe, however being in spec...

? Concept for question 1 and 2 1 new
question says, x takes same value as y. then variance of x given y value, is variance of y (which is equal to...

Word of dautio n 2

A & B: Hint in Understanding X|X 2
I thought X|X = X in the abstract, since conditioning a random variable on itself (as a random variable) d...

Be be careful with (d) 8
Conditioning notation is tricky here. I made a mistake.

Lecture is a red herring for d) 1
In the preceding lecture they gave this formula and stressed how the conditioning should be presented i...

? Question on point (a) 5
Questions (a) and (b) ask about the variance of X given Y, being X=Y. But the answer assumes the statem...

Question (a): confused with notation. 12
Community TA

Some notation questions 2