19. Exercise: Joint CDFs

Exercises due Mar 13, 2020 05:29 IST *Completed*

Exercise: Joint CDFs

3/3 points (graded)

a) Is it always true that if $x < x'$, then $F_{X,Y}(x,y) \leq F_{X,Y}(x',y)$?

Yes ✔ **Answer:** Yes

b) Suppose that the random variables X and Y are jointly continuous and take values on the unit square, i.e., $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Is $F_{X,Y}(x,y) = (x+2y)^2/9$ a legitimate joint CDF? *Hint:* Consider $F_{X,Y}(0,1)$.

No ✔ **Answer:** No

c) As above, suppose that the random variables X and Y are jointly continuous and take values on the unit square, i.e., $0 \leq x \leq 1$ and $0 \leq y \leq 1$. The joint CDF on that set is of the form $xy(x+y)/2$. Find an expression for the joint PDF which is valid for (x,y) in the unit square. Enter an algebraic function of x and y using standard notation.

x+y ✔ **Answer:** x+y

STANDARD NOTATION

Solution:

a) Since $x < x'$, the event $\{X \leq x, Y \leq y\}$ is a subset of the event $\{X \leq x', Y \leq y\}$, and therefore $F_{X,Y}(x,y) = \mathbf{P}(X \leq x, Y \leq y) \leq \mathbf{P}(X \leq x', Y \leq y) = F_{X,Y}(x',y)$.

b) Since the random variables are nonnegative, we have $F_{X,Y}(0,1) = \mathbf{P}(X \leq 0 \text{ and } Y \leq 1) = \mathbf{P}(X = 0 \text{ and } Y \leq 1) \leq \mathbf{P}(X = 0) = 0$, where the last equality holds because X is a continuous random variable. But zero is different from $(0 + 2 \cdot 1)^2/9$. Therefore, we do not have a legitimate joint CDF.

c) The joint CDF is of the form $x^2y/2 + y^2x/2$. The partial derivative with respect to x is $xy + y^2/2$. Taking now the partial derivative with respect to y, we obtain $x + y$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 5: Continuous random variables:Lec. 9: Conditioning on an event; Multiple r.v.'s / 19. Exercise: Joint CDFs

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? Legitimate CDF function Criteria 2
Could the instructor or one of the TAs state here the conditions of legitimate CDF function, please?

? Clarification over the solution of question b 2
Hey, On the question b) on the solution it states the following (I'm using letters so I don't give the answer...

c 2
For question part c, should we always start the derivative with respect to x first then y. How do know if le...

? Part B 7
Can someone explain what "where the last equality holds because X is a continuous random variable" m...

Is this a valid alternative line of reasoning? 4
Would an alternative approach be finding the joint PDF (by taking the partial derivative of the CDF first w...

? Valid CDF - Criterion 2
What, in general, is the criteria for a valid CDF?