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Mid Term due Apr 22, 2020 05:29 IST *Completed*

Problem 2

2.0/3.0 points (graded)

We are given a stick that extends from 0 to x. Its length, x, is the realization of an exponential random variable X, with mean 1. We break that stick at a point Y that is uniformly distributed over the interval $[0, x]$.

1. Write down the (fully specified) joint PDF $f_{X,Y}(x, y)$ of X and Y.

   For $0 < y \le x$:

   $$f_{X,Y}(x, y) = \boxed{\text{e^(-x)*(1/x)}} \qquad \checkmark \text{ Answer: e^(-x)/x}$$

   $$e^{-x} \cdot \left(\frac{1}{x}\right)$$

2. Find $\mathrm{Var}\big(\mathbf{E}\,[Y \mid X]\big)$.

   $$\mathrm{Var}\big(\mathbf{E}\,[Y \mid X]\big) =$$

   $\boxed{1/4}$ $\checkmark$ **Answer:** 1/4

3. We do not observe the value of X, but are told that $Y = 2.2$. Find the MAP estimate of X based on $Y = 2.2$.

   MAP estimate of X: $\boxed{2}$ ✖ **Answer:** 2.2

STANDARD NOTATION

Solution:

1.

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$f_{Y|X}(y|x) = \begin{cases} \frac{1}{x}, & 0 < y \leq x, \\ 0, & \text{otherwise.} \end{cases}$$

Using $f_{X,Y}(x,y) = f_X(x) f_{Y|X}(y|x)$, we get the joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{e^{-x}}{x}, & 0 < y \leq x, \\ 0, & \text{otherwise.} \end{cases}$$

2. Conditioned on $X = x$, Y is uniform on $[0, x]$. Thus, $\mathbf{E}[Y|X] = X/2$. This gives

$$\begin{aligned} \text{Var}(\mathbf{E}[Y|X]) &= \text{Var}\left(\frac{X}{2}\right) \\ &= \frac{1}{4}\text{Var}(X) \\ &= \frac{1}{4}, \end{aligned}$$

since X is exponential with parameter 1.

3. The nonzero portion of the joint PDF, e^{-x}/x, is a decreasing function in x. Thus the MAP rule assigns to $Y = y$ the smallest x where the joint PDF is nonzero, which is y itself. Thus when $Y = 2.2$, the MAP rule assigns the estimate 2.2.

Submit   You have used 3 of 3 attempts

🛈 Answers are displayed within the problem

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💬 Grading anomaly ?   5

💬 Graph I made to find the solution for the third question.   4

❓ Technical Problems
Staff, Although I submitted my answers for this question before ending the exam, it seems that it did not...   3

💬 My answers to Q2 (MidTerm2)   17

❓ Part 3
Is there any chance that f_Y(y) is an exponential integral (Ei(...)) ?   1 new   7

💬 third variable required ... does it matter what we use
If X is the realization of an exponential, clearly we don't want to use y as the parameter since it's already ...   2

💬 urgent
fellow learners there is similar problem(breaking of a stick) in the lecture videos in which Professor expla...   2

❓ What does it mean 'x , is the realization of an exponential random variable'?
Does it mean x = fx(k) = exp(-k) for k positive.   2 new   4

❓ Part 2
I get so worked up in my head over exams. This is new to me, by the way, never happened in the past. I ...   2

☑ Standart notation
How to express $f_{X,Y}(x,y)$= in standard notation? When I write f_{X,Y}(x,y) the standard notation does not ...   2

☑ Is there a way I can contact the TA about extending the deadline of the exam?
Hi TAs and course support team, I have been having medical condition and would like to provide you wit...   2