



1. Defective Coin

Problem Set due Apr 8, 2020 05:29 IST **Completed**

Problem 1. Defective Coin

3/3 points (graded)

A defective coin minting machine produces coins whose probability of Heads is a random variable Q with PDF

$$f_Q(q) = \begin{cases} 5q^4, & \text{if } q \in [0, 1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is tossed repeatedly, with successive tosses assumed to be independent. Let A be the event that the first toss of this coin results in Heads, and let B be the event that the second toss of this coin results in Heads.

1.

$\mathbf{P}(A) =$ **✓ Answer: 5/6**
(Your answer should be a number.)

2. Find the conditional PDF of Q given event A . Express your answer in terms of q using standard notation.

For $0 \leq q \leq 1$, $f_{Q|A}(q) =$ **✓ Answer: 6*q^5**

3.

$\mathbf{P}(B | A) =$ **✓ Answer: 6/7**
(Your answer should be a number.)



Solution:

1. To calculate $\mathbf{P}(A)$, we use the continuous version of the total probability theorem:

$$\mathbf{P}(A) = \int_0^1 \mathbf{P}(A \mid Q = q) f_Q(q) dq = \int_0^1 q \cdot (5q^4) dq = \left[\frac{5}{6} q^6 \right]_0^1 = \frac{5}{6}.$$

2. Using Bayes' rule,

$$\begin{aligned} f_{Q|A}(q) &= \frac{\mathbf{P}(A \mid Q = q) f_Q(q)}{\mathbf{P}(A)} \\ &= \begin{cases} \frac{q \cdot (5q^4)}{5/6}, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise,} \end{cases} \\ &= \begin{cases} 6q^5, & \text{if } 0 \leq q \leq 1, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

3. Again, we use the continuous version of the total probability theorem:

$$\begin{aligned} \mathbf{P}(B \mid A) &= \int_0^1 \mathbf{P}(B \mid A, Q = q) f_{Q|A}(q) dq \\ &= \int_0^1 \mathbf{P}(B \mid Q = q) f_{Q|A}(q) dq \\ &= \int_0^1 q(6q^5) dq \\ &= 6/7. \end{aligned}$$

The second equality holds because for a given value q of Q , the events A and B are (conditionally) independent.

Submit

You have used 1 of 3 attempts



i Answers are displayed within the problem

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? Wrong answer marked right?

1

💬 Idea/Note sharing for Prob.7a

1

? Staff-Info

Dear Professor, I kindly ask you to extend the deadline by 1 day if possible. Thanks.

3

? for Question 1

if $p_Q(q) = 5q^4$ for $q=1$, then $p(Q \mid q=1) = 5$? How could a probability equal to 5?

3

? I can't understand how to tackle the first problem.

How calculate $P(a)$, that is the probability that the first coin of the coin resulted in Head. I think it is ju...

5

? In #2, are we using a constant as $p_{X|Q}$?

When I do the integration, I end up with the original dist#. Is this somewhat a "word" problem, in that...

2

? How do I find $P(A \text{ conditioned on } Q)$?

I got the first part of the problem right, but that's not helping me with the rest of it, although I think if...

5

? Why independence of tosses (multiplication probability rule) doesn't apply in part 3.?

13

💬 The solved problems actually help!

I was lost in my approach, but after when I had fully watched the solved problem and applied some g...

3

? There are no solved Problems for section 7a ?

There are no solved problems before the problem set for this section? Thanks.

3

💬 Enrichment: Laplace's Rule of Succession

Pierre Simon de Laplace used similar methods to estimate the probability of the sun rising tomorrow ...

1

💬 is $P(A|Q(A|q))$ equals to $f_Q(q)$?

given $Q=q, 0 \leq q \leq 1, P(A) = 5 \cdot q^4$?

3

? Number of problems

Are there Six OR Five problems for this problem set? The syllabus indicate Six, but I can only see Five.





Go to study the solved problem section if you get stuck here.

3

I found the TAs explanation is extremely helpful

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