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Mid Term due Apr 22, 2020 05:29 IST *Completed*

Problem 1(a)

1/1 point (graded)

Suppose that X, Y, and Z are independent, with $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 2$, and $\mathbf{E}[X^2] = \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 5$.

Find $\mathrm{cov}(XY, XZ)$.

(Enter a numerical answer.)

$\mathrm{cov}(XY, XZ) =$ 4

Answer: 4

Solution:

$$\begin{aligned}\mathrm{cov}(XY, XZ) &= \mathbf{E}[(XY)(XZ)] - \mathbf{E}[XY]\,\mathbf{E}[XZ] \\ &= \mathbf{E}[X^2YZ] - \mathbf{E}[X]\,E[Y]\,\mathbf{E}[X]\,\mathbf{E}[Z] \\ &= \mathbf{E}[X^2]\,\mathbf{E}[Y]\,\mathbf{E}[Z] - \mathbf{E}[X]^2\mathbf{E}[Y]\,\mathbf{E}[Z] \\ &= 5 \times 2 \times 2 - 4 \times 2 \times 2 \\ &= 4.\end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Problem 2. Problem 1(b)

1.3333333333333333/2.0 points (graded)

Let X be a standard normal random variable. Another random variable is determined as follows. We flip a fair coin (independent from X). In case of Heads, we let $Y = X$. In case of Tails, we let $Y = -X$.

1. Is Y normal? Justify your answer.

- ( ) yes ✔
- (•) no
- ( ) not enough information to determine

✖

2. Compute $\mathrm{Cov}\,(X, Y)$.

$\mathrm{Cov}\,(X, Y) =$ 0 ✔ **Answer:** 0

Are X and Y independent?

- ( ) yes
- (•) no
- ( ) not enough information to determine

✔

Scroll down: There is one more problem below!

Solution:

1. Y is normal, since

$$\begin{aligned} F_Y(y) &= \frac{1}{2}P(X \leq y) + \frac{1}{2}P(-X \leq y) \\ &= \frac{1}{2}P(X \leq y) + \frac{1}{2}P(X \geq -y) \\ &= \frac{1}{2}P(X \leq y) + \frac{1}{2}P(X \leq y) \\ &= F_X(y). \end{aligned}$$

In the third line, we used the symmetry of the standard normal random variable.

2. X and Y is uncorrelated, since

$$\begin{aligned} \mathbf{E}[XY] - \mathbf{E}[X]\mathbf{E}[Y] &= \mathbf{E}[XY] \\ &= \frac{1}{2}\mathbf{E}[X^2] - \frac{1}{2}\mathbf{E}[X^2] \\ &= 0. \end{aligned}$$

X and Y are **not** independent.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Problem 3. Problem 1(c)

0.0/2.0 points (graded)
Find $P(X+Y \leq 0)$.

$P(X+Y \leq 0) =$ 1 ✖ **Answer:** 3/4

Solution:

First, observe that $X+Y$ has a symmetric distribution, that is,

$$P(X+Y \leq c) = P(X+Y \geq -c),$$

for any c. This is because with probability $\frac{1}{2}$, $X+Y=0$, and with probability $\frac{1}{2}$, $X+Y$ is a normal of variance 4.
Thus,

$$\begin{aligned} P(X+Y=0) &= \frac{1}{2}, \\ P(X+Y\neq 0) &= \frac{1}{2}, \\ P(X+Y<0) &= P(X+Y>0) = \frac{1}{4}. \end{aligned}$$

This gives $P(X+Y\leq 0) = P(X+Y<0) + P(X+Y=0) = 3/4$.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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- An alternative way to solve problem 3. 1
- How to understand the variable Y 1
  I know this is kinda late but I will ask it anyway. I find really confusing that fY(y) = fX(y) This statement l...
- Confused about the answer of Problem 3 16
  The answers seems very plausible and correct in terms of applying total probability theorem, howeve...
- Isn't the distribution of X identical to -X? 5
- Distribution of X + Y 16
  I think I can envision the CDF... like the the CDF of a normal distribution but with a 1/2 unit jump at 0. ...
- Probably a stupid question but I'm tired and I cannot see where we got the 1/4 in the solution to the third question 4
  Could someone please help me, thank you!
- [Staff]Question about problem 3.1(c) solution.

Hi. 1)Given following statement >This is because with probability 1/2 , X+Y=0 But isn't probability of co... 3

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Instructions were very clear about number of "attempts" for entering in the answer, so this isn't a co...

Failed to submit Problem 2 (1b) 1

Dear staff, I determined the correct answer for this problem and I thought I submitted the answer. I o...

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[Staff] Parts 2.2 and 3: different answers depending on order of coin flip and X taking a value? 2

[@staff]Fundamentals of Statistics 8

The next course of this micromasters, Fundamentals of Statistics, starts on May 11th, 2020 whereas t...