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8. Exercise: A criterion for independence

Exercises due Feb 28, 2020 05:29 IST Completed

Exercise: A criterion for independence

1/1 point (graded)

Suppose that the conditional PMF of X, given Y=y, is the same for every y for which $p_Y(y)>0$. Is this enough to guarantee independence?

Yes 🕶

✓ Answer: Yes

Solution:

The condition given means that when I tell you the value of Y, the conditional PMF of X will be the same. Thus, the value of Y makes no difference, and, intuitively, we have independence.

For a formal argument, let $c\left(x\right)=p_{X\mid Y}\left(x\mid y\right)$; we can define $c\left(x\right)$ this way (without a dependence on y) since we are assuming that $p_{X\mid Y}\left(x\mid y\right)$ is the same for all y. Now,

$$p_{X,Y}\left(x,y
ight)=p_{Y}\left(y
ight)p_{X\mid Y}\left(x\mid y
ight)=p_{Y}\left(y
ight)c\left(x
ight).$$

Summing over all y, we obtain

$$p_{X}\left(x
ight)=\sum_{y}p_{X,Y}\left(x,y
ight)=\sum_{y}p_{Y}\left(y
ight)c\left(x
ight)=c\left(x
ight).$$

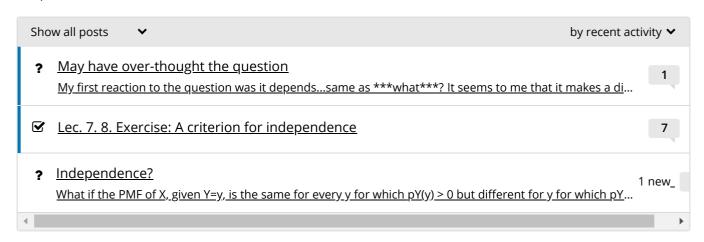
Therefore, $c\left(x\right)=p_{X}\left(x\right)$. It follows that $p_{X,Y}\left(x,y\right)=p_{X\mid Y}\left(x\mid y\right)p_{Y}\left(y\right)=c\left(x\right)p_{Y}\left(y\right)=p_{X}\left(x\right)p_{Y}\left(y\right)$, which establishes independence.

1 Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 4: Discrete random variables:Lec. 7: Conditioning on a random variable; Independence of r.v.'s / 8. Exercise: A criterion for independence



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