10. Exercise: Steady-state behavior

None due May 29, 2020 05:29 IST

Exercise: Steady-state behavior

2 points possible (ungraded)

In the previous video, we have seen that for a homogeneous discrete-time Markov chain with m states and one aperiodic recurrent class, the steady-state probabilities $\pi_1, \ldots, \pi_m$ can be found as the unique solution to the balance equations

$$\pi_j = \sum_k \pi_k p_{kj} \qquad j = 1, \ldots, m,$$

together with the normalization equation $\sum_{j=1}^{m} \pi_j = 1$.

In order to derive this system of equations, we have used one type of recursion for $r_{ij}(n)$. Inspired by the fact that there are many ways to write such recursions, let us see if similar other balance equations can be obtained. For each of the following systems of equations, decide whether, when combined with the normalization equation, it also has the steady-state probabilities as the unique solution.

1. $\pi_j = \sum_k p_{ik} \pi_j \qquad j = 1, \ldots, m$

   Select an option **Answer:** No

2. $\pi_j = \sum_k \pi_k r_{kj}(2) \qquad j = 1, \ldots, m$

   Select an option **Answer:** Yes

Solution:

1. No. Since $\sum_k p_{ik} = 1$ for all i, each of these m equations simply say that $\pi_j = \pi_j$, which does not even have a unique solution, not to mention a unique solution that also gives the correct steady-state probabilities.

2. Yes. We obtain the given system of equations by taking the limit as n goes to infinity on both sides of $r_{ij}(n) = \sum_k r_{ik}(n-2)\, r_{kj}(2)$.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

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I got both wrong, and I am still not sure what is being asked. 4