6. Correlation coefficients

Problem Set due Apr 1, 2020 05:29 IST *Completed*

Problem 6. Correlation coefficients

3/3 points (graded)

Consider random variables X, Y and Z, which are assumed to be pairwise uncorrelated (i.e., X and Y are uncorrelated, X and Z are uncorrelated, and Y and Z are uncorrelated). Suppose that

- $\mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 0$,
- $\mathbf{E}[X^2] = \mathbf{E}[Y^2] = \mathbf{E}[Z^2] = 1$,

Find the correlation coefficients $\rho(X-Y, X+Y)$, $\rho(X+Y, Y+Z)$, and $\rho(X, Y+Z)$.

1. $\rho(X-Y, X+Y) =$ 0 ✔ **Answer:** 0
2. $\rho(X+Y, Y+Z) =$ 1/2 ✔ **Answer:** 0.5
3. $\rho(X, Y+Z) =$ 0 ✔ **Answer:** 0

Solution:

1. We have

$$\operatorname{cov}(X-Y, X+Y) = \mathbf{E}\big[(X-Y)(X+Y)\big] - \mathbf{E}[X-Y]\,\mathbf{E}[X+$$

$$
\begin{aligned}
&= \mathbf{E}\left[X^2 - Y^2\right] - 0 \\
&= \mathbf{E}\left[X^2\right] - \mathbf{E}\left[Y^2\right] \\
&= 0.
\end{aligned}
$$

Hence, $\rho\left(X - Y, X + Y\right) = 0.$

2. Since X and Y are uncorrelated, with zero means, we have $\mathbf{E}\left[XY\right] = \operatorname{cov}\left(X, Y\right) = 0$. Similarly, we have $\mathbf{E}\left[XZ\right] = 0$ and $\mathbf{E}\left[YZ\right] = 0$. Hence,

$$
\begin{aligned}
\operatorname{cov}\left(X + Y, Y + Z\right) &= \mathbf{E}\left[\left(X + Y\right)\left(Y + Z\right)\right] - \mathbf{E}\left[X + Y\right]\mathbf{E}\left[Y + Z\right] \\
&= \mathbf{E}\left[XY + XZ + Y^2 + YZ\right] \\
&= \mathbf{E}\left[Y^2\right] \\
&= 1.
\end{aligned}
$$

Also,

$$
\begin{aligned}
\mathsf{Var}\left(X + Y\right) &= \mathbf{E}\left[\left(X + Y\right)^2\right] - \left(\mathbf{E}\left[X + Y\right]\right)^2 \\
&= \mathbf{E}\left[X^2 + 2XY + Y^2\right] - 0 \\
&= 2.
\end{aligned}
$$

Similarly, $\mathsf{Var}\left(Y + Z\right) = 2.$

Therefore, $\rho\left(X + Y, Y + Z\right) = \frac{\operatorname{cov}(X+Y, Y+Z)}{\sqrt{\mathsf{Var}(X+Y)\mathsf{Var}(Y+Z)}} = \frac{1}{2}.$

3.

$$
\begin{aligned}
\operatorname{cov}\left(X, Y + Z\right) &= \mathbf{E}\left[X\left(Y + Z\right)\right] - \mathbf{E}\left[X\right]\mathbf{E}\left[Y + Z\right] \\
&= \mathbf{E}\left[XY + YZ\right] - 0 \\
&= 0.
\end{aligned}
$$

Hence, $\rho\left(X, Y + Z\right) = 0.$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Problem Set 6 / 6. Correlation coefficients

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Intuition for part 1 Is there an intuition for this part? The answer seems like it could have an intuitive explanation. P/S: it loo...	4
Hint on 2? Not sure what I'm doing wrong but I got incorrect answer for point 2). I use formula with expectations to ...	1 new
0 denominator in correlation coeff. How it works the definition in the cases where the denominator is zero (0 variance) ?	2
How can this be solved without knowing if the X, Y and Z are independent? Given that uncorrelated does not necessarily mean the variables are independent, how can we compute...	4
Hint	1
Hint: Bilinearity of Covariance It really helps if you take in consideration that the covariance has the distributive property. In that way y...	2
How to find the expected value of a product? As in a previous problem, I'm stuck on trying to find the expected value of a product, and not a complica...	3
Did the lectures cover expected value of a sum? As above.	2