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3. The PDF of the maximum

Problem Set due Apr 1, 2020 05:29 IST Completed

Problem 3. The PDF of the maximum

3/3 points (graded)

Let X and Y be independent random variables, each uniformly distributed on the interval [0,1].

1. Let $Z=\max\{X,Y\}$. Find the PDF of Z . Express your answer in terms of z using standard notation.

For
$$0 < z < 1$$
:

$$f_{Z}\left(z
ight)=$$
 2*z $lacksquare$ Answer: 2*z $2\cdot z$

2. Let $Z=\max\{2X,Y\}$. Find the PDF of Z. Express your answer in terms of z using standard notation.

For
$$0 < z < 1$$
:

$$f_{Z}\left(z
ight)=egin{bmatrix} z & & & \\ \hline z & & & \\ \hline z & & & \\ \hline \end{array}$$
 Answer: z

For
$$1 < z < 2$$
:



STANDARD NOTATION

Solution:

Recall that the CDF of a random variable U distributed uniformly on the interval $\left[0,1\right]$ is given by

$$F_{U}\left(u
ight)=\left\{egin{array}{ll} 0, & ext{if } u<0, \ u, & ext{if } 0\leq u\leq 1, \ 1, & ext{if } u>1. \end{array}
ight.$$

1. Let $Z=\max\{X,Y\}$. For $z\in(0,1)$,

$$egin{aligned} F_Z(z) &= \mathbf{P} \, (Z \leq z) \ &= \mathbf{P} \, (X \leq z ext{ and } Y \leq z) \ &= F_X \, (z) \, F_Y \, (z) \ &= z^2 \end{aligned}$$

Hence, $f_{Z}\left(z
ight)=2z$, for $z\in\left(0,1
ight)$.

2. Let $Z = \max\{2X, Y\}$.

$$F_{Z}(z) = \mathbf{P}\left(Z \leq z\right) = \mathbf{P}\left(2X \leq z \text{ and } Y \leq z\right) = F_{X}\left(z/2\right) F_{Y}\left(z\right).$$

Hence, for
$$0 < z < 1$$
, $F_Z(z) = (z/2) \cdot z = z^2/2$, and $f_Z(z) = z$. For $1 < z < 2$, $F_Z(z) = (z/2) \cdot 1 = z/2$, and $f_Z(z) = 1/2$.

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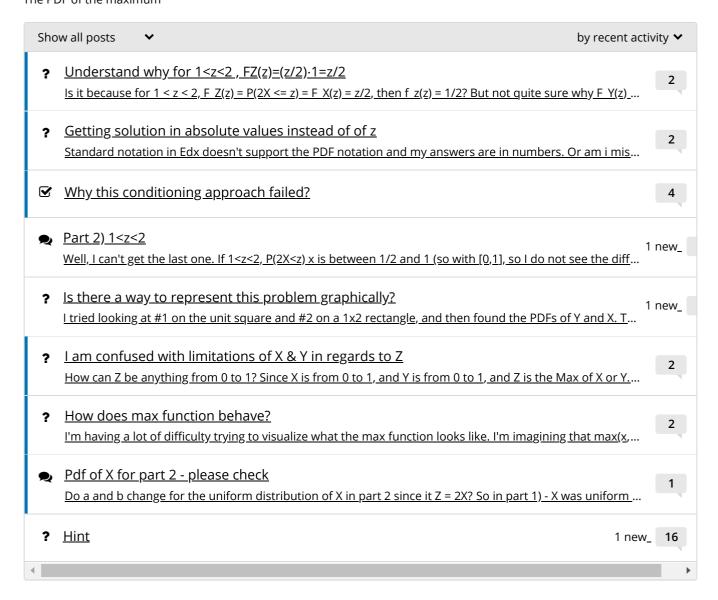
You have used 3 of 3 attempts

1 Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Problem Set 6 / 3. The PDF of the maximum



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