10. Exercise: Discrete unknown and continuous observation

Exercises due Apr 8, 2020 05:29 IST *Completed*

Exercise: Discrete unknown and continuous observation

2/2 points (graded)

Similar to the last example, suppose that $X = \Theta + W$, where Θ is equally likely to take the values -1 and 1, and where W is standard normal noise, independent of Θ. We use the estimator $\widehat{\Theta}$, with $\widehat{\Theta} = 1$ if $X > 0$ and $\widehat{\Theta} = -1$ otherwise. (This is actually the MAP estimator for this problem.)

a) Let us assume that the true value of Θ is 1. In this case, our estimator makes an error if and only if W has a low (negative) value. The conditional probability of error given the true value of Θ is 1, that is, $\mathbf{P}\left(\widehat{\Theta} \neq 1 \mid \Theta = 1\right)$, is equal to

- ◉ $\Phi(-1)$
- ○ $\Phi(0)$
- ○ $\Phi(1)$

✔

where Φ is the standard normal CDF.

b) For this problem, the overall probability of error is easiest found using the formula

- ○ $\mathbf{P}\left(\widehat{\Theta} \neq \Theta\right) = \int \mathbf{P}\left(\widehat{\Theta} \neq \Theta \mid X = x\right) f_X(x)\, dx$
- ◉ $\mathbf{P}\left(\widehat{\Theta} \neq \Theta\right) = \sum_\theta \mathbf{P}\left(\widehat{\Theta} \neq \theta \mid \Theta = \theta\right) p_\Theta(\theta)$

✔

Solution:

a) We have

$$\begin{aligned}\mathbf{P}\left(\widehat{\Theta} \neq 1 \mid \Theta = 1\right) &= \mathbf{P}\left(\Theta + W \leq 0 \mid \Theta = 1\right) = \mathbf{P}\left(1 + W \leq 0 \mid \Theta = 1\right) \\ &= \mathbf{P}\left(1 + W \leq 0\right) = \mathbf{P}\left(W \leq -1\right) = \Phi\left(-1\right).\end{aligned}$$

b) Similar to part (a), $\mathbf{P}\left(\widehat{\Theta} \neq \theta \mid \Theta = \theta\right)$ is easy to calculate for either choice of $\theta = -1$ or $\theta = 1$. For this reason, the second formula is easy to implement.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

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Topic: Unit 7: Bayesian inference:Lec. 14: Introduction to Bayesian inference / 10. Exercise: Discrete unknown and continuous observation

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- Can someone help me explain why the question's statement is true? — Let us assume that the true value of Θ is 1 . In this case, our estimator makes an error if and only if W ha... — 3
- Possible typo error in question a) — 1 new — 5
- STAFF - Grammar question b) — Question b) should either read: "For this problem, the overall probability of error is most easily found usi... — 2
- Will the overall probability of error be equal to 0.5? — If this were right, it would seem a bad estimator to me if the overal probabilty of error was this high. Isnt... — 5
- How do I tackle the second question? — Why if we are talking about a continuous case, can we use discrete? — 2
- Woah — Doesn't this require us to integrate a knarley expontential function? (1/sqrt(2-pi) sigma * exp^x^2/2) ??? ... — 3

- [x] Notation in a)

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