4. LLMS estimation with random sums

Problem Set due Apr 15, 2020 05:29 IST Completed

Problem 4. LLMS estimation with random sums

4/4 points (graded)

Let N be a random variable with mean $\mathbf{E}[N] = m$, and $\text{Var}(N) = v$; let $A_1, A_2, \ldots$ be a sequence of i.i.d random variables, all independent of N, with mean 1 and variance 1; let $B_1, B_2, \ldots$ be another sequence of i.i.d. random variables, all independent of N and of $A_1, A_2, \ldots$, also with mean 1 and variance 1. Let $A = \sum_{i=1}^{N} A_i$ and $B = \sum_{i=1}^{N} B_i$.

1. Find the following expectations using the law of iterated expectations. Express each answer in terms of m and v, using standard notation.

   $\mathbf{E}[AB] =$

   m^2+v

   $m^2 + v$

   Answer: m^2+v

   $\mathbf{E}[NA] =$

   m^2+v

   $m^2 + v$

   Answer: m^2+v

2. Let $\hat{N} = c_1 A + c_2$ be the LLMS estimator of N given A. Find c_1 and c_2 in terms of m and v.

   $c_1 =$

   v/(m+v)

   $\frac{v}{m+v}$

   Answer: v/(m+v)

$c_2 =$

m^2/(m+v)

Answer: m^2/(m+v)

$\frac{m^2}{m+v}$

STANDARD NOTATION

Solution:

1. We begin by finding $\mathbf{E}[AB]$.

$$\begin{aligned}\mathbf{E}[AB] &= \mathbf{E}[(A_1+\cdots+A_N)(B_1+\cdots+B_N)] \\ &= \mathbf{E}[\mathbf{E}[(A_1+\cdots+A_N)(B_1+\cdots+B_N) \mid N]] \\ &= \mathbf{E}[\mathbf{E}[(A_1+\cdots+A_N) \mid N]\,\mathbf{E}[(B_1+\cdots+B_N) \mid N]] \\ &= \mathbf{E}[N\mathbf{E}[A_1]\,N\mathbf{E}[B_1]] \\ &= \mathbf{E}[N^2] \\ &= \mathsf{Var}(N) + (\mathbf{E}[N])^2 \\ &= m^2 + v.\end{aligned}$$

   Similarly,

$$\begin{aligned}\mathbf{E}[NA] &= \mathbf{E}[\mathbf{E}[N(A_1+\cdots+A_N) \mid N]] \\ &= \mathbf{E}[N\mathbf{E}[A_1+\cdots+A_N \mid N]] \\ &= \mathbf{E}[N(N\mathbf{E}[A_1])] \\ &= \mathbf{E}[N^2] \\ &= m^2 + v.\end{aligned}$$

2. A is the sum of a random number, N, of independent and identically distributed random variables $A_1, \ldots, A_N$. Therefore,

$$\mathbf{E}[A] = \mathbf{E}[\mathbf{E}[A \mid N]] = \mathbf{E}[\mathbf{E}[A_1]\,N] = m,$$

   and

$$\mathsf{Var}(A) = \mathsf{Var}(A_i)\,\mathbf{E}[N] + (\mathbf{E}[A_i])^2\mathsf{Var}(N) = m + v.$$

Similarly, $\mathbf{E}[B] = m$, and $\mathsf{Var}(B) = m + v$. Furthermore,

$$\begin{aligned} \operatorname{cov}(N, A) &= \mathbf{E}[NA] - \mathbf{E}[N]\,\mathbf{E}[A] \\ &= (m^2 + v) - m^2 \\ &= v. \end{aligned}$$

Finally,

$$\begin{aligned} \hat{N} &= \mathbf{E}[N] + \frac{\operatorname{cov}(N, A)}{\mathsf{Var}(A)}(A - \mathbf{E}[A]) \\ &= m + \frac{v}{m+v}(A - m) \\ &= \frac{m^2}{m+v} + \frac{v}{m+v}A. \end{aligned}$$

Submit You have used 3 of 4 attempts

Answers are displayed within the problem

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var(A) 1

Meaning of i.i.d? 4
Problem says > ... be a sequence of i.i.d random variables I was wondering what is **i.i.d**? Sorry, I skip...

Part 1 variance 3
I am struggling with this question, including the first part. My answer only involves m, yet is wrong. But I ...

Result of E[AB | N]?

Result of E[AB|N]? 7

Need some hints 9
Problem at first looked very easy but I am completely stuck. I used law of iterated expectations, consider...

Sleight of hand? 5

No calculation hint for 2 5
I solve it just by using results from last question, for c1: answer should be some combinations of varianc...

part 2 is a lot harder than it looks 4
wow, this problem..... I managed to solve it after looking through multiple weeks of videos and using up ...

General advice for this problem 2
This problem is pretty straightforward if you apply with care the results from Unit 6. My advice is, if you ...

Why is Cov(N,A) not 0 due to each 'A' being Ind. of N? 5
I know that must be the case, but what's the intuition behind it?

Recomendation for part 1 1

Hints 9

Product of independent random variables conditionally independent? 2