13. Exercise: Independence and variances

Exercises due Feb 28, 2020 05:29 IST *Completed*

Exercise: Independence and variances

3/3 points (graded)

The pair of random variables (X,Y) is equally likely to take any of the four pairs of values $(0,1)$, $(1,0)$, $(-1,0)$, $(0,-1)$. Note that X and Y each have zero mean.

a) Find $\mathbf{E}[XY]$.

$\mathbf{E}[XY] =$ 0 **Answer:** 0

b) For this pair of random variables (X,Y), is it true that $\mathsf{Var}(X+Y) = \mathsf{Var}(X) + \mathsf{Var}(Y)$?

Yes **Answer:** Yes

c) We know that if X and Y are independent, then $\mathsf{Var}(X+Y) = \mathsf{Var}(X) + \mathsf{Var}(Y)$. Is the converse true? That is, does the condition $\mathsf{Var}(X+Y) = \mathsf{Var}(X) + \mathsf{Var}(Y)$ imply independence?

No **Answer:** No

Solution:

a) At each possible outcome, we have $XY = 0$, and therefore $\mathbf{E}[XY] = 0$.

b) Since the random variables have zero mean, $\mathbf{E}[X+Y] = 0$, $\mathsf{Var}(X) = \mathbf{E}[X^2]$, and $\mathsf{Var}(Y) = \mathbf{E}[Y^2]$. Combining this with the result from part (a), we conclude that

$$\mathsf{Var}(X+Y) = \mathbf{E}[(X+Y)^2] - (\mathbf{E}[X+Y])^2$$

$$
\begin{aligned}
&= \mathbf{E}\left[(X+Y)^2\right] \\
&= \mathbf{E}\left[X^2\right] + 2\mathbf{E}\left[XY\right] + \mathbf{E}\left[Y^2\right] \\
&= \mathbf{E}\left[X^2\right] + \mathbf{E}\left[Y^2\right] \\
&= \mathsf{Var}\,(X) + \mathsf{Var}\,(Y)\,.
\end{aligned}
$$

c) We have here an example of two random variables that satisfy the condition $\mathsf{Var}\,(X+Y) = \mathsf{Var}\,(X) + \mathsf{Var}\,(Y)$. But these random variables are not independent. For example, the information that $X = 1$ tells us that the value of Y must be zero.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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? Why the term "($\mathbf{E}[X+Y]$)^2" is zero?
Why the second term "($\mathbf{E}[X+Y]$)^2" equals zero in Var($X+Y$) =$\mathbf{E}$[($X+Y$)^2]−($\mathbf{E}[X+Y]$)^2 ?? 4 new_ 7

? Deadline
I cannot submit my answer. The official date is Feb 28, 2020, 00:59 CET. The current time is Feb 28, 2020,... 1

? c) Through the equations, I am getting converse to be true. Is what i am thinking right?
If Var(X+Y)=Var(X)+Var(Y), then E[X].E[Y] term is 0, which means at least one of them is zero. So this show... 5

1a independence?
I have been using as a test for independence that p(x,y)=p(x)p(y), but if you look at X and Y, they can take... 6

Just an observation
It just interesting that in the a) the answer will be the same no matter what is the distribution of the pro... 2