6. Upper and lower bounds on the probability of intersection

Problem Set due Feb 5, 2020 05:29 IST *Completed*

Problem 6. Upper and lower bounds on the probability of intersection

2/2 points (graded)

Given two events A, B with $\mathbf{P}(A) = 3/4$ and $\mathbf{P}(B) = 1/3$, what is the smallest possible value of $\mathbf{P}(A \cap B)$? The largest? That is, find a and b such that,

$$a \leq \mathbf{P}(A \cap B) \leq b,$$

holds and any value in the closed interval [a,b] is possible.

$a =$ 1/12 ✔ **Answer:** 1/12

$b =$ 1/3 ✔ **Answer:** 1/3

Solution:

First, we begin by recalling that, $\mathbf{P}(A \cap B)$ obeys,

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) + \mathbf{P}(B) - \mathbf{P}(A \cup B).$$

Now, for the lower bound, note that, $\mathbf{P}(A \cup B) \leq 1$, since the probability of any event can be at most one. Hence,

$$\mathbf{P}(A \cap B) \geq \mathbf{P}(A) + \mathbf{P}(B) - 1 = \frac{1}{12}.$$

Next, for the upper bound, we use the fact that, $A \cap B \subset A$ and $A \cap B \subset B$. Hence,

$$\mathbf{P}(A \cap B) \leq \mathbf{P}(A) \quad \text{and} \quad \mathbf{P}(A \cap B) \leq \mathbf{P}(B).$$

In particular, $\mathbf{P}(A \cap B)$ must be less than or equal to the smallest of $\mathbf{P}(A)$ and $\mathbf{P}(B)$, which means,

$$\mathbf{P}(A \cap B) \leq \frac{1}{3}.$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 1: Probability models and axioms:Problem Set 1 / 6. Upper and lower bounds on the probability of intersection

Show all posts by recent activity

? Lower bound
Hello, I put the answer thinking about the lower bound thinking of A and B being disjoint, but that didn't ... 4

? [Staff]Right Answer Bug. Coarse grader issue.
Hi, I have put a=0.08 but the system has shown that it is wrong. What is happening? 9

Upper bound
Hey classmates, How did you go about finding upper bound? I did it by guessing. how about analytically ? 8 new 12

? My upper bound answer was wrong, but isn't it the case where...
B is contained within A? B is a subset of A. That makes the answer to the upper bound almost trivial. Ho... 6

Answers

Does anyone know when the answers will be provided? 2

Video 2
Does anyone know what video this relates to?

lower bound [a] 17
shouldn't the lower bound [a] represent the case where the event A and B are disjoint?

Recommendation 5
Ideally, I think this is too soon for set 1. I just started this saturday and had to skip a lot of the lecture to c...

Is there is difference between "P" in normal font and "P" as displayed in this problem? 3
Hi, everyone! Probably a silly question: but is there a difference between "P" in normal font (for example:...

Answers at some point for revision 2
Will will get the solutions to these problems to help us revise. I found the answers to the after lecture qui...

Answer number input format 5
To what decimal precision should the answer be inputted with?