6. Exercise: LMS estimation error

Exercises due Apr 15, 2020 05:29 IST *Completed*

Exercise: LMS estimation error

3.0/3.0 points (graded)

As in the previous exercise, let Θ be the bias of a coin, i.e., the probability of Heads at each toss. We assume that Θ is uniformly distributed on $[0,1]$. Let K be the number of Heads in 9 independent tosses. We have seen that the LMS estimate of K is $\mathbf{E}[K \mid \Theta = \theta] = n\theta$.

a) Find the conditional mean squared error $\mathbf{E}\left[\left(K - \mathbf{E}[K \mid \Theta = \theta]\right)^2 \mid \Theta = \theta\right]$ if $\theta = 1/3$.

2 ✔ **Answer:** 2

b) Find the overall mean squared error of this estimation procedure.

3/2 ✔ **Answer:** 1.5

Solution:

a) This is the variance of the conditional distribution of K. Since the conditional distribution is binomial with parameters $n = 9$ and $\theta = 1/3$, the conditional variance is $9\,(1/3)\,(2/3) = 2$.

b) This is the average of the conditional variance, averaged over all possible values of the observation Θ, which has a uniform distribution:

$$\int_0^1 f_\Theta(\theta)\,\mathsf{Var}(K \mid \Theta = \theta)\,d\theta = \int_0^1 9\theta(1-\theta)\,d\theta$$

$$= \left(9\frac{1}{2}\theta^2 - 9\frac{\theta^3}{3} \right) \Big| 0_1$$
$$= 4.5 - 3$$
$$= 1.5.$$

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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Methodology needs to be improved This is a very great course, but the methodology needs to be improved. Contents are becoming very abs...	9
Hint for both questions- (a) and (b) For those who are struggling like I did. The first part simply uses the formula for the variance of the bino...	1
? Part 2 Oy. So many things, I'll throw them out. I wish I could move on to the next problems, but my brain gets s...	2
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I honestly just don't remember var properties off the top of my head I know what these two things mean based on the video, but could someone please direct me to the lectu...	1 new
I'm lost ... where do I start So I'll just think out loud and hopefully someone can point me in the right direction. OK, I got the last on...	2 new
Motivation Despite getting average grades in the lectures and problem sets. I usually have to think hard to solve pro...	2
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