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8. Exercise: Sample mean bounds

Exercises due May 1, 2020 05:29 IST Completed

Exercise: Sample mean bounds

2/2 points (graded)

By the argument in the last video, if the X_i are i.i.d. with mean μ and variance σ^2 , and if $M_n=\left(X_1+\cdots+X_n\right)/n$, then we have an inequality of the form

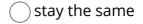
$$\mathbf{P}ig(|M_n-\mu|\geq\epsilonig)\leq rac{a\sigma^2}{n},$$

for a suitable value of a.

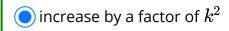
a) If $\epsilon=0.1$, then the value of a is: 100

b) If we change $\epsilon=0.1$ to $\epsilon=0.1/k$, for $k\geq 1$ (i.e., if we are interested in k times higher accuracy), how should we change n so that the value of the upper bound does not change from the value calculated in part (a)?

n should



 \bigcirc increase by a factor of k



 \bigcirc decrease by a factor of k



none of the above



Solution:

a) Chebyshev's inequality yields

$$\mathbf{P}ig(|M_n-\mu|\geq\epsilonig)\leqrac{\sigma^2}{n\epsilon^2},$$

so that
$$a=1/\epsilon^2=1/0.1^2=100$$
.

b) In order to keep the same upper bound, the term $n\epsilon^2$ in the denominator needs to stay constant. If we reduce ϵ by a factor of k, then ϵ^2 gets reduced by a factor of k^2 . Thus, n will have to be increased by a factor of k^2 .

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You have used 1 of 3 attempts

1 Answers are displayed within the problem

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Watch the next segment if you're stuck.
The following segment will make this question MUCH clearer if, like me, you didn't find it obvious at first!

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