1. Normal random variables

Problem Set due Mar 13, 2020 05:29 IST *Completed*

Problem 1. Normal random variables

5/5 points (graded)

Let X and Y be two normal random variables, with means 0 and 3, respectively, and variances 1 and 16, respectively. Find the following, using the standard normal table. Express your answers to an accuracy of **3 decimal places**.

Standard Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \leq z)$, where Z is a standard normal random variable, for z between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that $\Phi(1.71) = .9564$. When z is negative, the value of $\Phi(z)$ can be found using the formula $\Phi(z) = 1 - \Phi(-z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830

1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

*For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

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1.

$\mathbf{P}(X > -1) =$ 0.841 **Answer:** 0.841

2.

$\mathbf{P}(X \leq -2)$ 0.023 **Answer:** 0.023

3. Let $V = (4 - Y)/3$. Find the mean and the variance of V.

$\mathbf{E}[V] =$ 0.333 **Answer:** 1/3

$\text{Var}(V) =$ 1.778 **Answer:** 16/9

4.

$\mathbf{P}(-2 < Y \leq 2) =$ 0.296 **Answer:** 0.2957

Solution:

1. Since the distribution of X is symmetric around 0, we have,

$$\mathbf{P}(X > -1) = \mathbf{P}(X < 1) = \Phi(1) = 0.841.$$

2. Using symmetry again,

$$\mathbf{P}(X \leq -2) = \mathbf{P}(X > 2) = 1 - \mathbf{P}(X < 2) = 1 - \Phi(2) = 0.023.$$

3. We have $\mathbf{E}[V] = 4/3 - \mathbf{E}[Y]/3 = 1/3$, and $\text{Var}(V) = \frac{1}{3^2}\text{Var}(16) = 16/9$.

4. By standardizing Y, and using the normal table, we have

$$\begin{aligned}
\mathbf{P}(-2 < Y \leq 2) &= \mathbf{P}\left(\frac{-2-3}{4} \leq \frac{Y-3}{4} \leq \frac{2-3}{4}\right) \\
&= \mathbf{P}(-5/4 \leq Z \leq -1/4) \\
&= \mathbf{P}(1/4 \leq Z \leq 5/4) \\
&= \Phi(5/4) - \Phi(1/4) \\
&\approx 0.8944 - 0.5987 \\
&= 0.2957,
\end{aligned}$$

where, Z is a standard normal random variable.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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Help on #3? I don't know which rules to apply here ... is V even a normal random variable? I think so ... my best guess is that V shif...	9