14. Exercise: Natural estimators

Exercises due May 1, 2020 05:29 IST   *Completed*

Exercise: Natural estimators

3/3 points (graded)

The random variables X_i are i.i.d. and satisfy $\mathbf{E}\left[X_i^2\right] = \theta$. Use a natural estimator to calculate an estimate of θ based on the values $X_1 = 1$, $X_2 = 3$, $X_3 = -1$, $X_4 = 2$, $X_5 = 0$.

| 3 |

✔ **Answer:** 3

In order to calculate confidence intervals around your estimator, you need information on the variance of your estimator. This variance is determined by $\mathbf{E}\left[X_i^2\right]$ and $\mathbf{E}\left[X_i^a\right]$ for some other power a. What is the value of a?

$a =$ | 4 |

✔ **Answer:** 4

If you do not have any prior knowledge about the value of $\mathbf{E}\left[X_i^a\right]$, can you estimate it based on the available data?

Yes ⌄   ✔ **Answer:** Yes

Solution:

A natural estimator is

$$\frac{1}{5}\sum_{i=1}^{5} X_i^2 = \frac{1}{5}(1 + 9 + 1 + 4 + 0) = 3.$$

To find the variance of the estimator, you need the variance of X_i^2. Since $\text{Var}\left(X_i^2\right) = \mathbf{E}\left[X_i^4\right] - \left(\mathbf{E}\left[X_i^2\right]\right)^2$, you need to know $\mathbf{E}\left[X_i^4\right]$. This quantity can be estimated using the natural estimator
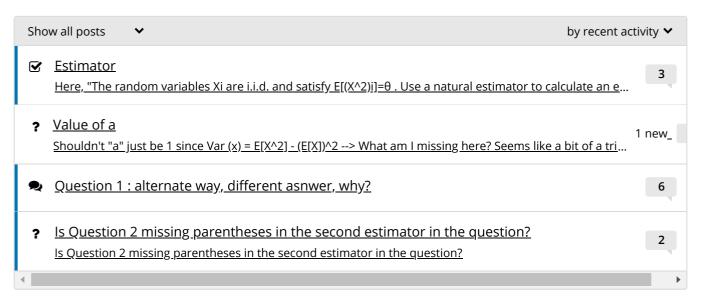
$$\frac{1}{n}\sum_{i=1}^{n}X_i^4.$$

Submit   You have used 2 of 3 attempts

ℹ   Answers are displayed within the problem

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☑ **Estimator**
Here, "The random variables Xi are i.i.d. and satisfy E[(X^2)i]=θ . Use a natural estimator to calculate an e...   **3**

? **Value of a**
Shouldn't "a" just be 1 since Var (x) = E[X^2] - (E[X])^2 --> What am I missing here? Seems like a bit of a tri...   **1 new_**

💬 **Question 1 : alternate way, different asnwer, why?**   **6**

? **Is Question 2 missing parentheses in the second estimator in the question?**
Is Question 2 missing parentheses in the second estimator in the question?   **2**

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