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## 3. PMF, expectation, and variance

Problem Set due Feb 28, 2020 05:29 IST Completed

Problem 3. PMF, expectation, and variance

6.0/6.0 points (graded)

The random variables X and Y have the joint PMF

$$p_{X,Y}\left(x,y
ight)=\left\{egin{aligned} c\cdot\left(x+y
ight)^{2}, & ext{if }x\in\left\{1,2,4
ight\} ext{ and }y\in\left\{1,3
ight\},\ 0, & ext{otherwise}. \end{aligned}
ight.$$

All answers in this problem should be numerical.

1. Find the value of the constant c.

2. Find  $\mathbf{P}\left(Y < X\right)$ .

3. Find **P** (Y = X).

4. Find the following probabilities.

(*Hint*: To avoid double jeopardy with later problem sets, the answers are  $\frac{74}{128}$ ,  $\frac{34}{128}$ ,  $\frac{20}{128}$ , 0, not necessarily in that order.)



$$P(X = 1) =$$
 20/128
 Answer: 20/128

  $P(X = 2) =$ 
 34/128
 Answer: 34/128

  $P(X = 3) =$ 
 0
 Answer: 0

  $P(X = 4) =$ 
 74/128
 Answer: 74/128

5. Find the expectations  $\mathbf{E}\left[X\right]$  and  $\mathbf{E}\left[XY\right]$ .

6. Find the variance of X.

$$Var(X) = \begin{bmatrix} 13/9 \end{bmatrix}$$
 Answer: 47/32

## **Solution:**

1. From the joint PMF, there are  $\operatorname{six}(x,y)$  pairs with nonzero probability mass. These pairs are (1,1),(1,3),(2,1),(2,3),(4,1),(4,3). Because the probability of the entire sample space must equal 1, we have:

$$c(1+1)^2 + c(1+3)^2 + c(2+1)^2 + c(2+3)^2 + c(4+1)^2 + c(4+3)^2 = 1.$$

Solving for c, we get  $c=\frac{1}{128}$ .

2. There are three possible outcomes for which y < x: (2,1), (4,1), (4,3).

$$\mathbf{P}\left(Y < X
ight) = p_{X,Y}\left(2,1
ight) + p_{X,Y}\left(4,1
ight) + p_{X,Y}\left(4,3
ight) = rac{9}{128} + rac{25}{128} + rac{49}{128} = rac{83}{128}.$$

3. There is only one possible outcome for which y=x: (1,1).



$$\mathbf{P}\left(Y=X
ight)=p_{X,Y}\left(1,1
ight)=rac{4}{128}.$$

4. We use the formula  $p_{X}\left(x
ight)=\sum_{y}p_{X,Y}\left(x,y
ight).$ 

For example,  $p_{X}\left(2
ight)=p_{X,Y}\left(2,1
ight)+p_{X,Y}\left(2,3
ight)=rac{34}{128}.$  More generally, we find that

$$p_{X}(x) = egin{cases} 20/128, & ext{if } x = 1, \ 34/128, & ext{if } x = 2, \ 74/128, & ext{if } x = 4, \ 0, & ext{otherwise}. \end{cases}$$

5. We have

$$\mathbf{E}\left[X
ight] = \sum_{x} x p_{X}\left(x
ight) = 1 \cdot rac{20}{128} + 2 \cdot rac{34}{128} + 4 \cdot rac{74}{128} = 3.$$

Using the expected value rule,

$$egin{align} \mathbf{E}\left[XY
ight] &= \sum_{x} \sum_{y} xyp_{X,Y}\left(x,y
ight) \ &= 1 \cdot rac{4}{128} + 2 \cdot rac{9}{128} + 4 \cdot rac{25}{128} + 3 \cdot rac{16}{128} + 6 \cdot rac{25}{128} + 12 \cdot rac{49}{128} \ &= rac{227}{32}. \end{split}$$

6. The variance of a random variable X can be computed as  $\mathbf{E}\left[X^2\right]-(\mathbf{E}\left[X\right])^2$  or as  $\mathbf{E}\left[(X-\mathbf{E}\left[X\right])^2\right]$ . We use the second approach here. We have

$$\mathsf{Var}(X) = (1-3)^2 \frac{20}{128} + (2-3)^2 \frac{34}{128} + (4-3)^2 \frac{74}{128} = \frac{47}{32}.$$

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You have used 4 of 5 attempts

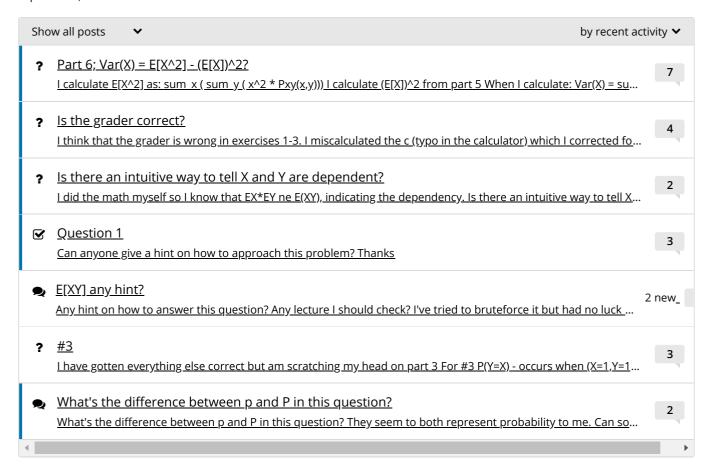


**1** Answers are displayed within the problem

## Discussion

**Hide Discussion** 

**Topic:** Unit 4: Discrete random variables:Problem Set 4 / 3. PMF, expectation, and variance



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