



## 11. Exercise: The effect of a stronger signal

Exercises due Apr 8, 2020 05:29 IST Completed

### Exercise: The effect of a stronger signal

1/1 point (graded)

For the model  $X = \Theta + W$ , and under the usual independence and normality assumptions for  $\Theta$  and  $W$ , the mean squared error of the LMS estimator is

$$\frac{1}{(1/\sigma_0^2) + (1/\sigma_1^2)},$$

where  $\sigma_0^2$  and  $\sigma_1^2$  are the variances of  $\Theta$  and  $W$ , respectively.

Suppose now that we change the observation model to  $Y = 3\Theta + W$ . In some sense the "signal"  $\Theta$  has a stronger presence, relative to the noise term  $W$ , and we should expect to obtain a smaller mean squared error. Suppose  $\sigma_0^2 = \sigma_1^2 = 1$ . The mean squared error of the original model  $X = \Theta + W$  is then  $1/2$ . In contrast, the mean squared error of the new model  $Y = 3\Theta + W$  is

✓ Answer: 0.1

*Hint:* Do not solve the problem from scratch. Think of an alternative observation model in which you observe  $Y' = \Theta + (W/3)$ .

#### Solution:

Since  $Y'$  is just  $Y$  scaled by a factor of  $1/3$ ,  $Y'$  carries the same information as  $Y$ , so that  $\mathbf{E}[\Theta | Y] = \mathbf{E}[\Theta | Y']$ . Thus, the alternative observation model  $Y' = \Theta + (W/3)$  will lead to the same estimates and will have the same mean squared error as the unscaled



model  $Y = 3\Theta + W$ . In the equivalent  $Y'$  model, we have a noise variance of  $1/9$  and therefore the mean squared error is

$$\frac{1}{\frac{1}{1} + \frac{1}{1/9}} = \frac{1}{10}.$$

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You have used 3 of 3 attempts

**i** Answers are displayed within the problem

## Discussion

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☒ Effect of a stronger signal

Hi, The same kind of problem is seen in Ex. 8 - Multiple observations, more general model, where  $X=2 \cdot T \dots$

3

? How do we get  $Y'$  from  $Y$

1

? Clarification

Are we to assume that the prior on theta in the  $Y$  model is normal with 0 mean and variance 1?

2

? Prior/posterior variances?

I definitely need to re-learn this material... The hint is:  $Y' = \Theta + (W/3)$ , which I understand. In this situation, I...

2

Would the  $Y'$  model relate in the same way to the  $Y$  model for any invertible function  $h$  with  $Y' = h(Y)$ ?

I tried to formulate my question so as to not reveal too much from the solution.

1

☒ Why is it wrong to apply the formula on  $Y$  with  $\text{var}(\Theta)=9$ ?

2 new\_ 7

? variance of the noise

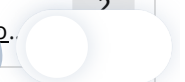
why is the variance  $1/9$ ?

2

From scratch approach

I have already done the question with the hint approach, and I understand why they are equivalent. Ho...

2





From scratch team

2

I suppose is due to a lack of understanding or skills on my part, but every time it says, "don't do it from s...

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