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Final Exam due May 20, 2020 05:29 IST Completed

Problem 3

3.0/3.0 points (graded)

Let X_1, X_2, X_3 be i.i.d. Binomial random variables with parameters $n = 2$ and $p = 1/2$. Define two new random variables

$$Y_1 = X_1 - X_3,$$

$$Y_2 = X_2 - X_3.$$

We further introduce indicator random variables $Z_i \in \{0, 1\}$ with $Z_i = 1$ if and only if $Y_i = 0$ for $i = 1, 2$.

Calculate the covariance of Y_1 and Y_2 .

(Give an exact answer or a decimal accurate to at least 3 decimal places.)

$\text{Cov}(Y_1, Y_2) =$ ✓ Answer: 1/2

Calculate the variance of Z_1 . (Give an exact answer or a decimal accurate to at least 3 decimal places.)

$\text{Var}(Z_1) =$ ✓ Answer: 15/64

Calculate the covariance of Z_1 and Z_2 . (Give an exact answer or a decimal accurate to at least 3 decimal places.)

$\text{Cov}(Z_1, Z_2) =$ ✓ Answer: 1/64

[STANDARD NOTATION](#)

Solution:



- Since X_i are independent, $\text{Cov}(Y_1, Y_2) = \text{Var}(X_3) = np(1-p) = 1/2$.

- Z_1 is Bernoulli with parameter

$$p = \mathbf{P}(Z_1 = 1) = \mathbf{P}(Y_1 = 0) = \mathbf{P}(X_1 = X_3) = (1/4)^2 + (1/2)^2 + (1/4)^2 = 3/8.$$

The variance is

$$\text{Var}(Z_1) = p(1-p) = 15/64.$$

- The covariance of Z_1, Z_2 is

$$\begin{aligned} \text{Cov}(Z_1, Z_2) &= \mathbf{E}[Z_1 Z_2] - \mathbf{E}[Z_1] \mathbf{E}[Z_2] \\ &= \mathbf{P}(X_1 = X_2 = X_3) - p^2 \quad \text{where } p = 3/8 \\ &= (1/4)^3 + (1/2)^3 + (1/4)^3 - (3/8)^2 = 1/64. \end{aligned}$$

Submit

You have used 2 of 3 attempts

i Answers are displayed within the problem

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✓ [Please help to understand Cov\(Z1,Z2\)](#)

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✓ [P\(X₁ = X₂ = X₃\)](#)

[Can someone show me how to calculate this like in the solution?](#)

7

? [\[STAFF\] The questions are not graded](#)

[Hi Staff, This question is not graded for me even though it shows I have used 1 out of 3 attempts, and also a green t...](#)

2

💬 [Sadness](#)

[I had found and saved the correct answers, but forgot to submit them.](#)

2

? [Cannot see my grade on this task](#)

[Hi, Staff! I submitted my answer but it is not graded. 'You have used 1 of 3 attempts' Response is not marked as cor...](#)

2

✓ [Parameter for Z1 - Can you advise where this approach went wrong?](#)

[Hi, I took a different approach to get parameter for Z but was wrong. I'd appreciate any advise to tell me where wen](#)

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<input checked="" type="checkbox"/> <u>How is $p = 3/8$?</u> <u>Can someone shed some light on the solution?</u>	3
<input type="checkbox"/> <u>My answers for Q3 of the Final</u> <u>a. $1/2$ b. $15/64$ c. $1/64$</u>	14
<input checked="" type="checkbox"/> <u>What "i.i.d" means?</u> <u>Let X_1, X_2, X_3 be "i.i.d." Binomial random variables. I do not understand this expression.</u>	3
<input checked="" type="checkbox"/> <u>Which unit/topic is referred here?</u> <u>I am concerned because I use somewhat "manual" tools (i.e. a tree of events) to calculate Bernoulli probability and t...</u>	1 new_

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