3. The PDF of the maximum

Problem Set due Apr 1, 2020 05:29 IST *Completed*

Problem 3. The PDF of the maximum

3/3 points (graded)

Let X and Y be independent random variables, each uniformly distributed on the interval $[0, 1]$.

1. Let $Z = \max\{X, Y\}$. Find the PDF of Z. Express your answer in terms of z using standard notation.

   For $0 < z < 1$:

   $f_Z(z) =$ 2*z ✔ **Answer:** 2*z

   $2 \cdot z$

2. Let $Z = \max\{2X, Y\}$. Find the PDF of Z. Express your answer in terms of z using standard notation.

   For $0 < z < 1$:

   $f_Z(z) =$ z ✔ **Answer:** z

   z

   For $1 < z < 2$:

   $f_Z(z) =$ 0.5 ✔ **Answer:** 0.5

   0.5

STANDARD NOTATION

Solution:

Recall that the CDF of a random variable U distributed uniformly on the interval $[0,1]$ is given by

$$F_U(u) = \begin{cases} 0, & \text{if } u < 0, \\ u, & \text{if } 0 \leq u \leq 1, \\ 1, & \text{if } u > 1. \end{cases}$$

1. Let $Z = \max\{X, Y\}$. For $z \in (0,1)$,

$$\begin{aligned} F_Z(z) &= \mathbf{P}(Z \leq z) \\ &= \mathbf{P}(X \leq z \text{ and } Y \leq z) \\ &= F_X(z)\, F_Y(z) \\ &= z^2 \end{aligned}$$

   Hence, $f_Z(z) = 2z$, for $z \in (0,1)$.

2. Let $Z = \max\{2X, Y\}$.

$$F_Z(z) = \mathbf{P}(Z \leq z) = \mathbf{P}(2X \leq z \text{ and } Y \leq z) = F_X(z/2)\, F_Y(z).$$

   Hence, for $0 < z < 1$, $F_Z(z) = (z/2) \cdot z = z^2/2$, and $f_Z(z) = z$.
   For $1 < z < 2$, $F_Z(z) = (z/2) \cdot 1 = z/2$, and $f_Z(z) = 1/2$.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Problem Set 6 / 3.
The PDF of the maximum

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- [Understand why for 1<z<2 , FZ(z)=(z/2)·1=z/2] 2
  Is it because for 1 < z < 2, F_Z(z) = P(2X <= z) = F_X(z) = z/2, then f_z(z) = 1/2? But not quite sure why F_Y(z) ...
- [Getting solution in absolute values instead of of z] 2
  Standard notation in Edx doesn't support the PDF notation and my answers are in numbers. Or am i mis...
- [Why this conditioning approach failed?] 4
- [Part 2) 1<z<2] 1 new_
  Well, I can't get the last one. If 1<z<2, P(2X<z) x is between 1/2 and 1 (so with [0,1], so I do not see the diff...
- [Is there a way to represent this problem graphically?] 1 new_
  I tried looking at #1 on the unit square and #2 on a 1x2 rectangle, and then found the PDFs of Y and X. T...
- [I am confused with limitations of X & Y in regards to Z] 2
  How can Z be anything from 0 to 1? Since X is from 0 to 1, and Y is from 0 to 1, and Z is the Max of X or Y....
- [How does max function behave?] 2
  I'm having a lot of difficulty trying to visualize what the max function looks like. I'm imagining that max(x,...
- [Pdf of X for part 2 - please check] 1
  Do a and b change for the uniform distribution of X in part 2 since it Z = 2X? So in part 1) - X was uniform ...
- [Hint] 1 new_ 16