8. Exercise: n-step recursion

None due May 29, 2020 05:29 IST

Exercise: n-step recursion

3 points possible (ungraded)

Let $r_{ij}(n) = \mathbf{P}(X_n = j \mid X_0 = i)$ be the n-step transition probability of a given homogeneous discrete-time Markov chain with m states. We have shown that $r_{ij}(n)$ satisfies the following recursion for $n \geq 2$: $r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1)\, p_{kj}$. For each of the following, decide whether it is also a valid recursion formula for $r_{ij}(n)$.

1. $r_{ij}(n) = \sum_{k=1}^{m} p_{ik} r_{kj}(n-1)$ for $n \geq 2$

   Select an option **Answer:** Yes

2. $r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-2)\, r_{kj}(2)$ for $n \geq 3$

   Select an option **Answer:** Yes

3. $r_{ij}(n) = \sum_{k=1}^{m} \sum_{\ell=1}^{m} r_{ik}(n-2)\, p_{k\ell} p_{\ell j}$ for $n \geq 3$

   Select an option **Answer:** Yes

Solution:

1. Yes. The recursion considers a one-step transition from i to any state k, followed by an $(n-1)$-step transition from k to j.

2.

Yes. The recursion considers an $(n-2)$-step transition from i to any state k, followed by a 2-step transition from k to j.

3. Yes. The recursion considers an $(n-2)$-step transition from i to any state k, followed by a one-step transition from k to any state ℓ, followed by a one-step transition from ℓ to j.

Submit You have used 0 of 1 attempt

Answers are displayed within the problem

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? Could $rij(n)$ also be written like this? 2

one-step from l to j 3
if it is only one-step from l to j, why the summation of l has a range from 1 to m?