12. Exercise: Continuous unknown and observation

Exercises due Apr 8, 2020 05:29 IST *Completed*

Exercise: Continuous unknown and observation

4/4 points (graded)

Let Θ and X be jointly continuous nonnegative random variables. A particular value x of X is observed and it turns out that $f_{\Theta|X}(\theta \,|\, x) = 2e^{-2\theta}$, for $\theta \geq 0$.

The following facts may be useful: for an exponential random variable Y with parameter λ, we have $\mathbf{E}[Y] = 1/\lambda$ and $\mathbf{Var}(Y) = 1/\lambda^2$.

a) The LMS estimate (conditional expectation) of Θ is

1/2 ✔ **Answer:** 0.5

b) The conditional mean squared error $\mathbf{E}\left[\left(\Theta - \widehat{\Theta}_{\mathrm{LMS}}\right)^2 \,|\, X = x\right]$ is

1/4 ✔ **Answer:** 0.25

c) The MAP estimate of Θ is

0 ✔ **Answer:** 0

d) The conditional mean squared error $\mathbf{E}\left[\left(\Theta - \widehat{\Theta}_{\mathrm{MAP}}\right)^2 \,|\, X = x\right]$ is

1/2 ✔ **Answer:** 0.5

Solution:

a) The posterior PDF is exponential with parameter 2. The LMS estimate is the mean of this distribution, which is 1/2.

b) Since $\widehat{\Theta}_{\text{LMS}}$ is the conditional mean, the mean squared error is the conditional variance, that is, the variance of an exponential random variable with parameter 2, and is equal to 1/4.

c) The posterior PDF, which is exponential, is largest at zero.

d) Since $\widehat{\Theta} = 0$, the conditional mean squared error is the second moment of the exponential distribution (that is, of the form $\mathbf{E}\left[Y^2\right]$, where Y is exponential with parameter 2). Using the formula $\mathbf{E}\left[Y^2\right] = \mathsf{Var}\left(Y\right) + \left(\mathbf{E}\left[Y\right]\right)^2$, we obtain

$$\mathbf{E}\left[Y^2\right] = \frac{1}{4} + \left(\frac{1}{2}\right)^2 = \frac{1}{2}.$$

Note that the LMS estimator results in a smaller mean squared error.

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Answers are displayed within the problem

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? About a)
Why is the LMS estimate the mean of this distribution? LMS is least mean square, how does directly usin... 2

? Part d
Why is the conditional mean squared error the "second moment of the exponential distribution", and w... 2 new

Why use MAP at all if LMS performs better?
As far as I understand, the LMS estimator always minimises the MSE, so why would I every use the MAP 12

help on a
Sorry, did he actually explain how to d a) (LMS). I saw formulas, but nothing corresponding to what we h... 3

Part (c) MAP estimate
From lecture, it says the MAP estimate of theta is the value of theta which maximizes the value of the Co... 7

Confused about given f
Isn't f (given in the second sentence of the problem statement) the PMF of possible thetas? So shouldn't ... 2

Confusion over conditional PDF 2

No integration is needed for all the questions. 5

Need some help on b
I think that I can leverage E[Y]=1/λ, but I need some help on understanding how to determine E[(Θ−Θ_L... 5

How to calculate a) hint? 3