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## 4. Exercise: Estimator properties

Exercises due May 1, 2020 05:29 IST Completed

Exercise: Estimator properties

3/4 points (graded)

We estimate the unknown mean  $\theta$  of a random variable X (where X has a finite and positive variance) by forming the sample mean  $M_n=(X_1+\cdots+X_n)/n$  of n i.i.d. samples  $X_i$  and then forming the estimator

$$\widehat{\Theta}=M_n+rac{1}{n}.$$

Is this estimator unbiased?

No	~	✓ Answer: No
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Is this estimator consistent?

Yes 🕶	✓ Answer: Yes
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Consider now a different estimator,  $\widehat{\Theta}_n=X_1$  , which ignores all but the first measurement.

Is this estimator unbiased?

Yes 🕶	<b>✓ Answer:</b> Yes
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Is this estimator consistent?





## **Solution:**

We have  $\mathbf{E}\left[\widehat{\Theta}_n\right]=\theta+(1/n)\neq\theta$ , so it is not unbiased. On the other hand,  $M_n$  converges (in probability) to  $\theta$ , and 1/n converges to zero. So, their sum,  $\widehat{\Theta}_n=M_n+(1/n)$  also converges (in probability) to  $\theta$ , and the estimator is consistent.

The second estimator is unbiased, because  $\mathbf{E}\left[\widehat{\Theta}_{n}\right] = \mathbf{E}\left[X_{1}\right] = \theta$ . But it is not consistent. Its value stays the same (equal to  $X_{1}$ ) for all n and therefore cannot converge to  $\theta$ , unless  $X_{1}$  is guaranteed to be equal to  $\theta$ . But this is impossible since X has positive variance.

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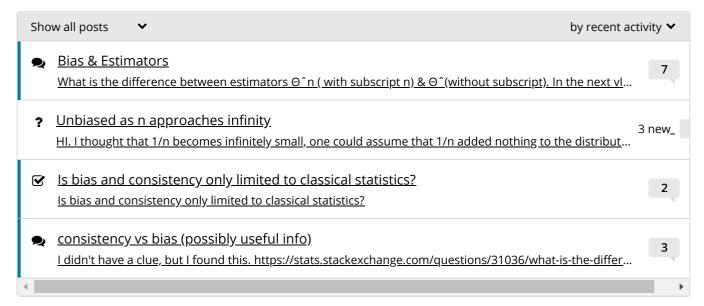
You have used 1 of 1 attempt

**1** Answers are displayed within the problem

## Discussion

**Hide Discussion** 

**Topic:** Unit 8: Limit theorems and classical statistics:Lec. 20: An introduction to classical statistics / 4. Exercise: Estimator properties



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