4.

Mid Term due Mar 4, 2020 05:29 IST *Completed*

Conditional Independence 1

2/4 points (graded)

Suppose that we have a box that contains two coins:

1. A fair coin: $\mathbf{P}(H) = \mathbf{P}(T) = 0.5$.
2. A two-headed coin: $\mathbf{P}(H) = 1$.

A coin is chosen at random from the box, i.e. either coin is chosen with probability $1/2$, and tossed twice. Conditioned on the identity of the coin, the two tosses are independent.

Define the following events:

- Event A: first coin toss is H.
- Event B: second coin toss is H.
- Event C: two coin tosses result in HH.
- Event D: the fair coin is chosen.

For the following statements, decide whether they are true or false.

1. A and B are independent.

(●) True

( ) False ✔

✖

2. A and C are independent.

- ○ True
- ◉ False ✔

✔

3. A and B are independent given D.

- ◉ True ✔
- ○ False

✔

4. A and C are independent given D.

- ◉ True
- ○ False ✔

✖

Solution:

1. False. Since we do not know whether it is a fair coin or the two-headed one when the coin is being tossed, getting a Heads during one toss increases our belief the the coin is the two-headed one, so that also increases our belief that the other toss also results in a Heads. Or we can also verify by definition:
$\mathbf{P}(A \cap B) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot 1 = \frac{5}{8} \neq \frac{9}{16} = \frac{3}{4} \cdot \frac{3}{4} = \mathbf{P}(A)\mathbf{P}(B)$.

2. False. $\mathbf{P}(A \cap C) = \mathbf{P}(C) \neq \mathbf{P}(A)\mathbf{P}(C)$.

3. True. Conditioned on D, A and B becomes the outcome of two independent fair coin tosses.

4. False. $\mathbf{P}(A \cap C|D) = \mathbf{P}(C|D) \neq \mathbf{P}(A|D)\mathbf{P}(C|D)$.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Conditional Independence 2

1/2 points (graded)

1. Suppose three random variables X, Y, Z have a joint distribution

$$\mathbf{P}_{X,Y,Z}(x,y,z) = \mathbf{P}_X(x)\,\mathbf{P}_{Z|X}(z \mid x)\,\mathbf{P}_{Y|Z}(y \mid z).$$

Then, X and Y are independent given Z.

- (●) True
- ( ) False

✔

2. Suppose random variables X and Y are independent given Z, then the joint distribution must be of the form

$$\mathbf{P}_{X,Y,Z}(x,y,z) = h(x,z)\,g(y,z),$$

where h, g are some functions.

- ( ) True ✔
- (●) False

✖

Solution:

1. True. Using $\mathbf{P}_{X,Y,Z}(x,y,z) = \mathbf{P}_X(x)\,\mathbf{P}_{Z|X}(z|x)\,\mathbf{P}_{Y|Z}(y|z)$, we have

$$\begin{aligned}\mathbf{P}_{X,Y|Z}(x,y|z) &= \frac{\mathbf{P}_{X,Y,Z}(x,y,z)}{\mathbf{P}_Z(z)} \\ &= \frac{\mathbf{P}_X(x)\,\mathbf{P}_{Z|X}(z|x)\,\mathbf{P}_{Y|Z}(y|z)}{\mathbf{P}_Z(z)} \\ &= \frac{\mathbf{P}_X(x)\,\mathbf{P}_{Z|X}(z|x)}{\mathbf{P}_Z(z)}\mathbf{P}_{Y|Z}(y|z) \\ &= \mathbf{P}_{X|Z}(x|z)\,\mathbf{P}_{Y|Z}(y|z),\end{aligned}$$

which shows X and Y are conditionally independent given Z.

2. True. Since X and Y are conditionally independent given Z, we have

$$\begin{aligned}\mathbf{P}_{X,Y,Z}(x,y,z) &= \mathbf{P}_Z(z)\,\mathbf{P}_{X,Y|Z}(x,y|z) \\ &= \mathbf{P}_Z(z)\,\mathbf{P}_{X|Z}(x|z)\,\mathbf{P}_{Y|Z}(y|z) \\ &= h(x,z)\,g(y,z),\end{aligned}$$

by letting $h(x,z) := \mathbf{P}_Z(z)\,\mathbf{P}_{X|Z}(x|z)$, $g(y,z) := \mathbf{P}_{Y|Z}(y|z)$. (In fact, by generalizing the argument for the part 1, we can show X and Y are conditionally independent given Z if and only if $\mathbf{P}_{X,Y,Z}(x,y,z) = h(x,z)\,g(y,z)$ for some h, g.)

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

Hide Discussion

Topic: Exam 1:Exam 1 / 4.

Show all posts by recent activity

- Automatic grading correct for Conditional Independence 2 question 2? 7
  I answered True, which is consistent with the solution. However this was marked incorrect. Can you p...
- Incorrectly graded? 16
- Q4.2.2 is False, and is graded correctly

Q4.2.2 is False, and is graded correctly. 12

Midterm 4.2.2 40

Staff

? [STAFF] Incorrect Gradding Pls Fix my mark 5

Hello Staff, My answer for Question 2 is correct as the answer shown. However, it is marked as incorr...

? "Grader error" resolution? 5

When is this issue of "grader error" going to be resolved?

STAFF Bug in grader for Q4.2.2 45

While I had many of the same issues in determining the answer as others- I finally had decided on 'Tr...

? Conditional Independence 2 Q2: Explanation shows True but I did not get graded even after answer true. 6

TA , Could you please check

? STAFF > MUST question 2.2, about conditional independence of P_XYZ(x,y,z) = h(x,z) g(y,z), be excluded from the exam? 25

The last question, #2.2 about conditional independence, has levels of ambiguity which significantly u...

? Conditional Independence 2 2

I've marked the last question - conditional independence question 2 as True, and it's marked incorrec...

Question 422 is an Unprocessable Entity 1

Question 4.2.2 is an [Unprocessable Entity][1]. I got this question right, but, admittedly, both true and...

? Unclear on interpretation of last question 3

I understand that this was discussed previously, but I am submitting this as a new post as I still wasn't...