13. Exercise: Bayes' rule and the false-positive puzzle

Exercises due Feb 12, 2020 05:29 IST *Completed*

Exercise: Bayes' rule and the false-positive puzzle

4.0/4.0 points (graded)

A test for a certain rare disease is assumed to be correct 95% of the time: if a person has the disease, the test result is positive with probability 0.95, and if the person does not have the disease, the test result is negative with probability 0.95. A person drawn at random from a certain population has probability 0.001 of having the disease.

1. Find the probability that a random person tests positive. (This answer will require an accuracy of 4 decimal places.)

   0.0509 ✔ **Answer:** 0.0509

2. Given that the person just tested positive, what is the probability he actually has the disease?

   0.0186 ✔ **Answer:** 0.01866

Solution:

Let A be the event that the person has the disease, and B the event that the test result is positive.

1. The desired probability is

$$\mathbf{P}(B) = \mathbf{P}(A)\,\mathbf{P}(B \mid A) + \mathbf{P}(A^c)\,\mathbf{P}(B \mid A^c) = 0.001 \cdot 0.95 + 0.999 \cdot 0.05 = 0.0509.$$

2. The desired probability is

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B \mid A)}{\mathbf{P}(B)} = \frac{0.001 \cdot 0.95}{0.0509} \approx 0.01866.$$

Note that even though the test was assumed to be fairly accurate, a person who has tested positive is still very unlikely (probability less than 2%) to have the disease. The explanation is that when testing 1000 people, we expect about 1 person to have the disease (and most likely test positive), but also expect about $1000 \cdot 0.999 \cdot 0.05 \approx 50$ people to test positive without having the disease. Hence, when we see a positive test, it is about 50 times more likely to correspond to one of the 50 false positives.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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Is there a way to visulize this?		4
What does it mean that the probability of being sick given that the test tells you that is low?	I don't know how to interpret this, I mean, the test has very high accuracy, and despite this fact, the probability of ...	2 new
What am I doing wrong?	This problem is driving me crazy because it seems so simple but I can't get the right answer. I drew a tree diagram ...	8 new
Is the answer correct on 1.?	The solution says "Let A be the event that the person has the disease, and B the event that the test result is positiv...	2
Believe in your capacity to find the result !	I was almost giving up when I told myself, come on, nothing is blocking you from succeeding except yourself ! So I ...	2
Tip [Bayes]	In resources I saw this link, it's an example very helpful to solve this problem. http://vassarstats.net/bayes.html#co...	5
Breaking this question down to pieces		1
communication channel analogy	1. initial message - probability of disease in the population 2. received message - test results 3. communication cha...	2
visual animation for Bayes' Theorem	I was having a little trouble with this concept and found this at Brown University. https://seeing-theory.brown.edu/...	3
Thanks to those who asked about decimal places	I would have assumed four decimal places (for consistency), but it was nice to have you guys tread the way.	1
why the first value given		

A test for a certain rare disease is assumed to be correct 95% of the time - why is this given 1

Got it right upto 4 decimals by only using the 'Radar intuition' (didn't even have to look up Bayes Rule!) 1

<-----Spoiler alert: hints here, but no answers-----> Honestly! I just drew out the tree - has disease, no disease, tests ...

Doesn't state precision on 2nd question, so I got it wrong 2

I put the second question as a fraction(as it didn't state precision) and got it wrong :(Don't make the same mistak