6. Bayes' rule

Problem Set due Mar 13, 2020 05:29 IST *Completed*

Problem 6. Bayes' rule

1/1 point (graded)

Let K be a discrete random variable with PMF

$$p_K(k) = \begin{cases} 1/4, & \text{if } k = 1, \\ 1/2, & \text{if } k = 2, \\ 1/4, & \text{if } k = 3, \\ 0 & \text{otherwise.} \end{cases}$$

Conditional on $K = 1, 2$, or 3, random variable Y is exponentially distributed with parameter 1, $1/2$, or $1/3$, respectively.

Using Bayes' rule, find the conditional PMF $p_{K|Y}(k \mid y)$. Which of the following is the correct expression for $p_{K|Y}(2|y)$, when $y \geq 0$?

- $\dfrac{e^{-y/2}}{e^{-y} + e^{-y/2} + \frac{1}{3}e^{-y/3}}$
- $\dfrac{e^{-y}}{\frac{1}{3}e^{-y} + e^{-y/2} + \frac{1}{3}e^{-y/3}}$
- $\dfrac{e^{-y/2}}{\frac{1}{3}e^{-y} + \frac{1}{3}e^{-y/2} + \frac{1}{3}e^{-y/3}}$

$$\frac{e^{-y/3}}{e^{-y}+e^{-y/2}+\frac{1}{3}e^{-y/3}}$$

✔

Solution:

Applying Bayes' rule, we have

$$p_{K|Y}(k \mid y) = \frac{p_K(k)\, f_{Y|K}(y \mid k)}{f_Y(y)}.$$

By the total probability theorem,

$$\begin{aligned} f_Y(y) &= \sum_k p_K(k)\, f_{Y|K}(y \mid k) \\ &= p_K(1)\, f_{Y|K}(y \mid 1) + p_K(2)\, f_{Y|K}(y \mid 2) + p_K(3)\, f_{Y|K}(y \mid 3) \\ &= \frac{1}{4}e^{-y} + \frac{1}{2}\cdot\frac{1}{2}e^{-y/2} + \frac{1}{4}\cdot\frac{1}{3}e^{-y/3} \\ &= \frac{1}{4}e^{-y} + \frac{1}{4}e^{-y/2} + \frac{1}{12}e^{-y/3}. \end{aligned}$$

Hence, for $k = 2$, we have,

$$\begin{aligned} p_{K|Y}(2 \mid y) &= \frac{p_K(2)\, f_{Y|K}(y \mid 2)}{f_Y(y)} \\ &= \frac{\frac{1}{4}e^{-y/2}}{\frac{1}{4}e^{-y} + \frac{1}{4}e^{-y/2} + \frac{1}{12}e^{-y/3}} \\ &= \frac{e^{-y/2}}{e^{-y}+e^{-y/2}+\frac{1}{3}e^{-y/3}}. \end{aligned}$$

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You have used 1 of 2 attempts

Answers are displayed within the problem

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? Coefficient on numerator and denominator 3

Shouldnt there be an coefficient of 1/4 on the numerator and the 1/2 second term of the denominator? ...

Feeling stupid 3

I got the right answer, but I don't quite know how. I think I got the numerator, but then the denominator ...