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## 11. Exercise: Total probability theorem

Exercises due Feb 12, 2020 05:29 IST Completed

Exercise: Total probability theorem

2.0/2.0 points (graded)

We have an infinite collection of biased coins, indexed by the positive integers. Coin i has probability  $2^{-i}$  of being selected. A flip of coin i results in Heads with probability  $3^{-i}$ . We select a coin and flip it. What is the probability that the result is Heads? The geometric sum

formula may be useful here:  $\sum_{i=1}^{\infty} \alpha^i = rac{lpha}{1-lpha}$  , when |lpha| < 1 .

The probability that the result is Heads is: 1/5

**Answer:** 0.2

## **Solution:**

We think of the selection of coin i as scenario/event  $A_i$ . By the total probability theorem, for the case of infinitely many scenarios,

$$\mathbf{P}\left(\text{Heads}\right) = \sum_{i=1}^{\infty} \mathbf{P}\left(A_{i}\right) \mathbf{P}\left(\text{Heads}|A_{i}\right) = \sum_{i=1}^{\infty} 2^{-i} 3^{-i} = \sum_{i=1}^{\infty} \left(1/6\right)^{i} = \frac{1/6}{1 - (1/6)} = \frac{1}{5}.$$

Submit

You have used 1 of 3 attempts

**1** Answers are displayed within the problem

Discussion

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2	Fractions  I know this is a very basic math question and probably stupid, but I graduated back in 2000where can	4
2	<u>a Hint</u>	7
?	Anyone can give some hints on the solution?  Appreciate if anyone could give some hints on the solution.	12
?	Struggling with the wording of the question?  I'm reading this very literally which is throwing me off. It asks us that if we select a coin out of an infinite	5
Ą	<u>Checking your answer</u> <u>One tip: I found I was able to check my answer using MS Excel by constructing the infinite sum using the</u>	1
2	Rats, I threw out 2 off the cuff answers without thinking too much. Stupid. Now I have only one real thoughtful response left.:-( Just wanted to s	4
?	Probability of the complements? or the other biased coins?  HI. This makes me think we need to be able to consider what happens when coin i is not selected. Shoul	2
Q	Approach Feedback  OK. Here is the logic I am using. Would appreciate any feedback. The probability of selecting the biased c	7
2	A Major Hint  Let us consider coin 3 for making things easier. $P(C3) = 1/(2^3) P(H C3) = 1/(3^3)$ so they are asking $P(H)$	2
Q	If coin count = 1, then its selection = 1/2?  I'm a little bothered by the idea of a universe where the selection of a coin from just one option is 50 / 50.	3
Q	<u>Use excel to understand the problem</u> <u>I used excel to lay out the problem. My first column was an index, the second for the probability of selec</u>	4
?	What about the unbiased coin?  Hi, if the Chance of selecting a biased coin is 1/2 then the selection of an unbiased coin should be 1/2 as	3
?	interpretation  The probability of a flip of a coin ( C i ) resulting in Head(H), does it mean P(C i n H) or P(H/C i).	3

