1. Steady-state convergence

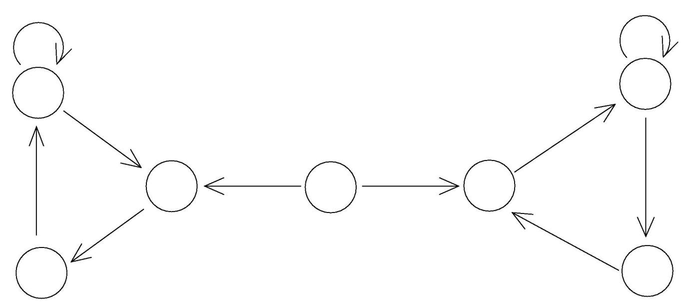
Problem Set due May 29, 2020 05:29 IST

Problem 1. Steady-state convergence

6 points possible (ungraded)

Let $X_0, X_1, \ldots$ be a Markov chain, and let $r_{ij}(n) \equiv \mathbf{P}(X_n = j \mid X_0 = i)$.

1. Consider the Markov chain represented below. The circles represent distinct states, while the arrows correspond to positive (one-step) transition probabilities.



For this Markov chain, determine whether each of the following statements is true or false.

(a) For every i and j, the sequence $r_{ij}(n)$ converges, as $n \to \infty$, to a limiting value π_j, which does not depend on i.

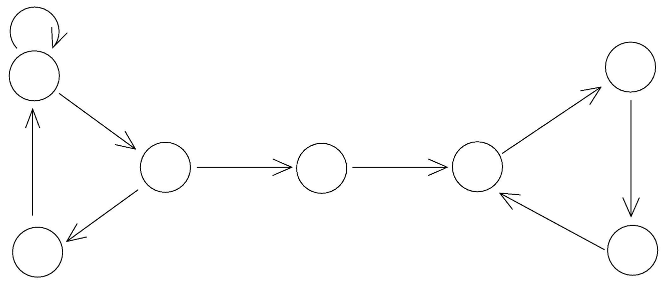
Select an option **Answer:** False

(b) Statement (a) is true, and $\pi_j > 0$ for every state j.

Select an option **Answer:** False

2.

Consider the Markov chain represented below. The circles represent distinct states, while the arrows correspond to positive (one-step) transition probabilities.



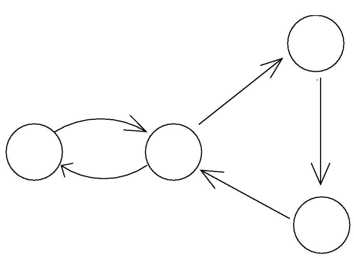
(a) For every i and j, the sequence $r_{ij}(n)$ converges, as $n \to \infty$, to a limiting value π_j, which does not depend on i.

Select an option **Answer:** False

(b) Statement (a) is true, and $\pi_j > 0$ for every state j.

Select an option **Answer:** False

3. Consider the Markov chain represented below. The circles represent distinct states, while the arrows correspond to positive (one-step) transition probabilities.



(a) For every i and j, the sequence $r_{ij}(n)$ converges, as $n \to \infty$, to a limiting value π_j, which does not depend on i.

Select an option **Answer:** True

(b) Statement (a) is true, and $\pi_j > 0$ for every state j.

Select an option **Answer:** True

Solution:

1. There are two recurrent classes, both aperiodic. The probability of ending up in each depends on the initial state, and hence π_j's do not exist.
   (a) False
   (b) False

2. The right-most three states are the only recurrent states and form a recurrent class. However, the recurrent class is periodic with period 3 and thus the occupancy probabilities in that class do not converge. The occupancy probabilities in the left-most four states converge to zero.
   (a) False
   (b) False

3. The chain consists of one recurrent class and no transient states. One way to see that the recurrent class is aperiodic is to note that from the center state one can reach every state of the chain in exactly four steps.
   (a) True
   (b) True

Submit You have used 0 of 3 attempts

Answers are displayed within the problem

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Topic: Unit 10: Markov chains:Problem Set 10 / 1. Steady-state convergence

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Notes and Hints
The first Markov chain has two recurrent classes. The second Markov chain only has one recurrent class ...

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