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## 1. Marie gives away children toys

Problem Set due May 13, 2020 05:29 IST Completed

Problem 1. Marie gives away children toys

7/7 points (graded)

Marie distributes toys for toddlers. She makes visits to households and gives away one toy

probability of the door being answeresidence is $1/3.$ Assume that the $\epsilon$	ered is $3/4$ , and the probability that there is a toddler in events "Door answered" and "Toddler in residence" are elated to different households are independent.
1. What is the probability that so	she has not distributed any toys by the end of her second
9/16	<b>✓ Answer:</b> 9/16
2. What is the probability that she gives away the first toy on her fourth visit?	
27/256	<b>✓ Answer:</b> 27/256
G	ay her second toy on her fifth visit, what is the conditional away her third toy on her eighth visit?
9/64	<b>✓ Answer:</b> 9/64

4. What is the probability that she will give away the second toy on her fourth visit?

**✓ Answer:** 27/256 27/256

Given that she has not given away her second toy by her third visit, what is the conditional probability that she will give away her second toy on her fifth visit?

1/8 **Answer:** 1/8

6. We will say that Marie "needs a new supply"" immediately **after** the visit on which she gives away her last toy. If she starts out with three toys, what is the probability that she completes at least five visits before she needs a new supply?

997/1024 **Answer:** 243/256

7. If she starts out with exactly six toys, what is the expected value of the number of houses with toddlers that Marie visits without leaving any toys (because the door was not answered) before she needs a new supply?

24/12 **✓ Answer:** 2

## Solution:

A successful (i.e., the door is answered, and a toddler is present in the residence) visit ("trial//) occurs with probability  $p=\frac{3}{4}\cdot\frac{1}{3}=\frac{1}{4}$ .

1. This is the probability that the first two trials were failures, which happens with probability

$$(1-p)(1-p)=rac{3}{4}\cdotrac{3}{4}=rac{9}{16}.$$

2. She gives away her first toy on her fourth visit if and only if the first three trials are failures, and the last (fourth) trial is a success. This happens with probability

$$(1-p)(1-p)(1-p)p = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{27}{256}.$$

3.



The given event of interest happens if and only if the sixth and seventh trials are failures, and the eighth trial is a success. Using the fresh start property of Bernoulli process, this happens with probability

$$(1-p)\,(1-p)\,p=rac{3}{4}\cdotrac{3}{4}\cdotrac{1}{4}=rac{9}{64}.$$

4. We are interested in the probability that the second success time,  $Y_2$  is equal to 4, namely,  ${f P}\,(Y_2=4)$ . Here,  $Y_2$  has a Pascal PMF and

$$\mathbf{P}\left(Y_2=4
ight)=inom{4-1}{2-1}p^2(1-p)^{4-2}=3\cdot\left(rac{1}{4}
ight)^2\cdot\left(rac{3}{4}
ight)^2=rac{27}{256}.$$

5. We are given the event that  $\{Y_2>3\}$  and are asked to find the conditional probability of the event  $\{Y_2=5\}$ . Note that the possible values of  $Y_2$  are  $2,3,\ldots$  and therefore,

$$\mathbf{P}(Y_2 > 3) = 1 - \mathbf{P}(Y_2 = 2) - \mathbf{P}(Y_2 = 3).$$

We then have,

$$egin{aligned} \mathbf{P}\left(Y_2=5\mid Y_2>3
ight) &= rac{\mathbf{P}\left(Y_2=5\cap Y_2>3
ight)}{\mathbf{P}\left(Y_2>3
ight)} \ &= rac{\mathbf{P}\left(Y_2=5
ight)}{\mathbf{P}\left(Y_2>3
ight)} \ &= rac{inom{5-1}{2-1}inom{1}{4}inom{5-2}{4}inom{5-2}{1-p_{Y_2}\left(2
ight)-p_{Y_2}\left(3
ight)} \ &= rac{rac{27}{256}}{1-rac{1}{4}\cdotrac{1}{4}-2\cdotrac{3}{4}\cdotrac{1}{4}\cdotrac{1}{4}} \ &= rac{rac{27}{256}}{rac{27}{22}} = rac{1}{8}, \end{aligned}$$



This is the probability that the time  $Y_3$  of the third success is greater than or equal to 5:

$$egin{align} \mathbf{P}\left(Y_3 \geq 5
ight) &= 1 - \mathbf{P}\left(Y_3 \leq 4
ight) = 1 - \sum_{\ell=3}^4 inom{\ell-1}{3-1} inom{1}{4}^3 inom{3}{4}^{\ell-3} \ &= 1 - inom{3}{2} rac{1}{64} \cdot rac{3}{4} - inom{2}{2} rac{1}{64} = rac{243}{256}. \end{split}$$

7. In this part, we are only considering the visits to houses with toddlers. At each such visit, either (i) the door is answered ("success"), which happens with probability 3/4, and a toy is given, or (ii) the door is not answered ("failure") which happens with probability 1/4. We wish to determine the expected number of failures until the 6th success. The expected number of trials up to and including the 6th success is  $6/\left(3/4\right)=8$ . The number of failures is the number of such visits minus the number of successes, namely, 6. Therefore, the expected number of failures is 8-6=2.

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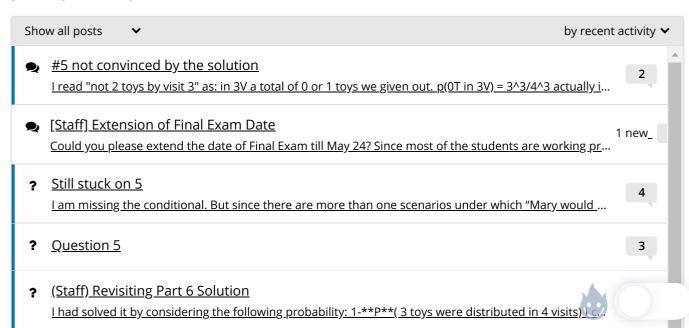
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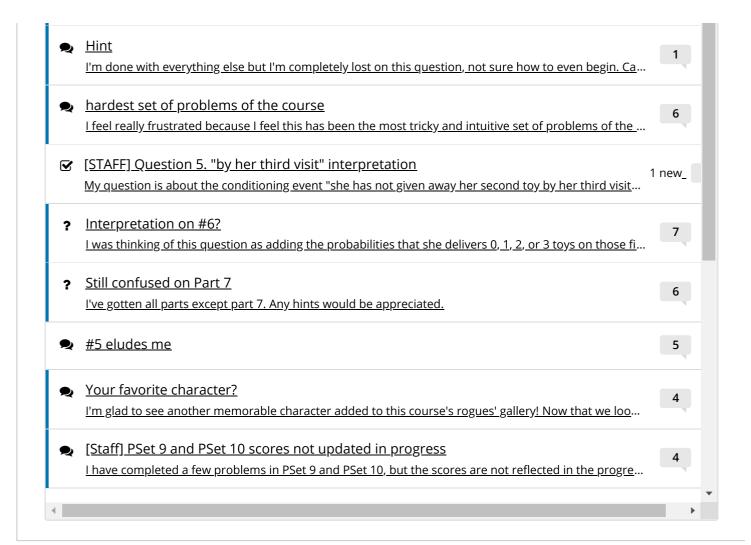
**1** Answers are displayed within the problem

## Discussion

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