15. Exercise: The hat problem

Exercises due Feb 28, 2020 05:29 IST *Completed*

Exercise: The hat problem

2.0/2.0 points (graded)

Consider the hat problem, with $n = 10$. What is the expected value of $X_3 X_6 X_7$?

$$\mathbf{E}\left[X_3 X_6 X_7\right] = \boxed{\text{1/720}} \qquad \checkmark \text{ Answer: 0.00139}$$

Solution:

By symmetry, this is the same as $\mathbf{E}\left[X_1 X_2 X_3\right]$. Since the product $X_1 X_2 X_3$ is either zero or one, this is the same as

$$\mathbf{P}\left(X_1 X_2 X_3 = 1\right) = \mathbf{P}\left(X_1 = 1\right) \cdot \mathbf{P}\left(X_2 = 1 \mid X_1 = 1\right) \cdot \mathbf{P}\left(X_3 = 1 \mid X_1 = X_2 = 1\right).$$

By thinking in terms of the sequential description of the process, we have seen that $\mathbf{P}\left(X_1 = 1\right) = 1/10$ and $\mathbf{P}\left(X_2 = 1 \mid X_1 = 1\right) = 1/9$. By a similar argument, given that the first two people obtained their own hats, the third person is faced with 8 hats, one of which is his/her own, and has probability $\mathbf{P}\left(X_3 = 1 \mid X_1 = X_2 = 1\right) = 1/8$ of picking it. Thus, the final answer is $(1/10) \cdot (1/9) \cdot (1/8)$.

Submit   You have used 2 of 3 attempts

ℹ Answers are displayed within the problem

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☑ **How symmetry applies here?**

    👤 Community TA

14 new **26**

💬 **Another approach to this problem**

There are a total of n! possibilities. Out of those n! possibilities, in how many of those does persons 3, 6 and ...

3

☑ **Why don't we take into account other peoples' actions?**

2

💬 **Solution to this problem is not complete to my opinion**

I solved this problem rather intuitively and struggled to understand the solution. I think it should state some...

2 new

? **Hint on how to do this problem?**

I've watched the previous lecture several times. But am struggling. Is the question, what is the expected valu...

3

? **Clarification for answer grading**

Hi I have got the solution and correct till 4 decimal places. The answer showed 5th decimal place and graded...

7

💬 **Wrong answer displayed**

Answer is giving Probability directly of hats whereas questions asks Expectation .Is Probability is same as exp...

4

? **Guidance please...**

Is this problem asking for the expected value of the product of the three rv's, X3, X6, X7 ? If so, what is the m...

5 new

? **I'm of two minds here....**

If we are assuming that X_i are independent (as was discussed in lecture), can this not be a question of count...

3

💬 **Independence of RVs**

As stated in the lecture, these RVs are NOT independent right? I.e. X3, X6 and X7 are NOT independent?

2