4

Mid Term due Apr 22, 2020 05:29 IST   *Completed*

Problem 4(a)

2/2 points (graded)

A random variable X is generated as follows. We flip a coin. With probability p, the result is Heads, and then X is generated according to a PDF $f_{X|H}$ which is uniform on $[0, 1]$. With probability $1 - p$ the result is Tails, and then X is generated according to a PDF $f_{X|T}$ of the form

$$f_{X|T}(x) = 2x, \qquad \text{if } x \in [0, 1].$$

(The PDF is zero everywhere else.)

1. What is the (unconditional) PDF $f_X(x)$ of X?

   For $0 \le x \le 1$:

   $f_X(x) =$

   | (2*x)+(p)-(2*p*x) | ✔

   Answer: p+2*x*(1-p)

   $$(2 \cdot x) + (p) - (2 \cdot p \cdot x)$$

2. Calculate $\mathbf{E}[X]$.

$$\mathbf{E}[X] =$$

(4-p)/6   ✔ **Answer:** 2/3-p/6

$\dfrac{4-p}{6}$

Solution:

1. By the analog of the total probability theorem for PDFs, we have for $x \in [0, 1]$:

$$f_X(x) = p \cdot 1 + (1 - p) \cdot (2x).$$

Thus,

$$f_X(x) = \begin{cases} p + 2x(1 - p), & 0 \le x \le 1, \\ 0, & \text{otherwise} \end{cases}$$

2. Using the total expectation theorem,

$$\begin{aligned}
\mathbf{E}[X] &= p\mathbf{E}[X|H] + (1 - p)\mathbf{E}[X|T] \\
&= \frac{1}{2}p + (1 - p)\int_0^1 x(2x)\,dx \\
&= \frac{1}{2}p + \frac{2}{3}(1 - p) \\
&= \frac{2}{3} - \frac{p}{6}.
\end{aligned}$$

Submit   You have used 3 of 3 attempts

ℹ Answers are displayed within the problem

Problem 4(b)

2/2 points (graded)
We now wish to estimate the result of the coin toss, based on the value of X.

1. Find $P\left(\text{Tails}|X = 1/4\right)$.

$P\left(\text{Tails}|X = 1/4\right) =$

> (1-p)/(1+p)

✔ **Answer:** (1-p)/(1+p)

$$\frac{1-p}{1+p}$$

2. The MAP rule decides in favor of Heads if $X < a$ and in favor of Tails if $X > a$. What is a?

$a =$

> p/(2*(1-p))

✔ **Answer:** p/(2*(1-p))

$$\frac{p}{2\cdot(1-p)}$$

Solution:

1. Using the Bayes' rule,

$$
\begin{aligned}
P\left(\text{Tails}\,|\,X = 1/4\right) &= \frac{P\left(\text{Tails}\right) f_{X|T}\left(\frac{1}{4}\right)}{f_X\left(\frac{1}{4}\right)} \\
&= \frac{(1-p)\frac{1}{2}}{p + \frac{1}{2}(1-p)} \\
&= \frac{1-p}{1+p}.
\end{aligned}
$$

2. For an observation $X = x$ for some $x \in [0,1]$, we judge in favor of Heads when

$$\frac{p f_{X|H}\left(x\right)}{f_X\left(x\right)} > \frac{(1-p) f_{X|T}\left(x\right)}{f_X\left(x\right)};$$

or, ignoring the denominator term, when $p > (1-p)\,2x$. Equivalently, when

$$x < \frac{p}{2(1-p)}.$$

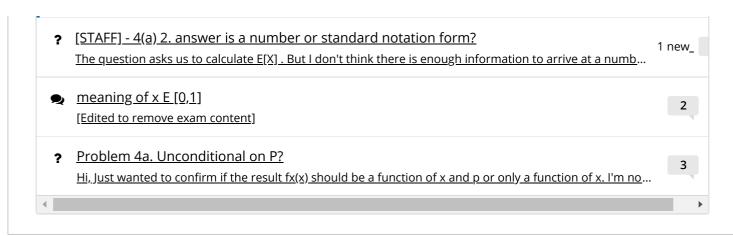Submit   You have used 2 of 3 attempts

ℹ Answers are displayed within the problem

Error and Bug Reports/Technical Issues

Topic: Exam 2:Exam 2 / 4

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💬 **STAFF: Q4b 2 Answer is right**   2
Hi Staff, can you please review Q4B #2 for me? I've done this problem flawlessly and my answer to the q...

💬 **Thought 4(b) was looking for a definitive estimate, not formula?**   4
I initially came up with the correct answers for both parts of 4.b, but then disregarded them as understo...

💬 **My answers for Q4 (Midterm 2)**   14

💬 **[STAFF]points for problem 3**   1
is Problem 3 of 2.5 points OR 3 if 2.5 then total points of all problems is 17.5

? **How come the exams, even the online ones are turning my dumb mode on? - -** 1
got 3/4 wrong there just because i was looking at probability 2x and calculating for probability x^2

💬 **[STAFF] Bug Question 4(a) Submited and not graded**   1
Hi, I saw that other users had the same problem. I submited the question, but it was not graded. I know I...

💬 **[STAFF] Missed to submit answers of 4(a). They are saved though. Please help submit.** 2
[STAFF] Missed to submit answers of 4(a). They are saved though. Please help submit.

? **[Staff] Problem 4(b) shows as not submitted yet I have the right answer saved** 1
So I don't know whose mistake this was and I have no way to prove I pressed the submit button. But I di...

💬 **[Staff]Time Limit**   2
Finishing this in 48 hrs is challenging considering that most people who are doing this course are also w...

? **[STAFF] BUG PROBLEM 4A.2**
Hello Staff, I had another answer and submitted but when I clicked on submit the answer got changed a...

? [STAFF] - 4(a) 2. answer is a number or standard notation form?

The question asks us to calculate E[X] . But I don't think there is enough information to arrive at a numb...

1 new

💬 meaning of x E [0,1]

[Edited to remove exam content]

2

? Problem 4a. Unconditional on P?

Hi, Just wanted to confirm if the result fx(x) should be a function of x and p or only a function of x. I'm no...

3