6. True or False

Problem Set due Feb 28, 2020 05:29 IST *Completed*

Problem 6. True or False

2/8 points (graded)

For each of the following statements, state whether it is true (meaning, always true) or false (meaning, not always true):

1. Let X and Y be two binomial random variables.

   (a) If X and Y are independent, then $X+Y$ is also a binomial random variable.

   True ✖ **Answer:** False

   (b) If X and Y have the same parameters, n and p, then $X+Y$ is a binomial random variable.

   True ✖ **Answer:** False

   (c) If X and Y have the same parameter p, and are independent, then $X+Y$ is a binomial random variable.

   True ✔ **Answer:** True

2. Suppose that, $\mathbf{E}[X]=0$. Then, $X=0$.

   True ✖ **Answer:** False

3. Suppose that, $\mathbf{E}[X^2]=0$. Then, $\mathbf{P}(X=0)=1$.

   False ✖ **Answer:** True

4.

Let X be a random variable. Then, $\mathbf{E}[X^2] \geq \mathbf{E}[X]$.

True ✖ **Answer:** False

5. Suppose that, X is a random variable, taking positive integer values, which satisfies $\mathbf{E}[(X-6)^2] = 0$. Then, $p_X(4) = p_X(5)$.

False ✖ **Answer:** True

6. Suppose that $\mathbf{E}[X] \geq 0$. Then, $X \geq 0$ with probability 1, i.e., $\mathbf{P}(X \geq 0) = 1$.

False ✔ **Answer:** False

Solution:

1. (a) False. Intuitively, X corresponds to independent coin flips of a coin with a certain bias, and Y corresponds to independent coin flips of another coin, which need not have the same bias as the first coin. Throughout the overall sequence of coin flips, the bias is not kept constant, and so we are in a different situation from the one modeled by binomial random variables.

   For a concrete (and extreme) counter-example, suppose that X and Y are independent Bernoulli random variables, with parameters 0.9 and 0.1, respectively. In particular, they are both binomial with $n=1$. The sum $X+Y$ takes values in $\{0,1,2\}$. So, if it were binomial, it would need to have a parameter n equal to 2. The parameter p of such a binomial would have to satisfy $\mathbf{E}[X+Y] = 2p$. Since $\mathbf{E}[X+Y] = \mathbf{E}[X] + \mathbf{E}[Y] = 0.9 + 0.1 = 1$, we would require $p = 1/2$. This would then imply that $\mathbf{P}(X+Y=2) = p^2 = 1/4$. However, we can check that

   $$\mathbf{P}(X+Y=2) = \mathbf{P}(X=1) \cdot \mathbf{P}(Y=1) = 0.9 \cdot 0.1 \neq 1/4.$$

   The contradiction shows that $X+Y$ is not binomial.

   (b) False. If X and Y have the same parameters, n and p, $X+Y$ is not necessarily a binomial random variable. For example, if the random variables X and Y are dependent and $X=Y$, then the random variable $X+Y$ has zero probability at all odd values of n. Therefore, $X+Y$ is not binomial.

(c) True. We may interpret $X+Y$ as the number, X, of Heads in some independent tosses of a coin, plus the number, Y, of Heads in some additional independent tosses of the **same** coin. Therefore, $X+Y$ is binomial.

2. False. Consider a random variable with

$$p_X(x) = \begin{cases} 1/2, & \text{if } x = 1, \\ 1/2, & \text{if } x = -1. \end{cases}$$

We have $\mathbf{E}[X] = 0$, but X takes nonzero values.

3. True. Suppose that X satisfies $\mathbf{E}[X^2] = 0$ but $\mathbf{P}(X=0) \neq 1$. Then, $\mathbf{P}(X=w) > 0$ for some $w \neq 0$. It would follow that $\mathbf{E}[X^2] \geq w^2 \cdot \mathbf{P}(X=w) > 0$, which would contradict the assumption that $\mathbf{E}[X^2] = 0$.

4. False. Let X be deterministic and equal $1/2$. Then $\mathbf{E}[X^2] = 1/4$, while $\mathbf{E}[X] = 1/2 > \mathbf{E}[X^2]$.

5. True. Since, $\mathbf{E}[(X-6)^2] = 0$, and since $(X-6)^2 \geq 0$, we obtain that $(X-6)^2$ must be equal to 0, with probability 1, namely, $p_X(6) = 1$. Hence, $p_X(4) = 0 = p_X(5)$.

6. False. Suppose X is 1 or -1, with equal probability. Then $\mathbf{E}[X] = 0$, but $\mathbf{P}(X \geq 0) = 1/2 \neq 1$.

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You have used 1 of 1 attempt

Answers are displayed within the problem

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? Q5 Solution Doubt: How can (X-6)^2 = 0?

[In the solution of Q5, it is stated that since E[(X-6)^2] = 0, (X-6)^2 = 0. Failing to understand this. Can s...] 2

? [dont understand question 1]
[In the explanation of question 1a says "The parameter p of such a binomial would have to satisfy E[X...] 1

[Unable to post answers]
[I appear to be receiving errors due to quiz state - is the quiz closed? Why did it close at 7:00pm EST of ...] 3

[1.c. what's the intuition?]
[My intuition was that the sum of two binomial variables would not be binomial anymore. for instance...] 1 new_

[Question 4: Is X considered to be integer or real?]
[Is X considered to be integer or real?] 2

? [Not sure about 1b.]
[I thought equal rate (p) was enough to determine the status of their sum. Why would sample size mat...] 2

? [For Question 2-6 , Is X a binomial random variable?]
[Do we assume X as a random variable or binomial random variable?] 4

[Question 1. How can I check if a random variable is binomial?]
[What is enough to consider a random variable a binomial one?] 1 new_ 4

[Random Variable X in questions 2 and 3. Still binomial?]
[The statement that X is a binomial random variable falls under question 1, but it's unclear if it still app...] 2

[Explanation on Question 5]
[Can I get some hint on Question 5 on how to approach it?] 3

[Question 5 ?]
[I'm not sure what question 5 is asking for - is there a relation between the E [(X - 6)^2] being Zero and ...] 2

[Any help with the question 6?]
[We know from the lecture that if X is greater or equal to 0, then E[X] will be greater or equal to 0. And ...] 3

? [How do you figure out why you were wrong?] 2

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