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8. Exercise: n-step recursion

None due May 29, 2020 05:29 IST

Exercise: n-step recursion

3 points possible (ungraded)

Let $r_{ij}(n)=\mathbf{P}\left(X_n=j\mid X_0=i\right)$ be the n-step transition probability of a given homogeneous discrete-time Markov chain with m states. We have shown that $r_{ij}(n)$ satisfies the following recursion for $n\geq 2$: $r_{ij}(n)=\sum_{k=1}^m r_{ik}\left(n-1\right)p_{kj}$. For each of the following, decide whether it is also a valid recursion formula for $r_{ij}(n)$.

$$^{1.}$$
 $r_{ij}\left(n
ight)=\sum_{k=1}^{m}p_{ik}r_{kj}\left(n-1
ight)$ for $n\geq2$

Select an option **→ Answer:** Yes

$$^{2.}$$
 $r_{ij}\left(n
ight)=\sum_{k=1}^{m}r_{ik}\left(n-2
ight)r_{kj}\left(2
ight)$ for $n\geq3$

Select an option ➤ **Answer:** Yes

3.
$$r_{ij}\left(n
ight) = \sum_{k=1}^{m} \sum_{\ell=1}^{m} r_{ik}\left(n-2
ight) p_{k\ell} p_{\ell j}$$
 for $n \geq 3$

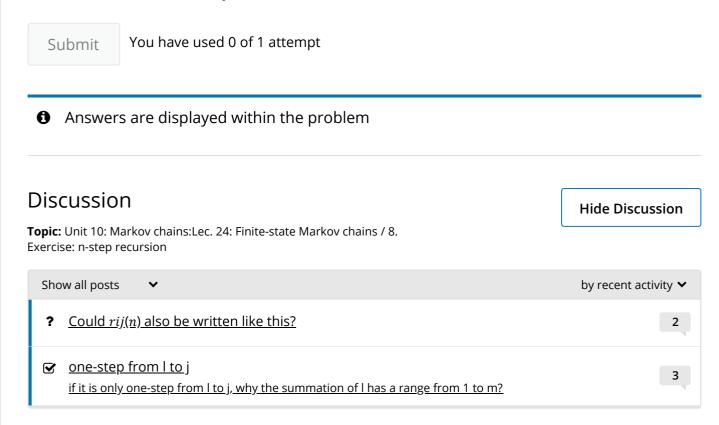
Select an option ➤ Answer: Yes

Solution:

1. Yes. The recursion considers a one-step transition from i to any state k, followed by an (n-1)-step transition from k to j.

Yes. The recursion considers an (n-2)-step transition from i to any state k, followed by a 2-step transition from k to j.

3. Yes. The recursion considers an (n-2)-step transition from i to any state k, followed by a one-step transition from k to any state ℓ , followed by a one-step transition from ℓ to j.



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