



## 5. Arrivals during overlapping time intervals

Problem Set due May 13, 2020 05:29 IST Past Due

### Problem 5. Arrivals during overlapping time intervals

3 points possible (graded)

Consider a Poisson process with rate  $\lambda$ . Let  $N$  be the number of arrivals in  $(0, t]$  and  $M$  be the number of arrivals in  $(0, t + s]$ , where  $t > 0, s \geq 0$ .

In each part below, your answers will be algebraic expressions in terms of  $\lambda, t, s, m$  and/or  $n$ . Enter "lambda" for  $\lambda$  and use "exp()" for exponentials. Do **not** use "fac()" or "!" for factorials. Follow standard notation.

1. For  $0 \leq n \leq m$ , the conditional PMF  $p_{M|N}(m | n)$  of  $M$  given  $N$  is of the form  $\frac{a}{b!}$  for suitable algebraic expressions in place of  $a$  and  $b$ .

$a =$

Answer:  $\text{lambda}^{(m-n)} * s^{(m-n)} * \text{exp}(-\text{lambda} * s)$

$b =$

Answer:  $m-n$

2.



$$E[NM] =$$

Answer:  $\lambda t \lambda s + \lambda t + (\lambda t)^2$

STANDARD NOTATION

### Solution:

1. To find  $P_{M|N}(m | n)$ , we assume there are  $n$  arrivals in the first  $t$  time units, and we are looking for the probability that there are  $m - n$  arrivals in the subsequent  $s$  time units. This follows a Poisson distribution with parameter  $\lambda s$ :

$$p_{M|N}(m | n) = \frac{(\lambda s)^{m-n} e^{-\lambda s}}{(m-n)!}, \quad \text{for } m \geq n \geq 0.$$

2. We can rewrite the expectation as

$$\begin{aligned} \mathbf{E}[NM] &= \mathbf{E}[N(M - N) + N^2] \\ &= \mathbf{E}[N] \mathbf{E}[M - N] + \mathbf{E}[N^2] \\ &= (\lambda t)(\lambda s) + \left( \text{var}(N) + (\mathbf{E}[N])^2 \right) \\ &= (\lambda t)(\lambda s) + \lambda t + (\lambda t)^2, \end{aligned}$$

where the second equality is obtained because of the independence of the number of arrivals,  $N$  and  $M - N$ , during disjoint time intervals.

Submit

You have used 0 of 3 attempts

**i** Answers are displayed within the problem

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? For Staff: bug after submitting 3rd attempt; can you pls fix  
I got 2/3 points (1st question right) after second submission. After my 3rd attempt, I still got Q2 wrong; h...

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💬 Hints for all

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? Hint on 2?  
I`m really stuck on 2. Any small hint would be appreciated. Thanks!

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