11. Exercise: CI's via the CLT

Exercises due May 1, 2020 05:29 IST *Completed*

Exercise: CI's via the CLT

2/2 points (graded)

The sample mean estimate $\widehat{\Theta}$ of the mean of a random variable with variance 1, based on 100 samples, happened to be 22. The 80% confidence interval provided by the CLT is of the form $[a, b]$, with:

$a =$ 21.872 ✔ **Answer:** 21.872

$b =$ 22.128 ✔ **Answer:** 22.128

Your answers should include at least 2 decimal digits.

$$\left[\widehat{\Theta} - \frac{1.96\sigma}{\sqrt{n}}, \widehat{\Theta} + \frac{1.96\sigma}{\sqrt{n}}\right].$$

You may want to refer to the normal table (below). For your reference, if we had 95% instead of 80%, the confidence interval would be of the form

$$\left[\widehat{\Theta} - \frac{1.96\sigma}{\sqrt{n}}, \widehat{\Theta} + \frac{1.96\sigma}{\sqrt{n}}\right].$$

Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \leq z)$, where Z is a standard normal random variable, for z between 0 and $3.49.$ For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to $0.01,$ so that $\Phi(1.71) = .9564.$ When z is negative, the value of $\Phi(z)$ can be found using the formu
$\Phi(z) = 1 - \Phi(-z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	[illegible]	[illegible]

3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

* For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

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Solution:

The number 1.96 for the 95% confidence interval was chosen because we wanted to have 2.5% probability at either tail of the normal, and using the fact $\Phi(1.96) = 0.975$. In this case, we want to have 10% probability at each tail, and we need to find a value z such that $\Phi(z) = 0.9$. From the normal table, the closest choice is $z = 1.28$. We therefore obtain

$$\left[\widehat{\Theta} - \frac{1.28\sigma}{\sqrt{n}}, \widehat{\Theta} - \frac{1.28\sigma}{\sqrt{n}}\right],$$

or

$$[22 - 1.28/10, 22 + 1.28/10] = [21.872, 22.128].$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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Ex: CI's via the CLT

The problem doesn't match with the definition of CI , ie; the interval [a,b] should be random, which was clearly stated... 5

? The solution is mistyped

I noticed that the solution is mistypes, the second term should be plus instead of minus as it shown. 5

Confidence interval for a realisations of the sample mean

Hi, from the lecture I know that It does not make sense to report a confidence interval [a,b] where a,b are constant. I… 1

Possible bug in solution checking?

On my first attempt I made a silly mistake that meant both answers were off by about 0.038. However, the system m… 1