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Final Exam due May 20, 2020 05:29 IST *Completed*

Problem 5 (a)

2/2 points (graded)

Note: You may find the following useful for this problem. For i.i.d. random variables X_i and a nonnegative random variable N that is independent of all the X_i's:

$$\mathbf{E}\left[\sum_{i=1}^{N} X_i\right] = \mathbf{E}[N]\,\mathbf{E}[X_1]$$

$$\mathrm{Var}\left[\sum_{i=1}^{N} X_i\right] = \mathbf{E}[N]\,\mathrm{Var}[X_1] + (\mathbf{E}[X_1])^2\,\mathrm{Var}(N).$$

Each phone call by Ali consumes an amount of time that follows an exponential distribution with mean 5 minutes. The number of different phone calls Ali makes on any given day has a Poisson distribution with mean 3. Assume that a single call always falls within a single day (no calls continue past midnight).

Further, suppose that the number of phone calls that Ali makes on different days are independent, and that the lengths of the phone calls are also independent of each other. For simplicity, also assume that different phone calls never overlap and that there are 30 days in each given month.

Let X be the total number of minutes Ali spends on the phone during one month.

Find $\mathbf{E}(X)$.

$\mathbf{E}(X) =$ 450 ✔ **Answer:** 450

Find $\mathrm{Var}(X)$.

$\text{Var}(X) =$ 4500 ✔ **Answer:** 4500

STANDARD NOTATION

Scroll Down: There is one more problem below.

Solution:

Let $i = 1, \ldots, 30$ be the day of the month. Let N_i be the number of phone calls Alice makes during day i. Let $X_{i,j}$ be the number of minutes Alice spends on the jth phone call on day i, with $j = 1, \ldots, N_i$. Then, we can write

$$X = \sum_{i=1}^{30} \sum_{j=1}^{N_i} X_{i,j}.$$

Recall that N_i follows a Poisson distribution with mean 3 calls, and $X_{i,j}$ follows an exponential distribution with mean 5 minutes. Moreover, these random variables are independent to each other. So using linearity of expectation,

$$\mathbb{E}(X) = \sum_{i=1}^{30} \mathbb{E}\left(\sum_{j=1}^{N_i} X_{i,j}\right) = \sum_{i=1}^{30} \left(\mathbb{E}(N_i)\,\mathbb{E}(X_{i,1})\right) = 30 \cdot 5 \cdot 3 = 450 \text{ minutes}.$$

As for the variance,

$$\begin{aligned} \text{Var}(X) &= \sum_{i=1}^{30} \text{Var}\left(\sum_{j=1}^{N_i} X_{i,j}\right) \\ &= \sum_{i=1}^{30} \left(\mathbb{E}(N_i)\,\text{Var}(X_{i,1}) + (\mathbb{E}(X_{i,1}))^2 \text{Var}(N_i)\right) \\ &= 30\left(3 \cdot 25 + 5^2 \cdot 3\right) = 4500 \text{ minutes}^2. \end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Problem 5 (b)

2.0/2.0 points (graded)

Using the central limit theorem and a standard normal table or calculator, find the probability that the total **number of phone calls** Ali makes during an entire year (12 months of 30 days each) is between 1100 and 1200.

(Note that in this part of the question, you are asked about the number of phone calls, not the number of minutes.)

(Give an answer accurate to at least 3 decimals.)

0.2712 ✔ **Answer:** 0.2708

Solution:

Let $N_{i,j}$ be the number of phone calls Alice makes on the ith day of the jth month in the year. Then the total number of phone calls can be written as

$$N_{tol} = \sum_{j=1}^{12} \sum_{i=1}^{30} N_{i,j}.$$

Since 360 is large, we can say N_{tol} approximately follows a normal distribution with mean $\mu = 360 \cdot \mathbb{E}(N_{1,1}) = 1080$ and variance $\sigma^2 = 360 \cdot \mathbf{Var}(N_{1,1}) = 1080$. Hence,

$$\begin{aligned}
\mathbb{P}(1100 \leq N_{tol} \leq 1200) &= \mathbb{P}\left(\frac{20}{\sqrt{1080}} \leq \frac{N_{tol} - 1080}{\sqrt{1080}} \leq \frac{120}{\sqrt{1080}}\right) \\
&= \Phi\left(\frac{120}{\sqrt{1080}}\right) - \Phi\left(\frac{20}{\sqrt{1080}}\right) \\
&\approx \Phi(3.65) - \Phi(0.61) \\
&= .99987 - .72907 = 0.2708.
\end{aligned}$$

STANDARD NOTATION

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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- [STAFF] 5b - Why does the problem require an answer accurate at 3 decimals but don't use the 1/2 correction? — 57
  Dear Staff, The problem don't suggest using the 1/2 correction, but they don't tell that you are not all...
- **Edited** The de Moivre-Laplace 1/2-correction is not applicable for problem 5b — 25
- BETWEEN and the 1/2 CORRECTION and NORMAL TABLE/CALCULATOR — 31
- [Staff] Requesting for the consideration of Question 5-B....for upto 2 decimal points to be correct... — 1
  Dear Staffs, Can we get credit for the answer with 2 decimal points accuracy? I can see a number of u...
- @Staff:unfair to mark answers with half correct wrong — 1
  Given that we have been taught half correct it is unfair to mark it wrong unless you specify that it can ...
- 5b - unfair grading (closer approximations marked as wrong) — 3
  How can it be that finding a **better** approximation by using the 1/2 correction (which was ***tau...
- My answers — 13
  5a 450, 4500 5b 0.278
- [Staff/TA] 1/2 correction should be considered as correct answer too — 5
  As number of phone call is integer r.v. I have applied 1/2 correction. So the probability will be little de...
- 5b 0.2708 Vs 0.2741 — 2
  Dear staff, It is very unfortunate to see that rounding is given such an importance instead of testing t...
- Question about 5 (a) 2 - why wouldn't this be 69750? — 8
- I rolled the dice and got the "right" answer. — 2
  I agree with the complaints here. Through different interpretations of the problem, I came up with an...
- [Staff] Regarding problem 5b

? I have following points: 1. Exact calculation for this problem using python is not feasible on PC with li... 2

? [STAFF] Grading of Problem 5 (b) looks problematic 2