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5. Arrivals during overlapping time intervals

Problem Set due May 13, 2020 05:29 IST Past Due

Problem 5. Arrivals during overlapping time intervals

3 points possible (graded)

Consider a Poisson process with rate λ . Let N be the number of arrivals in (0,t] and M be the number of arrivals in (0,t+s], where $t>0,s\geq 0$.

In each part below, your answers will be algebraic expressions in terms of λ,t,s,m and/or n . Enter "lambda" for λ and use "exp()" for exponentials. Do **not** use "fac()" or "!" for factorials. Follow standard notation.

1. For $0 \leq n \leq m$, the conditional PM	$\in p_{M N}\left(m\mid n ight)$ of M given N is of the form $rac{a}{b!}$
for suitable algebraic expressions in	place of a and b .

a =	
Answer: lambda^(m-n)*s^(m-n)*exp(-lambda*s)	
b =	Answer: m-n

2.



$$E\left[NM
ight]=$$

Answer: lambda*t*lambda*s+lambda*t+(lambda*t)^2

STANDARD NOTATION

Solution:

1. To find $P_{M|N}\left(m\mid n\right)$, we assume there are n arrivals in the first t time units, and we are looking for the probability that there are m-n arrivals in the subsequent s time units. This follows a Poisson distribution with parameter λs :

$$p_{M|N}\left(m\mid n
ight)=rac{\left(\lambda s
ight)^{m-n}e^{-\lambda s}}{\left(m-n
ight)!},\quad ext{for }m\geq n\geq 0.$$

2. We can rewrite the expectation as

$$egin{array}{lll} \mathbf{E}\left[NM
ight] &=& \mathbf{E}\left[N\left(M-N
ight)+N^2
ight] \ &=& \mathbf{E}\left[N
ight]\mathbf{E}\left[M-N
ight]+\mathbf{E}\left[N^2
ight] \ &=& \left(\lambda t
ight)\left(\lambda s
ight)+\left(\mathrm{var}\left(N
ight)+\left(\mathbf{E}\left[N
ight]
ight)^2
ight) \ &=& \left(\lambda t
ight)\left(\lambda s
ight)+\lambda t+\left(\lambda t
ight)^2, \end{array}$$

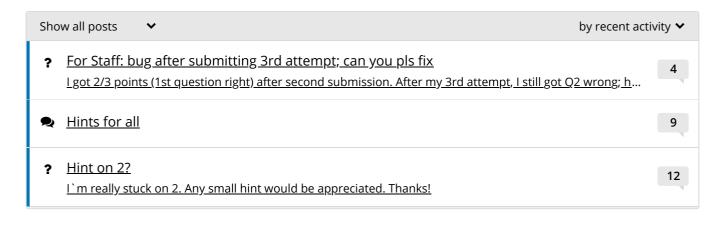
where the second equality is obtained because of the independence of the number of arrivals, N and M-N, during disjoint time intervals.

Submit

You have used 0 of 3 attempts

1 Answers are displayed within the problem

Topic: Unit 9: Bernoulli and Poisson processes:Problem Set 9 / 5. Arrivals during overlapping time intervals



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