7. Exercise: Expected value rule and total expectation theorem

Exercises due Mar 13, 2020 05:29 IST *Completed*

Exercise: Expected value rule and total expectation theorem

6/8 points (graded)

Let X, Y, and Z be jointly continuous random variables. Assume that all conditional PDFs and expectations are well defined. E.g., when conditioning on $X=x$, assume that x is such that $f_X(x)>0$. For each one of the following formulas, state whether it is true for all choices of the function g or false (i.e., not true for all choices of g).

1. $\mathbf{E}[g(Y)\mid X=x]=\int g(y)\, f_{Y|X}(y\mid x)\, dy$

True ✔ **Answer:** True

2. $\mathbf{E}[g(y)\mid X=x]=\int g(y)\, f_{Y|X}(y\mid x)\, dy$

False ✔ **Answer:** False

3. $\mathbf{E}[g(Y)]=\int \mathbf{E}[g(Y)\mid Z=z]\, f_Z(z)\, dz$

True ✔ **Answer:** True

4. $\mathbf{E}[g(Y)\mid X=x, Z=z]=\int g(y)\, f_{Y|X,Z}(y\mid x,z)\, dy$

True ✔ **Answer:** True

5. $\mathbf{E}\left[g(Y) \mid X=x\right] = \int \mathbf{E}\left[g(Y) \mid X=x, Z=z\right] f_{Z|X}(z \mid x)\, dz$

True ✔ **Answer:** True

6. $\mathbf{E}\left[g(X,Y) \mid Y=y\right] = \mathbf{E}\left[g(X,y) \mid Y=y\right]$

False ✖ **Answer:** True

7. $\mathbf{E}\left[g(X,Y) \mid Y=y\right] = \mathbf{E}\left[g(X,y)\right]$

True ✖ **Answer:** False

8. $\mathbf{E}\left[g(X,Z) \mid Y=y\right] = \int g(x,z)\, f_{X,Z|Y}(x,z \mid y)\, dy$

False ✔ **Answer:** False

Solution:

1. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$.

2. False. Here the quantity inside the expectation, $g(y)$, is a number (not a random variable). The left-hand side is a function of y, whereas on the right-hand side, y, is a dummy variable that gets integrated away. So, the formula is wrong on a purely syntactical basis (the left-hand side depends on y, while the right-hand side does not).

3. True. This is the total expectation theorem, where we condition on the events $Z=z$.

4. True. This is the usual expected value rule, applied to a conditional model where we are given that $X=x$ and $Z=z$.

5. True. This is the same total expectation theorem as in the third part, except that everything is calculated within a conditional model in which event $X=x$ is known to have occurred.

6. True. When we condition on $Y=y$, we know the value of Y, and we can replace $g(X,Y)$ by $g(X,y)$.

7.

False. Given that $Y = y$, we need to somehow take into account the conditional distribution of X, whereas the right-hand side is determined by the unconditional PDF of X.

8. False. The left-hand side is a function of y, whereas the right-hand side (after y is integrated out) is a function of x and z. The correct form (expected value rule, in a conditional model) is:

$$\mathbf{E}\big[g(X,Z)\,|\,Y=y\big] = \int\int g(x,z)\, f_{X,Z|Y}(x,z\,|\,y)\; dx\, dz.$$

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

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Topic: Unit 5: Continuous random variables:Lec. 10: Conditioning on a random variable; Independence; Bayes' rule / 7. Exercise: Expected value rule and total expectation theorem

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- ? Question 5 — I need more explanation for question 5 please — 1 new_ 4
- I'm not sure who writes those problems but thank you so much. — Although I got some of them wrong, but it cleared many issues I had with notations and conditioning . — 4
- ? How to prove part 6 mathematically? — I'm struggling to lose the second integral in the LHS to get to RHS. Any help? (deadline is over so we can ... — 5
- ? Difficulty to interpret hypothesis — "X , Y, and Z be jointly continuous random variables" Does it mean that the pairs X and Y, Y and Z, Z and ... — 2
- ? RHS of 6 and 7 — To me, the right hand side of parts 6 and 7 is equivalent, but I might be missing some notation nuance. I... — 1 new_
- (Staff) Serious issue with (2) — I have given this section a good thought, and I could get all right except 2. My initial choice was what you — 4

☑	number 2 - isn't the expected value of a constant is a constant?	4
?	Part 7	2
?	Part 5. Why expectation and an extra variable in the integral?	4
?	#5 Can help to explain on the derivation on #5?	3
	Hint for #5 Course -> Unit 4: Discrete random variables -> Lec. 7: Conditioning on a random variable; Independence ...	1