3. PMF, expectation, and variance

Problem Set due Feb 28, 2020 05:29 IST *Completed*

Problem 3. PMF, expectation, and variance

6.0/6.0 points (graded)

The random variables X and Y have the joint PMF

$$p_{X,Y}(x,y) = \begin{cases} c\cdot(x+y)^2, & \text{if } x \in \{1,2,4\} \text{ and } y \in \{1,3\}, \\ 0, & \text{otherwise.} \end{cases}$$

All answers in this problem should be numerical.

1. Find the value of the constant c.

   $c =$ 1/128 ✔ **Answer:** 1/128

2. Find $\mathbf{P}(Y < X)$.

   $\mathbf{P}(Y < X) =$ 83/128 ✔ **Answer:** 83/128

3. Find $\mathbf{P}(Y = X)$.

   $\mathbf{P}(Y = X) =$ 4/128 ✔ **Answer:** 4/128

4. Find the following probabilities.

   (*Hint*: To avoid double jeopardy with later problem sets, the answers are $\frac{74}{128}$, $\frac{34}{128}$, $\frac{20}{128}$, 0, not necessarily in that order.)

$\mathbf{P}(X=1) =$ 20/128 ✔ **Answer:** 20/128

$\mathbf{P}(X=2) =$ 34/128 ✔ **Answer:** 34/128

$\mathbf{P}(X=3) =$ 0 ✔ **Answer:** 0

$\mathbf{P}(X=4) =$ 74/128 ✔ **Answer:** 74/128

5. Find the expectations $\mathbf{E}[X]$ and $\mathbf{E}[XY]$.

$\mathbf{E}[X] =$ 384/128 ✔ **Answer:** 3

$\mathbf{E}[XY] =$ 7.22 ✔ **Answer:** 227/32

6. Find the variance of X.

Var $(X) =$ 13/9 ✔ **Answer:** 47/32

Solution:

1. From the joint PMF, there are six (x, y) pairs with nonzero probability mass. These pairs are $(1,1), (1,3), (2,1), (2,3), (4,1), (4,3)$. Because the probability of the entire sample space must equal 1, we have:

$$c(1+1)^2 + c(1+3)^2 + c(2+1)^2 + c(2+3)^2 + c(4+1)^2 + c(4+3)^2 = 1.$$

Solving for c, we get $c = \frac{1}{128}$.

2. There are three possible outcomes for which $y < x$: $(2,1), (4,1), (4,3)$.

$$\mathbf{P}(Y < X) = p_{X,Y}(2,1) + p_{X,Y}(4,1) + p_{X,Y}(4,3) = \frac{9}{128} + \frac{25}{128} + \frac{49}{128} = \frac{83}{128}.$$

3. There is only one possible outcome for which $y = x$: $(1,1)$.

$$\mathbf{P}(Y = X) = p_{X,Y}(1,1) = \frac{4}{128}.$$

4. We use the formula $p_X(x) = \sum_y p_{X,Y}(x,y)$.

   For example, $p_X(2) = p_{X,Y}(2,1) + p_{X,Y}(2,3) = \frac{34}{128}$. More generally, we find that

$$p_X(x) = \begin{cases} 20/128, & \text{if } x = 1, \\ 34/128, & \text{if } x = 2, \\ 74/128, & \text{if } x = 4, \\ 0, & \text{otherwise.} \end{cases}$$

5. We have

$$\mathbf{E}[X] = \sum_x x p_X(x) = 1 \cdot \frac{20}{128} + 2 \cdot \frac{34}{128} + 4 \cdot \frac{74}{128} = 3.$$

   Using the expected value rule,

$$\begin{aligned} \mathbf{E}[XY] &= \sum_x \sum_y xy p_{X,Y}(x,y) \\ &= 1 \cdot \frac{4}{128} + 2 \cdot \frac{9}{128} + 4 \cdot \frac{25}{128} + 3 \cdot \frac{16}{128} + 6 \cdot \frac{25}{128} + 12 \cdot \frac{49}{128} \\ &= \frac{227}{32}. \end{aligned}$$

6. The variance of a random variable X can be computed as $\mathbf{E}[X^2] - (\mathbf{E}[X])^2$ or as $\mathbf{E}[(X - \mathbf{E}[X])^2]$. We use the second approach here. We have

$$\text{Var}(X) = (1-3)^2 \frac{20}{128} + (2-3)^2 \frac{34}{128} + (4-3)^2 \frac{74}{128} = \frac{47}{32}.$$

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You have used 4 of 5 attempts

Answers are displayed within the problem

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- Part 6; Var(X) = E[X^2] - (E[X])^2? — 7
  I calculate E[X^2] as: sum_x (sum_y (x^2 * Pxy(x,y))) I calculate (E[X])^2 from part 5 When I calculate: Var(X) = su...
- Is the grader correct? — 4
  I think that the grader is wrong in exercises 1-3. I miscalculated the c (typo in the calculator) which I corrected fo...
- Is there an intuitive way to tell X and Y are dependent? — 2
  I did the math myself so I know that EX*EY ne E(XY), indicating the dependency. Is there an intuitive way to tell X...
- Question 1 — 3
  Can anyone give a hint on how to approach this problem? Thanks
- E[XY] any hint? — 2 new_
  Any hint on how to answer this question? Any lecture I should check? I've tried to bruteforce it but had no luck ...
- #3 — 3
  I have gotten everything else correct but am scratching my head on part 3 For #3 P(Y=X) - occurs when (X=1,Y=1...
- What's the difference between p and P in this question? — 2
  What's the difference between p and P in this question? They seem to both represent probability to me. Can so...