11. Exercise: The effect of a stronger signal

Exercises due Apr 8, 2020 05:29 IST *Completed*

Exercise: The effect of a stronger signal

1/1 point (graded)

For the model $X = \Theta + W$, and under the usual independence and normality assumptions for Θ and W, the mean squared error of the LMS estimator is

$$\frac{1}{(1/\sigma_0^2) + (1/\sigma_1^2)},$$

where σ_0^2 and σ_1^2 are the variances of Θ and W, respectively.

Suppose now that we change the observation model to $Y = 3\Theta + W$. In some sense the "signal" Θ has a stronger presence, relative to the noise term W, and we should expect to obtain a smaller mean squared error. Suppose $\sigma_0^2 = \sigma_1^2 = 1$. The mean squared error of the original model $X = \Theta + W$ is then $1/2$. In contrast, the mean squared error of the new model $Y = 3\Theta + W$ is

1/10 ✔ **Answer:** 0.1

Hint: Do not solve the problem from scratch. Think of an alternative observation model in which you observe $Y' = \Theta + (W/3)$.

Solution:

Since Y' is just Y scaled by a factor of $1/3$, Y' carries the same information as Y, so that $\mathbf{E}[\Theta \mid Y] = \mathbf{E}[\Theta \mid Y']$. Thus, the alternative observation model $Y' = \Theta + (W/3)$ will lead to the same estimates and will have the same mean squared error as the unscal

model $Y = 3\Theta + W$. In the equivalent Y' model, we have a noise variance of $1/9$ and therefore the mean squared error is

$$\frac{1}{\frac{1}{1} + \frac{1}{1/9}} = \frac{1}{10}.$$

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Answers are displayed within the problem

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Topic: Unit 7: Bayesian inference:Lec. 15: Linear models with normal noise / 11. Exercise: The effect of a stronger signal

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- Effect of a stronger signal
  Hi, The same kind of problem is seen in Ex. 8 - Multiple observations, more general model, where X=2* T... 3
- How do we get Y' from Y 1
- Clarification
  Are we to assume that the prior on theta in the Y model is normal with 0 mean and variance 1? 2
- Prior/posterior variances?
  I definitely need to re-learn this material... The hint is: $Y'=\Theta+(W/3)$, which I understand. In this situation, I... 2
- Would the Y' model relate in the same way to the Y model for any invertible function h with Y' = h(Y)?
  I tried to formulate my question so as to not reveal too much from the solution. 1
- Why is it wrong to apply the formula on Y with var(Θ)=9? 2 new 7
- variance of the noise
  why is the variance 1/9? 2
- From scratch approach
  I have already done the question with the hint approach, and I understand why they are equivalent. Ho... 2

From scratch team

I suppose is due to a lack of understanding or skills on my part, but every time it says, "don't do it from s...

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