2. Find the limits

Problem Set due May 1, 2020 05:29 IST *Completed*

Problem 2. Find the limits

3/3 points (graded)

Let S_n be the number of successes in n independent Bernoulli trials, where the probability of success at each trial is $1/3$. Provide a numerical value, to a precision of 3 decimal places, for each of the following limits. You may want to refer to the standard normal table.

Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \le z)$, where Z is a standard normal random variable, for z between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that $\Phi(1.71) = .9564$. When z is negative, the value of $\Phi(z)$ can be found using the formula $\Phi(z) = 1 - \Phi(-z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	[illegible]	[illegible]

1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

*For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

Hide

1.

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{n}{3} - 10 \leq S_n \leq \frac{n}{3} + 10\right) =$$

0

✔ **Answer:** 0

2.

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{n}{3}-\frac{n}{6} \le S_n \le \frac{n}{3}+\frac{n}{6}\right) =$$

1

Answer: 1

3.

$$\lim_{n\to\infty} \mathbf{P}\left(\frac{n}{3}-\frac{\sqrt{2n}}{5} \le S_n \le \frac{n}{3}+\frac{\sqrt{2n}}{5}\right) =$$

0.4514

Answer: 0.4514

Solution:

First, notice that $S_n = X_1 + \cdots + X_n$, where the X_i are independent Bernoulli random variables with parameter $1/3$. Hence, $\mathbf{E}[S_n] = n/3$, and $\textsf{Var}(S_n) = 2n/9$.

1. Fix an $\epsilon > 0$. No matter how small ϵ is, we have, for sufficiently large n, $\epsilon\sqrt{n} > 10$. For any such large enough n,

$$\begin{aligned}
\mathbf{P}\left(\frac{n}{3}-10 \le S_n \le \frac{n}{3}+10\right) &\le \mathbf{P}\left(\frac{n}{3}-\epsilon\sqrt{n} \le S_n \le \frac{n}{3}+\epsilon\sqrt{n}\right)\\
&= \mathbf{P}\left(-\epsilon\sqrt{n} \le S_n - \frac{n}{3} \le \epsilon\sqrt{n}\right)\\
&= \mathbf{P}\left(-\frac{\epsilon\sqrt{n}}{\sqrt{2n/9}} \le \frac{S_n-\frac{n}{3}}{\sqrt{2n/9}} \le \frac{\epsilon\sqrt{n}}{\sqrt{2n/9}}\right)\\
&= \mathbf{P}\left(-\frac{3}{\sqrt{2}}\epsilon \le \frac{S_n-\frac{n}{3}}{\sqrt{2n/9}} \le \frac{3}{\sqrt{2}}\epsilon\right).
\end{aligned}$$

By the Central Limit Theorem,

$$\lim_{n\to\infty} \mathbf{P}\left(-\frac{3}{\sqrt{2}}\epsilon \le \frac{S_n-\frac{n}{3}}{\sqrt{2n/9}} \le \frac{3}{\sqrt{2}}\epsilon\right) = \Phi\left(\frac{3}{\sqrt{2}}\epsilon\right) - \Phi\left(-\frac{3}{\sqrt{2}}\epsilon\right).$$

Since this is true for every $\epsilon > 0$, it is also true in the limit as $\epsilon \downarrow 0$. The final answer then follows from the fact that,

$$\lim_{\epsilon\downarrow 0}\left[\Phi\left(\frac{3}{\sqrt{2}}\epsilon\right) - \Phi\left(-\frac{3}{\sqrt{2}}\epsilon\right)\right] = \Phi(0) - \Phi(0) = 0.$$

2. The given event, after some algebraic manipulations, is equivalent to the following event:

$$\left|\frac{S_n}{n} - \frac{1}{3}\right| \leq \frac{1}{6}.$$

Since $\mathbf{E}\left[S_n/n\right] = n/3$, by the weak law of large numbers, the probability of the event above converges to 1 as $n \to \infty$.

3. By the Central Limit Theorem,

$$\begin{aligned}\lim_{n\to\infty} \mathbf{P}\left(\frac{n}{3} - \frac{\sqrt{2n}}{5} \leq S_n \leq \frac{n}{3} + \frac{\sqrt{2n}}{5}\right) &= \mathbf{P}\left(\left|\frac{S_n - \frac{n}{3}}{\sqrt{2n/9}}\right| \leq \frac{\sqrt{2n}/5}{\sqrt{2n/9}}\right)\\ &= \Phi\left(0.6\right) - \Phi\left(-0.6\right)\\ &\approx 0.4514.\end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 8: Limit theorems and classical statistics:Problem Set 8 / 2. Find the limits

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- Solution of question 2 — 3
  In question 2 isn't it that E[Sn/n] = 1/3 ?
- Lost in translation (somewhat) — 4
- still stuck — 3
  I've been through all the comments below. I'm still stuck at this one... I did the whole calculation with the Normalisati...
- Once again about CLT (question for everyone who has got the correct answer) — 5
- Any hints for this one folks ? — 5
  maybe i am overthinking or not using all the info available on the problems
- Hint — 2

Hint for Q3

Hello Does anyone have a hint for Q3? Thanks

5

Part 2 limit

For part 2, I’m getting the limit of the probability = infinity as n goes to infinity? Doesn’t seem right because I’m assum...

4

How to deal with root(n) in denominator?

6

I don't get how CDF looks like here...

Hi! Maybe that's because I am tired and it almost 6 am here, but still I don't understand how does the CDF of nubmer...

1