8. Exercise: Confidence interval interpretation

Exercises due May 1, 2020 05:29 IST *Completed*

Exercise: Confidence interval interpretation

3/4 points (graded)

Every day, I try to estimate an unknown parameter using a fresh data set. I look at the data and then I use some formulas to calculate a 70% confidence interval, $[\widehat{\Theta}^-, \widehat{\Theta}^+]$, based on the day's data.

Are the following statements accurate?

Over the next 100 days, I expect that the unknown parameter will be inside the confidence interval about 70 times.

Yes ✔ **Answer:** Yes

If today's confidence interval is $[0.41, 0.47]$, there is probability 70% that the unknown parameter is inside this confidence interval.

No ✔ **Answer:** No

Out of 100 days on which the confidence interval happens to be $[0.41, 0.47]$, I expect that the unknown parameter will be inside the confidence interval about 70 times.

Yes ✖ **Answer:** No

Today, I decided to use a Bayesian approach, by viewing the unknown parameter, denoted by Θ, as a continuous random variable and assuming a prior PDF for Θ. I observe a specific value x, calculate the posterior $f_{\Theta|X}(\cdot \mid x)$, and find out that

$$\int_{0.41}^{0.47} f_{\Theta|X}(\theta \,|\, x)\, d\theta = 0.70.$$

Am I allowed to say that there is probability 70% that the unknown parameter is inside the (Bayesian) confidence interval $[0.41, 0.47]$?

Yes ✔ **Answer:** Yes

Solution:

The first statement is true. The confidence interval is a random interval and has probability 0.70 of capturing the true value of the unknown parameter. Using the frequency interpretation of probabilities, we expect about 70 successful captures.

The second statement is false. The value of the parameter is not random. Conditional on the confidence interval being $[0.41, 0.47]$, the event "the unknown parameter is inside the confidence interval" does not involve anything random, and so its probability cannot be 0.70.

The third statement may appear to be closer to the first one rather than the second one. However, the same explanation as for the second statement applies.

The fourth case involves the conceptually different setting of Bayesian inference. Here, Θ is a random variable, and

$$0.70 = \int_{0.41}^{0.47} f_{\Theta|X}(\theta \,|\, x)\, d\theta = \mathbf{P}\big(\Theta \in [0.41, 0.47] \mid X = x\big),$$

is indeed the (conditional) probability that Θ belongs to the interval $[0.41, 0.47]$.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

Hide Dis

Topic: Unit 8: Limit theorems and classical statistics:Lec. 20: An introduction to classical statistics / 8. Exercise: Confidence interval interpretation

Show all posts | by recent activity

- CI
  What if the unknown parameter were random in the problem : " If today's confidence interval is [0.41... — 5
- Still can't understand the 2nd :(
  Suppose I took 100 measurements each with its own 70% confidence interval. Then I take one of thos... — 4 new
- The misused term in the 4th statement
  The "Bayesian confidence interval" doesn't makes sense (the misused term in the 4th statement), bec... — 1
- Bayesian confidence interval?
  I do not recall in prior lecture the mention of Bayesian confidence interval. Is probability equivalent to... — 3
- Bayesian Approach
  The confidence interval shouldn't correspond to an unconditional probability? — 1
- Simple way of understanding
  Hi guys, it might be late but I still want to share my understanding with you and please tell me if I am ... — 1
- Third question
  I didn't understand the answer given. Can anyone offer an alternative explanation (without giving the ... — 3 new
- 4th statement? Bayesian Approach.
  I understand that in classical statistics, theta is a fixed number with one true value. Hence, for a fixed ... — 2
- Third statement: English
  Is this trying to say, that we did compute the confidence interval independently on 100 different days ... — 1 new
- Hint: don't get too hung up on the terminology for the last statement
  Focus on whether or not it makes sense conceptually. Personally, I got focused on the term "confiden... — 1
- CI "trapping" true parameter
  Hi. It may be that I did not understand the question (I often end up reading things then thinking the o... — 2
- [Staff] First and the second statement — 1
- Hint
  I found this key insight useful after extensively reading [this thread][1] originally posted by sharov. In ... — 1