# 3. LLMS estimation

Problem Set due Apr 15, 2020 05:29 IST Completed

## Problem 3. LLMS estimation

3/3 points (graded)

Let $X = U + W$ with $\mathbf{E}[U] = m$, Var $(U) = v$, $\mathbf{E}[W] = 0$, and Var $(W) = h$. Assume that $U$ and $W$ are independent.

1. The LLMS estimator of $U$ based on $X$ is of the form $\hat{U} = aX + b$. Find $a$ and $b$. Express your answers in terms of $m$, $v$, and $h$ using standard notation.

   $a =$ v/(v+h) ✔ **Answer:** v/(v+h)

   $\frac{v}{v+h}$

   $b =$ m*h/(v+h) ✔ **Answer:** m*h/(v+h)

   $\frac{m \cdot h}{v+h}$

2. We now further assume that $U$ and $W$ are normal random variables and then construct $\hat{U}_{LMS}$, the LMS estimator of $U$ based on $X$, under this additional assumption. Would $\hat{U}_{LMS}$ be identical to $\hat{U}$, the LLMS estimator developed without the additional normality assumption in Part 1?

   Yes ✔ **Answer:** Yes

STANDARD NOTATION

**Solution:**

1. In order to write the LLMS estimator we need to find $\mathbf{E}[X]$, Var $(X)$, and cov $(U, X)$. We have

$$\mathbf{E}[X] = \mathbf{E}[U + W] = \mathbf{E}[U] + \mathbf{E}[W] = \mathbf{E}[U] = m,$$

$$\begin{aligned}
\mathsf{Var}(X) &= \mathsf{Var}(U+W) \\
&= \mathsf{Var}(U) + \mathsf{Var}(W) \qquad \text{since } U \text{ and } W \text{ are independent} \\
&= v+h, \\
\mathrm{cov}(U,X) &= \mathbf{E}[UX] - \mathbf{E}[U]\,\mathbf{E}[X] \\
&= \mathbf{E}[U(U+W)] - m^2 \\
&= \mathbf{E}[U^2] + \mathbf{E}[U]\,\mathbf{E}[W] - m^2 \qquad \text{since } U \text{ and } W \text{ are independent} \\
&= \mathbf{E}[U^2] - m^2 \\
&= \mathbf{E}[U^2] - (\mathbf{E}[U])^2 \\
&= \mathsf{Var}(U) = v.
\end{aligned}$$

Substituting these results into the formula for the LLMS estimator yields

$$\hat{U} = m + \frac{v}{v+h}(X-m)\,.$$

2. We know that the LMS estimator of $U$ based on $X$, under the normality assumption we have introduced, is linear in $X$. Therefore, it coincides with the LLMS estimator.

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

## Discussion

Hide Discussion

**Topic:** Unit 7: Bayesian inference:Problem Set 7b / 3. LLMS estimation

Show all posts by recent activity

[Staff]Answer to Q2 doesn't coincide with the question.
Hi. In general term the question is "Do we need normality assumptions for ...". And answer should state something "... 2
Community TA

Hint: Check Lec17 exersice 8 or 13
It's pretty much the same thing. Here is a link to 13: "https://courses.edx.org/courses/course-v1:MITx+6.431x+1T202... 2

Hint for Q1 8

Hint for Q2
Review the last part of lecture 17, "14. Example with multiple observations". 4

Hint

Use the formula given in Lecture Lecture 17. "LLMS Example": E(theta) + Cov(theta, X)/ Var(X) *(X - E(X)) Apply this equ...

1