3. Shuttles

Problem Set due May 13, 2020 05:29 IST *Past Due*

Problem 3. Shuttles

5 points possible (graded)

In parts 1, 3, 4, and 5 below, your answers will be algebraic expressions. Enter "lambda" for λ and "mu" for μ. Follow standard notation.

1. Shuttles bound for Boston depart from New York every hour on the hour (e.g., at exactly one o"clock, two o'clock, etc.). Passengers arrive at the departure gate in New York according to a Poisson process with rate λ per hour. What is the expected number of passengers on any given shuttle? (Assume that everyone who arrives between two successive shuttle departures boards the shuttle immediately following his/her arrival. That is, shuttles are big enough to accommodate all arriving passengers.)

   Answer: lambda

2. For this and for the remaining parts of this problem, suppose that the shuttles are not operating on a deterministic schedule. Rather, their interdeparture times are independent and exponentially distributed with common parameter μ per hour. Furthermore, shuttle departures are independent of the process of passenger arrivals. Is the sequence of shuttle departures a Poisson process?

   Select an option

   Answer: Yes it is a Poisson process

3. Let us say that an "event" occurs whenever a passenger arrives or a shuttle departs. What is the expected number of "events" that occur in any one-hour interval?

Answer: mu+lambda

4. If a passenger arrives at the gate and sees 2λ people waiting (assume that 2λ is an integer), what is his/her expected waiting time until the next shuttle departs?

Answer: 1/mu

5. Find the PMF, $p_N(n)$, of the number, N, of people on any given shuttle. Assume that $\lambda = 20$ and $\mu = 2$.
For $n \geq 0$,

$p_N(n) =$

Answer: (2/22)*(20/22)^n

STANDARD NOTATION

Solution:

1. The number of people who arrive within an hour is Poisson-distributed with parameter λ, and its expected value is λ.

2. If the interdeparture times for the shuttles are independent and exponentially distributed with common parameter μ, then shuttle departures form a Poisson process with rate μ per hour.

3. Here, we are merging two independent Poisson processes, which results in a Poisson process of rate $\mu + \lambda$ per hour. Therefore, the expected number of "events" occurring in one hour will be $\mu + \lambda$.

4. The number of people waiting conveys some information on the time since the last departure. On the other hand, the memorylessness property of the Poisson process of shuttle departures implies that this number is independent of the time un

next departure. Thus, the expected waiting time is just $1/\mu$, irrespective of how many people are waiting.

5. Each "event" has probability $\lambda/(\lambda+\mu)$ of being a passenger arrival ("failure") and probability $\mu/(\lambda+\mu)$ of being a shuttle departure ("success"). Furthermore, different events are independent. The number of passengers on a shuttle is the number of failures until the first success and is distributed as $K-1$, where K is a geometric random variable with parameter $\mu/(\lambda+\mu)$. Thus, the PMF of the number of people on the shuttle is

$$p_N(n) = \left(\frac{\lambda}{\lambda+\mu}\right)^n \left(\frac{\mu}{\lambda+\mu}\right), \quad n = 0, 1, \ldots.$$

Assuming that $\lambda = 20$, and $\mu = 2$, this becomes

$$p_N(n) = \left(\frac{20}{22}\right)^n \left(\frac{2}{22}\right), \quad n = 0, 1, \ldots.$$

Submit You have used 0 of 4 attempts

Answers are displayed within the problem

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? Question 5 Confused about Intuition 2

Little hint 15

If you struggle with question 5, take a look at Additional theoretical material section for this unit.

relation between Poisson and exponential 2

If exponential models time to the next event, but Poisson the number events in a fixed period of time, ar

? [Staff] Fundamentals of Statistics
I have already earned verified certificate for Fundamentals of Statistics, and hope to earn verified certific... 2

Hint for 5.
Hi. 1)[Lecture][1] 23.7 would be very helpful here. We can simulate as two merged processes -- passange... 4
Community TA

? Hint for 4
Could someone give a hint for Question 4. Not sure I'm understanding it correctly. 4

? mu in 2. 5

Part 4 - information to solve this depends on other parts (1-2)
The assumption about the randomness of the interarrival times was mentioned only in point 2, and usu... 1

? #5
Hi. I watched the suggested video (thrice!), but I am still not getting the correct answer. The situation in t... 2