



6. Tossing a triple of coins

Problem Set due May 1, 2020 05:29 IST Completed

Problem 6. Tossing a triple of coins

8/8 points (graded)

We have a red coin, for which $\mathbf{P}(\text{Heads}) = 0.4$, a green coin, for which $\mathbf{P}(\text{Heads}) = 0.5$, and a yellow coin, for which $\mathbf{P}(\text{Heads}) = 0.6$. The flips of the same or of different coins are independent. For each of the following situations, determine whether the random variable N can be approximated by a normal.

If yes, enter the mean and variance of N . If not, enter 0 in both of the corresponding answer boxes.

1. Let N be the number of Heads in 300 tosses of the red coin.

mean:

✓ Answer: 120

Variance:

✓ Answer: 72

2. Let N be the number of Heads in 300 tosses. At each toss, one of the three coins is selected at random (either choice is equally likely), and independently from everything else.

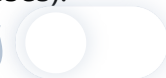
mean:

✓ Answer: 150

variance:

✓ Answer: 75

3. Let N be the number of Heads in 100 tosses of the red coin, followed by 100 tosses of the green coin, followed by 100 tosses of the yellow coin (for a total of 300 tosses).



mean:

✓ Answer: 150

variance:

✓ Answer: 73

4. We select one of the three coins at random: each coin is equally likely to be selected. We then toss the selected coin 300 times, independently, and let N be the number of Heads.

mean:

✓ Answer: 0

variance:

✓ Answer: 0

Solution:

For each of the following parts let X_i be a random variable that takes value 1 if the i th toss is Heads and takes value 0 otherwise.

1. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter 0.4. Here, $\mathbf{E}[N] = 300 \cdot 0.4 = 120$ and $\text{Var}(N) = 300 \cdot 0.4 \cdot (1 - 0.4) = 72$.
2. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter 0.5. Here $\mathbf{E}[N] = 300 \cdot 0.5 = 150$ and $\text{Var}(N) = 300 \cdot 0.5 \cdot (1 - 0.5) = 75$.
3. Let $Y_1 = \sum_{i=1}^{100} X_i$, $Y_2 = \sum_{i=101}^{200} X_i$, and $Y_3 = \sum_{i=201}^{300} X_i$, such that $N = Y_1 + Y_2 + Y_3$. The CLT applies and Y_1 can be approximated by a normal because the X_i for $i = 1, \dots, 100$ are independent and identically distributed Bernoulli random variables with parameter 0.4. Using a similar argument, Y_2 , and Y_3 can also be approximated by normal random variables. Since Y_1, Y_2 , and Y_3 are all independent, we conclude that N can also be approximated by a normal. Here,

$$\mathbf{E}[N] = \mathbf{E}[Y_1] + \mathbf{E}[Y_2] + \mathbf{E}[Y_3] = 100 \cdot 0.4 + 100 \cdot 0.5 + 100 \cdot 0.6 = 150.$$

Similarly,



$$\begin{aligned}
 \text{Var}(N) &= \text{Var}(Y_1) + \text{Var}(Y_2) + \text{Var}(Y_3) \\
 &= 100 \cdot 0.4 \cdot (1 - 0.4) + 100 \cdot 0.5 \cdot (1 - 0.5) + 100 \cdot 0.6 \cdot (1 - 0.6) \\
 &= 73.
 \end{aligned}$$

4. The CLT does not apply in this case as N is approximately a mixture of three normals.

Submit

You have used 1 of 3 attempts

i Answers are displayed within the problem


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-  I don't get why the law of total variance doesn't work in #2 though.

Using the formula on the Wikipedia page, for example, I get 673, which is obviously not the right answer. ht...


3
-  Qu for Staff - Grades for second submission not counted


Hi, I resubmitted answers to part 4 and my mark for the question was shown as 7/8 last night however my...


1
-  Hints for all

7
-  Similarities to Assignment Problem


This problem has structural similarities to problem: CLT Applicability, in Lecture 19.

1
-  [Staff] Law of total variance for Q2


 Community TA

17
-  Help with the 4.

For the mean I've used the law of total expectations. First calculating the number of heads conditioned to t...

12
-  [Staff] Could the deadline be delayed by one day?

This week's project on the *Machine Learning with Python* course was very demanding and time consumi...

1
-  Law of Total Variance for Question 2 ?

I'm trying to solve question #2 with Law of Total Variance but without success for the moment... Based on t...

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