8. Exercise: Counting committees

Exercises due Feb 19, 2020 05:29 IST *Completed*

Exercise: Counting committees

2.0/2.0 points (graded)

We start with a pool of n people. A chaired committee consists of $k \geq 1$ members, out of whom one member is designated as the chairperson. The expression $k\binom{n}{k}$ can be interpreted as the number of possible chaired committees with k members. This is because we have $\binom{n}{k}$ choices for the k members, and once the members are chosen, there are then k choices for the chairperson. Thus,

$$c = \sum_{k=1}^{n} k\binom{n}{k}$$

is the total number of possible chaired committees of any size.

Find the value of c (as a function of n) by thinking about a different way of forming a chaired committee: first choose the chairperson, then choose the other members of the committee. The answer is of the form

$$c = (\alpha + n^{\beta})\, 2^{\gamma n + \delta}.$$

What are the values of α, β, γ, and δ?

$\alpha =$ 0 ✔ **Answer:** 0

$\beta =$ 1 ✔ **Answer:** 1

$\gamma =$ 1 ✔ **Answer:** 1

$\delta =$ -1 ✔ **Answer:** -1

Solution:

We first choose the chairperson, for which there are n choices, and then choose an arbitrary subset of the remaining $n-1$ people, who will be the remaining committee members. For example, this arbitrary subset could be the empty set, which would mean that the committee is of size 1: only the chairperson. There are 2^{n-1} possible subsets of a set with $n-1$ elements, and so there are 2^{n-1} ways of choosing the remaining committee members. Thus, an alternative expression for the number of possible chaired committees of any size is $n2^{n-1}$, from which we can extract the values of α, β, γ, and δ.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

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- Kindly give me change to do this excercise because in last few days i have fever — 1
  Kindly give me change to do this excercise because in last few days i have fever
- last minute hint — 2
  it took me a while before I could finally get it. Eventually, I started from the c = (alpha +...) formula. An...
- Easy way to approach the question — 1
  This question is much easier if you at first ignore all the given formulas and just think about how to s...
- k members or k-1 members in the committee? — 2
  Once 1 member has been chosen as a chairperson, don't I need to pick k-1 people for the committee...
- Holy mother of convoluted answer... — 9
  40 minutes going mad about that, just to find this answer, jesus christ... If you're still suffering, my hin...
- I'm not sure how this relates to what we just learned

This is the algebra related to what we just did, but does not really relate to what we were actually disc... 3

This question would have been much simpler, if it is worded this way 1
Different ways to form a committee: **one way:** We start with a pool of n people. A chaired co...

It's simple, don't be stressed, but be careful 1
In the last minute of the previous video you can understand how to proceed in this equality. But you ...

It is easier than you think 1
At first I thought it was really difficult, but then I start to took the first person who is the chairman. Hi...

got the first part but how to calculate values of alpha, beta, gamma... 1 new
I got the value of c as a function of n. fairly simple with the help of revisiting the lecture still i'm lost at ...

If you are stuck, refer to Example 1.31 of the textbook 7
If you are stuck, refer to Example 1.31 of the textbook. You will know what I mean when you see it.

Don't get intimidated 2
This is kind of an "uh, duh!" once you get it.

Key observation 2