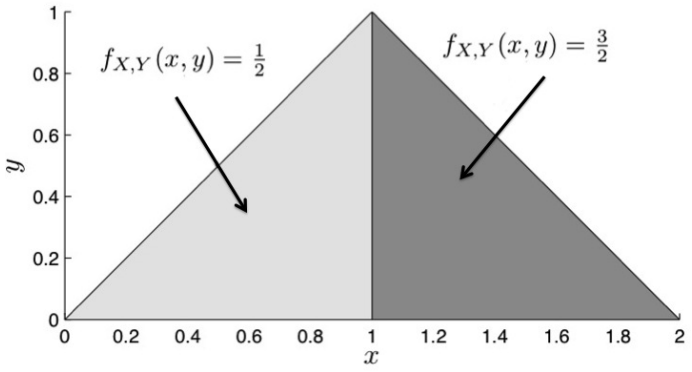
7. A joint PDF on a triangular region

Problem Set due Mar 13, 2020 05:29 IST *Completed*

Problem 7. A joint PDF on a triangular region

10/10 points (graded)

This figure below describes the joint PDF of the random variables X and Y. These random variables take values in $[0, 2]$ and $[0, 1]$, respectively. At $x = 1$, the value of the joint PDF is $1/2$.



1. Are X and Y independent?

- ◯ Yes
- ◉ No

✔

2. Find $f_X(x)$. Express your answers in terms of x, using the standard notation.

If $0 < x \leq 1$:

$f_X(x) =$

x/2

✔ **Answer:** x/2

$\frac{x}{2}$

If $1 < x < 2$:

$f_X(x) =$

-3*x/2+3

✔ **Answer:** 3-(3*x)/2

$-\frac{3 \cdot x}{2} + 3$

If $x < 0$ or $x \geq 2$:

$f_X(x) =$ 0 ✔ **Answer:** 0

0

3. Find $f_{Y|X}(y \mid 0.5)$.

If $0 < y < 1/2$:

$f_{Y|X}(y \mid 0.5) =$ 2 ✔ **Answer:** 2

If $y < 0$ or $y > 1/2$:

$f_{Y|X}(y \mid 0.5) =$ 0 ✔ **Answer:** 0

4. Find $f_{X|Y}(x \mid 0.5)$.

If $1/2 < x < 1$:

$f_{X|Y}(x \mid 0.5) =$ 1/2 ✔ **Answer:** 1/2

If $1 < x < 3/2$:

$f_{X|Y}(x \mid 0.5) =$ 3/2 ✔ **Answer:** 3/2

If $x < 1/2$ or $x > 3/2$:

$f_{X|Y}(x \mid 0.5) =$ 0 ✔ **Answer:** 0

5. Let $R = XY$ and let A be the event that $\{X < 0.5\}$. Find $\mathbf{E}[R|A]$.

$\mathbf{E}[R \mid A] =$ 1/16 ✔ **Answer:** 1/16

STANDARD NOTATION

Solution:

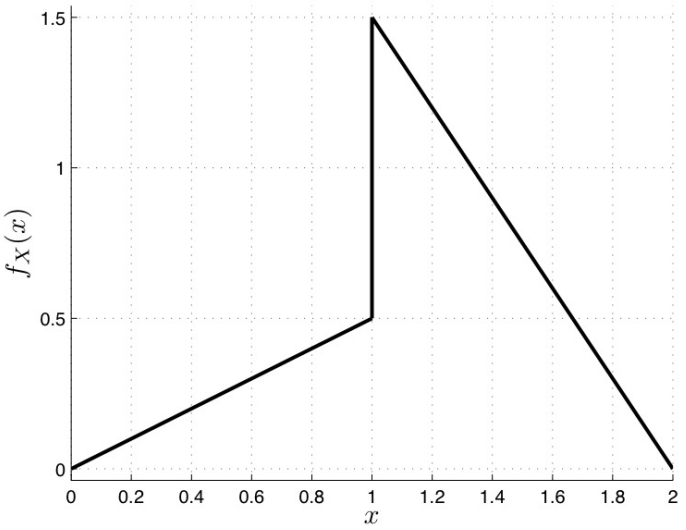
1. In order for X and Y to be independent, the value of X should not give any information about Y. But if X is smaller than say 0.5, then we can infer that $Y < 0.5$.

   In other words, $f_{Y|X}(y \mid 0.5) \neq f_Y(y)$. Therefore, X and Y are not independent.

2. Using the formula $f_X(x) = \int f_{X,Y}(x,y)\,dy$, we have,

$$f_X(x) = \begin{cases} \int_0^x \frac{1}{2}\,dy, & \text{if } 0 < x \leq 1, \\ \int_0^{2-x} \frac{3}{2}\,dy, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise,} \end{cases}$$
$$= \begin{cases} x/2, & \text{if } 0 < x \leq 1, \\ -3x/2 + 3, & \text{if } 1 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

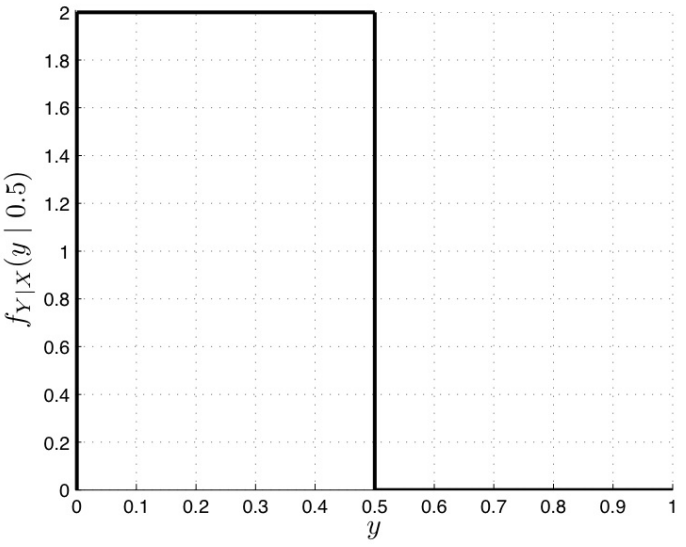
   A plot of the PDF is shown below:



3. Given that $X = 0.5$, Y is uniformly distributed between 0 and $1/2$. Thus,

$$f_{Y|X}\left(y \mid 0.5\right) = \begin{cases} 2, & \text{if } 0 \leq y \leq 1/2, \\ 0, & \text{otherwise.} \end{cases}$$

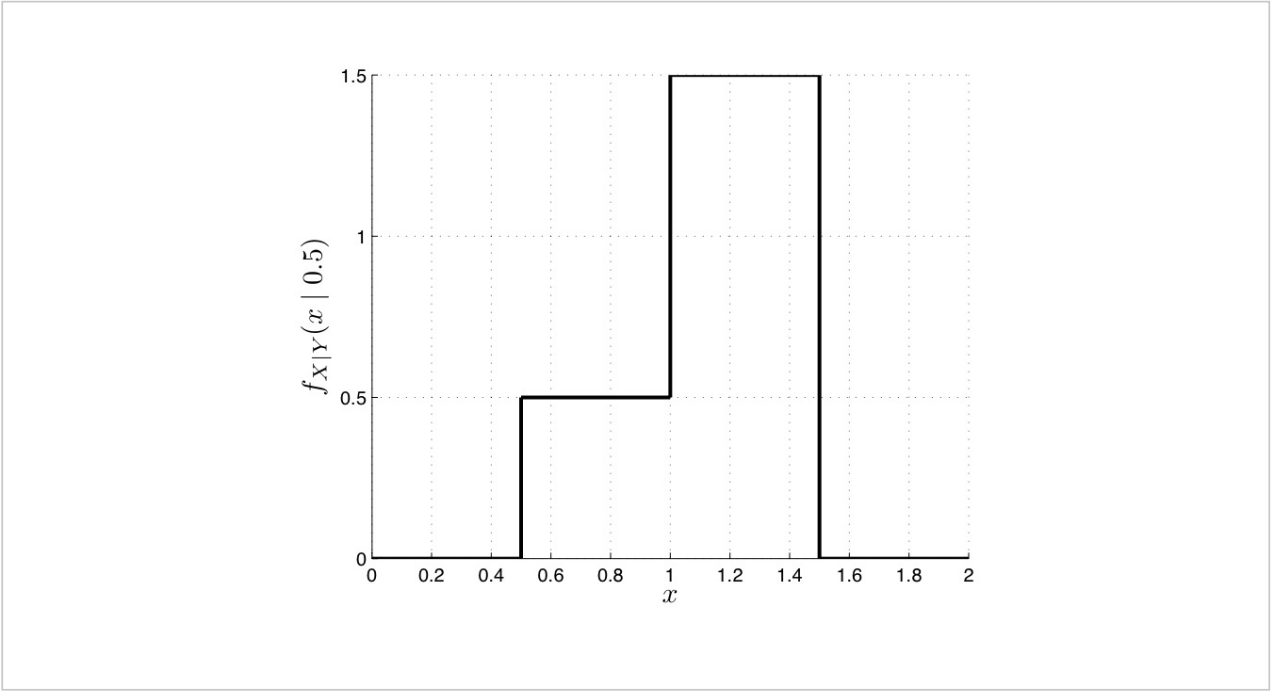
A plot of the conditional PDF is shown below:



4. Given that $Y = 0.5$, the conditional distribution of X is piecewise constant:

$$f_{X|Y}\left(x \mid 0.5\right) = \begin{cases} 1/2, & \text{if } 1/2 \leq x \leq 1, \\ 3/2, & \text{if } 1 < x \leq 3/2, \\ 0, & \text{otherwise.} \end{cases}$$

A plot of the conditional PDF is shown below:



5. Under event A, the pair (X, Y) takes values in a triangular region with sides of length $1/2$, and area $1/8$. The conditional point PDF is uniform, so that $f_{X,Y|A}\left(x, y\right) = 8$ on that set. The conditional expectation is

$$\begin{aligned} \mathbf{E}\left[R \mid A\right] &= \mathbf{E}\left[XY \mid A\right] \\ &= \int\int xy f_{X,Y|A}\left(x, y\right)\ dx\ dy \\ &= \int_0^{0.5} \int_y^{0.5} 8xy\ dx\ dy \\ &= 1/16. \end{aligned}$$

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You have used 3 of 5 attempts

Answers are displayed within the problem

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Not understanding 3 (reviewing for exam) I don't understand getting the marginal PDF of Y here. I intuitively guessed 2 since the total probability of the "slice" had...	2
E[X|A] and E[Y|A]	1
7.4	10
The marginal distribution of y Now that the answers are here, can anybody show me how to derive the marginal distribution of y from the joint distrib...	6
7.4 Can anybody provide a solution to 7.4 using integrals? I understand it intuitively, but even after reading through all the ...	2
How do we know the conditional PDF is uniform/piecewise constant in 7.3 and 7.4? I struggled with integrations on these two problems, but the solutions don't show any integrations. Did we need to inte...	8
Understanding of part 5	5
Why is the integral for Q2 part B from 0 to 2x-3 please? Still having trouble understanding how to determine integral range for marginal PDF. Any guide appreciated.	2
The same bug keeps happening	4
How many hours did everybody spend on P-set 5? How many hours did everybody spend on P-set 5?	21
[Staff] - I have got some of the answer correct for above problem and have saved it as well , but when i tried submitting it exam got closed [Staff] - I have got some of the answer correct for above problem and have saved it as well , but when i tried submitting ...	2
Additional study material The classes and Solved Exercises we had on Unit 5 was not enough for me to have a good comprehension about all exe...	4
7.4 Limits of Integration for fY(0.5) Having stipulated the limits for 0.5 < x < 1 and 1 < x < 1.5 in the 1st and 2nd part of question respectively, why the integr...	7
ran out of time	2