6. Tossing a triple of coins

Problem Set due May 1, 2020 05:29 IST *Completed*

Problem 6. Tossing a triple of coins

8/8 points (graded)

We have a red coin, for which $\mathbf{P}(\text{Heads}) = 0.4$, a green coin, for which $\mathbf{P}(\text{Heads}) = 0.5$, and a yellow coin, for which $\mathbf{P}(\text{Heads}) = 0.6$. The flips of the same or of different coins are independent. For each of the following situations, determine whether the random variable N can be approximated by a normal.

If yes, enter the mean and variance of N. If not, enter 0 in both of the corresponding answer boxes.

1. Let N be the number of Heads in 300 tosses of the red coin.

   mean: 120 ✔ **Answer:** 120

   Variance: 72 ✔ **Answer:** 72

2. Let N be the number of Heads in 300 tosses. At each toss, one of the three coins is selected at random (either choice is equally likely), and independently from everything else.

   mean: 150 ✔ **Answer:** 150

   variance: 75 ✔ **Answer:** 75

3. Let N be the number of Heads in 100 tosses of the red coin, followed by 100 tosses of the green coin, followed by 100 tosses of the yellow coin (for a total of 300 tosses).

mean: 150 ✔ **Answer:** 150

variance: 73 ✔ **Answer:** 73

4. We select one of the three coins at random: each coin is equally likely to be selected. We then toss the selected coin 300 times, independently, and let N be the number of Heads.

mean: 0 ✔ **Answer:** 0

variance: 0 ✔ **Answer:** 0

Solution:

For each of the following parts let X_i be a random variable that takes value 1 if the ith toss is Heads and takes value 0 otherwise.

1. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter 0.4. Here, $\mathbf{E}[N] = 300 \cdot 0.4 = 120$ and $\mathsf{Var}(N) = 300 \cdot 0.4 \cdot (1 - 0.4) = 72$.

2. $N = \sum_{i=1}^{300} X_i$. The CLT applies and N can be approximated by a normal because the X_i are independent and identically distributed Bernoulli random variables with parameter 0.5. Here $\mathbf{E}[N] = 300 \cdot 0.5 = 150$ and $\mathsf{Var}(N) = 300 \cdot 0.5 \cdot (1 - 0.5) = 75$.

3. Let $Y_1 = \sum_{i=1}^{100} X_i$, $Y_2 = \sum_{i=101}^{200} X_i$, and $Y_3 = \sum_{i=201}^{300} X_i$, such that $N = Y_1 + Y_2 + Y_3$. The CLT applies and Y_1 can be approximated by a normal because the X_i for $i = 1, \ldots, 50$ are independent and identically distributed Bernoulli random variables with parameter 0.4. Using a similar argument, Y_2, and Y_3 can also be approximated by normal random variables. Since Y_1,Y_2, and Y_3 are all independent, we conclude that N can also be approximated by a normal. Here,

$$\mathbf{E}[N] = \mathbf{E}[Y_1] + \mathbf{E}[Y_3] + \mathbf{E}[Y_3] = 100 \cdot 0.4 + 100 \cdot 0.5 + 100 \cdot 0.6 = 150.$$

Similarly,

$$\begin{aligned}\mathrm{Var}\left(N\right) &= \mathrm{Var}\left(Y_1\right) + \mathrm{Var}\left(Y_2\right) + \mathrm{Var}\left(Y_3\right) \\ &= 100 \cdot 0.4 \cdot (1 - 0.4) + 100 \cdot 0.5 \cdot (1 - 0.5) + 100 \cdot 0.6 \cdot (1 - 0.6) \\ &= 73.\end{aligned}$$

4. The CLT does not apply in this case as N is approximately a mixture of three normals.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

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- I don't get why the law of total variance doesn't work in #2 though. — 3
  Using the formula on the Wikipedia page, for example, I get 673, which is obviously not the right answer. ht...
- Qu for Staff - Grades for second submission not counted — 1
  Hi, I resubmitted answers to part 4 and my mark for the question was shown as 7/8 last night however my ...
- Hints for all — 7
- Similarities to Assignment Problem — 1
  This problem has structural similarities to problem: CLT Applicability, in Lecture 19.
- [Staff] Law of total variance for Q2 — 17
  Community TA
- Help with the 4. — 12
  For the mean I've used the law of total expectations. First calculating the number of heads conditioned to t...
- [Staff] Could the deadline be delayed by one day? — 1
  This week's project on the *Machine Learning with Python* course was very demanding and time consumi...
- Law of Total Variance for Question 2 ? — 6
  I'm trying to solve question #2 with Law of Total Variance but without success for the moment... Based on t...