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Cross Validated

Sum of a random number of r.v.'s [closed]

Asked 8 months ago Active 8 months ago Viewed 136 times

2

Closed. This question is off-topic. It is not currently accepting answers.

Want to improve this question? Update the question so it's on-topic for Cross Validated.

Closed 9 months ago.

A fair coin is flipped independently until the first Heads is observed. Let the random variable **K** be the number of tosses until the first Heads is observed plus 1. For example, if we see **TTTHTH**, then **K=5**. For **k=1,2,…,K** , let X_k be a continuous random variable that is uniform over the interval **[0,5]**. The X_k are independent of one another and of the coin flips.

Let X=$\sum_{k=1}^{k} X_k$. Find the mean and variance of **X** . You may use the fact that the mean and variance of a geometric random variable with parameter **p** are $\frac{1}{p}$ and $\frac{(1-p)}{(p^2)}$ respectively.

1. What is $E[X]$?
2. What is $Var[X]$?

My solution:

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$E[X_k] = \frac{1}{2} \times 5 = \frac{5}{2}$

$E[X] = E[N] \times E[X_k] = 2 \times \frac{5}{2} = 5$

$Var[N] = \frac{(1-p)}{p^2} = 2$

$Var[X_k] = \frac{1}{12} \times 5^2 = \frac{25}{12}$

probability self-study random-variable geometric-distribution

asked Jul 13 '19 at 3:57
Joseph Calvin
41 2

Avoid cross-posting: math.stackexchange.com/q/3291498/321264. Better to choose where you want to ask. – StubbornAtom Jul 13 '19 at 4:39

I saw this question on this site (stats.stackexchange.com) hours ago. – user158565 Jul 13 '19 at 4:50

Please decide which site you want to ask your question on, & delete the other version. Cross-posting is against SE policy & it wastes a lot of people's time. – gung - Reinstate Monica ♦ Jul 13 '19 at 12:06

I'm voting to close this question as off-topic because it is cross-posted on Mathematics. – gung - Reinstate Monica ♦ Jul 13 '19 at 12:06

2 Answers

Active Oldest Votes

2

Firstly, I assume $N = K$ in your solutions. The expected value and variance of X can be found via Law of Iterated Expectation (LIE) and Law of Total Variance (LTV):

$$E[X] = E[E[X|K]], \quad \text{var}(X) = E[\text{var(X|K)}] + \text{var}(E[X|K])$$

For the expectation, your approach is correct, but it can be found via LIE:

$$E[X|K] = KE[X_k] \rightarrow E[KE[X_k]] = E[K]E[X_k]$$

You just need to correct your expectation for K: $E[K] = 1/p + 1$, since it is of the form $1 + Y$, where Y is a geometric RV with parameter p. Also, note that $\text{var}(K) = \text{var}(1 + Y) = \text{var}(Y) = (1 - p)/p^2$ as yours.

For the variance, we need $\text{var}(X|K) = \text{var}(\sum X_k|K) = K\,\text{var}(X_k)$, and by LTV:

$$\begin{aligned}\text{var}(X) &= E[K\,\text{var}(X_k)] + \text{var}(KE[X_k]) \\ &= \text{var}(X_k)E[K] + E[X_k]^2\,\text{var}(K)\end{aligned}$$

The rest is substitution.

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24.2k 1 15 37

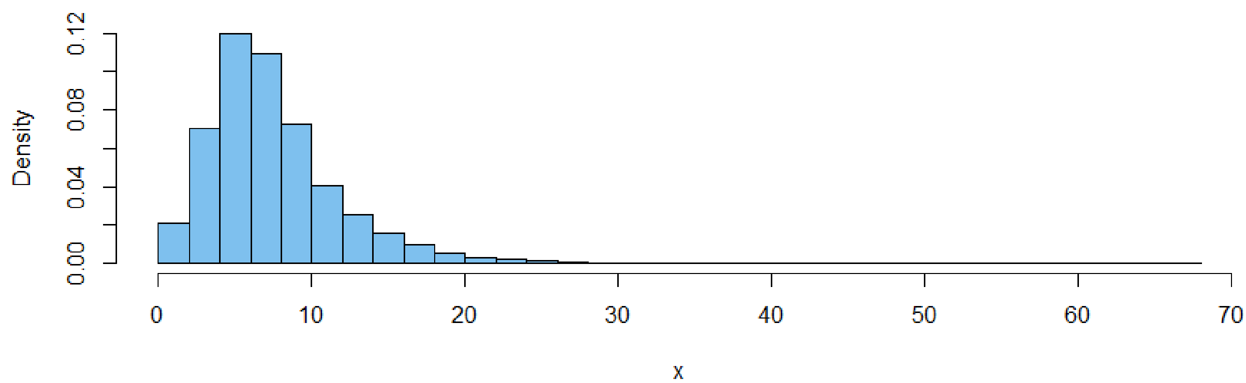
1

Formulas for the mean and variance of X are stated and derived here. Notice that there are two components to $V(X)$.

If you're interested in *distribution* of X you can do a simulation. In R, the a geometric random variable counts the number of failures before the first success; this is the *second* version in Wikipedia. With a million iterations one can expect about 3 significant digits of accuracy.

```
set.seed(712)
x = replicate(10^6,  sum(runif(rgeom(1,.5)+2,0,5)))
mean(x);  var(x)
[1] 7.497338     # aprx E(X) = 15/2
[1] 18.73797     # aprx V(X) = 75/4
mean(x < 10)
[1] 0.787989     # aprx P(X<10) = 0.788 +/- 0.001
2*sd(x < 10)/10^3
[1] 0.0008174656 # aprx 95% marg of sim err

hist(x, prob=T, br=30, col="skyblue2", main="")
```



edited Jul 13 '19 at 7:10

answered Jul 13 '19 at 5:59

BruceET

18k 1 11 37

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