8. Exercise: A criterion for independence

Exercises due Feb 28, 2020 05:29 IST *Completed*

Exercise: A criterion for independence

1/1 point (graded)

Suppose that the conditional PMF of X, given $Y = y$, is the same for every y for which $p_Y(y) > 0$. Is this enough to guarantee independence?

Yes ✔ **Answer:** Yes

Solution:

The condition given means that when I tell you the value of Y, the conditional PMF of X will be the same. Thus, the value of Y makes no difference, and, intuitively, we have independence.

For a formal argument, let $c(x) = p_{X|Y}(x \mid y)$; we can define $c(x)$ this way (without a dependence on y) since we are assuming that $p_{X|Y}(x \mid y)$ is the same for all y. Now,

$$p_{X,Y}(x,y) = p_Y(y)\, p_{X|Y}(x \mid y) = p_Y(y)\, c(x).$$

Summing over all y, we obtain

$$p_X(x) = \sum_y p_{X,Y}(x,y) = \sum_y p_Y(y)\, c(x) = c(x).$$

Therefore, $c(x) = p_X(x)$. It follows that $p_{X,Y}(x,y) = p_{X|Y}(x \mid y)\, p_Y(y) = c(x)\, p_Y(y) = p_X(x)\, p_Y(y)$, which establishes independence.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 4: Discrete random variables:Lec. 7: Conditioning on a random variable; Independence of r.v.'s / 8. Exercise: A criterion for independence

Show all posts by recent activity

- ? May have over-thought the question — 1
  My first reaction to the question was it depends...same as ***what***? It seems to me that it makes a di...
- Lec. 7. 8. Exercise: A criterion for independence — 7
- ? Independence? — 1 new_
  What if the PMF of X, given Y=y, is the same for every y for which pY(y) > 0 but different for y for which pY...