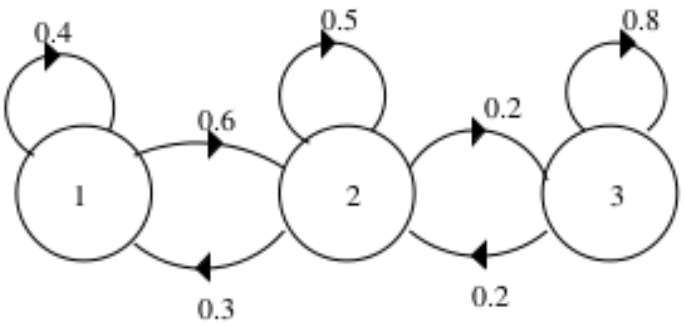
16. Exercise: Birth and death

None due May 29, 2020 05:29 IST

Exercise: Birth and death

5 points possible (ungraded)

Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following probabilities, assuming that when we start observing the chain, it is already in steady-state.



1. The steady-state probabilities for each state.

$\pi_1 =$ **Answer:** 0.2

$\pi_2 =$ **Answer:** 0.4

$\pi_3 =$ **Answer:** 0.4

2. The probability that the first transition we observe is a birth.

Answer: 0.2

3. The probability that the first change of state we observe is a birth.

Answer: 0.36

Solution:

1. The local balance equations take the form $0.6\pi_1 = 0.3\pi_2$ and $0.2\pi_2 = 0.2\pi_3$. Together with the normalization equation, we get $\pi_1 = 1/5$, $\pi_2 = \pi_3 = 2/5$.

2. We observe a birth if (i) we are in state 1 and the next transition is from 1 to 2, or (ii) we are in state 2 and the next transition is from 2 to 3. Hence, the desired probability is $\pi_1 p_{12} + \pi_2 p_{23} = 1/5$.

3. Note that a self-transition is not a change of state. If the state is 1, which happens with probability $1/5$, the first change of state is certain to be a birth. If the state is 2, which happens with probability $2/5$, the next change of state is to either 1 or 3. The probability that it is to 3 (i.e., a birth) is $p_{23}/\left(p_{21} + p_{23}\right) = 0.2/\left(0.3 + 0.2\right) = 2/5$. Finally, if the state is 3, the probability that the first change of state is a birth is equal to 0 since 3 is the highest state. Thus, the probability that the first change of state that we observe is a birth is equal to $\left(1/5\right)\left(1\right) + \left(2/5\right)\left(2/5\right) = 9/25$.

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Answers are displayed within the problem

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Hint for Q3

"Change of state" means we need to change state, so 1 transitions back to 1 is not a "Change of state". T..