4. Serap and her umbrella

Problem Set due Feb 12, 2020 05:29 IST *Completed*

Problem 4. Serap and her umbrella

4.0/6.0 points (graded)

Before leaving for work, Serap checks the weather report in order to decide whether to carry an umbrella. On any given day, with probability 0.2 the forecast is “rain" and with probability 0.8 the forecast is “no rain". If the forecast is “rain", the probability of actually having rain on that day is 0.8. On the other hand, if the forecast is “no rain", the probability of actually raining is 0.1.

1. One day, Serap missed the forecast and it rained. What is the probability that the forecast was “rain"?

   2/3 ✔ **Answer:** 0.66667

2. Serap misses the morning forecast with probability 0.2 on any day in the year. If she misses the forecast, Serap will flip a fair coin to decide whether to carry an umbrella. (We assume that the result of the coin flip is independent from the forecast and the weather.) On any day she sees the forecast, if it says “rain" she will always carry an umbrella, and if it says “no rain" she will not carry an umbrella. Let U be the event that “Serap is carrying an umbrella", and let N be the event that the forecast is “no rain". Are events U and N independent?

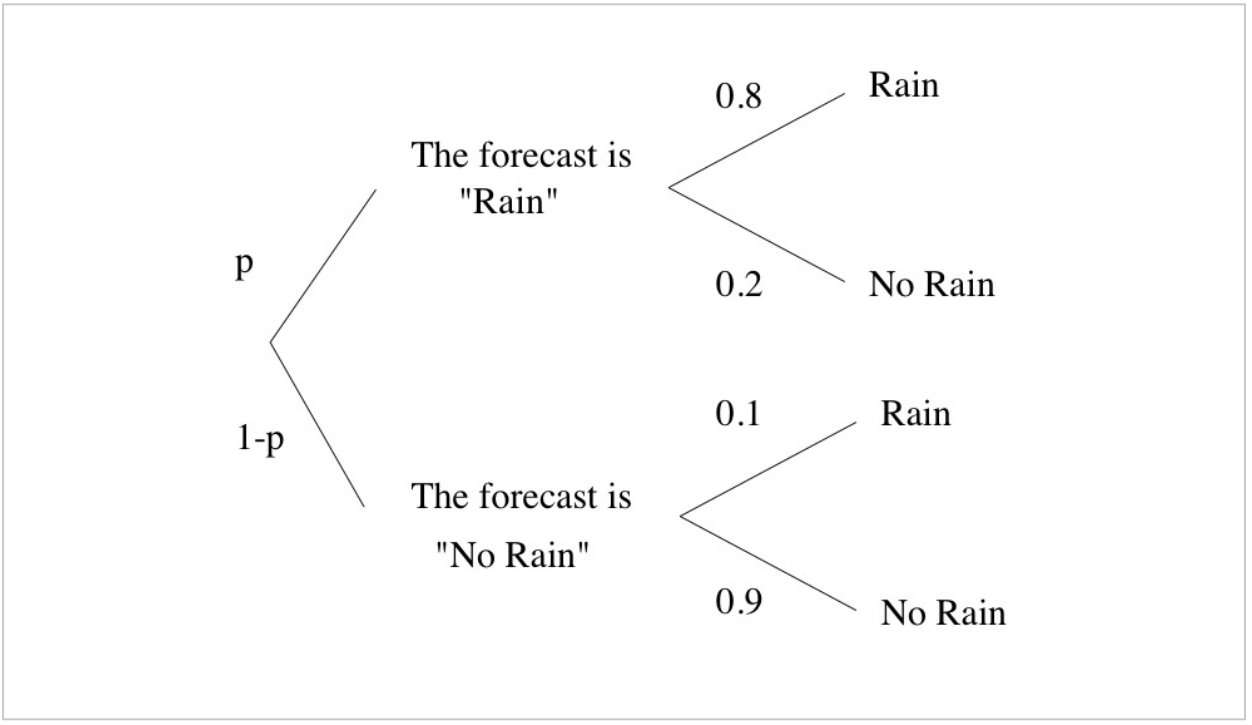
   No ✔ **Answer:** No

3. Serap is carrying an umbrella and it is not raining. What is the probability that she saw the forecast?

   ✖ **Answer:** 0.29630

Solution:

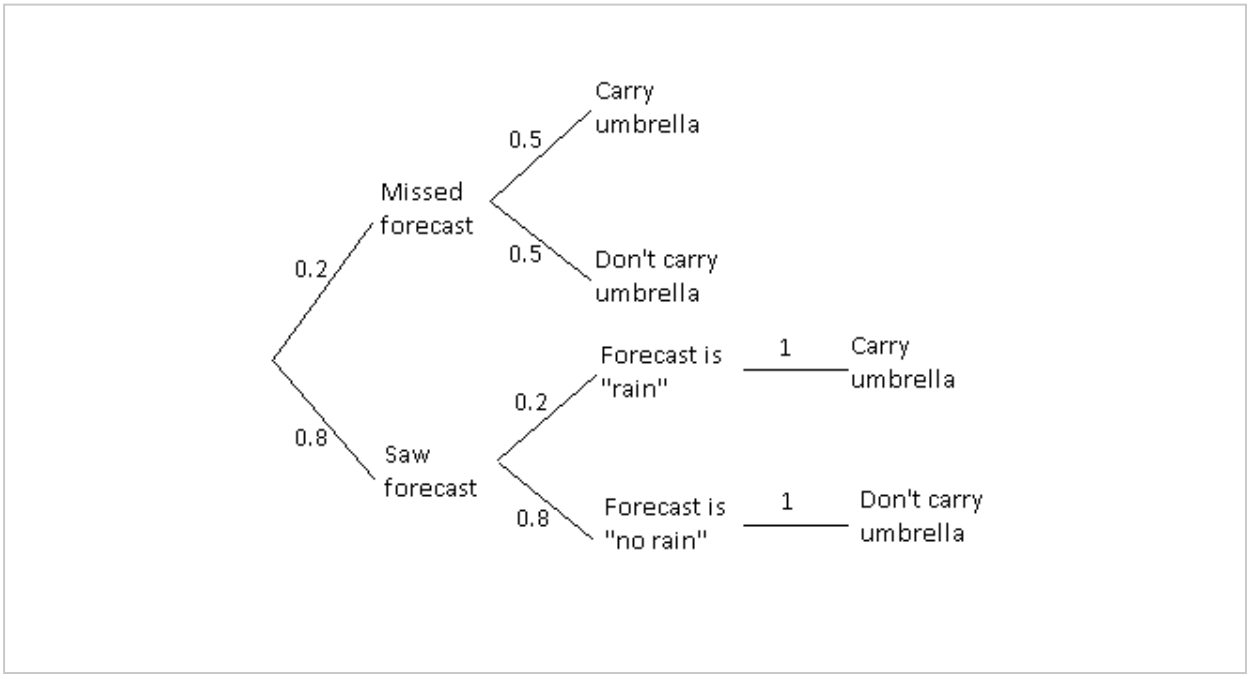
1. We can represent the scenario using the following tree diagram, where $p = 0.2$ is the probability that the forecast is “rain".



Let A be the event that the forecast was “rain", and let B be the event that it rained. Then,

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A)\,\mathbf{P}(B \mid A)}{\mathbf{P}(B)} = \frac{(0.2)(0.8)}{(0.2)(0.8) + (0.8)(0.1)} = \frac{2}{3}.$$

2. The tree diagram in this case is the following:



$$\begin{aligned}
\mathbf{P}(U) &= (0.2)(0.5) + (0.8)(1)(0.2) = 0.26 \\
\mathbf{P}(N) &= 0.8 \\
\mathbf{P}(U \cap N) &= \mathbf{P}(U \cap N | \text{Missed forecast})\,\mathbf{P}(\text{Missed forecast}) \\
&\quad + \mathbf{P}(U \cap N | \text{Saw forecast})\,\mathbf{P}(\text{Saw forecast}) \\
&= (0.5 \cdot 0.8)(0.2) + (0)(0.8) = 0.08
\end{aligned}$$

Since $\mathbf{P}(U \cap N) \neq \mathbf{P}(U)\,\mathbf{P}(N)$, the two events are not independent.

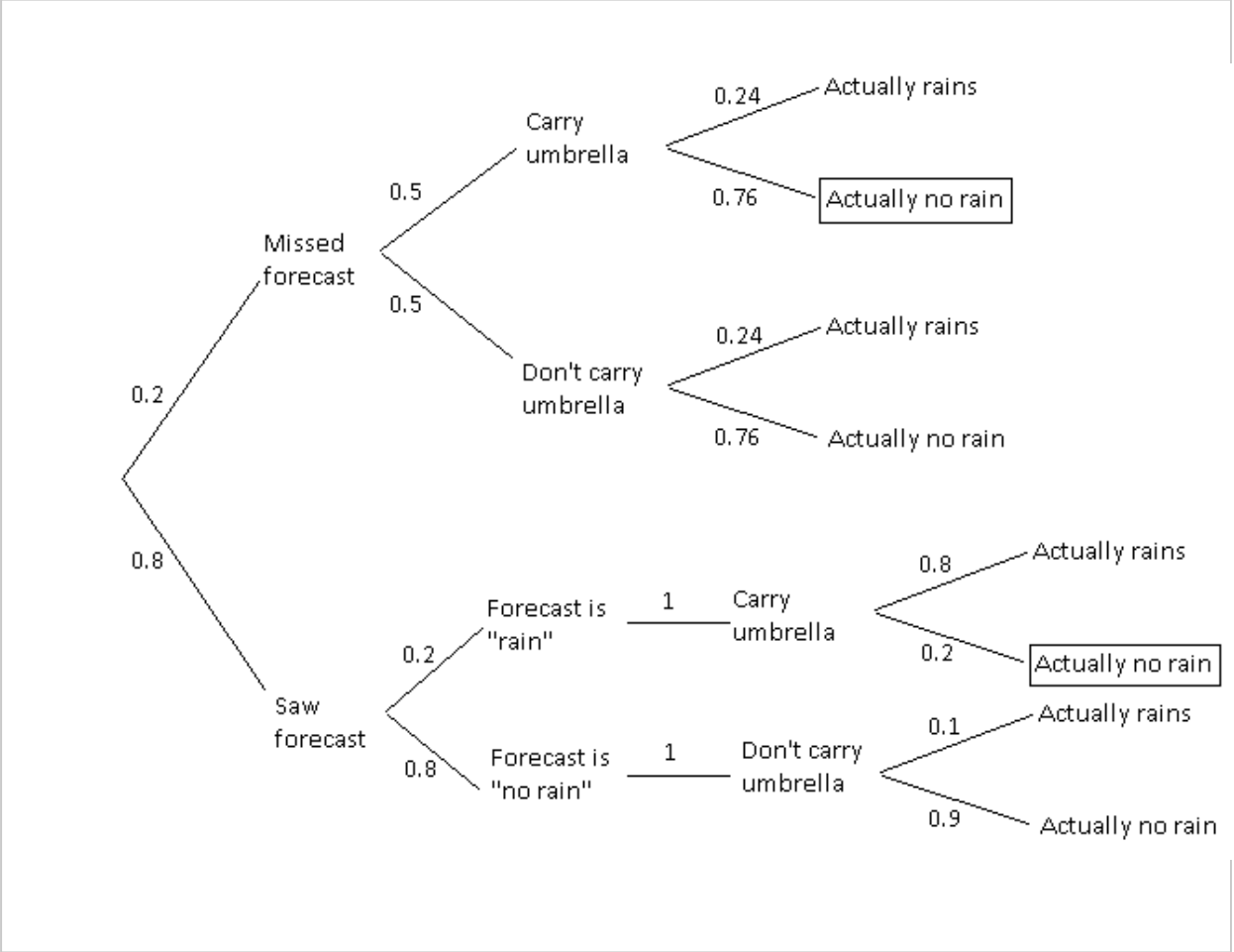
3. Let us first find the probability that it actually rains:

$$\mathbf{P}(\text{Actually rains}) = (0.8)(0.2) + (0.1)(0.8) = 0.24.$$

Thus,

$$\mathbf{P}(\text{Actually no rain}) = 1 - 0.24 = 0.76.$$

We can then extend the tree from part 2 as follows:



The event that Serap is carrying an umbrella and it is not raining corresponds to the two boxed cases in the figure above, of which the lower one is the one where Serap saw the forecast. Therefore, the desired probability is

$$\begin{aligned}&\mathbf{P}\left(\text{Saw forecast} \mid \text{Umbrella and actually no rain}\right)\\&= \frac{(0.8)\,(0.2)\,(1)\,(0.2)}{(0.8)\,(0.2)\,(1)\,(0.2) + (0.2)\,(0.5)\,(0.76)}\\&= \frac{8}{27}.\end{aligned}$$

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You have used 1 of 5 attempts

Answers are displayed within the problem

Discussion

Topic: Unit 2: Conditioning and independence:Problem Set 2 / 4. Serap and her umbrella

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I couldn't solve #3 along util....	I realize we need to use the information in #2.	2
Part 3	In general and taking example of part 3 , how do we know if the question is asking to: calculate proba...	5
Tree Diagram for Part 3	I'm thinking of setting up a tree diagram for Part 3 but I'm not sure how start setting up the tree. Sho...	5
Struggling w/ #1 - Do you need missed forecast for equation	I tried using total probability it rained plus probability the forecast said rain.	2
How many decimal places for part C?	I believe I have answered the question correctly (I came to the same solution others have hinted at h...	4
Q3 wrong answer, can you check my probs?		1
Probability zero	Doesn't carrying an umbrella when it's not raining have a probability of zero? I'm stuck on that for q3...	6
Hint Q3	So it took me a while to figure out how to do it, so sharing some some Valuable hints. When extendin...	1 new
Part 3 depends on Part 2?	I do not understand if I need to use additional information from Part 2 to to solve Part 3. It does not s...	4
Part 3 - Carrying an umbrella and seeing the forecast	Hi all, The second part tell us that first there is a probability of Serap missing the forecast. If she miss...	12
Am i correct in saying Q2 cannot be answered mathematically?	As its not given if the coin is fair or biased. In other words we must answer this intuitively? or it must ...	4
any hint for first 1?	I got correct 2nd and 3rd but I don't understand what is wrong with the first one	4
implication of independence/dependence in Bayes rule.	I got Q2 and Q3 correct but I did not mathematically carried out my response of Q2 in Q3. I just solve...	2