6.

Mid Term due Mar 4, 2020 05:29 IST *Completed*

For all problems on this page, use the following setup:

Let N be a positive integer random variable with PMF of the form

$$p_N(n) = \frac{1}{2} \cdot n \cdot 2^{-n}, \qquad n = 1, 2, \ldots .$$

Once we see the numerical value of N, we then draw a random variable K whose (conditional) PMF is uniform on the set $\{1, 2, \ldots, 2n\}$.

Joint PMF

0/1 point (graded)

Write down an expression for the joint PMF $p_{N,K}(n,k)$.

For $n = 1, 2, \ldots$ and $k = 1, 2, \ldots, 2n$:

$p_{N,K}(n,k) =$ 2*n*n ✖ **Answer:** (1/2)^(n+2)

$2 \cdot n \cdot n$

STANDARD NOTATION

Solution:

We are given that:

$$p_{K|N}(k \mid n) = \frac{1}{2n}, \qquad k = 1, 2, \ldots, 2n. \tag{7.2}$$

By definition:

$$p_{N,K}(n,k) = p_{K|N}(k \mid n)\, p_N(n) = \frac{1}{2n}\frac{1}{2} \cdot n \cdot 2^{-n} = \left(\frac{1}{2}\right)^{n+2}, \qquad n = 1, 2, \ldots, \quad k = 1, 2, \ldots, 2n \quad (7.3)$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Marginal Distribution

0.0/1.5 points (graded)

Find the marginal PMF $p_K(k)$ as a function of k. For simplicity, provide the answer **only for the case when k is an even number**. (The formula for when k is odd would be slightly different, and you do not need to provide it).

Hint: You may find the following helpful: $\sum_{i=0}^{\infty} r^i = \dfrac{1}{1-r}$ for $0 < r < 1$.

For $k = 2, 4, 6, \ldots$:

$p_K(k) =$ 1 ✖ **Answer:** (1/2)^(k/2+1)

1

STANDARD NOTATION

Solution:

Solution

Observe that in the infinite sum $p_K(k) = \sum_{n=1}^{\infty} p_{N,K}(n,k)$ only the terms from $n = k/2$ and above have non-zero probability. Indeed, $K = k = 4$ has probability 0 if $n < k/2 = 4/2 = 2$.

Hence:

$$\begin{aligned}
p_K(k) &= \sum_{n=k/2}^{\infty} p_{N,K}(n,k) = \sum_{n=k/2}^{\infty} \left(\frac{1}{2}\right)^{n+2} \\
&= \sum_{n=k/2}^{\infty} \left(\frac{1}{2}\right)^{n+2} = \frac{1}{4} \sum_{n=k/2}^{\infty} \left(\frac{1}{2}\right)^{n} \\
&= \frac{1}{4}\left[\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n} - \sum_{0}^{k/2-1} \left(\frac{1}{2}\right)^{n}\right] \\
&= \frac{1}{4}\left[\frac{1}{1-\frac{1}{2}} - \frac{1-\left(\frac{1}{2}\right)^{k/2-1+1}}{1-\frac{1}{2}}\right]
\end{aligned}$$

$$= (\frac{1}{2})^{k/2+1} \qquad \text{for} k = 2, 4, \ldots$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discrete PMFs

0/2 points (graded)

Let A be the event that K is even. Find $P(A|N=n)$ and $P(A)$.

$P(A \mid N=n) =$ 0 0 ✗ **Answer:** 1/2

$P(A) =$ 1 1 ✗ **Answer:** 1/2

STANDARD NOTATION

Solution:

Let A be then event that K is even. We need to check whether $P(A \mid N=n) = P(A)$ is true for the event A to be independent of N.
Now because $p_{K|N}(k \mid n)$ is uniform over the $2n$-size set $\{1, 2, \ldots, 2n\}$ and there are exactly n even numbers in this set, we have that:

$$P(A \mid N=n) = \frac{n}{2n} = \frac{1}{2}, \qquad n \geq 1. \tag{7.4}$$

Intuitively, knowledge of n does not affect the beliefs about A, and we have independence. A full, formal argument goes as follows:

$$\begin{aligned} P(A) &= \sum_{n=1}^{\infty} P(A \mid N=n) P(N=n) \\ &= \frac{1}{2} \sum_{n=1}^{\infty} P(N=n) = \frac{1}{2}, \end{aligned}$$

where the last step follows because PMFs always sum to 1. So, $P(A \mid N=n) = P(A)$, for all n.
Equivalently, $P(A \text{ and } N=n) = P(A \mid N=n) \cdot P(N=n) = P(A) \cdot P(N=n)$, for all n, whi[illegible] defining property of independence.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Independence 2

0.5/0.5 points (graded)

Is the event A independent of N?

- (selected) yes
- no
- not enough information to determine

Solution:

See solution to problem above.

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Error and Bug Reports/Technical Issues

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So disappointed in this class
For the record, I've loved the lectures, and got a lot out of them. They were clear, informative and I took copious notes (63 pages ... 69

[STAFF] Passing grade
I only got 51% on the mid term; I understand that the pass grade is 60% at least. So do I still a chance of passing if I can 'drag' my ... 8

please, need full explanation of 2nd question (marginal distribution) :-) 4

Bug
Sometimes when I type in "Sigma," I get the summation symbol, sometimes just the word "Sigma." I've tried restarting Chrome, b... 7

No Score on Answers
Hi Staff, I submitted the answers (which look correct), but received no score on all my exam answers. Wonder if you can help on t... 5

Uniform from 1 to 2n

In Q1 -- uniform must have to be = 1/(2n -1), because the origin is 1 (not zero). Could you check it please?. Thks | 2

? Can't understand answer on marginals | 7

Error in the problem | 2
Dear Staff, I have one question related to last problem of the exam. If we sum up the probabilities of the joint PMF we get 1/4, ho...

Ambiguous problem statement | 5

? About the question regarding "Independence 2" | 9
"Is the event A independent of N?" - is it same as asking if the two events are independent or is it different? Also, are the two eve...

? Can I have partial credit for Q6, Discrete PMFs question? | 2

? Solution to Descrete PMF's | 2
The solution says "Let A be then event that K is even. We need to check whether $P(A|N=n)=P(A)$ is true for the event A to be indep...

[STAFF] What am I missing here... | 8

Connecting these questions to the course material | 2
Can anyone point to the text readings, lecture slides/transcripts, problems, lecture that cover the material this question convers?