1. Tosses of a biased coin

Problem Set due Feb 28, 2020 05:29 IST Completed

Problem 1. Tosses of a biased coin

7/7 points (graded)

Consider 10 independent tosses of a biased coin with the probability of Heads at each toss equal to p, where $0 < p < 1$.

1. Let A be the event that there are 6 Heads in the first 8 tosses. Let B be the event that the 9th toss results in Heads.

   Find $\mathbf{P}(B \mid A)$ and express it in terms of p using standard notation. (You can click on the “STANDARD NOTATION" button below.)

   p

   p

   ✔ **Answer:** p

2. Find the probability that there are 3 Heads in the first 4 tosses and 2 Heads in the last 3 tosses. Express your answer in terms of p using standard notation. Remember not to use ! or combinations in your answer.

   12*p^5*(1-p)^2

   $12 \cdot p^5 \cdot (1-p)^2$

   ✔ **Answer:** 12*p^5*(1-p)^2

3. Given that there were 4 Heads in the first 7 tosses, find the probability that the 2nd Heads occurred at the 4th toss. Give a numerical answer.

   9/35

   ✔ **Answer:** 9/35

4.

We are interested in calculating the probability that there are 5 Heads in the first 6 tosses and 3 Heads in the last 5 tosses. Give the exact numerical values of a, b, c, d that would match the answer $ap^7(1-p)^3 + bp^c(1-p)^d$.

$a =$ 30 ✔ **Answer:** 30

$b =$ 4 ✔ **Answer:** 4

$c =$ 8 ✔ **Answer:** 8

$d =$ 2 ✔ **Answer:** 2

STANDARD NOTATION

Solution:

1. Event A refers to the first 8 tosses and event B refers to the 9th toss. Since tosses are independent, the 9th toss is independent of the first 8 tosses, and so events A and B are independent. Thus, $\mathbf{P}(B \mid A) = \mathbf{P}(B) = p$.

2. Let C be the event "3 Heads in the first 4 tosses" and let D be the event "2 Heads in the last 3 tosses". Since there is no overlap in the tosses involved in events C and D, these two events are independent. Therefore,

$$\begin{aligned}\mathbf{P}(C \cap D) &= \mathbf{P}(C)\,\mathbf{P}(D) \\ &= \binom{4}{3} p^3 (1-p) \cdot \binom{3}{2} p^2 (1-p) \\ &= 12p^5(1-p)^2.\end{aligned}$$

3. Let E be the event "4 Heads in the first 7 tosses" and let F be the event "2nd Heads occurred on the 4th toss". We are asked to find $\mathbf{P}(F \mid E) = \mathbf{P}(F \cap E)/\mathbf{P}(E)$.

   The event $F \cap E$ occurs if there is 1 Heads in the first 3 tosses, Heads on the 4th toss, and 2 Heads in the next 3 tosses. Thus, we have

$$\begin{aligned}\mathbf{P}(F \mid E) &= \frac{\mathbf{P}(F \cap E)}{\mathbf{P}(E)} \\ &= \frac{\binom{3}{1} p(1-p)^2 \cdot p \cdot \binom{3}{2} p^2 (1-p)}{\binom{7}{4} p^4 (1-p)^3}\end{aligned}$$

$$= \frac{\binom{3}{1} \cdot 1 \cdot \binom{3}{2}}{\binom{7}{4}}$$

$$= \frac{9}{35}.$$

Alternatively, we can solve this problem by counting. We are given that 4 Heads occurred in the first 7 tosses. Each sequence of 7 tosses with 4 Heads is equally likely, and so the discrete uniform probability law can be used here. There are $\binom{7}{4}$ elements in E. For the event $E \cap F$, there are $\binom{3}{1}$ ways to arrange 1 Heads in the first 3 tosses, 1 way to arrange the 2nd Heads in the 4th toss, and $\binom{3}{2}$ ways to arrange 2 Heads in the next 3 tosses. Therefore,

$$\mathbf{P}(F \mid E) = \frac{\binom{3}{1} \cdot 1 \cdot \binom{3}{2}}{\binom{7}{4}} = \frac{9}{35}.$$

4. Let G be the event “5 Heads in the first 6 tosses" and let H be the event “3 Heads in the last 5 tosses". These two events are not independent as there is some overlap in the tosses, namely, the 6th toss. To compute the probability of interest, we partition the set $G \cap H$ into two (disjoint) subsets by considering separately the two possible results of the 6th toss:

$$\begin{aligned} G \cap H = \; & \{\text{4 Heads in tosses 1-5, 6th toss is Heads, 2 Heads in tosses 7-10}\} \\ \cup \; & \{\text{5 Heads in tosses 1-5, 6th toss is Tails, 3 Heads in tosses 7-10}\}. \end{aligned}$$

Therefore,

$$\begin{aligned} \mathbf{P}(G \cap H) &= \binom{5}{4} p^4 (1-p)^1 \cdot p \cdot \binom{4}{2} p^2 (1-p)^2 \\ &\quad + \binom{5}{5} p^5 \cdot (1-p) \cdot \binom{4}{3} p^3 (1-p) \\ &= 30p^7(1-p)^3 + 4p^8(1-p)^2. \end{aligned}$$

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You have used 3 of 5 attempts

Answers are displayed within the problem

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- ? Q2. the answer doesn't include the fact of the (first) or (last) tosses — 1
  Shouldn't we get the probability P(having 3 Heads out of 4 tosses) * P(having those 4 tosses be the first out...
- HW Due Date — 7 new_
  Both the syllabus and calendar say this is due on the 27th, whereas the course says its due on the 25th. Ple...
- [Staff] Due date preponed!? — 4
  Staff: I checked yesterday that the due date for problem set 4 was 2/28/2020 18:59 EST. I came back to sub...
- ? [STAFF] Understanding of Part 4: where is the mistake? — 6
- ? Question 1 Clarification for B — 3
  Perhaps its obvious but I have a silly question. Does B mean only 9th toss is head and none are head befor...
- [STAFF] Wrong due date shown for student!! Please do something — 9
  I appeal for this problem set . it says due date 28th not 27. That is so misleading and made me not able to ...
- STAFF-Part 3 — 9 new_ 13
  Hello, can u guide me how to solve n3.
- At least so many tosses? or exactly so many tosses — 3
  Part 2 asks "Find the probability that there are 3 Heads in the first 4 tosses and 2 Heads in the last 3 tosses...
- ? I get the sense that binomial coefficient is needed here, but the use of Standard Notation seems to suggest otherwise? — 2
  It seems to me that we need to use the binomial coefficient here (ie n choose k), but since we are explicitly ...
- Part 4 — 2 new_
  Any hints? My current path doesn't remotely resemble the form given, and I'm at a loss as to what other o...
- q2. Are there explanations available? I want to see it after the due date. — 2
  I have found the right answer for this question, but I don't agree with it because I don't understand how th...
- Do we need to consider all 10 tosses for all parts? — 4
  For instance, for part 1: Do we consider - 6 Heads in first 8 tosses 9th toss is a Heads AND 2 possible scena...
- ? part 4 - stuck! — 3
  for the beginning part of the value: A * P^7 * (1-p) ^ 3 . the exponents ^7 and ^3 imply a combination of 7