MATHEMATICS

What is the pdf of $Y = \log X$?

Asked 7 years, 5 months ago Active 7 years, 5 months ago Viewed 4k times

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Let X be a standard exponential random variable, and $Y = \log X$.

(a) Find DIRECTLY the c.d.f of Y and use it to calculate the density of Y.

(b) Find DIRECTLY the p.d.f of Y.

So far, I did:

(a) $F_Y(y) = P(\log X \leq y) = P(X \leq e^y) = F_X(e^y) f_Y(y) = F'_Y(y) = F'_X(e^y) = [f_X(e^y)][e^y]$
$= (e^{-e^y})(e^y) = e^{y-e^y}$

(b) $f_Y(y) = P(\log X = y) = P(X = e^y) = f_X(e^y) = e^{-e^y}$

How come the answers I got for part a and b are not the same. What did I do wrong?

probability-distributions

edited Oct 22 '12 at 6:03 mrs

asked Oct 22 '12 at 4:09 woaini

I can't seem to figure out how to use the MathJax syntax correctly. – woaini Oct 22 '12 at 4:10

1 Answer

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Because PDFs don't work like this. The issue is you need to change of variables on the probability $P(Y = y) = f_Y(y)dy$, and not just the density. More precisely, if $y = \log(x)$ then $dy = e^{-y}dx$, so $P(Y = y) = f_Y(y)dy = P(\log X = y) = f_X(e^y)dx = f_X(e^y)e^y dy$ so that you recover your answer from the first part.

answered Oct 22 '12 at 5:00

Alex R.
28.4k 1 28 60

Sorry but the identity $P(Y = y) = f_Y(y)dy$ is a monstrosity, which should be avoided at all cost. – Did Oct 22 '12 at 5:43

If y = log x, then shouldn't dy = (1/x) dx? – woaini Oct 22 '12 at 6:04

@woaini $y = \log x \implies e^y = x \implies e^y dy = dx \implies dy = e^{-y}dx$. Both are correct, since $\frac{1}{x} = e^{-y}$. – Robert Mastragostino Oct 22 '12 at 6:20

Just for future reference I believe that you need to use the pdf $f_{X>0}$ since $Y = \log(X)$ is only defined when $X > 0$. So $f_Y(y) = e^y f_{X>0}(e^y) = 2e^2 f_X(e^y)$. This pdf will integrate to 1 over $\mathbb{R}$ however $f_X(e^y)e^y$ will only integrate to $\frac{1}{2}$ over $\mathbb{R}$. You could also consider $Y = \log|X|$ and here we would avoid needing to condition $X > 0$ and we would again see that $f_Y(y) = 2e^y f_X(e^y)$ – alpastor Apr 12 '19 at 15:24