5. Exercise: Normal unknown and additive noise

Exercises due Apr 8, 2020 05:29 IST *Completed*

Exercise: Normal unknown and additive noise

4/4 points (graded)

As in the last video, let $X = \Theta + W$, where Θ and W are independent normal random variables and W has mean zero.

a) Assume that W has positive variance. Are X and W independent?

No ✔ **Answer:** No

b) Find the MAP estimator of Θ based on X if $\Theta \sim N(1, 1)$ and $W \sim N(0, 1)$, and evaluate the corresponding estimate if $X = 2$.

$\hat{\theta} =$ 1.5 ✔ **Answer:** 1.5

c) Find the MAP estimator of Θ based on X if $\Theta \sim N(0, 1)$ and $W \sim N(0, 4)$, and evaluate the corresponding estimate if $X = 2$.

$\hat{\theta} =$ 2/5 ✔ **Answer:** 0.4

d) For this part of the problem, suppose instead that $X = 2\Theta + 3W$, where Θ and W are standard normal random variables. Find the MAP estimator of Θ based on X under this model and evaluate the corresponding estimate if $X = 2$.

$\hat{\theta} =$ 4/13 ✔ **Answer:** 0.30769

Solution:

a) They are not independent. This is intuitively clear because W has an effect on X. Another way to see it is that we have (by independence of Θ and W) that $\mathbf{E}[\Theta W] = \mathbf{E}[\Theta]\,\mathbf{E}[W] = 0$, which leads to

$$\mathbf{E}[XW] = \mathbf{E}[(\Theta + W)\,W] = \mathbf{E}[W^2] \neq 0 = \mathbf{E}[X]\,\mathbf{E}[W],$$

which in turn implies that X and W are not independent.

b) If we focus on the terms that involve θ, the posterior is of the form

$$c(x)\,e^{-(\theta-1)^2/2}e^{-(x-\theta)^2/2}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $(\hat{\theta} - 1) + (\hat{\theta} - x) = 0$, or $\hat{\theta} = (1 + x)/2$, which, when $x = 2$, evaluates to $3/2$.

c) If we focus on the terms that involve θ, the posterior is of the form

$$c(x)\,e^{-\theta^2/2}e^{-(x-\theta)^2/(2\cdot 4)}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $\hat{\theta} + (\hat{\theta} - x)/4 = 0$, or $\hat{\theta} = x/5$, which, when $x = 2$, evaluates to $2/5$.

d) Note that conditional on $\Theta = \theta$, the random variable X is normal with mean 2θ and variance 9. If we focus on the terms that involve θ, the posterior is of the form

$$c(x)\,e^{-\theta^2/2}e^{-(x-2\theta)^2/(2\cdot 9)}.$$

To find the MAP estimate, we set the derivative with respect to θ of the exponent to zero, so that $\hat{\theta} + 2(2\hat{\theta} - x)/9 = 0$, or $\hat{\theta} = 2x/13$, which, when $x = 2$, evaluates to $4/13$.

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You have used 3 of 3 attempts

Answers are displayed within the problem

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- **?** Having trouble understanding distribution of X in part D — 3
  So from the lectures and the answers it seems that x takes 2*theta as a mean and var(3W) as varianc...
- **?** Ex : Normal unknown & additive noise — 1
  Hi, The assumption in (a) that W has a positive variance - is it actually required , since it is already give...
- separating the linear effects on μ and var — 2
  I just want to confirm the separate effects. If X = 2Θ, then μ_x would be 2Θ, right, and var(X) would be ...
- [STAFF] independence question (a) reasoning — 5
  For question a is it correct to notice that Var (f(X|w)) = Var(θ) while var (X) = Var(θ) + Var(W) Var(W)...
- **?** hint on question a — 2
  I don't really get the point of question a) ? What are they trying to tell us here? at a first glance, a varia...
- **?** Are we finding the estimate or the estimator? — 1
  I thought, being that x is taking on a particular value, that we are finding a "number" for theta (and so...
- **?** part d, need help with understanding the prior — 2 new_ 8
- **?** Help, please. I must not understand at all. — 5
  Hi. I got all wrong except the yes/no. I really think I am on the right track, but I can't be positive, I gues...
- **?** having difficulty calculating the posterior — 2
  Im not able to calculate the posterior of (b) please help
- **?** How the variance of Θ is plugged in? — 2 new_
  I suppose that after taking a derivative in a case, when **X = Θ + W** and the mean of Θ equals to ze...
- W / noise — 2
  Hi, Forgive me if I missed this somewhere. W~N(mu,sigma^2). However, when we calculate f_X|Theta,...
- Hint for b — 2
  Hello I need a hint for b. I thought that with Θ~N(1,1) , the θ^hat would be one higher than in the exa...

What about distribution of Θ and W in (d)?

3