7. Exercise: Independence

Exercises due Feb 28, 2020 05:29 IST *Completed*

Exercise: Independence

5/5 points (graded)

Let X, Y, and Z be discrete random variables.

a) Suppose that Z is identically equal to 3, i.e., $\mathbf{P}(Z=3)=1$. Is X guaranteed to be independent of Z?

Yes

✔ **Answer:** Yes

b) Would either of the following be an appropriate definition of independence of the pair (X,Y) from Z?

- $p_{X,Y,Z}(x,y,z) = p_X(x)\, p_Y(y)\, p_Z(z)$, for all x,y,z

  No

  ✔ **Answer:** No

- $p_{X,Y,Z}(x,y,z) = p_{X,Y}(x,y)\, p_Z(z)$, for all x,y,z

  Yes

  ✔ **Answer:** Yes

c) Suppose that X,Y,Z are independent. Is it true that X and Y are independent?

Yes

✔ **Answer:** Yes

d) Suppose that X,Y,Z are independent. Is it true that (X,Y) is independent from Z?

Yes

✔ **Answer:** Yes

Solution:

a) Since Z is deterministic, the value of Z does not provide any information, and so, intuitively, we have independence. For a formal argument, suppose that $z \neq 3$. Then, $p_{X,Z}(x,z) = 0 = p_X(x)\,p_Z(z)$. And for $z=3$, $p_{X,Z}(x,3) = \mathbf{P}(X=x, Z=3) = \mathbf{P}(X=x) = \mathbf{P}(X=x)\cdot 1 = p_X(x)\,p_Z(3)$, so that the definition of independence is satisfied.

b) The second definition is correct, because it says that events of the form $\{X=x \text{ and } Y=y\}$ are independent from events of the form $\{Z=z\}$. On the other hand, the first imposes the stronger requirement that X is also independent of Y.

c) Intuitively, since X, Y, Z are independent, none of the random variables provides information about the others. For a formal argument,

$$p_{X,Y}(x,y) = \sum_z p_{X,Y,Z}(x,y,z) = \sum_z p_X(x)\,p_Y(y)\,p_Z(z) = p_X(x)\,p_Y(y)\sum_z p_Z(z) = p_X(x)\,p_Y(y).$$

d) Intuitively, the value of the pair (X, Y) provides no information about the random variable Z. We will verify that the appropriate definition of independence of (X, Y) from Z from part (b) is satisfied. We first use independence of X, Y, Z, and then the fact, from part (c), that $p_X(x)\,p_Y(y) = p_{X,Y}(x,y)$, to obtain

$$p_{X,Y,Z}(x,y,z) = p_X(x)\,p_Y(y)\,p_Z(z) = p_{X,Y}(x,y)\,p_Z(z),$$

as desired.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

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Topic: Unit 4: Discrete random variables:Lec. 7: Conditioning on a random variable; Independence of r.v.'s / 7. Exercise: Independence

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Hint for a: 1
Think of X as simply a yes or no question. If Z != 3, probability of X happening also dissapears. However, does Z = 3 guarrantee that X ...

I did not understand the formal argument in the explanation for question a) 3
In particular, I did not understand this passage: $pX,Z(x,z)=0=pX(x)pZ(z)$. Why should it be zero? Could someone please explain this to ...

Question b, equation 1 6
Hi guys, I don't understand why pX,Y(x,y) is not equal to pX(x).pY(y). Can anyone yelp me with that?

Can someone please clarify Part b 3

Mistook "appropriate definition" for "sufficient condition" in b.1 1
while appropriate definition means it defines the scenario, I assumed the question to be "does the following statement make indepe...

The meaning of "appropriate definition" in b) [STAFF] 3 new_
Is a definition that is true but not the most lax (i.e. holds stricter qualities than that would be required) considered "appropriate"? Isn...

part a counter example 2
Is not it possible to construct X as a function of Z,so they will be dependent?

Question 1: What if intersection is empty? 3
Hi! I was wondering about the case where the distribution of X and Z have no intersection? Am I missing something, or are we not su

? A random variable defined as a pair of other random variables? 2

? Is X guaranteed to be independent of Z?
I'm having trouble reconciling the intuitive definition of independence with the formal explanation in the solution. What if the rando… 7 new_

? Independence of (X, Y) from Z
I was a little bit confused what this meant by (X, Y). Is (X, Y) viewed as a random variable mapping from the sample space to a 2-tuple? 3