8. Exercise: CLT practice

Exercises due May 1, 2020 05:29 IST *Completed*

Exercise: CLT practice

6.0/6.0 points (graded)

The random variables X_i are i.i.d. with mean 2 and standard deviation equal to 3. Assume that the X_i are nonnegative. Let $S_n = X_1 + \cdots + X_n$.

Use the CLT to find good approximations to the following quantities. You may want to refer to the normal table. In parts (a) and (b), give answers with 4 decimal digits.

Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \leq z)$, where Z is a standard normal random variable, for z between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that $\Phi(1.71) = .9564$. When z is negative, the value of $\Phi(z)$ can be found using the formula $\Phi(z) = 1 - \Phi(-z)$.

z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.859[illegible]	[illegible]1

1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

* For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

Hide

a) $\mathbf{P}\left(S_{100} \leq 245\right) \approx$ 0.9332 ✔ **Answer:** 0.9332

b) We let N (a random variable) be the first value of n for which S_n exceeds 119.

$\mathbf{P}(N > 49) \approx$ 0.8413 Answer: 0.8413

c) What is the largest possible value of n for which we have $\mathbf{P}(S_n \leq 128) \approx 0.5$?

$n =$ 64 Answer: 64

Solution:

We will use Z_n to refer to the standardized random variable $(S_n - 2n) / (3\sqrt{n})$.

a) We have

$$\mathbf{P}(S_{100} \leq 245) = \mathbf{P}\left(\frac{S_{100} - 2 \cdot 100}{3 \cdot \sqrt{100}} \leq \frac{245 - 2 \cdot 100}{3 \cdot \sqrt{100}}\right) = \mathbf{P}(Z_n \leq 1.5) \approx 0.9332.$$

b) The event $N > 49$ is the same as the event $S_{49} \leq 119$. Its probability is

$$\mathbf{P}(S_{49} \leq 119) = \mathbf{P}\left(\frac{S_{49} - 2 \cdot 49}{3 \cdot \sqrt{49}} \leq \frac{119 - 2 \cdot 49}{3 \cdot \sqrt{49}}\right) = \mathbf{P}(Z_n \leq 1) \approx 0.8413.$$

c) We want n such that

$$0.5 \approx \mathbf{P}(S_n \leq 128) = \mathbf{P}\left(\frac{S_n - 2n}{3\sqrt{n}} \leq \frac{128 - 2n}{3\sqrt{n}}\right) = \Phi\left(\frac{128 - 2n}{3\sqrt{n}}\right).$$

But since $0.5 = \Phi(0)$, we must have $(128 - 2n) / (3\sqrt{n}) = 0$, so that $n = 128/2 = 64$.

A faster way to see the answer is to note that since the normal is symmetric around its mean, the relation $\mathbf{P}(S_n \leq 128) \approx 0.50$ tells us that 128 should be equal to the mean, $2n$, of S_n.

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

Topic: Unit 8: Limit theorems and classical statistics:Lec. 19: The Central Limit Theorem (CLT) / 8. Exercise: CLT practice

Hide Discussion

Show all posts

by recent activity

- Ex. - CLT Practice
  Hi, Kindly explain the comment on solution of (c) : 'A faster way to see the answer is to note that since the normal is s... 3
- CDF of a positive RV
  I have gotten all three correct. But as the question states that X is a oi\\positive RV, should we not be subtracting Nor... 6
- Point c)
  On this problem should I add 4 decimal digits or just simply the integer that fits the best? 2
- confusion in problem 8(b)
  Isn't P(N>49) is the probability that number of Xis exceed 49 whose sum is less than or equal to 119. which must be e... 6
- Any hints for c
  For C I did almost the same as in example 3, but it does not look like I got close to the answer. Anything I should cons... 2
- Typo in b) ?
  Because the mean is 2, b) seems to contradict itself. Per the example with the containers in the preceding video (unle... 6
- Hint - Q(2b)
  The entire exercise is just as explained in the lecture "CLT example" .. however note that in the lecture ***n*** did n... 1
- Q(c): Why "the largest possible value" of n? 4