



## 14. Exercise: Counting partitions

Exercises due Feb 19, 2020 05:29 IST Completed

### Exercise: Counting partitions

3.0/3.0 points (graded)

We have 9 distinct items and three persons. Alice is to get 2 items, Bob is to get 3 items, and Charlie is to get 4 items.

1. As just discussed, this can be done in  $\frac{a!}{b! 3! 4!}$  ways. Find  $a$  and  $b$ .

$a =$   ✓ Answer: 9

$b =$   ✓ Answer: 2

2. A different way of generating the desired partition is as follows. We first choose 2 items to give to Alice. This can be done in  $\binom{c}{d}$  different ways. Find  $c$  and  $d$ . (There are 2 possible values of  $d$  that are correct. Enter the smaller value.)

$c =$   ✓ Answer: 9

$d =$   ✓ Answer: 2

3. Having given 2 items to the Alice, we now give 3 items to Bob. This can be done in  $\binom{e}{f}$  ways. Find  $e$  and  $f$ . (There are 2 possible values of  $f$  that are correct. Enter the smaller value.)

$e =$   ✓ Answer: 7

$f =$   ✓ Answer: 3



Verify that the answer from part 1 agrees with the answer that you get by combining parts 2 and 3.

**Solution:**

1. By the multinomial formula,  $a = 9$  and  $b = 2$ .
2. We want the number of ways of choosing 2 items out of 9 items. This is the number of 2-element subsets of a 9-element set, so that  $c = 9$  and  $d = 2$ .
3. We have 7 remaining items out of which we need to choose 3. Hence,  $e = 7$  and  $f = 3$ .

From part 1, the number of ways of splitting up the 9 items between Alice, Bob, and Charlie in the specified manner is  $\frac{9!}{2!3!4!}$ .

In parts 2 and 3, we calculate this answer in a different way. Let us now verify that the two methods produce the same answer.

From part 2, we can first give Alice her 2 items in  $\binom{9}{2} = \frac{9!}{2!7!}$  ways. Then, from part 3, we can give Bob his 3 items from the remaining 7 items in  $\binom{7}{3} = \frac{7!}{3!4!}$  ways. Finally, Charlie's 4 items are exactly the 4 items that remain, so there is only 1 way to give him his items. Combining these steps, we have a total of

$$\frac{9!}{2!7!} \cdot \frac{7!}{3!4!} \cdot 1 = \frac{9!}{2!3!4!}$$

ways, which agrees with the answer from part 1.

Submit

You have used 3 of 3 attempts

**i** Answers are displayed within the problem

## Discussion

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💬 Big number, wow!  
Was anyone else surprised by how big the actual amount of partitions was? Not sure its in the spirit of th...

6

? Solution to problem all the possible subsets of a set containing  $n$  elements  
Though this a question related to previous section and most of the fellow learners have moved ahead, L...

3

💬 Textbook  
Got the textbook.... it really really helps. Been tending towards overcomplicating problems because of d...

2

? What if the items were NOT distinct?  
Would the formulas change in any way if the items weren't distinct but (partly) similar? Let's say out of th...

1

💬 Do not overcomplicate your answers  
On my first attempt went head first to put  $a!$  as the answer to 1.a - they are just asking for "a" - don't ove...

1

✓ Why are there 2 possible values of  $d$  &  $f$  that are correct?  
I got the correct answers, but I am curious to know how there could be other possible correct values for...

8

💬 Sum of  $n$  first naturals  
Hello, I was checking the validity of my results by brute force counting and realized that  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$  f...

1

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