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5. Exercise: Continuous convolution

Exercises due Mar 25, 2020 05:29 IST Completed

Exercise: Continuous convolution

2/2 points (graded)

When calculating the convolution of two PDFs, one must be careful to use the appropriate limits of integration. Suppose that X and Y are nonnegative random variables. In particular, $f_X(x)$ is equal to some positive function $h_X(x)$ for $x \geq 0$ and is zero for x < 0. Similarly, $f_Y(y)$ is equal to some positive function $h_Y(y)$ for $y \geq 0$, and is zero for y < 0. Then, the convolution integral $\int_{-\infty}^{\infty} f_X(x) \, f_Y(z-x) \, dx$ is of the form

$$\int_{a}^{b}h_{X}\left(x
ight) h_{Y}\left(z-x
ight) \,dx,$$

for suitable choices of a and b determined by z. Fix some $z \geq 0$. Find a and b. (Your answer can be an algebraic function of z.)

Solution:

The integrand is equal to $h_X\left(x\right)h_Y\left(z-x\right)$ only for those choices of x for which the arguments of the functions h_X and h_Y are nonnegative; that is, when $x\geq 0$ and $z-x\geq 0$, which yields $0\leq x\leq z$. Thus, we should only integrate from 0 to z.

Graphically, the PDF of X extends from 0 to ∞ . Also, when we flip the PDF of Y, the resulting PDF extends from $-\infty$ to 0, and when we shift to the right it by z, it will extend from $-\infty$ to z. Thus the two PDFs that we need to multiply in the convolution integral.

overlap only for values from 0 to z.

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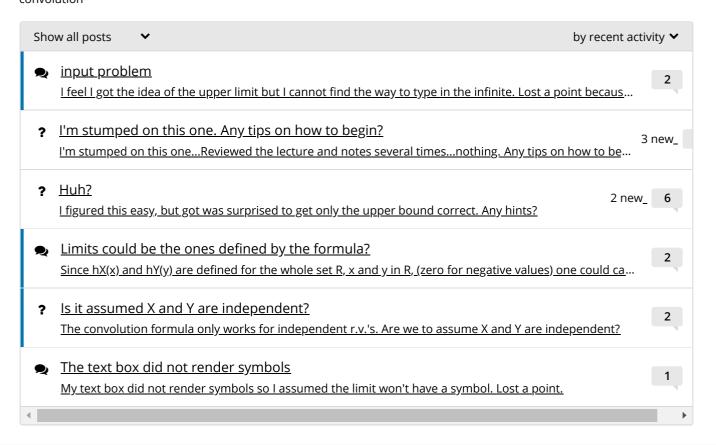
You have used 2 of 3 attempts

1 Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Lec. 12: Sums of independent r.v.'s; Covariance and correlation / 5. Exercise: Continuous convolution



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