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18. Exercise: Correlation properties

Exercises due Mar 25, 2020 05:29 IST Completed

Exercise: Correlation properties

6/6 points (graded)

As in the preceding example, let Z, V, and W be independent random variables with mean 0 and variance 1, and let X=Z+V and Y=Z+W. We have found that $\rho\left(X,Y\right)=1/2$.

a) It follows that:

$$ho\left(X,-Y
ight)=$$
 -1/2 $ho\left(X,-Y
ight)=$ Answer: -0.5

$$ho\left(-X,-Y
ight)=$$
 1/2 ho Answer: 0.5

b) Suppose that X and Y are measured in dollars. Let X' and Y' be the same random variables, but measured in cents, so that X'=100X and Y'=100Y. Then,

$$ho\left(X',Y'
ight)= \center{1/2}$$
 $ightharpoons$ Answer: 0.5

c) Suppose now that $ilde{X}=3Z+3V+3$ and $ilde{Y}=-2Z-2W$. Then

$$ho\left(ilde{X}, ilde{Y}
ight)=
hotarrow -1/2$$
 $ightharpoonup$ Answer: -0.5

d) Suppose now that the variance of ${\it Z}$ is replaced by a very large number. Then

$$ho\left(X,Y
ight)$$
 is close to $ho\left(X,Y
ight)$



e) Alternatively, suppose that the variance of Z is close to zero. Then

Solution:

We saw that a linear transformation $x\mapsto ax+b$ of a random variable does not change the value of the correlation coefficient, except for a possible sign change if the coefficient a is negative. Note that in the case of $\rho\left(-X,-Y\right)$, we have two sign changes, hence no sign change.

For the last two parts, if Z has a very large variance, then the terms V and W become insignificant, and $\rho\left(X,Y\right)\approx\rho\left(Z,Z\right)=1.$ And if Z has very small variance, then X and Y are approximately independent, so that $\rho\left(-X,-Y\right)\approx0.$ (These conclusions can also be justified by an exact calculation.)

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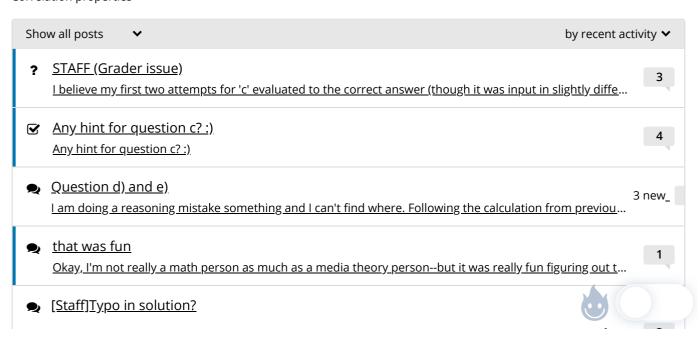
You have used 3 of 3 attempts

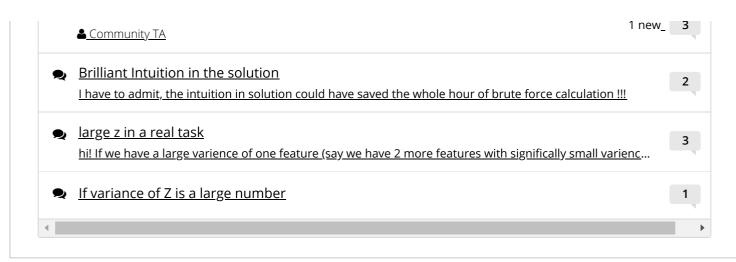
1 Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Lec. 12: Sums of independent r.v.'s; Covariance and correlation / 18. Exercise: Correlation properties





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