1. The PDF of the logarithm of X

Problem Set due Apr 1, 2020 05:29 IST *Completed*

Problem 1. The PDF of the logarithm of X

7/7 points (graded)

Let X be a non-negative random variable. Find the PDF of the random variable $Y = \ln X$ for each of the following cases:

1. For general f_X, $f_Y(y) =$

- (●) $f_X(e^y)\, e^y$
- ( ) $\dfrac{f_X(e^y)}{e^y}$
- ( ) $\dfrac{f_X(\ln y)}{y}$
- ( ) none of the above

✔

2. When $f_X(x) = \begin{cases} 1/4, & \text{if } 2 < x \leq 6, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y \leq b, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$ and the values of a and b using standard notation.

$g(y) =$ 1/4*e^y ✔ **Answer:** (e^y)/4

$\frac{1}{4} \cdot e^y$

$a =$ ln(2) ✔ **Answer:** 0.6931

$b =$ ln(6) ✔ **Answer:** 1.791

3. When $f_X(x) = \begin{cases} 2(x-1), & \text{if } 1 < x \leq 2, \\ 0, & \text{otherwise,} \end{cases}$

we have $f_Y(y) = \begin{cases} g(y), & \text{if } a < y \leq b, \\ 0, & \text{otherwise.} \end{cases}$

Give a formula for $g(y)$, and the values of a and b, using standard notation.

$g(y) =$

2*(e^y-1)*e^y

✔ **Answer:** 2*exp(2*y)-2*exp(y)

$2 \cdot (e^y - 1) \cdot e^y$

$a =$ ln(1) ✔ **Answer:** 0

$b =$ ln(2) ✔ **Answer:** 0.6931

STANDARD NOTATION

Solution:

1. $f_Y(y) = f_X(e^y)\, e^y$. Note that $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(\ln X \leq y) = \mathbb{P}(X \leq e^y) = F_X(e^y)$. Differentiating both sides with respect to y and using the chain rule, we obtain

$$f_Y(y) = f_X(e^y)\, e^y.$$

2. For X between 2 and 6, $Y = \ln(X)$ takes values between $\ln(2)$ ad $\ln(6)$. By applying the formula found in the previous part, we obtain

$$f_Y(y) = \begin{cases} \dfrac{e^y}{4}, & \text{if } \ln 2 < y \leq \ln 6, \\ 0, & \text{otherwise.} \end{cases}$$

3. For X between 1 and 2, $Y = \ln(X)$ takes values between 0 ad $\ln(2)$. By applying the formula in part (1), we obtain

$$f_Y(y) = \begin{cases} 2e^{2y} - 2e^y, & \text{if } 0 < y \leq \ln 2, \\ 0, & \text{otherwise.} \end{cases}$$

Submit You have used 4 of 5 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Problem Set 6 / 1. The PDF of the logarithm of X

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Didn't even touch this homework. I'm taking a goose-egg on this one. I would guess I'm not the only one. (Okay, not quite a goose-egg. I ran through and pick...	2
I have no idea where to start on 2 and 3 I solved 1. I'm just struggling with what steps I need to take for questions 2 and 3. I simply have no idea. Please don't respo...	4
application of chain rule In nearly there on part 1, but unfortunately I'm struggling with rusty Calculus. I'll try to keep this general. I have a CDF(Y) in ...	2
Thank you for extending the deadline Work has been really busy due to the outbreak. I was worried that I needed to drop out, so this was a very welcome lifeline...	2
Is ln(x) the log with base 10 or log with base e? See title.	2
Accepted answer of 3.b problematic I think it should be something different.	5
[Staff] Just a Clarification on the Deadline for PSet6 The course overview shows that the deadline for the PSet6 is 25th March. Just to be clear that it isn't 18th March.	3
Is it Y=ln (X) everywhere? Hello, Do we consider y=ln(x) also for question 2 and 3?	2