6. Random incidence under Erlang interarrivals

Problem Set due May 13, 2020 05:29 IST *Past Due*

Problem 6. Random incidence under Erlang interarrivals

2 points possible (graded)

A factory produces copper wires. A blue mark is placed on a very long length of copper. The wire is then cut into pieces. The lengths of different pieces are independent, and the length of each piece is distributed according to the same PDF $f_X(x)$. Let R be length of the piece including the blue mark. Determine the expected value of R in each of the following cases.

In each part below, express your answer in terms of μ using standard notation. Enter "mu" for μ.

1. Suppose that $f_X(x) = \begin{cases} \mu e^{-\mu x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

$\mathbf{E}[R] =$

Answer: 2/mu

2. Suppose that $f_X(x) = \begin{cases} \frac{\mu^4 x^3 e^{-\mu x}}{6}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

$\mathbf{E}[R] =$

Answer: 5/mu

STANDARD NOTATION

Solution:

1. The lengths of the pieces of copper wire are independent and exponentially distributed with parameter μ. As explained in lecture, due to the memorylessness of the exponential, the distribution of the length of the piece of copper wire containing the blue mark is a second order Erlang. Thus, $\mathbf{E}[R] = 2\mathbf{E}[X] = 2/\mu$.

2. Here, X is Erlang of order 4. Think of sections on the copper wire, each exponentially distributed with parameter μ. We then interpret each piece as **four** consecutive sections of exponentially distributed lengths. The piece of the copper wire with the blue mark will have the mark in one of these four sections. By the standard random incidence analysis, the expected length of that section will be $2/\mu$. However, the piece of copper wire containing the blue mark also consists of three other sections, each of which is exponentially distributed with parameter μ, and hence, have an expected length of $1/\mu$. Thus, the total expected length of the piece of copper wire containing the blue mark is $2/\mu + 1/\mu + 1/\mu + 1/\mu = 5/\mu$.

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Answers are displayed within the problem

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Topic: Unit 9: Bernoulli and Poisson processes:Problem Set 9 / 6. Random incidence under Erlang interarrivals

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Hint for all	5
Relevant materiel Pg 322 of the text; "The random incidence paradox". Solved problem 7; "Random incidence under Erland...	9
Question 2 Would it be possible to have some help? I used the same reasoning as in Lecture 23 Video 14 but I don't ...	2