2

Final Exam due May 20, 2020 05:29 IST   *Completed*

Problem 2

2.0/2.0 points (graded)

Let X and Y be independent random variables with zero means, and variances 1 and 2, respectively. Let $U = X + Y$ and $V = X + 2Y$.

Find the coefficients a and b of the Linear Least Mean Squares (LLMS) estimator $\widehat{V}_L = aU + b$ of V based on U.

$a =$ | 5/3 | ✔ **Answer:** 5/3

$b =$ | 0 | ✔ **Answer:** 0

Solution:

We have the general formula:

$$\widehat{V}_L = \mathbf{E}\left[V\right] + \frac{\text{cov}\left(V, U\right)}{\text{Var}\left(U\right)}\left(U - \mathbf{E}\left[U\right]\right). \tag{16.1}$$

First, note

$$
\begin{aligned}
\mathbf{E}\left[V\right] &= \mathbf{E}\left[X + 2Y\right] \\
&= \mathbf{E}\left[X\right] + 2\mathbf{E}\left[Y\right] \\
&= 0 + 0 \\
&= 0.
\end{aligned}
$$

Similarly,

$$\mathbf{E}[U] = \mathbf{E}[X+Y]$$
$$= \mathbf{E}[X] + \mathbf{E}[Y]$$
$$= 0.$$

We also have by independence of X, Y:

$$\mathsf{Var}(U) = \mathsf{Var}(X) + \mathsf{Var}(Y) = 3.$$

We now compute the covariance $\mathrm{cov}(V, U)$:

$$\mathrm{cov}(V, U) = \mathbf{E}[VU] - \mathbf{E}[V]\mathbf{E}[U]$$
$$= \mathbf{E}[(X+Y)(X+2Y)]$$
$$= \mathbf{E}[X^2] + 3\mathbf{E}[XY] + 2\mathbf{E}[Y^2]$$
$$= \mathsf{Var}(X) + 3\mathbf{E}[X]\mathbf{E}[Y] + 2\mathsf{Var}(Y)$$
$$= 1 + 0 + 4$$
$$= 5.$$

Plugging in this information into the first equation above, we finally get:

$$\widehat{V}_L = \frac{5}{3}U.$$

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ⓘ Answers are displayed within the problem

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My answers for Q2 of the Final

a. 5/3 b. 0

9

? fraction or decimal

for a non terminating rational number, should we enter the fraction or round it off(how many decimal pl...

3