3. Hypothesis test with a continuous observation

Problem Set due Apr 8, 2020 05:29 IST *Completed*

Problem 3. Hypothesis test with a continuous observation

3/3 points (graded)

Let Θ be a Bernoulli random variable that indicates which one of two hypotheses is true, and let $\mathbf{P}(\Theta = 1) = p$. Under the hypothesis $\Theta = 0$, the random variable X has a normal distribution with mean 0, and variance 1. Under the alternative hypothesis $\Theta = 1$, X has a normal distribution with mean 2 and variance 1.

Consider the MAP rule for deciding between the two hypotheses, given that $X = x$.

1. Suppose for this part of the problem that $p = 2/3$. The MAP rule can choose in favor of the hypothesis $\Theta = 1$ if and only if $x \geq c_1$. Find the value of c_1.

   $c_1 =$ 2/3 ✔ **Answer:** 0.6534

2. For this part, assume again that $p = 2/3$. Find the conditional probability of error for the MAP decision rule, given that the hypothesis $\Theta = 0$ is true.

   $\mathbf{P}(\mathrm{error} \mid \Theta = 0) =$ 0.2578 ✔ **Answer:** 0.2578

3. Find the overall (unconditional) probability of error associated with the MAP rule for $p = 1/2$.

   0.1587 ✔ **Answer:** 0.1587

You may want to consult to standard normal table.

Normal Table

The entries in this table provide the numerical values of $\Phi(z) = \mathbf{P}(Z \leq z)$, where Z is a standard normal random variable, for z between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that $\Phi(1.71) = .9564$. When z is negative, the value of $\Phi(z)$ can be found using the formu[illegible]
$\Phi(z) = 1 - \Phi(-z)$.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	[illegible]	[illegible]

3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

* For $z \geq 3.50$, the probability is greater than or equal to $.9998$.

Hide

Solution:

1. For $0 < p < 1$, we can choose in favor of the hypothesis $\Theta = 1$ if and only if

$$\begin{aligned} f_{X|\Theta}(x \mid 1)\, p_\Theta(1) &\geq f_{X|\Theta}(x|0)\, p_\Theta(0) \\ \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(x-2)^2\right)\cdot p &\geq \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right)\cdot(1-p) \\ \frac{x^2}{2} - \frac{(x-2)^2}{2} &\geq \ln\frac{1-p}{p} \\ x &\geq 1 + \frac{1}{2}\ln\frac{1-p}{p}. \end{aligned}$$

For $p = 2/3$, this threshold corresponds to $c_1 = 1 - (\ln 2)/2 \approx 0.6534$.

2. Under the hypothesis $\Theta = 0$, an error occurs if we decide $\Theta = 1$. Therefore,

$$\begin{aligned} \mathbf{P}(\text{error}|\Theta = 0) &= \mathbf{P}(X \geq c_1 \mid \Theta = 0) \\ &= 1 - \mathbf{P}(X < c_1 \mid \Theta = 0) \\ &\approx 1 - \Phi(0.65) \\ &\approx 0.2578, \end{aligned}$$

since under $\Theta = 0$, X is a standard normal random variable.

3. With $p = 1/2$, the threshold becomes 1. Therefore, we decide $\Theta = 1$, whenever $x \geq 1$, and decide $\Theta = 0$, whenever $x < 1$. f

$$\begin{aligned} \mathbf{P}(\text{error}) &= \mathbf{P}(\text{error}|\Theta = 0)\, p_\Theta(0) + \mathbf{P}(\text{error}|\Theta = 1)\, p_\Theta(1) \\ &= \mathbf{P}(X \geq 1 \mid \Theta = 0)\frac{1}{2} + \mathbf{P}(X < 1 \mid \Theta = 1)\frac{1}{2} \end{aligned}$$

$$
\begin{aligned}
&= \frac{1-\Phi(1)}{2} + \frac{\mathbf{P}(X-2<-1 \mid \Theta=1)}{2} \\
&= \frac{1-\Phi(1)}{2} + \frac{1-\Phi(1)}{2} \\
&= 1-\Phi(1) \\
&\approx 1-0.8413 = 0.1587.
\end{aligned}
$$

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 7: Bayesian inference:Problem Set 7a / 3. Hypothesis test with a continuous observation

Show all posts by recent activity

Reminder
I think that somebody forgets that the students are average people. I read the book, I watched the lectures and I h... 23

[Staff] Possible error in solution to Question 1.
Starting with line 3 of the solution for question 1. x^2/2 - (x-2)^2 >= ln((1-p)/p) x^2 - (x-2)^2 >= 2 * ln((1-p)/p) 2*x - 4 ... 4

Will there be more materials to learn? Will the mid-term be so difficult?
Sorry I have to get a lot of external help to get through this problem set. Will the mid-term be same difficult? Will t... 7

[Staff] Need the solutions for the Problem Set After deadline
Need solutions for the problem set after deadline since most of the questions are complex(too difficult). And it wil... 3

Staff, I can't answer anymore.
Hi. it's being a really long week for me and I thought that I have until the midnight of today (or 23.59h of 8 of April)... 1

what is c1 ??? unexpected notation
He was using c's in the lectures to denote a constant before the exponential (typically = 1/sigma * rad(2-pi) but so... 2

[Staff] Uncertainty in regard to the decimals
Since the correct answers to the exercises are not available yet I can't still determine whether there's an issue wit... 3

Do I understand correctly parts 2 and 3? 6

For people who need a hint to start
This question is easy once you figure out where to look: 1. As others have pointed out you need to follow the form... 1

This link is a good example for this problem http://www.cs.haifa.ac.il/~rita/ml_course/lectures/Bayesian_Decision.pdf

? [Do you need the textbook to solve these questions?](#)
These questions seem to require some additional reading other than the lecture videos i.e lecture 14 and lecture ...
5

[Hint based on my experience](#)
Hello everyone. Sometimes when I am facing a tough problem like this one and I get stuck, I come here to the disc...
4

[Hint for starter](#)
How does the MAP rule can help deciding between two hypotheses? Is there an exercise or a video that can help ?
5