14. Exercise: The time of the kth arrival

Exercises due May 13, 2020 05:29 IST *Completed*

Exercise: The time of the kth arrival

1/2 points (graded)

Let Y_k be the time of the kth arrival in a Poisson process with parameter $\lambda = 1$. In particular, $\mathbf{E}[Y_k] = k$.

Is it true that $\mathbf{P}(Y_k \geq k) = 1/2$ for any finite k?

Yes ✖ **Answer:** No

Is it true that $\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = 1/2$?

Yes ✔ **Answer:** Yes

Solution:

Consider the special case of $k = 1$. Then, $\mathbf{P}(Y_1 \geq 1) = e^{-1} \neq 1/2$.

When k is large, the central limit theorem applies because Y_k is the sum of k i.i.d. (exponential) random variables. Its (standardized) distribution is approximately normal, hence approximately symmetric around its mean. More formally, using the fact that the variance of an exponential with parameter 1 is 1, we have

$$\lim_{k\to\infty} \mathbf{P}(Y_k \geq k) = \lim_{k\to\infty} \mathbf{P}\left(\frac{Y_k - k}{\sqrt{k}} \geq 0\right) = \Phi(0) = \frac{1}{2},$$

where Φ is is the standard normal CDF.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

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?	Where is my mistake? Help!!!! Apart from the CLT suggested by the answer, I solved this question in the following way but I cannot see ...	2
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?	Why The weight of Yk above it's expectation is not 1/2??? Isn't the pdf of Yk balanced on the point of expectation, I understand that the expectation is like center o...	5
	Does the probability change if we use "strictly greater" instead of "greater or equal"?	1
	best way to verify general impression My gut says that if the expectation is here, then half the PMF is above that and the other half is below. B...	1
?	When k approach to infinity using large number theorem Using large number theorem: Y_k/k=(T_1+T_2+...)/k=1 when k goes to infinity So the P(Y_k>=k) should 1. ...	7