5. Probabilities on a continuous sample space

Problem Set due Feb 5, 2020 05:29 IST *Completed*

Problem 5. Probabilities on a continuous sample space

6/6 points (graded)

Alice and Bob each choose at random a real number between zero and one. We assume that the pair of numbers is chosen according to the uniform probability law on the unit square, so that the probability of an event is equal to its area.

We define the following events:

A = {The magnitude of the difference (for any two real numbers x and y, the value $|x-y|$) of the two numbers is greater

B = {At least one of the numbers is greater than $1/4$}

C = {The sum of the two numbers is 1}

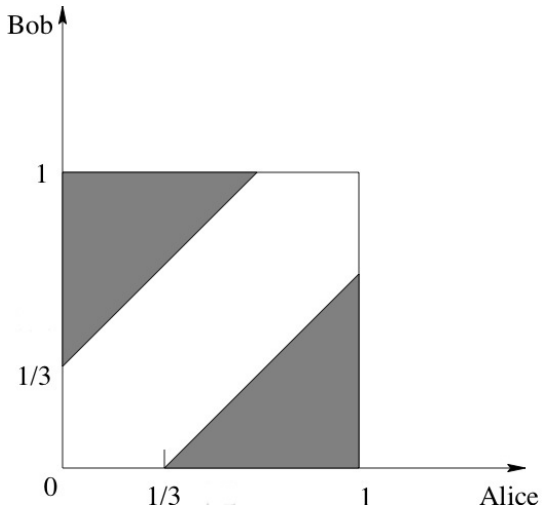
D = {Alice's number is greater than $1/4$}

Find the following probabilities:

1. $\mathbf{P}(A) =$ 4/9 ✔ **Answer:** 0.44444
2. $\mathbf{P}(B) =$ 15/16 ✔ **Answer:** 0.9375
3. $\mathbf{P}(A \cap B) =$ 4/9 ✔ **Answer:** 0.44444
4. $\mathbf{P}(C) =$ 0 ✔ **Answer:** 0
5. $\mathbf{P}(D) =$ 3/4 ✔ **Answer:** 0.75
6. $\mathbf{P}(A \cap D) =$ 89/288 ✔ **Answer:** 0.30903

Solution:

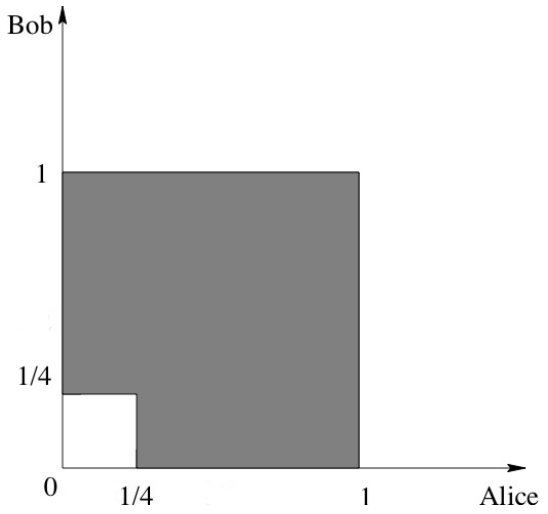
1. We have the following figure, where the axes represent Alice and Bob"s choices, and the shaded areas (the two triangles) represent points where Alice"s and Bob's choices differ by more than $1/3$.



Under the uniform probability law, the probability of the event is its area. Using the formula for the area of a triangle, we find

$$\mathbf{P}(A) = 2 \cdot \frac{(2/3)^2}{2} = \boxed{4/9}.$$

2. The set of points for which at least one of the numbers is greater than $1/4$ is the shaded region in the following figure:

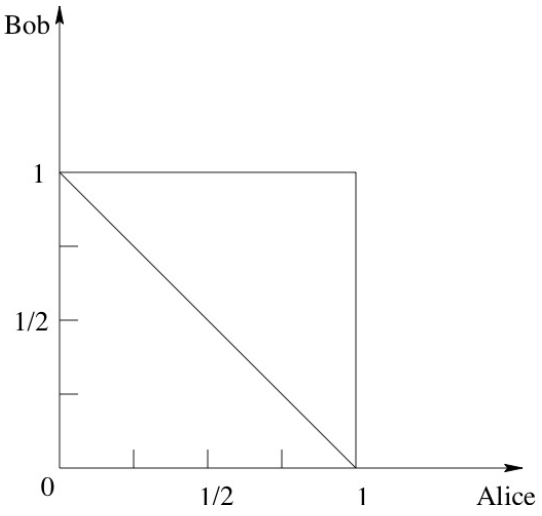


Its probability is:

$$\begin{aligned}
\mathbf{P}(B) &= 1 - \mathbf{P}(\text{both numbers are less than or equal to } 1/4) \\
&= 1 - \text{Area of unshaded square} \\
&= 1 - 1/4 \cdot 1/4 \\
&= 1 - 1/16 \\
&= \boxed{15/16}.
\end{aligned}$$

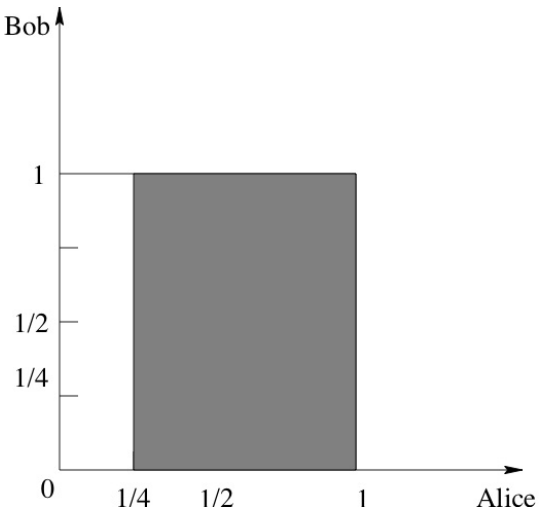
3. Event A is a subset of event B, so that $A \cap B = A$. Thus, $\mathbf{P}(A \cap B) = \mathbf{P}(A) = \boxed{4/9}$.

4. The set of points where the sum of the two numbers is 1 is the diagonal of slope -1 shown in the next figure.



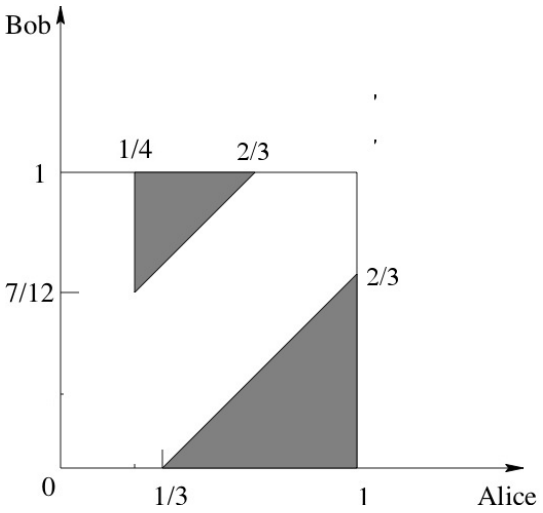
Since it is a line segment, it has zero area and $\mathbf{P}(C) = \boxed{0}$.

5. Event D is the shaded region in the figure below.



Thus, $\mathbf{P}(D) = \text{area of shaded region} = 1 \cdot (3/4) = \boxed{3/4}$.

6. By intersecting the shaded areas associated with events A and D, we obtain the shaded region shown below.



Then,

$$\begin{aligned}\mathbf{P}\left(A \cap D\right) &= \text{area of shaded region} \\ &= 2/3 \cdot 2/3 \cdot 1/2 + 5/12 \cdot 5/12 \cdot 1/2 \\ &= 2/9 + 25/288 \\ &= \boxed{89/288}.\end{aligned}$$

Submit You have used 1 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 1: Probability models and axioms:Problem Set 1 / 5. Probabilities on a continuous sample space

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- More practice problems definitely would have helped — 9
  I feel more than out of my depth here. I watched all the videos, did all the provided problems throughout the lecture, read over the provided chapter excerpts, and...
- Solving the Problem with Python — 3
  Hi All, Hope someone finds this approach useful. Most programming languages have packages that allow you to pick a number at random from a uniform distribu...
- Magnitude of difference — 2 new — 5
- Was the statement of Event A wrongly described ? I think so! — 1
  The statement said this: {The magnitude of the difference (for any two real numbers x and y, the value $|x-y|$) of the two numbers is greater than 1/3} It refers to t...
- Last one was tough.. — 3
  It was fun having to think through so much for solving something.. Guess I am very rusty with basic maths. Even after spending a lot of time, couldn't get the last o...
- Trouble conceiving A — 5
  Hi all, I'm having trouble drawing A to get the shape to calculate, any advice?
- Are 3 and 6 solvable via the Conditional Probability Law? — 1
  It seems that they should expand to something like "the probability of A * the probability of B given A". I think I'm just not understanding how to apply them prope...
- The last one was hardest one

the last one was hardest one, so I solved it in an engineering way, but I don't know to solve it completely by using prob law. however, it give me the correct result. 4 new 13

? Need help with A∩B and A∩D
I can picture them geometrically - area of B is almost identical to the solved problem of Romeo/Juliet and B/D is just a rectangle. But I cannot calculate the area bec... 4

Solutions after the due date?
Hey Admin, I still couldn't solve 1,3 and 6 after three attempts. Could we please have the solutions to these problems after the due date? Cheers, JK 1

[Staff] request for 1 more attempt for this question
Dear Staffs, I could solve all other questions in this homework exercise correctly except the intersection questions on Problem 5. Actually, I exhausted my attempts... 1

I get stuck on number 6 while others are correct
I get difficult to answer no 6, whereas my answers for numbers 1 and 5 are correct. I use a graph to calculate all the questions like on tutorial solved problem Unif... 1 new

Understanding C
Any hints or suggestion for building an understanding of C? For the other cases, I used a discrete set to help visualize solutions that I later found with geometry, bu... 1 new