11. Exercise: Independence and CDFs

Exercises due Mar 13, 2020 05:29 IST *Completed*

Exercise: Independence and CDFs

1/2 points (graded)

a) Suppose that X and Y are independent. Is it true that their joint CDF satisfies $F_{X,Y}(x,y) = F_X(x)\,F_Y(y)$, for all x and y?

Yes ✔ **Answer:** Yes

b) Suppose that $F_{X,Y}(x,y) = F_X(x)\,F_Y(y)$, for all x and y. Is it true that X and Y are independent?

Hint: Recall the formula $f_{X,Y}(x,y) = (\partial^2/\partial x \partial y)\,F_{X,Y}(x,y)$.

No ✘ **Answer:** Yes

Solution:

a) Yes. We have

$$\begin{aligned} F_{X,Y}(x,y) &= \mathbf{P}(X \le x, Y \le y) \\ &= \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(x,y)\,dx\,dy \\ &= \int_{-\infty}^{x} f_X(x)\,dx \int_{-\infty}^{y} f_Y(y)\,dy \\ &= F_X(x)\,F_Y(y)\,. \end{aligned}$$

b) True. Using the formula in the hint, we find that

$$
\begin{aligned}
f_{X,Y}(x,y) &= \frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) \\
&= \frac{\partial^2}{\partial x \partial y} F_X(x) F_Y(y) \\
&= \frac{\partial}{\partial x} F_X(x) \frac{\partial}{\partial y} F_Y(y) \\
&= f_X(x) f_Y(y),
\end{aligned}
$$

and therefore we have independence.

Submit You have used 1 of 1 attempt

Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 5: Continuous random variables:Lec. 10: Conditioning on a random variable; Independence; Bayes' rule / 11. Exercise: Independence and CDFs

Show all posts by recent activity

? Part B Answer: Why didn't we use product rule for differentiation ?
I am trying not disclose the answer here. I expected an expansion with the product rule in solution step ... 4

I thought I was doing OK in this course.
Then a bunch of 1-shot questions come along where I'm essentially guessing simply because I don't und... 2 new_

? Solution for b)
Where does the first step (the second row) of the solution come from? 4

How about the case when the cdf's are not differentiable ?
How to deal with the case when the cdf's are not differentiable everywhere in the proof for part b ? How ... 4

? Where does the hint in part b) come from? 4