



Course > Unit 6: ... > Lec. 13:... > 11. Exe...

11. Exercise: Conditional variance definition

Exercises due Mar 25, 2020 05:29 IST Completed

Exercise: Conditional variance definition

3/5 points (graded)

For each one of the following statements, indicate whether it is true or false.

(a) If X=Y (i.e., the two random variables always take the same values), then $\operatorname{Var}(X\,|\,Y)=0.$



(b) If X=Y (the two random variables always take the same values), then ${\sf Var}\,(X\,|\,Y)={\sf Var}\,(X).$

(c) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X\,|\,Y)$ takes the value

$$\mathbf{E}[(X - \mathbf{E}[X | Y = y])^2 | Y = y].$$

True

Answer: True

(d) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X\,|\,Y)$ takes the value

$$\mathbf{E}[(X - \mathbf{E}[X|Y])^2 | Y = y.]$$

True ✓ Answer: True



(e) If Y takes on the value y, then the random variable $\operatorname{\sf Var}(X\,|\,Y)$ takes the value

$$\mathbf{E}[(X - \mathbf{E}[X])^2 | Y = y.]$$

False	~
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✓ Answer: False

Solution:

- (a) Conditioned on Y, X is deterministic, and $\text{Var}\left(X\,|\,Y=y\right)=0$. This implies that the random variable $\text{Var}\left(X\,|\,Y\right)$ is identically equal to zero. Thus, the statement is true.
- (b) False, because the previous statement is true.
- (c) This statement is just the definition of the numerical value of the conditional variance. We are in a universe where the event Y=y is known to have occurred, and every expectation is replaced by the corresponding conditional expectation.
- (d) The outer expectation places us in a universe where Y=y. Given this information, the value of the random variable $\mathbf{E}\left[X\,|\,Y\right]$ becomes a known quantity, equal to $\mathbf{E}\left[X\,|\,Y=y\right]$. Thus, this statement is equivalent to the preceding one and is true.
- (e) This is false, because all expectations should be conditional on the universe (Y=y) within which we are working. For a concrete counterexample, suppose that X is zero-mean and that Y=X. Then, as in part (a), ${\sf Var}\,(X\,|\,Y=y)=0$. On the other hand, since ${\bf E}\,[X]=0$, we have

$$\mathbf{E} ig[(X - \mathbf{E} \, [X])^2 \, | \, Y = y ig] = \mathbf{E} \, [X^2 \, | \, Y = y] = \mathbf{E} \, [Y^2 \, | \, Y = y] = y^2.$$

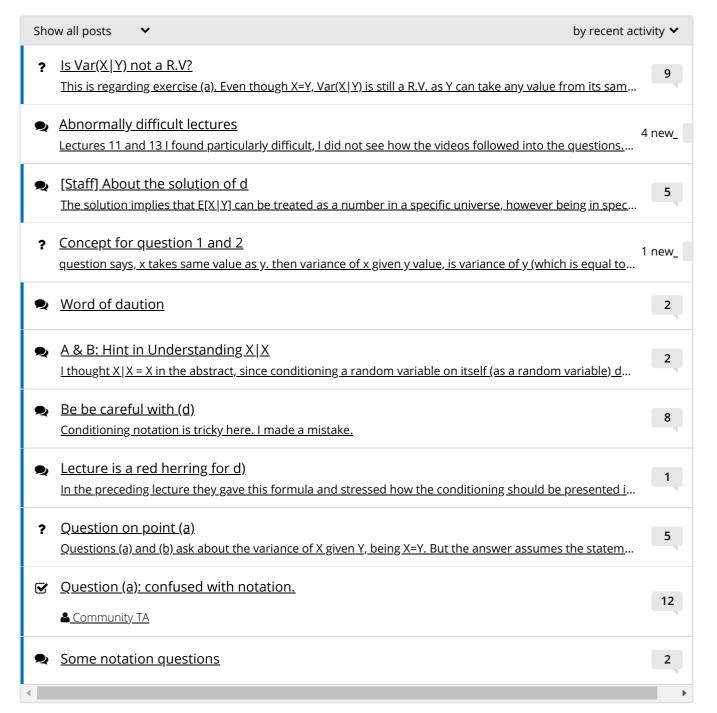
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You have used 1 of 1 attempt

1 Answers are displayed within the problem

Discussion

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