



[Course](#) > [Unit 3:...](#) > [Proble...](#) > 2. A thr...

2. A three-sided die

Problem Set due Feb 19, 2020 05:29 IST Completed

Problem 2. A three-sided die, part 1

1.0/1.0 point (graded)

The newest invention of the 6.431x staff is a three-sided die. On any roll of this die, the result is 1 with probability $1/2$, 2 with probability $1/4$, and 3 with probability $1/4$.

Consider a sequence of six independent rolls of this die.

Find the probability that exactly two of the rolls result in a 3.

☒ $\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$

☐ $\binom{6}{2} \left(\frac{1}{4}\right)^2$

☐ $\binom{6}{2} \left(\frac{1}{4}\right)^2 \binom{6}{4} \left(\frac{3}{4}\right)^4$

☐ $\binom{6}{2} \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^2$



Solution:



Each roll is an independent trial with probability $1/4$ of resulting in a 3 (a "success"). The probability of exactly 2 successes in 6 trials is given by the binomial probabilities, with $n = 6$, $k = 2$, and $p = 1/4$:

$$\binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$$

Submit

You have used 1 of 2 attempts

i Answers are displayed within the problem

Problem 2. A three-sided die, part 2

3.0/3.0 points (graded)

- Given that exactly two of the six rolls resulted in a 1, find the probability that the first roll resulted in a 1.

Note: Your answer should be a number. Do not enter "!" or combinations in your answer.

1/3

✓ Answer: 0.33333

- We are told that exactly three of the rolls resulted in a 1 and exactly three rolls resulted in a 2. Given this information, find the probability that the six rolls resulted in the sequence (1, 2, 1, 2, 1, 2).

Note: Your answer should be a number. Do not enter "!" or combinations in your answer.

1/20

✓ Answer: 0.05

- The conditional probability that exactly k rolls resulted in a 3, given that at least one roll resulted in a 3, is of the form:

$$\frac{1}{1 - (c_1/c_2)^{c_3}} \binom{c_3}{k} \left(\frac{1}{c_2}\right)^k \left(\frac{c_1}{c_2}\right)^{c_3-k}, \quad \text{for } k = 1, 2, \dots, 6.$$



Find the values of the constants c_1 , c_2 , and c_3 :

| | | |
|---------|--------------------------------|-------------|
| $c_1 =$ | <input type="text" value="3"/> | ✓ Answer: 3 |
| $c_2 =$ | <input type="text" value="4"/> | ✓ Answer: 4 |
| $c_3 =$ | <input type="text" value="6"/> | ✓ Answer: 6 |

Solution:

1. The probability of obtaining a 1 on a single roll is $1/2$, and the probability of obtaining a 2 or 3 on a single roll is also $1/2$. For the purposes of solving this problem, we treat obtaining a 2 or a 3 as an equivalent result. This way all possible sequences with entries "a 1" or "not a 1" are equally likely, and we can solve the problem using the counting methods, applied to the conditional model.

We know that there are $\binom{6}{2}$ ways of rolling exactly two 1's. Of these $\binom{6}{2}$ ways, exactly $\binom{5}{1} = 5$ ways result in a 1 on the first roll, since we can place the other 1 in any of the five remaining rolls. The rest of the rolls must be either 2 or 3. Thus the probability that the first roll is a 1 given that exactly two rolls resulted in a 1 is $\frac{5}{\binom{6}{2}} = \frac{1}{3}$.

2. We have

$$\mathbf{P}(121212 | \text{exactly three 1s and three 2s}) = \frac{\mathbf{P}(121212)}{\mathbf{P}(\text{exactly three 1s and three 2s})}.$$

Any particular sequence of three 1s and three 2s has the same probability: $\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3$. There are $\binom{6}{3}$ possible sequences with exactly three 1s and three 2s, of which exactly one sequence is 121212. Therefore,

$$\mathbf{P}(121212 | \text{exactly three 1s and three 2s}) = \frac{\left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3}{\binom{6}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3} = \frac{1}{20}.$$

3. Let A be the event that at least one roll results in a 3. Then,
 $\mathbf{P}(A) = 1 - \mathbf{P}(\text{no rolls resulted in a 3}) = 1 - \left(\frac{3}{4}\right)^6.$

Let B be the event that there were exactly k rolls that resulted in a 3, where $k \in \{1, 2, \dots, 6\}$. Note that $\mathbf{P}(B) = \binom{6}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{6-k}$.



Note also that $B \subset A$. Thus, the desired probability is:

$$\begin{aligned}\mathbf{P}(B | A) &= \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(A)} \\ &= \frac{\mathbf{P}(B)}{\mathbf{P}(A)} \\ &= \frac{1}{1 - (3/4)^6} \binom{6}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{6-k} \quad \text{for } k = 1, 2, \dots, 6.\end{aligned}$$

Submit

You have used 2 of 4 attempts

i Answers are displayed within the problem

Discussion

Hide Discussion

Topic: Unit 3: Counting: Problem Set 3 / 2. A three-sided die

Show all posts ▼

by recent activity ▼

? Q2- denominator value: What is the probability of exactly 3 1's & 3 2's?
Where am I going wrong in understanding the concept? :(Here is what I thought - P(A) = probability of 3 1's...

6

💬 Problem 2.1

1

💬 Hint for Problem Set 3 Problem 2.3

1

? To Staff: Error in grading
Hi - I had entered questions to part 2 #'s 2 and 3 correctly, and I went to answer my final input for Part 2...

3

? Where is the answer for the question 2#1
I don't see answer for the question 1 in the second part

1

? PROBLEM 2 PART 2
I am using **multinomial** way to solve this, what we have learned in last video. But not getting. Is I am...

6 new_

💬 Part 2 #3 simplification
A useful hint is that the first part '1/1-(c1/c2)^c3' is the denominator, and this is a conditional probability...

2

✓ Are we still assuming that there are 6 rolls in Part 2 Q3?
Are we still assuming that there are 6 rolls in Part 2 Q3 (the one about finding the values for the c's), or is...

2



Part 2 question 2
After 2 failed attempts, I figured out why the multinomial/sequence formula just was not working. I think ...

4

? Binomial Vs. Multinomial
Can someone please explain why the binomial model seems to be working to answer this question (as o...

2

Hint
Please note that this information: The newest invention of the 6.431x staff is a three-sided die. On any rol...

4

? Potential hint?
Is there a way that you, the staff, can pre-view a hint that I would like to contribute, to make sure that the...

1

? It is misleading
It is misleading the fact that you present the both exercises together thinking that probabilities apply to b...

6

