10. Exercise: Conditional variance II

Exercises due Mar 25, 2020 05:29 IST *Completed*

Exercise: Conditional variance II

3/3 points (graded)

The random variable Q is uniform on $[0,1]$. Conditioned on $Q = q$, the random variable X is Bernoulli with parameter q.

(a) The conditional variance, $\mathsf{Var}(X \mid Q)$, is equal to:

- ○ $1/4$
- ○ $q(1-q)$
- ● $Q(1-Q)$
- ○ q^2
- ○ Q^2

✔

(b) Recall that a uniform random variable on $[0,1]$ has a variance of $1/12$ and also satisfies $\mathbf{E}[Q^2] = 1/3$. Then:

$\mathsf{Var}\big(\mathbf{E}[X \mid Q]\big) =$ 1/12 ✔ **Answer:** 0.08333

$\mathbf{E}\big[\mathsf{Var}(X \mid Q)\big] =$ 1/6 ✔ **Answer:** 0.16667

Solution:

(a) We know that $\mathsf{Var}\left(X \,|\, Q=q\right) = q\,(1-q)$, for all $q \in [0,1]$, which translates into the abstract statement $\mathsf{Var}\left(X \,|\, Q\right) = Q\,(1-Q)$.

(b) Since $\mathbf{E}\left[X \,|\, Q\right] = Q$, we have $\mathsf{Var}\big(\mathbf{E}\left[X \,|\, Q\right]\big) = \mathsf{Var}\left(Q\right) = 1/12$.

Since $\mathsf{Var}\left(X \,|\, Q\right) = Q\,(1-Q)$, we have

$$\mathbf{E}\big[\mathsf{Var}\left(X \,|\, Q\right)\big] = \mathbf{E}\big[Q\,(1-Q)\big] = \mathbf{E}\left[Q\right] - \mathbf{E}[Q^2] = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

Submit You have used 2 of 3 attempts

Answers are displayed within the problem

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Topic: Unit 6: Further topics on random variables:Lec. 13: Conditional expectation and variance revisited; Sum of a random number of independent r.v.'s / 10. Exercise: Conditional variance II

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? viral explosion 1
Suppose a person who is a carrier of an infectious virus enters a city with a population of 100k. How lon...

? Due date for these problems 23:59UTC ? But I cannot submit at 03:30 EDT which is 07:30 UTC. 3
I understand that the due date for these Unit 6 problems is 23:59UTC. But I cannot submit at 03:30 EDT ...

numbers of expressions 3
My brain is fried from all these nestings of formulas and random variables, and am frankly losing sight o...

Straightforward approach works as well 3
You can integrate if you don't trust your intuition solving part (b). But I must admit that the authors' solu...

? how to begin ? 1 new 6
Hi , can someone please help me with how to approach this problem? I am lost here....

Hint, second part

In the last question, we have solved E[X/Q], and now we have the value of the variance of Q. In the first p... 1

? rrd question

I could work out the first 2, but I am struggling with the 3rd. Is E[Var(X|Q]=Var(X)-Var(E[X|Q])? If so what i... 5