7. Sum of a random number of r.v.'s

Problem Set due Apr 1, 2020 05:29 IST *Completed*

Problem 7. Sum of a random number of r.v.'s

2/2 points (graded)

A fair coin is flipped independently until the first Heads is observed. Let the random variable K be the number of tosses until the first Heads is observed **plus 1**. For example, if we see TTTHTH, so that the first head is observed after 4 tosses, then $K = 4 + 1 = 5$. For $k = 1, 2, \ldots, K$, let X_k be a continuous random variable that is uniform over the interval $[0, 5]$. The X_k are independent of one another and of the coin flips. Let $X = \sum_{k=1}^{K} X_k$. Find the mean and variance of X. You may use the fact that the mean and variance of a geometric random variable with parameter p are $1/p$ and $(1-p)/p^2$, respectively.

$\mathbf{E}[X] =$ 7.5 ✔ **Answer:** 7.5

$\mathsf{Var}(X) =$ 75/4 ✔ **Answer:** 18.75

Solution:

Since X_k is uniform over $[0, 5]$, we have $\mathbf{E}[X_k] = 5/2$ and $\mathsf{Var}(X_k) = 5^2/12 = 25/12$.

Note that $K - 1$ is the number of tosses until the first Heads, and is therefore geometric with parameter $p = 1/2$. In particular, $\mathbf{E}[K-1] = 2$ and $\mathrm{Var}(K-1) = 2$, which implies that $\mathbf{E}[K] = 3$ and $\mathsf{Var}(K) = 2$.

Since $X = \sum_{k=1}^{K} X_k$ is the sum of a random number of independent and identically distributed random variables, we have

$$\mathbf{E}[X] = \mathbf{E}[X_1]\,\mathbf{E}[K] = \frac{5}{2} \cdot 3 = 15/2,$$

and

$$\mathsf{Var}(X) = \mathsf{Var}(X_1)\,\mathbf{E}[K] + (\mathbf{E}[X_1])^2 \mathsf{Var}(K) = \frac{25}{12} \cdot 3 + \frac{25}{4} \cdot 2 = 75/4.$$

Submit You have used 3 of 3 attempts

Answers are displayed within the problem

Discussion

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Topic: Unit 6: Further topics on random variables:Problem Set 6 / 7.
Sum of a random number of r.v.'s

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- Wald's Identity was not discussed in lecture, but it is very intuitive, it is a key hint, and start point — 2
  Wald's identity is the starting point to solve this. E[X_1 + · · · + X_N] = E[E(X1 + · · · + X_N | N)] = E[N*E[...
- STAFF - Why is the variable (K-1) the number of tosses until the first Heads? — 6
  Hi. I know that there are too many students to answer individual questions from students, however, I ...
- [STAFF] — 5
  My answers were marked correct for question 7 but it has not been reflected in progress chart. I belie...
- Can someone help me with the definition of X_k? — 2
  The solution makes perfect sense mathematically but I could not understand what is X_k and why the...
- K as RV+1, versus a RV~Geo(p) — 8
  HI, I haven't gone through this all the way, but I think this is a relevant question. Are we thinking of K ...
- [STAFF] Unable to see the solutions — 2
  Hi Staffs, I'm unable to see the solutions to all of the questions in this problem set. Please let me kno...
- I got seriously tricked by the TTTHTH example...

- What I thought: K is number of tosses it takes to get 2 heads (i.e Sum of 2 Geometric r.v) - What was ...

Pls notice: "observed plus 1" — 1
i have missed this condition for the first time

[STAFF] Let Xk be a continuous random variable that is uniform over the interval [0,5] -> or [0,K]? — 7

At a later time. — 1
Hi Staff. Because I just have completed around 11% of the course, I will not qualify for a certificate. Th...

[Staff][Solved] Something wrong with the total credit? — 5
[Solved] I got all the problems in this set right, but my total credit is 90% (as 26/29). Would you please ...

Hint please — 8
Confused a bit by understanding this problem definition. How did you approach this problem?

[Staff] is k from 1..K or 2..K? — 7