1.- $X \sim Gamma(\kappa, \nu)$ $\kappa = 14$ $E(X) = \frac{\kappa}{\nu}$ $\sigma_x^2 = \frac{\kappa}{\nu^2} = E(X^2) - E(X)^2$

$Y|X=x \sim Exp(1/x)$ $\nu = 0.6$

$E(Y|X) = \frac{1}{\left(\frac{1}{x}\right)} = X$

$Cor(X,Y) = \frac{COV(X,Y)}{\sigma_y \cdot \sigma_y}$

$E(X^2) = \frac{\kappa}{\nu^2} + \frac{\kappa^2}{\nu^2}$

$Var(X) = \frac{\kappa}{\nu^2}$ $Var(Y/X) = \frac{1}{\left(\frac{1}{X}\right)^2}$

$COV(X,Y) = E(X \cdot Y) - E(X) \cdot E(Y)$

$E[X E(Y|X)] - E(X) \cdot E[E(Y|X)]$

$E[X \cdot X] - E(X) \cdot E(X)$

$= \frac{\kappa + \cancel{\kappa^2}}{\nu^2} - \cancel{\frac{\kappa}{\nu} \cdot \frac{\kappa}{\nu}}$

$Var(Y) = Var(E(Y|X)) + E(Var(Y|X))$

$= Var(X) + E(X^2)$

$Var(Y) = \frac{\kappa}{\nu^2} + \frac{\kappa + \kappa^2}{\nu^2}$

$COV = \frac{\kappa}{\nu^2}$

$CR = \frac{COV}{\sigma_x \cdot \sigma_y} = 0,25$ ENR

2.- X: Leng

Y: Mat

Z: Ciencias

$P(0,17X + 0,58Y + 0,25Z > 700)$

$\sigma = \sum\sum \rho_{ij} \sigma_{x_i} \sigma_{x_j} \frac{dg}{dx_i} \frac{dg}{dx_j}$

$dX = [0.17, 0.58, 0.25]$

3.- X: Clientes

$X \sim Poisson(100)$

p: Compra

$p = 0.2$

$n = 30$

Compra $\sim$ Poisson (20)

$\mu = \frac{1}{20}$

Poisson (20) $E(20) = 20$ $\sigma_{P20} = 20$

$P(\bar{X}_n \cdot p > 20)$

$\overline{Pois(20)}_n \sim N(\mu, \sigma/\sqrt{n})$

$1 - pnorm(20, 20, 20/\sqrt{30})$

4.- $X \sim Gamma(\kappa, \nu)$ $\mu_x = \frac{\kappa}{\nu} = 24$ $\sigma_x = \frac{\kappa}{\nu^2} = 24$ $\kappa = 24$ $\nu = 1$

$Y = 1/x$

$Y = g(\mu_x) + (X - \mu_x)\frac{dg}{dX} + \frac{1}{2}(X - \mu_x)^2 \frac{d^2g}{dX^2}$

$\frac{dg}{dX} = -X^{-2}$

$\frac{d^2g}{dX^2} = -1(+2)X^{-3}$

$E(g(X)) = g(\mu_x) + \frac{g''(\mu_x)}{2} \cdot \sigma_x^2$

$= \frac{1}{24} + \frac{(\cancel{2} \cdot (24)^{\cancel{-3}\,-1})}{\cancel{2}} \cdot \cancel{24^2}$

$\frac{1}{24} + \frac{1}{24} = \frac{1}{12}$

5.— $X \sim Exp(\nu)$

$\mu_{exp} = \frac{1}{\nu}$

$n = 30$

$\nu = 0.4$

$Sesgo = E(\hat{\nu}) - \nu$

$E\left(\frac{1}{\bar{x}}\right)$

$E\left(\frac{1}{\bar{x}}\right) \approx g(\mu_x)$

$\hat{\nu} = \frac{1}{\bar{x}}$

$\bar{x} \sim Gama(n, n\nu)$

$g(\mu_x) = \frac{n}{\sum X_i} = \frac{n}{\sum \frac{1}{\nu}} = \frac{\cancel{n}}{\cancel{n} \cdot \frac{1}{\nu}} = \nu$

Estimación de primer grado ‼

$Sesgo = E(\bar{\nu}) - \nu = \nu - \nu = 0 \rightarrow$ No sirve ‼

$g(x) = \left(\frac{1}{n}\sum X_i\right)^{-1}$ $\quad g' = -1\left(\frac{1}{n}\sum X_i\right)^{-2} \cdot \frac{1}{n}$ $\quad g'' = 2\left(\frac{1}{n}\sum X_i\right)^{-3} \cdot \frac{1}{n^2} \rightarrow (\mu_x) = 2\left(\cancel{\frac{1}{n}} \cdot \cancel{n} \cdot \frac{1}{\nu}\right)^{-3} \cdot \frac{1}{n^2}$

$\frac{2\nu^3}{n^2}$

g

$E(\nu) \approx \underbrace{g(\mu_x)}_{\nu} + \frac{g''(\mu_x)}{2} \cdot \sigma_x^2$

$\nu + \frac{2\cancel{\nu^3}}{n^2} \frac{1}{\cancel{\nu^2}}$

$\hookrightarrow R$

$\cancel{\theta^2} + \cancel{2\theta} - \cancel{2\theta^2} + \cancel{\theta^2} - \cancel{2\theta} + 1$

6.— $p_X(x) = \begin{cases} \theta^2 & x = -1 \\ 2\theta(1-\theta) & x = 0 \\ (1-\theta)^2 & x = 1 \end{cases}$

$\theta^2 + 2\theta(1-\theta) + (1-\theta)^2 = 1$

$\cancel{\theta^2} + \cancel{2\theta} - \cancel{2\theta^2} + \cancel{\theta^2} - \cancel{2\theta} + 1 = 1$

$\hookrightarrow$ Infinitas soluciones ‼

no

$E(X) = (-1)\theta^2 + \cancel{0 \cdot \theta(1-\theta)}^{0} + (1-\theta)^2 = \sum X \cdot p_X(x)$

$= -\cancel{\theta^2} + \cancel{\theta^2} - 2\theta + 1$

$\bar{X} = -2\theta + 1$

$2\theta = 1 - \bar{X}$

$\hat{\theta} = \frac{1 - \bar{X}}{2}$

$\bar{X} = (1-\theta)^2 \cdot 2 + \cancel{0 \cdot 2\theta(1-\theta)}^{0}$

$= 2(\theta^2 - 2\theta + 1)$

$\bar{x} = 2\theta^2 - 4\theta + 2$???

$L = \prod f_X(x)$

7.— $L = \theta^2 \cdot 2\theta(1-\theta) \cdot (1-\theta)^2$

$\ln(L) = 2\ln(\theta) + \ln(2\theta) + \ln(1-\theta) + 2\ln(1-\theta)$

$\frac{d\ln(L)}{d\theta} = \frac{2}{\theta} + \frac{1}{\cancel{2}\theta}\cancel{2} + \frac{1}{1-\theta} \cdot (-1) + \frac{2}{1-\theta} \cdot (-1)$

$= \frac{3}{\theta} + \frac{1}{1-\theta} = 0$

$3(1-\theta) = \theta$

$3 - 3\theta = \theta$

$3 = 4\theta \rightarrow \hat{\theta} = \frac{3}{4}$