Dinkelbach Algorithm for SBPO Challenge 2025

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The Problem

In warehouse operations, picking each order individually is inefficient. Instead, we group compatible orders into **waves** so that their items can be collected together through shorter and more efficient routes. The goal is to decide which orders should form the next wave to maximize **picking productivity** — that is, to collect as many products as possible while visiting as few aisles as needed.

Let:

- O = set of all pending orders.
- I_o = set of items requested in order $o \in O$.
- A = set of aisles in the warehouse.
- $A_i \subseteq A$ = aisles containing item i.
- u_{oi} = units of item i requested by order o.
- u_{ai} = units of item i available in aisle a.
- LB, UB = lower and upper bounds on the total number of items in the wave.

We want to select:

$$O' \subseteq O \quad \text{(orders in the wave)}, \qquad A' \subseteq A \quad \text{(aisles to visit)}$$

so as to maximize the ratio between collected units and visited aisles:

$$\max_{O', A'} \frac{\sum_{o \in O'} \sum_{i \in I_o} u_{oi}}{|A'|}$$

subject to:

$$\sum_{o \in O'} \sum_{i \in I_o} u_{oi} \geq LB \tag{1}$$

$$\sum_{o \in O'} \sum_{i \in I_o} u_{oi} \leq UB \tag{2}$$

$$\sum_{o \in O'} u_{oi} \leq \sum_{a \in A'} u_{ai}, \quad \forall i \in I_o,\ o \in O' \tag{3}$$

A pair (O', A') satisfying (1)–(3) defines a feasible wave, and the optimal wave maximizes the productivity ratio above.

An Exact Parametric Approach

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- **Mauris tempor** risus nulla, sed ornare
- **Libero tincidunt** a duis congue vitae
- **Dui ac pretium** morbi justo neque, ullamcorper

Eget augue porta, bibendum venenatis tortor.

Warm start

The Dinkelbach algorithm can benefit from a high quality initial solution, and thus we consider two simple greedy strategies to obtain them. The first one prioritizes picking aisles of a big *size* (i.e. those $a \in A$ that maximize $\sum_{i \in I_o} u_{ai}$) while the second one prioritizes aisles with high *diversity* (i.e. those $a \in A$ that maximize $|\{i \in I_o : u_{ai} > 0\}|$). As seen in Figure 1 optimal solutions have these type of aisles.

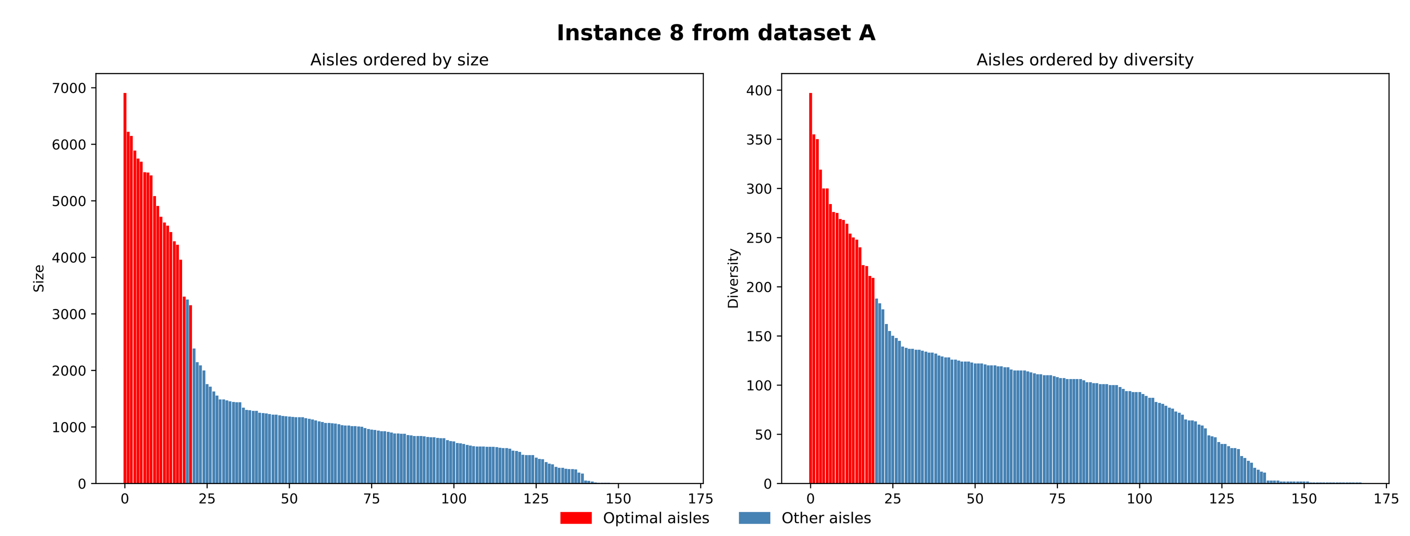

Figure 1. Aisles sorted by size and diversity and optimal aisles for instance 8 from dataset A.

Our algorithm fixes, for each possible k, the first k aisles sorted based on each of these two criteria, and then picks orders greedily sorting them by size. Our implementation is efficient with complexity almost linear in the input size, and usually finds solutions 10%-close to the optimal one in the order of seconds, as can be seen in Figure 2. In most cases the greedy approach based on diversity beats the one based on size.

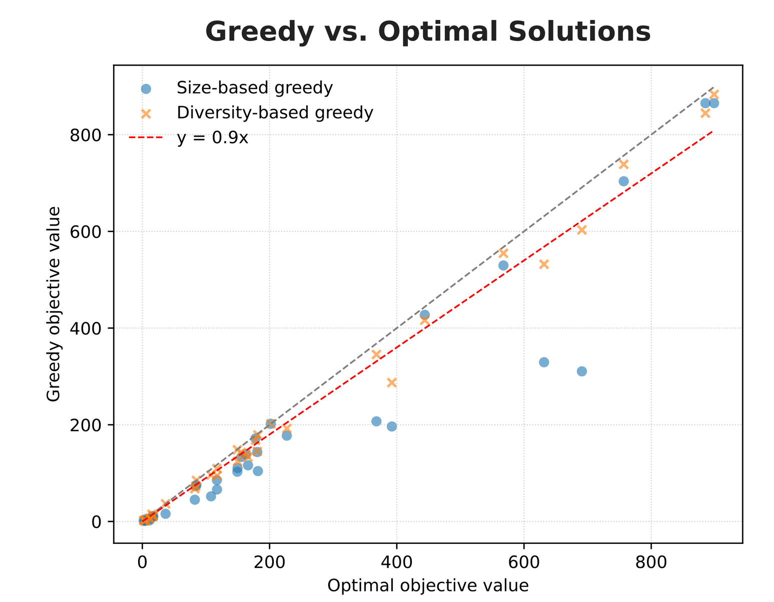

Figure 2. Comparison between the optimal values and the results given by the greedy algorithms.

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Foo	13.37	384,394	α
Bar	2.17	1,392	β
Baz	3.14	83,742	δ
Qux	7.59	974	γ

Table 1. A table caption.

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References

[1] Claude E. Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, 1948.