Homework 2

-Horseshoe Vortex Method-

By Ignacio Anguita
Ariadna Fernandez
Mario Gayete
Nuria Martin
Jeremi Sanchez
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1 Validation

1.1 Grid convergence

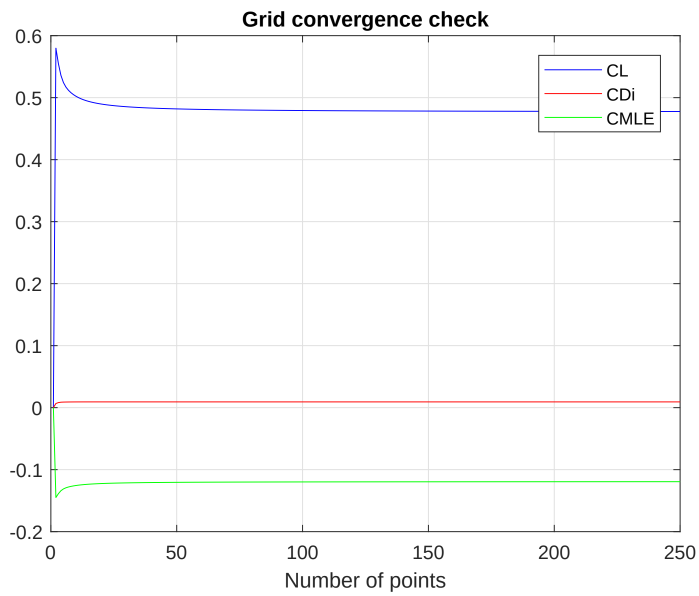

Figure 1: Graph of the grid convergence.

In this plot we can observe that for around 50 points Cl,Cdi and Cmle are stabilized. So, we are going to use 51 points (50 panels) for all the validations.

1.2 Problem 1

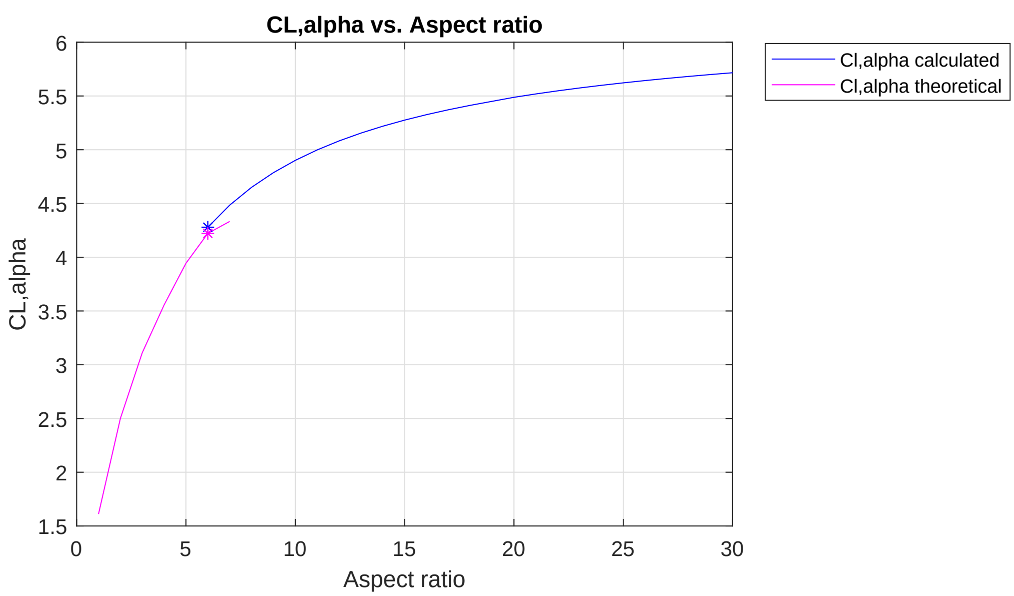

Figure 2: Graph of lift slope in front of aspect ratio.

If the variation of $C_{l\alpha}$ is studied, it is possible to observe that both, calculated and theoretical results from Katz-Plotkin [2] converge in a similar distribution around 6. If, in a more concrete comparation, the point of A=6 is studied, the value of $C_{l\alpha}$ calculated is 4.279, while the value of $C_{l\alpha}$ theoretical is 4.222 (approximated value taken from the figure 12.16). These are close values given that the error is only about 1%. It is also remarkable that $C_{l\alpha}$ presents a higher variation for small values of A that for higher values.

1.3 Problem 2

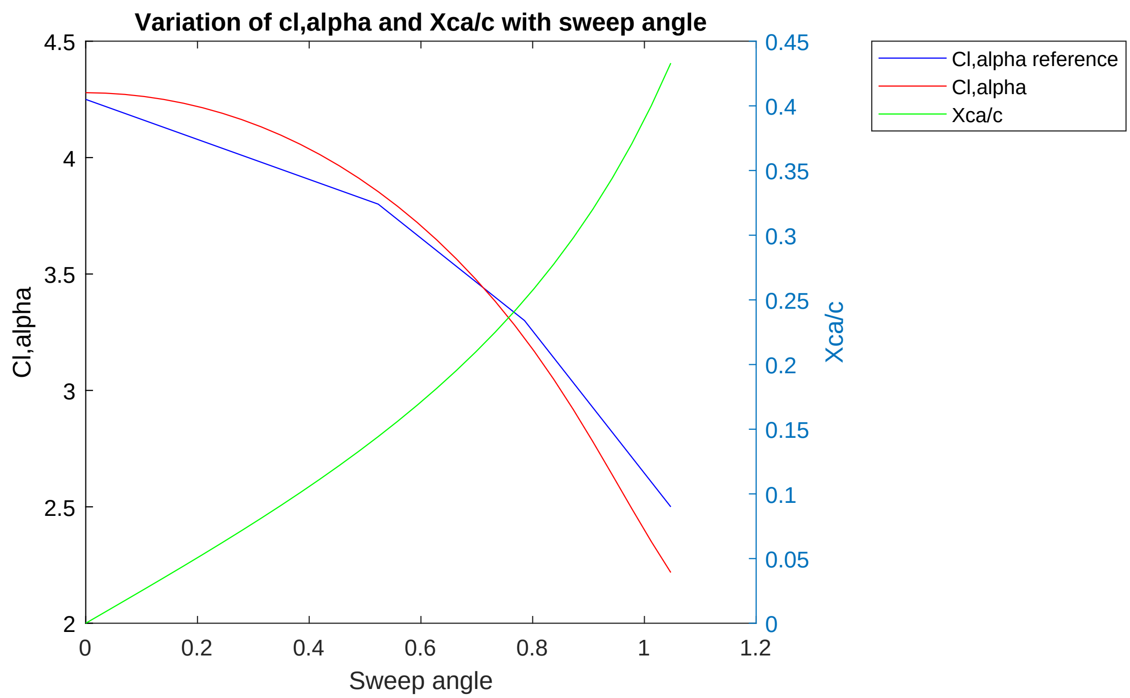

Figure 3: Graph of lift slope and aerodynamic centre position for different sweep angles(in rad).

In this plot are represented, on the left, the variation of $C_{l\alpha}$ with the sweep angle and, on the right, the variation of the aerodynamic center in percentage of the chord. About the variation of the $C_{l\alpha}$ is appreciable that in this case the $C_{l\alpha}$ goes in decrease with the increase of the sweep angle. This is the opposite relation that $C_{l\alpha}$ had with the aspect ratio. At the same time, the variation of the aerodynamic center is directly related to the variation of the sweep angle. For a sweep angle of 0°, the aerodynamic center is located in 0, while for a value of sweep angle of 20°, the position of the aerodynamic center tends to 10% of the chord.

1.4 Problem 3

In the plot below we are studying the variation of Cl for A=10,C_L=0.25 and no sweep, for different taper ratios. As we can observe, the more the taper ratio increases,the bigger is the C_l in the root and smaller in the tips.

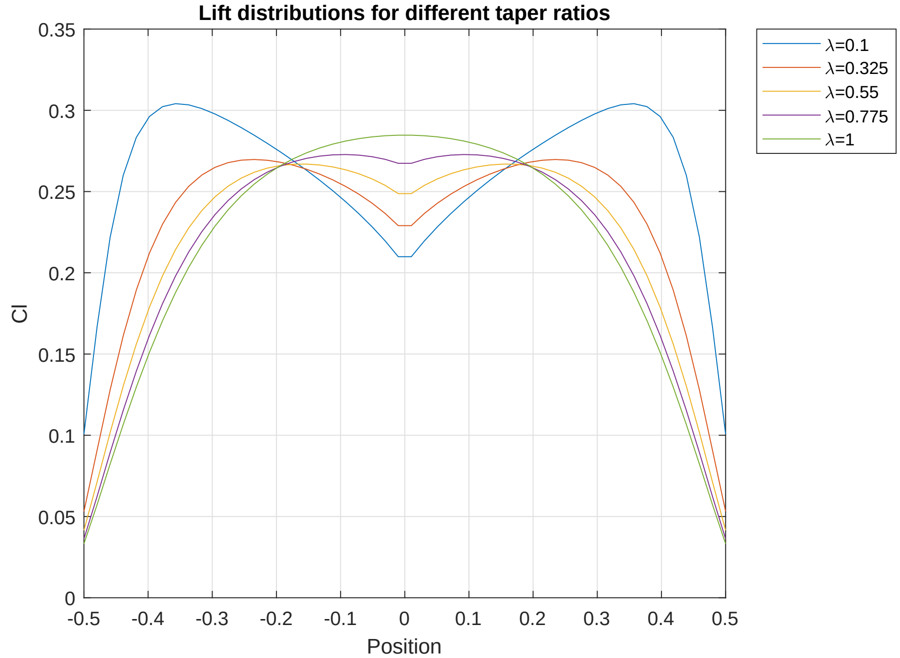

Figure 4: Graph of lift distributions for different taper ratios.

1.5 Problem 4

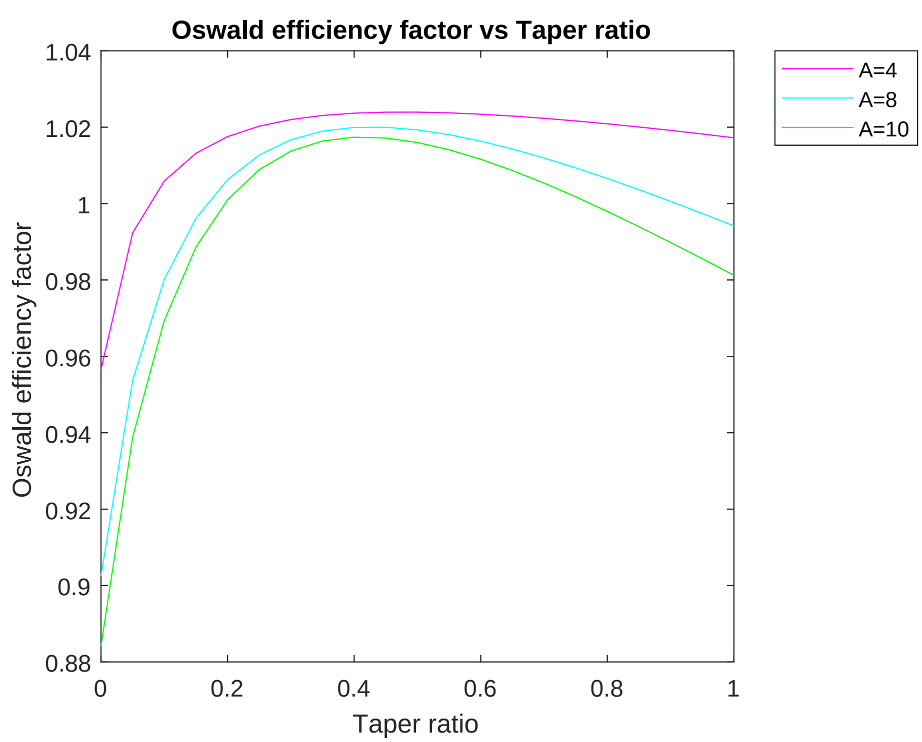

Figure 5: Graph of the Oswald efficiency factor in front of taper ratio for different aspect ratios.

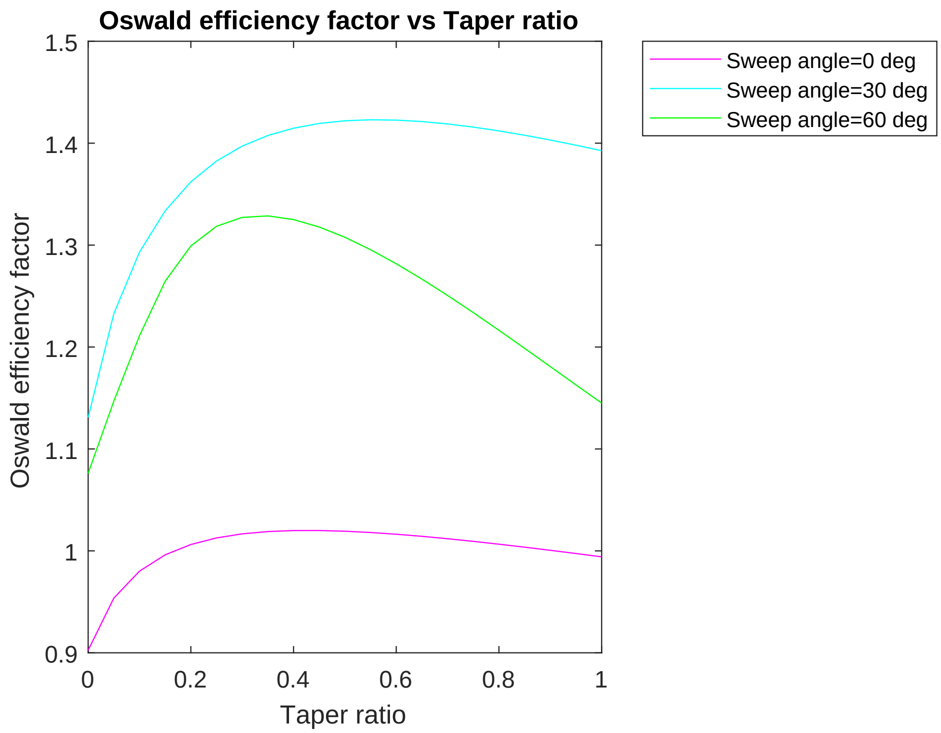

Figure 6: Graph of the Oswald efficiency factor in front of the taper ratio for different sweep angles.

These two plots represent the variation of the Oswald efficiency factor for different taper ratios, aspect ratios and sweep angles.
In the first graph it is observed that for smaller aspect ratios, the Oswald factor is bigger. Regarding taper ratios, there is an optimal taper ratio at λ =0.4, but Oswald factor is much bigger in hight taper ratios rather than in low ones. In the second graph the Oswald's factor with sweep angle of 0 ° is normal, but when we observe this factor with 30° and 60 ° the results are not the expected ones. This is probably due to the fact that C_{di} is very small, and any little mistakes produced by the numerical results of the horseshoe vortex method may result in a big error of the calculation of the C_{di} and so of the Oswald factor.We were unable to find whether was an error of the horshoe method (because it is not an enough good aproximation) or if it was an error of our code.Nevertheless those big Oswald factors are not a realistic (or good) result.

1.6 Problem 5

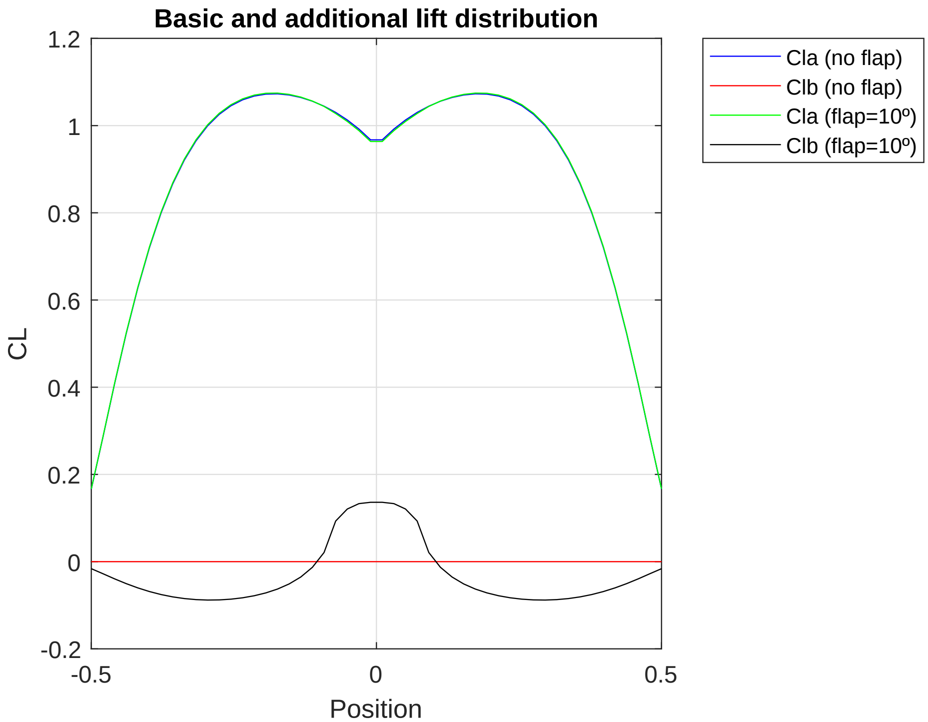

Figure 7: Graph of basic and additional lift distribution for different flap deflections.

Flap deflection	C_{m0}	x_{ac}	$C_{L,max}$	α_{max} (in $^\circ$)
0 $^\circ$	-0.5	0.0509	1.3982	5.0465
10 $^\circ$	-0.5	0.0509	1.8384	5.0913

Table 1: Numerical results for different flap deflections.

In this problem we suppose A=8, λ =0.5, ε_t =0, Δ_{50} =0 and NACA 2408. Of the first graph we observe that Cla is the same for both flap positions, this is because Cla only depends on the wing planform. Meanwhile, Clb changes when deflecting the flap because it depends on zero-lift angle. In the table we can see different numerical results for C_{m0},$x_{c/4}$,$C_{L,max}$ and α_{max} for two flap deflections.C_{m0} and $x_{c/4}$ are the same for both because $x_{c/4}$ only depends of the geometry of the wing and in C_{m0} the difference is very small. In the second graph there is represented the lift distribution for both flap deflections. C_L with the flap deflected is bigger than for η =0.Also, we can observe that in the root chord tere is a sudden decrease of C_L, and around this position we have the maximum C_L.

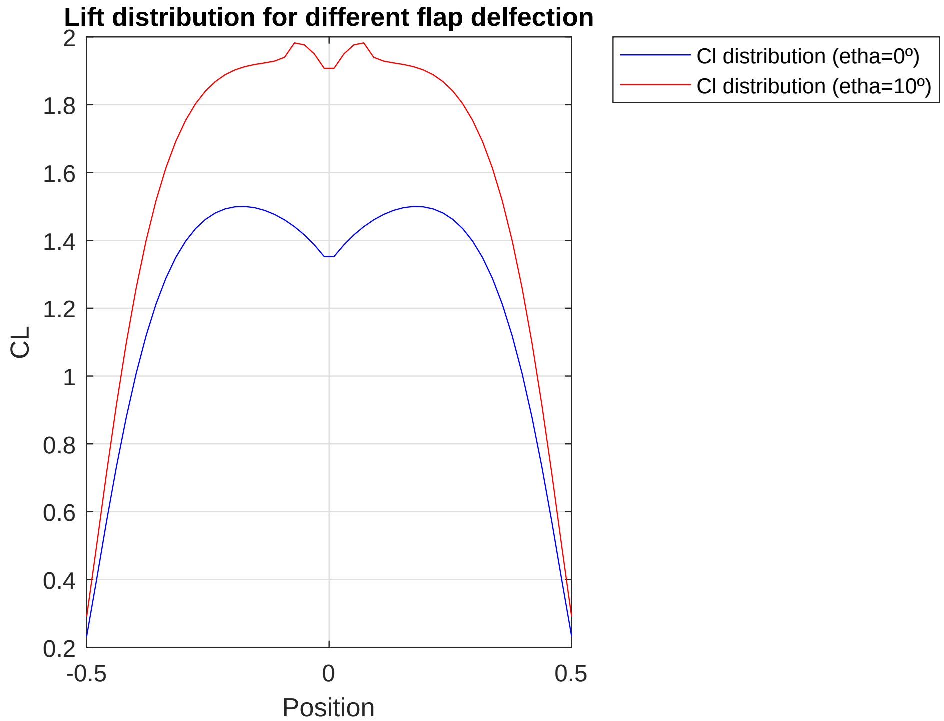

Figure 8: Graph of lift distribution for different flap deflection.

1.7 Problem 6

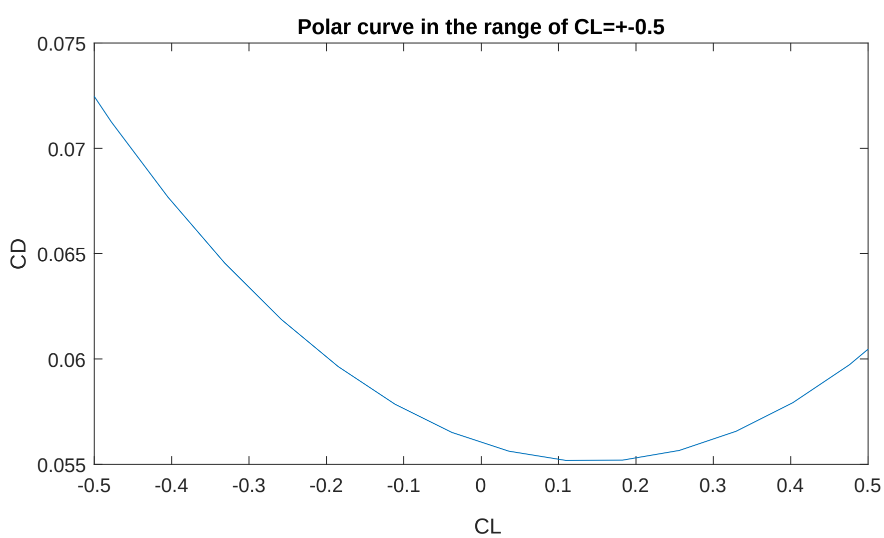

Figure 9: Graph of the polar curve for a NACA 2408.

In problem 6 we have used the same wing as in problem 5. To obtain this polar curve we have extracted some data of [1] as Cdp for a range of ± 0.5 C_L. Then we have make the operation $C_D = C_{Di} + C_{DP}$ to plot the final curve. In the graph it can be seen that there is a C_D minimum in C_L=0.15

2 Results discussion

Based on the results presented above, both taper ratio and sweep have a negative effect on the structure. A big taper ratio reduces the lift on the tips (although the C_l is smaller), which means that the bending moment is augmented. A big sweep angle increases the lift in the tips, increasing also the bending moment. There is a taper ratio optimal to reduce the Oswald factor, though lowering the taper ratio reduces the downwash near the tips and makes the lift distribution more elliptical. This also means that the stall behaviour is worse. An increase of the sweep angle results in a reduction of the $C_{l\alpha}$ and an increase if the Oswald factor (this depends on the angle, it is not linear).

3 Code

3.1 Constants

```
%% Constants
N=51;% Number of points
b=1;% Span of the wing
P_Distribution=1; % Type of discretization 0 for linear, 1 for full cosine
A=8; % Aspect ratio
Taper_ratio=1; %Taper Ratio
Quarter_chord_sweep_angle=degtorad(0); %Quarter Chord sweep angle
Dihedral=degtorad(0); %Dihedral angle
alpha_zero_lift_initial=degtorad(0); %alpha zero lift in the root
alpha_zero_lift_final=degtorad(0); %alpha zero lift in the tail
Total_twist=degtorad(0);%total twist in the tail, we supose positive twist in when it is wash-out
S=b^2/A;%Surface of the wing
rho=1;%Density of the air
Alpha=deg2rad(6);
v_inf_mod=1;%module of the free flow velocity
v_inf=[v_inf_mod*cos(Alpha) 0 v_inf_mod*sin(Alpha)];%flow velocity
flap_initial_position=b/2*[0 1/2]; %initial flap's postion (y-potition),we consider symmetrical wing=
flap_final_position=b/2*[0.25 0.75]; %final flap's postion (y-potition), we consider symmetrical wing=
etha=deg2rad(0); %flap deflection
E=0.2; %flap's adimensional chord
theta_h=acos(2*E-1);
```

3.2 Main

```
%% %% COMPUTATION OF INCREMENT OF ALPHA ZERO LIFT DUE TO THE FLAP
eff_factor_t=-(1-theta_h/pi+sin(theta_h)/pi); %therical efficiency factor
%Real efficiency factor:
if etha<deg2rad(10)
    eff_factor_r=eff_factor_t;
elseif etha==deg2rad(10)
    eff_factor_r=eff_factor_t*0.8;
elseif etha>deg2rad(10) && etha<deg2rad(20) %we caproximate to linear variation in each
%discretization of the correction factor
    eff_factor_r=eff_factor_t*(0.7-((deg2rad(20)-etha)*(-0.1))/(deg2rad(20)-deg2rad(10)));
elseif etha==deg2rad(20)
    eff_factor_r=eff_factor_t*0.7;
end
A_alpha_l0=eff_factor_r*etha;
%% GEOMETRY Discretization
%X(N,1) points of which is divided the span
if P_Distribution==0
    x(:,1)=lineal(b,N); %we stablish the coordinates x of each airfoil's point that
    % makes the panel (Linear Dist.)
elseif P_Distribution==1
    x(:,1)=full_cosine(b,N); %we stablish the coordinates x of each airfoil's point that
    % makes the panel (Full Cosine Dist.)
else
    output ERROR
end
%Calculation of the Cr and Ct
[Cr,Ct]=Cr_Ct_Calculation(b,A,Taper_ratio);
%Calculation of the Vortex Points
[x_vortex_1]= Vortex_points_1(x(:,1),Cr,Quarter_chord_sweep_angle,Dihedral,N);
[x_vortex_A]=Vortex_points_2(x(:,1),x_vortex_1,b,N,Dihedral);
[x_vortex_D]=Vortex_points_3(x(:,1),x_vortex_1,b,N,Dihedral);
x_vortex_B=[x_vortex_1(:,1),x_vortex_A(:,2),x_vortex_A(:,3)];
x_vortex_C=[x_vortex_1(:,1),x_vortex_D(:,2),x_vortex_D(:,3)];%Calculation of the control points
[x_control]= Control_points(x_vortex_1,Cr,Ct,b,N,Dihedral);%Calculation of the normal vectors
```

```
[normal_vector]=Normal_vector(alpha_zero_lift_initial,alpha_zero_lift_final,Total_twist,
x_control,Dihedral,N,b,flap_initial_position,flap_final_position,A_alpha_l0);
%% Solver
[gamma,induced_velocity_w]=gamma_solver(x_vortex_A,x_vortex_B,x_vortex_C,x_vortex_D,x_control,
normal_vector,v_inf,N,x_vortex_1);
A_Lift=Lifts(x,gamma,rho,v_inf_mod,N); %lift generated by each section of the wing
Lift=ones(1,N-1)*A_Lift; %total lift generated by the wing
CL=Lift/(0.5*rho*v_inf_mod^2*S); %lift coefficient of the wing
Cly=Cl_y(A_Lift,rho,v_inf_mod,x_vortex_B,x_vortex_C,Cr,Ct,b,N,x); %lift coefficient of each
%section
M_LE=Moment_LE(A_Lift,x_vortex_1,N); %leading edge moment
CMLE=CM_LE(M_LE,rho,S,v_inf_mod,Cr,Taper_ratio); %leading edge moment coefficient
[alpha_ind,CDi,Di]=induced_alpha_drag(gamma,induced_velocity_w,A_Lift,v_inf_mod,S,N,x );
function [ x ] = lineal(b,N)
x=zeros(N,1);
x(1,1)=-b/2;
for i=2:N %Calculation of the points
    x(i,1)=b/(N-1)+x(i-1,1);
end
end
function [x] = full_cosine(b,N)
i=1:N;
x=-b/2 +(b/2)*(1-cos((i-1)*pi/(N-1)));
end
function [Cr,Ct] = Cr_Ct_Calculation(b,A,Taper_ratio)
Cr=2*b/(A*(1+Taper_ratio));%Calculation of the Cr
Ct=Cr*(Taper_ratio); %Calculation of the Ct
end
function[x_control] = Control_points(x_vortex_1 ,Cr,Ct,b,N,Dihedral)%Calculation of the 3/4 tangent chord line
for i=1:N-1
    section_chord(i)=Cr-(Cr-Ct)*abs(x_vortex_1(i,2))/(0.5*b);
end

x_control=zeros(N-1,3);%Definition of the matrix
for i=1:N-1%Calculation of the second component (which is the same of the vortex_1)
x_control(i,2)=x_vortex_1(i,2);
end
for i=1:N-1%Calculation of the first componente
x_control(i,1)=x_vortex_1(i,1)+0.5*section_chord(i);
end
if(Dihedral~=0)%in case of dihedral we put a Z coordinate different than 0
for i=1:N-1
x_control(i,3)=sin(Dihedral)*abs(x_control(i,2));
end
end
end
function[x_vortex_1] = Vortex_points_1(x,Cr,Quarter_chord_sweep_angle,Dihedral,N)
x_vortex_1=zeros(N-1,3);%Definition of the matrix
for i=1:N-1%Calculation of the second component (which is in the middle of two points)
    x_vortex_1(i,2)=(x(i,1)+x(i+1,1))/2;
end
for i=1:N-1%Calculation of the first component, using the quarter chord sweep angle
    x_vortex_1(i,1)=0.25*Cr+abs(x_vortex_1(i,2))*tan(Quarter_chord_sweep_angle);
end
if(Dihedral~=0)%in case of dihedral we put a Z coordinate different than 0
    for i=1:N-1
        x_vortex_1(i,3)=sin(Dihedral)*abs(x_vortex_1(i,2));
    end
end
end
function[x_vortex_2] = Vortex_points_2(x,x_vortex_1,b,N,Dihedral)
x_vortex_2=zeros(N-1,3);%Definition of the matrix
for i=1:N-1%Calculation of the second component
    x_vortex_2(i,2)=x(i);
end
for i=1:N-1%Calculation of the first component
    x_vortex_2(i,1)=(x_vortex_1(i,1))+20*b;
end
if(Dihedral~=0)
    for i=1:N-1%in case of dihedral we put a Z coordinate different than 0
        x_vortex_2(i,3)=sin(Dihedral)*abs(x_vortex_2(i,2));
    end
end
end
function[x_vortex_3] = Vortex_points_3(x,x_vortex_1,b,N,Dihedral)
x_vortex_3=zeros(N-1,3);%Definition of the matrix
for i=1:N-1%Calculation of the second component
    x_vortex_3(i,2)=x(i+1);
end
for i=1:N-1%Calculation of the first component
    x_vortex_3(i,1)=(x_vortex_1(i,1))+20*b;
```

```
end
if(Dihedral~=0)
    for i=1:N-1 %in case of dihedral we put a Z coordinate different than 0
        x_vortex_3 (i,3)=sin(Dihedral)*abs(x_vortex_3(i,2));
    end
end
end
function[normal_vector]=Normal_vector(alpha_zero_lift_initial,alpha_zero_lift_final,Total_twist,
x_control,Dihedral, N,b, flap_ini,flap_fin,A_alpha_l0)
%First we define the function of the zero lift line, where m is the slope and n the initial value.
m=((alpha_zero_lift_final-Total_twist-alpha_zero_lift_initial)*2/b);
n=alpha_zero_lift_initial;
rotation_matrix=[1 0 0; 0 cos(Dihedral) -sin(Dihedral); 0 sin(Dihedral) cos(Dihedral)];
%definition of the rotation matrix
normal_vector=zeros(N-1,3);%definition of the normal vector matrix
verification_vector=zeros(N-1,1); %we define this vector to see which points have
%the influence of the flap
for i=1:N-1 %in this loop we find which control points have the influence of the flap
    for j=1:length(flap_ini)
        if abs(x_control(i,2))>flap_ini(j) && abs(x_control(i,2))<flap_fin(j)
            verification_vector(i)=1;
        end
    end
end
for i=1:N-1 %in this loop we calculate the first component (the x)of the normal vector and the
% third component (the z)
    if verification_vector(i)==0
        normal_vector(i,1)=-sin(m*abs(x_control(i,2))+n);%it is the sinus of the angle of the ZLL
        normal_vector(i,3)=cos(m*abs(x_control(i,2))+n);%it is the cosine of the angle of the ZLL
    else
        normal_vector(i,1)=-sin(m*abs(x_control(i,2))+n+A_alpha_l0);
        normal_vector(i,3)=cos(m*abs(x_control(i,2))+n+A_alpha_l0);
    end
end
if(Dihedral~=0)%in case of Dihedral we apply a rotation matrix
for i=1:N-1
    normal_vector(i,:)=normal_vector(i,:)*rotation_matrix;
end
end
end
function [ vector,modul ] = vector_r( x,y )
vector=(x-y)'; %we calculate the vector that goes from point y to x
modul=sqrt(vector'*vector); %we calculate the modul of this vector
end
function [ vel_induced ] = induced_velocity_line( x1, x2, xp )
%Computation of the induced velocity by a single vortex line
[r0,mod_r0]=vector_r(x2,x1);
[r1,mod_r1]=vector_r(xp,x1);
[r2,mod_r2]=vector_r(xp,x2);
ror=cross(r1,r2);
mod_ror=sqrt(ror'*ror);
vel_induced=(1/(4*pi))*((r0/mod_ror)'*(r1/mod_r1-r2/mod_r2))*ror/mod_ror;
if (mod_r1<=1e-6 || mod_r2<=1e-6 || mod_ror<=1e-6)
   vel_induced=zeros(3,1);
end
end
function [gamma,w_induced ] = gamma_solver(xA,xB,xC,xD,xp,vn,v_inf,N,x )
M=N-1;
w_induced=zeros(M,M);
A=zeros(M,M);
B=zeros(M,1);
for i=1:M
   for j=1:M
      vel_induced_AB=induced_velocity_line(xA(j,:),xB(j,:),xp(i,:));
      vel_induced_BC=induced_velocity_line(xB(j,:),xC(j,:),xp(i,:));
      vel_induced_CD=induced_velocity_line(xC(j,:),xD(j,:),xp(i,:));
      vel_induced=vel_induced_AB+vel_induced_BC+vel_induced_CD; %induced velocity on
      %1 by horseshoe vortex j
      A(i,j)=vel_induced'*vn(i,:)'; %influence coefficient a(i,j)
      w_induced_AB=induced_velocity_line(xA(j,:),xB(j,:),x(i,:));
      w_induced_CD=induced_velocity_line(xC(j,:),xD(j,:),x(i,:));
      w_induced(i,j)=w_induced_AB(3)+w_induced_CD(3);
   end
   B(i)=-v_inf*vn(i,:)';
end
gamma=A\B;
end
function[Lift] = Lifts(x,gamma,rho,u,N)
Lift=zeros(N-1,1);
for i=1:N-1
    Lift(i)=abs(rho*(x(i+1)-x(i)))*gamma(i)*u;
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```
end
end
function [Cl]=  Cl_y(Lift ,rho ,u,x_vortex_B ,x_vortex_C ,Cr,Ct,b,N,x)
%Calculation of every panel surface
S_section=zeros(N-1,1);
partial_chord=zeros(N,1); %chord of each division
for i=1:N
  partial_chord(i)=Cr-(Cr-Ct)/(b/2)*abs(x(i));
end
partial_chord(N)=Ct;
for i=1:N-1
     S_section(i)=(partial_chord(i)+partial_chord(i+1))*0.5*abs(x_vortex_C(i,2)-x_vortex_B(i,2));
end
%Calculation of every Cl
Cl=zeros(N-1,1);
for i=1:N-1
   Cl(i)=Lift(i)/(0.5*rho*u*u*S_section(i));
end
end
function [Leading_Edge_Moment]=Moment_LE(Lift ,x_vortex_1 ,N)
%Calculation of the Leading edge moment with the aproximations:
%cos(alpha)=1, Xle=0;
Leading_Edge_Moment=0;
for i=1:N-1
     Leading_Edge_Moment=Leading_Edge_Moment-(Lift(i)*x_vortex_1(i,1));
end
end
function [CMLE]=  CM_LE(Leading_Edge_Moment,rho,S,u,Cr,Taper_ratio)
%Calculation of the mean chord line
mean_chord=(2/3)*Cr*(1+Taper_ratio+Taper_ratio^2)/(1+Taper_ratio);
%Calculation of the CM_LE
CMLE=Leading_Edge_Moment/(0.5*rho*u*u*S*mean_chord);
end
function [ alpha_ind , Cdi, Di ] = induced_alpha_drag(gamma,w_wake,A_Lift,u_inf,S,N,x )
alpha_ind=zeros(N-1,1);
Cdi=0;
Di=0;
for i=1:N-1
     for j=1:N-1
          alpha_ind_partial(j)=gamma(j)*w_wake(i,j);
     end
     alpha_ind(i)=-1/u_inf*sum(alpha_ind_partial);
     Di=Di+A_Lift(i)*alpha_ind(i);
     Cdi=Cdi+gamma(i)*abs(x(i+1)-x(i))*alpha_ind(i);
end
Cdi=2*Cdi/(u_inf*S);
end
```

References

[1] Abbot, Ira H.; Doenhoff, Albert E. von. " Theory of Wing Sections: including a summary of airfoil data." *New York: Dover,1959. p.475*

[2] Katz, Joseph; Plotkin, Allen " Low-Speed Aerodynamics." *Cambridge Aerospace Series,13 p.348*