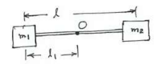
5.3



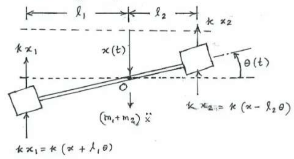
Assume masses m_1 and m_2 to be point masses.

Let O denote the C.G. of the system located at a distance of l_1 from mass m_1 so that

$$m_1 l_1 = (l - l_1)\, m_2 = l_2\, m_2$$

$$\text{or} \quad l_1 = (l - l_1)\frac{m_2}{m_1} \; ; \quad l_2 = l - l_1 \qquad (1)$$

Free body diagram :



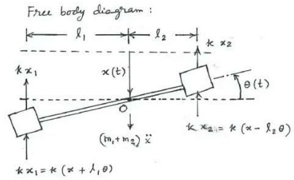
Equations of motion:

$$(m_1 + m_2)\ddot{x} = -2k(x + l_1\theta) - 2k(x - l_2\theta) \qquad (1)$$

$$J_0\ddot{\theta} = 2k(x - l_2\theta)\cdot l_2 - 2k(x + l_1\theta)\cdot l_1 \qquad (2)$$

$$\text{or} \quad l_1 = (l - l_1)\frac{m_2}{m_1} \; ; \quad l_2 = l - l_1 \qquad (1)$$

Free body diagram :



Equations of motion:

$$(m_1 + m_2)\ddot{x} = -2k(x + l_1\theta) - 2k(x - l_2\theta) \qquad (1)$$

$$J_0\ddot{\theta} = 2k(x - l_2\theta)\cdot l_2 - 2k(x + l_1\theta)\cdot l_1 \qquad (2)$$

where

(m_1+m_2) denotes the total mass acting through O

$J_o = (m_1 l_1^2 + m_2 l_2^2)$ indicates of mass moment of inertia of the system.

Eqs. (1) and (2) can be written in matrix form as

$$\begin{bmatrix} (m_1+m_2) & 0 \\ 0 & (m_1 l_1^2 + m_2 l_2^2) \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4k & (2k l_1 - 2k l_2) \\ (2k l_1 - 2k l_2) & (2k l_2^2 + 2k l_1^2) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (3)$$

When $m_1 = 50$ kg, $m_2 = 200$ kg, $k = 1000$ N/m and $l = 1$ m,
Eq. (1) gives

$$l_1 = (1 - l_1)\frac{200}{50} = 4 - 4 l_1$$

or $l_1 = 0.8$ m; $l_2 = 0.2$ m

Eq. (3) becomes:

$$\begin{bmatrix} 250 & 0 \\ 0 & 40 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 4000 & 1200 \\ 1200 & 1360 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (4)$$

By assuming harmonic solutions

$$x(t) = X \cos(\omega t + \phi) \qquad (5)$$

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$$= \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (3)$$

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By assuming harmonic solutions

$$\left.\begin{aligned} x(t) &= X \cos(\omega t + \phi) \\ \theta(t) &= \Theta \cos(\omega t + \phi) \end{aligned}\right\} \qquad (5)$$

Eq. (4) can be written as

$$-\begin{bmatrix}250\omega^2 & 0\\ 0 & -40\omega^2\end{bmatrix}\begin{Bmatrix}X\\ \Theta\end{Bmatrix}+\begin{bmatrix}4000 & 1200\\ 1200 & 1360\end{bmatrix}\begin{Bmatrix}X\\ \Theta\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (6)$$

i.e.

$$\begin{bmatrix}(4000-250\omega^2) & 1200\\ 1200 & (1360-40\omega^2)\end{bmatrix}\begin{Bmatrix}X\\ \Theta\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (7)$$

For a nontrivial solution of X and Θ, the following condition is to be satisfied:

$$\begin{vmatrix}4000-250\omega^2 & 1200\\ 1200 & 1360-40\omega^2\end{vmatrix}=0 \quad (8)$$

Eq. (8) is the frequency equation which can be expanded as:

$$(0.01\,\omega^4-0.5\,\omega^2+4)\,10^6=0$$

or $$0.01\,\omega^4-0.5\,\omega^2+4=0 \quad (9)$$

The roots of Eq. (9) are

$$\omega^2=\frac{+0.5\pm\sqrt{0.25-0.16}}{0.02}=\frac{0.5\pm0.3}{0.02}=10,\ 40$$

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$$\begin{bmatrix}(4000-250\omega^2) & 1200\\ 1200 & (1360-40\omega^2)\end{bmatrix}\begin{Bmatrix}X\\ \Theta\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \quad (7)$$

For a nontrivial solution of X and Θ, the following condition is to be satisfied:

$$\begin{vmatrix}4000-250\omega^2 & 1200\\ 1200 & 1360-40\omega^2\end{vmatrix}=0 \quad (8)$$

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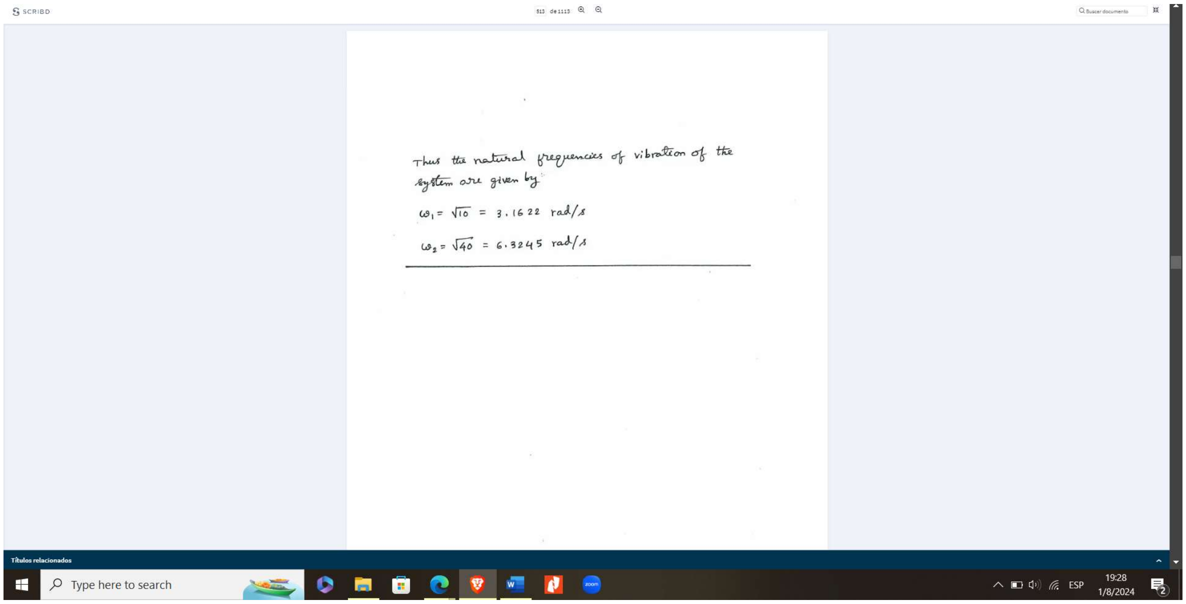


Thus the natural frequencies of vibration of the system are given by

$\omega_1 = \sqrt{10} = 3.1622$ rad/s

$\omega_2 = \sqrt{40} = 6.3245$ rad/s

the inertia effect of the pulley.

31. A steel bar is fixed at the upper end and carried a concentrated mass of 20 kg at its lower end. The bar has a cross-section of 10 mm × 15 mm and a length of 10 m. Determine the natural frequency for the longitudinal vibration neglecting the mass of the bar. Take $E = 210$ GPa for steel.

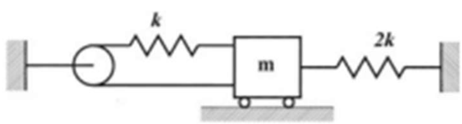
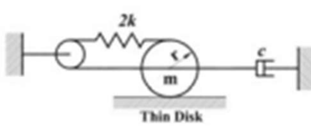


Figure P4.30



(b)

Determine the damping constant for the system shown in Figure E4.13 such that it is critically damped. Given $k = 1000\,\mathrm{N/m}$ and $m = 20\,\mathrm{kg}$. Assume that the disk is thin and rolls without slip.

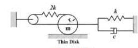


Figure E4.13

Solution

If the disk is displaced toward right by x, then right end of the left spring will be displaced by $2x$ and the left end of the same spring will be displaced by x. Therefore,

the relative displacement between two ends of the left spring will be $3x$. Then the compression in the right spring will be x. Then total potential energy of the system can be determined as

$$V = \frac{1}{2}(2k)(3x)^2 + \frac{1}{2}(k)(x)^2 = \frac{1}{2}(19k)x^2$$

Therefore, equivalent stiffness of the system is given by

$$k_{eq} = 19k$$

Total kinetic energy of the system can be determined as

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\left(\frac{\dot{x}}{r}\right)^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{x}}{r}\right)^2 = \frac{1}{2}\left(\frac{3}{2}m\right)\dot{x}^2$$

Therefore, equivalent mass of the system is given by

$$m_{eq} = \frac{3}{2}m$$

Therefore, critical damping constant of the system can be determined as

$$c = c_c = 2\sqrt{k_{eq}m_{eq}} = 2\sqrt{19k \times \frac{3}{2}m}$$
$$= 2\sqrt{19 \times 1000 \times \frac{3}{2} \times 20} = 1509.9668\,\mathrm{N\,s/m}$$

Example 4.14