In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt

from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.stattools import adfuller

from ma_model import ma_series
from ar_model import ar_series
```

Constants setup

In [ ]:
```
BURNIN = 500
N = 5000
SEED = 42
```

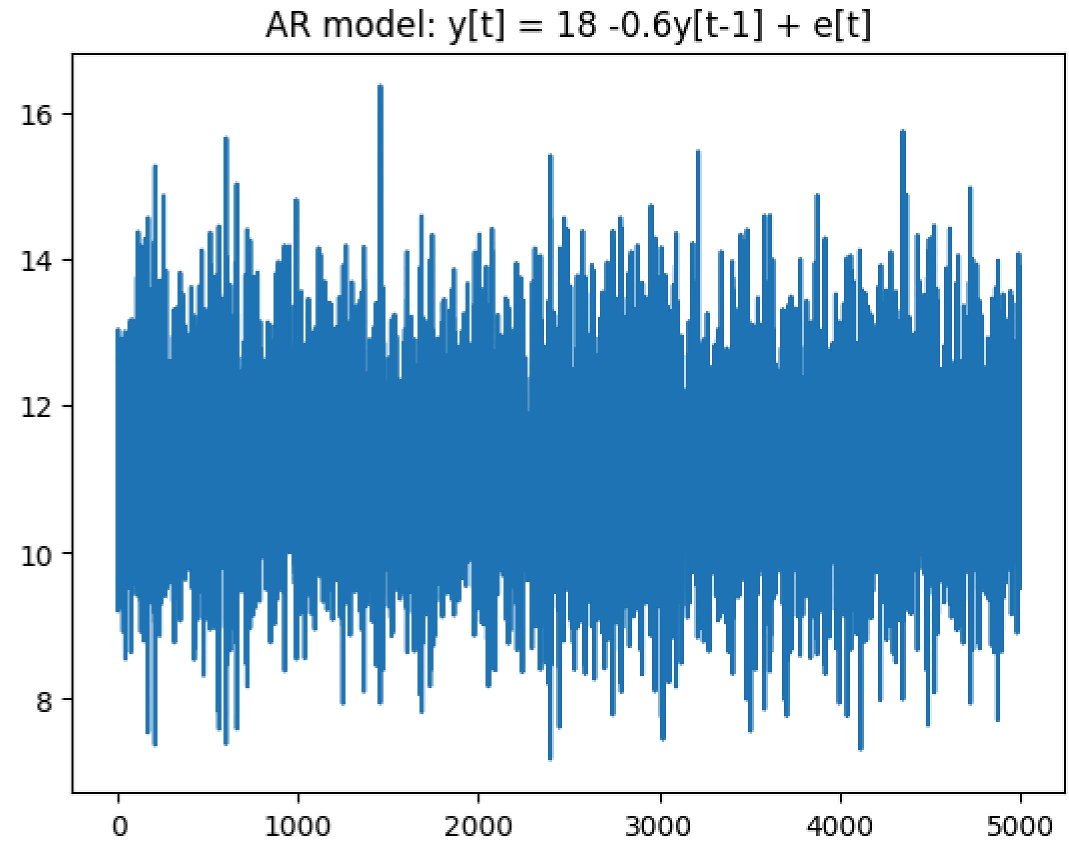
Correlations in AR model

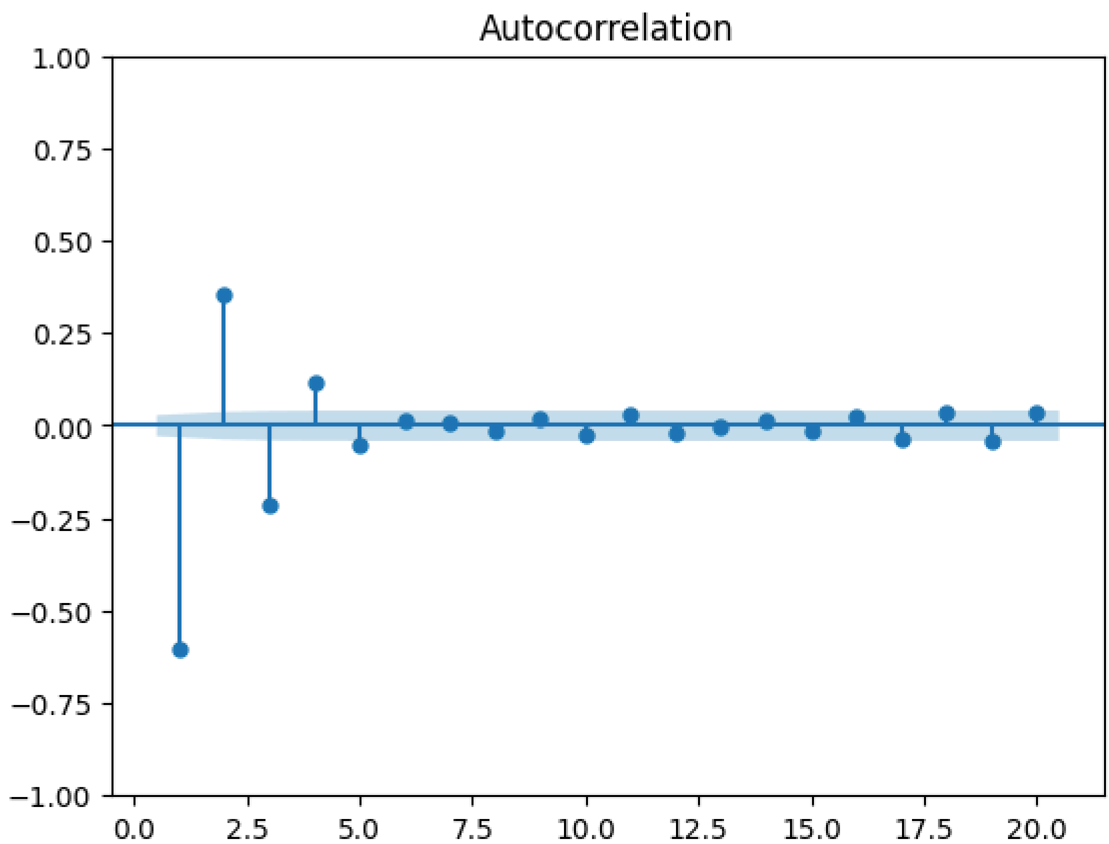
In [ ]:
```
ar_series_1, ar_formula_1 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([-0.6]), seed=SEED)

plt.title(ar_formula_1)
plt.plot(ar_series_1)
fig = plot_acf(ar_series_1, lags=20, zero=False)
fig = plot_pacf(ar_series_1, lags=20, zero=False)
acorr_ljungbox(ar_series_1, lags=20, return_df=True)
```

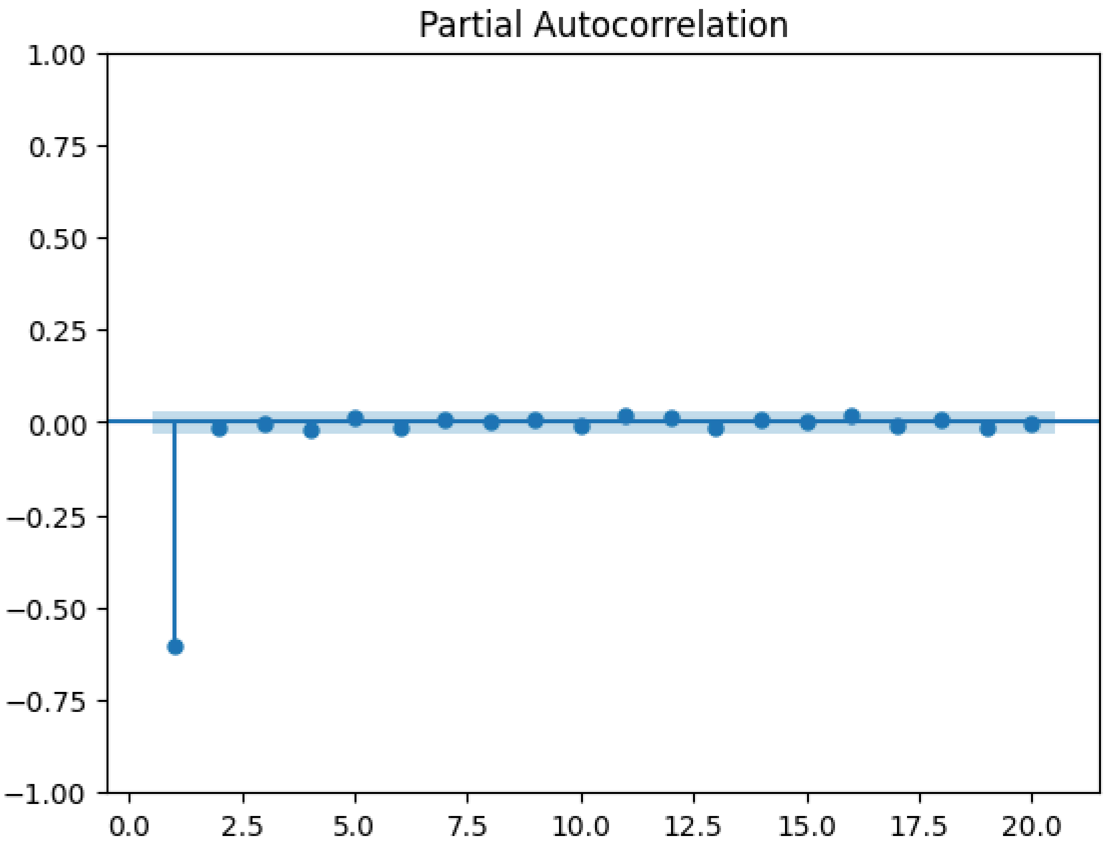
```
AR model: y[t] = 18 -0.6y[t-1] + e[t]
```

Out[ ]:

	lb_stat	lb_pvalue
1	1826.691767	0.0
2	2457.314658	0.0
3	2680.744180	0.0
4	2745.886276	0.0
5	2759.040234	0.0
6	2759.780959	0.0
7	2760.340900	0.0
8	2761.579899	0.0
9	2763.715950	0.0
10	2766.282141	0.0
11	2770.698410	0.0
12	2772.212735	0.0
13	2772.214016	0.0
14	2773.041093	0.0
15	2774.122868	0.0
16	2777.749506	0.0
17	2783.438981	0.0
18	2790.330182	0.0
19	2798.411716	0.0
20	2803.689824	0.0


AR model: y[t] = 18 -0.6y[t-1] + e[t]
16
14
12
10
8
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



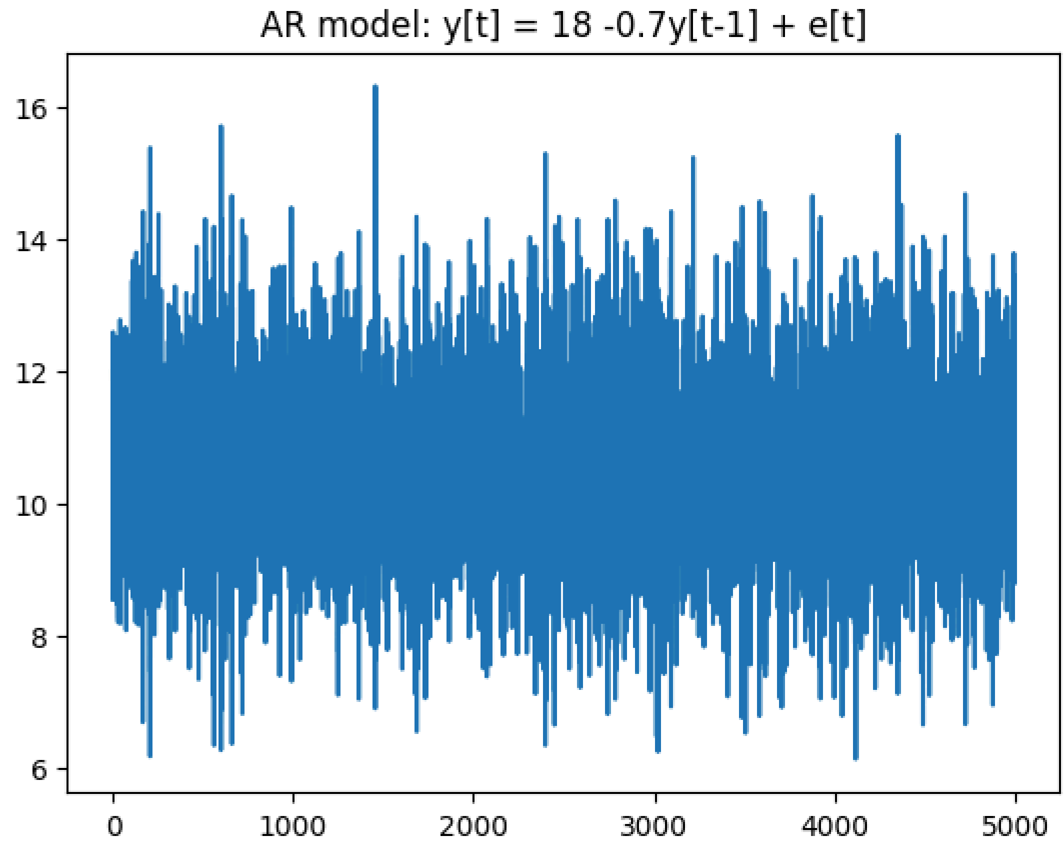
In [ ]:

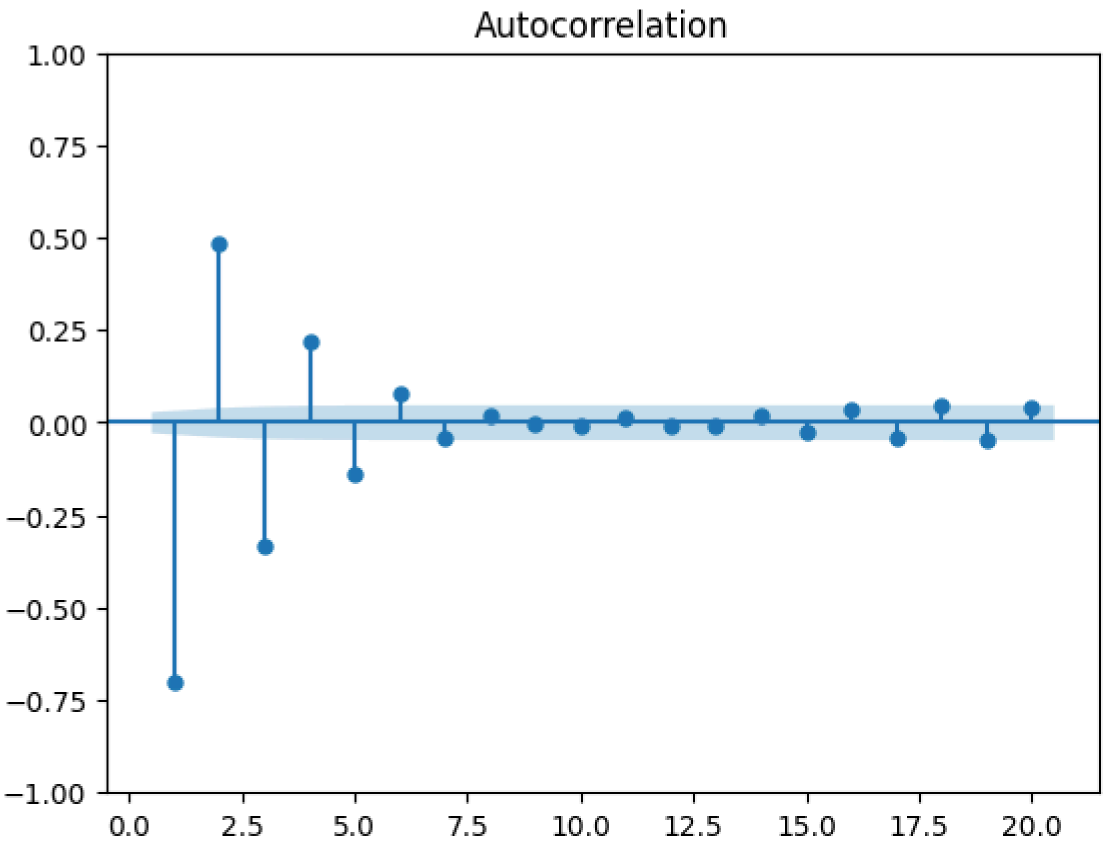
```
ar_series_2, ar_formula_2 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([-0.7]), seed=SEED)

plt.title(ar_formula_2)
plt.plot(ar_series_2)
fig = plot_acf(ar_series_2, lags=20, zero=False)
fig = plot_pacf(ar_series_2, lags=20, zero=False)
acorr_ljungbox(ar_series_2, lags=20, return_df=True)
```

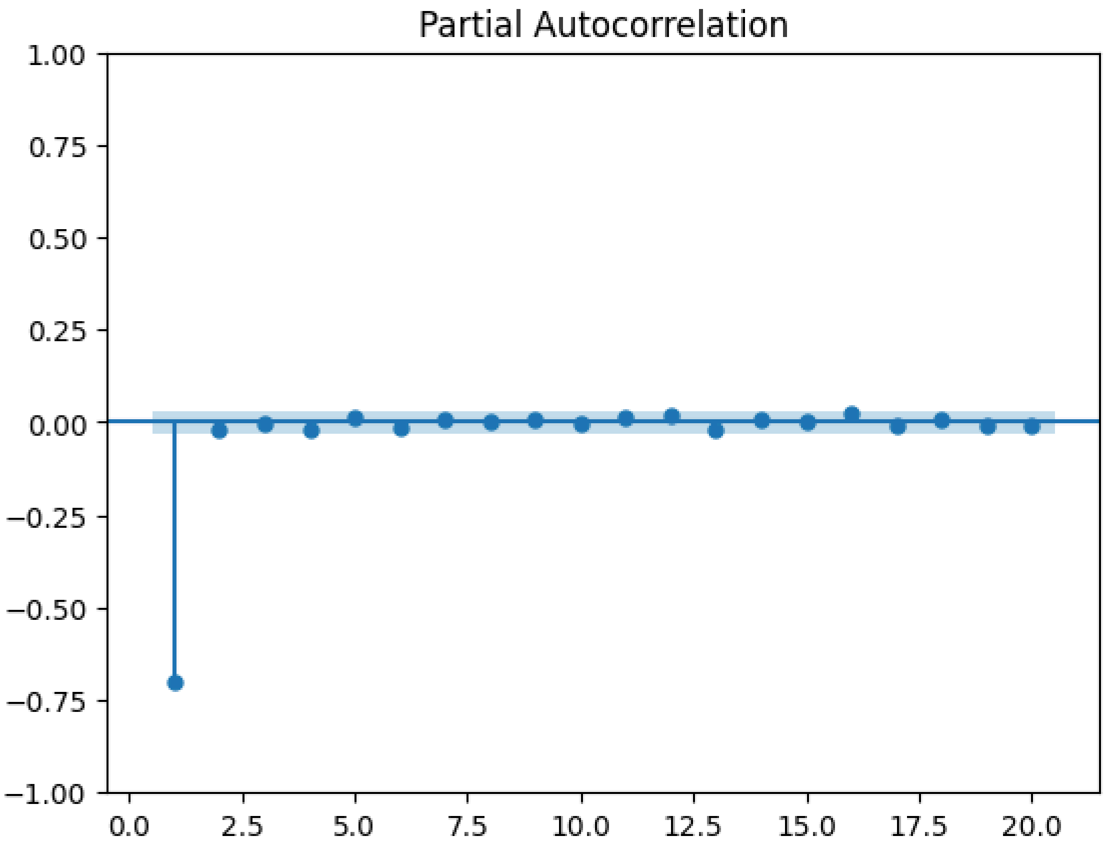
```
AR model: y[t] = 18 -0.7y[t-1] + e[t]
```

Out[ ]:

	lb_stat	lb_pvalue
1	2457.161149	0.0
2	3618.631995	0.0
3	4171.256316	0.0
4	4410.741539	0.0
5	4503.375498	0.0
6	4533.555290	0.0
7	4540.866228	0.0
8	4542.458437	0.0
9	4542.497862	0.0
10	4542.678069	0.0
11	4543.788489	0.0
12	4544.032652	0.0
13	4544.284701	0.0
14	4545.944316	0.0
15	4548.386706	0.0
16	4553.932734	0.0
17	4562.044687	0.0
18	4571.686722	0.0
19	4582.347496	0.0
20	4590.199335	0.0


AR model: y[t] = 18 -0.7y[t-1] + e[t]
16
14
12
10
8
6
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



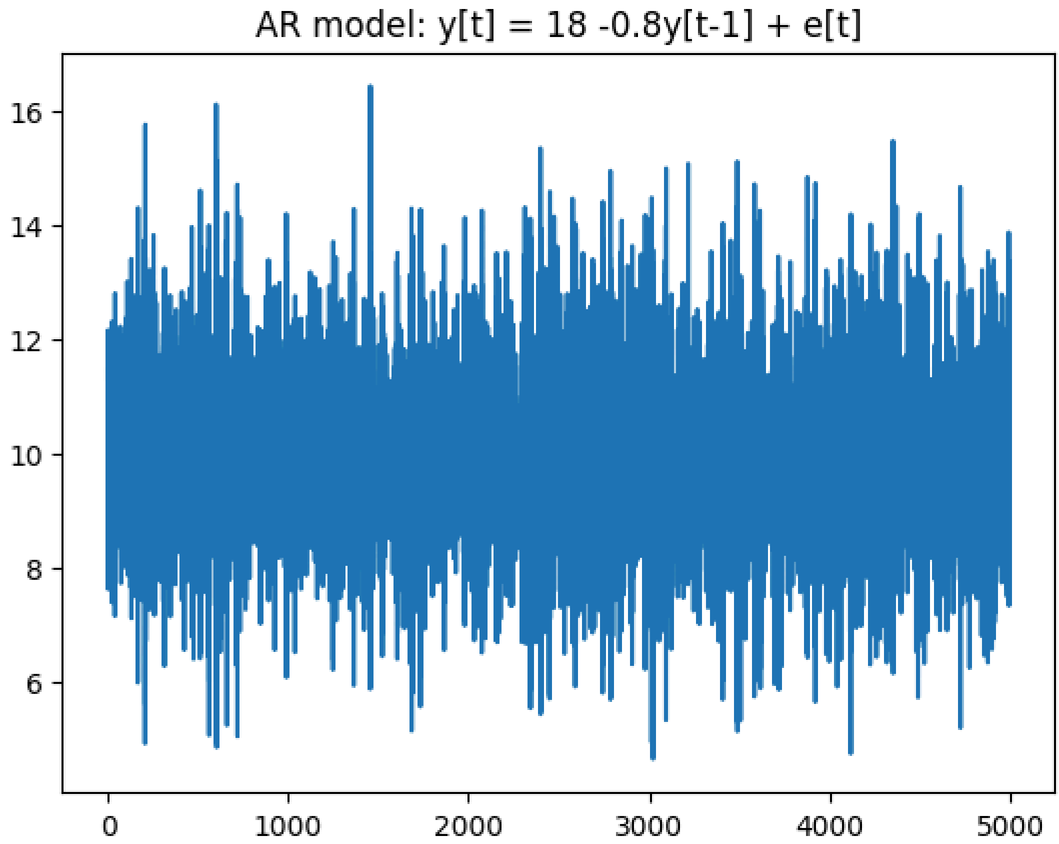
In [ ]:

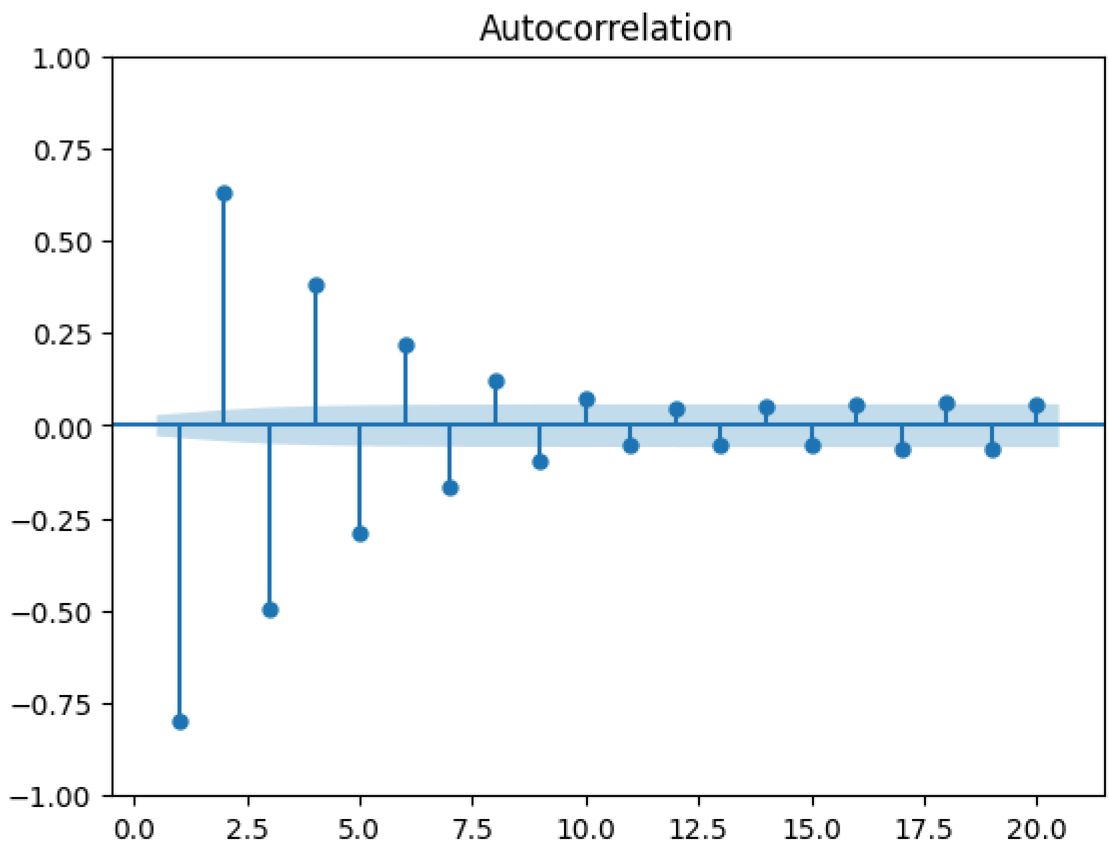
```
ar_series_3, ar_formula_3 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([-0.8]), seed=SEED)

plt.title(ar_formula_3)
plt.plot(ar_series_3)
fig = plot_acf(ar_series_3, lags=20, zero=False)
fig = plot_pacf(ar_series_3, lags=20, zero=False)
acorr_ljungbox(ar_series_3, lags=20, return_df=True)
```

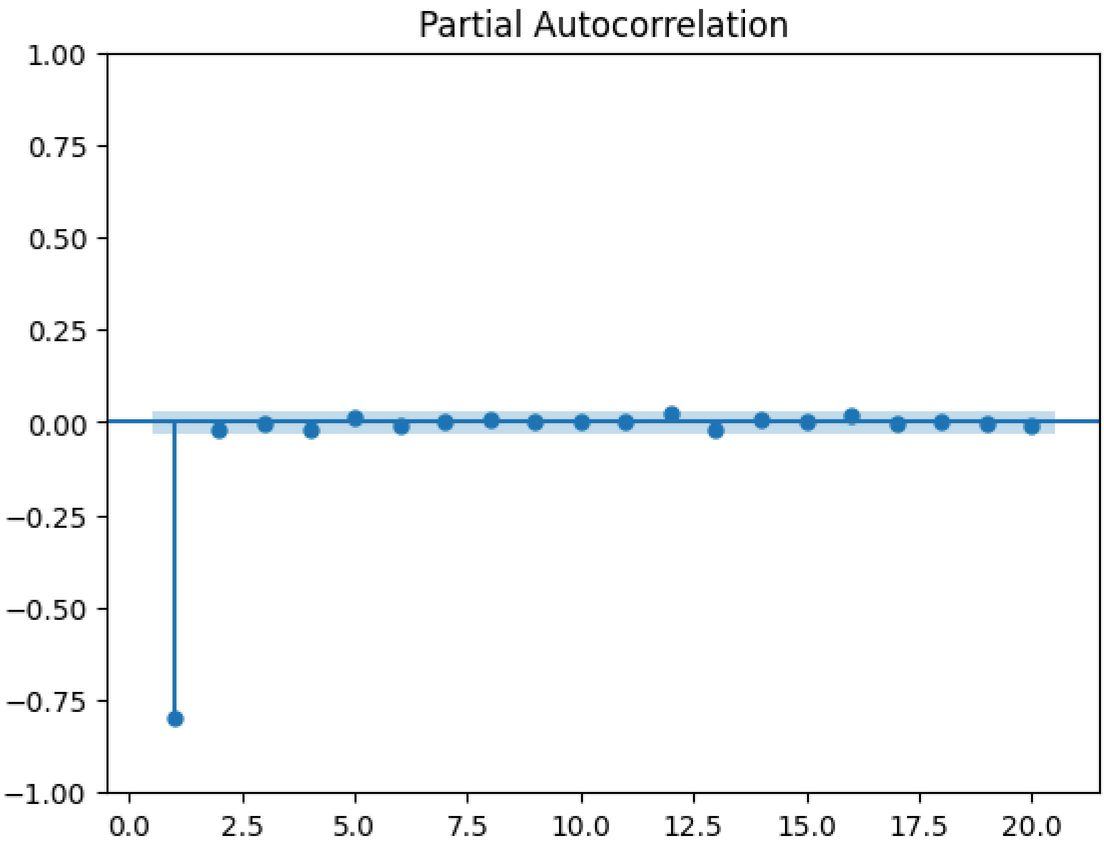
```
AR model: y[t] = 18 -0.8y[t-1] + e[t]
```

Out[ ]:

	lb_stat	lb_pvalue
1	3179.440748	0.0
2	5153.628248	0.0
3	6381.691046	0.0
4	7116.220653	0.0
5	7542.085628	0.0
6	7782.280436	0.0
7	7915.230636	0.0
8	7992.213703	0.0
9	8035.684801	0.0
10	8060.743535	0.0
11	8074.438005	0.0
12	8086.052397	0.0
13	8099.027810	0.0
14	8113.513082	0.0
15	8127.766880	0.0
16	8144.869785	0.0
17	8163.670219	0.0
18	8182.816382	0.0
19	8201.503074	0.0
20	8216.212460	0.0


AR model: y[t] = 18 -0.8y[t-1] + e[t]
16
14
12
10
8
6
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

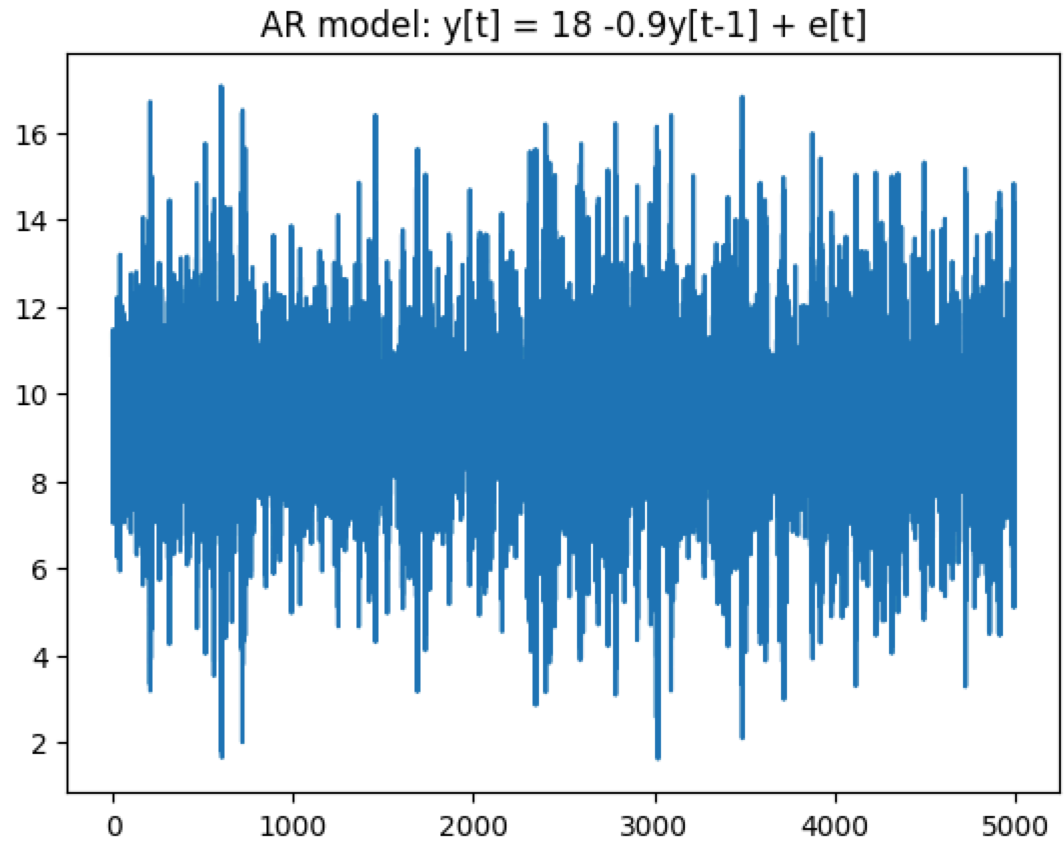


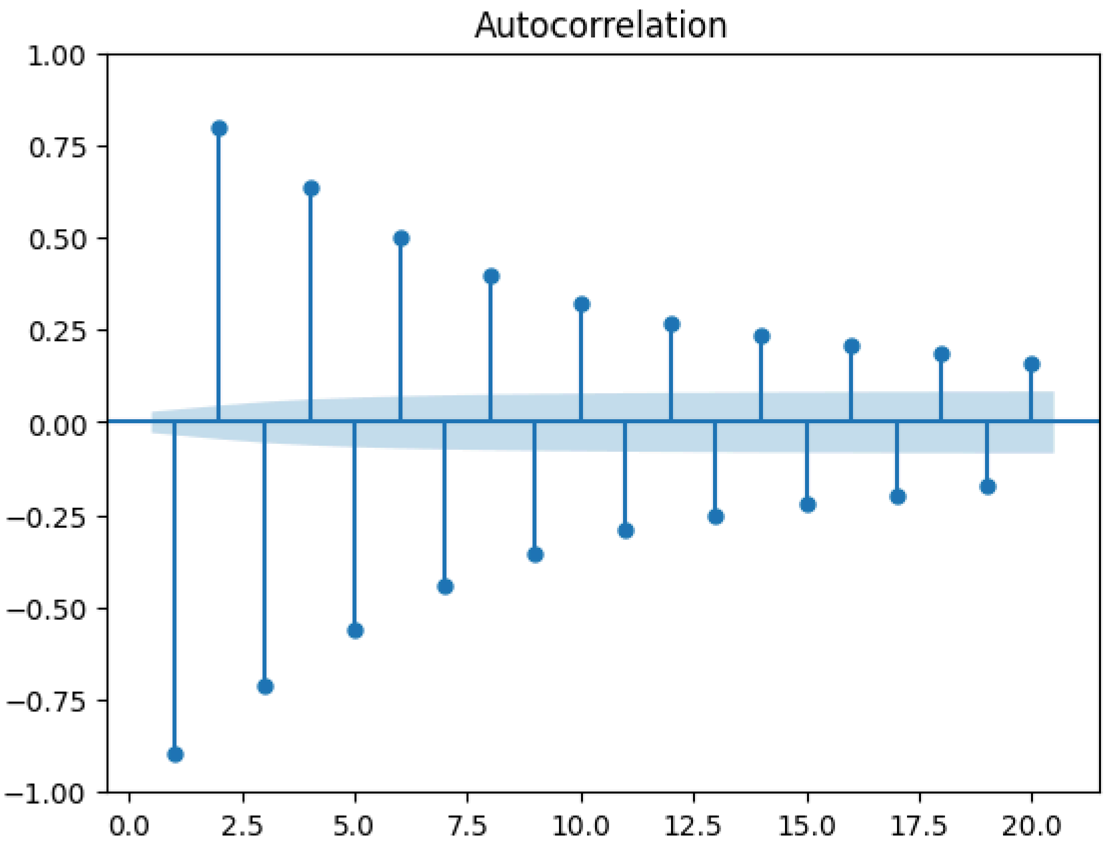
In [ ]:
```
ar_series_4, ar_formula_4 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([-0.9]), seed=SEED)

plt.title(ar_formula_4)
plt.plot(ar_series_4)
fig = plot_acf(ar_series_4, lags=20, zero=False)
fig = plot_pacf(ar_series_4, lags=20, zero=False)
acorr_ljungbox(ar_series_4, lags=20, return_df=True)
```

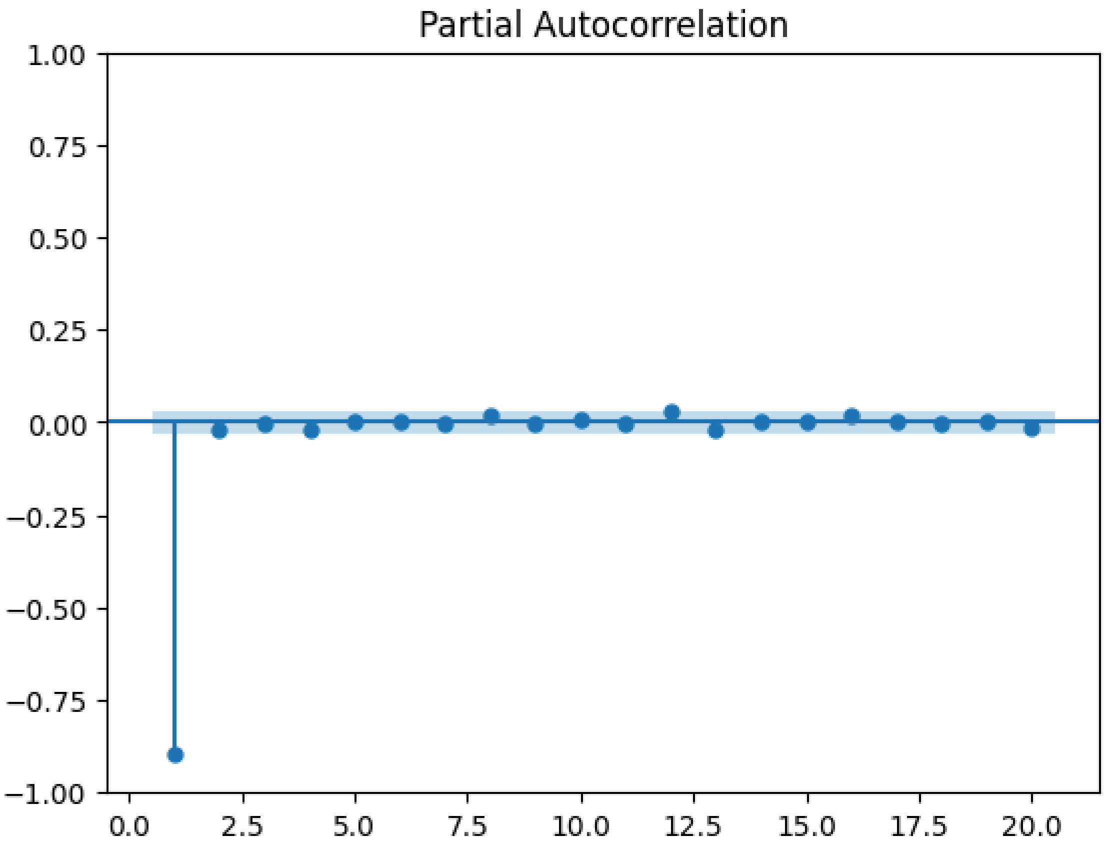
```
AR model: y[t] = 18 -0.9y[t-1] + e[t]
```

Out[ ]:

	lb_stat	lb_pvalue
1	4015.633406	0.0
2	7210.041258	0.0
3	9757.877470	0.0
4	11768.875817	0.0
5	13350.496811	0.0
6	14594.535265	0.0
7	15577.489727	0.0
8	16366.875291	0.0
9	17002.895048	0.0
10	17520.624081	0.0
11	17943.235603	0.0
12	18303.651164	0.0
13	18619.721890	0.0
14	18898.234681	0.0
15	19140.985213	0.0
16	19358.459385	0.0
17	19552.042707	0.0
18	19722.395963	0.0
19	19870.878391	0.0
20	19995.088049	0.0


AR model: y[t] = 18 -0.9y[t-1] + e[t]
16
14
12
10
8
6
4
2
0
1000
2000
3000
4000
5000

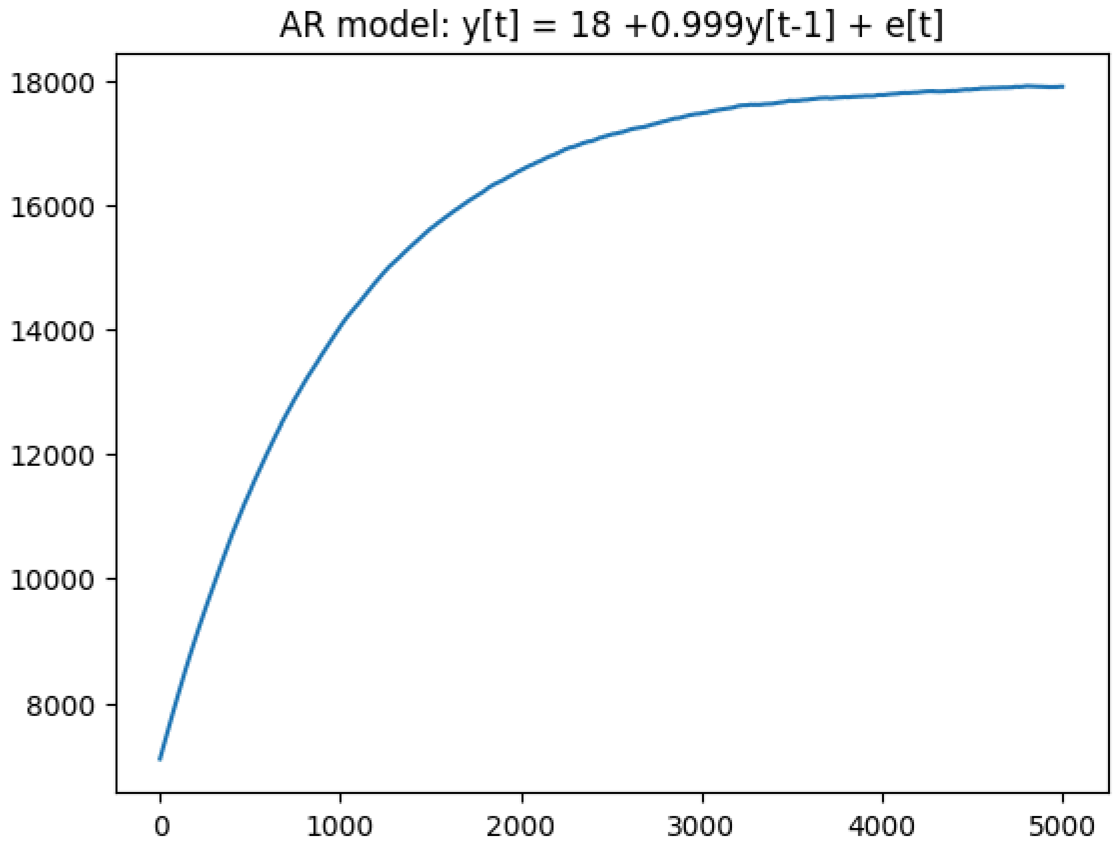

Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

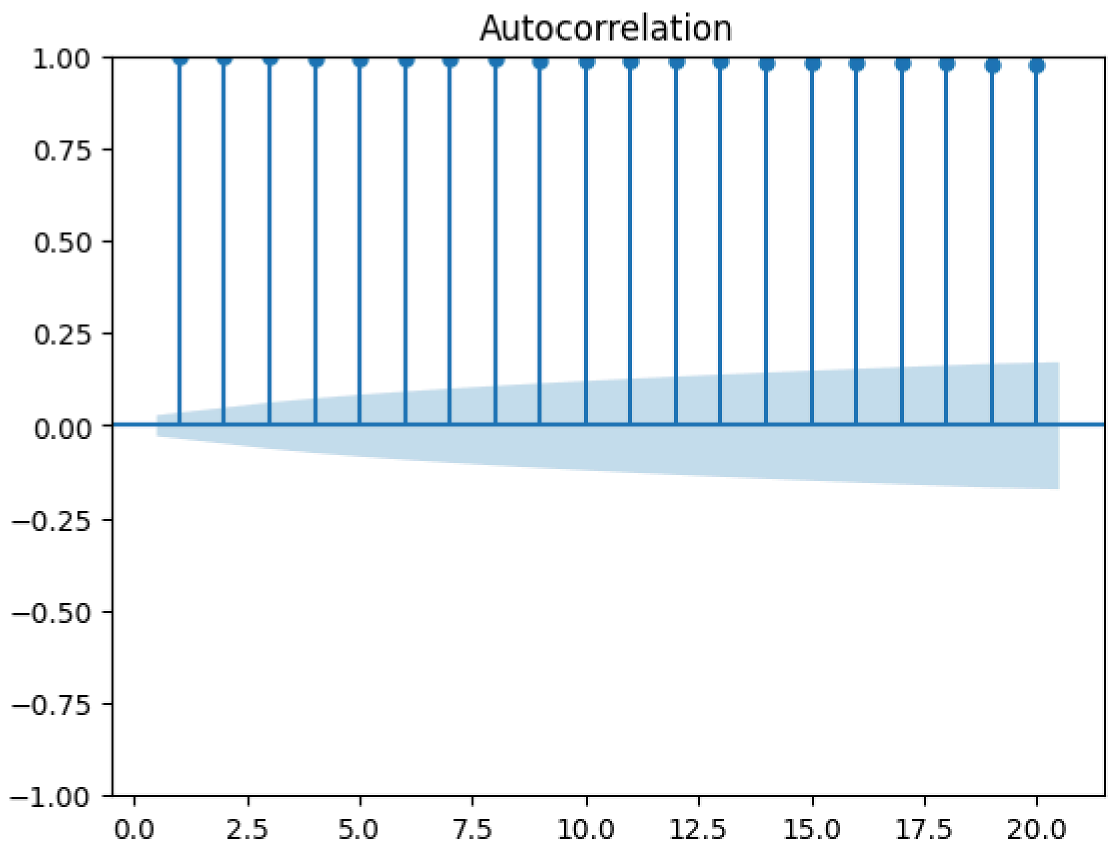


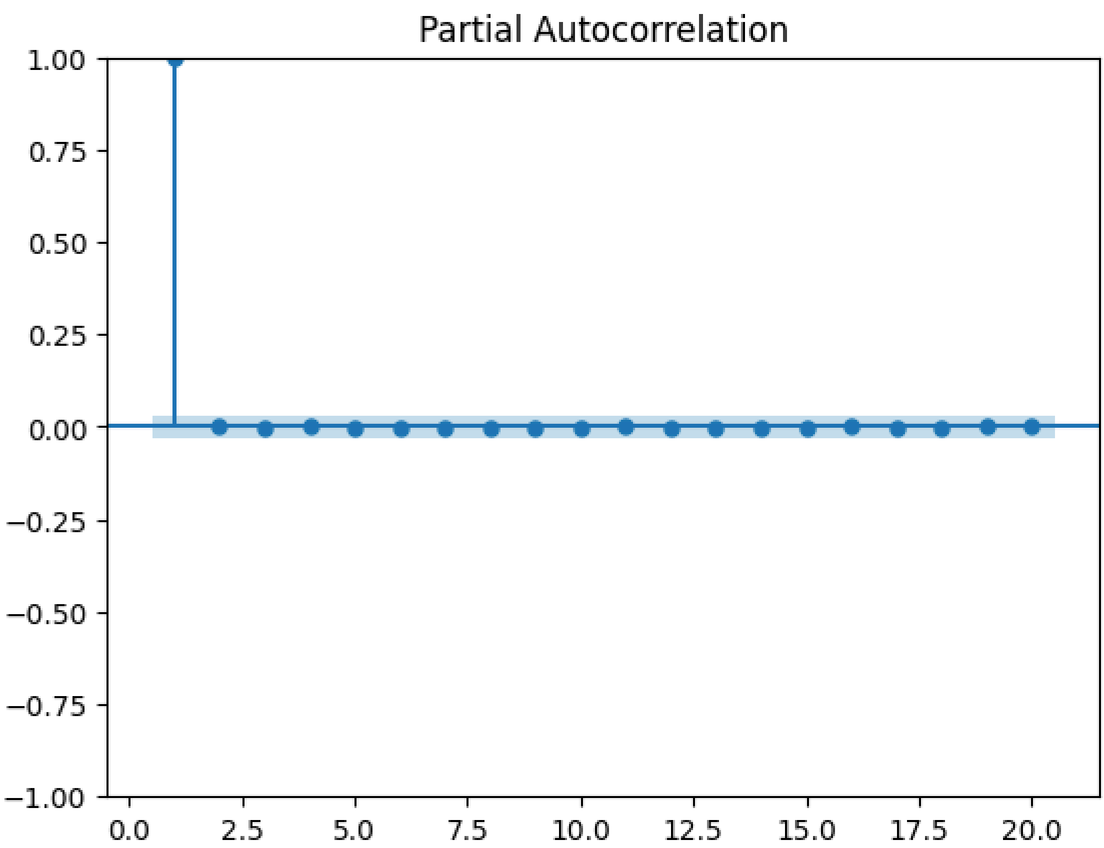
Is stationary if |theta| > 1?

In [ ]:
```
ar_series_5, ar_formula_5 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([0.999]), seed=SEED)

plt.title(ar_formula_5)
plt.plot(ar_series_5)
fig = plot_acf(ar_series_5, lags=20, zero=False)
fig = plot_pacf(ar_series_5, lags=20, zero=False)
```

```
AR model: y[t] = 18 +0.999y[t-1] + e[t]
```


AR model: y[t] = 18 +0.999y[t-1] + e[t]
18000
16000
14000
12000
10000
8000
0
1000
2000
3000
4000
5000

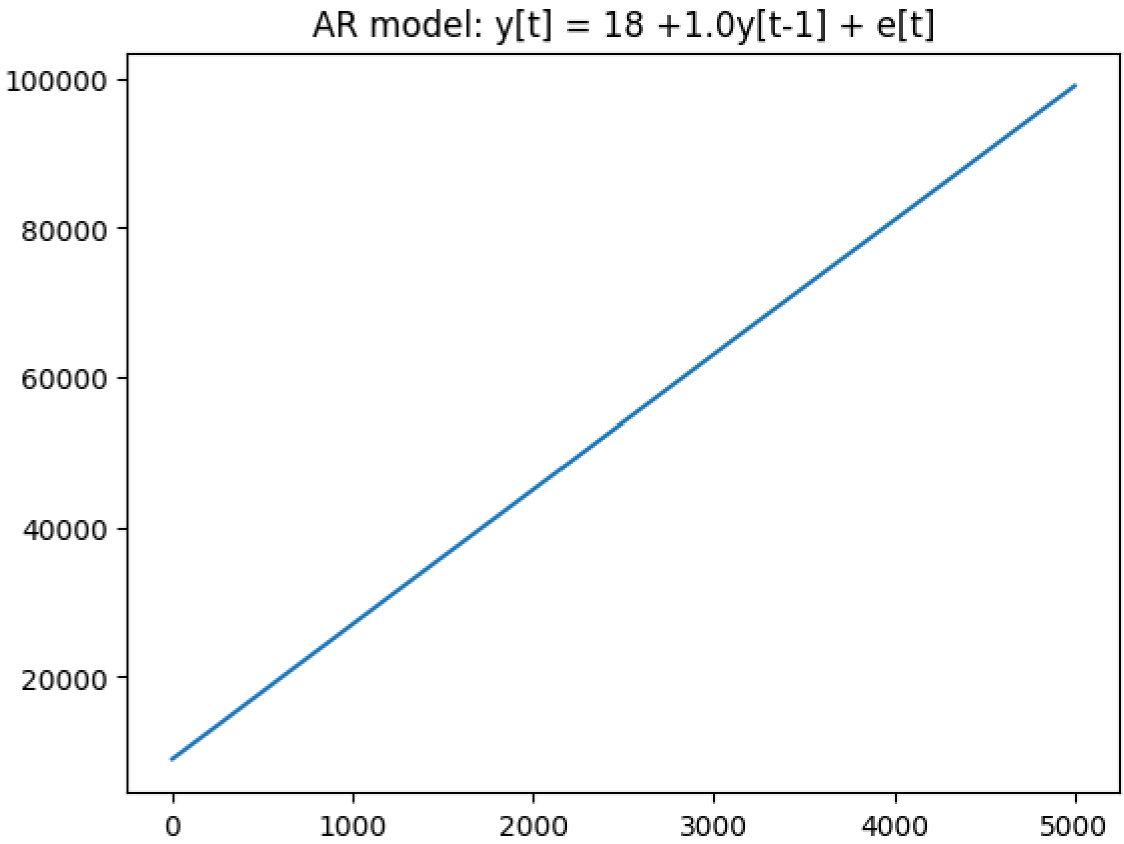

Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

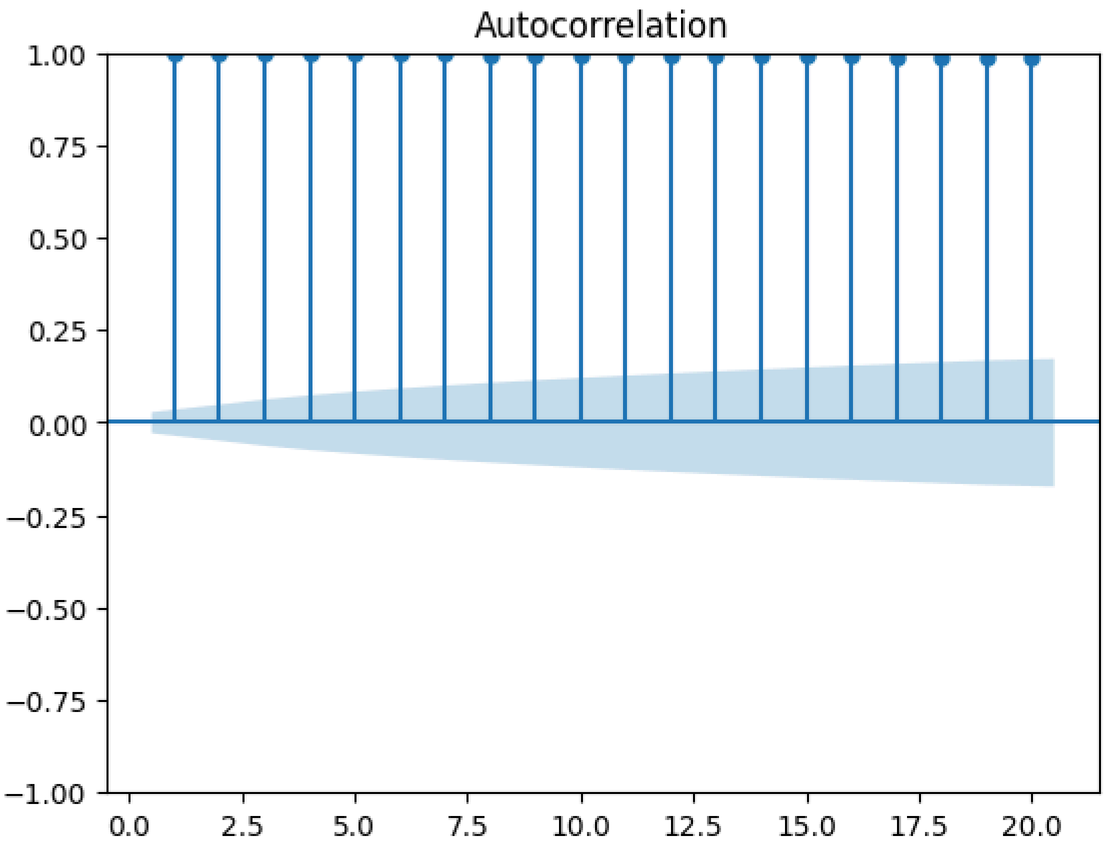


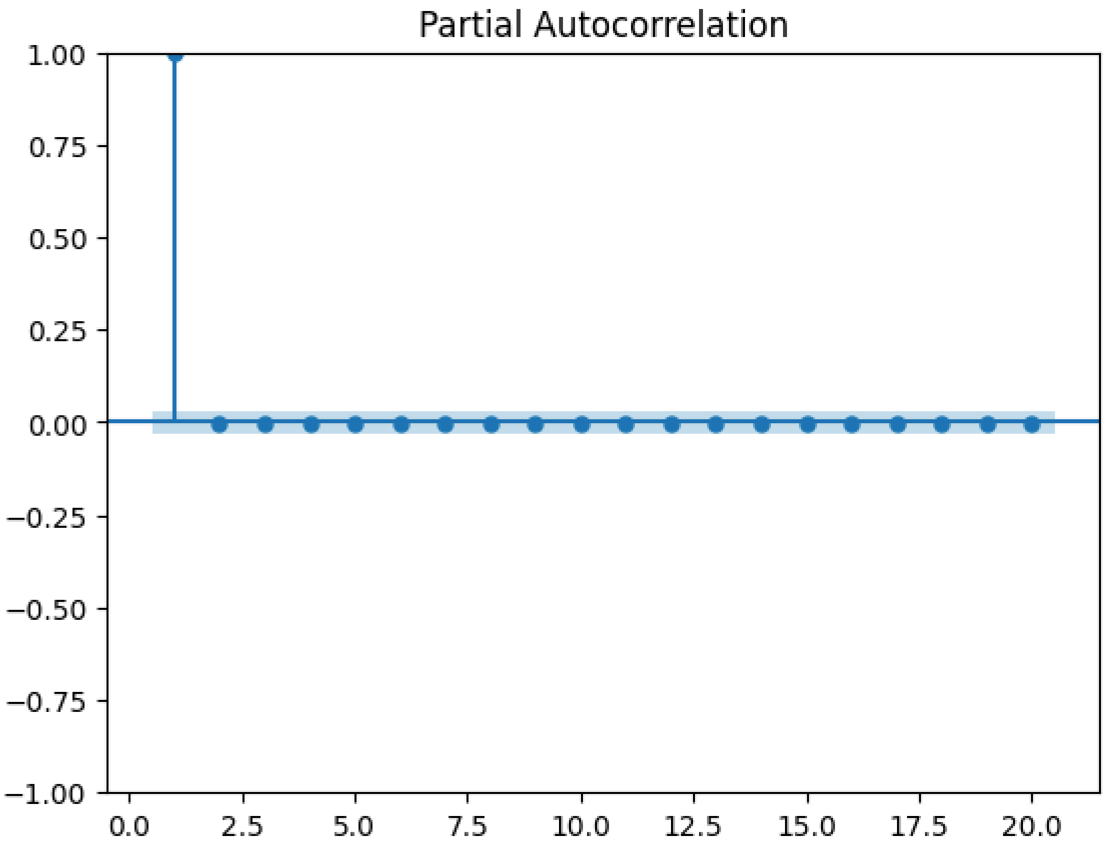
In [ ]:

```
ar_series_5, ar_formula_5 = ar_series(burnin=BURNIN, n=N, c=18,
c=np.array([1.0]), seed=SEED)

plt.title(ar_formula_5)
plt.plot(ar_series_5)
fig = plot_acf(ar_series_5, lags=20, zero=False)
fig = plot_pacf(ar_series_5, lags=20, zero=False)
```

```
AR model: y[t] = 18 +1.0y[t-1] + e[t]
```


AR model: y[t] = 18 +1.0y[t-1] + e[t]
100000
80000
60000
40000
20000
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



We can observe that if theta get closer to one then the series becomes more and more non stationary and even becomes a linear function.

AR(2) model

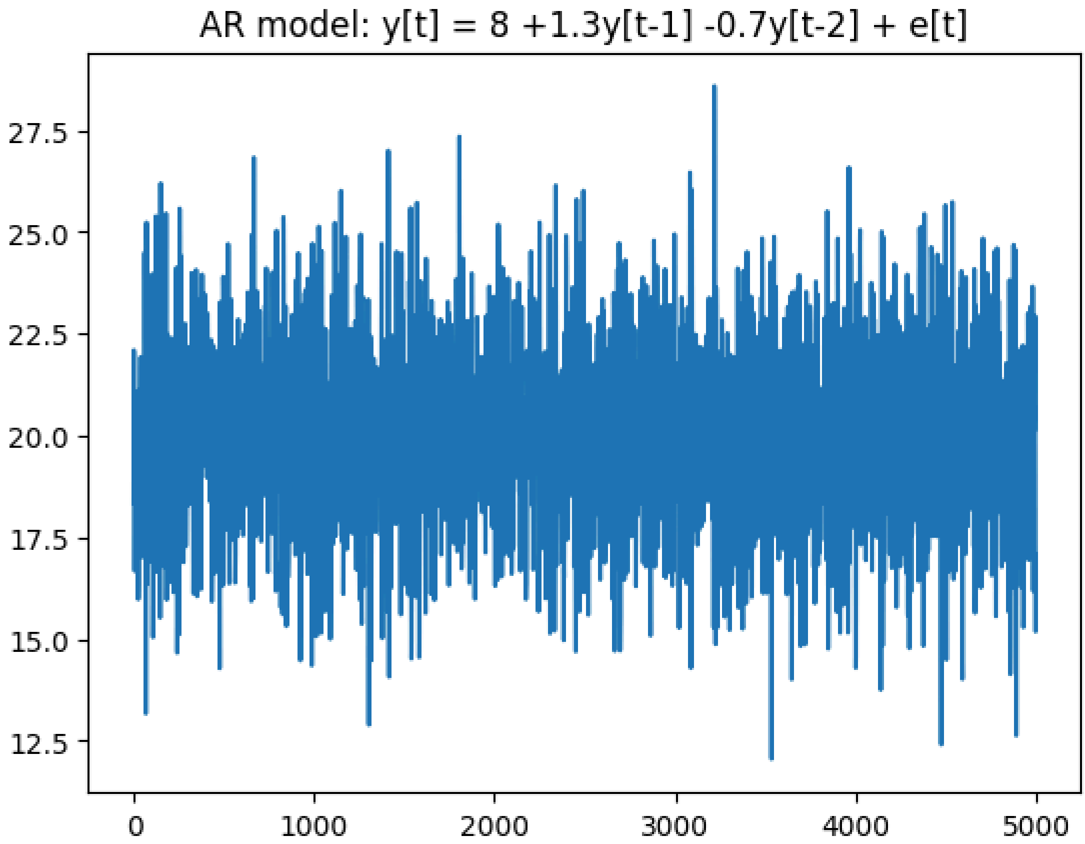
In [ ]:

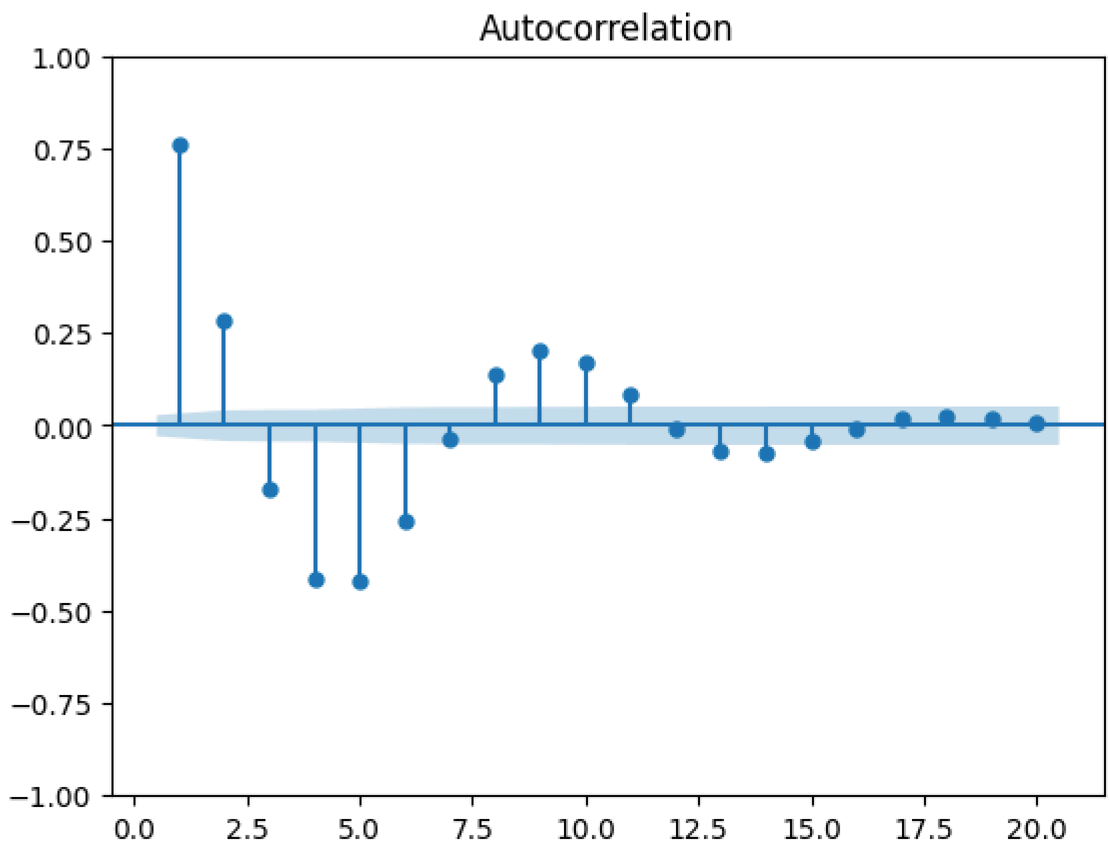
```
ar2_series, ar2_formula = ar_series(burnin=BURNIN, n=N, c=8, o=np.array([1.3,
-0.7]), seed=SEED)

plt.title(ar2_formula)
plt.plot(ar2_series)
fig = plot_acf(ar2_series, lags=20, zero=False)
fig = plot_pacf(ar2_series, lags=20, zero=False)
acorr_ljungbox(ar2_series, lags=[20], return_df=True)
```

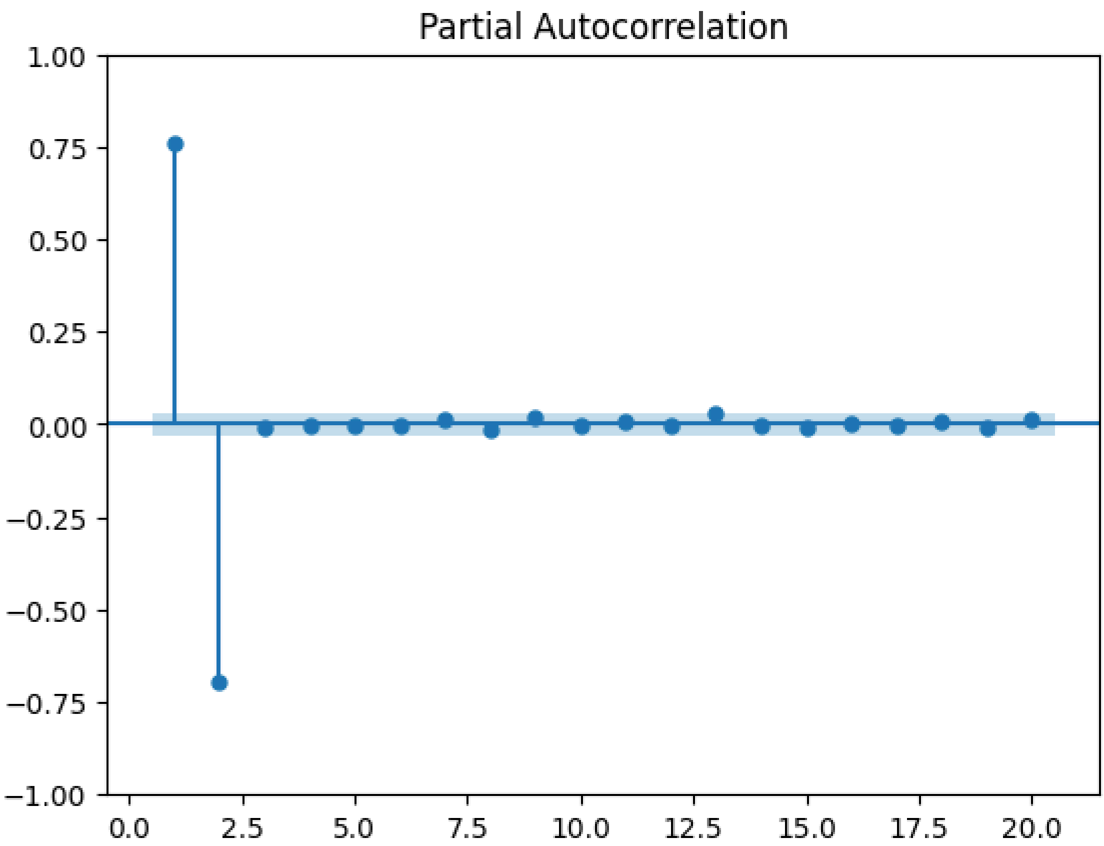
```
AR model: y[t] = 8 +1.3y[t-1] -0.7y[t-2] + e[t]
```

Out[ ]:

	lb_stat	lb_pvalue
20	6055.81025	0.0


AR model: y[t] = 8 +1.3y[t-1] -0.7y[t-2] + e[t]
27.5
25.0
22.5
20.0
17.5
15.0
12.5
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



Correlations in MA model

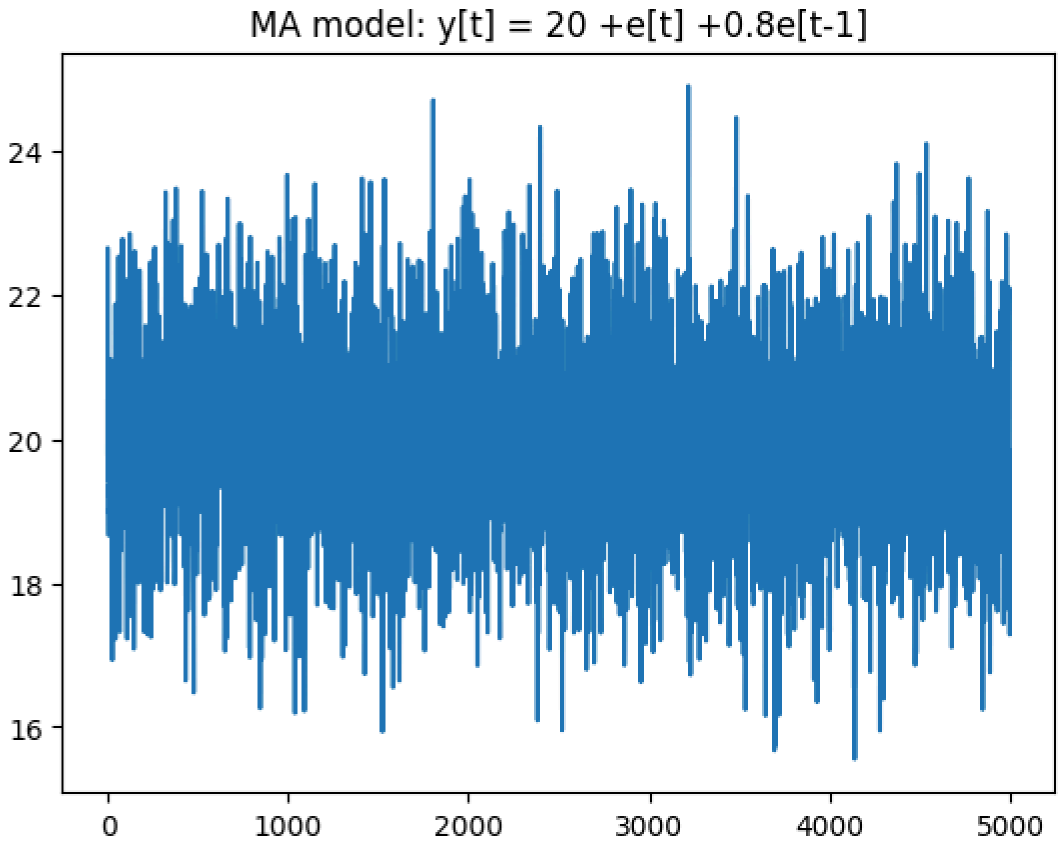
In [ ]:
```
ma_series_1, ma_formula_1 = ma_series(burnin=BURNIN, n=N, c=20,
Q=np.array([0.8]), seed=SEED)
```

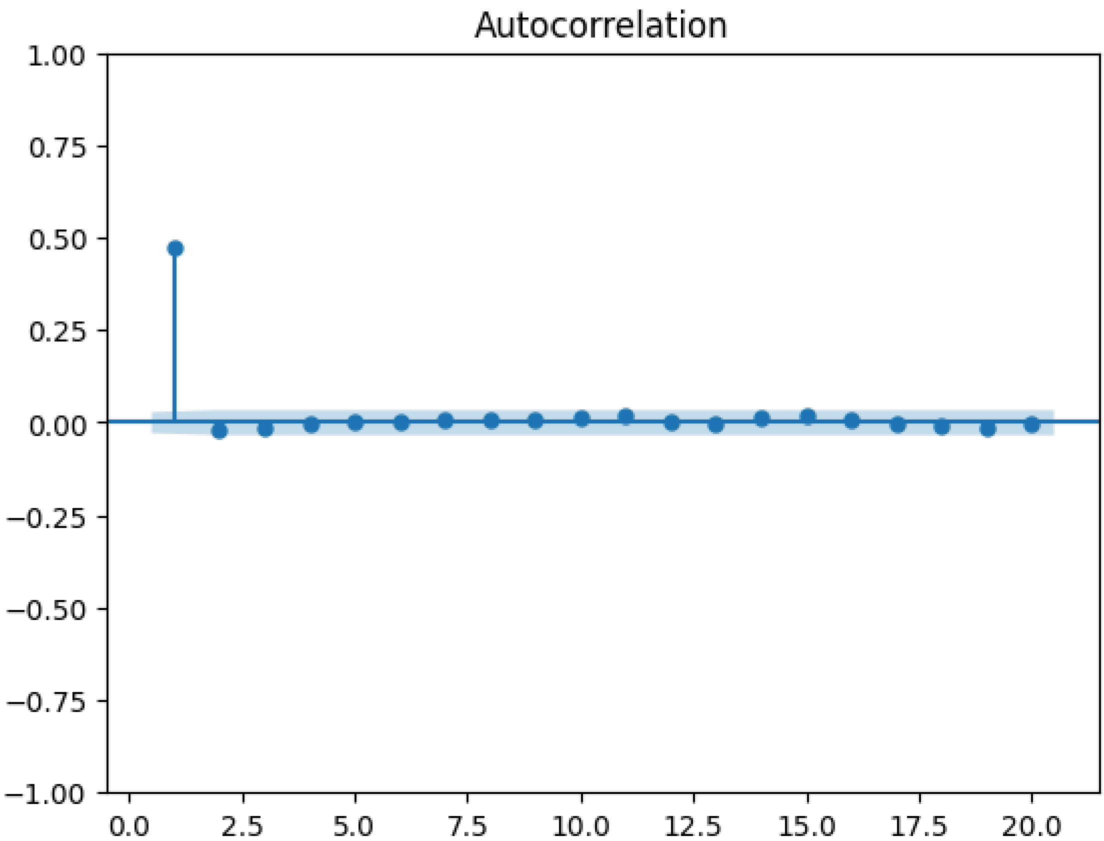
```
MA model: y[t] = 20 +e[t] +0.8e[t-1]
```

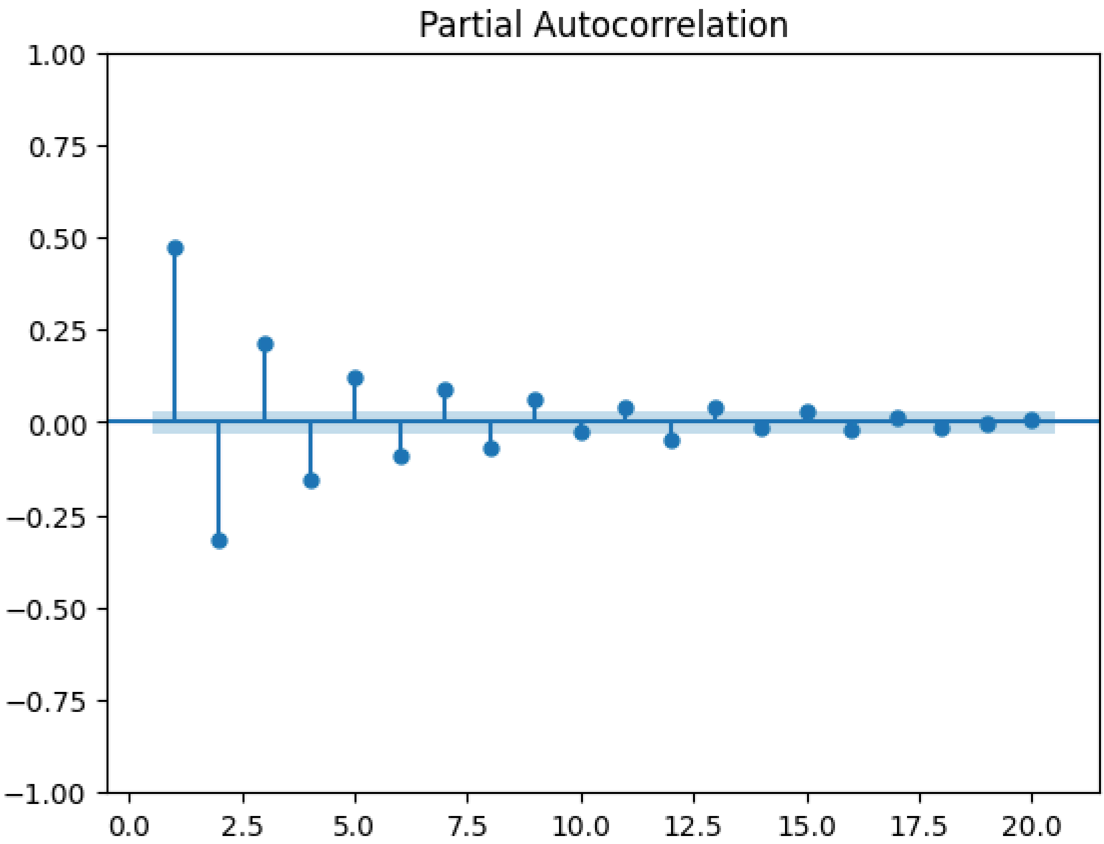
In [ ]:
```
plt.title(ma_formula_1)
plt.plot(ma_series_1)
fig = plot_acf(ma_series_1, lags=20, zero=False)
fig = plot_pacf(ma_series_1, lags=20, zero=False)
acorr_ljungbox(ma_series_1, lags=20, return_df=True)
```

Out[ ]:

	lb_stat	lb_pvalue
1	1129.790574	1.107010e-247
2	1131.975101	1.565777e-246
3	1133.188786	2.294328e-245
4	1133.236673	4.729765e-244
5	1133.265791	8.354605e-243
6	1133.274595	1.317230e-241
7	1133.847202	1.420233e-240
8	1134.251392	1.533295e-239
9	1134.399712	1.751250e-238
10	1135.685184	1.068713e-237
11	1137.975499	3.752398e-237
12	1137.977906	3.903029e-236
13	1138.042038	3.762776e-235
14	1138.970869	2.268680e-234
15	1140.740172	8.695084e-234
16	1141.255433	5.983041e-233
17	1141.258149	5.130238e-232
18	1141.447332	3.889263e-231
19	1142.279535	2.086163e-230
20	1142.366340	1.571817e-229


MA model: y[t] = 20 +e[t] +0.8e[t-1]
24
22
20
18
16
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

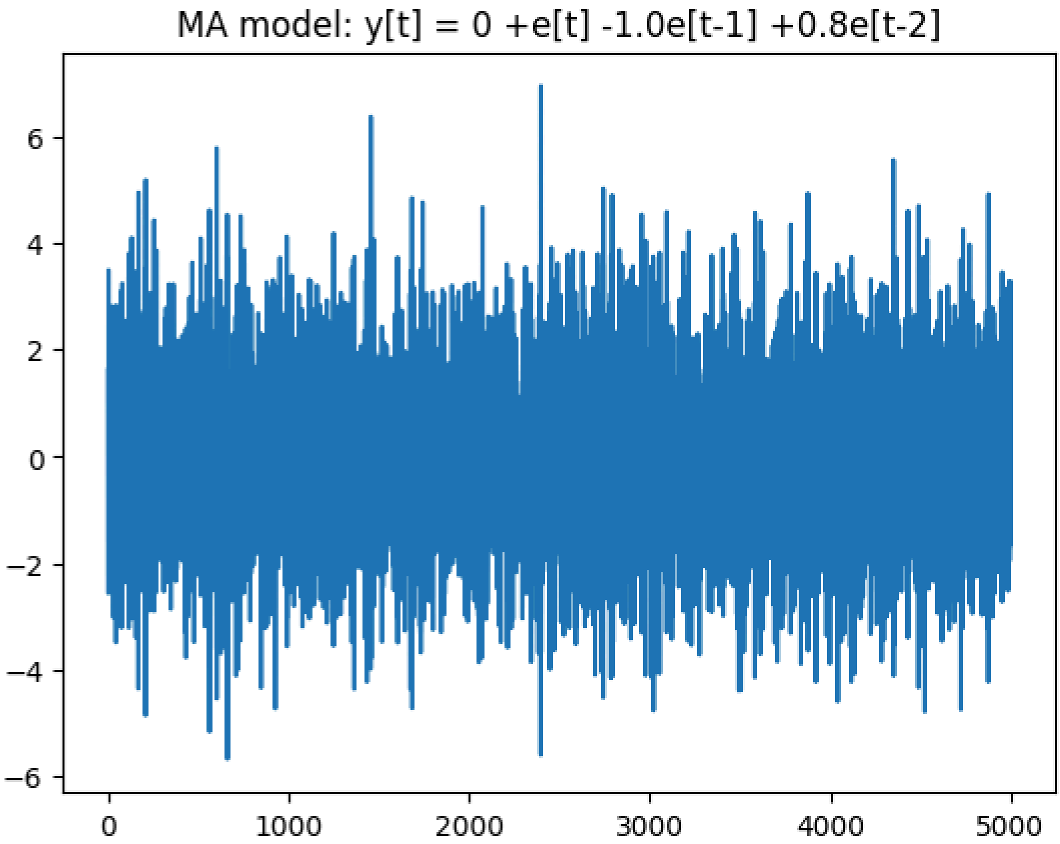


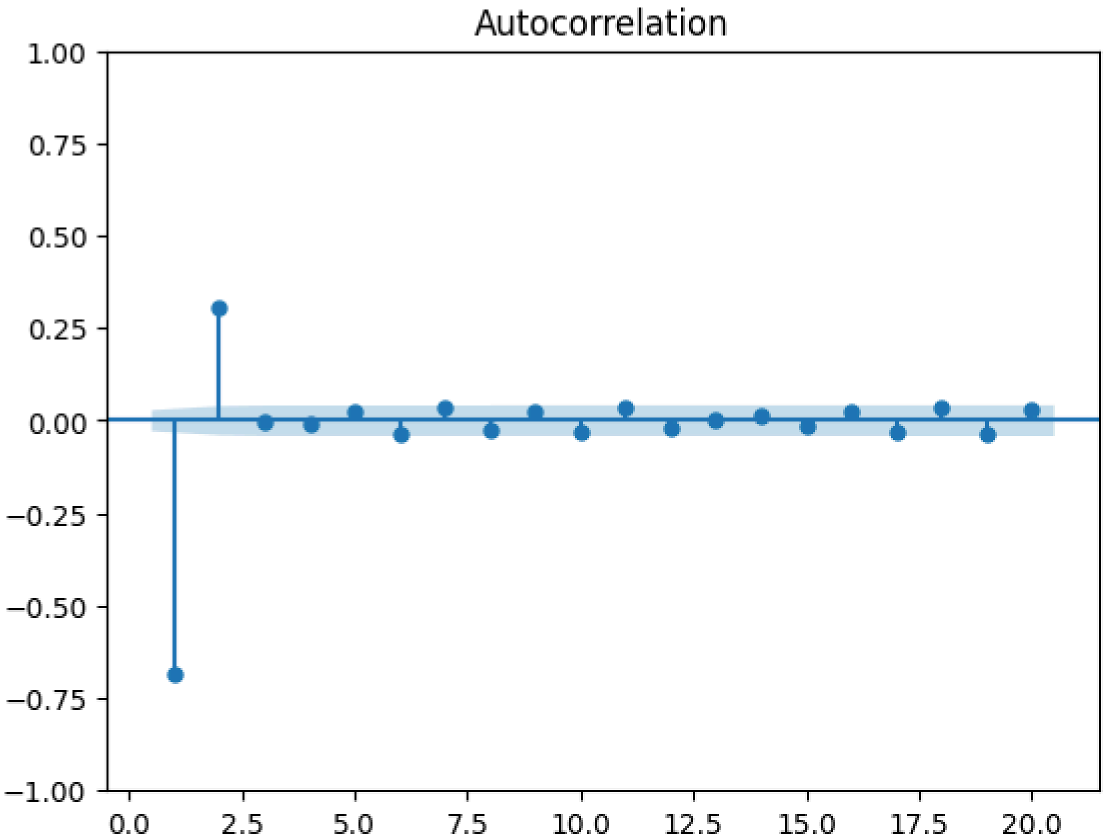
In [ ]:
```
ma_series_2, ma_formula_2 = ma_series(burnin=BURNIN, n=N, c=0, Q=np.array([-1, 0.8]), seed=SEED)

plt.title(ma_formula_2)
plt.plot(ma_series_2)
fig = plot_acf(ma_series_2, lags=20, zero=False)
fig = plot_pacf(ma_series_2, lags=20, zero=False)
acorr_ljungbox(ma_series_2, lags=20, return_df=True)
```

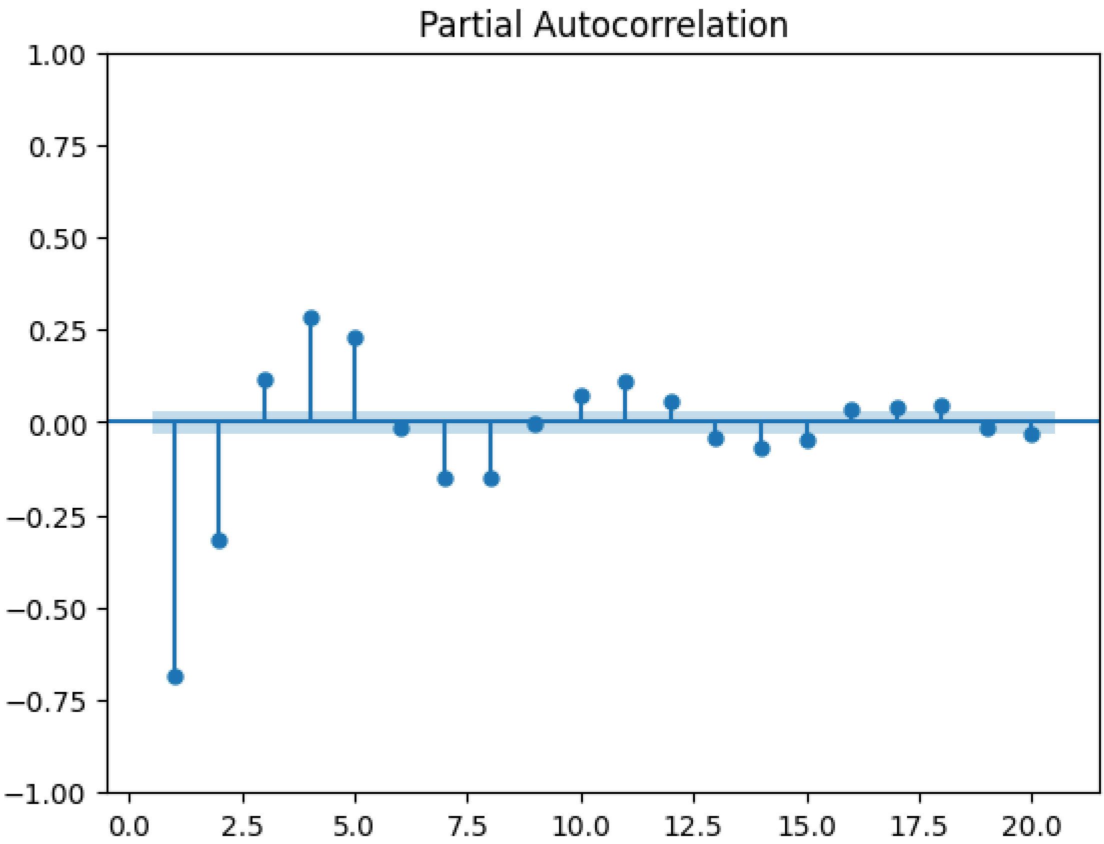
```
MA model: y[t] = 0 +e[t] -1.0e[t-1] +0.8e[t-2]
```

Out[ ]:

	lb_stat	lb_pvalue
1	2359.502107	0.0
2	2820.291391	0.0
3	2820.301705	0.0
4	2820.839185	0.0
5	2823.601056	0.0
6	2829.526616	0.0
7	2835.963860	0.0
8	2839.452515	0.0
9	2842.469889	0.0
10	2846.285594	0.0
11	2851.851298	0.0
12	2853.786300	0.0
13	2853.803853	0.0
14	2854.645509	0.0
15	2855.868856	0.0
16	2858.372140	0.0
17	2863.102816	0.0
18	2870.138150	0.0
19	2876.604159	0.0
20	2880.845252	0.0


MA model: y[t] = 0 +e[t] -1.0e[t-1] +0.8e[t-2]
6
4
2
0
−2
−4
−6
0
1000
2000
3000
4000
5000


Autocorrelation
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0



Comparison of AR and MA models

We can observe that ar models tend to have bigget autocorrelation and low partial autocorrelation. On the other hand, ma models behave the opposite way.

From the above we can conclude that usage of AR models is more suitable if we want to capture the direct influence of previous values on the current one. On the other hand, MA models are better in capturing the effects of past points without considerating possible fluctuations from closest points.