46211 Offshore Wind Energy - Fall 2021
Assignment 5: Dynamic model for a simplified spar floater supporting the DTU 10MW wind turbine

Introduction

The fifth assignment concerns a dynamic model of a simplified spar floater supporting the DTU 10MW wind turbine [1]. The assignment consists of three parts: model formulation, dynamic analysis and modification of the blade pitch controller.

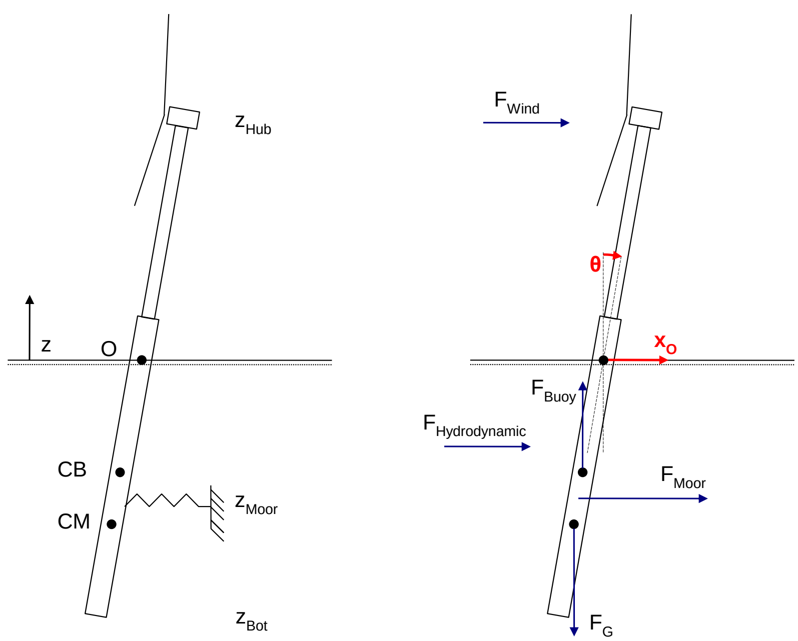

Figure 1: Definition sketch.

Recommended time plan and report length

To ensure a proper progression and validation of the results, a recommended time plan is given below. We expect a maximum of 20 pages plus the code appendix.

Part 1: To be done 12 November and the following days
Part 2: To be done 16–19 November and the following days
Part 3: To be done 23–26 November and the following days

Submission deadline: 29 November 2021 23:59

Part 1: Model formulation

The first part deals with the *analytical* formulation of the model.

Consider the floating wind turbine in figure 1. It consists of a spar buoy, a tower, a rotor and a nacelle, which are all rigidly connected. In this simplified model, 2D motion is assumed with 2 degrees of freedom: surge and pitch. The wind turbine is moored by a simple linear spring, which supplies a horizontal mooring force at the attachment point whenever its horizontal position differs from the resting position. The wind turbine is subject to an external wind load acting horizontally at the hub height, and the spar is subject to hydrodynamic forces from linear waves, added mass and damping. The parameters defining the model are given below.

$$
\begin{aligned}
M_{\text{Floater}} &= 1.0897 \cdot 10^{7} \ \text{kg} \\
z_{\text{CM,Floater}} &= -105.95 \ \text{m} \\
I_{\text{CM,Floater}} &= 1.1627 \cdot 10^{10} \ \text{kg m}^2 \\
M_{\text{Tower}} &= 5.4692 \cdot 10^{5} \ \text{kg} \\
z_{\text{CM,Tower}} &= 56.40 \ \text{m} \\
I_{\text{CM,Tower}} &= 4.2168 \cdot 10^{8} \ \text{kg m}^2 \\
M_{\text{Nacelle}} &= 446036 \ \text{kg} \\
M_{\text{Rotor}} &= 227962 \ \text{kg} \\
z_{\text{Hub}} &= 119 \ \text{m} \\
D_{\text{Spar}} &= 11.2 \ \text{m} \\
z_{\text{Bot}} &= -120 \ \text{m} \\
C_m &= 1.0 \\
C_D &= 0.6 \\
\rho_{\text{Water}} &= 1025 \ \text{kg/m}^3 \\
g &= 9.81 \ \text{m/s}^2 \\
h &= 320 \ \text{m} \\
K_{\text{Moor}} &= 66700 \ \text{N/m} \\
z_{\text{Moor}} &= -60 \ \text{m}
\end{aligned}
$$

A dynamic model for the surge and pitch motion of the point of flotation (O) can be written as

$$\mathbf{M}\begin{bmatrix}\ddot{x}_0\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}F_{\text{Moor}}+\tilde{F}_{\text{Hydrodynamic}}+F_{\text{Wind}}\\ \tau_{\text{Buoy}}+\tau_{\text{Moor}}+\tilde{\tau}_{\text{Hydrodynamic}}+\tau_{\text{Wind}}\end{bmatrix}. \tag{1}$$

In the model formulation, the surge and pitch deflections can be assumed small, therefore small-angle approximations can be used for θ.

1. Calculate the total mass M_{Tot}, the position of the center of mass $z_{\text{CM,Tot}}$, and the moment of inertia (around point O) $I_{\text{O,Tot}}$ for the whole structure.

2. Using a sketch, express the local surge displacement $x(z)$ as function of (x_0, θ).

3. Express F_{Moor} and τ_{Moor} as function of (x_0, θ).

4. Express the *restoring* moment from gravity and hydrostatic forces τ_{Buoy} as function of (x_0, θ).

5. Express the integrated hydrodynamic force and moment $\tilde{F}_{\text{Hydrodynamic}}$ and $\tilde{\tau}_{\text{Hydrodynamic}}$ in terms of (x_0, θ) and their time derivatives, given that the horizontal hydrodynamic force for a slice of the spar buoy of height $\mathrm{d}z$ is

$$\begin{aligned}\mathrm{d}\tilde{F}_{\text{Hydrodynamic}}=\rho_{\text{water}}C_m A\left(\dot{u}(z)-\ddot{x}(z)\right)\mathrm{d}z+\rho_{\text{water}}A\dot{u}(z)\mathrm{d}z\\ +\tfrac{1}{2}\rho_{\text{water}}C_D D(u(z)-\dot{x}(z))|u(z)-\dot{x}(z)|\mathrm{d}z.\end{aligned} \tag{2}$$

Here A, D are the cross-sectional area and diameter of the spar buoy at the up-right position.

6. Re-cast the model to the form

$$\left(\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}+\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\right)\begin{bmatrix}\ddot{x}_0\\ \ddot{\theta}\end{bmatrix}+\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}\begin{bmatrix}x_0\\ \theta\end{bmatrix}=\begin{bmatrix}F_{\text{Hydrodynamic}}+F_{\text{Wind}}\\ \tau_{\text{Hydrodynamic}}+\tau_{\text{Wind}}\end{bmatrix}. \tag{3}$$

Give analytical expressions and numerical values for all the elements in $\mathbf{M}$, $\mathbf{A}$ and $\mathbf{C}$. Further, give analytical expressions for all the hydrodynamic forcing terms at the right-hand side.
Hint: The elements of $\mathbf{M}$ can be found from the lecture slides and from the answer to question 1. The elements of $\mathbf{A}$ and $\mathbf{C}$ can be identified from the answers to questions 3, 4 and 5.

7. Using the homogeneous version of (3), estimate the natural frequencies for surge and pitch.

Part 2: Dynamic analysis

The second part concerns the *numerical* implementation of the model and analysis of the response to wind and waves. The test runs follow selected load cases in [2].

General Hints: When calculating response time series to external loads, a length of 600 s + initial transient should be considered, with the transient time at least 600 s. The analysis (e.g. PSD, std) should *not* include the transient. It is also a good idea to present the results in this part using subplots with time series (left) and PSD (right) of wind speed at the hub, free-surface elevation, floater surge and floater pitch. The cut-off frequency for the wind and wave spectra can be taken as $f_{\text{highcut}} = 0.5$ Hz. The frequency resolution $df = 1/T_{\text{Dur}}$ must be used where T_{Dur} is the duration of the signal. It is highly recommended to specify the times for the ODE solver by choosing $t_{\text{span}} = [0 : dt : T_{\text{dur}} - dt]$ with $dt = 0.1$ s, and to pre-compute the time series of wave kinematics and wind speed with a finer time step of $dt = 0.05$ s.

8. Re-write (3) to a first-order system by introducing the state vector

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} x_0 \\ \theta \\ \dot{x}_0 \\ \dot{\theta} \end{bmatrix}. \tag{4}$$

The first-order system must have the form

$$\frac{\mathrm{d}\underline{q}}{\mathrm{d}t} = \underline{f}(t, \underline{q}). \tag{5}$$

Express $\underline{f}(t, \underline{q})$ in terms of the quantities in (3).

9. Write a MATLAB program that solves (5) using the MATLAB `ode4` solver. Initially, the hydrodynamic forcing and wind forcing can be left out. Introduce a linear damping term $B_{11}\dot{x}_0$ with $B_{11} = 2 \cdot 10^5$ N/(m/s) into the left-hand side of the surge equation in (3) and include this in the MATLAB program.

10. Solve the equations of motion for two decay tests in surge and pitch, respectively:

(a) First, solve with an initial condition of $\underline{q} = [1\ 0\ 0\ 0]^T$ for a time period of 600 s.

(b) Next, solve for the initial condition $\underline{q} = [0\ 0.1\ 0\ 0]^T$.

Present time series and PSD of surge and pitch for both decay tests. Demonstrate by analysis of the results that the numerical model has the same natural frequencies as found in question 7. Are the surge and pitch motions coupled?

11. Include the hydrodynamic forcing terms. For simplicity, the hydrodynamic forcing is only computed up to the still water level. No Wheeler stretching is therefore needed in this assignment.

(a) Repeat the decay tests of question 10 and compare the time series. Discuss the results.

(b) Now consider a pitch decay test with an initial condition of $\underline{q} = [0\ 1\ 0\ 0]^T$ and the hydrodynamic forcing terms disabled. Compare the results to the same pitch decay test with the hydrodynamic forcing terms enabled. Discuss the results. Is the model still reliable for this decay case?

12. Compute the response for a regular wave of $H = 6$ m and $T = 10$ s. The wave kinematics for the wave forcing can be computed at the up-right position of the spar buoy. Present time series and PSD for surge and pitch and discuss their behaviour. Note that the initial condition

should be changed from the one associated with the decay tests.

13. Include wind forcing in the model. The wind force at the hub is determined from a C_T coefficient in three steps as follows:

- First, a mean thrust force is calculated as

$$\bar{F}_{\text{wind}} = \frac{1}{2}\rho_{\text{air}} A_{\text{Rotor}} C_T(V_{10\text{ min}}) V_{10\text{ min}}^2. \tag{6}$$

- Next, a time-varying thrust time series is computed as

$$\tilde{F}_{\text{wind}}(t) = \frac{1}{2}\rho_{\text{air}} A_{\text{Rotor}} C_T(V_{\text{rel}}) V_{\text{rel}} \left|V_{\text{rel}}\right|. \tag{7}$$

- Finally, an ad-hoc reduction of the dynamic part is applied to compensate for the spatial variation of turbulence across the rotor,

$$F_{\text{wind}}(t) = \bar{F}_{\text{wind}} + f_{\text{red}}(V_{10\text{ min}}) \left(\tilde{F}_{\text{wind}} - \bar{F}_{\text{wind}}\right). \tag{8}$$

Here

$$\begin{aligned}
V_{\text{rel}} &= V_{\text{hub}} - \dot{x}_{\text{hub}} \\
C_T &= \begin{cases} C_{T0} & V_{\text{rel}} \leq V_{\text{Rated}} \\ C_{T0} \exp\left[-a(V_{\text{rel}} - V_{\text{Rated}})^b\right] & V_{\text{rel}} > V_{\text{Rated}} \end{cases} \\
C_{T0} &= 0.81 \\
a &= 0.5 \\
b &= 0.65 \\
V_{\text{Rated}} &= 11.4 \text{ m/s} \\
\rho_{\text{air}} &= 1.22 \text{ kg/m}^3 \\
D_{\text{Rotor}} &= 178 \text{ m}
\end{aligned}$$

Further, the empirical reduction factor f_{red} serves to correct the standard deviation of the resulting thrust force, and is given by

$$f_{\text{red}}(V_{10\text{ min}}) = \begin{cases} 0.54 & \text{for } V_{10\text{ min}} < V_{\text{rated}} \\ 0.54 + 0.027(V_{10\text{ min}} - V_{\text{rated}}) & \text{for } V_{10\text{ min}} > V_{\text{rated}} \end{cases} \tag{9}$$

The numerical constants are all extracted or derived from the DTU 10MW reference wind turbine specifications [1].

Repeat the results from question 12 with addition of a steady wind of 8 m/s at the hub height. Compare the results to those from question 12. How does the steady wind affect the std of the wave-induced motions and the transient?

14. Repeat the calculations from question 13, but this time with irregular waves (JONSWAP spectrum with $H_s = 6$ m, $T_p = 10$ s, $\gamma = 3.3$). Present the results as time series and PSD for surge and pitch. Analyse the power spectra: which motions are induced by the wind and which are induced by the waves? What causes the spectral peaks observed?

15. Extend the wind forcing to include a Kaimal wind spectrum. The following formulation of the spectral density function for the fluctuating part of the wind speed can be used:

$$S_{\text{Wind}} = \frac{4I^2 V_{10\,\text{min}} l}{\left(1 + 6\frac{f\,l}{V_{10\,\text{min}}}\right)^{5/3}}. \tag{10}$$

Compute the motion of the wind turbine subject to irregular waves (JONSWAP spectrum with $H_s = 6$ m, $T_p = 10$ s, $\gamma = 3.3$) and turbulent wind (Kaimal wind climate of $V_{10\,\text{min}} = 8$ m/s, $I = 0.14, l = 340.2$ m). Present the results as time series and PSD for surge and pitch. Analyse the power spectra: which motions are induced by the wind and which are induced by the waves? What causes the spectral peaks observed?

Part 3: Adaption of pitch control for dynamic stability

The third part consist of adapting the controller to the floating configuration and investigating the dynamic stability.

16. Consider a load case of steady wind with no turbulence and no waves. Compute the response for (a) $V_{10\,\text{min}} = 10$ m/s and (b) $V_{10\,\text{min}} = 16$ m/s. Explain why the floater pitch motion is different for the two cases.

17. For a real wind turbine the adjustment of C_T to V_{rel} happens through dynamic control of the blade pitch. As this involves mechanical motion, the adjustment is associated with a time lag. In the present simplified model, the process can be represented by the following differential equation for C_T, where γ is a constant:

$$\frac{\mathrm{d}C_T}{\mathrm{d}t} = -\gamma \left(C_T - C_T(V_{\text{rel}})\right). \tag{11}$$

Include C_T as a fifth element of the state vector $\underline{q}$ and include (11) in the dynamic model. Demonstrate that for $\gamma = 2$, the new model gives similar results as in question 16(b).

18. Experiment with γ and suggest a value that makes the floater pitch response stable. What would this γ parameter represent in a real wind turbine? What is the consequence of choosing γ too small? Hint: think not only in terms of floater pitch stability.

References

[1] C. Bak et al. *Description of the DTU 10 MW reference wind turbine*. Tech. rep. No. I-0092, DTU Wind Energy, 2013.

[2] J. Jonkman and W. Musial. *Offshore Code Comparison Collaboration (OC3) for IEA Task 23: Offshore wind technology and deployment*. Tech. rep. No. NREL/TP-5000-48191, National Renewable Energy Laboratory, 2010.