Short exercises Lecture 5
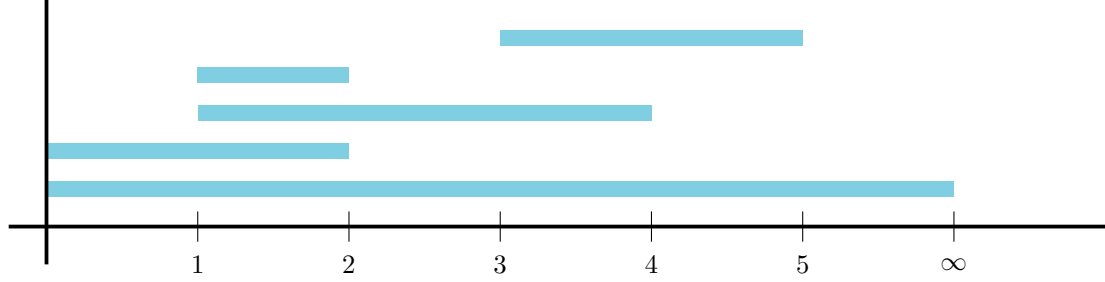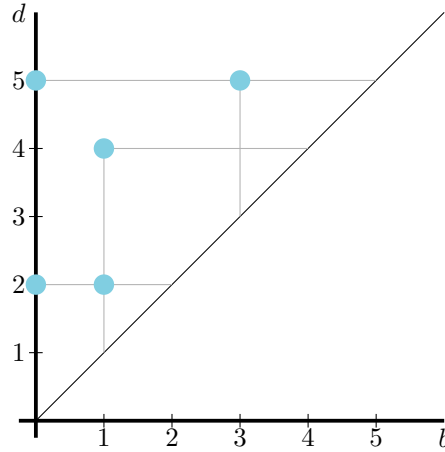
Exercise 1. Draw a persistence diagram and a landscape for the following persistence module over a field $\mathbb{F}$:
$$\mathbb{F}[0, \infty) \oplus \mathbb{F}[0, 2) \oplus \mathbb{F}[1, 4) \oplus \mathbb{F}[1, 2) \oplus \mathbb{F}[3, 5).$$

Solution. The barcode for this persistence module is the following:

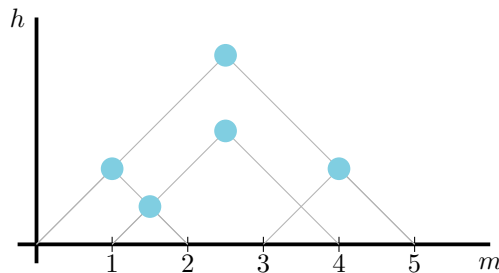The persistece diagram associated to the barcode above is:

Notice that the infinite ray $[0, \infty)$ is represented by the point $(0, 5)$.
Finally, the landscape diagram. The transformation from (b, d) coordinates to (m, h) coordinates is the following:

$(\mathbf{b}, \mathbf{d})$	$(\mathbf{m}, \mathbf{h})$
$(0, 5)$	$(2.5, 2.5)$
$(0, 2)$	$(1, 1)$
$(1, 4)$	$(2.5, 1.5)$
$(1, 2)$	$(1.5, 0.5)$
$(3, 5)$	$(4, 1)$

Then the landscape:

We coded a small program in `Mathematica` to compute the λ_i of the landscape.

```
In[1]:=  Λ[t_, b_, d_] := Max[{0, Min[{t - b, d - t}]}]
        λ[t_, k_, bd_] := Module[{l = {}, i = 1},
          For[i = 1, i ≤ Length[bd], i++, AppendTo[l, Λ[t, bd[[i]][[1]], bd[[i]][[2]]]]];
          Return[Sort[l, (#1 > #2) &][[k]]]]
```

```
In[3]:=  bd = {{0, 5}, {0, 2}, {1, 4}, {1, 2}, {3, 5}};
```
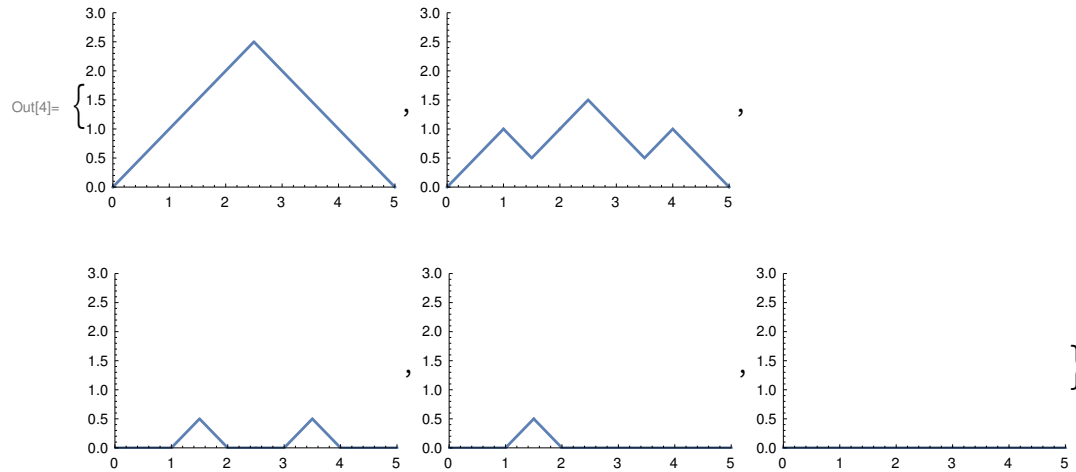
```
In[4]:=  Table[Plot[λ[k, i, bd], {k, 0, 5}, PlotRange → {{0, 5}, {0, 3}}], {i, 1, 5}]
```

Out[4]=


```
In[5]:=  Plot[{λ[k, 1, bd], λ[k, 2, bd], λ[k, 3, bd], λ[k, 4, bd], λ[k, 5, bd]},
          {k, 0, 5}, PlotLegends → {"λ₁", "λ₂", "λ₃", "λ₄", "λ₅"}]
```

Out[5]=