```
In [1]: from scipy.spatial.distance import directed_hausdorff
        import gudhi as gd
        import matplotlib.pyplot as plt
```

Consider the following point clouds in $\mathbb{R}^2$:

$$X = \{(0.81, 2.87), (2.15, 1.18), (3.19, 3.62), (4.17, 2.01), (5.32, 4.88), (6.21, 3.13)\},$$

$$Y = \{(0.75, 2.80), (2.33, 1.25), (3.28, 3.66), (4.15, 2.15), (5.24, 4.78), (6.34, 3.12)\}.$$

```
In [2]: X = [(0.81, 2.87),(2.15, 1.18),(3.19, 3.62),(4.17, 2.01),(5.32, 4.88),(6.21, 3.13)]
        Y = [(0.75, 2.80),(2.33, 1.25),(3.28, 3.66),(4.15, 2.15),(5.24, 4.78),(6.34, 3.12)]
```

(a) Compute the Hausdorff distance $d_H(X, Y)$.

```
In [3]: directed_hausdorff(X,Y)[0]

Out[3]: 0.19313207915827985
```

So, $d_H(X, Y) = 0.19313207915827985$.

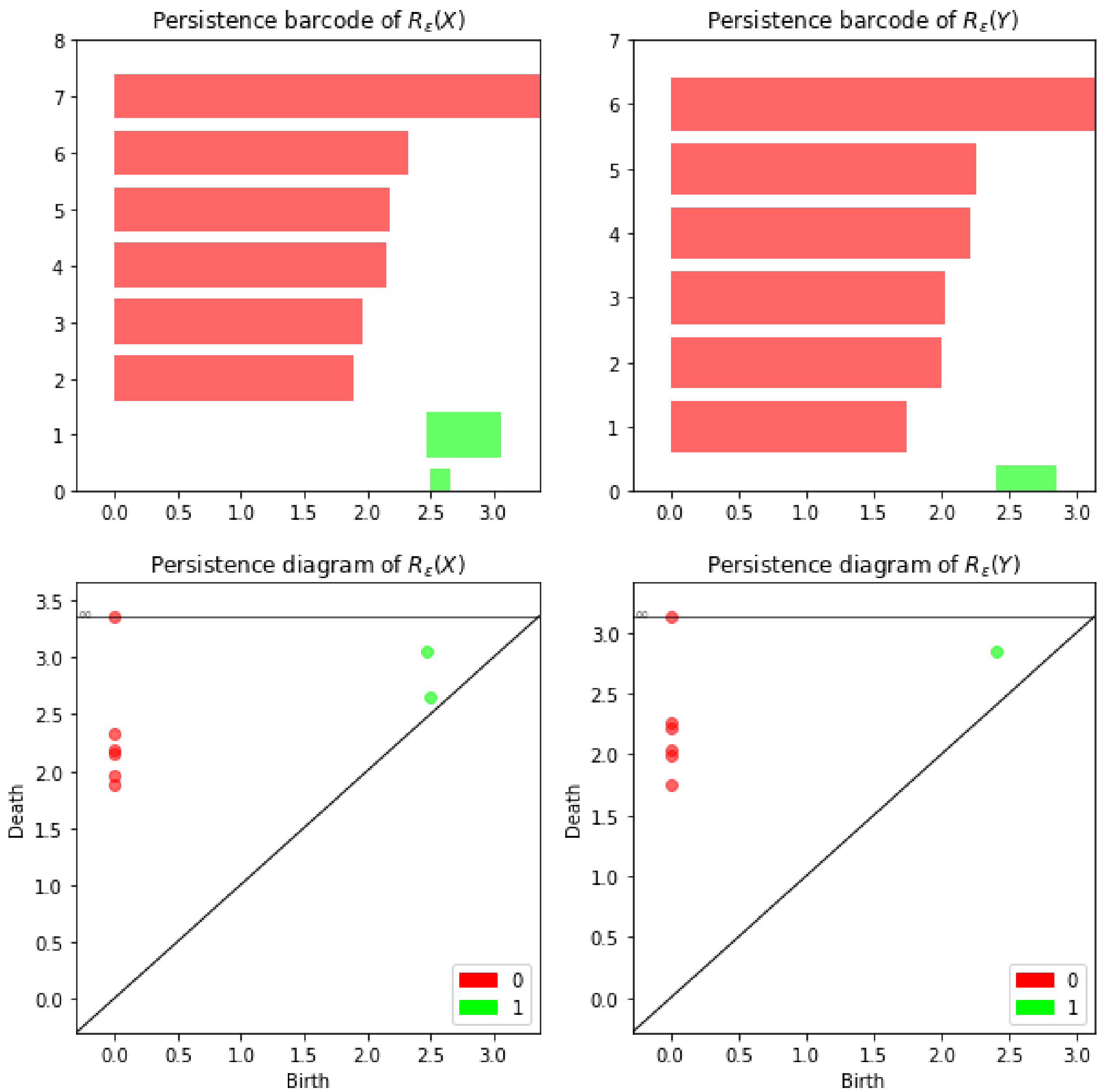
(b) Prove that $W_\infty(D(X), D(Y)) < 2d_H(X, Y)$, where $W_\infty(D(X), D(Y))$ is the bottleneck distance between the Vietoris-Rips persistence diagrams of X and Y.

Let us now compute the bottleneck distance using `GUDHI`. First, let us have a look at the persistence barcodes and diagrams.

```
In [4]: RX = gd.RipsComplex(points=X)
        st_X = RX.create_simplex_tree(max_dimension=len(X))
        diag_X = st_X.persistence(min_persistence=0.01)

        RY = gd.RipsComplex(points=Y)
        st_Y = RY.create_simplex_tree(max_dimension=len(Y))
        diag_Y = st_Y.persistence(min_persistence=0.01)

        plt.figure(figsize=(10,10))
        plt.subplot(2,2,1)
        gd.plot_persistence_barcode(diag_X, legend=False)
        plt.title(r'Persistence barcode of $R_\varepsilon(X)$')
        plt.subplot(2,2,2)
        gd.plot_persistence_barcode(diag_Y, legend=False)
        plt.title(r'Persistence barcode of $R_\varepsilon(Y)$')
        plt.subplot(2,2,3)
        gd.plot_persistence_diagram(diag_X, legend=True)
        plt.title(r'Persistence diagram of $R_\varepsilon(X)$')
        plt.subplot(2,2,4)
        gd.plot_persistence_diagram(diag_Y, legend=True)
        plt.title(r'Persistence diagram of $R_\varepsilon(Y)$')
        plt.show()
```



Now let us compute the Bottleneck distance between $D(X)$ and $D(Y)$:

```
In [5]: persistence_points_X = [x[1] for x in diag_X]
        persistence_points_Y = [y[1] for y in diag_Y]
        gd.bottleneck_distance(persistence_points_X, persistence_points_Y)
```

```
Out[5]: 0.21256525302337526
```

So, $W_\infty(D(X), D(Y)) = 0.21256525302337526$, which is clearly smaller than $2d_H(X, Y)$.