3d orthogonal ($A,\ B$ are linear transformations)

$$A = \left\{ \begin{array}{ll} L(\boldsymbol{e_x}) = & A_{xx}\boldsymbol{e_x} + A_{yx}\boldsymbol{e_y} + A_{zx}\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & A_{xy}\boldsymbol{e_x} + A_{yy}\boldsymbol{e_y} + A_{zy}\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & A_{xz}\boldsymbol{e_x} + A_{yz}\boldsymbol{e_y} + A_{zz}\boldsymbol{e_z} \end{array} \right\}$$

$$\operatorname{mat}(A) = \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

$$\det(A) = A_{xz}\left(A_{yx}A_{zy} - A_{yy}A_{zx}\right) - A_{yz}\left(A_{xx}A_{zy} - A_{xy}A_{zx}\right) + A_{zz}\left(A_{xx}A_{yy} - A_{xy}A_{yx}\right)$$

$$\overline{A} = \left\{ \begin{array}{ll} L(\boldsymbol{e_x}) = & A_{xx}\boldsymbol{e_x} + A_{xy}\boldsymbol{e_y} + A_{xz}\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & A_{yx}\boldsymbol{e_x} + A_{yy}\boldsymbol{e_y} + A_{yz}\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & A_{zx}\boldsymbol{e_x} + A_{zy}\boldsymbol{e_y} + A_{zz}\boldsymbol{e_z} \end{array} \right\}$$

$$\operatorname{Tr}(A) = A_{xx} + A_{yy} + A_{zz}$$

$$A(e_x \wedge e_y) = \left(A_{xx}A_{yy} - A_{xy}A_{yx}\right)\boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(A_{xx}A_{zy} - A_{xy}A_{zx}\right)\boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(A_{yx}A_{zy} - A_{yy}A_{zx}\right)\boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$A(e_x) \wedge A(e_y) = \left(A_{xx}A_{yy} - A_{xy}A_{yx}\right)\boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(A_{xx}A_{zy} - A_{xy}A_{zx}\right)\boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(A_{yx}A_{zy} - A_{yy}A_{zx}\right)\boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$A + B = \left\{ \begin{array}{ll} L(\boldsymbol{e_x}) = & \left(A_{xx} + B_{xx}\right)\boldsymbol{e_x} + \left(A_{yx} + B_{yx}\right)\boldsymbol{e_y} + \left(A_{zx} + B_{zx}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & \left(A_{xy} + B_{xy}\right)\boldsymbol{e_x} + \left(A_{yy} + B_{yy}\right)\boldsymbol{e_y} + \left(A_{zy} + B_{zy}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & \left(A_{xz} + B_{xz}\right)\boldsymbol{e_x} + \left(A_{yz} + B_{yz}\right)\boldsymbol{e_y} + \left(A_{zz} + B_{zz}\right)\boldsymbol{e_z} \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L(\boldsymbol{e_x}) = & \left(A_{xx}B_{xx} + A_{xy}B_{yx} + A_{xz}B_{zx}\right)\boldsymbol{e_x} + \left(A_{yx}B_{xx} + A_{yy}B_{yx} + A_{yz}B_{zx}\right)\boldsymbol{e_y} + \left(A_{zx}B_{xx} + A_{zy}B_{yx} + A_{zz}B_{zx}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & \left(A_{xx}B_{xy} + A_{xy}B_{yy} + A_{xz}B_{zy}\right)\boldsymbol{e_x} + \left(A_{yx}B_{xy} + A_{yy}B_{yy} + A_{yz}B_{zy}\right)\boldsymbol{e_y} + \left(A_{zx}B_{xy} + A_{zy}B_{yy} + A_{zz}B_{zy}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & \left(A_{xx}B_{xz} + A_{xy}B_{yz} + A_{xz}B_{zz}\right)\boldsymbol{e_x} + \left(A_{yx}B_{xz} + A_{yy}B_{yz} + A_{yz}B_{zz}\right)\boldsymbol{e_y} + \left(A_{zx}B_{xz} + A_{zy}B_{yz} + A_{zz}B_{zz}\right)\boldsymbol{e_z} \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{ll} L(\boldsymbol{e_x}) = & \left(A_{xx} - B_{xx}\right)\boldsymbol{e_x} + \left(A_{yx} - B_{yx}\right)\boldsymbol{e_y} + \left(A_{zx} - B_{zx}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & \left(A_{xy} - B_{xy}\right)\boldsymbol{e_x} + \left(A_{yy} - B_{yy}\right)\boldsymbol{e_y} + \left(A_{zy} - B_{zy}\right)\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & \left(A_{xz} - B_{xz}\right)\boldsymbol{e_x} + \left(A_{yz} - B_{yz}\right)\boldsymbol{e_y} + \left(A_{zz} - B_{zz}\right)\boldsymbol{e_z} \end{array} \right\}$$

2d general ($A,\ B$ are linear transformations)

$$A = \left\{ \begin{array}{ll} L(\boldsymbol{e_u}) = & A_{uu}\boldsymbol{e_u} + A_{vu}\boldsymbol{e_v} \\ L(\boldsymbol{e_v}) = & A_{uv}\boldsymbol{e_u} + A_{vv}\boldsymbol{e_v} \end{array} \right\}$$

$$\det(A) = A_{uu}A_{vv} - A_{uv}A_{vu}$$

$$\operatorname{Tr}(A) = \frac{(e_u \cdot e_u)(e_v \cdot e_v)A_{uu}}{(e_u \cdot e_u)(e_v \cdot e_v) - (e_u \cdot e_v)^2} + \frac{(e_u \cdot e_u)(e_v \cdot e_v)A_{vv}}{(e_u \cdot e_u)(e_v \cdot e_v) - (e_u \cdot e_v)^2} - \frac{(e_u \cdot e_v)^2 A_{uu}}{(e_u \cdot e_u)(e_v \cdot e_v) - (e_u \cdot e_v)^2} - \frac{(e_u \cdot e_v)^2 A_{vv}}{(e_u \cdot e_u)(e_v \cdot e_v) - (e_u \cdot e_v)^2}$$

$$A(e_u \wedge e_v) = \left(A_{uu}A_{vv} - A_{uv}A_{vu}\right)\boldsymbol{e_u} \wedge \boldsymbol{e_v}$$

$$A(e_u) \wedge A(e_v) = \left(A_{uu}A_{vv} - A_{uv}A_{vu}\right)\boldsymbol{e_u} \wedge \boldsymbol{e_v}$$

$$B = \left\{ \begin{array}{ll} L(\boldsymbol{e_u}) = & B_{uu}\boldsymbol{e_u} + B_{vu}\boldsymbol{e_v} \\ L(\boldsymbol{e_v}) = & B_{uv}\boldsymbol{e_u} + B_{vv}\boldsymbol{e_v} \end{array} \right\}$$

$$A + B = \left\{ \begin{array}{ll} L(\boldsymbol{e_u}) = & \left(A_{uu} + B_{uu}\right)\boldsymbol{e_u} + \left(A_{vu} + B_{vu}\right)\boldsymbol{e_v} \\ L(\boldsymbol{e_v}) = & \left(A_{uv} + B_{uv}\right)\boldsymbol{e_u} + \left(A_{vv} + B_{vv}\right)\boldsymbol{e_v} \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L(\boldsymbol{e_u}) = & \left(A_{uu}B_{uu} + A_{uv}B_{vu}\right)\boldsymbol{e_u} + \left(A_{vu}B_{uu} + A_{vv}B_{vu}\right)\boldsymbol{e_v} \\ L(\boldsymbol{e_v}) = & \left(A_{uu}B_{uv} + A_{uv}B_{vv}\right)\boldsymbol{e_u} + \left(A_{vu}B_{uv} + A_{vv}B_{vv}\right)\boldsymbol{e_v} \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{ll} L(\boldsymbol{e_u}) = & \left(A_{uu} - B_{uu}\right)\boldsymbol{e_u} + \left(A_{vu} - B_{vu}\right)\boldsymbol{e_v} \\ L(\boldsymbol{e_v}) = & \left(A_{uv} - B_{uv}\right)\boldsymbol{e_u} + \left(A_{vv} - B_{vv}\right)\boldsymbol{e_v} \end{array} \right\}$$

$$a \cdot \overline{A}(b) - b \cdot \underline{A}(a) = 0$$

4d Minkowski spaqce (Space Time)

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\underline{T} = \left\{ \begin{array}{ll} L(\boldsymbol{e_t}) = & T_{tt}\boldsymbol{e_t} + T_{xt}\boldsymbol{e_x} + T_{yt}\boldsymbol{e_y} + T_{zt}\boldsymbol{e_z} \\ L(\boldsymbol{e_x}) = & T_{tx}\boldsymbol{e_t} + T_{xx}\boldsymbol{e_x} + T_{yx}\boldsymbol{e_y} + T_{zx}\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & T_{ty}\boldsymbol{e_t} + T_{xy}\boldsymbol{e_x} + T_{yy}\boldsymbol{e_y} + T_{zy}\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & T_{tz}\boldsymbol{e_t} + T_{xz}\boldsymbol{e_x} + T_{yz}\boldsymbol{e_y} + T_{zz}\boldsymbol{e_z} \end{array} \right\}$$

$$\overline{T} = \left\{ \begin{array}{ll} L(\boldsymbol{e_t}) = & T_{tt}\boldsymbol{e_t} - T_{tx}\boldsymbol{e_x} - T_{ty}\boldsymbol{e_y} - T_{tz}\boldsymbol{e_z} \\ L(\boldsymbol{e_x}) = & -T_{xt}\boldsymbol{e_t} + T_{xx}\boldsymbol{e_x} + T_{xy}\boldsymbol{e_y} + T_{xz}\boldsymbol{e_z} \\ L(\boldsymbol{e_y}) = & -T_{yt}\boldsymbol{e_t} + T_{yx}\boldsymbol{e_x} + T_{yy}\boldsymbol{e_y} + T_{yz}\boldsymbol{e_z} \\ L(\boldsymbol{e_z}) = & -T_{zt}\boldsymbol{e_t} + T_{zx}\boldsymbol{e_x} + T_{zy}\boldsymbol{e_y} + T_{zz}\boldsymbol{e_z} \end{array} \right\}$$

$$\operatorname{tr}(\underline{T}) = T_{tt} + T_{xx} + T_{yy} + T_{zz}$$

$$a \cdot \overline{T}(b) - b \cdot \underline{T}(a) = 0$$