$$\mathbf{v} \cdot \nabla = v^{x} \frac{\partial}{\partial x} + v^{y} \frac{\partial}{\partial y} + v^{z} \frac{\partial}{\partial z}$$

$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

$$\mathbf{v} \cdot \nabla f = v^{x} \partial_{x} f + v^{y} \partial_{y} f + v^{z} \partial_{z} f$$

$$\nabla^{2} f = \partial_{x}^{2} f + \partial_{y}^{2} f + \partial_{z}^{2} f$$

$$= (\partial^{2} A^{x} + \partial^{2} A^{x} + \partial^{2} A^{x}) e_{x} + (\partial^{2} A^{y} + \partial^{2} A^{y}) e_{x} + (\partial^{2} A^{z} + \partial^{2} A^{z} + \partial^{2} A^{z}) e_{x}$$

 $X = xe_x + ue_y + ze_z$

 $\mathbf{v} = v^x \mathbf{e}_x + v^y \mathbf{e}_y + v^z \mathbf{e}_z$ $\mathbf{A} = A^x \mathbf{e}_x + A^y \mathbf{e}_y + A^z \mathbf{e}_z$

$$\nabla^{2} \mathbf{A} = \left(\partial_{x}^{2} A^{x} + \partial_{y}^{2} A^{x} + \partial_{z}^{2} A^{x}\right) \mathbf{e}_{x} + \left(\partial_{x}^{2} A^{y} + \partial_{y}^{2} A^{y} + \partial_{z}^{2} A^{y}\right) \mathbf{e}_{y} + \left(\partial_{x}^{2} A^{z} + \partial_{y}^{2} A^{z} + \partial_{z}^{2} A^{z}\right) \mathbf{e}_{z}$$

$$\bar{\nabla} \cdot v = v^{x} \frac{\partial}{\partial x} + v^{y} \frac{\partial}{\partial y} + v^{z} \frac{\partial}{\partial z}$$

$$\mathbf{X} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\bar{\nabla} \cdot v = v^x \frac{\partial}{\partial x} + v^y \frac{\partial}{\partial y} + v^z \frac{\partial}{\partial z}$$

$$\mathbf{X} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\bar{\nabla} \cdot \mathbf{Y} = 3 + x \frac{\partial}{\partial x} + y \frac{\partial}{\partial z} + z \frac{\partial}{\partial z}$$

$$\mathbf{X} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\bar{\nabla} \cdot \mathbf{X} = 3 + x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\mathbf{X} \cdot \nabla - \bar{\nabla} \cdot \mathbf{X} = -3$$

$$\bar{\nabla} \cdot \mathbf{X} = 3 + x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z}$$

$$\mathbf{X} \cdot \nabla - \bar{\nabla} \cdot \mathbf{X} = -3$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{2}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2 \tan(\theta)} \frac{\partial}{\partial \theta} + r^{-2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$$

$$\mathbf{X} \cdot \nabla - \bar{\nabla} \cdot \mathbf{X} = -3$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{2}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2 \tan(\theta)} \frac{\partial}{\partial \theta} + r^{-2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$$

$$(\mathbf{\nabla}^2) \cdot \mathbf{r} = \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial r^2} \mathbf{r} + \frac{\partial^2}{\partial \theta} \mathbf{r} + \frac{\partial^2}{\partial \theta} \mathbf{r} + \frac{\partial^2}{\partial \theta} \mathbf{r} + \frac{\partial^2}{\partial \theta} \mathbf{r}$$

$$\nabla^{2} = \nabla \cdot \nabla = \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial}{\partial r^{2}} + \frac{1}{r^{2} \tan(\theta)} \frac{\partial}{\partial \theta} + r^{-2} \frac{\partial}{\partial \theta^{2}} + \frac{1}{r^{2} \sin^{2}(\theta)} \frac{\partial}{\partial \phi^{2}}$$

$$(\nabla^{2}) f = \frac{1}{r^{2}} \left(r^{2} \partial_{r}^{2} f + 2r \partial_{r} f + \partial_{\theta}^{2} f + \frac{\partial}{\partial \theta} f + \frac{\partial^{2} f}{\sin^{2}(\theta)} \right)$$

 $\nabla \cdot (\nabla f) = \frac{1}{r^2} \left(r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right)$