$$(u, v) \rightarrow (r, \theta, \phi) = [1, u, v]$$

Unit Sphere Manifold:

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin^{2}{\left (u \right)} \end{bmatrix}$$

$$a = a^{u} \boldsymbol{e_{u}} + a^{v} \boldsymbol{e_{v}}$$

$$f = f^{u} \boldsymbol{e_{u}} + f^{v} \boldsymbol{e_{v}}$$

$$\nabla = \boldsymbol{e_{u}} \frac{\partial}{\partial u} + \frac{1}{\sin^{2}{\left (u \right)}} \boldsymbol{e_{v}} \frac{\partial}{\partial v}$$

$$a \cdot \nabla = a^{u} \frac{\partial}{\partial u} + a^{v} \frac{\partial}{\partial v}$$

$$\left (a \cdot \nabla \right) \boldsymbol{e}_{u} = \frac{a^{v}}{\tan{\left (u \right)}} \boldsymbol{e_{v}}$$

$$\left (a \cdot \nabla \right) \boldsymbol{e}_{v} = - \frac{a^{v}}{2} \sin{\left (2 u \right)} \boldsymbol{e_{u}} + \frac{a^{u}}{\tan{\left (u \right)}} \boldsymbol{e_{v}}$$

$$\left (a \cdot \nabla \right) f = \left (a^{u} \partial_{u} f^{u} - \frac{a^{v} f^{v}}{2} \sin{\left (2 u \right)} + a^{v} \partial_{v} f^{u}\right) \boldsymbol{e_{u}} + \left (\frac{a^{u} f^{v}}{\tan{\left (u \right)}} + a^{u} \partial_{u} f^{v} + \frac{a^{v} f^{u}}{\tan{\left (u \right)}} + a^{v} \partial_{v} f^{v}\right) \boldsymbol{e_{v}}$$

Tensors on the Unit Sphere

$$V = a_{1}^{u} V_{u} + a_{1}^{v} V_{v}$$

$$T = T_{uu} a_{1}^{u} a_{2}^{u} + T_{uv} a_{1}^{u} a_{2}^{v} + T_{vu} a_{1}^{v} a_{2}^{u} + T_{vv} a_{1}^{v} a_{2}^{v}$$

Tensor Contraction

$$T[1,2] = T_{uu} + \frac{T_{vv}}{\sin^{2}{\left (u \right)}}$$

Tensor Evaluation

$$T(a,b) = a^{u} b^{u} T_{uu} + a^{u} b^{v} T_{uv} + a^{v} b^{u} T_{vu} + a^{v} b^{v} T_{vv}$$

$$T(a,b+c) = a^{u} b^{u} T_{uu} + a^{u} b^{v} T_{uv} + a^{u} c^{u} T_{uu} + a^{u} c^{v} T_{uv} + a^{v} b^{u} T_{vu} + a^{v} b^{v} T_{vv} + a^{v} c^{u} T_{vu} + a^{v} c^{v} T_{vv}$$

$$T(a,\alpha b) = \alpha a^{u} b^{u} T_{uu} + \alpha a^{u} b^{v} T_{uv} + \alpha a^{v} b^{u} T_{vu} + \alpha a^{v} b^{v} T_{vv}$$

Geometric Derivative With Respect To Slot

$$\nabla_{a_{1}} T = \left (a_{2}^{u} T_{uu} + a_{2}^{v} T_{uv}\right) \boldsymbol{e_{u}} + \frac{a_{2}^{u} T_{vu} + a_{2}^{v} T_{vv}}{\sin^{2}{\left (u \right)}} \boldsymbol{e_{v}}$$

$$\nabla_{a_{2}} T = \left (a_{1}^{u} T_{uu} + a_{1}^{v} T_{vu}\right) \boldsymbol{e_{u}} + \frac{a_{1}^{u} T_{uv} + a_{1}^{v} T_{vv}}{\sin^{2}{\left (u \right)}} \boldsymbol{e_{v}}$$

Covariant Derivatives

$$\begin{aligned} \mathcal{D}V = & \partial_{u} V_{u} a_{1}^{u} a_{2}^{u} \\ & + \left (- \frac{V_{v}}{\tan{\left (u \right)}} + \partial_{v} V_{u}\right) a_{1}^{u} a_{2}^{v} \\ & + \left (- \frac{V_{v}}{\tan{\left (u \right)}} + \partial_{u} V_{v}\right) a_{1}^{v} a_{2}^{u} \\ & + \left (\frac{V_{u}}{2} \sin{\left (2 u \right)} + \partial_{v} V_{v}\right) a_{1}^{v} a_{2}^{v} \end{aligned}$$

$$\begin{aligned}\mathcal{D}T =&\partial_u T_{uu} a_1^u a_2^u a_3^u + \left(-\frac{T_{uv}}{\tan(u)} - \frac{T_{vu}}{\tan(u)} + \partial_v T_{uu}\right) a_1^u a_2^u a_3^v + \left(-\frac{T_{uv}}{\tan(u)} + \partial_u T_{uv}\right) a_1^u a_2^v a_3^u \\ &+ \left(\frac{T_{uu}}{2}\sin(2u) - \frac{T_{vv}}{\tan(u)} + \partial_v T_{uv}\right) a_1^u a_2^v a_3^v + \left(-\frac{T_{vu}}{\tan(u)} + \partial_u T_{vu}\right) a_1^v a_2^u a_3^u + \left(\frac{T_{uu}}{2}\sin(2u) - \frac{T_{vv}}{\tan(u)} + \partial_v T_{vu}\right) a_1^v a_2^u a_3^v \\ &+ \left(-\frac{2T_{vv}}{\tan(u)} + \partial_u T_{vv}\right) a_1^v a_2^v a_3^u + \left(\frac{T_{uv}}{2}\sin(2u) + \frac{T_{vu}}{2}\sin(2u) + \partial_v T_{vv}\right) a_1^v a_2^v a_3^v\end{aligned}$$

$$\mathcal{D}T[1,3](a) = \frac{a^u T_{uu}\sin(2u)}{2\sin^2(u)} - \frac{a^u T_{vv}}{\sin^2(u)\tan(u)} + a^u \partial_u T_{uu} + \frac{a^u \partial_v T_{vu}}{\sin^2(u)} - \frac{a^v T_{uv}}{\tan(u)} + \frac{a^v T_{uv}\sin(2u)}{2\sin^2(u)} + \frac{a^v T_{vu}\sin(2u)}{2\sin^2(u)} + a^v \partial_u T_{uv} + \frac{a^v \partial_v T_{vv}}{\sin^2(u)}$$

1-D Manifold On Unit Sphere:

$$\nabla = \frac{1}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} \boldsymbol{e_s} \frac{\partial}{\partial s}$$

$$\nabla g = \frac{\partial_s g}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} \boldsymbol{e_s}$$

$$\nabla \cdot \boldsymbol{h} = \frac{1}{\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2} \left(\left(\sin^2(u^s)(\partial_s v^s)^2 + (\partial_s u^s)^2\right)\partial_s h^s + \left(\sin^2(u^s)\partial_s v^s \partial_s^2 v^s + \frac{\partial_s u^s}{2}\sin(2u^s)(\partial_s v^s)^2 + \partial_s u^s \partial_s^2 u^s\right) h^s\right)$$