$$\boldsymbol{X} = x\boldsymbol{e_x} + y\boldsymbol{e_y} + z\boldsymbol{e_z}$$

$$\boldsymbol{v} = v^x\boldsymbol{e_x} + v^y\boldsymbol{e_y} + v^z\boldsymbol{e_z}$$

$$\boldsymbol{A} = A^x\boldsymbol{e_x} + A^y\boldsymbol{e_y} + A^z\boldsymbol{e_z}$$

$$\boldsymbol{v}\cdot\nabla = v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y} + v^z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boldsymbol{v}\cdot\nabla f = v^x\partial_x f + v^y\partial_y f + v^z\partial_z f$$

$$\nabla^2 f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\nabla^2\boldsymbol{A} = \left(\partial_x^2 A^x + \partial_y^2 A^x + \partial_z^2 A^x\right)\boldsymbol{e_x} + \left(\partial_x^2 A^y + \partial_y^2 A^y + \partial_z^2 A^y\right)\boldsymbol{e_y} + \left(\partial_x^2 A^z + \partial_y^2 A^z + \partial_z^2 A^z\right)\boldsymbol{e_z}$$

$$\bar{\nabla}\cdot v = v^x\frac{\partial}{\partial x} + v^y\frac{\partial}{\partial y} + v^z\frac{\partial}{\partial z}$$

$$\boldsymbol{X}\cdot\nabla = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$$

$$\bar{\nabla}\cdot\boldsymbol{X} = 3 + x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$$

$$\boldsymbol{X}\cdot\nabla - \bar{\nabla}\cdot\boldsymbol{X} = -3$$

$$\nabla^2 = \nabla\cdot\nabla = \frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2\tan(\theta)}\frac{\partial}{\partial\theta} + r^{-2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2}{\partial\phi^2}$$

$$\left(\nabla^2\right)f = \frac{1}{r^2}\left(r^2\partial_r^2 f + 2r\partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)}\right)$$

$$\nabla\cdot(\nabla f) = \frac{1}{r^2}\left(r^2\partial_r^2 f + 2r\partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)}\right)$$