$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$(\nabla^2) f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\nabla \cdot (\nabla f) = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{2}{r} \frac{\partial}{\partial r} + \frac{\cos(\theta)}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial r^2} + r^{-2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2}{\partial \phi^2}$$

$$\left(\nabla^{2}\right)f = \frac{1}{r^{2}}\left(r^{2}\partial_{r}^{2}f + 2r\partial_{r}f + \partial_{\theta}^{2}f + \frac{\partial_{\theta}f}{\tan\left(\theta\right)} + \frac{\partial_{\phi}^{2}f}{\sin^{2}\left(\theta\right)}\right)$$

$$\nabla \cdot (\nabla f) = \frac{1}{r^2} \left(r^2 \partial_r^2 f + 2r \partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)} \right)$$