$$
\begin{aligned}
v =& v^x \boldsymbol{e_x} \\
&+ v^y \boldsymbol{e_y} \\
&+ v^z \boldsymbol{e_z}
\end{aligned}
$$

$$
V^x \boldsymbol{e_x} + V^y \boldsymbol{e_y} + V^z \boldsymbol{e_z}
$$

$$
\begin{aligned}
\nabla V =& (\partial_x V^x + \partial_y V^y + \partial_z V^z) \\
&+ (-\partial_y V^x + \partial_x V^y)\, \boldsymbol{e_x} \wedge \boldsymbol{e_y} \\
&+ (-\partial_z V^x + \partial_x V^z)\, \boldsymbol{e_x} \wedge \boldsymbol{e_z} \\
&+ (-\partial_z V^y + \partial_y V^z)\, \boldsymbol{e_y} \wedge \boldsymbol{e_z}
\end{aligned}
$$

$$
\begin{aligned}
\nabla V =& (\partial_x V^x + \partial_y V^y + \partial_z V^z) \\
&+ (-\partial_y V^x + \partial_x V^y)\, \boldsymbol{e_x} \wedge \boldsymbol{e_y} + (-\partial_z V^x + \partial_x V^z)\, \boldsymbol{e_x} \wedge \boldsymbol{e_z} + (-\partial_z V^y + \partial_y V^z)\, \boldsymbol{e_y} \wedge \boldsymbol{e_z}
\end{aligned}
$$

$$
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
$$