$$(u,v) \rightarrow (r,\theta,\phi) = [1,u,v]$$

Unit Sphere Manifold:

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin^{2}(u) \end{bmatrix}$$

$$a = a^{u}\boldsymbol{e_{u}} + a^{v}\boldsymbol{e_{v}}$$

$$f = f^{u}\boldsymbol{e_{u}} + f^{v}\boldsymbol{e_{v}}$$

$$\nabla = \boldsymbol{e_{u}}\frac{\partial}{\partial u} + \frac{1}{\sin^{2}(u)}\boldsymbol{e_{v}}\frac{\partial}{\partial v}$$

$$a\cdot\nabla = a^{u}\frac{\partial}{\partial u} + a^{v}\frac{\partial}{\partial v}$$

$$(a\cdot\nabla)\,\boldsymbol{e}_{u} = \frac{a^{v}}{\tan(u)}\boldsymbol{e_{v}}$$

$$(a\cdot\nabla)\,\boldsymbol{e}_{v} = -\frac{a^{v}}{2}\sin(2u)\boldsymbol{e_{u}} + \frac{a^{u}}{\tan(u)}\boldsymbol{e_{v}}$$

$$(a\cdot\nabla)\,f = \left(a^{u}\partial_{u}f^{u} - \frac{a^{v}f^{v}}{2}\sin(2u) + a^{v}\partial_{v}f^{u}\right)\boldsymbol{e_{u}} + \left(\frac{a^{u}f^{v}}{\tan(u)} + a^{u}\partial_{u}f^{v} + \frac{a^{v}f^{u}}{\tan(u)} + a^{v}\partial_{v}f^{v}\right)\boldsymbol{e_{v}}$$

Tensors on the Unit Sphere

$$V = a_{1}^{u}V_{u} + a_{1}^{v}V_{v}$$

$$T = T_{uu}a_{1}^{u}a_{2}^{u} + T_{uv}a_{1}^{u}a_{2}^{v} + T_{vu}a_{1}^{v}a_{2}^{u} + T_{vv}a_{1}^{v}a_{2}^{v}$$

Tensor Contraction

$$T[1,2] = T_{uu} + \frac{T_{vv}}{\sin^{2}(u)}$$

Tensor Evaluation

$$T(a,b) = a^{u}b^{u}T_{uu} + a^{u}b^{v}T_{uv} + a^{v}b^{u}T_{vu} + a^{v}b^{v}T_{vv}$$

$$T(a,b+c) = a^{u}b^{u}T_{uu} + a^{u}b^{v}T_{uv} + a^{u}c^{u}T_{uu} + a^{u}c^{v}T_{uv} + a^{v}b^{u}T_{vu} + a^{v}b^{v}T_{vv} + a^{v}c^{u}T_{vu} + a^{v}c^{v}T_{vv}$$

$$T(a,\alpha b) = \alpha a^{u}b^{u}T_{uu} + \alpha a^{u}b^{v}T_{uv} + \alpha a^{v}b^{u}T_{vu} + \alpha a^{v}b^{v}T_{vv}$$

Geometric Derivative With Respect To Slot

$$\nabla_{a_{1}}T = \left(a_{2}^{u}T_{uu} + a_{2}^{v}T_{uv}\right)\boldsymbol{e_{u}} + \frac{a_{2}^{u}T_{vu} + a_{2}^{v}T_{vv}}{\sin^{2}(u)}\boldsymbol{e_{v}}$$

$$\nabla_{a_2} T = \left(a_1^u T_{uu} + a_1^v T_{vu}\right) \boldsymbol{e_u} + \frac{a_1^u T_{uv} + a_1^v T_{vv}}{\sin^2\left(u\right)} \boldsymbol{e_v}$$

Covariant Derivatives

$$\begin{aligned}\mathcal{D}V = & \partial_u V_u a_1^u a_2^u \\ & + \left(-\frac{V_v}{\tan\left(u\right)} + \partial_v V_u\right) a_1^u a_2^v \\ & + \left(-\frac{V_v}{\tan\left(u\right)} + \partial_u V_v\right) a_1^v a_2^u \\ & + \left(\frac{V_u}{2}\sin\left(2u\right) + \partial_v V_v\right) a_1^v a_2^v\end{aligned}$$

$$\begin{aligned}\mathcal{D}T = & \partial_u T_{uu} a_1^u a_2^u a_3^u + \left(-\frac{T_{uv}}{\tan\left(u\right)} - \frac{T_{vu}}{\tan\left(u\right)} + \partial_v T_{uu}\right) a_1^u a_2^u a_3^v + \left(-\frac{T_{uv}}{\tan\left(u\right)} + \partial_u T_{uv}\right) a_1^u a_2^v a_3^u \\ & + \left(\frac{T_{uu}}{2}\sin\left(2u\right) - \frac{T_{vv}}{\tan\left(u\right)} + \partial_v T_{uv}\right) a_1^u a_2^v a_3^v + \left(-\frac{T_{vu}}{\tan\left(u\right)} + \partial_u T_{vu}\right) a_1^v a_2^u a_3^u + \left(\frac{T_{uu}}{2}\sin\left(2u\right) - \frac{T_{vv}}{\tan\left(u\right)} + \partial_v T_{vu}\right) a_1^v a_2^u a_3^v \\ & + \left(-\frac{2T_{vv}}{\tan\left(u\right)} + \partial_u T_{vv}\right) a_1^v a_2^v a_3^u + \left(\frac{T_{uv}}{2}\sin\left(2u\right) + \frac{T_{vu}}{2}\sin\left(2u\right) + \partial_v T_{vv}\right) a_1^v a_2^v a_3^v\end{aligned}$$

$$\mathcal{D}T[1,3](a) = \frac{a^u T_{uu}\sin\left(2u\right)}{2\sin^2\left(u\right)} - \frac{a^u T_{vv}}{\sin^2\left(u\right)\tan\left(u\right)} + a^u \partial_u T_{uu} + \frac{a^u \partial_v T_{vu}}{\sin^2\left(u\right)} - \frac{a^v T_{uv}}{\tan\left(u\right)} + \frac{a^v T_{uv}\sin\left(2u\right)}{2\sin^2\left(u\right)} + \frac{a^v T_{vu}\sin\left(2u\right)}{2\sin^2\left(u\right)} + a^v \partial_u T_{uv} + \frac{a^v \partial_v T_{vv}}{\sin^2\left(u\right)}$$