$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\left(\nabla^2\right) f = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\nabla \cdot (\nabla f) = \partial_x^2 f + \partial_y^2 f + \partial_z^2 f$$

$$\nabla^2 = \nabla \cdot \nabla = \frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2} + \frac{1}{r^2 \tan(\theta)}\frac{\partial}{\partial \theta} + r^{-2}\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2(\theta)}\frac{\partial^2}{\partial \phi^2}$$

$$\left(\nabla^2\right) f = \frac{1}{r^2}\left(r^2 \partial_r^2 f + 2r\partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)}\right)$$

$$\nabla \cdot (\nabla f) = \frac{1}{r^2}\left(r^2 \partial_r^2 f + 2r\partial_r f + \partial_\theta^2 f + \frac{\partial_\theta f}{\tan(\theta)} + \frac{\partial_\phi^2 f}{\sin^2(\theta)}\right)$$