$$\boldsymbol{a} \cdot (\boldsymbol{bc}) = -(a \cdot c)\,\boldsymbol{b} + (a \cdot b)\,\boldsymbol{c}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) = -(a \cdot c)\,\boldsymbol{b} + (a \cdot b)\,\boldsymbol{c}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) = (a \cdot d)\,\boldsymbol{b} \wedge \boldsymbol{c} - (a \cdot c)\,\boldsymbol{b} \wedge \boldsymbol{d} + (a \cdot b)\,\boldsymbol{c} \wedge \boldsymbol{d}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) + \boldsymbol{c} \cdot (\boldsymbol{a} \wedge \boldsymbol{b}) + \boldsymbol{b} \cdot (\boldsymbol{c} \wedge \boldsymbol{a}) = 0$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b}) = 3\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{d}) - \boldsymbol{d}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}) = 4\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \cdot (\boldsymbol{c} \wedge \boldsymbol{d}) = -(a \cdot c)\,(b \cdot d) + (a \cdot d)\,(b \cdot c)$$

$$((\boldsymbol{a} \wedge \boldsymbol{b}) \cdot \boldsymbol{c}) \cdot \boldsymbol{d} = -(a \cdot c)\,(b \cdot d) + (a \cdot d)\,(b \cdot c)$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \times (\boldsymbol{c} \wedge \boldsymbol{d}) = -(b \cdot d)\,\boldsymbol{a} \wedge \boldsymbol{c} + (b \cdot c)\,\boldsymbol{a} \wedge \boldsymbol{d} + (a \cdot d)\,\boldsymbol{b} \wedge \boldsymbol{c} - (a \cdot c)\,\boldsymbol{b} \wedge \boldsymbol{d}$$