

$$g_{ij} = \begin{bmatrix} (a \cdot a) & (a \cdot b) & (a \cdot c) & (a \cdot d) \\ (a \cdot b) & (b \cdot b) & (b \cdot c) & (b \cdot d) \\ (a \cdot c) & (b \cdot c) & (c \cdot c) & (c \cdot d) \\ (a \cdot d) & (b \cdot d) & (c \cdot d) & (d \cdot d) \end{bmatrix}$$

$$a \cdot (bc) = -(a \cdot c) \mathbf{b} + (a \cdot b) \mathbf{c}$$

$$a \cdot (b \wedge c) = -(a \cdot c) \mathbf{b} + (a \cdot b) \mathbf{c}$$

$$a \cdot (b \wedge c \wedge d) = (a \cdot d) \mathbf{b} \wedge \mathbf{c} - (a \cdot c) \mathbf{b} \wedge \mathbf{d} + (a \cdot b) \mathbf{c} \wedge \mathbf{d}$$

$$a \cdot (b \wedge c) + c \cdot (a \wedge b) + b \cdot (c \wedge a) = 0$$

$$a(b \wedge c) - b(a \wedge c) + c(a \wedge b) = 3a \wedge b \wedge c$$

$$a(b \wedge c \wedge d) - b(a \wedge c \wedge d) + c(a \wedge b \wedge d) - d(a \wedge b \wedge c) = 4a \wedge b \wedge c \wedge d$$

$$(a \wedge b) \cdot (c \wedge d) = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$((a \wedge b) \cdot c) \cdot d = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$(a \wedge b) \times (c \wedge d) = -(b \cdot d) \mathbf{a} \wedge \mathbf{c} + (b \cdot c) \mathbf{a} \wedge \mathbf{d} + (a \cdot d) \mathbf{b} \wedge \mathbf{c} - (a \cdot c) \mathbf{b} \wedge \mathbf{d}$$