PSI - Plasma-Surface Interaction

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Course: Numeric Linear Algebra
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Project Proposal

Motivation

- Application: tokamak and plasmatron.
- Difficulties: Hard to obtaine accurate experimental data.
- Solution: Modeling provides the ability to solve this problem numerically.

Problem

Obtain the stationary solution of PSI(Plasma-Surface Interaction) solving nonlinear system.

Method

Matrix sweep method, Newton's method, Multigrid.

Literature

- *E. Meier, U. Shumlak*
  A general nonlinear fluid model for reacting plasma- neutral mixtures.
  Physics of Plasma. 2012. – 19. – 072508 (11 p).
- *Benilov M.S.,*
  The ion flux from a thermal plasma to a surface.
  J. Phys. D, Appl. Phys. – 1995. – 28. – P. 286-294.
- *M.S. Almeida, M.S. Benilov, G.V. Naidis*
  Simulation of the layer of non- equilibrium ionization in a high-pressure argon plasma with multiply-charged ions
  Ibid. – 2000. –33. – P. 960–967.

Problem statement

Data

$$T, \gamma, P, j$$

T — temperature of plasma,
γ — the ratio of iron atoms to argon ones in surface,
P — pressure
j — current density

Demand

Building the model

$$f : (T, \gamma, P, j) \mapsto \hat{\mathbf{n}},$$

$\mathbf{n}$ — vector of consentration in each point of grid.

Physical model.

Mass concervation

$$n_1 = n_4 + n_5 + 2n_6$$

$$\frac{dn_i v_i}{dx} = -S_i$$

$$\sum_{\gamma=1}^{6} n_\gamma T_\gamma = \frac{P_0}{k_B},$$

Momentum concervation

$$\sum_{\gamma=1}^{6} q_\gamma n_\gamma v_\gamma = j$$

$$n_2 v_2 + n_4 v_4 = 0$$

$$n_3 v_3 + n_5 v_5 + n_6 v_6 = G_{Fe}$$

$$\frac{d(p_\gamma + m_\gamma n_\gamma v_\gamma^2)}{dx} = E q_\gamma n_\gamma + \sum_{\theta \neq \gamma} P_{\gamma\theta} + R_\gamma$$

Dimensional parametars in SI

$$n_0 = 10^{23}, \quad L_0 = 10^{-4}, \quad v_0 = 10^3, \quad \alpha_0 = n_0 t_0, \quad \beta = n_0^2 t_0$$

Final equation

$$\frac{d\mathbf{K}}{dx} = \frac{d}{dx}(A^{-1} D \frac{d\mathbf{n}}{dx} + A^{-1} D^{tem} \frac{d\mathbf{T}}{dx} + (A^{-1}\mathbf{J} + A^{-1}\mathbf{C})) = \mathbf{S}$$

Apximation of differential operators

$$\frac{d}{dx}S_{(k-2)m}\frac{dn_k}{dx} -> n_k^{i-1}\frac{S_{(k-2)m}^{i-\frac{1}{2}}}{h_i(0.5(h_{i-1}+h_i))} - n_k^i\left(\frac{S_{(k-2)m}^{i-\frac{1}{2}}}{h_i0.5(h_{i-1}+h_i)} + \frac{S_{(k-2)m}^{i+\frac{1}{2}}}{h_i0.5(h_{i+1}+h_i)}\right) + n_k^{i+1}\frac{S_{(k-2)m}^{i+\frac{1}{2}}}{h_i0.5(h_{i+1}+h_i)}$$

Gauss-Newton's method

System of non-linear equations

$$W\mathbf{x} + \mathbf{b} - \mathbf{f}(\mathbf{x}) = 0 = \mathbf{A}(x)$$

Newton's iteration

$$\Delta\mathbf{x} = \left(\frac{\partial \mathbf{A}(x)}{\partial \mathbf{x}}\right)^{-1} \mathbf{A}(x)$$

$$\left(\frac{\partial \mathbf{A}(x)}{\partial \mathbf{x}}\right) \approx W + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}$$

Solve by matrix sweep method(Thomas' block method)

$$(W - J)\Delta\mathbf{x} = -\mathbf{A}(x)$$

$$\mathbf{x} := \mathbf{x} + \Delta\mathbf{x}$$

Non-linear Multigrid method

- $$\mathbf{r}^{2h} = I_h^{2h}(\mathbf{f}^h - A^h(\mathbf{v}^h))$$
  $$\mathbf{v}^{2h} = I_h^{2h}\mathbf{v}^h,$$
  I_h^{2h} — operator of interpolation from fine-grid to coarse-grid.

- Solve the coarse-grid problem
  $$A^{2h}(u^{2h}) = A^{2h}(v^{2h}) + r^{2h}$$
- Error
  $$\mathbf{e}^{2h} = \mathbf{u}^{2h} - \mathbf{v}^{2h}$$
  .
- $$\mathbf{v}^h \longleftarrow \mathbf{v}^h + I_{2h}^h\mathbf{e}^{2h}$$
  .

Computational experimet

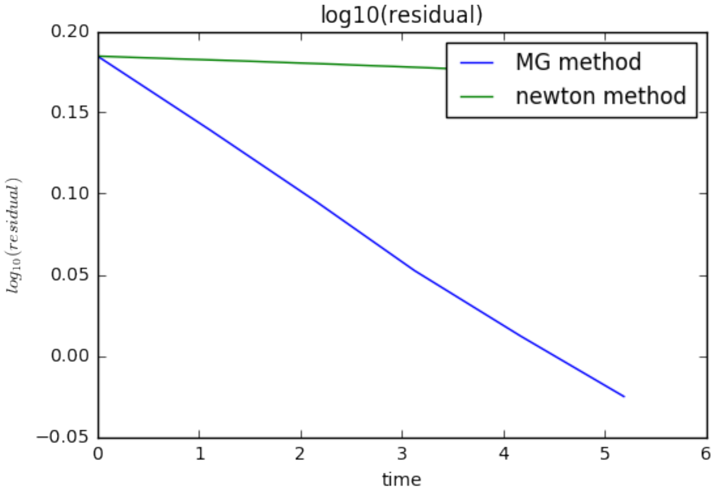

Рис.: Comparing MG and Newton's Method

Computational experimet

$0 — e$, $1 — Ar$, $2 — Fe$, $3 — Ar^+$, $4 — Fe^+$, $5 — Fe^{++}$

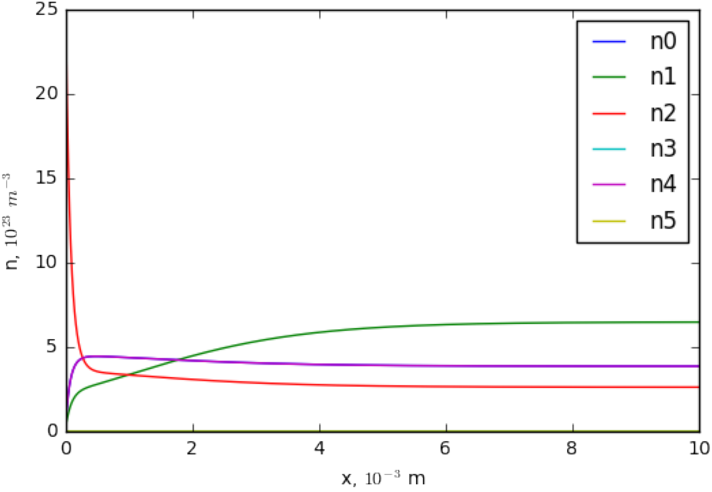

Рис.: Distribution of concentration

Computational experimet

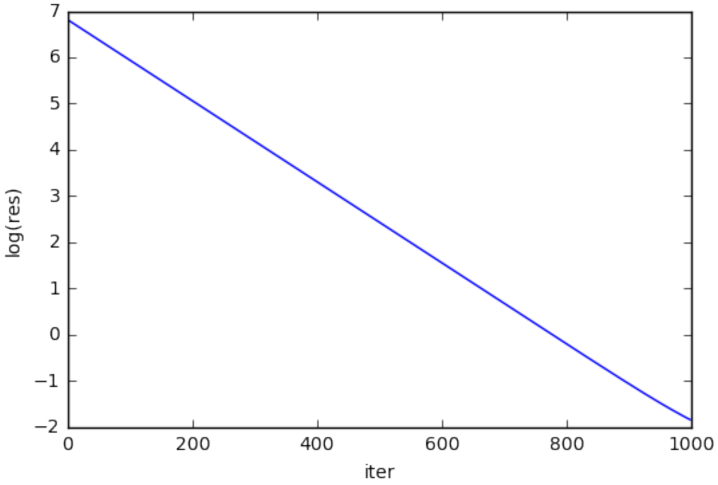

Рис.: Convergence

Computational experimet

0 — e, 1 — Ar, 2 — Fe, 3 — Ar^+, 4 — Fe^+, 5 — Fe^{++}

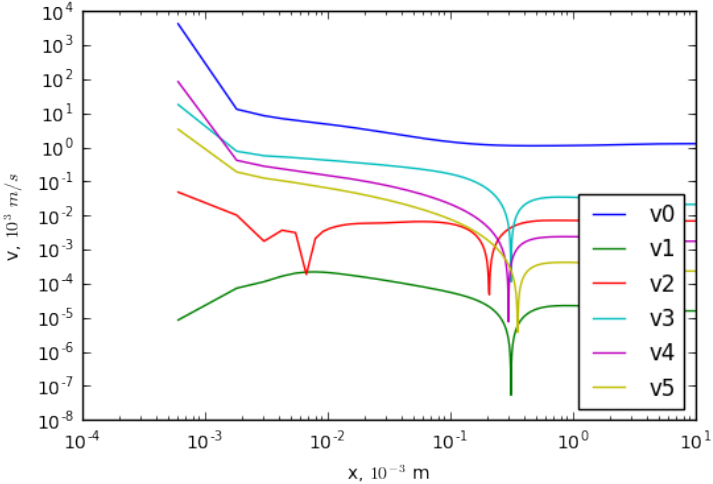

Рис.: Distribution of absolute value of velocity

Contribution

- Lenar Siraziev: testing Newton's method
- Darina Dvinskikh: Making presenation, Generation non-linear grid, obtaining analitical jacobian of vector-function,
- Nikolay Vaulin: multigrid and Thomas' block algorithm
- Alexey Ignatov: problem statement, proposal, matrices generation, approximation differential equations

Conclusion

- Created methods for processing tridioganal sparse block matrices
- Compared two methods: Multigrid has better converges compared to Newton's method
- Obtained the stationary solution which is correspond to physical concept of PSI