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MIMO Wireless

Practice Set 3

Problem 1 Lets the transmit symbol vector $\tilde{\mathbf{X}} = [1+j, -1+j]$ (which is not normalized). Find received symbol vector $\mathbf{Y}$ (ignore the the noise) and $\mathbf{G}_{ZF}$, where

$$\mathbf{H} = \begin{bmatrix} -0.8256 + 0.8831i & 0.8025 + 0.4420i \\ -0.2678 + 0.3977i & 1.2196 - 0.0895i \end{bmatrix}$$

Verify your results.

Problem 2 *Diversity Gain*
We are given a $(1 \times M)$ SIMO channel with each diversity branch modeled as

$$y_i = \sqrt{E_s} h_i s + n_i, \;\; i = 1, \cdots, M$$

where n_i is additive ZMCSCG noise with variance N_0.

a) With Maximum Ratio Combining (MRC), find the received SNR (η).

b) Assuming Maximum Likelihood (ML) detection at the receiver, show that we can calculate the probability of symbol error P_e as

$$P_e \leq \overline{N}_e Q\left(\sqrt{\frac{\rho d_{min}^2 \|h\|^2}{2}}\right)$$

with $\overline{N}_e$ being the average number of nearest neighbors and d_{min} being the minimum distance between points in the underlying scalar constellation.

Problem 3 *Dominant eigenmode transmission.*
For an $M_T \times M_R$ MIMO system with CSIT, consider a transmission scheme that uses all available transmit power in the strongest mode.

a) Show that the expected array gain is given by

$$AG = \mathcal{E}\{\lambda_{max}\}.$$

b) Using the result from (a) and the Chernoff bound, derive lower and upper bounds for the average probability of error $\overline{P}_e$ in the high SNR regime. Assume an independently Rayleigh-fading channel matrix. The following may be useful:

$$\|\mathbf{H}\|_F^2 = \sum_{i=1}^{r} \lambda_i$$

$$\frac{\sum_{i=1}^{r} \lambda_i}{r} \leq \lambda_{max} \leq \sum_{i=1}^{r} \lambda_i$$

Here, r is the rank of $\mathbf{H}\mathbf{H}^H$.

c) What can we conclude about the diversity performance of dominant mode transmission?

Problem 4 *Comparison of Diversity Schemes (MIMO)*
We need to send reliable data to a receiver through a Rayleigh fading channel and want to compare three different diversity schemes. Sweep the SNR from 0 to 15 dB and assume independently Rayleigh fading channels. We are using BPSK modulation, i.e., $d_{\min} = 2$ and $\overline{N}_e = 1$.

a) Assume a SISO link where the received signal y is given as $y = \sqrt{E_s}hs + n$ with noise power $\mathcal{E}\{|n|^2\} = N_0$. Plot the average probability of error $\overline{P}_e$ over $\rho = E_s/N_0$. (Recall that $Q(x) = 1/2 \cdot \operatorname{erfc}(x/\sqrt{2})$.)

b) On the same graph, plot the average probability of error $\overline{P}_e$ over ρ assuming a 2×2 MIMO system with no channel knowledge and Alamouti coding.

c) Finally, plot $\overline{P}_e$ over ρ assuming a 2×2 MIMO system with dominant eigenmode transmission.

Problem 5 Consider a $M \times M$ MIMO channel with ZMCSCG elements of unit variance.

a) For $M = 2$, plot ergodic channel capacity and 10% outage capacity over SNR with and without channel state information at the transmitter. How does channel state information affect the capacities at low and high SNR, respectively?

b) For SNR $= 10\,\mathrm{dB}$, plot ergodic capacity over the number of antennas M, for $M \in \{2, 4, 6, 8\}$. How does the value of channel state information change with M?