August 8, 2016

MIMO Wireless
Practice Set 1

Problem 1 A transmitter with height of 6m has the transmit power of 42 dBm.

a) A receiver is placed 2km away from the transmitter with the height of 2m from the ground. what is the received power in dBm?

b) What is the range of the transmitter if receiver sensitivity is -90dBm?

c) Analyze the above mentioned system with different distances ($100m \leq d \leq 10km$).

Problem 2 This problem demonstrates the duality of uncorrelatedness and WSS (wide sense stationarity).

a) Given that we have a WSS channel i.e.

$$E_t\left[p(\tau,t)p^*(\tau,t+\Delta t)\right] = R_t(\tau,\Delta t)$$

and we define the delay-Doppler frequency channel description as

$$U(\tau,\nu) = \int_{-T_u/2}^{T_u/2} p(\tau,t)e^{-j2\pi\nu t}dt$$

Show that as $T_u \to \infty$

$$E_\nu\left[U(\tau,\nu_1)U^*(\tau,\nu_2)\right] = 0, \text{ for } \nu_1 \neq \nu_2$$

i.e. the channel at different Doppler frequencies is uncorrelated.

b) Given that the scatterers contributing to the delay spread in a channel have independent fading i.e.

$$E_\tau\left[p(\tau_1,t)p^*(\tau_2,t)\right] = 0, \text{ for } \tau_1 \neq \tau_2$$

show that

$$E\left[P(f,t)P^*(f+\Delta f,t)\right] = R_f(\Delta f,t)$$

Problem 3 Generate a SISO rayleigh channel in MATLAB for 10^5 samples and plot its histogram.

Problem 4 Generate a SISO Rician channel in MATLAB for 10^5 samples and compare its histogram with Rayleigh with different K values ($K = \{-20,-10,0,10,20\}$).

Problem 5 Generate $2X2$ MIMO channel (rayleigh) in MATLAB for 1000 time slots.

Problem 6 Generate $2X2$ correlated MIMO channel in MATLAB for the following correlation matrix for 1000 time slots.

$$\mathbf{R_t} = \begin{bmatrix} 1 & 0.76e^{0.17j*pi} \\ 0.76e^{0.17j*pi} & 1 \end{bmatrix}$$

$$\mathbf{R_r} = \begin{bmatrix} 1 & 0.6e^{0.23j*pi} \\ 0.6e^{0.23j*pi} & 1 \end{bmatrix}$$

Problem 7 Generate $2X2$ MIMO channel (Rician) in MATLAB for 1000 time slots (ignore the phase components).

Problem 8 We are given the 2-tap delay spread MIMO channel, $\mathbf{H}_1 g(\tau) + \mathbf{H}_2 g(\tau - \tau_1)$ where $g(\tau)$ is the pulse shaping function.

a) Write an analytical expression for the resulting channel sampled at symbol spacing T_s for sample times $t \in \{0, T_s, \ldots, NT_s\}$.

b) Generate the sampled MIMO channel for the 2x2 MIMO system with the following parameters. Do this for $N = 4$ with symbol spacing T_s then fractional spacing $T_s/2$. Do this for the two cases $\tau_1 = T_s$ and $\tau_1 = T_s/4$.

$$H_1 = \begin{bmatrix} 0.9 & 0.75 \\ 0.6 & 0.8 \end{bmatrix}, H_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}$$

For the pulse shaping filter use a raised cosine with 30% excess bandwidth ($\beta = 0.3$) and T_s symbol spacing:

$$g(t) = \frac{\operatorname{sinc}(t/T_s)\cos(\pi\beta t/T_s)}{1 - 4\beta^2 t^2/T_s^2}$$