MIMO Wireless
Practice Set 4

Problem 1 Consider a 2×1 MISO channel. For each case, state the capacity-optimal transmit covariance matrix $\mathbf{\Sigma}$. Assume a transmit power constraint of P.

a) The transmitter has no channel state information.

b) The transmitter knows the magnitude channel gains $|h_1|$ and $|h_2|$, but does not know the channel phase information.

c) The channel $[h_1, h_2]$ is known exactly to the transmitter.

Problem 2 Consider a 2×2 MIMO transmission channel without channel state information at the transmitter.

a) Assuming Rayleigh fading with Kronecker channel correlation, plot ergodic capacity over SNR for

$$\mathbf{R}_t = \mathbf{I} \qquad\qquad \mathbf{R}_r = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix},$$

and $\rho \in \{0, 0.2, 0.8\}$.

b) Assuming uncorrelated Ricean fading, plot ergodic capacity over SNR for K-factors of 0, 1, and 10. Assume that the fixed part of the channel is given by

$$\overline{\mathbf{H}} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Problem 3 *Alamouti Code and Channel Estimation Errors*
The Alamouti code uses an estimate of the channel to perform ML decoding. In this question, we will study the effect of channel estimation errors. Assume we have a 2×1 MISO system with Alamouti space-time coding and we model the channel as

$$\mathbf{h} = \begin{bmatrix} h_1 + \epsilon_1, & h_2 + \epsilon_2 \end{bmatrix}$$

where ϵ_1 and ϵ_2 are ZMCSCG variables with variance σ_ϵ. After Alamouti receiver processing (using h_1 and h_2), the received signal can be modeled as

$$\widehat{\mathbf{y}} = \tilde{\mathbf{y}}_{signal} + \tilde{\mathbf{y}}_{mismatch} + \tilde{\mathbf{n}}$$

a) Derive expressions for the three terms $\tilde{\mathbf{y}}_{signal}$, $\tilde{\mathbf{y}}_{mismatch}$, and $\tilde{\mathbf{n}}$.

b) How can we characterize the main diagonal and off-diagonal terms in the expression for $\tilde{\mathbf{y}}_{mismatch}$?

c) Based on your results from (a) derive an expression for the SNR assuming input energy E_s and noise variance N_0.

d) Using the following parameters, plot the BER for Alamouti Coding.

- SNR = 15 dB
- Modulation = BPSK
- Decoder - Binary ML detector (Simple slicing)
- Channel - 1×2 ZMCSCG MISO channel
- Estimation error, ZMCSCG variable with variance from 0 to 1 in steps of 0.1

What conclusions can be drawn from from your plots?

Problem 4 *Spatial Multiplexing*
In this problem, we consider the per stream mutual information for horizontal and diagonal encoding. Consider a MIMO channel with $M_t = M_r = 2$ with perfect CSI at the receiver and no channel state information at the transmitter. The channel input-output equation is given as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

where $\mathbf{x} \in C^{M_t}$ is the input vector, $\mathbf{H} \in C^{M_r \times M_t}$ is the channel, $\mathbf{n} \in C^{M_r}$ is the thermal noise, and $\mathbf{y} \in C^{M_r}$ is the output vector. The transmitter has a power constraint of E_s and the thermal noise at the receiver is distributed as $\mathcal{CN}(0, \sigma^2 \mathbf{I}_{M_r})$. Assume a linear zero-forcing receiver. Next, let us suppose that the channel (unknown to the transmitter) is randomly selected to be

$$\mathbf{H} = \begin{bmatrix} 1 & 0.5 + i \\ -0.25 & -i \end{bmatrix}.$$

a) First, suppose that horizontal encoding with successive cancellation is used on this channel. First we decode the data stream transmitted from the first antenna while treating the stream from the second antenna as noise. After successful detection, subtract the first stream and decode the second stream with only thermal noise. What is the mutual information of each of the two streams when decoded in this way?

b) Now, suppose that diagonal encoding is used with two streams (illustrated below). Each stream will spend an equal amount of time being transmitted from each antenna. The first stream is decoded treating the second stream as noise and the second stream is decoded with only thermal noise. What are the mutual informations of the two streams in this case?

$$\begin{bmatrix} S_{11} & S_{21} & 0 \\ 0 & S_{12} & S_{22} \end{bmatrix}$$

Here, S_{11} is the first half of the output of the first stream, S_{12} is the second half of the output of the first stream. S_{21} is the first half of the output of the second stream, S_{22} is the second half of the output of the second stream.

Problem 5 *(Channel correlations)*   Consider a 2×2 MIMO channel and recall that $\mathbf{R} = \mathcal{E}(\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^T)$. While the matrix $\mathbf{R}$ is the most general way to capture channel correlation, it is often difficult to work with. Instead, the Kronecker model $\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{H}_w\mathbf{R}_T^{1/2}$ is often used, where the entries of $\mathbf{H}_w$ are iid ZMCSCG. In this problem, we consider the underlying assumptions of the Kronecker model. Let us assume

$$\mathbf{R}_T = \begin{bmatrix} 1 & \alpha \\ \alpha^* & 1 \end{bmatrix} \text{ and } \mathbf{R}_R = \begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix}$$

a) Compute $\mathbf{R} = \mathbf{R}_T^T \otimes \mathbf{R}_R$.

b) How does the correlation between two transmit antennas depend on the choice of receive antenna? How does the correlation between two receive antennas depend on the choice of transmit antenna? What can you say about the physical properties of the random channel H for this identity to hold?