Practice Set 1 Solutions

Problem 1

a) Assume $G_t = G_r = 1$

$$\begin{aligned} P_t(watts) &= 10^{(42-30)/10} = 15.8489 \\ P_r &= P_t\left(\frac{h_t h_r}{d^2}\right) = 1.4264e^{-10} \\ P_r(dBm) &= 10log_{10}(P_r) + 30 = -68.4576dBm \end{aligned}$$

b) Set $P_r(dBm) = -90dBm$ and find d which is $6.9km$

c)

```
clc
close all

PdBm=42;
P_W=10^((PdBm-30)/10)
ht=6;
hr=2;
d=[100:500:10e3];

Pr=P_W*(ht*hr./d.^2).^2;

Pr_dBm=10*log10(Pr)+30;

figure
plot(d,Pr_dBm,'-sb')
xlabel('Distance(m)')
ylabel('Received power(dBm)')
```

See Figure 1 for output.

Problem 2

a)

$$\begin{aligned} E\left[U(\tau,\nu_1)U^*(\tau,\nu_2)\right] &= E\left[\int_{-T_u/2}^{T_u/2}\int_{-T_u/2}^{T_u/2} p(\tau,t)p^*(\tau,t')e^{-j2\pi(\nu_1 t-\nu_2 t')}dtdt'\right] \\ &= \int_{-T_u/2}^{T_u/2}\int_{-T_u/2}^{T_u/2} E\left[p(\tau,t)p^*(\tau,t')\right]e^{-j2\pi(\nu_1 t-\nu_2 t')}dtdt' \end{aligned}$$

by the stationarity assumption we have:

$$E\left[U(\tau,\nu_1)U^*(\tau,\nu_2)\right] = \int_{-T_u/2}^{T_u/2}\int_{-T_u/2}^{T_u/2} R_t(\tau,t'-t)e^{-j2\pi(\nu_1 t-\nu_2 t')}dtdt'$$

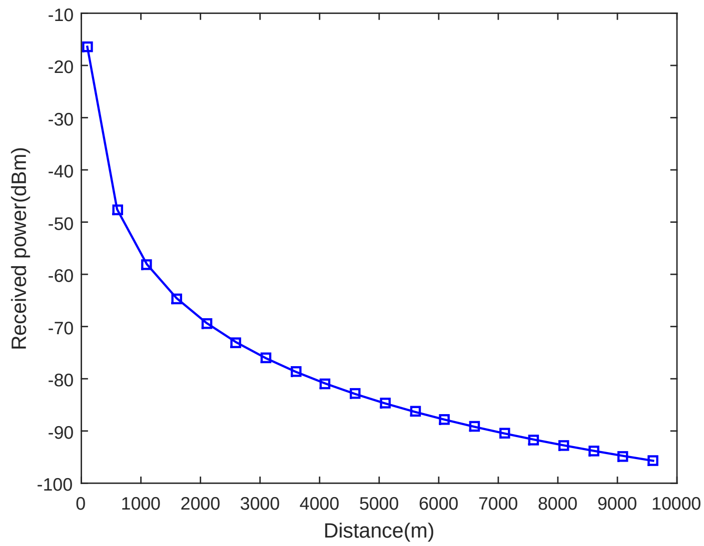

Figure 1: Distance Vs Received power (dBm)

Let $\Delta t = t' - t$ and $T_u \to \infty$:

$$\begin{aligned} E\left[U(\tau,\nu_1)U^*(\tau,\nu_2)\right] &= \int_{\mathbb{R}}\int_{\mathbb{R}} R_t(\tau,\Delta t)e^{-j2\pi(\nu_1 t-\nu_2\Delta t-\nu_2 t)}dtd\Delta t \\ &= \mathcal{F}^{-1}_{\Delta t}\left\{R_t(\tau,\Delta t)\right\}(\nu_2)\int_{\mathbb{R}} e^{-j2\pi(\nu_1-\nu_2)t}dt \end{aligned}$$

where $\mathcal{F}^{-1}_k\{f(k)\}(x) = \int_{-\infty}^{\infty} f(k)e^{2\pi ikx}dk$ is the Inverse Fourier Transform operator with respect to the appropriate variable. The last integral is just the Fourier Transform of a complex exponential evaluated at zero frequency: $\delta(f+\nu_1-\nu_2)|_{f=0}$. Thus,

$$E\left[U(\tau,\nu_1)U^*(\tau,\nu_2)\right] = \mathcal{F}^{-1}_{\Delta t}\left\{R_t(\tau,\Delta t)\right\}(\nu_2)\delta(\nu_1-\nu_2)$$

And we are done!

b)

$$\begin{aligned} E\left[P(f,t)P^*(f+\Delta f,t)\right] &= \int_{\mathbb{R}}\int_{\mathbb{R}} E\left[p(\tau_1,t)p^*(\tau_2,t)\right]e^{j2\pi[f\tau_1-(f+\Delta f)\tau_2]}d\tau_1 d\tau_2 \\ &= \int_{\mathbb{R}} E\left[|p(\tau_2,t)|^2\right]e^{-j2\pi\Delta f\tau_2}d\tau_2 \\ &= R_f(\Delta f,t) \end{aligned}$$

The last equality follows since the integral above it is only a function of Δf.

Problem 3

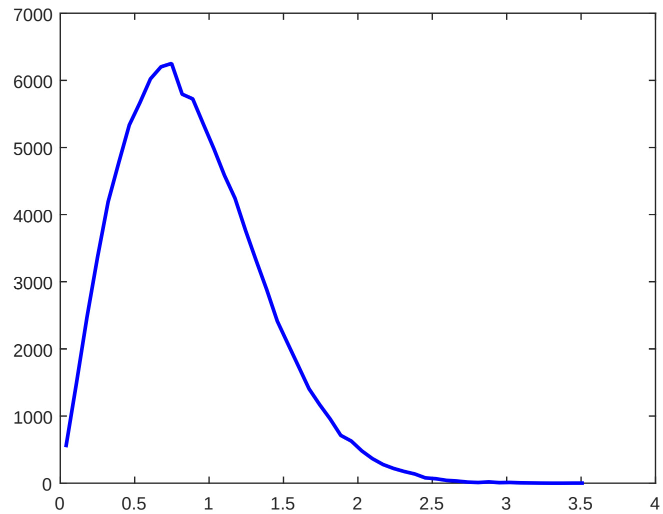

Figure 2: Raleigh pdf

```
clc
close all
L=1e5;
Hw = (randn(1,L)+j*randn(1,L))/sqrt(2);
figure;
% hist(abs(H),50);
[temp,x]=hist(abs(Hw),50);
plot(x,temp,['b-'],'LineWidth',2), hold on
```

See Figure 2 for output.

Problem 4

```
K_dB=[-20,-10, 0, 10, 20];
K=10.^(K_dB./10);
mr=['-k' '-m' '-c' '-r' '-g'];
for i=1:length(K)
    H = sqrt(K(i)/(K(i)+1)) + sqrt(1/(K(i)+1)).*Hw;
    [temp,x]=hist(abs(H),50);
    plot(x,temp,mr(i),'LineWidth',1.2); hold on
end
```

See Figure 3 for output.

Problem 5

```
nT=2;
```

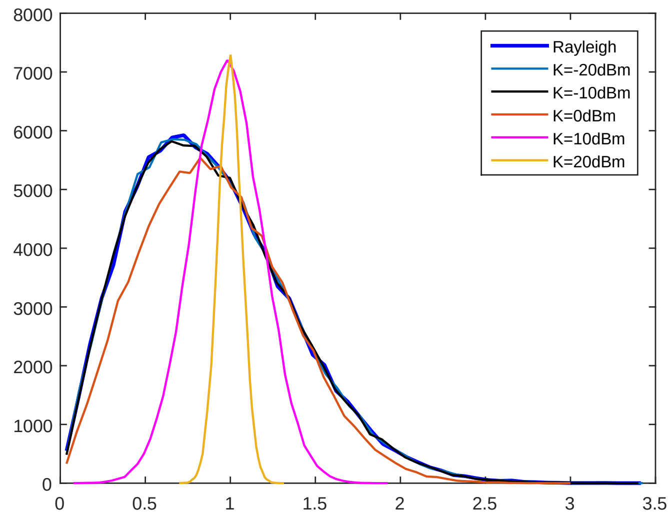

Figure 3: Rician and rayleigh pdfs

```
nR=2;
N=1000;
H_iid = sqrt(2)*(randn(nR,nT,N)+j*randn(nR,nT,N))
```

Problem 6

```
clc
close all
clear all
Rt=[1                       0.76*exp(0.17j*pi)
   0.76*exp(-0.17j*pi)   1                    ];
Rr=[1                       0.6*exp(0.23j*pi)
   0.6*exp(-0.23j*pi)   1                     ];

nT=2;
nR=2;
N=1000;
Hw=sqrt(1/2)*(randn(nT*nR,N)+j*randn(nT*nR,N));
% R = chol(kron(Rt,Rr))';
R = sqrtm(sqrt(kron(Rt,Rr)));

H_crr=zeros(nR,nT,N);
for i=1:N
    tmp=R*Hw(:,i);
    H_crr(:,:,i)=reshape(tmp,nR,nT);
end
H_crr
```

Problem 7

```
K_dB=20;
K=10^(K_dB/10);
H_bar=ones(nR,nT,N);
% H_bar(1,2,:)=-1
H_ric = sqrt(K/(K+1)).*H_bar + sqrt(1/(K+1)).*H_w;
```

Problem 8

a)

$$\begin{aligned}\mathbf{H}[k] &= \mathbf{H}_1 g(kT_s) + \mathbf{H}_2 g(kT_s - \tau_1) \\ &= \frac{\operatorname{sinc}(k)\cos(\pi\beta k)}{1-4\beta^2k^2}\mathbf{H}_1 + \frac{\operatorname{sinc}(k-\tau_1/T_s)\cos(\pi\beta(k-\tau_1/T_s))}{1-4\beta^2(k-\tau_1/T_s)^2}\mathbf{H}_2\end{aligned}$$

b) The following script produces the desired results:

```
clear all
H1 = [0.9 0.75;0.6 0.8];
H2 = [0.4 0.2;0.3 0.1];

%delay spread, in multiples of the symbol spacing
tau = [1 .25];

%sample spacing (2 cases)
ss = [1 .5];

beta = 0.3;

for t = 1:length(tau),
    for s = 1:length(ss),
        %Generate the sample times
        ks = [0:ss(s):4];

        for l = 1:length(ks),
            k = ks(l);
            x = sinc(k)*cos(pi*beta*k)/(1-4*beta^2*k^2)*H1+...
```

```
                sinc(k-tau(t))*cos(pi*beta*(k-tau(t)))...
                /(1-4*beta^2*(k-tau(t))^2)*H2;
            H(:,:,l) = x;
        end
        s = sprintf(' Delay = %d Ts and sampling rate = %d Ts: ',...
            tau(t),ss(s));
        disp(s)
        H
    end
end
```