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MIMO Wireless

Practice Set 2

Problem 1 For the given channel vector **H** calculate capacity for SIMO and MISO with $E_S/N_0 = 20dB$.

$$\mathbf{H} = \begin{bmatrix} 0.5464 + 0.2294i \\ -1.2666 + 0.2736i \end{bmatrix}$$

Problem 2 For the given channel Matrix **H** calculate capacity for MIMO 2X2 Channel with $E_S/N_0 = 25dB$.

$$\mathbf{H} = \begin{bmatrix} -0.8256 + 0.8831i & 0.8025 + 0.4420i \\ -0.2678 + 0.3977i & 1.2196 - 0.0895i \end{bmatrix}$$

Problem 3 Compare the CDF for 2X2 and 4X4 MIMO channel.

Problem 4 Plot the SNR Vs Capacity curve and compare with the following links.

a) 1X2 SIMO

b) 2X1 MISO

c) 2X2 MIMO

Problem 5 Compare the correlated and iid channel impact in capacity.

Problem 6 Analyze the impact of LOS in capacity.

Problem 7 Check the orthogonality of the following matrix.

$$\mathbf{X} = \begin{bmatrix} 1+j & 1+j \\ -1+j & 1-j \end{bmatrix}$$

Problem 8 Lets the Transmit symbol vector $\tilde{\mathbf{X}} = [1+j, -1+j]$. For 2X1 MISO system with channels of $h_1 = 0.2362 - 1.0244j$ and $h_2 = -0.3048 + 0.2587j$, perform transmit and receive operations using the Alamouti scheme.

Problem 9 *(MISO array gain)* Consider a MISO channel $y = \mathbf{H}\mathbf{x} + n$ with two transmit antennas, channel matrix $\mathbf{H} = [h_1, h_2]$, and noise power $\mathcal{E}(|n|^2) = N_0$. The total available transmit power over all antennas is P ($\mathcal{E}[\mathbf{x}^H\mathbf{x}] \leq P$). Suppose we want to transmit the zero-mean, unit variance random signal s.

a) Assume $h_1 = h_2 = 1$. For simplicity, we decide to transmit s directly from both antennas using $\mathbf{x} = a[s, s]^T$, with appropriate scaling factor $a > 0$. Find a that maximizes receiver signal to noise ratio SNR_{MISO}. Compare this to the achievable signal to noise ratio SNR_{SISO} in a SISO system with channel gain $H = 1$ and the same power constraint. Compute the *transmit array gain*, i.e., the SNR difference in dB between MISO and SISO.

b) What is the array gain if $h_1 = h_2 = -1$ and $\mathbf{x} = a[s, s]^T$?

c) Now let $h_1 = 1$ and $h_2 = -1$. Find the array gain with $\mathbf{x} = a[s, s]^T$. Find (a_1, a_2) such that $\mathbf{x} = s[a_1, a_2]^T$ maximizes array gain. What does this imply about the role of CSIT for transmit array gain?

Problem 10 *(Singular Values of* $\mathbf{H}$*)*

a) Generate 10,000 ZMCSCG channels of dimensions 4×4 and plot the empirical probability density function (pdf) of the minimum eigenvalue of $\mathbf{H}\mathbf{H}^H$.

b) In the lecture notes, an analytical expression is given for this pdf. Plot this analytical expression in the same diagram to verify that the distribution of part (a) matches the theory.

c) What significance does this have for the capacity of a MIMO system?