Thank you for your interest for this project. I hope we can learn knowledge and provide help to each other about optimization methods/ python language/ finance/actuarial science.

1. The goal of this project:
   1. Be able to perform reserve and related calculation (do what the current actuarial software can do)
   2. Be able to calculate risks regarding assets/ liabilities
   3. Be able to perform simulations for different scenarios, be able to perform automatic management actions for those simulations.
   4. Be able to manage assets and calculate all those related indicators(if we can)

Basic concepts about finance:

1. Time value of money

Let us suppose that the current interest rate is 5%. This means that if you save 1 dollar at the bank today (time 0), you will get 1.05 dollar at time 1. So, we may think that 1.05 dollar at time 1 has approximately the same value as 1 dollar today. We call this concept “present value” (PV) (the value of any future money for the current date). i.e.

PV(1.05 dollar at time 1)=1.05\*(1/1.05)=1

The first 1.05 is the money at time1. The 1/1.05 is called the discount factor, s.t. the final results show the equivalent value of the cash flow (1.5 dollar at time 1) if we are at time 0 (the amount we are willing to lend at time 0 if we are able to get back 1.5 dollar at time 1).

Therefore, what is the value of getting 1 dollar at time 2, if we are at time 0? The answer is in the following.

PV(1 dollar at time 2)= 1\* (1/1.05)^2

Its value at time 1 is 1/1.05. Its value at time 0 is obtained by multiplying a discount factor of one year (1/1.05) to its value at time 1. Therefore, it is 1/1.05^2.

Simple Bond calculation:

Let us say that there is a bond which gives you 5 dollar every year, for 3 year. After that, it will give you back 100 dollar at the end of 3rd year. What is the amount of money you are willing to pay today for such bond?

Ans:

PV(bond)=5/1.05+5/1.05^2+5/1.05^3+100/1.05^3

Explanation:

5 dollars 1st year, 5 dollars 2nd year, 105 dollars 3rd year.

Values of different components can be added together.

Some notes about interest rate.

Interest rates may be changed by time. For example, the interest rate from time 0 to time 1(denoted as f(0,1)) is 5%, 2nd year f(1,2) is 6%, 3rd year f(2,3) is 7%. Then the value of the bond in the previous example will be:

PV(bond)=5/1.05+5/1.05/1.06+5/1.05/1.06/1.07+100/1.05/1.06/1.07

The PV is actually a recursive formula:

PV(time t)=PV(time t+1)/(1+f(t,t+1))+cashflow(t)

PV(time t) means the money you are willing to pay at time t if you can get all the stated money at time t or after.

Example,

(105 dollar paid at time 3)

(5 dollar paid at time 2, plus discounted value for PV at time 3

(similar ideas)

Note that no money is paid at time 0. The PV for time 0 has already been shown in the previous section.

1. Basic structure of the model:
   1. general model.

Nearly all the variables in financial world are related to time, and there are many of them. Therefore, it is a common practice to build a “general model”(I am sure there is a specific name for this in the programming community). Financial/ actuarial people will write scripts about their models. For example (using the above bond example)

def Int\_Rate(t):

blablabla

return f(t,t+1)

def Cash\_Flow(t):

blablabla

return the\_amount\_of\_money\_will\_be\_received\_at\_time\_t

def PV(t):

return PV(t+1)/(1+Int\_Rate(t))+Cash\_Flow(t)

Each of the above definitions are called a “variable” by me. Some people also called them “cells”. Because there are so many variables, it is a good idea to build a “general model” s.t. we can manage/ add/delete/modify each variable. The “general model” should be able to do the following:

* + 1. Read scripts (eg. the definitions of functions Int\_Rate(t), Cash\_Flow(t) and PV(t) shown above) and identify all the variables (Int\_Rate, Cash\_Flow, PV).
    2. Be able to do calculations based on these variables (e.g. we need to calculate and records the results for PV(t) for all 0<=t<=100, t is an integer) .(this is the part where numba and all those optimizations are needed, I will explain more about this in the following sections)
    3. Separate variables into groups. Be able to perform some special actions to each group.
    4. Show the dependent structure between variables (this is the part where c3 or related MRO algorithm will be used, also we need to study how to visualize these directed graphs). For example, PV is dependent on Int\_Rate and Cash\_Flow.
    5. Be able to show the calculation results of these variables in an elegant way, s.t. people can trace the errors easily (this is the part where clever ideas about visualization are needed).
    6. be able to store all the results/components of this model to a database/file that can save spaces.
    7. excel API that can obtain the results easily from the database.
    8. multi-thread or process
    9. be able to stop or restart in a nice way (I expect we can do this)

1. Basic actuarial/insurance knowledge:
   1. policy

When a person buys an insurance contract, this contract is called a “policy”. Each policy has a key called “policy number (policy ID)”. Sometimes, each customer also has a key called “customer ID(or number)”. These customers are called “policyholders”

* 1. Common cash flows related to policies.

As shown in the contract, the customer will pay “Premium”(say $1000) to the insurance company at the beginning of each period (to make it simple, let us say he will pay premiums at the beginning of each year.) He will continue to pay premiums for the period of “Premium term”(say 5 years). After the premium term, he will no longer pay any premiums to the insurance company.

During the “policy term”(say 10 years), insurance companies will have an obligation for the customer. If the customer is dead during these 10 years (if he has paid premiums normally), the insurance companies will need to pay “Death Benefits”(say $1000000) to the customer’s wife/husband or any related people.

Let me show you a simple example. Say the “premium term” is 1 year, “policy term” is also 1 year, then the insurance company (insurer)

* + 1. At time 0, the insurer gets premium ($1000) from the insured (the customer).
    2. At time 0, the insurer pays expenses and commissions ($100) to the agents and staffs. (so now $900 left)
    3. The probability of death for the customer in that year is 0.05%. The expected benefit paid at time 1 will be 0.05%\*death benefit=0.05%\*$1000000=$500
    4. At time 1, the insurer will expect to gain interest equals to $900\*interest rate=900\*0.05=45
    5. The insurer will expect to gain “Gross profit” at time 1=money left+ interest gain –expected death benefit paid=900+45-500
  1. concepts of reserve

Say the customer’s premium term is only 1 year, but the policy term is 100years. At time 0, the customer pays all the premium. The insurer of course cannot recognize this premium as profit. The insurer has an obligation to pay death benefits for the next 100 years, if the customer is dead at any time. Therefore, the insurance company need to hold a “reserve” as a liability. This “Reserve” is (conceptually) the amount of money that the insurer owes its customers. For example, at time 0, point 3.2.3 may be the “Reserve” that an insurer needs to hold, which is the amount of money it owes the customer. In the reality, there are many ways to calculate this reserve, under various principles and regulation philosophies. The “reserve” defined in laws has to be more conservative, so as to protect the policyholders(customers)

* 1. actuarial present value (APV)

it is more easy to explain it using the above example (3.2.3).

What money are you willing to pay at time 0 s.t. you will get $1000000 if you die in the next year?

Ans: APV for death benefit at time 0

=Death benefit \* expected death rate\* discount rate

=$1000000\*0.05%\*(1/1.05)

we can divide the above question into 2 parts:

* + 1. What money are you willing to pay at time 1 (if you are a very very rational man that is risk-neutral), s.t. you will get $1000000 if you die in the next year?

Ans: $1000000\*0.05%

* + 1. What money are you willing to pay at time 0, if you will get $1000000\*0.05% at time 1?

Ans: $1000000\*0.05%\*(1/1.05)

What if your premium term is 2 years, and policy term is also 2 years?

The APV of the premium you paid at time 0

=APV(premium paid at time 0) + APV(premium paid at time 1)

=premium paid at time 0 + premium paid at time 1 \* (1/(1+interest rate))\*survival rate of the first year

survival rate of the first year=1-death rate=0.95

The reason to add this survival rate, is because if you die the first year, you dont need to pay premium the second year, and already can get your death benefit.

The APV of the death benefit you paid at time 1

=APV(benefit paid if dying first year) + APV(benefit paid if dying second year)

=death benefit\*death rate of first year\* (1/(1+interest rate))+ death benefit\*survival rate of first year\*death rate of second year\* (1/(1+interest rate))^2

Notes: if he dies first year, the death benefits will be paid at time 1. if he dies second year, he must first survive the first year, that is the reason we will multiply “survival rate of first year” in the second component.

1. More notes about the models
   1. You may expect that there are several hundred variables, more than 10000 policyholders(customers). For each variable, we may project the results in a monthly basis or annual basis. If annual basis is used, it means that you need to calculate and record the results of selected variables from t=0 to t=100 (let’s say each person live for 100 years after they buy the insurance product). Also some new regulations ask insurance companies to do stochastic simulations (>1000 scenarios)

Therefore, we must be able to call efficiently/ record efficiently the following number of functions:

around 500 variables\*10000 policyholders\*100 (for t)\*1000 scenarios

* 1. All the above things I have mentioned are all related to liability side of an insurance company. If we are able to achieve this (which is quite hard), we may switch to asset side of the company, and do CAPM/APT and all those kinds of stuff. But that is not the main goal of this project.

1. miscellaneous topics

I think the best usage of our model is:

* + 1. for academic use
    2. for educational use
    3. for insurance startups who don’t have too much resources for a model that is too large and complicated
    4. as a prototype for more mature models
    5. for insurance/ financial industry in the developing world.