Objetivo: Equações $\varphi(x)$ e $y(x)$
Calcular φ_A e y_A

1) Sistema de referência

2) Escrever a eq. $M(x)$:

$$M(x) = -P \cdot x$$

3) Subst. $M(x)$ na EDLE:

$$\frac{d^2 y}{dx^2} = \frac{d\varphi}{dx} = -\frac{M(x)}{EI} \longrightarrow \frac{d^2 y}{dx^2} = \frac{d\varphi}{dx} = +\frac{P}{EI} \cdot x$$

4) 1ª integração $\longrightarrow \varphi(x)$:

$$\frac{d\varphi}{dx} = +\frac{P}{EI} \cdot x \longrightarrow \int d\varphi = \int +\frac{P}{EI} x \, dx + C_1$$

$$\varphi(x) = +\frac{P}{EI} \cdot \frac{x^2}{2} + C_1 \qquad \boxed{1}$$

5) 2ª integração $\longrightarrow y(x)$:

$$\frac{dy}{dx} = \varphi \longrightarrow dy = \varphi \, dx$$

$$y(x) = \int dy = \int \left\{ +\frac{P}{EI} \cdot \frac{x^2}{2} + C_1 \right\} dx + C_2$$

$$y(x) = +\frac{P}{EI} \cdot \frac{x^3}{6} + C_1 \cdot x + C_2 \qquad \boxed{2}$$

6) Calcular C_1 e C_2 aplicando as condições de contorno do problema

Engastamento

Em $x = L = x_B$

$$Em \quad x = L = x_B$$

$$\begin{cases} \varphi_B = 0 \quad CC1 \\ y_B = 0 \quad CC2 \end{cases}$$

↳ Subst. CC1 na eq(1):

$$\varphi(x=L) = \varphi_B = 0 = + \frac{P}{EI}\frac{(L)^2}{2} + C_1$$

$$\therefore \quad C_1 = -\frac{1}{2}\frac{PL^2}{EI}$$

↳ Subst. CC2 na eq(2):

$$y(x=L) = y_B = 0 = + \frac{P}{EI}\frac{(L)^3}{6} - \frac{1}{2}\frac{PL^2}{EI}\cdot(L) + C_2$$

$$\therefore \quad C_2 = +\frac{1}{3}\frac{PL^3}{EI}$$

7) Escrever as eq. finais:

$$\varphi(x) = \frac{P}{EI}\cdot\left(\frac{x^2}{2} - \frac{L^2}{2}\right) \qquad [rad]$$

$$y(x) = \frac{P}{EI}\cdot\left(\frac{x^3}{6} - \frac{L^2}{2}\cdot x + \frac{L^3}{3}\right) \qquad \begin{array}{c}[mm]\\ [cm]\\ [m]\end{array}$$

8) Calcular φ_A e y_A

$$x_A = 0 \rightarrow Ponto \ A$$

Portanto:

$$\varphi(x=0) = \varphi_A = -\frac{1}{2}\frac{PL^2}{EI} \qquad [rad]$$

$$\varphi(x=0) = \varphi_A = -\frac{1}{2}\frac{PL}{EI} \qquad [\text{rad}]$$

$$y(x=0) = y_A = +\frac{1}{3}\frac{PL^3}{EI}$$