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1-)

$$f(x,y) = \sqrt{x^2 - 2y}$$

a-) $\sqrt{x^2 - 2y} \geq 0$

$x^2 - 2y \geq 0$

$x^2 \geq 2y$

$$\boxed{\text{Dom } f(x,y) = \{x, y \in \mathbb{R}^2 / x^2 \geq 2y\}}$$

b-) $f(x,y) = \sqrt{x^2 - 2y}$

$f(x,y) \geq 0$

$\text{Im } f(x,y) \geq 0$

$$\boxed{\text{Im } f = \mathbb{R}^+}$$

c-) $P = \left(2, -\frac{5}{2}\right)$

$f(x,y) = \sqrt{x^2 - 2y}$

$c = \sqrt{x^2 - 2y}$

$c^2 = x^2 - 2y$

$c^2 = (2)^2 - 2\left(\frac{-5}{2}\right)$

$c^2 = 4 + 5$

$c = 3$

$$\boxed{\sqrt{x^2 - 2y} = 3}$$

d-) $y = \frac{x^2 - 9}{2}$

$y' = \frac{dy}{dx}\left(\frac{x^2 - 9}{2}\right)$

$y' = \frac{2x}{2} \Rightarrow y' = x$

$\vec{V}_{tg} = \begin{bmatrix} x & \frac{x^2 - 9}{2} \end{bmatrix}^t$

$\vec{V}_{tg}' = \begin{bmatrix} 1 & x \end{bmatrix}^t$

$$\boxed{\vec{V}_{tg}' = \begin{bmatrix} 1 & 2 \end{bmatrix}^t}$$

$y - y_0 = y'(x - x_0)$

$y + \frac{5}{2} = 2(x - 2)$

$y + \frac{5}{2} = 2x - 4$

$y = 2x - 4 - \frac{5}{2}$

$y = 2x - \frac{8}{2} - \frac{5}{2}$

$$\boxed{y = 2x - \frac{13}{2}}$$

e-) $\vec{\nabla} f(x,y) = \begin{bmatrix} F_x & F_y \end{bmatrix}^t$

$$F_x(2, -5/2) = \frac{2x}{2\sqrt{x^2 - 2y}} = \frac{2 \cdot 2}{2\sqrt{2^2 - 2\left(\frac{-5}{2}\right)}} = \frac{2}{\sqrt{4+5}}$$

$$F_x = \frac{2}{3}$$

$$F_y(2, -5/2) = \frac{1 \cdot (-\cancel{2})}{\cancel{2} \cdot \sqrt{x^2 - 2y}} = \frac{-1}{\sqrt{4+5}} = \frac{-1}{\sqrt{9}}$$

$$F_y = \frac{-1}{3} \longrightarrow \boxed{\vec{\nabla} f(x,y) = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \end{bmatrix}^t}$$

f-) Valor máximo $= \|\vec{\nabla} f(x,y)\|$

$$\|\vec{\nabla} f(x,y)\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2}$$

$$= \sqrt{\frac{4}{9} + \frac{1}{9}}$$

$$= \sqrt{5/9}$$

$$\boxed{\|\vec{\nabla} f(x,y)\| = \frac{1}{3}\sqrt{5}}$$

g-) $V_{lg} \cdot \vec{\nabla} f(x,y) = 0$

$$\begin{bmatrix} 1 & 2 \end{bmatrix}^t \cdot \begin{bmatrix} 2/3 \\ -1/3 \end{bmatrix} = 0$$

$$\frac{2}{3} - \frac{2}{3} = 0$$

$$0 = 0 \ (V)$$

Produto escalar entre os vetores igualou a 0

Portanto eles são ortogonais