Математический анализ
(25.05.20)

№ 7.3.28

$P(x) = x^4 - x^3 + 5x^2 - 4x + 1 \quad , x = 1$

$$P(x) = P(x_0) + \frac{P'(x_0)}{1!}(x - x_0) + \frac{P''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{P^{(n)}(x_0)}{n!}(x - x_0)^n + o((x-x_0)^n)$$

2) $P(1) = 1^4 - 1^3 + 5 \cdot 1^2 - 4 \cdot 1 + 1 = 2$

3) $P'(x) = (x^4 - x^3 + 5x^2 - 4x + 1)' = 4x^3 - 3x^2 + 10x - 4$

$P'(1) = 4 \cdot 1^3 - 3 \cdot 1^2 + 10 \cdot 1 - 4 = 7$

4) $P''(x) = (4x^3 - 3x^2 + 10x - 4)' = 12x^2 - 6x + 10$

$P''(1) = 12 \cdot 1^2 - 6 \cdot 1 + 10 = 16$

5) $P'''(x) = (12x^2 - 6x + 10)' = 24x - 6$

$P'''(1) = 24 \cdot 1 - 6 = 18$

6) $P^{(4)}(x) = (24x - 6)' = 24$

$P^{(4)}(1) = 24.$

7) $P^{(5)}(x) = (24)' = 0$ => не буде відмінно ненульового степеневого члена.

8) $P(x) = P(1) + \frac{P'(1)}{1!} \cdot (x-1) + \frac{P''(1)}{2!}(x-1)^2 + \frac{P'''(1)}{3!}(x-1)^3 + \frac{P^{(4)}(1)}{4!}(x-1)^4 =$

$= 2 + \frac{7}{1!}(x-1) + \frac{16}{2!}(x-1)^2 + \frac{18}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4 =$

$= 2 + 7(x-1) + 8(x-1)^2 + 3(x-1)^3 + (x-1)^4$

№ 7.5.29

$f(x) = x^3 + 4x^2 - 6x - 8$, $x_0 = -1$

1) $x_0 = -1$

2) $f(-1) = (-1)^3 + 4(-1)^2 - 6(-1) - 8 = 1$

3) $f'(x) = 3x^2 + 8x - 6$

4) $f'(-1) = 3\cdot(-1)^2 + 8(-1) - 6 = -11$

5) $f''(x) = 6x + 8$

6) $f''(-1) = 6\cdot(-1) + 8 = 2$

7) $f'''(x) = 6$

8) $f'''(-1) = 6$

9) $f^{(4)}(x) = 0 \Rightarrow$ нет остаточного члена

10) $f(x) = f(-1) + \frac{f'(-1)}{1!}(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f'''(-1)}{3!}(x+1)^3 =$

$= 1 - \frac{11}{1!}(x+1) + \frac{2}{2!}(x+1)^2 + \frac{6}{3!}(x+1)^3 = 1 - 11(x+1) + (x+1)^2 + (x+1)^3$

№ 7.5.32

$f(x) = x^5 - 3x^4 + 7x + 2$, $x_0 = 2$

1) $x_0 = 2$

2) $f(2) = (2)^5 - 3(2)^4 + 7\cdot 2 + 2 = 0$

3) $f'(x) = 5x^4 - 12x^3 + 7$

$f'(2) = 5\cdot(2)^4 - 12(2)^3 + 7 = -9$

4) $f''(x) = 20x^3 - 36x^2$

$f''(2) = 160 - 144 = 16$

5) $f'''(x) = 60x^2 - 72x$

$f'''(2) = 60\cdot 4 - 72\cdot 2 = 240 - 144 = 96$

6) $f^{(4)}(x) = 120x - 72$ $\quad f^{(4)}(2) = 240 - 72 = 168$

7) $f^{(5)}(x) = 120$ $\quad f^{(5)}(2) = 120$

8) $f^{(6)}(x) = 0 \Rightarrow$ нет остаточного числа

в) $P(x) = 0 + \frac{-8}{1!}(x-2) + \frac{16}{2!}(x-2)^2 + \frac{96}{3!}(x-2)^3 + \frac{168}{4!}(x-2)^4 + \frac{120}{5!}(x-2)^5$
$= -8(x-2) + 8(x-2)^2 + 16(x-2)^3 + 7(x-2)^4 + (x-2)^5$

17.3.3

① $f(x) = \frac{1}{x}$, $x_0 = 1$

1) $x_0 = 1$

2) $f(1) = \frac{1}{1} = 1$

3) $f'(x) = -\frac{1}{x^2}$

$f'(1) = -1$

4) $f''(x) = \frac{2}{x^3}$

$f''(1) = \frac{2}{1} = 2$

5) $f'''(x) = -\frac{6}{x^4}$

$f'''(1) = -6 = -2 \cdot 3 \cdot 1 = -1 \cdot 2 \cdot 3$

6) $f^{(4)}(x) = 2 \cdot 3 \cdot 4 \cdot \frac{1}{x^5}$

$f^{(4)}(1) = 1 \cdot 2 \cdot 3 \cdot 4$

7) $f^{(5)}(x) = (2 \cdot 3 \cdot 4 \cdot x^{-5})' = -2 \cdot 3 \cdot 4 \cdot 5 \cdot \frac{1}{x^6}$

$f^{(5)}(1) = -1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$

8) $f^{(n)}(1) = (-1)^n \cdot n!$

$f(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4 +$
$+ \frac{f^{(n)}(1)}{n!}(x-1)^n + o((x-1)^n) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \ldots$
$+ (-1)^n (x-1)^n + o((x-1)^n)$

② $f(x) = \operatorname{arctg} x$, до $o(x^3)$

1) По формуле Маклорена $\Rightarrow x_0 = 0$

2) Вычислить до $o(x^3) \Rightarrow f(0), f'(0), f''(0)$

3) $f(0) = \operatorname{arctg}(0) = 0$

4) $f'(x) = (\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$f'(0) = \frac{1}{1+0^2} = 1$

5) $f''(x) = \left(\frac{1}{1+x^2}\right)' = \left((1+x^2)^{-1}\right)' = -1 \cdot (1+x^2)^{-1-1} \cdot (1+x^2)' =$
$= -\frac{1}{(1+x^2)^2} \cdot (0+2x) = -\frac{2x}{(1+x^2)^2}$

$f''(0) = -\frac{2 \cdot 0}{(1+0)^2} = 0$

6) $f'''(x) = \left(-\frac{2x}{(1+x^2)^2}\right)' = \frac{(-2x)'(1+x^2)^2 - (-2x)\left((1+x^2)^2\right)'}{\left((1+x^2)^2\right)^2} =$

$= \frac{-2\cdot(1+x^2)^2 + 2x\cdot 2\,(1+x^2)\cdot(0+2x)}{(1+x^2)^4} = \frac{-2(1+x^2)^2 + 8x^2(1+x^2)}{(1+x^2)^4} =$

$= \frac{6x^2-2}{(1+x^2)^3}$

$f'''(0) = \frac{6\cdot 0^2-2}{(1+0^2)^3} = -\frac{2}{1} = -2$

7) $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + o(x^3) =$

$= 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{-2}{3!}x^3 + o(x^3) = x - \frac{1}{3}x^3 + o(x^3)$