

N 7.3.19

$$\lim_{x \rightarrow \infty} x^2 \cdot e^{-x} = \left[ \frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{(x^2)'}{(e^x)'} = \lim_{x \rightarrow \infty} \frac{(2x)'}{(e^x)'} = \lim_{x \rightarrow \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$

N 7.3.20

$$\begin{aligned} \lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{\sinh x} \right) &= [\infty - \infty] = \lim_{x \rightarrow 0} \frac{\sinh x - x}{x \cdot \sinh x} = \left[ \frac{0}{0} \right] = \\ &= \lim_{x \rightarrow 0} \frac{(\sinh x - x)'}{(x \cdot \sinh x)'} = \lim_{x \rightarrow 0} \frac{(\cosh - 1)'}{(\sinh x + x \cosh x)'} = \lim_{x \rightarrow 0} \frac{-\sinh x}{2 \cosh x - 2 \sinh x} = \\ &= \frac{0}{2 - 0} = \frac{0}{2} = 0 \end{aligned}$$

N 7.3.21

$$\begin{aligned} \lim_{x \rightarrow \infty} x(e^{\frac{1}{x}} - 1) &= [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x}} - 1}{\frac{1}{x}} = \left[ \frac{0}{0} \right] = \lim_{x \rightarrow \infty} \frac{(e^{\frac{1}{x}} - 1)'}{(\frac{1}{x})'} = \\ &= \lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x}} \cdot (-\frac{1}{x^2})}{(-\frac{1}{x^2})} = \lim_{x \rightarrow \infty} e^{\frac{1}{x}} = e^{\frac{1}{\infty}} = e^0 = 1 \end{aligned}$$

N 7.3.22

$$\begin{aligned} \lim_{x \rightarrow 1} \left( \frac{1}{1-x^3} - \frac{1}{1-x^5} \right) &= [\infty - \infty] = \lim_{x \rightarrow 1} \frac{(x^3 - x^5)'}{((1-x^3)(1-x^5))'} = \\ &= \lim_{x \rightarrow 1} \frac{3x^2 - 5x^4}{-3x^2 + 3x^5 - 2x - 2x^4} = \lim_{x \rightarrow 1} \frac{3x^2 - 5x^4}{5x^4 - 3x^2 - 2x} = \frac{3 - 5}{5 - 3 - 2} = \frac{1}{0} = \infty \end{aligned}$$