(15.06.20)

№6.4.38

$$\lim_{x\to 0} \frac{\sin^2(3x)}{\sin^2(2x)} = \left[\frac{0}{0}\right] = \left(\lim_{x\to 0} \frac{\sin(3x)}{\sin(2x)}\right)^2 =$$

$$= \left(\lim_{x\to 0} \frac{\sin(3x)\cdot x}{\sin(2x)\cdot x}\right)^2 = \left(\lim_{x\to 0} \frac{\frac{\sin(3x)}{x}}{\frac{\sin(2x)}{x}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2{,}25$$

№6.4.39

$$\lim_{x\to 0} \frac{tg(2x)}{\sin(5x)} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\frac{\sin(2x)}{\cos(2x)}}{\sin(5x)} = \lim_{x\to 0} \frac{\sin(2x)}{\cos(2x)\sin(5x)} =$$

$$= \lim_{x\to 0} \frac{1}{\cos(2x)} \cdot \lim_{x\to 0} \frac{\sin(2x)}{\sin(5x)} = \frac{1}{1} \cdot \lim_{x\to 0} \frac{\frac{\sin(2x)}{x}}{\frac{\sin(5x)}{x}} = \frac{2}{5} = 0{,}4.$$

№6.4.40

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] = \left[1-\cos\alpha = 2\sin^2\frac{\alpha}{2}\right] =$$

$$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{x^2} = \lim_{x\to 0} \left(2\cdot \frac{\sin\frac{x}{2}}{x} \cdot \frac{\sin\frac{x}{2}}{x}\right) =$$

$$= 2\cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2} = 0{,}5$$

№6.4.41

$$\lim_{x\to 0} x\cdot ctg x = [0\cdot\infty] = \lim_{x\to 0} \left(x\cdot \frac{\cos x}{\sin x}\right) =$$

$$= \lim_{x\to 0}(\cos x)\cdot \lim_{x\to 0}\left(\frac{x}{\sin x}\right) = 1\cdot \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right) = \frac{1}{1} = 1.$$

№ 6.4.42

$$\lim_{x\to 0}\frac{\operatorname{arctg}2x}{x}=\left[\frac{0}{0}\right]=\lim_{y\to 0}\frac{y}{\frac{1}{2}\operatorname{tg}y}=\lim_{y\to 0}\frac{y}{\frac{1}{2}\frac{\sin y}{\cos y}}=$$

$$=\lim_{y\to 0}\frac{y\cdot 2\cdot\cos y}{\sin y}=\lim_{y\to 0}(2\cos y)\cdot\lim_{y\to 0}\frac{y}{\sin y}=2\cdot 1=2$$

№ 6.4.43

$$\lim_{x\to 0}\frac{\cos 5x-\cos 3x}{x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{-2\cdot\sin 4x\cdot\sin x}{x^2}=$$

$$=\lim_{x\to 0}(-2)\cdot\lim_{x\to 0}\frac{\sin 4x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{x}=-2\cdot 4\cdot 1=-8$$

№ 6.4.44

$$\lim_{x\to 1}\frac{\sin(6\pi x)}{\sin(\pi x)}=\left[\frac{0}{0}\right]=\lim_{y\to 0}\frac{\sin(6\pi(y+1))}{\sin(\pi(y+1))}=\lim_{y\to 0}\frac{\sin(6\pi y+6\pi)}{\sin(\pi y+\pi)}=$$

$$=\lim_{y\to 0}\frac{\sin(6\pi y)}{-\sin(\pi y)}=\lim_{y\to 0}\frac{\sin(6\pi y)\cdot y}{-\sin(\pi y)\cdot y}=-\frac{6\pi}{\pi}=-6$$

№ 6.4.45

$$\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\operatorname{tg}4x}=\left[\frac{0}{0}\right]=\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\frac{\sin 4x}{\cos 4x}}=\lim_{x\to\frac{\pi}{2}}(\cos 4x)\cdot\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\sin 4x}=$$

$$=1\cdot\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\sin 4x}=\lim_{y\to 0}\frac{\sin(2(y+\frac{\pi}{2}))}{\sin(4(y+\frac{\pi}{2}))}=\lim_{y\to 0}\frac{\sin(2y+\pi)}{\sin(4y+2\pi)}=$$

$$=\lim_{y\to 0}\frac{-\sin 2y}{\sin 4y}=\lim_{y\to 0}\frac{-\sin 2y\cdot y}{\sin 4y\cdot y}=-\frac{2}{4}=-0{,}5$$