Математический анализ
(11.05.20)

№7.3.11.

1) $\lim\limits_{x\to 0} \dfrac{x^3}{x-\sin x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 0} \dfrac{(x^3)'}{(x-\sin x)'} = \lim\limits_{x\to 0} \dfrac{3x^2}{1-\cos x} = \lim\limits_{x\to 0} \dfrac{6x}{\sin x} =$

$= 6 \lim\limits_{x\to 0} \dfrac{x}{\sin x} = 6 \cdot \lim\limits_{x\to 0} \dfrac{1}{\frac{\sin x}{x}} = 6 \cdot 1 = 6.$

№7.3.12

$\lim\limits_{x\to 2} \dfrac{x^3+x-10}{x^3-3x-2} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 2} \dfrac{3x^2+1}{3x^2-3} = \dfrac{12+1}{12-3} = \dfrac{13}{9}$

№7.3.14

$\lim\limits_{x\to 0} \dfrac{e^x-1}{\sin x} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 0} \dfrac{e^x}{\cos x} = \dfrac{1}{1} = 1$

№7.3.13

$\lim\limits_{x\to 1} \dfrac{\ln x}{x-1} = \left[\dfrac{0}{0}\right] = \lim\limits_{x\to 1} \dfrac{\frac{1}{x}}{1} = \dfrac{1}{1} = 1$

№7.3.15

$\lim\limits_{x\to \infty} \dfrac{\ln x}{x} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to \infty} \dfrac{\frac{1}{x}}{1} = \lim\limits_{x\to \infty} \dfrac{1}{x} = \dfrac{1}{\infty} = 0$

№7.3.16

$\lim\limits_{x\to \infty} \dfrac{e^x}{x^3} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to \infty} \dfrac{e^x}{3x^2} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to \infty} \dfrac{e^x}{6x} = \lim\limits_{x\to \infty} \dfrac{e^x}{6} = \dfrac{\infty}{6} = \infty$

№7.3.17.

$\lim\limits_{x\to 0} \dfrac{\ln x}{\operatorname{ctg} 2x} = \left[\dfrac{\infty}{\infty}\right] = \lim\limits_{x\to 0} \dfrac{\frac{1}{x}}{\left(-\frac{1}{\sin^2(2x)}\right)\cdot 2} = \lim\limits_{x\to 0} \dfrac{-\sin^2(2x)}{2x} = -\dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{\sin^2(2x)}{x} =$

$= -\dfrac{1}{2} \lim\limits_{x\to 0} \dfrac{4\sin(2x)\cos(2x)}{1} = -2 \lim\limits_{x\to 0} \left(\sin(2x)\cos(2x)\right) = -\lim\limits_{x\to 0} \left(\sin(4x)\right) = 0$

№7.3.18.

$\lim\limits_{x\to 0+0} x \ln x = [0\cdot\infty] = \lim\limits_{x\to 0+0} \dfrac{\ln x}{\frac{1}{x}} = \lim\limits_{x\to 0+0} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0+0} (-x) = 0$

) $\lim\limits_{x\to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1}\right) = [\infty - \infty] = \lim\limits_{x\to 1} \dfrac{x-1-\ln x}{(x-1)\ln x} = \left[\dfrac{0}{0}\right] =$

$= \lim\limits_{x\to 1} \dfrac{1-\frac{1}{x}}{\ln x + 1 - \frac{1}{x}} = \lim\limits_{x\to 1} \dfrac{\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} = \dfrac{\frac{1}{1^2}}{\frac{1}{1}+\frac{1}{1^2}} = \dfrac{1}{2}$