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Подгруппа №1

Интегралы и диф. уравнение (14.12.20)

№8.2.6

$$\int \frac{\ln^5 x\,dx}{x} = \left[\, t = \ln x \Rightarrow dt = d(\ln x) = \frac{1}{x}dx \,\right] =$$

$$= \int t^5\,dt = \frac{t^6}{6} + C = \frac{\ln^6 x}{6} + C$$

№8.2.7

$$\int \frac{\sin x\,dx}{\cos x + 1} = \left[\begin{array}{l} t = \cos x + 1 \Rightarrow dt = d(\cos x + 1) = -\sin x\,dx \\ -dt = \sin x\,dx \end{array}\right] =$$

$$= -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos x + 1| + C$$

№8.2.8

$$\int \frac{x^2\,dx}{x^3+1} = \left[\begin{array}{l} t = x^3 + 1 \Rightarrow dt = d(x^3+1) = 3x^2\,dx \\ \frac{dt}{3} = x^2\,dx \end{array}\right] =$$

$$= \frac{1}{3}\int \frac{dt}{t} = \frac{\ln|x|}{3} + C = \frac{\ln|x^3+1|}{3} + C$$

№8.2.9

$$\int \frac{\operatorname{arctg} x\,dx}{x^2+1} = \left[\, t = \operatorname{arctg} x \Rightarrow dt = d(\operatorname{arctg} x) = \frac{dx}{1+x^2} \,\right] =$$

$$= \int t\,dt = \frac{t^2}{2} + C = \frac{\operatorname{arctg}^2 x}{2} + C$$

№ 8.2.11

$$\int \frac{4x+3}{\sqrt{x^2-5}}dx = \int\left(\frac{4x}{\sqrt{x^2-5}} - \frac{3}{\sqrt{x^2-5}}\right)dx = 4\cdot\int\frac{x\,dx}{\sqrt{x^2-5}} + 3\int\frac{dx}{\sqrt{x^2-6}} =$$

$$= \left[\begin{matrix} t = x^2-5 \Rightarrow dt = 2x\,dx \\ \frac{dt}{2} = x\,dx \end{matrix}\right] = 4\int\frac{dt}{2\sqrt{t}} + 3\int\frac{dx}{\sqrt{x^2-5}} =$$

$$= 2\int\frac{dt}{\sqrt{t}} + 3\ln\left|x+\sqrt{x^2-5}\right| + C = 4\sqrt{t} + 3\ln\left|x+\sqrt{x^2-5}\right| + C =$$

$$= 4\sqrt{x^2-5} + 3\ln\left|x+\sqrt{x^2-5}\right| + C$$

№ 8.2.12

$$\int e^{\sin^2 x}\sin 2x\,dx = \left[t = \sin^2 x,\ dt = 2\sin x\cos x\,dx\right] =$$

$$= \int e^t dt = e^t + C = e^{\sin^2 x} + C$$

№ 8.2.13

$$\int\frac{1-2\sin x}{\cos^2 x}dx = \int\left(\frac{1}{\cos^2 x} - \frac{2\sin x}{\cos^2 x}\right)dx = \int\frac{dx}{\cos^2 x} - 2\int\frac{\sin x\,dx}{\cos^2 x} =$$

$$= \int\frac{dx}{\cos^2 x} - 2\int\frac{dt}{t^2} = \int\frac{dx}{\cos^2 x} + 2\int t^{-2}dt =$$

$$= \operatorname{tg} x - 2\cdot t^{-1} + C = \operatorname{tg} x - \frac{2}{t} + C = \operatorname{tg} x - \frac{2}{\cos x} + C$$

№ 8.2.14

$$\int\frac{3x-4}{x^2-4}dx = 3\int\frac{x\,dx}{x^2-4} - 4\int\frac{dx}{x^2-4} = \left[\begin{matrix} t = x^2-4 \Rightarrow dt = \\ d(x^2-4) = 2x\,dx \quad \frac{dt}{2} = x\,dx \end{matrix}\right] =$$

$$= \frac{3}{2}\int\frac{dt}{t} - 4\int\frac{dx}{x^2-2^2} = \frac{3\ln|t|}{2} - \ln\left|\frac{x-2}{x+2}\right| + C =$$

$$= \frac{3\ln|x^2-4|}{2} - \ln\left|\frac{x-2}{x+2}\right| + C$$