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Подгруппа №1

(19.02.21)

№8.4.9.

$$\int \sqrt[3]{x} \cdot \sqrt[3]{1+3\sqrt[3]{x^2}}\,dx = x^{\frac{1}{3}}\left(1+3x^{\frac{2}{3}}\right)^{\frac{1}{3}}dx = \left[\begin{matrix} m=\frac{1}{3},\ n=\frac{2}{3} \\ p=\frac{1}{3},\ a=1,\ b=3 \end{matrix}\right.$$

1) $p=\frac{1}{3} \notin Z$

2) $\frac{m+1}{n} = \frac{\frac{1}{3}+1}{\frac{2}{3}} = 2 \in Z$

$1+3x^{\frac{2}{3}} = t^k$, $k=3$, т.к. знаменатель дроби $p=\frac{1}{3} \Rightarrow k=3$

$\Rightarrow d(1+3x^{\frac{2}{3}}) = d(t^3) \Rightarrow (1+3x^{\frac{2}{3}})'_x\,dx = (t^3)'_t\,dt \Rightarrow$

$\Rightarrow 2x^{-\frac{1}{3}}dx = 3t^2dt \Rightarrow$

$= (x^{\frac{1}{3}})^2 = \frac{t^3-1}{3} \Rightarrow x^{\frac{1}{3}} = \frac{1}{\sqrt{3}}\sqrt{t^3-1} \Rightarrow dx = \frac{3}{2}t^2\,\frac{1}{\sqrt{3}}\sqrt{t^3-1}\,dt =$

$= \frac{\sqrt{3}}{2}t^2\sqrt{t^3-1}\,dt\ \Big] =$

$$= \int \frac{1}{\sqrt{3}} \cdot \sqrt{t^3-1} \cdot \sqrt[3]{t^3} \cdot \frac{\sqrt{3}}{2} t^2 \sqrt{t^3-1}\, dt = \frac{1}{2}\int t^2 (t^3-1) t\, dt =$$

$$= \frac{1}{2}\int t^3 (t^3-1)\, dt = \frac{1}{2}\int t^6 dt - \frac{1}{2}\int t^3 dt =$$

$$= \frac{1}{14} t^7 - \frac{1}{8} t^4 + C = \left[1 + 3x^{\frac{2}{3}} = t^3 \Rightarrow t = \sqrt[3]{1+3\sqrt[3]{x^2}} \right]$$

$$= \frac{1}{14}\left(\sqrt[3]{1+3\sqrt[3]{x^2}}\right)^7 - \frac{1}{8}\left(\sqrt[3]{1+3\sqrt[3]{x^2}}\right)^4 + C =$$

$$= \frac{1}{14} \cdot \sqrt[3]{(1+3\sqrt[3]{x^2})^7} - \frac{1}{8} \sqrt[3]{(1+3\sqrt[3]{x^2})^4} + C$$

N 8.4.2.

$$\int \frac{\sqrt[3]{x}\, dx}{\sqrt[3]{x^2} - \sqrt{x}} = \left[\begin{matrix} n = 3,\ q = 2 \Rightarrow k = \text{НОК}(3;2) = 6 \\ x = t^6 \Rightarrow dx = d(t^6) \Rightarrow dx = 6t^5 dt \end{matrix} \right] =$$

$$= \int \frac{\sqrt[3]{t^6} \cdot 6t^5 dt}{\sqrt[3]{(t^6)^2} - \sqrt{t^6}} = 6\int \frac{t^2 \cdot t^5\, dt}{t^4 - t^3} = 6\int \frac{t^7\, dt}{t^3(t-1)} = 6\int \frac{t^4\, dt}{t-1} =$$

$$= \left[\begin{array}{l|l} t^4 + 0t^3 + 0t^2 + 0t + 0 & t-1 \\ \hline t^4 - t^3 & t^3 + t^2 + t + 1 \\ \quad t^3 + 0t^2 & \\ \quad t^3 - t^2 & \\ \qquad t^2 + 0t & \\ \qquad t^2 - t & \\ \qquad\quad t - 0 & \\ \qquad\quad t - 1 & \\ \qquad\qquad 1 & \end{array} \right] =$$

$$= 6\int \frac{(t^3+t^2+t+1)(t-1)+1}{t-1} dt = 6\int (t^3+t^2+t+1)\, dt + 6\int \frac{dt}{t-1} =$$

$$= 6\int t^3 dt + 6\int t^2 dt + 6\int t\, dt + 6\int dt + 6\int \frac{dt}{t-1} =$$

$$= \frac{6t^4}{4} + \frac{6t^3}{3} + \frac{6t^2}{2} + 6t + 6\ln|t-1| + C =$$

$$= \frac{3t^4}{2} + 2t^3 + 3t^2 + 6t + 6\ln|t-1| + C = [t = \sqrt[6]{x}] =$$

$$= \frac{3\sqrt[6]{x^4}}{2} + 2\sqrt[6]{x^3} + 3\sqrt[6]{x^2} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C =$$

$$= \frac{3\sqrt[3]{x^2}}{2} + 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C$$

№8.4.3

$$\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}} = \left[\begin{matrix} n=2 \quad q=4 \\ k = \text{НОК}(2,4)=4 \end{matrix} \;\; x = t^4 \Rightarrow t = \sqrt[4]{x} \quad dx = 4t^3\,dt \right] =$$

$$= \int \frac{4t^3\,dt}{\sqrt{t^4}+\sqrt[4]{t^4}} = 4\int \frac{t^3\,dt}{t^2+t} = 4\int \frac{t^2\,dt}{t+1} = \Big[$$

$$\begin{array}{r|l} t^2+0t+0 & t+1 \\ \hline -(t^2+t) & t-1 \\ -t+0 & \\ -(-t-1) & \\ \hline 1 & \end{array}$$

$$\Big] = 4\int \frac{(t+1)(t-1)+1}{t+1}\,dt =$$

$$= 4\int (t-1)\,dt + 4\int \frac{dt}{t+1} = 4\int t\,dt - 4\int dt + 4\int \frac{dt}{t+1} =$$

$$= 2t^2 - 4t + 4\ln|t+1| + C =$$

$$= 2\sqrt[4]{x^2} - 4\sqrt[4]{x} + 4\ln|\sqrt[4]{x}+1| + C =$$

$$= 2\sqrt{x} - 4\sqrt[4]{x} + 4\ln(\sqrt[4]{x}+1) + C$$

№8.4.5.

$$\int \frac{dx}{\sqrt[3]{(2x+1)^2} - \sqrt{2x+1}} = \left[\begin{matrix} n=3, \; q=2 \\ k=6 \end{matrix} \Rightarrow 2x+1 = t^6 \Rightarrow 2dx = 6t^5\,dt \Rightarrow dx = 3t^5\,dt\right]$$

$$= 3\int \frac{t^5\,dt}{\sqrt[3]{t^{12}} - \sqrt{t^6}} = 3\int \frac{t^5\,dt}{t^4 - t^3} = 3\int \frac{t^2\,dt}{t-1} =$$

$$= 3\int \frac{t^2-1+1}{t-1}\,dt = 3\int \frac{(t-1)(t+1)}{t-1}\,dt + 3\int \frac{dt}{t-1} =$$

$$= 3\int (t+1)\,dt + 3\int \frac{dt}{t-1} = \frac{3t^2}{2} + 3t + 3\ln|t-1| + C =$$

$$= \frac{3\sqrt[6]{(2x+1)^2}}{2} + 3\sqrt[6]{2x+1} + 3\ln|\sqrt[6]{2x+1} - 1| + C =$$

$$= \frac{3\sqrt[3]{2x+1}}{2} + 3\sqrt[6]{2x+1} + 3\ln|\sqrt[6]{2x+1} - 1| + C$$

№3.4.6.

$$\int \frac{dx}{1-\sqrt[3]{x+1}} = \left[\begin{matrix} k=3 \Rightarrow k=3 \\ x+1 = t^3 \Rightarrow dx = 3t^2 dt \end{matrix} \right] = \int \frac{3t^2 dt}{1+t} =$$

$$= 3\int \frac{t^2 dt}{t+1} = 3\int \frac{(t^2-1+1)dt}{t+1} = 3\int \frac{(t-1)(t+1)dt}{t+1} + 3\int \frac{dt}{t+1} =$$

$$= 3\int t\,dt - 3\int dt + 3\int \frac{dt}{t+1} = \frac{3}{2}t^2 - 3t + 3\ln|t+1| + C =$$

$$= \left[t = \sqrt[3]{x+1} \right] = \frac{3}{2}\sqrt[3]{(x+1)^2} - 3\sqrt[3]{x+1} + 3\ln|\sqrt[3]{x+1}+1| + C$$

№3.4.8.

$$\int \frac{\sqrt{x}}{x^2\sqrt{x-1}}dx = \int x^{-\frac{3}{2}}(x-1)^{-\frac{1}{2}}dx = \left[1)\ p = -\frac{1}{2} \notin \mathbb{Z}, \right.$$

$$2)\ \frac{-\frac{3}{2}+1}{1} = -\frac{1}{2} \notin \mathbb{Z},\quad 3)\ \frac{-\frac{3}{2}+1}{1} - \frac{1}{2} = -1 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow -1x^{-1} + 1 = t^2 \Rightarrow t^2 = -\frac{1}{x} + 1 = \frac{x-1}{x} \Rightarrow t = \sqrt{\frac{x-1}{x}} \Rightarrow$$

$$\left. \Rightarrow x = -\frac{1}{t^2-1} \Rightarrow dx = -\frac{2t}{(t^2-1)^2}dt \right] =$$

$$= \int \left(\frac{1}{1-t^2}\right)^{-\frac{3}{2}} \left(\frac{1}{1-t^2} - 1\right)^{-\frac{1}{2}} \frac{2t\,dt}{(1-t^2)^2} =$$

$$= \int (1-t^2)^{\frac{3}{2}-2} \left(\frac{t^2}{1-t^2}\right)^{-\frac{1}{2}} 2t\,dt = \int \frac{2t}{\sqrt{1-t^2}} \cdot \frac{\sqrt{1-t^2}}{t}dt =$$

$$= 2\int dt = 2t + C = \frac{2\sqrt{x-1}}{\sqrt{x}} + C$$

№3.4.10

$$\int \sqrt{x}(1+\sqrt[3]{x})^4 dx = \int x^{\frac{1}{2}}(1+x^{\frac{1}{3}})^4 dx = \left[m = \frac{1}{2},\ n = \frac{1}{3},\ p = 4 \right.$$

$$\left. 1)\ p = 4 \in \mathbb{Z} \Rightarrow k = 6 \Rightarrow dx = 6t^5 dt \Rightarrow x = t^6 \Rightarrow t = \sqrt[6]{x} \right] =$$

$$= \int \sqrt{t^6}(1+\sqrt[3]{t^6})^4 6t^5 dt = 6\int t^3(1+t^2)^4 t^5 dt =$$

$$= 6\int t^8(1+t^2)^4 dt = 6\int t^8(t^8 + 4t^6 + 6t^4 + 4t^2 + 1)dt =$$

$= 6\int (t^{16}+4t^{14}+6t^{12}+4t^{10}+t^{8})\,dt =$

$= 6\int t^{16}dt + 24\int t^{14}dt + 36\int t^{12}dt + 24\int t^{10}dt + 6\int t^{8}dt =$

$= \frac{6}{17}t^{17} + \frac{24}{15}t^{15} + \frac{36}{13}t^{13} + \frac{24}{11}t^{11} + \frac{6}{9}t^{9} + C =$

$= \frac{6}{17}(\sqrt[6]{x})^{17} + \frac{24}{15}(\sqrt[6]{x})^{15} + \frac{36}{13}(\sqrt[6]{x})^{13} + \frac{24}{11}(\sqrt[6]{x})^{11} + \frac{2}{3}(\sqrt[6]{x})^{9} + C =$

$= \frac{6}{17}x^2\sqrt[6]{x^5} + \frac{24}{15}x^2\sqrt[6]{x^3} + \frac{36}{13}x^2\sqrt[6]{x} + \frac{24}{11}x\sqrt[6]{x^5} + \frac{2}{3}x\sqrt[6]{x^3} + C =$

$= \frac{6x^2\sqrt[6]{x^5}}{17} + \frac{24x^2\sqrt{x}}{15} + \frac{36x^2\sqrt[6]{x}}{13} + \frac{24x\sqrt[6]{x^5}}{11} + \frac{2x\sqrt{x}}{3} + C$

№8.4.11

$\int \frac{dx}{x^4\sqrt{x^2+1}} = \int x^{-4}(1+x^2)^{-\frac{1}{2}}dx = [\, m=-4,\ n=2,\ p=-\frac{1}{2}$

$1)\ p=-\frac{1}{2} \notin \mathbb{Z},\ 2)\ \frac{-4+1}{2}=-\frac{3}{2} \notin \mathbb{Z}\quad 3)\ -\frac{3}{2}-\frac{1}{2}=-\frac{4}{2}=-2 \in \mathbb{Z} \Rightarrow$

$\Rightarrow x^{-2}+1=t^2 \Rightarrow (1+x^2)=x^2(\frac{1}{x^2}+1)=x^2(x^{-2}+1) \Rightarrow$

$\Rightarrow x=\sqrt{\frac{1}{t^2-1}} \Rightarrow dx=\left(\left(\frac{1}{t^2-1}\right)^{\frac{1}{2}}\right)'_t dt \Rightarrow$

$\Rightarrow [\, \frac{1}{2}\left(\frac{1}{t^2-1}\right)^{-\frac{1}{2}}\left(\frac{-2t}{(t^2-1)^2}\right) = -\frac{1}{2}\frac{\sqrt{t^2-1}}{1}\left(\frac{-2t}{(t^2-1)^2}\right) =$

$= -t(t^2-1)^{\frac{1}{2}-\frac{4}{2}} = -t(t^2-1)^{-\frac{3}{2}} = -\frac{t}{(t^2-1)^{\frac{3}{2}}}\ ] =$

$= \int x^{-4}(x^2(x^{-2}+1))^{-\frac{1}{2}}dx = -\int (t^2-1)^2\left(\frac{t^2}{t^2-1}\right)^{-\frac{1}{2}}\frac{t\,dt}{(t^2-1)^{\frac{3}{2}}} =$

$= -\int (t^2-1)^{\frac{1}{2}}(t^2-1)^{\frac{1}{2}}dt = -\int (t^2-1)dt = -\int t^2dt + \int dt =$

$= -\frac{t^3}{3} - t + C = [\, t=\sqrt{\frac{x^2+1}{x^2}} = \frac{\sqrt{x^2+1}}{x}\ ] =$

$= -\frac{\sqrt{(x^2+1)^3}}{3x^3} + \frac{\sqrt{x^2+1}}{x} + C = \frac{3x^2\sqrt{x^2+1}-(x^2+1)\sqrt{x^2+1}}{3x^3} + C =$

$= \frac{\sqrt{x^2+1}(2x^2-1)}{3x^3} + C$