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Подгруппа №1

Интегралы и диф уравнения (21.12.20)

№8.2.16

$$\int\sqrt{9-x^2}\,dx = \left[\begin{array}{l} x=3\sin t \Rightarrow 9-x^2 = 9-(3\sin t)^2 = \\ = 9-9\sin^2 t = 9\cos^2 t \Rightarrow dx = 3\cos t\,dt \end{array}\right] =$$

$$= \int(9-9\sin^2 t)\,3\cos t\,dt = \int 3\cos t\cdot 3\cos t\,dt = \int 9\cos^2 t\,dt =$$

$$= 9\int\frac{1+\cos 2t}{2}\,dt = \frac{9}{2}\int(1+\cos 2t)\,dt =$$

$$= \frac{9}{2}\int dt + \frac{9}{2}\int\cos 2t\,dt = \frac{9}{2}\left(t+\frac{1}{2}\sin 2t\right)+C =$$

$$= \frac{9}{2}\left(\arcsin\frac{x}{3}-\frac{1}{2}\sin\left(2\arcsin\frac{x}{3}\right)\right)+C =$$

$$= \frac{9}{2}\arcsin\frac{x}{3}+\frac{9}{4}\sin\left(2\arcsin\frac{x}{3}\right)+C =$$

$$= \left[\begin{array}{l} \sin\left(2\arcsin\frac{x}{3}\right) = 2\sin\left(\arcsin\frac{x}{3}\right)\cos\left(\arcsin\frac{x}{3}\right) = \\ = 2\frac{x}{3}\cdot\sqrt{1-\left(\frac{x}{3}\right)^2} = 2\frac{x}{3}\sqrt{1-\frac{x^2}{9}} = \frac{2x}{9}\sqrt{9-x^2} \end{array}\right] =$$

$$= \frac{9}{2}\arcsin\frac{x}{3}+\frac{9}{4}\cdot\frac{2x}{9}\sqrt{9-x^2}+C = \frac{9}{2}\arcsin\frac{x}{3}+\frac{x}{2}\sqrt{9-x^2}+C$$

№8.2.17

$$\int\frac{dx}{x\sqrt{x+1}} = \left[x+1=t^2 \Rightarrow x=t^2-1 \Rightarrow dx=2t\,dt\right] =$$

$$= \int\frac{2t\,dt}{(t^2-1)\sqrt{t^2}} = \int\frac{2t\,dt}{(t^2-1)t} = \int\frac{2\,dt}{t^2-1} = 2\int\frac{dt}{t^2-1} =$$

$$= \ln\left|\frac{t-1}{t+1}\right|+C = \ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C$$

№8.2.18

$$\int x\sqrt{2-x}\,dx = \left[\begin{array}{l} 2-x=t^2 \Rightarrow t=\sqrt{2-x} \quad dx=-2t\,dt \\ -x=t^2-2 \Rightarrow x=2-t^2 \end{array}\right] =$$

$$= \int(2-t^2)\cdot\sqrt{t^2}\,(-2t\,dt) = \int(2-t^2)\,t\,(-1)\,t\,dt =$$

$$= \int (2\cdot t\cdot(-2)\cdot t - t^2\cdot t\cdot(-2)\cdot t)\,dt = \int(-4t^2+2t^4)\,dt =$$

$$= -4\int t^2\,dt + 2\int t^4\,dt = \frac{2t^5}{5} - \frac{4t^3}{3} + C =$$

$$= \frac{2}{5}\left(\sqrt{2-x}\right)^5 - \frac{4}{3}\left(\sqrt{2-x}\right)^3 + C = \frac{2\sqrt{(2-x)^5}}{5} - \frac{4\sqrt{(2-x)^3}}{3} + C$$

№ 8.2.19

$$\int \frac{\sqrt{x}\,dx}{x+16} = \left[\begin{array}{l} x = t^2 \Rightarrow t = \sqrt{x} \\ dx = 2t\,dt \end{array}\right] = \int \frac{t\cdot 2t\,dt}{t^2+16} =$$

$$= 2\int \frac{t^2\,dt}{t^2+16} = 2\int \frac{t^2+16-16}{t^2+16}\,dt = 2\int\left(1 - \frac{16}{t^2+16}\right)dt =$$

$$= 2\int dt - 32\int \frac{dt}{t^2+16} = 2t - 8\,\mathrm{arctg}\,\frac{t}{4} + C =$$

$$= 2\sqrt{x} - 8\,\mathrm{arctg}\,\frac{\sqrt{x}}{4} + C$$

№ 8.2.21

$$\int x\sin x\,dx = \left[\begin{array}{l} u = x \Rightarrow u' = 1 \\ v' = \sin x \Rightarrow v = -\cos x \end{array}\right] = x(-\cos x) - \int(-\cos x)\,dx =$$

$$= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C = \sin x - x\cos x + C$$