№7.3.25

$$\lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}=[1^\infty]$$

$$y=(\cos 2x)^{\frac{1}{x^2}};\quad \ln y=\ln\left((\cos 2x)^{\frac{1}{x^2}}\right)$$

$$\ln y=\frac{1}{x^2}\ln\cos 2x \qquad \lim_{x\to 0}(\ln y)=\lim_{x\to 0}\left(\frac{1}{x^2}\ln\cos 2x\right)$$

$$\lim_{x\to 0}\frac{1}{x^2}\ln\cos 2x=\lim_{x\to 0}\frac{\ln\cos 2x}{x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(\ln\cos 2x)'}{(x^2)'}=$$

$$=\lim_{x\to 0}\frac{\frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2}{2x}=\lim_{x\to 0}\frac{-\operatorname{tg}2x}{x}=\left[\frac{0}{0}\right]=$$

$$=-\lim_{x\to 0}\frac{(\operatorname{tg}2x)'}{x'}=-\lim_{x\to 0}\frac{2\cdot\frac{1}{\cos^2 x}}{1}=-\lim_{x\to 0}\frac{2}{\cos^2 x}=-\frac{2}{1}=-2$$

$$\lim_{x\to 0}(\ln y)=\ln\left(\lim_{x\to 0}y\right)$$

$$\ln\left(\lim_{x\to 0}y\right)=-2\Rightarrow\lim_{x\to 0}y=e^{-2}\Rightarrow\lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}=e^{-2}$$

№7.3.26.

$$\lim_{x\to 0}\left(\frac{1}{x}\right)^{x^2}=[\infty^0]$$

$$y=\left(\frac{1}{x}\right)^{x^2};\quad \ln y=\ln\left(\frac{1}{x}\right)^{x^2};\quad \ln y=x^2\ln\left(\frac{1}{x}\right)$$

$$\lim_{x\to 0}(\ln y)=\lim_{x\to 0}x^2\ln\left(\frac{1}{x}\right)$$

$$\lim_{x\to 0} x^2 \ln\left(\frac{1}{x}\right) = [0\cdot\infty] = \lim_{x\to 0} \frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{x^2}} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0} \frac{\left(\ln\left(\frac{1}{x}\right)\right)'}{(x^{-2})'} = \lim_{x\to 0} \frac{x\cdot\left(-\frac{1}{x^2}\right)}{-2\cdot x^{-3}} = \lim_{x\to 0} \frac{x^3}{2x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{3x^2}{2} = \frac{0}{2} = 0$$

$$\lim_{x\to 0}(\ln y) = \ln\left(\lim_{x\to 0} y\right) \Rightarrow$$

$$\Rightarrow \ln\left(\lim_{x\to 0} y\right) = 0 \Rightarrow \lim_{x\to 0} y = e^0 \Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)^{x^2} = 1$$

№7.3.27

$$\lim_{x\to 0} x^{\frac{1}{1+\ln x}} = [0^0]$$

$$y = x^{\frac{1}{1+\ln x}}; \quad \ln y = \ln x^{\frac{1}{1+\ln x}}, \quad \ln y = \frac{1}{1+\ln x}\ln x$$

$$\lim_{x\to 0}(\ln y) = \lim_{x\to 0}\left(\frac{1}{1+\ln x}\cdot \ln x\right)$$

$$\lim_{x\to 0} \frac{1}{1+\ln x}\ln x = \lim_{x\to 0} \frac{\ln x}{1+\ln x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0} \frac{(\ln x)'}{(1+\ln x)'} =$$

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to 0} 1 = 1$$

$$\lim_{x\to 0}(\ln y) = \ln\left(\lim_{x\to 0} y\right) \Rightarrow$$

$$\Rightarrow \ln\left(\lim_{x\to 0} y\right) = 1 \Rightarrow \lim_{x\to 0} y = e^1 \Rightarrow \lim_{x\to 0} x^{\frac{1}{1+\ln x}} = e$$