13.04.20

№7.1.1

def $y=f(x)$, y'

1) $y=3x^2$, $\Delta y=f(x+\Delta x)-f(x) \Rightarrow \Delta y=3(x+\Delta x)^2-3x^2=$
$3(x^2+2x\cdot\Delta x+\Delta x^2)-3x^2=3x^2+6x\Delta x+\Delta x^2-3x^2=$
$=6x\Delta x+\Delta x^2=3\Delta x(2x+\Delta x)$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{3\Delta x(2x+\Delta x)}{\Delta x}=\left[\frac{0}{0}\right]=$$

$$=\lim_{\Delta x\to 0}3(2x+\Delta x)=3\left(\lim_{x\to 0}(2x)+\lim_{x\to 0}\Delta x\right)=3(2x+0)=6x$$

2) $y=\sin x$

$\Delta y=\sin(x+\Delta x)-\sin x=$ $\quad \sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$=2\sin\frac{x+\Delta x-x}{2}\cos\frac{x+\Delta x+x}{2}=$ $\quad \sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$

$$=2\sin\frac{\Delta x}{2}\cos\frac{2x+\Delta x}{2}=2\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)$$

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{2\cdot\sin\frac{\Delta x}{2}\cos\left(x+\frac{\Delta x}{2}\right)}{\Delta x}=$$

$$=\lim_{\Delta x\to 0}(\cos(x+\Delta x))\cdot\lim_{\Delta x\to 0}\frac{2\cdot\sin\left(\frac{\Delta x}{2}\right)}{\Delta x}=$$

$$=\cos x\cdot 1=\cos x$$

№7.1.6.

$f'(x)$ - ?, базовый уровень.

1) $f(x) = \frac{9}{\sqrt[3]{x^2}} - 5^{x+1} = 9 \cdot x^{-\frac{2}{3}} - 5^{x+1} =$

$= (9 \cdot x^{-2/3})'_x - (5^{x+1})'_x = 9 \cdot (x^{-2/3})'_x - (5^{x+1})'_{x+1} \cdot (x+1)'_x =$

$= 9(-\frac{2}{3}) x^{-2/3-1} - 5^{x+1} \ln 5 \cdot (x'_x + 1'_x) = -6x^{-5/3} - 5^{x+1} \ln 5$

$\cdot (1x^{+1} + 0) = -6x^{-5/3} - 5^{x+1} \ln 5 \cdot 1 = -6x^{-5/3} - 5^{x+1} \ln 5$

2) $f(x) = (x^4 - x)(3tgx - 1)$

$f'(x) = ((x^4 - x)(3tgx - 1))' = (x^4 - x)'(3tgx - 1) + (x^4 - x)(3tgx - 1)' =$

$= (4x^3 - 1)(3tgx - 1) + (x^4 - x) \cdot \frac{3}{\cos^2 x}$

№7.1.27

1) $y = \sin^2 x$

$y'_x = (\sin^2 x)'_x = ((\sin x)^2)'_x = 2\sin x \cos x = \sin 2x$.

2) $y = \ln(arctg\, 3x)$

$y' = (\ln(arctg\, 3x))'_x = (\ln(arctg(3x)))'_x =$

$= [(\ln\square)', (arctg\,\square)', (3x)'] = [\ln x' = \frac{1}{x}, arctg\, x' = \frac{1}{1+x^2}] =$

$= \frac{1}{arctg\, 3x} \cdot \frac{1}{1+9x^2} \cdot 3 = \frac{3}{(1+9x^2)\, arctg(3x)}$

№7.1.58

$$(\ln y)'_x = \frac{y'}{y} \Rightarrow y' = y\cdot(\ln y)'_x$$

1) $y = x^{\sin x}$

$$\ln y = \ln x^{\sin x}; \quad \ln y = \sin x \ln x; \quad (\ln y)' = (\sin x \cdot \ln x)';$$

$$\frac{y'}{y} = (\sin x)'\cdot \ln x + \sin x(\ln x)'$$

$$\frac{y'}{y} = \cos x \ln x + \frac{\sin x}{x} \Rightarrow y' = y\left(\cos x \ln x + \frac{\sin x}{x}\right) \Rightarrow$$

$$\Rightarrow y' = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$$

2) $y = \dfrac{(x-1)^3\sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$

$$\ln y = \ln \frac{(x-1)^3\sqrt{x+2}}{\sqrt[3]{(x+1)^2}}; \quad \ln y = \ln(x-1)^3 + \ln\sqrt{x+2} - \ln\sqrt[3]{(x+1)^2}$$

$$\ln y = 3\ln(x-1) + \frac{1}{2}\ln(x+2) - \frac{2}{3}\ln(x+1)$$

$$(\ln y)' = \left(3\ln(x-1) + \frac{1}{2}\ln(x+2) - \frac{2}{3}\ln(x+1)\right)'$$

$$\frac{y'}{y} = \frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)} \Rightarrow$$

$$\Rightarrow y' = \frac{(x-1)^3\sqrt{x+2}}{\sqrt[3]{(x+1)^2}}\left(\frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)}\right)$$