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Подгруппа 1

Лекция (02.11.20)

№11.4.1

$z = e^{x^2+y^2} \quad x = a\cdot\cos t \quad y = a\cdot\sin t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

1) $z = e^{x^2+y^2} = e^{(a\cdot\cos t)^2 + (a\cdot\sin t)^2} = e^{a^2(\cos^2 t + \sin^2 t)} = e^{a^2\cdot 1} = e^{a^2}$

Тогда

$$\frac{dz}{dt} = (e^{a^2})'_t = 0$$

2) $\frac{\partial z}{\partial x} = (e^{x^2+y^2})'_x = e^{x^2+y^2}(x^2+y^2)'_x = e^{x^2+y^2}\cdot 2x$

$\frac{\partial z}{\partial y} = (e^{x^2+y^2})'_y = e^{x^2+y^2}(x^2+y^2)'_y = e^{x^2+y^2}\cdot 2y$

$\frac{dx}{dt} = (a\cdot\cos t)'_t = a(-\sin t) = -a\sin t$

$\frac{dy}{dt} = (a\cdot\sin t)'_t = a\cos t$

Тогда

$$\frac{dz}{dt} = 2x \cdot e^{x^2+y^2} \cdot (-a\sin t) + 2y \cdot e^{x^2+y^2} \cdot a\cos t =$$

$$= 2a\cos t \cdot e^{(a\cos t)^2+(a\sin t)^2} \cdot (-a)\sin t +$$

$$+ 2a\sin t \cdot e^{(a\cos t)^2+(a\sin t)^2} \cdot a\cos t =$$

$$= -a^2\sin(2t) \cdot e^{a^2(\cos^2 t+\sin^2 t)} + a^2\sin(2t) \cdot e^{a^2(\cos^2 t+\sin^2 t)} =$$

$$= -a^2\sin(2t)\, e^{a^2} + a^2\sin(2t)\, e^{a^2} = 0$$

№ 11.4.2

$z = x^5 + 2xy - y^3 \qquad x = \cos 2t \qquad y = \operatorname{arctg} t$

$\frac{dz}{dt} - ?$

1) $\frac{\partial z}{\partial x} = (x^5 + 2xy - y^3)'_x = 5x^4 + 2y - 0 = 5x^4 + 2y$

$\frac{\partial z}{\partial y} = (x^5 + 2xy - y^3)'_y = 2x - 3y^2$

2) $\frac{dx}{dt} = (\cos 2t)'_t = -\sin 2t\,(2t)'_t = -2\sin 2t$

$\frac{dy}{dt} = (\operatorname{arctg} t)'_t = \frac{1}{1+t^2}$

3) $\frac{dz}{dt} = (5x^4 + 2y)\cdot(-2\sin 2t) + (2x - 3y^2)\frac{1}{1+t^2} =$

$= -2(5x^4 + 2y)\sin 2t + (2x - 3y^2)\frac{1}{1+t^2}$

№ 11.4.3

$z = xy + xyv + yuv$

$x = \sin t \quad y = \ln t \quad u = e^t \quad v = \operatorname{arctg} t$

$\frac{dz}{dt} - ?$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial u}\frac{du}{dt} + \frac{\partial z}{\partial v}\frac{dv}{dt}$$

1) $\frac{\partial z}{\partial x} = (xy + xyv + yuv)'_x = y + yv + 0 = y + yv$

$\frac{\partial z}{\partial y} = x + xv + uv$

$\frac{\partial z}{\partial u} = yv$

$\frac{\partial z}{\partial v} = xy + yu$

2) $\frac{dx}{dt} = \cos t \quad \frac{dy}{dt} = \frac{1}{t} \quad \frac{du}{dt} = e^t \quad \frac{dv}{dt} = \frac{1}{1+t^2}$

3) $\frac{dz}{dt} = (y + yv)\cdot\cos t + (x + xv + uv)\cdot\frac{1}{t} + yv\cdot e^t +$

$+ (xy + yu)\left(\frac{1}{1+t^2}\right) = y(1+v)\cos t + \frac{x + xv + uv}{t} +$

$+ yv\cdot e^t + \frac{y(x+u)}{1+t^2}$

№ 11.4.11

$z = 3^{x^2} \cdot \operatorname{arctg} y \quad x = \frac{u}{v} \quad y = uv$

$\frac{\partial z}{\partial u} - ? \quad \frac{\partial z}{\partial v} - ?$

1) $\frac{\partial z}{\partial x} = (3^{x^2} \cdot \operatorname{arctg} y)'_x = 3^{x^2} \cdot \ln 3 \cdot (x^2)'_x \operatorname{arctg} y =$
$= 3^{x^2} \cdot \ln 3 \cdot 2x \cdot \operatorname{arctg} y = 3^{x^2} \cdot 2x \cdot \ln 3 \cdot \operatorname{arctg} y$

$\frac{\partial z}{\partial y} = (3^{x^2} \cdot \operatorname{arctg} y)'_y = 3^{x^2} \cdot \frac{1}{1+y^2}$

2) $\frac{\partial x}{\partial u} = x'_u = \left(\frac{u}{v}\right)'_u = \frac{1}{v}$

$\frac{\partial x}{\partial v} = x'_v = \left(\frac{u}{v}\right)'_v = -\frac{u}{v^2}$

$\frac{\partial y}{\partial u} = (uv)'_u = v \cdot 1 = v$

$\frac{\partial y}{\partial v} = (uv)'_v = u \cdot 1 = u$

3) $\frac{\partial z}{\partial u} = 3^{x^2} \cdot 2x \cdot \ln 3 \cdot \operatorname{arctg} y \cdot \frac{1}{v} + 3^{x^2} \cdot \frac{1}{1+y^2} \cdot v$

$\frac{\partial z}{\partial v} = 3^{x^2} \cdot 2x \cdot \ln 3 \cdot \operatorname{arctg} y \left(-\frac{u}{v^2}\right) + 3^{x^2} \cdot \frac{1}{1+y^2} \cdot u$

№ 4.12

$$z = \frac{x^2}{y} \qquad x = u - 2v \qquad y = 2u + v$$

1) $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$

$dx = \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv$

$dy = \frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv$

2) $\frac{\partial z}{\partial x} = \left(\frac{x^2}{y}\right)'_x = \frac{2x}{y} \qquad \frac{\partial z}{\partial y} = -\frac{x^2}{y^2}$

3) $\frac{\partial x}{\partial u} = 1 \qquad \frac{\partial x}{\partial v} = -2 \qquad \frac{\partial y}{\partial u} = 2 \qquad \frac{\partial y}{\partial v} = 1$

$dx = du - 2dv \qquad dy = 2du + dv$

4) $$dz = \frac{2x}{y}(du - 2dv) - \frac{x^2}{y^2}(2du + dv) =$$

$$= \frac{x}{y} \cdot 2 \cdot \left(1 - \frac{x}{y}\right) du - \frac{x}{y}\left(4 + \frac{x}{y}\right) dv =$$

$$= \frac{x}{y}\left(2\left(1 - \frac{x}{y}\right) du - \left(4 + \frac{x}{y}\right) dv\right) =$$

$$= \frac{u - 2v}{2u + v}\left(2\left(1 - \frac{u - 2v}{2u + v}\right) du - \left(4 + \frac{u - 2v}{2u + v}\right) dv\right) =$$

$$= \frac{u-2v}{2u+v} \cdot \frac{1}{2u+v}\left(2(2u+v-(u-2v))du + (4(2u+v)-(u-2v))dv\right) =$$

$$= \frac{u-2v}{(2u+v)^2}\left(2(u+3v)du + (9u+2v)dv\right)$$

№ 11.4.4

$z = x^2 + y^2 + xy \quad x = a\sin t \quad y = a\cos t.$

$\frac{dz}{dt} - ? \qquad \frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}.$

1) $\frac{\partial z}{\partial x} = (x^2+y^2+xy)'_x = 2x+y$

$\frac{\partial z}{\partial y} = (x^2+y^2+xy)'_y = 2y+x$

2) $\frac{dx}{dt} = (a\sin t)'_t = a\cos t$

$\frac{dy}{dt} = (a\cos t)'_t = -a\sin t$

3) $\frac{dz}{dt} = a\cos t\,(2x+y) - a\sin t\,(2y+x) =$

$= a(2x+y)\cos t - a(2y+x)\sin t$

№ 11.4.5

$z = \cos(2t + 4x^2 - y), \quad x = \frac{1}{t}, \quad y = \frac{\sqrt{t}}{\ln t}$

№11.4.5

$z = \cos(2t + 4x^2 - y) \quad x = \frac{1}{t} \quad y = \frac{\sqrt{t}}{\ln t}$

1) $\frac{\partial z}{\partial x} = -\sin(2t + 4x^2 - y) \cdot 8x$

$\frac{\partial z}{\partial y} = -\sin(2t + 4x^2 - y)(-1) = \sin(2t + 4x^2 - y)$

$\frac{\partial z}{\partial t} = -\sin(2t + 4x^2 - y) 2$

2) $\frac{dx}{dt} = -\frac{1}{t^2} \qquad \frac{dy}{dt} = \frac{(\sqrt{t})' \ln t - \sqrt{t}(\ln t)'}{\ln^2 t} =$

$= \frac{\frac{1}{2\sqrt{t}} \ln t - \sqrt{t} \frac{1}{t}}{\ln^2 t} = \frac{\frac{\ln t}{2\sqrt{t}} - \frac{\sqrt{t}}{t}}{\ln^2 t} =$

$= \frac{t \ln t - 2t}{2t\sqrt{t} \ln^2 t} = \frac{\ln t - 2}{2\sqrt{t} \ln^2 t}$

3) $\frac{dz}{dt} = -\sin(2t + 4x^2 - y) 2 - \sin(2t + 4x^2 - y)\left(-\frac{8x}{t^2}\right) +$

$+ \sin(2t + 4x^2 - y)\left(\frac{\ln t - 2}{2\sqrt{t} \ln^2 t}\right) =$

$= -\sin(2t + 4x^2 - y)\left(2 - \frac{8x}{t^2} - \frac{\ln t - 2}{2\sqrt{t} \ln^2 t}\right)$

№11.4.6

$z = x^2 y^3 u \quad x = t \quad y = t^2 \quad u = \sin t$

1) $\frac{\partial z}{\partial x} = 2xy^3u \quad \frac{\partial z}{\partial y} = 3x^2y^2u \quad \frac{\partial z}{\partial u} = x^2y^3$

2) $\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t \quad \frac{du}{dt} = \cos t$

3) $\frac{dz}{dt} = 2xy^3u + 6x^2y^2u\,t + x^2y^3\cos t$

№11.4.7

$z = e^{xy} \ln(x+y) \quad x = t^3 \quad y = 1 - t^3$

1) $e^{t^3(1-t^3)} \ln(t^3 + 1 - t^3) = e^{t^3 - t^6} \ln 1 =$
$= e^{t^3 - t^6} \cdot 0 = 0$

2) $\frac{dz}{dt} = 0$

№11.4.8

$z = xy\,\mathrm{arctg}(xy) \quad x = t^2 + 1 \quad y = t^3$

1) $\frac{\partial z}{\partial x} = (xy)'\,\mathrm{arctg}(xy) + xy(\mathrm{arctg}(xy))' =$
$= y\,\mathrm{arctg}(xy) + \frac{xy^2}{1+x^2y^2}$

$$\frac{\partial z}{\partial y} = x\,\mathrm{arctg}(xy) + \frac{x^2 y}{1+x^2y^2}$$

2) $\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2$

3)

$$\frac{dz}{dt} = \left(y\,\mathrm{arctg}(xy) + \frac{xy^2}{1+x^2y^2}\right)2t + \left(x\,\mathrm{arctg}(xy) + \frac{x^2y}{1+x^2y^2}\right)3t^2$$

№ 11.4.9

$z = e^{2x-3y} \qquad x = \mathrm{tg}\, t \qquad y = t^2 - t$

1) $\frac{\partial z}{\partial x} = e^{2x-3y} \cdot 2 \qquad \frac{\partial z}{\partial y} = e^{2x-3y} \cdot (-3)$

2) $\frac{dx}{dt} = \frac{1}{\cos^2 t} \qquad \frac{dy}{dt} = 2t - 1$

3) $\frac{dz}{dt} = e^{2x-3y}\left(\frac{2}{\cos^2 t} - 3(2t-1)\right)$

№ 11.4.10

$z = x^y \qquad x = \ln t \qquad y = \sin t$

1) $\frac{\partial z}{\partial x} = yx^{y-1} \qquad \frac{\partial z}{\partial y} = x^y \ln x$

2) $\frac{dx}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = \cos t$

9) $\frac{dz}{dt} = \frac{y x^{y-1}}{t} + x^{y} \ln x \cos t$