Войтенко Игорь Александрович

Подгруппа №1

№8.3.8.

$$\int \frac{3x+5}{(x^2-2x+17)^2}dx = \left[\begin{array}{l} A=3,\ B=5,\ p=-2,\ q=17,\ n=2 \\ x^2-2x+17 \Rightarrow D=-64<0 \Rightarrow \end{array} \right.$$

$$\left. \Rightarrow 3x+5 = 4(2x-2)+(5+8) = 4(2x-2)+13 \right] =$$

$$= 4\int \frac{(2x-2)dx}{(x^2-2x+17)^2} + 13\int \frac{dx}{(x^2-2x+17)^2} = \left[\begin{array}{l} 1)\ t = x^2-2x+17 \Rightarrow dt=(2x-2)dx \\ 2)\ y = x-1 \Rightarrow dy = dx \end{array} \right.$$

$$\left. a=\sqrt{17-1}=4 \right] = 4\int \frac{dt}{t^2} + 13\int \frac{dy}{(y^2+4^2)^2} = 4\int t^{-2}dt +$$

$$+13\left(\frac{1}{2(2-1)4^2} \cdot \frac{y}{y^2+4^2} + \frac{1}{16} \cdot \frac{2\cdot 2-3}{2\cdot 2-2} \cdot \int \frac{dy}{y^2+4^2} \right) =$$

$$= 4\int t^{-2}dt + 13\left(\frac{y}{32(y^2+16)} + \frac{1}{32}\int \frac{dy}{y^2+4^2} \right) =$$

$$= 4\left(-\frac{1}{t}\right) + 13\left(\frac{y}{32(y^2+16)} + \frac{1}{32} \cdot \frac{1}{4}\,\mathrm{arctg}\,\frac{y}{4} \right) + C =$$

$$= \frac{13}{32}\left(\frac{x-1}{x^2-2x+17} + \frac{1}{4}\,\mathrm{arctg}\,\frac{x-1}{4} \right) - \frac{4}{x^2-2x+17} + C$$

№8.3.2.

$$\int \frac{4dx}{x+3} = 4\int \frac{dx}{x+3} = [A=4;\ a=-3] =$$

$$= A\ln|x-a| + C = 4\ln|x+3| + C$$

№8.3.3.

$$\int \frac{dx}{(x-1)^5} = [A=1,\ a=1,\ k=5] = \frac{A}{1-k} \cdot \frac{1}{(x-a)^{k-1}} + C$$

$$= -\frac{1}{4} \frac{1}{(x-1)^4} + C = -\frac{1}{4(x-1)^4} + C$$

№8.3.4.

$$\int \frac{11dx}{(x+2)^3} = [A=11;\ a=-2,\ k=3] =$$

$$= -\frac{11}{2} \cdot \frac{1}{(x+2)^2} + C = -\frac{11}{2(x+2)^2} + C$$

№8.3.6.

$$\int \frac{dx}{x^2+10x+29} = \left[\begin{array}{l} y = x + \frac{p}{2} \Rightarrow y = x+5 \Rightarrow \\ \Rightarrow dy = dx \end{array} \right] =$$

$$= \left[a = \sqrt{29-25} = 2,\ x^2+10x+29 = y^2+2^2 \right] =$$

$$= \int \frac{dy}{y^2+2^2} = \frac{1}{2}\operatorname{arctg}\frac{y}{2} + C = \frac{1}{2}\operatorname{arctg}\frac{x+5}{2} + C$$

№8.3.6.

$$\int \frac{(x+6)dx}{x^2-2x+17} = \left[\begin{array}{l} A=1;\ B=6;\ p=-2;\ q=17 \\ x^2-2x+17=0 \Rightarrow D=-64<0 \end{array} \Rightarrow \right.$$

$$\left. \Rightarrow x+6 = \frac{1}{2}(2x-2)+\left(6-\frac{1\cdot(-2)}{2}\right) = \frac{1}{2}(2x-2)+(6+1) \right] =$$

$$= \frac{1}{2}\int \frac{(2x-2)dx}{x^2-2x+17} + 7\int \frac{dx}{x^2-2x+17} = \left[\begin{array}{l} 1)\ t = x^2-2x+17 \Rightarrow dt = (2x-2)dx \\ 2)\ y = x-1 \Rightarrow dy = dx \end{array} \right.$$

$$\left. a = \sqrt{17-1} = 4 \right] = \frac{1}{2}\int \frac{dt}{t} + 7\int \frac{dy}{y^2+4^2} =$$

$$= \frac{1}{2}\ln|t| + 7\cdot\frac{1}{4}\operatorname{arctg}\frac{y}{4} + C =$$

$$= \frac{1}{2}\ln|x^2-2x+17| + \frac{7}{4}\operatorname{arctg}\frac{x-1}{4} + C$$

№8.3.7.

$$\int \frac{(4x-1)dx}{x^2+x+1} = \left[\begin{array}{l} A=4;\ B=-1;\ p=1;\ q=1 \\ x^2+x+1=0 \Rightarrow D=-3<0 \end{array} \Rightarrow \right.$$

$$\left. \Rightarrow 4x-1 = \frac{4}{2}(2x+1)+\left(-1-\frac{4\cdot 1}{2}\right) = 2(2x+1)+(-3) \right] =$$

$$= 2\int \frac{(2x+1)dx}{x^2+x+1} - 3\int \frac{dx}{x^2+x+1} = \left[\begin{array}{l} 1)\ t = x^2+x+1 \Rightarrow dt = (2x+1)dx \Rightarrow \\ 2)\ y = x+\frac{1}{2} \end{array} \right.$$

$$\left. \Rightarrow a = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2} \right] = 2\int \frac{dt}{t} - 3\int \frac{dy}{y^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= 2\ln|t| - \frac{3}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{y}{\frac{\sqrt{3}}{2}} + C =$$

$$= 2\ln|x^2+x+1| - 2\sqrt{3}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + C$$