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Подгруппа № 1

Лекция (19.10.20)

1) Частные приращение:

$\Delta_x z = f(x+\Delta x\,; y) - f(x, y)$

$\Delta_y z = f(x; y+\Delta y) - f(x, y)$

2) Полное приращение:

$\Delta z = f(x+\Delta x, y+\Delta y) - f(x, y)$

3) $\Delta z \neq \Delta_x z + \Delta_y z$

4) Частные производные:

$z'_x = \lim\limits_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x}$; $\frac{\partial z}{\partial x}$; $\frac{\partial f}{\partial x}(x;y)$; $\frac{\partial}{\partial x} z$;

$\frac{\partial}{\partial x} f$, $\frac{\partial}{\partial x} f(x;y)$

$z'_y = \lim\limits_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y}$, $\frac{\partial z}{\partial y}$; $\frac{\partial f}{\partial y}(x;y)$, $\frac{\partial}{\partial y} z$;

$\frac{\partial}{\partial y} f$, $\frac{\partial}{\partial y} f(x;y)$

№11.3.1

$z = xy^2 - \frac{x}{y}$ $\quad \Delta_x z, \Delta_y z, \Delta z$ - ?

$M_0(3; -2)$, $\Delta x = 0{,}1$, $\Delta y = -0{,}05$

1) $M_0(3; -2)$

] $x_0 = 3$, $y_0 = -2$

Тогда

$x = x_0 + \Delta x = 3 + 0{,}1 = 3{,}1$ $\quad y = y_0 + \Delta y = -2 - 0{,}05 = -2{,}05$

Тогда $M_1(3{,}1; -2{,}05)$

2) $z(M_0)=z(3;-2)=3\cdot(-2)^2-\frac{3}{-2}=12+1,5=13,5$

$z(x_0+\Delta x; y_0)=z(3,1;-2)=3,1\cdot(-2)^2-\frac{3,1}{-2}=$

$=12,4+1,55=13,95$

$z(x_0;y_0+\Delta y)=z(3;-2,05)=3\cdot(-2,05)^2-\frac{3}{-2,05}=$

$=3\cdot4,2025+\frac{3}{2,05}\approx12,6075+1,4634=14,0709\approx$

$\approx14,07$

$z(M_1)=z(x,y)=3,1\cdot(-2,05)^2-\frac{3,1}{-2,05}=$

$=3,1\cdot4,2025+\frac{3,1}{2,05}\approx13,0278+1,5122=$

$=14,54$

3) $\Delta_x z=z(x_0+\Delta x;y_0)-z(x_0,y_0)=13,95-13,5=$

$=0,45$

$\Delta_y z=z(x_0;y_0+\Delta y)-z(x_0;y_0)=14,07-13,5=$

$=0,57$

4) $\Delta z=z(x_0+\Delta x;y_0+\Delta y)-z(x_0;y_0)=$

$=14,54-13,5=1,04$

№11.3.2

$z=x^2y$, $M_0(1;2)$ $\Delta x=0,1$; $\Delta y=-0,2$

1) $M_0(1;2)\Rightarrow x_0=1$, $y_0=2\Rightarrow$

$x=x_0+\Delta x=1+0,1=1,1$

$y=y_0+\Delta y=2-0,2=1,8$

2) $z(x_0, y_0) = 1^2 \cdot 2 = 2$

$z(x_0 + \Delta x; y_0) = (1,1)^2 \cdot 2 = 1,21 \cdot 2 = 2,42$

$z(x_0; y_0 + \Delta y) = 1 \cdot 1,8 = 1,8$

$z(x_0 + \Delta x; y_0 + \Delta y) = 1,21 \cdot 1,8 = 2,178$

3) $\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 2,42 - 2 = 0,42$

$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 1,8 - 2 = -0,2$

4) $\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) =$

$= 2,178 - 2 = 0,178$

Дифференциал функции

$dz = \underbrace{z'_x dx}_{d_x z} + \underbrace{z'_y dy}_{d_y z}$

частные дифференциалы

$dx = \Delta x$, $dy = \Delta y$

$z'_x = f'_x(x; y)$, $f'_x(x_0; y_0)$

$z'_y = f'_y(x; y)$, $f'_y(x_0; y_0)$

dz – полный дифференциал

$$f(x_0 + \Delta x; y_0 + \Delta y) \approx f(x_0; y_0) + f'_x(x_0; y_0) \cdot \Delta x + f'_y(x_0; y_0) \cdot \Delta y$$

Линеаризация функции $z = f(x, y)$ в окрестности точки $M_0(x_0; y_0)$

№ 11.3.9

$$z=\frac{x}{y^3}+\frac{y}{x^3}-\frac{1}{6x^2y} \qquad z'_x-?\ ,\ z'_y-?$$

$$z'_x=\left(\frac{x}{y^3}+\frac{y}{x^3}-\frac{1}{6x^2y}\right)'_x=\frac{1}{y^3}(x)'+y\cdot(x^{-3})'-\frac{1}{6y}(x^{-2})'=$$

$$=\frac{1}{y^3}\cdot 1+y\cdot(-3)\cdot x^{-4}-\frac{1}{6y}\cdot(-2)\cdot x^{-3}=$$

$$=\frac{1}{y^3}-\frac{3y}{x^4}+\frac{1}{3x^3y}$$

$$z'_y=\left(\frac{x}{y^3}+\frac{y}{x^3}-\frac{1}{6x^2y}\right)'_y=x\cdot(y^{-3})'+\frac{1}{x^3}\cdot(y)'-\frac{1}{6x^2}\cdot(y^{-1})'=$$

$$=-\frac{3x}{y^4}+\frac{1}{x^3}+\frac{1}{6x^2y^2}$$

№ 11.3.10

$$z=\frac{x^2-2xy}{y^2+2xy+1} \qquad z'_x,\ z'_y-?$$

$$z'_x=\left(\frac{x^2-2xy}{y^2+2xy+1}\right)'_x=\frac{(x^2-2xy)'_x\cdot(y^2+2xy+1)-(x^2-2xy)\cdot(y^2+2xy+1)'_x}{(y^2+2xy+1)^2}=$$

$$=\frac{(2x-2y)(y^2+2xy+1)-(x^2-2xy)\cdot 2y}{(y^2+2xy+1)^2}$$

$$z'_y=\frac{-2x(y^2+2xy+1)-(x^2-2xy)(2y+2x)}{(y^2+2xy+1)^2}$$

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$z = \cos\left(\frac{x^2+y^2}{x^3+y^3}\right)$ $z'_x,\ z'_y,\ \left(\frac{\partial z}{\partial x}\right) d_x z,\ d_y z,\ dz - ?$

1) $z'_x = \left(\cos\frac{x^2+y^2}{x^3+y^3}\right)'_x = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot$

$\cdot\left(\frac{x^2+y^2}{x^3+y^3}\right)'_x = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{2x(x^3+y^3)-(x^2+y^2)3x^2}{(x^3+y^3)^2} =$

$= \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}$

$z'_y = \ldots = \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}$

2) $d_x z = z'_x dx =$

$= \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}dx$

$d_y z = z'_y dy =$

$= \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}dy$

3) $dz = d_x z + d_y z =$

$= \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}dx +$

$+ \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\sin\frac{x^2+y^2}{x^3+y^3}dy =$

$$= \frac{1}{(x^3+y^3)^2} \sin\frac{x^2+y^2}{x^3+y^3}\left(3x^2(x^2+y^2) - 2x(x^3+y^3)dx - 3y^2(x^2+y^2) + 2y(x^3+y^3)\right)$$

№ 11.5.17

$$u = \frac{x}{\sqrt{y^2+z^2}} \qquad du - ?$$

$$du = u'_x dx + u'_y dy + u'_z dz$$

$$u'_x = \left(\frac{x}{\sqrt{y^2+z^2}}\right)'_x = \frac{1}{\sqrt{y^2+z^2}}$$

$$u'_y = \left(\frac{x}{\sqrt{y^2+z^2}}\right)'_y = \frac{-xy}{\sqrt{(y^2+z^2)^3}}$$

$$u'_z = \frac{-xz}{\sqrt{(y^2+z^2)^3}}$$

$$du = \frac{1}{\sqrt{y^2+z^2}} - \frac{xy+xz}{\sqrt{(y^2+z^2)^3}}$$