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Подгруппа №1

Часть 2 (25.01.21)

№8.2.33

$$\int \cos(6x+1)dx = \left[\begin{array}{l} 6x+1 = t \\ dt = d(6x+1) \Rightarrow dt = 6dx \Rightarrow \frac{1}{6}dt = dx \end{array}\right] =$$
$$= \int \cos t \, \frac{1}{6} dt = \frac{1}{6}\int \cos t \, dt = \frac{\sin t}{6} + C = \frac{\sin(6x+1)}{6} + C$$

№8.2.34

$$\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = \left[\begin{array}{l} 5x-2 = t \\ dt = d(5x-2) \Rightarrow dt = 5dx \Rightarrow \frac{1}{5}dt = dx \end{array}\right] =$$
$$= \frac{1}{5}\int \frac{dt}{\sqrt[3]{t^4}} = \frac{1}{5}\int t^{-\frac{4}{3}} dt = \frac{1}{5} \cdot \frac{t^{-\frac{4}{3}+1}}{-\frac{4}{3}+1} + C =$$
$$= \frac{1}{5} \cdot \frac{t^{-\frac{1}{3}}}{-\frac{1}{3}} + C = -\frac{3}{5\sqrt[3]{t}} + C = -\frac{3}{5\sqrt[3]{5x-2}} + C$$

№8.2.35

$$\int \frac{\sqrt{\operatorname{tg} x}\,dx}{\cos^2 x} = \left[\begin{array}{l} \operatorname{tg} x = t \\ dt = d(\operatorname{tg} x) \Rightarrow dt = \frac{dx}{\cos^2 x} \end{array}\right] =$$
$$= \int \sqrt{t}\,dt = \int t^{\frac{1}{2}} dt = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C =$$
$$= \frac{2\sqrt{t^3}}{3} + C = \frac{2\sqrt{\operatorname{tg}^3 x}}{3} + C$$

№8.2.36

$$\int \frac{e^x\,dx}{e^{2x}+9} = \left[\begin{array}{l} e^x = t \\ dt = d(e^x) \Rightarrow dt = e^x dx \end{array}\right] = \int \frac{dt}{t^2+3^2} =$$
$$= \frac{1}{3}\operatorname{arctg}\frac{t}{3} + C = \frac{1}{3}\operatorname{arctg}\frac{e^x}{3} + C$$

№8.2.37

$$\int \frac{x^5\,dx}{\sqrt{x^6+7}} = \left[\begin{array}{l} x^6+7 = t \\ dt = d(x^6+7) \Rightarrow dt = 6x^5 dx \Rightarrow \frac{1}{6}dt = x^5 dx \end{array}\right] =$$
$$= \frac{1}{6}\int \frac{dt}{\sqrt{t}} = \frac{2\sqrt{t}}{6} + C = \frac{\sqrt{x^6+7}}{3} + C$$

№8.2.38

$$\int \frac{dx}{\arccos x \cdot \sqrt{1-x^2}} = \left[\begin{array}{l} \arccos x = t \\ dt = d(\arccos x) \Rightarrow dt = -\frac{dx}{\sqrt{1-x^2}} \end{array} \right]$$

$$= -\int \frac{dt}{t} = -\ln t + C = -\ln(\arccos x) + C$$

№8.2.39

$$\int \frac{(2x+3)dx}{(x^2+3x-1)^4} = \left[\begin{array}{l} x^2+3x-1 = t \\ dt = (2x+3)dx \end{array} \right] = \int \frac{dt}{t^4} =$$

$$= \int t^{-4} dt = \frac{t^{-4+1}}{-4+1} + C = \frac{t^{-3}}{-3} + C = -\frac{1}{3t^3} + C =$$

$$= -\frac{1}{3(x^2+3x-1)^3} + C$$

№8.2.40

$$\int \cos^{11} 2x \sin 2x \, dx = \left[\begin{array}{l} \cos 2x = t \\ dt = d(\cos 2x) \Rightarrow dt = -2\sin 2x dx \end{array} \right]$$

$$= -\frac{1}{2} \int t^{11} dt = -\frac{1}{2} \cdot \frac{t^{12}}{12} + C = -\frac{t^{12}}{24} + C = -\frac{\cos^{12} 2x}{24} + C$$

№8.2.41

$$\int \frac{7^{\sqrt{x}} dx}{\sqrt{x}} = \left[\begin{array}{l} \sqrt{x} = t \\ dt = d(\sqrt{x}) \Rightarrow 2dt = \frac{dx}{\sqrt{x}} \end{array} \right] = 2\int 7^t dt =$$

$$= \frac{2 \cdot 7^t}{\ln 7} + C = \frac{2 \cdot 7^{\sqrt{x}}}{\ln 7} + C = \frac{2}{\ln 7} \cdot 7^{\sqrt{x}} + C$$

№8.2.42

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx = \left[\begin{array}{l} \frac{1}{x} = t \\ dt = d(\frac{1}{x}) \Rightarrow -dt = \frac{dx}{x^2} \end{array} \right] = -\int e^t dt =$$

$$= -e^t + C = -e^{\frac{1}{x}} + C$$

№8.2.43

$$\int \frac{\ln 5x \, dx}{x} = \left[\begin{array}{l} \ln 5x = t \\ dt = d(\ln 5x) \Rightarrow dt = \frac{dx}{x} \end{array} \right] =$$

$$= \int t \, dt = \frac{t^{1+1}}{1+1} + C = \frac{t^2}{2} + C = \frac{\ln^2 5x}{2} + C$$

№8.2.44

$\int ctg\,x\,dx = \ln|\sin x| + C$

№8.2.45

$\int 4x\sqrt[3]{x^2+8}\,dx = \left[\begin{matrix} x^2+8 = t \\ dt = d(x^2+8) \Rightarrow 2dt = 4x\,dx \end{matrix}\right] =$

$= 2\int \sqrt[3]{t}\,dt = 2\cdot\frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3\sqrt[3]{t^4}}{2} + C =$

$= \frac{3\sqrt[3]{(x^2+8)^4}}{2} + C$

№8.2.46

$\int \frac{\cos x\,dx}{\sin^2 x} = \left[\begin{matrix} \sin x = t \\ dt = d(\sin x) \Rightarrow dt = \cos x\,dx \end{matrix}\right] =$

$= \int \frac{dt}{t^2} = \int t^{-2}dt = \frac{t^{-2+1}}{-2+1} + C = \frac{t^{-1}}{-1} + C =$

$= -\frac{1}{t} + C = -\frac{1}{\sin x} + C$

№8.2.47

$\int tg\,2x\,dx = \left[\begin{matrix} 2x = t \\ dt = d(2x) \Rightarrow dt = 2dx \Rightarrow \frac{1}{2}dt = dx \end{matrix}\right] =$

$= \frac{1}{2}\int tg\,t\,dt = \frac{1}{2}\cdot -\ln|\cos t| + C = -\frac{1}{2}\ln|\cos 2x| + C$

№8.2.48

$\int \frac{x\,dx}{x^4+1} = \left[\begin{matrix} x^2 = t \\ dt = d(x^2) \Rightarrow dt = 2x\,dx \Rightarrow \frac{1}{2}dt = x\,dx \end{matrix}\right] =$

$= \frac{1}{2}\int \frac{dt}{t^2+1^2} = \frac{1}{2}\,arctg\,x^2 + C$

№8.2.49.

$\int e^{-x^3}x^2\,dx = \left[\begin{matrix} -x^3 = t \\ dt = d(-x^3) \Rightarrow dt = -3x^2dx \Rightarrow -\frac{1}{3}dt = x^2dx \end{matrix}\right] =$

$= -\frac{1}{3}\int e^t dt = -\frac{e^t}{3} + C = -\frac{e^{-x^3}}{3} + C$

№8.2.50

$$\int \frac{x^2dx}{\sqrt{x^6-4}} = \left[\begin{matrix} x^3 = t \\ dt = 3x^2dx \Rightarrow \frac{1}{3}dt = x^2dx \end{matrix} \right] =$$

$$= \frac{1}{3}\int \frac{dt}{\sqrt{t^2-4}} = \frac{1}{3}\ln|t+\sqrt{t^2-4}| + C =$$

$$= \frac{1}{3}\ln|x^3+\sqrt{x^6-4}| + C$$

№8.2.51

$$\int (8\cos\frac{x}{3}-5)^2 \sin\frac{x}{3}dx = \left[\begin{matrix} 8\cos\frac{x}{3}-5 = t \\ dt = d(8\cos\frac{x}{3}-5) \Rightarrow dt = \frac{-8\sin\frac{x}{3}}{3}dx \end{matrix} \right.$$

$$\Rightarrow -\frac{3}{8}dt = \sin\frac{x}{3}dx \Big] = -\frac{3}{8}\int t^2dt = -\frac{3}{8}\cdot\frac{t^3}{3} + C =$$

$$= -\frac{t^3}{8} + C = -\frac{(8\cos\frac{x}{3}-5)^3}{8} + C$$

№8.2.52

$$\int \frac{(3x^2-2x+7)dx}{\sqrt{x^3-x^2+7x-2}} = \left[\begin{matrix} x^3-x^2+7x-2 = t \\ dt = (3x^2-2x+7)dx \end{matrix} \right] =$$

$$= \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{x^3-x^2+7x-2} + C$$

№8.2.53

$$\int x(2x+1)^{35}dx = \left[\begin{matrix} 2x+1 = t \Rightarrow x = \frac{t-1}{2} \\ dt = d(2x+1) \Rightarrow dt = 2dx \Rightarrow \frac{1}{2}dt = dx \end{matrix} \right.$$

$$= \int \frac{(t-1)t^{35}}{4}dt = \frac{1}{4}\int (t-1)t^{35}dt = \frac{1}{4}\int (t^{36}-t^{35})dt$$

$$= \frac{1}{4}\left(\int t^{36}dt - \int t^{35}dt\right) = \frac{1}{4}\left(\frac{t^{37}}{37} - \frac{t^{36}}{36}\right) + C =$$

$$= \frac{1}{4}\left(\frac{(2x+1)^{37}}{37} - \frac{(2x+1)^{36}}{36}\right) + C$$

№8.2.54

$$\int (x-2)\sqrt{x+4}\,dx = \left[\begin{matrix} x+4=t \Rightarrow x=t-4 \\ dt=d(x+4) \Rightarrow dt=dx \end{matrix}\right] =$$

$$= \int (t-6)\sqrt{t}\,dt = \int (t^{\frac{3}{2}} - 6\sqrt{t})dt = \int t^{\frac{3}{2}}dt - 6\int \sqrt{t}\,dt =$$

$$= \frac{t^{\frac{3}{2}+\frac{2}{2}}}{\frac{3}{2}+\frac{2}{2}} - 6\,\frac{t^{\frac{1}{2}+\frac{2}{2}}}{\frac{1}{2}+\frac{2}{2}} + C = \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 6\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2\sqrt{t^5}}{5} - 4\sqrt{t^3} + C = \frac{2\sqrt{(x+4)^5}}{5} - 4\sqrt{(x+4)^3} + C$$

№8.2.55

$$\int \frac{3\sqrt{x} - 2\cos\frac{1}{x^2}}{x^3}dx = 3\int \frac{\sqrt{x}}{x^3}dx - 2\int \frac{\cos\frac{1}{x^2}}{x^3}dx =$$

$$= 3\int x^{-\frac{5}{2}}dx - 2\int \frac{\cos\frac{1}{x^2}}{x^3}dx = \left[\begin{matrix} \frac{1}{x^2}=t \\ dt=d\left(\frac{1}{x^2}\right) \Rightarrow -\frac{dt}{2}=\frac{dx}{x^3} \end{matrix}\right] =$$

$$= 3\int x^{-\frac{5}{2}}dx + \int \cos t\,dt = 3\,\frac{x^{-\frac{5}{2}+\frac{2}{2}}}{-\frac{5}{2}+\frac{2}{2}} + \sin t + C =$$

$$= 3\,\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + \sin\frac{1}{x^2} + C = -\frac{2}{\sqrt{x^3}} + \sin\frac{1}{x^2} + C = \sin\frac{1}{x^2} - \frac{2}{\sqrt{x^3}} + C$$

№8.2.56

$$\int \frac{7x+2}{\sqrt{x^2+10}}dx = 7\int \frac{x\,dx}{\sqrt{x^2+10}} + 2\int \frac{dx}{\sqrt{x^2+10}} = \left[\begin{matrix} x^2+10=t \\ dt=2x\,dx \Rightarrow \frac{dt}{2}=x\,dx \end{matrix}\right] =$$

$$= \frac{7}{2}\int \frac{dt}{\sqrt{t}} + 2\int \frac{dx}{\sqrt{x^2+10}} = 7\sqrt{t} + 2\ln|x+\sqrt{x^2+10}| + C =$$

$$= 7\sqrt{x^2+10} + 2\ln|x+\sqrt{x^2+10}| + C$$

№8.2.57

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{e^x dx}{e^{2x}+1} = \left[\begin{matrix} e^x=t \\ dt=e^x dx \end{matrix}\right] = \int \frac{dt}{t^2+1} =$$

$$= \operatorname{arctg} t + C = \operatorname{arctg} e^x + C$$

№8.2.58

$$\int \frac{x+3}{x^2+3}dx = \int \frac{x\,dx}{x^2+3} + 3\int \frac{dx}{x^2+3} = \left[\begin{matrix} x^2+3=t \\ dt=2x\,dx \Rightarrow \frac{dt}{2} \end{matrix} \right.$$

$$= \frac{1}{2}\int \frac{dt}{t} + 3\int \frac{dx}{x^2+(\sqrt{3})^2} = \frac{\ln|t|}{2} + \frac{3}{\sqrt{3}} \operatorname{arctg}\frac{x}{\sqrt{3}} + C =$$

$$= \frac{\ln(x^2+3)}{2} + \frac{3}{\sqrt{3}} \operatorname{arctg}\frac{x}{\sqrt{3}} + C$$

№8.2.59

$$\int \frac{x+4\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx = \int \frac{x\,dx}{\sqrt{1-x^2}} + 4\int \frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}dx =$$

$$= \left[\begin{matrix} 1-x^2=t \\ dt=-2x\,dx \Rightarrow -\frac{dt}{2}=x\,dx \end{matrix} ; \begin{matrix} \arcsin x = u \\ du = d(\arcsin x) \Rightarrow du = \frac{dx}{\sqrt{1-x^2}} \end{matrix} \right]$$

$$= -\frac{1}{2}\int \frac{dt}{\sqrt{t}} + 4\int \sqrt{u}\,du = -\sqrt{t} + 4\,\frac{u^{\frac{1}{2}+\frac{1}{2}}}{\frac{1}{2}+\frac{2}{2}} + C =$$

$$= -\sqrt{t} + \frac{8\sqrt{u^3}}{3} + C = \frac{8}{3}\sqrt{\arcsin^3 x} - \sqrt{1-x^2} + C$$

№8.2.60

$$\int \frac{1-6x}{(x+1)(x-1)}dx = \int \frac{1-6x}{x^2-1}dx = \int \frac{dx}{x^2-1} - 6\int \frac{x\,dx}{x^2-1} =$$

$$= \left[\begin{matrix} x^2-1=t \\ dt=2x\,dx \Rightarrow \frac{dt}{2}=x\,dx \end{matrix} \right] = \int \frac{dx}{x^2-1} - \frac{6}{2}\int \frac{dt}{t} =$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - \frac{6}{2}\ln|t| + C = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - \frac{6}{2}\ln|x^2-1| + C =$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\ln|x^2-1| + C$$

№8.2.61

$$\int (\cos^2 x - \sin^2 x)\sqrt[3]{1+\sin 2x}\,dx = \int \cos 2x\sqrt[3]{1+\sin 2x}\,dx =$$

$$= \left[\begin{matrix} 1+\sin 2x = t \\ dt = 2\cos 2x\,dx \Rightarrow \frac{dt}{2} = \cos 2x\,dx \end{matrix} \right] = \frac{1}{2}\int \sqrt[3]{t}\,dt =$$

$$= \frac{1}{2}\cdot\frac{t^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} + C = \frac{1}{2}\,\frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3\sqrt[3]{t^4}}{8} + C = \frac{3\sqrt[3]{(1+\sin 2x)^4}}{8}$$

№8.2.62

$$\int \frac{e^{tgx} - 7\sin x + 5\sin 2x}{\cos^2 x}dx = \int \frac{e^{tgx}dx}{\cos^2 x} - 7\int \frac{\sin x dx}{\cos^2 x} + 10\int \frac{\sin x \cos x dx}{\cos^2 x}$$

$$= \int \frac{e^{tgx}dx}{\cos^2 x} - 7\int \frac{\sin x dx}{\cos^2 x} + 10\int \frac{\sin x dx}{\cos x} = \left[\begin{matrix} tgx = t; & \cos x = u \\ dt = \frac{dx}{\cos^2 x}; & -du = \sin x dx \end{matrix} \right] =$$

$$= \int e^t dt + 7\int \frac{du}{u^2} - 10\int \frac{du}{u} = e^t + 7\frac{u^{-2+1}}{-2+1} - 10\ln|u| + C =$$

$$= e^t - \frac{7}{u} - 10\ln|u| + C = e^{tgx} - \frac{7}{\cos x} - 10\ln|\cos x| + C$$

№8.2.63

$$\int \sqrt{16 - x^2}\,dx = \left[\begin{matrix} x = 4\sin t \\ dx = d(4\sin t) \Rightarrow dx = 4\cos t\,dt \end{matrix} \right] =$$

$$= \int \sqrt{16 - 16\sin^2 t}\,4\cos t\,dt = \int 4\cos t \cdot 4\cos t\,dt = 16\int \cos^2 t\,dt =$$

$$= 16\int \frac{1+\cos 2t}{2}dt = 8\int (1+\cos 2t)dt = 8\int dt + 8\int \cos 2t\,dt$$

$$= 8t + 4\sin 2t + C = \left[t = \arcsin\frac{x}{4} \right]$$

$$= 8\arcsin\frac{x}{4} + 8\sin\left(\arcsin\frac{x}{4}\right)\cos\left(\arcsin\frac{x}{4}\right) + C$$

$$= 8\arcsin\frac{x}{4} + 8\cdot\frac{x}{4}\cdot\sqrt{1-\frac{x^2}{16}} + C =$$

$$= 8\arcsin\frac{x}{4} + 2x\sqrt{\frac{16-x^2}{16}} + C = 8\arcsin\frac{x}{4} + \frac{x}{2}\sqrt{16-x^2} + C$$

№8.2.64

$$\int \frac{dx}{1+\sqrt{x}} = \left[\begin{matrix} x = t^2 \Rightarrow t = \sqrt{x} \\ dx = 2t\,dt \end{matrix} \right] = 2\int \frac{t\,dt}{1+t} = 2\int \frac{(1+t-1)dt}{1+t} =$$

$$= 2\int dt - 2\int \frac{dt}{1+t} = \left[\begin{matrix} 1+t = u \\ dt = du \end{matrix} \right] = 2\int dt - 2\int \frac{du}{u} =$$

$$= 2t - 2\ln|u| + C = 2t - 2\ln|1+t| + C =$$

$$= 2\sqrt{x} - 2\ln|1+\sqrt{x}| + C$$

№8.2.65

$$\int x\sqrt{x+3}\,dx = \left[\begin{matrix} x+3=t \Rightarrow x=t-3 \\ dx=dt \end{matrix}\right] = \int (t-3)\sqrt{t}\,dt =$$

$$= \int(\sqrt{t^3}-3\sqrt{t})dt = \int\sqrt{t^3}dt - 3\int\sqrt{t}\,dt =$$

$$= \frac{2\sqrt{t^5}}{5} - 2\sqrt{t^3} + C = \frac{2\sqrt{(x+3)^5}}{5} - 2\sqrt{(x+3)^3} + C$$

№8.2.66

$$\int\frac{dx}{(x+1)\sqrt{x}} = \left[\begin{matrix} x=t^2 \\ dx=2t\,dt \end{matrix}\right] = 2\int\frac{t\,dt}{(t^2+1)t} = 2\int\frac{dt}{t^2+1} =$$

$$= 2\,\mathrm{arctg}\,t + C = \left[\, t=\sqrt{x} \,\right] = 2\,\mathrm{arctg}\sqrt{x} + C$$

№8.2.67

$$\int\frac{x\,dx}{\sqrt{1-x}} = \left[\begin{matrix} 1-x=t \Rightarrow x=1-t \\ dx=-dt \end{matrix}\right] = \int\frac{(t-1)dt}{\sqrt{t}} =$$

$$= \int\sqrt{t}\,dt - \int\frac{dt}{\sqrt{t}} = \frac{2}{3}\sqrt{t^3} - 2\sqrt{t} + C =$$

$$= \frac{2\sqrt{(1-x)^3}}{3} - 2\sqrt{1-x} + C = \frac{2\sqrt{(1-x)^3} - 6\sqrt{1-x}}{3} + C =$$

$$= \frac{2\sqrt{1-x}\,(1-x-3)}{3} + C = \frac{2\sqrt{1-x}\,(-x-2)}{3} + C =$$

$$= C - \frac{2\sqrt{1-x}\,(x+2)}{3}$$

№8.2.68

$$\int\frac{x^2dx}{\sqrt{1-x^2}} = \left[\begin{matrix} x=\sin t \Leftrightarrow t=\arcsin x \\ dx=\cos t\,dt \end{matrix}\right] =$$

$$= \int\frac{\sin^2 t\cos t\,dt}{\cos t} = \int\sin^2 t\,dt = \int\frac{1-\cos 2t}{2}dt$$

$$= \frac{1}{2}\int dt - \frac{1}{2}\int\cos 2t\,dt = \frac{1}{2}t - \frac{1}{4}\sin 2t + C =$$

$$= \frac{1}{2}t - \frac{1}{2}\sin t\cos t + C = \frac{1}{2}\arcsin x - \frac{1}{2}\sin(\arcsin x)\ldots$$

$$= \frac{1}{2}\arcsin x - \frac{1}{2}x\sqrt{1-x^2} + C = \frac{1}{2}(\arcsin x - x\sqrt{1-x^2}) + C$$

№8.2.69

$$\int x\ln x\,dx = \left[\begin{matrix} u=\ln x \Rightarrow u'=\frac{1}{x} \\ v'=x \Rightarrow v=\frac{x^2}{2}\end{matrix}\right] = \frac{x^2\ln x}{2} - \int\frac{x^2\,dx}{2x} =$$

$$= \frac{x^2\ln x}{2} - \frac{1}{2}\int x\,dx = \frac{x^2\ln x}{2} - \frac{1}{2}\cdot\frac{x^2}{2} + C = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C =$$

$$= \frac{x^2}{4}(2\ln x - 1) + C$$

№8.2.70

$$\int(2x+3)\cos x\,dx = \left[\begin{matrix} u=2x+3 \Rightarrow u'=2 \\ v'=\cos x \Rightarrow v=\sin x\end{matrix}\right] =$$

$$= (2x+3)\sin x - \int\sin x\,2\,dx = (2x+3)\sin x - 2\int\sin x\,dx =$$

$$= (2x+3)\sin x + 2\cos x + C$$

№8.2.71

$$\int x\,\mathrm{sh}\,5x\,dx = \left[\begin{matrix} u=x \Rightarrow u'=1 \\ v'=\mathrm{sh}\,5x \Rightarrow v=\frac{\mathrm{ch}\,5x}{5}\end{matrix}\right] =$$

$$= \frac{x\,\mathrm{ch}\,5x}{5} - \int\frac{\mathrm{ch}\,5x\,dx}{5} = \frac{x\,\mathrm{ch}\,5x}{5} - \frac{1}{5}\int\mathrm{ch}\,5x\,dx =$$

$$= \frac{x\,\mathrm{ch}\,5x}{5} - \frac{\mathrm{sh}\,5x}{25} + C$$

№8.2.72

$$\int\frac{x\cos x\,dx}{\sin^3 x} = \left[\begin{matrix} u=x \Rightarrow u'=1 \\ v'=\frac{\cos x}{\sin^3 x} \Rightarrow\end{matrix}\right. \left[\int\frac{\cos x\,dx}{\sin^3 x} = \left[\begin{matrix} t=\sin x \\ dt=\cos x\,dx\end{matrix}\right] =\right.$$

$$\left.= \int\frac{dt}{t^3} = \frac{t^{-3+1}}{-3+1} + C = \frac{t^{-2}}{-2} + C = -\frac{1}{2t^2} + C = -\frac{1}{2\sin^2 x} + C\right] \Rightarrow$$

$$\left.\Rightarrow v'=\frac{\cos x}{\sin^3 x} \Rightarrow v = -\frac{1}{2\sin^2 x}\right] = -\frac{x}{2\sin^2 x} - \int -\frac{dx}{2\sin^2 x} =$$

$$= -\frac{x}{2\sin^2 x} + \frac{1}{2}\int\frac{dx}{\sin^2 x} = C - \frac{x}{2\sin^2 x} - \frac{1}{2}\,\mathrm{ctg}\,x$$

№8.2.73

$$\int x^2 \ln x = \left[\begin{matrix} u=\ln x \Rightarrow u'=\frac{1}{x} \\ v'=x^2 \Rightarrow v=\frac{x^3}{3} \end{matrix} \right] =$$

$$= \frac{x^3\ln x}{3} - \int \frac{x^3}{3}\cdot\frac{1}{x}dx = \frac{x^3\ln x}{3} - \frac{1}{3}\int x^2 dx =$$

$$= \frac{x^3\ln x}{3} - \frac{1}{3}\cdot\frac{1}{3}x^3 + C = \frac{x^3\ln x}{3} - \frac{x^3}{9} + C =$$

$$= \frac{x^3}{9}(3\ln x - 1) + C$$

№8.2.74

$$\int(x^2-4x+1)e^{-x}dx = \left[\begin{matrix} u=x^2-4x+1 \Rightarrow u'=2x-4 \\ v'=e^{-x} \Rightarrow v=-e^{-x} \end{matrix} \right] =$$

$$= -e^{-x}(x^2-4x+1) + \int e^{-x}(2x-4)dx =$$

$$= -e^{-x}(x^2-4x+1) + \int 2xe^{-x}dx - \int 4e^{-x}dx =$$

$$= -e^{-x}(x^2-4x+1) + 2\int xe^{-x}dx - 4\int e^{-x}dx =$$

$$= \left[\begin{matrix} \bar u = x \Rightarrow \bar u' = 1 \\ \bar v' = e^{-x} \Rightarrow \bar v = -e^{-x} \end{matrix} \right] = -e^{-x}(x^2-4x+1) + 2(-e^{-x}x + \int e^{-x}dx)$$

$$-4\int e^{-x}dx = -e^{-x}(x^2-4x+1) - 2e^{-x}x - 2e^{-x} + 4e^{-x} + C =$$

$$= -e^{-x}(x^2-4x+1) - 2e^{-x}x + 2e^{-x} + C =$$

$$= e^{-x}(-x^2+4x-1-2x+2) + C = e^{-x}(1+2x-x^2) + C$$

№8.2.75.

$$\int x^3e^x dx = \left[\begin{matrix} u=x^3 \Rightarrow u'=3x^2 \\ v'=e^x \Rightarrow v=e^x \end{matrix} \right] = x^3e^x - 3\int x^2e^x dx =$$

$$= \left[\begin{matrix} \bar u = x^2 \Rightarrow \bar u' = 2x \\ \bar v' = e^x \Rightarrow \bar v = e^x \end{matrix} \right] = x^3e^x - 3(x^2e^x - 2\int xe^x dx) =$$

$$= \left[\begin{matrix} \bar{\bar u} = x \Rightarrow \bar{\bar u}'=1 \\ \bar{\bar v}'=e^x \Rightarrow \bar{\bar v}=e^x \end{matrix} \right] = x^3e^x - 3(x^2e^x - 2(xe^x - \int e^x dx))$$

$$= x^3e^x - 3x^2e^x + 6xe^x + 6e^x + C = e^x(x^3-3x^2+6x+6)$$

№8.2.76

$$\int \frac{\arccos x\,dx}{\sqrt{1+x}} = \left[\begin{array}{l} u=\arccos x \Rightarrow u' = -\frac{1}{\sqrt{1-x^2}} \\ v'=\frac{1}{\sqrt{1+x}} \Rightarrow v = 2\sqrt{1+x} \end{array} \right] =$$

$$= 2\sqrt{1+x}\arccos x + \int \frac{2\sqrt{1+x}\,dx}{\sqrt{1-x^2}} =$$

$$= 2\sqrt{1+x}\arccos x + 2\int \frac{\sqrt{1+x}\,dx}{\sqrt{1-x}\sqrt{1+x}} =$$

$$= 2\sqrt{1+x}\arccos x + 2\int \frac{dx}{\sqrt{1-x}} =$$

$$= 2\sqrt{1+x}\arccos x - 4\sqrt{1-x} + C = 2(\sqrt{1+x}\arccos x - 2\sqrt{1-x}) + C$$

№8.2.77

$$\int \frac{\arcsin\sqrt{x}\,dx}{\sqrt{1-x}} = \left[\begin{array}{l} u=\arcsin\sqrt{x} \Rightarrow u' = \frac{1}{2\sqrt{x}\sqrt{1-x}} \\ v'=\frac{1}{\sqrt{1-x}} \Rightarrow v = -2\sqrt{1-x} \end{array} \right] =$$

$$= -2\sqrt{1-x}\arcsin\sqrt{x} + \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} - 2\sqrt{1-x}\arcsin\sqrt{x} + C =$$

$$= 2(\sqrt{x} - \sqrt{1-x}\arcsin\sqrt{x}) + C$$

№8.2.78

$$\int \frac{x^2\,dx}{(x^2-1)^2} = \int \frac{(x^2-1)+1\,dx}{(x^2-1)^2} = \int \frac{dx}{x^2-1} + \int \frac{dx}{(x^2-1)^2}$$

№8.2.79

$$\int \cos(\ln x)dx = \left[\begin{matrix} x = e^t \Rightarrow t = \ln x \\ dx = e^t dt \end{matrix}\right] =$$

$$= \int e^t \cos t\, dt = \left[\begin{matrix} u = \cos t \Rightarrow u' = -\sin t \\ v' = e^t \Rightarrow v = e^t \end{matrix}\right] =$$

$$= e^t \cos t + \int e^t \sin t\, dt = \left[\begin{matrix} u = \sin t \Rightarrow u' = \cos t \\ v' = e^t \Rightarrow v = e^t \end{matrix}\right] =$$

$$= e^t \cos t + e^t \sin t - \int e^t \cos t\, dt$$

$$\int e^t \cos t\, dt = e^t \cos t + e^t \sin t - \int e^t \cos t\, dt$$

$$2\int e^t \cos t\, dt = e^t \cos t + e^t \sin t + C$$

$$\int e^t \cos t\, dt = \frac{e^t(\cos t + \sin t)}{2} + C$$

$$\int \cos(\ln x)dx = \frac{x(\cos(\ln x) + \sin(\ln x))}{2} + C$$

№8.2.80

$$\int e^{3x}\cos^2 x\, dx = \left[\begin{matrix} u = \cos^2 x \Rightarrow u' = -2\sin x\cos x = -\sin 2x \\ v' = e^{3x} \Rightarrow v = \frac{e^{3x}}{3} \end{matrix}\right] =$$

$$= \frac{e^{3x}\cos^2 x}{3} + \int \frac{e^{3x}\sin 2x}{3}dx = \frac{e^{3x}\cos^2 x}{3} + \frac{1}{3}\int e^{3x}\sin 2x\, dx =$$

$$= \left[\begin{matrix} u = \sin 2x \Rightarrow u' = 2\cos 2x \\ v' = e^{3x} \Rightarrow v = \frac{e^{3x}}{3} \end{matrix}\right] = \frac{e^{3x}\cos^2 x}{3} + \frac{1}{3}\Big(\frac{e^{3x}\sin 2x}{3} -$$

$$- \int \frac{2e^{3x}\cos 2x\, dx}{3} = \frac{e^{3x}\cos^2 x}{3} + \frac{1}{3}\Big(\frac{e^{3x}\sin 2x}{3} - \frac{2}{3}\int \frac{e^{3x}\cos 2x}{1}$$

$$= \left[\begin{matrix} u = \cos 2x \Rightarrow u' = -2\sin 2x \\ v' = e^{3x} \Rightarrow v = \frac{e^{3x}}{3} \end{matrix}\right] = \frac{e^{3x}\cos^2 x}{3} + \frac{1}{3}\Big(\frac{e^{3x}\sin 2x}{3} -$$

$$- \frac{2}{3}\Big(\frac{e^{3x}\cos 2x}{3} + \frac{2}{3}\int e^{3x}\sin 2x\, dx\Big)$$

$$\int e^{3x}\sin 2x\, dx = \frac{3e^{3x}\sin 2x - 2e^{3x}\cos 2x}{13} + C$$

$$\int e^{3x}\cos^2 x\,dx = \frac{e^{3x}\cos^2 x}{3} + \frac{3e^{3x}\sin 2x - 2e^{3x}\cos 2x}{39} + C$$

№ 8.2.81

$$\int e^{\sqrt{x}}dx = \left[\begin{matrix} t^2 = x \\ dx = 2t\,dt \end{matrix}\right] = \int 2te^t dt = \left[\begin{matrix} u = 2t \Rightarrow u' = 2 \\ v' = e^t \Rightarrow v = e^t \end{matrix}\right] =$$

$$= 2te^t - 2\int e^t dt = 2te^t - 2e^t + C =$$

$$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C = 2e^{\sqrt{x}}(\sqrt{x} - 1) + C$$

№ 8.2.82

$$\int \frac{x\,dx}{\cos^2 x} = \left[\begin{matrix} u = x \Rightarrow u' = 1 \\ v' = \frac{1}{\cos^2 x} \Rightarrow v = \operatorname{tg} x \end{matrix}\right] = x\operatorname{tg} x - \int \operatorname{tg} x\,dx =$$

$$= x\operatorname{tg} x + \ln|\cos x| + C$$

№ 8.2.83

$$\int x^3 e^{x^2}dx = \left[\begin{matrix} x = \sqrt{t} \Rightarrow t = x^2 \\ dx = \frac{dt}{2\sqrt{t}} \end{matrix}\right] = \int \frac{t\sqrt{t}\,e^t\,dt}{2\sqrt{t}}$$

$$= \frac{1}{2}\int te^t dt = \left[\begin{matrix} u = t \Rightarrow u' = 1 \\ v' = e^t \Rightarrow v = e^t \end{matrix}\right] =$$

$$= \frac{1}{2}\left(te^t - \int e^t dt\right) = \frac{1}{2}(te^t - e^t) + C =$$

$$= \frac{e^t}{2}(t - 1) + C = \frac{e^{x^2}}{2}(x^2 - 1) + C$$

№ 8.2.84

$$\int \ln(x + \sqrt{x^2+1})dx = \left[\begin{matrix} \ln(x + \sqrt{x^2+1}) = u \Rightarrow u' = \frac{1}{\sqrt{x^2+1}} \\ v' = 1 \Rightarrow v = x \end{matrix}\right] =$$

$$= x\ln(x + \sqrt{x^2+1}) - \int \frac{x\,dx}{\sqrt{x^2+1}} = \left[\begin{matrix} x^2+1 = t \Rightarrow \\ dt = 2x\,dx \Rightarrow \frac{dt}{2} = x\,dx \end{matrix}\right] =$$

$$= x\ln(x + \sqrt{x^2+1}) - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\ln(x + \sqrt{x^2+1}) - \sqrt{t} + C =$$

$$= x\ln(x + \sqrt{x^2+1}) - \sqrt{x^2+1} + C$$

№8.2.85

$$\int \sin 2x \ln \sin^2 x \, dx = \int 2\sin x \cos x \ln(\sin x) dx =$$

$$= \begin{bmatrix} t = \sin x \\ dt = \cos x \, dx \end{bmatrix} = 2\int t \ln t \, dt = \begin{bmatrix} u = \ln t \Rightarrow u' = \frac{1}{t} \\ v' = t \Rightarrow v = \frac{t^2}{2} \end{bmatrix}$$

$$= 2\left(\frac{t^2 \ln t}{2} - \frac{1}{2}\int t \, dt\right) = t^2 \ln t - \frac{t^2}{2} + C =$$

$$= \sin^2 x \ln(\sin x) - \frac{\sin^2 x}{2} + C =$$

$$= \frac{\sin^2 x}{2}\left(2\ln(\sin x) - 1\right) + C$$

№8.2.86

$$\int x^2 \arccos 3x \, dx = \begin{bmatrix} u = \arccos 3x \Rightarrow u' = -\frac{3}{\sqrt{1-9x^2}} \\ v' = x^2 \Rightarrow v = \frac{x^3}{3} \end{bmatrix}$$

$$= \frac{x^3 \arccos 3x}{3} + \int \frac{3x^3 \, dx}{\sqrt{1-9x^2}} = \frac{x^3 \arccos 3x}{3} + 3\int \frac{x^3 \, dx}{\sqrt{1-9x^2}}$$

$$= \begin{bmatrix} 1 - 9x^2 = t & \Rightarrow x = \sqrt{\frac{1-t}{9}} & -\frac{dt}{18} = x \, dx \\ dt = -18x \, dx & & \end{bmatrix} =$$

$$= \frac{x^3 \arccos 3x}{3} - 3\int \frac{(1-t)\,dt}{18 \cdot 9\sqrt{t}} =$$

$$= \frac{x^3 \arccos 3x}{3} - \frac{1}{54}\int \frac{(1-t)\,dt}{\sqrt{t}} = \frac{x^3 \arccos 3x}{3} -$$

$$- \frac{1}{54}\left(\int \frac{dt}{\sqrt{t}} - \int \sqrt{t} \, dt\right) = \frac{x^3 \arccos 3x}{3} - \frac{1}{54}\left(2\sqrt{t} - \frac{2\sqrt{t^3}}{3}\right)$$

$$= \frac{x^3 \arccos 3x}{3} - \frac{\sqrt{t}}{27} + \frac{\sqrt{t^3}}{81} + C =$$

$$= \frac{x^3 \arccos 3x}{3} - \frac{\sqrt{1-9x^2}}{27} + \frac{\sqrt{(1-9x^2)^3}}{81} + C$$

№8.2.87.

$$\int x\sin\sqrt{x}dx = \left[\begin{matrix} t=\sqrt{x} \Rightarrow x=t^2 \\ dx=2t \end{matrix}\right] = \int 2t\, t^2 \sin t\, dt =$$

$$= 2\int t^3 \sin t\, dt = \left[\begin{matrix} u=t^3 \Rightarrow u'=3t^2 \\ v'=\sin t \Rightarrow v=-\cos t \end{matrix}\right] =$$

$$= 2(-t^3\cos t + 3\int t^2 \cos t\, dt) = \left[\begin{matrix} u=t^2 \Rightarrow u'=2t \\ v'=\cos t \Rightarrow v=\sin t \end{matrix}\right] =$$

$$= 2(-t^3\cos t + 3(t^2 \sin t - 2\int t\sin t\, dt)) = \left[\begin{matrix} u=t \Rightarrow u'=1 \\ v'=\sin t \Rightarrow v=-\cos t \end{matrix}\right] =$$

$$= 2(-t^3\cos t + 3(t^2\sin t - 2(-t\cos t + \int \cos t\, dt))) =$$

$$= -2t^3\cos t + 6t^2 \sin t + 12t\cos t - 12\sin t + C =$$

$$= -2\sqrt{x^3}\cos\sqrt{x} + 6x\sin\sqrt{x} + 12\sqrt{x}\cos\sqrt{x} - 12\sin\sqrt{x} + C =$$

$$= 2(6-x)\sqrt{x}\cos\sqrt{x} + 6(x-2)\sin\sqrt{x} + C$$

№8.2.88

$$\int \arcsin^2 x\, dx = \left[\begin{matrix} t=\arcsin x \Rightarrow x=\sin t \\ dt=\frac{dx}{\sqrt{1-x^2}} \end{matrix}\right] =$$

$$= \int t^2 \sqrt{1-\sin^2 t}\, dt = \int t^2 \cos t\, dt = \left[\begin{matrix} u=t^2 \Rightarrow u'=2t \\ v'=\cos t \Rightarrow v=\sin t \end{matrix}\right] =$$

$$= t^2 \sin t - 2\int t\sin t\, dt = \left[\begin{matrix} u=t \Rightarrow u'=1 \\ v'=\sin t \Rightarrow v=-\cos t \end{matrix}\right] =$$

$$= t^2\sin t - 2(-t\cos t + \int \cos t\, dt) =$$

$$= t^2 \sin t + 2t\cos t - 2\sin t + C =$$

$$= \arcsin^2 x\, \sin(\arcsin x) + 2\arcsin x\, \cos(\arcsin x) - 2\sin(\arcsin x) + C =$$

$$= x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C$$

№8.2.89.

$$\int \frac{\cos\sqrt{x}\,dx}{\sqrt{x}} = \left[\begin{matrix} x = t^2 \\ dx = 2t\,dt \end{matrix}\right] = 2\int \frac{\cos t \; t\,dt}{t} =$$

$$= 2\int \cos t\,dt = 2\sin t + C = 2\sin\sqrt{x} + C$$

№8.2.90

$$\int \operatorname{arctg}\sqrt{x}\,dx = \left[\begin{matrix} x = t^2 \Rightarrow t = \sqrt{x} \\ dx = 2t\,dt \end{matrix}\right] = 2\int \operatorname{arctg} t \; t\,dt =$$

$$= \left[\begin{matrix} u = \operatorname{arctg} t \Rightarrow u' = \frac{1}{1+t^2} \\ t\,dt \Rightarrow v = \frac{t^2}{2} \end{matrix}\right] =$$

$$= \frac{t^2 \operatorname{arctg} t}{2} - \frac{1}{2}\int \frac{t^2}{1+t^2}\,dt = \frac{t^2 \operatorname{arctg} t}{2} - \frac{1}{2}\left(\int dt - \int\frac{dt}{1+t^2}\right)$$

$$= \frac{t^2 \operatorname{arctg} t}{2} - \frac{t}{2} + \frac{1}{2}\operatorname{arctg} t + C =$$

$$= \frac{1}{2}\left(x \operatorname{arctg}\sqrt{x} - \sqrt{x} + \operatorname{arctg}\sqrt{x}\right) + C$$

№8.2.91

$$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + C$$

№8.2.92

$$\int \frac{\ln^2 x\,dx}{x\sqrt{3-\ln x}} = \left[\begin{matrix} \ln x = t \Rightarrow x = e^t \\ dx = e^t\,dt \end{matrix}\right] =$$

$$= \int \frac{e^t t^2\,dx}{e^t\sqrt{3-t}} = \int \frac{t^2\,dx}{\sqrt{3-t}} = \left[\begin{matrix} u = t^2 \Rightarrow u' = 2t \\ v' = \frac{1}{\sqrt{3-t}} \Rightarrow v = -2\sqrt{3-t} \end{matrix}\right] =$$

$$= -2t^2\sqrt{3-t} + 4\int t\sqrt{3-t}\,dt = \left[\begin{matrix} u = t \Rightarrow u' = 1 \\ v' = \sqrt{3-t} \Rightarrow v = -\frac{2\sqrt{(3-t)^3}}{3} \end{matrix}\right]$$

$$= -2t^2\sqrt{3-t} + 4\left(-\frac{2t\sqrt{(3-t)^3}}{3} + \frac{2}{3}\int\sqrt{(3-t)^3}\,dt\right) =$$

$$= -2t^2\sqrt{3-t} - \frac{8t(3-t)\sqrt{3-t}}{3} + \frac{8}{3}\left(-\frac{2(3-t)^2\sqrt{3-t}}{5}\right) + C =$$

$$= \frac{-30t^2\sqrt{3-t} - 40t(3-t)\sqrt{3-t} - 16(3-t)^2\sqrt{3-t}}{15} + C =$$

$$= -\frac{2\sqrt{5-t}\,(t^2+4t+24)}{5} + C =$$

$$= -\frac{2\sqrt{5-\ln x}\,(\ln^2 x + 4\ln x + 24)}{5} + C$$

№ 8.2.93.

$$\int \frac{e^{\operatorname{arctg} x} + 8x}{1+x^2}\,dx = \int \frac{e^{\operatorname{arctg} x}\,dx}{1+x^2} + 8\int \frac{x\,dx}{1+x^2} =$$

$$= \left[\begin{matrix} t = \operatorname{arctg} x \Rightarrow x = \operatorname{tg} t \\ dx = (x^2+1)\,dt \end{matrix}\right] = \int \frac{e^t (x^2+1)\,dt}{(x^2+1)} + 8\int \frac{x\,dx}{x^2+1} =$$

$$= \int e^t\,dt + 8\int \frac{x\,dx}{x^2+1} = \left[\begin{matrix} u = x^2+1 \\ du = 2x\,dx \Rightarrow \frac{du}{2} = x\,dx \end{matrix}\right] =$$

$$= \int e^t\,dt + 4\int \frac{du}{u} = e^t + 4\ln|u| + C =$$

$$= e^{\operatorname{arctg} x} + 4\ln|x^2+1| + C$$

№ 8.2.94.

$$\int \frac{3x + 5\sin\left(\frac{1}{e^x}\right)}{e^x}\,dx = 3\int \frac{x\,dx}{e^x} + 5\int \frac{\sin\left(\frac{1}{e^x}\right)dx}{e^x} = \left[\begin{matrix} t = \frac{1}{e^x} \\ dx = -e^x\,dt \end{matrix}\right] =$$

$$= 3\int \frac{x\,dx}{e^x} - 5\int \sin t\,dt = \left[\begin{matrix} u = x \Rightarrow u' = 1 \\ v' = e^{-x} \Rightarrow v = -e^{-x} \end{matrix}\right] =$$

$$= 5\cos t + 3\left(-xe^{-x} - \int -e^{-x}\,dx\right) = 5\cos\frac{1}{e^x} - 3xe^{-x} - 3e^{-x} + C =$$

$$= 5\cos\frac{1}{e^x} - 3e^{-x}(x+1) + C$$