№6.4.46

1) $\lim\limits_{x\to\infty}\left(1+\frac{k}{x}\right)^x = [1^\infty] = \lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x}{k}}\right)^x =$

$= \lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x}{k}}\right)^{\frac{x}{k}\cdot k} = \left(\lim\limits_{x\to\infty}\left(1+\frac{1}{\frac{x}{k}}\right)^{\frac{x}{k}}\right)^k =$

$= \left(\lim\limits_{y\to\infty}\left(1+\frac{1}{y}\right)^y\right)^k = e^k$

2) $\lim\limits_{x\to 0}\sqrt[x]{1+5x} = \lim\limits_{x\to 0}(1+5x)^{\frac{1}{x}} = [1^\infty] =$

$= \lim\limits_{y\to 0}(1+y)^{\frac{1}{y/5}} = \lim\limits_{y\to 0}(1+y)^{\frac{5}{y}} = \left(\lim\limits_{y\to 0}(1+y)^{\frac{1}{y}}\right)^5 = e^5$

3) $\lim\limits_{x\to\infty}\left(\frac{x+3}{x-2}\right)^x = \left[\left(\frac{\infty}{\infty}\right)^\infty\right] = \lim\limits_{x\to\infty}\left(\frac{x\left(1+\frac{3}{x}\right)}{x\left(1-\frac{2}{x}\right)}\right)^x =$

$= \lim\limits_{x\to\infty}\left(\frac{\left(1+\frac{3}{x}\right)}{\left(1-\frac{2}{x}\right)}\right)^x = \lim\limits_{x\to\infty}\frac{\left(1+\frac{3}{x}\right)^x}{\left(1-\frac{2}{x}\right)^x} = \frac{\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^x}{\lim\limits_{x\to\infty}\left(1+\frac{(-2)}{x}\right)^x} =$

$= \frac{e^3}{e^{-2}} = e^5$

4) $\lim\limits_{x\to 0}\frac{e^{2x}-1}{7x} = \left[\frac{0}{0}\right] = \lim\limits_{y\to 0}\frac{e^y-1}{7\cdot\frac{y}{2}} = \lim\limits_{y\to 0}\frac{e^y-1}{\frac{7}{2}\cdot y} =$

$= \lim\limits_{y\to 0}\frac{2}{7}\cdot\frac{e^y-1}{y} = \frac{2}{7}\cdot 1 = \frac{2}{7}$

№6.4.48

$\lim\limits_{x\to 0}\sqrt[2x]{1+3x} = \lim\limits_{x\to 0}(1+3x)^{\frac{1}{2x}} = [1^\infty] =$

$= \lim\limits_{y\to 0}(1+y)^{\frac{1}{2\cdot\frac{y}{3}}} = \left(\lim\limits_{y\to 0}(1+y)^{\frac{1}{y}}\right)^{\frac{3}{2}} = e^{\frac{3}{2}} = e^{1,5}$

№ 6.4.49

$$\lim_{x\to\infty}\left(\frac{x-5}{x-4}\right)^x=\left[\left(\frac{\infty}{\infty}\right)^\infty\right]=\lim_{x\to\infty}\left(\frac{1+\frac{(-5)}{x}}{1+\frac{4}{x}}\right)^x=\frac{\lim\limits_{x\to\infty}\left(1+\frac{(-5)}{x}\right)^x}{\lim\limits_{x\to\infty}\left(1+\frac{4}{x}\right)^x}=$$

$$=\frac{e^{-5}}{e^{4}}=e^{-9}$$

№ 6.4.50

$$\lim_{x\to0}\left(\frac{3+5x}{3+2x}\right)^{\frac{1}{x}}=[1^\infty]=\frac{\lim\limits_{x\to0}3\left(1+\frac{5}{3}x\right)^{\frac{1}{x}}}{\lim\limits_{x\to0}3\left(1+\frac{2}{3}x\right)^{\frac{1}{x}}}=$$

$$=\frac{\left(\lim\limits_{y\to0}(1+y)^{\frac{1}{y}}\right)^{\frac{5}{3}}}{\left(\lim\limits_{t\to0}(1+t)^{\frac{1}{t}}\right)^{\frac{2}{3}}}=\frac{e^{\frac{5}{3}}}{e^{\frac{2}{3}}}=e^{\frac{5}{3}-\frac{2}{3}}=e^{\frac{3}{3}}=e$$

№ 6.4.51.

$$\lim_{x\to2}\frac{e^x-e^2}{x-2}=\left[\frac{0}{0}\right]=\lim_{y\to0}\frac{e^{y+2}-e^2}{y+2-2}=\lim_{y\to0}\frac{e^2(e^y-1)}{y}=$$

$$=\lim_{y\to0}e^2\cdot\lim_{y\to0}\frac{e^y-1}{y}=e^2\cdot1=e^2$$

№ 6.4.52

$$\lim_{x\to\infty}\left(\frac{5-x}{6-x}\right)^{x+2}=\left[\left(\frac{\infty}{\infty}\right)^\infty\right]=\lim_{y\to\infty}\left(\frac{5-y+2}{6-y+2}\right)^y=$$

$$=\lim_{y\to\infty}\left(\frac{y-7}{y-8}\right)^y=\lim_{y\to\infty}\left(\frac{1+(-7)/y}{1+(-8)/y}\right)^y=\frac{e^{-7}}{e^{-8}}=e^{-7+8}=e$$

№6.4.53

$$\lim_{x\to e}\frac{\ln x-1}{x-e}=\left[\frac{0}{0}\right]=\lim_{x\to e}\frac{\ln\left(\frac{x}{e}\right)}{x-e}=$$

$$=\lim_{y\to 0}\frac{\ln\left(\frac{y+e}{e}\right)}{y+e-e}=\lim_{y\to 0}\frac{\ln\left(1+\frac{y}{e}\right)}{y}=$$

$$=\lim_{y\to 0}\frac{\ln(1+ye^{-1})}{y}=e^{-1}$$

№6.4.54

$$\lim_{x\to 0}(1-\sin x)^{\frac{1}{\sin x}}=[1^{\infty}]=\left(\lim_{y\to 0}(1+y)^{\frac{1}{y}}\right)^{-1}=e^{-1}$$

№6.4.55

$$\lim_{x\to\infty}x(\ln(x+3)-\ln x)=[\infty(\infty-\infty)]=$$

$$=\lim_{x\to\infty}x\left(\ln\left(\frac{x+3}{x}\right)\right)=\lim_{x\to\infty}x\left(\ln\left(1+\frac{3}{x}\right)\right)=$$

$$=\lim_{y\to 0}\frac{1}{y}\left(\ln\left(1+\frac{3}{1/y}\right)\right)=\lim_{y\to 0}\frac{\ln(1+3y)}{y}=3$$