Вариант 16 Войтенко Игорь

№1.

$f(x) = 0{,}5\cos x - 9$

$\cos x : E(\cos x) = [-1; 1]$

$0{,}5\cos x : E(0{,}5\cos x) = [-0{,}5; 0{,}5]$

Следовательно $E(f) = [-9{,}5; -8{,}5]$

Ответ: $E(f) = [-9{,}5; -8{,}5]$

№2

$$x_n = \frac{3n^2 - 5n + 16}{(-1)^n} + n$$

$$x_1 = \frac{3 \cdot 1^2 - 5 \cdot 1 + 16}{-1} + 1 = \frac{3 - 5 + 16}{-1} + 1 = -14 + 1 = -13$$

$$x_2 = \frac{3 \cdot 4 - 5 \cdot 2 + 16}{1} + 2 = \frac{12 - 10 + 16}{1} + 2 = 18 + 2 = 20$$

$$x_3 = \frac{3 \cdot 9 - 5 \cdot 3 + 16}{-1} + 3 = \frac{27 - 15 + 16}{-1} + 3 = -28 + 3 = -25$$

$$x_4 = \frac{3 \cdot 16 - 5 \cdot 4 + 16}{1} + 4 = \frac{48 - 20 + 16}{1} + 4 = 44 + 4 = 48$$

$$x_5 = \frac{3 \cdot 25 - 5 \cdot 5 + 16}{-1} + 5 = \frac{75 - 25 + 16}{-1} + 5 = -66 + 5 = -61$$

Ответ: $x_1 = -13$ $x_3 = -25$ $x_5 = -61$

$x_2 = 20$ $x_4 = 48$

№3.

1) $$\lim_{x\to -2} \frac{31x^3+248}{5x^3+5x^2+15x+50} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to -2} \frac{31(x^3+8)}{5x^3+5x^2+15x+50} =$$

= [Схема Горнера:

	5	5	15	50
-2	5	-5	25	0

$$(x^3+8) = (x+2)(x^2-2x+4)$$] =

$$= \lim_{x\to -2} \frac{31(x+2)(x^2-2x+4)}{(x+2)(5x^2-5x+25)} =$$

$$= \lim_{x\to -2} \frac{31(x^2-2x+4)}{5x^2-5x+25} = \frac{31(4+4+4)}{20+10+25} =$$

$$= \frac{31\cdot 12}{55} = \frac{372}{55}$$

2) $$\lim_{x\to 3} \frac{1-\sqrt{x-2}}{3-\sqrt{12-x}} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 3} \frac{(1-\sqrt{x-2})(1+\sqrt{x-2})(3+\sqrt{12-x})}{(3-\sqrt{12-x})(1+\sqrt{x-2})(3+\sqrt{12-x})} =$$

$$= \lim_{x\to 3} \frac{(1-(x-2))(3+\sqrt{12-x})}{(3-(12-x))(1+\sqrt{x-2})} =$$

$$= \lim_{x\to 3} \frac{(3-x)(3+\sqrt{12-x})}{(x-9)(1+\sqrt{x-2})} = \frac{(3-3)(3+3)}{(3-9)(1+1)} = 0$$

3) $\lim\limits_{x\to 0} \dfrac{\sin^3 x - tg x \sin^2 x}{x^3 \cdot \sin^2 x} = \left[\frac{0}{0}\right] =$

$= \lim\limits_{x\to 0} \dfrac{\sin^2 x(\sin x - tg x)}{\sin^2 x \; x^3} =$

$= \lim\limits_{x\to 0} \dfrac{\sin x - tg x}{x^3} = \lim\limits_{x\to 0} \dfrac{(\sin x)' - (tg x)'}{(x^3)'} =$

$= \lim\limits_{x\to 0} \dfrac{(\cos x)' - (1/\cos^2 x)'}{(3x^2)'} =$

$= \lim\limits_{x\to 0} \dfrac{(-\sin x)' + 2\left(\frac{\sin x}{\cos^3 x}\right)'}{(6x)'} =$

$= \lim\limits_{x\to 0} \dfrac{-\cos x + 2\left(\frac{(\sin x)'\cos^3 x - \sin x(\cos^3 x)'}{\cos^6 x}\right)}{6} =$

$= \lim\limits_{x\to 0} \dfrac{-\cos x + 2\left(\frac{\cos^4 x - 3\sin^2 x \cos^2 x}{\cos^6 x}\right)}{6} =$

$= \dfrac{-1 + 2\left(\frac{1 - 3\cdot 0 \cdot 1}{1}\right)}{6} = \dfrac{-1+2}{6} = \dfrac{1}{6}$

4) $$\lim_{x\to\infty}\left(\frac{11-x}{x+7}\right)^{5x}=\left[\left(\frac{\infty}{\infty}\right)^{\infty}\right]=$$

$$=\left(\lim_{x\to\infty}\left(\frac{11-x}{x+7}\right)^{x}\right)^{5}=\left(\lim_{x\to\infty}-\left(\frac{x-11}{x+7}\right)^{x}\right)^{5}=$$

$$=-\left(\lim_{x\to\infty}\left(\frac{x-11}{x+7}\right)^{x}\right)^{5}=-\left(\lim_{x\to\infty}\left(1+\frac{(-18)}{x+7}\right)^{x}\right)^{5}=$$

$$=-e^{-18\cdot 5}=-\frac{1}{e^{90}}$$

№4.

1) $y = \operatorname{arctg}^3 \ln \frac{\sqrt{8+x}}{x+2}$

$$\left(\left(\operatorname{arctg} \ln \frac{\sqrt{8+x}}{x+2}\right)^3\right)' = \frac{3\left(\ln \frac{\sqrt{8+x}}{x+2}\right)' \operatorname{arctg}^2\left(\ln \frac{\sqrt{8+x}}{x+2}\right)}{1+\left(\ln \frac{\sqrt{8+x}}{x+2}\right)^2} =$$

$$= \frac{3(x+2)\left(\left(\frac{\sqrt{8+x}}{x+2}\right)'\right) \operatorname{arctg}^2\left(\ln \frac{\sqrt{8+x}}{x+2}\right)}{\left(1+\ln^2 \frac{\sqrt{8+x}}{x+2}\right)\sqrt{8+x}} =$$

$$= \frac{3(x+2)\left(\frac{(\sqrt{8+x})'(x+2)-(\sqrt{8+x})(x+2)'}{(x+2)^2}\right) \operatorname{arctg}^2\left(\ln \frac{\sqrt{8+x}}{x+2}\right)}{\left(1+\ln^2 \frac{\sqrt{8+x}}{x+2}\right)\sqrt{8+x}} =$$

$$= \left[(\sqrt{8+x})' = \frac{1}{2\sqrt{8+x}}\right] =$$

$$= \frac{3(x+2)\left(\frac{\frac{x+2}{2\sqrt{8+x}} - \sqrt{8+x}}{(x+2)^2}\right) \operatorname{arctg}^2\left(\ln \frac{\sqrt{8+x}}{x+2}\right)}{\left(1+\ln^2 \frac{\sqrt{8+x}}{x+2}\right)\sqrt{8+x}}$$

2) $y = \left(\sqrt{x^2+10x}\right)^{arctg(x+x^2)}$

$\ln y = arctg(x+x^2)\ln\sqrt{x^2+10x} \Rightarrow$

$\frac{y'}{y} = \left(arctg(x+x^2)\frac{1}{2}\ln(x^2+10x)\right)' \Rightarrow$

$\frac{y'}{y} = \frac{\left(arctg(x+x^2)\ln(x^2+10x)\right)'}{2} \Rightarrow$

$\frac{y'}{y} = \frac{(x+x^2)' \cdot (x^2+10x)'}{2(1+(x+x^2)^2)(x^2+10x)} \Rightarrow$

$\frac{y'}{y} = \frac{(1+2x)(2x+10)}{2(1+(x+x^2)^2)(x^2+10x)} \Rightarrow$

$y' = \frac{\left(\sqrt{x^2+10x}\right)^{arctg(x+x^2)}(1+2x)(2x+10)}{2(1+(x+x^2)^2)(x^2+10x)}$

3) $\cos(4x+5y)+\dfrac{x^3+x^2}{y}=5x \Rightarrow$

$$\cos(4x+5y)+\frac{x^3+x^2}{y}-5x=0 \Rightarrow$$

$$\frac{dy}{dx}=\frac{\frac{df(x;y)}{dx}}{\frac{df(x;y)}{dy}} \Rightarrow$$

$$\left.\begin{aligned}\frac{df(x;y)}{dx}&=-4\sin(4x+5y)-5+\frac{3x^2+2x}{y}\\ \frac{df(x;y)}{dy}&=-5\sin(4x+5y)-\frac{x^3+x^2}{y^2}\end{aligned}\right| \Rightarrow$$

$$\frac{dy}{dx}=\frac{-4\sin(4x+5y)-5+\frac{3x^2+2x}{y}}{-5\sin(4x+5y)-\frac{x^3+x^2}{y^2}}$$