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Подгруппа №1

Лекция (12.10.20)

③ $\sin(3xy-7y)+\frac{x^2+3xy}{y^2}=2x+xy$

$\left(\sin(3xy-7y)+\frac{x^2+3xy}{y^2}\right)'_x=(2x+xy)'_x$

$(\sin(3xy-7y))'_x+\left(\frac{x^2+3xy}{y^2}\right)'_x=(2x)'_x+(xy)'_x$

$\cos(3xy-7y)(3xy-7y)'_x+\frac{(x^2+3xy)'_x\cdot y^2-(x^2+3xy)\cdot(y^2)'_x}{(y^2)^2}$

$=2\cdot(x)'_x+((x)'_x\cdot y+x\cdot(y)'_x)$

$\cos(3xy-7y)(3y+3xy'-7y')+\frac{(2x+3y+3xy')y^2-(x^2+3xy)\cdot(y^2)'_x}{y^4}$

$=2+y+xy'$

$\cos(3xy-7y)(3x-7)\cdot y'+\frac{3xy^2-2yx^2-6xy^2}{y^2}\cdot y'-xy'=$

$=2+y-\cos(3xy-7y)\cdot 3y-\frac{(2x+3y)y^2}{y^4}$

$$y'=\frac{2+y-3y\cdot\cos(3xy-7y)-\frac{2x+3y}{y^2}}{(3x-7)\cdot\cos(3xy-7y)+\frac{3xy+2x^2}{y^4}-x}$$

$dy = f'(x)dx$

$d^2y = f''(x)dx^2$

$d^3y = f'''(x)dx^3 = (d^2y)'dx$

4) $dy - ?$

$$y = \frac{(x^2+x+1)\cdot 7^x}{(x^3-5)\ln(x)}$$

$$y' = \frac{((x^2+x+1)\cdot 7^x)'((x^3-5)\ln x) - ((x^2+x+1)\cdot 7^x)((x^3-5)\ln x)'}{((x^3-5)\ln x)^2} =$$

$$= \frac{((x^2+x+1)' 7^x + (x^2+x+1)(7^x)')((x^3-5)\ln x) -}{(x^3-5)^2 \ln^2 x}$$

$$\frac{-((x^2+x+1)7^x)((x^3-5)'\ln x + (x^3-5)(\ln x)')}{(x^3-5)^2 \ln^2 x} =$$

$= ((2x+1)7^x + (x^2+x+1)7^x \ln 7)((x^3-5)\ln x) -$

$-((x^2+x+1)7^x)((3x^2)\ln x + (x^3-5)\frac{1}{x}) / (x^3-5)^2 \cdot \ln^2 x$

5) dy, d^2y, d^3y

1) $y = \sqrt[4]{x} \cdot (\ln x)$

$$y' = (\sqrt[4]{x}\ln x)' = \frac{1}{4}x^{\frac{1}{4}-1}\ln x + \sqrt[4]{x}\cdot\frac{1}{x} = \frac{\ln x}{4\cdot\sqrt[4]{x^3}} + \frac{\sqrt[4]{x}}{x} =$$

$$= \frac{\ln x}{4\sqrt[4]{x^3}} + \frac{1}{\sqrt[4]{x^3}} = \frac{4+\ln x}{4\sqrt[4]{x^3}} \; ; \; dy = \frac{4+\ln x}{4\sqrt[4]{x^3}}dx$$

$$y'' = \left(\frac{4 - \ln x}{4\sqrt[4]{x^3}}\right)' = \frac{\frac{1}{x}\cdot 4\cdot\sqrt[4]{x^3} - (4-\ln x)\cdot 4\cdot\frac{3}{4}\cdot x^{-\frac{1}{4}}}{16\cdot\left(x^{\frac{3}{4}}\right)^2} =$$

$$= \frac{\frac{4\sqrt[4]{x^3}}{x} - \frac{3(4-\ln x)}{\sqrt[4]{x}}}{16\cdot x^{3/2}} = \frac{4 - 12 - 3\ln x}{16\cdot x^{1/4}\cdot x^{3/2}} = \frac{-8-3\ln x}{16\sqrt[4]{x^7}}$$

$$d^2y = \frac{-8-3\ln x}{16\sqrt[4]{x^7}}\,dx^2$$

$$y''' = \left(\frac{-8-3\ln x}{16\sqrt[4]{x^7}}\right)' = \frac{-3\cdot\frac{1}{x}\cdot 16\cdot x^{\frac{7}{4}} - (-8-3\ln x)\cdot 16\cdot\frac{7}{4}\cdot x^{\frac{3}{4}}}{256\cdot x^{\frac{7}{2}}} =$$

$$= \frac{-48\, x^{3/4} + (8+3\ln x)\cdot 4\cdot 7\cdot x^{3/4}}{256\cdot x^{7/2}} =$$

$$= \frac{x^{3/4}\cdot 4\left(-12 + (8+3\ln x)\,7\right)}{256\cdot x^{7/2}} = \frac{44+21\ln x}{64\cdot\sqrt[4]{x^{11}}}$$

$$d^3y = \frac{44+21\ln x}{64\cdot\sqrt[4]{x^{11}}}\,dx^3$$

Д/з

5 2) $dy,\ d^2y,\ \ldots$ $\quad y = tg^2(x^5)$

$$y' = \left(tg^2(x^5)\right)' = \frac{10x^4\, tg(x^5)}{\cos^2(x^5)}$$

$$dy = \frac{10x^4\, tg(x^5)}{\cos^2(x^5)}$$

$$y'' = 10\left(\frac{x^4 tg(x^5)}{\cos^2(x^5)}\right)' = 10\cdot\frac{(x^4 tg(x^5))'\cos^2(x^5) - x^4 tg(x^5)(\cos^2(x^5))'}{\cos^4(x^5)} =$$

$$= 10\cdot\left(\frac{((x^4)' tg(x^5) + x^4(tg(x^5))')\cos^2(x^5) - x^4 tg(x^5)(\cos^2(x^5))'}{\cos^4(x^5)}\right) =$$

$$= 10\left(\frac{\left(4x^3 tg(x^5) + \frac{5x^8}{\cos^2(x^5)}\right)\cdot\cos^2(x^5) - x^4 tg(x^5)(-10x^4 \sin(x^5)\cos(x^5))}{\cos^4(x^5)}\right)$$

$$= \frac{40x^3 tg(x^5)}{\cos^2(x^5)} + \frac{50x^8}{\cos^4(x^5)} + \frac{100x^8 \sin(x^5) tg(x^5)}{\cos^3(x^5)}$$

$$d^2y = \frac{40x^3 tg(x^5)}{\cos^2(x^5)} + \frac{50x^8}{\cos^4(x^5)} + \frac{100x^8 \sin(x^5) tg(x^5)}{\cos^3(x^5)} dx^2$$

№6

1) $$\lim_{x\to 0} \frac{\ln(\sin(7x))}{\ln(5x)} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0} \frac{\frac{7\cos 7x}{\sin 7x}}{\frac{1}{x}} =$$

$$= \lim_{x\to 0} \frac{7x\cos 7x}{\sin 7x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{7\cos 7x - 49x\sin 7x}{7\cos 7x} =$$

$$= \frac{7\cdot 1 - 0}{7\cdot 1} = \frac{7}{7} = 1$$

2) $$\lim_{x\to 0} \frac{x^5}{x^2 - \sin x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{5x^4}{2x - \cos x} = \left[\frac{0}{0-1}\right] =$$

$$= \frac{0}{-1} = 0$$

3) $\lim\limits_{x\to 0+0} x\cdot \ln x = [0\cdot\infty] = \lim\limits_{x\to 0}\frac{\ln x}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] =$

$= \lim\limits_{x\to 0}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \left[\frac{\infty}{\infty}\right] = \lim\limits_{x\to 0} -\frac{x^2}{x} = \left[\frac{0}{0}\right] =$

$= \lim\limits_{x\to 0} -\frac{2x}{1} = -\frac{2\cdot 0}{1} = -\frac{0}{1} = 0$

4) $\lim\limits_{x\to 1}\left(\frac{1}{1-x^3} - \frac{1}{1-x^2}\right) = [\infty-\infty] =$

$= \lim\limits_{x\to 1}\left(\frac{1}{(1-x)(x^2+x+1)} - \frac{1}{(1-x)(1+x)}\right) =$

$= \lim\limits_{x\to 1}\left(\frac{1+x-x^2-x-1}{(1-x)(1+x)(x^2+x+1)}\right) = \lim\limits_{x\to 1}\frac{-x^2}{(1-x)(1+x)(x^2+x+1)}$

$= \frac{-1}{0\cdot 2\cdot 3} = -\frac{1}{0} = \infty$