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Подгруппа №1

Д.з (14.02.21)

№8.3.9

$$\int \frac{dx}{(x^2+1)^3} = \int \frac{dx}{(x^2+1^2)^3} = \frac{1}{2(k-1)a^2} \cdot \frac{x}{(x^2+a^2)^{k-1}} + \frac{1}{a^2} \cdot \frac{2k-3}{2k-2} \int \frac{dx}{(x^2+a^2)^{k-1}} =$$

$$= \frac{1}{4} \cdot \frac{x}{(x^2+1)^2} + \frac{3}{4} \int \frac{dx}{(x^2+1)^2} = \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{1}{2} \frac{x}{(x^2+1)} + \frac{1}{2} \int \frac{dx}{x^2+1}\right) =$$

$$= \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{x}{2(x^2+1)} + \frac{1}{2} \operatorname{arctg} \frac{x}{1}\right) + C =$$

$$= \frac{x}{4(x^2+1)^2} + \frac{3}{8}\left(\frac{x}{x^2+1} + \operatorname{arctg} x\right) + C$$

№8.3.10

$$\int \frac{dx}{(x^2-4x+29)^2} = \left[a = \sqrt{29-4} = 5 \right] =$$

$$= \int \frac{dx}{((x-2)^2+5^2)^2} = \left[\begin{matrix} x-2=y \\ dx=dy \end{matrix} \right] = \int \frac{dy}{(y^2+5^2)^2} =$$

$$= \frac{1}{50} \cdot \frac{y}{(y^2+25)} + \frac{1}{25} \cdot \frac{1}{2} \int \frac{dy}{y^2+5^2} = \frac{y}{50(y^2+25)} + \frac{1}{50}\left(\frac{1}{5} \operatorname{arctg} \frac{y}{5}\right) + C =$$

$$= \frac{1}{50}\left(\frac{x-2}{(x-2)^2+25} + \frac{1}{5} \operatorname{arctg} \frac{x-2}{5}\right) + C =$$

$$= \frac{1}{50}\left(\frac{x-2}{x^2-4x+29} + \frac{1}{5} \operatorname{arctg} \frac{x-2}{5}\right) + C$$

№8.3.11.

$$\int \frac{3x-2}{(x^2+6x+10)^2} dx = \left[\begin{matrix} A=3, \ B=-2, \ p=6, \ q=10, \ k=2 \\ x^2+6x+10=0 \Rightarrow D=-4<0 \end{matrix} \right. \Rightarrow$$

$$\Rightarrow 3x-2 = \frac{3}{2}(2x+6) + \left(-2 - \frac{3 \cdot 6}{2}\right) = \frac{3}{2}(2x+6) - 11 \Big] =$$

$$= \frac{3}{2} \int \frac{(2x+6)dx}{(x^2+6x+10)^2} - 11 \int \frac{dx}{(x^2+6x+10)^2} = \left[\begin{matrix} 1)\ t = x^2+6x+10 \Rightarrow dt = (2x+6)dx \\ 2)\ y = x+3 \Rightarrow dy = dx \end{matrix} \right] =$$

$$= \left[a = \sqrt{10-9} = 1 \right] = \frac{3}{2} \int \frac{dt}{t^2} - 11 \int \frac{dy}{(y^2+1^2)^2} =$$

$$= \frac{3}{2}\int t^{-2}dt - 11\int\frac{dy}{(y^2+1^2)^2} = -\frac{3}{2t} - 11\left(\frac{1}{2}\cdot\frac{y}{y^2+1} + \frac{1}{2}\int\frac{dy}{y^2+1}\right) + C =$$

$$= -\frac{3}{2t} - 11\left(\frac{y}{2(y^2+1)} + \frac{1}{2}\operatorname{arctg} y\right) + C =$$

$$= -\frac{3}{2(x^2+6x+10)} - \frac{11}{2}\left(\frac{x+3}{x^2+6x+10} + \operatorname{arctg}(x+3)\right) + C =$$

$$= C - \frac{11x+36}{2(x^2+6x+10)} - \frac{11}{2}\operatorname{arctg}(x+3)$$

№8.3.13

$$\int\frac{2x-3}{(x-5)(x+2)}dx = \left[\begin{array}{l}(x-5)(x+2)=0\\ x_1=5,\ x_2=-2\end{array};\ \frac{2x-3}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2} =\right.$$

$$= \frac{A(x+2)+B(x-5)}{(x-5)(x+2)} \Rightarrow 2x-3 = A(x+2) + B(x-5)$$

1) $2x-3 = Ax + 2A + Bx - 5B \Rightarrow 2x-3 = (A+B)x + (2A-5B)$

$$\begin{cases} A+B=2 \\ 2A-5B=-3 \end{cases} \quad \begin{cases} A=2-B \\ 4-2B-5B=-3 \end{cases} \quad \begin{cases} A=1 \\ B=1 \end{cases}$$

2) $2x-3 = A(x+2) + B(x-5)$

$x_1=5 \Rightarrow 10-3=7A \Leftrightarrow A=1$

$x_2=-2 \Rightarrow -7=-7B \Rightarrow B=1$

Тогда $\frac{2x-3}{(x-5)(x+2)} = \frac{1}{x-5} + \frac{1}{x+2} \Big] =$

$$= \int\frac{dx}{x-5} + \int\frac{dx}{x+2} = \ln|x-5| + \ln|x+2| + C =$$

$$= \ln|(x-5)(x+2)| + C$$

№8.3.14

$$\int \frac{x+2}{x^2-6x+5}dx = \int \frac{x+2}{(x-5)(x-1)}dx = \left[\begin{matrix} (x-5)(x-1)=0 \\ x_1=5,\ x_2=1 \end{matrix} \quad \frac{x+2}{(x-5)(x-1)} = \frac{A}{x-5} + \frac{B}{x-1} = \right.$$

$$= \frac{A(x-1)+B(x-5)}{(x-5)(x-1)} \Rightarrow x+2 = A(x-1)+B(x-5)$$

1) $x+2 = Ax - A + Bx - 5B \Rightarrow x+2 = (A+B)x + (-A-5B)$

$$\begin{cases} A+B=1 \\ -A-5B=2 \end{cases} \quad \begin{cases} A=1-B \\ -1+B-5B=2 \end{cases} \quad \begin{cases} A=\frac{7}{4} \\ B=-\frac{3}{4} \end{cases}$$

2) $x+2 = A(x-1)+B(x-5)$

$x_1=5 \Rightarrow 7=4A \Rightarrow A=\frac{7}{4}$

$x_2=1 \Rightarrow 3=-4B \Rightarrow B=-\frac{3}{4}$

Тогда $\frac{x+2}{(x-5)(x-1)} = \frac{\frac{7}{4}}{x-5} - \frac{\frac{3}{4}}{x-1} \Big] =$

$$= \frac{7}{4}\int \frac{dx}{x-5} - \frac{3}{4}\int \frac{dx}{x-1} = \frac{7}{4}\ln|x-5| - \frac{3}{4}\ln|x-1| + C$$

№ 8.3.15.

$$\int \frac{dx}{x^4+x^2} = \int \frac{dx}{x^2(x^2+1)} = \left[\begin{matrix} x^2(x^2+1)=0 \Rightarrow \\ x=0 \end{matrix} \quad \frac{1}{x^2(x^2+1)} = \frac{A}{x^2} + \frac{B}{x^2+1} \right.$$

$$= \frac{A(x^2+1)+Bx^2}{x^2(x^2+1)} \Rightarrow 1 = A(x^2+1) + Bx^2 .$$

1) $1 = Ax^2 + A + Bx^2 \Rightarrow 1 = (A+B)x^2 + A$

$$\begin{cases} A+B=0 \\ A=1 \end{cases} \Rightarrow \begin{cases} B=-1 \\ A=1 \end{cases}$$

2) $1 = A(x^2+1) + Bx^2$

$x=0 \Rightarrow 1 = A$

$x_1=1 \Rightarrow 1 = 2A + B \Rightarrow 1 = 2 + B \Rightarrow B=-1$

Тогда $\frac{1}{x^2(x^2+1)} = \frac{1}{x^2} - \frac{1}{x^2+1} \Big] =$

$$= \int \frac{dx}{x^2} - \int \frac{dx}{x^2+1} = -\frac{1}{x} - \operatorname{arctg} x + C =$$

$$= C - \frac{1}{x} - \operatorname{arctg} x$$

№ 8.3.16

$$\int \frac{x^5+x^4-8}{x^3-4x}\,dx = \Big[$$

$$\begin{array}{l|l} -\,x^5 + x^4 + 0x^3 + 0x^2 + 0x - 8 & x^3-4x \\ \underline{x^5 - 4x^3} & \overline{x^2+x+4} \\ \quad -\,x^4 + 4x^3 & \\ \quad \underline{x^4 - 4x^2} & \\ \qquad -\,4x^3 + 4x^2 & \\ \qquad \underline{4x^3 - 16x} & \\ \qquad\quad 4x^2 + 16x - 8 & \end{array}$$

$$\frac{4x^2+16x-8}{x^3-4x} + x^2 + x + 4 \quad \Big] =$$

$$= \int \frac{4x^2+16x-8}{x^3-4x}dx + \int x^2 dx + \int x\,dx + 4\int dx =$$

$$= \int \frac{4x^2+16x-8}{x(x^2-4)}dx + \frac{x^3}{3} + \frac{x^2}{2} + 4x + C = [$$

$$\frac{A}{x} + \frac{B}{x-2} - \frac{C}{x+2} = \frac{A(x^2-4)+Bx(x+2)+Cx(x-2)}{x(x-2)(x+2)} \Rightarrow$$

$$\Rightarrow 4x^2+16x-8 = A(x^2-4) + B(x^2+2x) + C(x^2-2x)$$

1) $4x^2+16x-8 = Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx$

$4x^2+16x-8 = (A+B+C)x^2+(2B-2C)x-4A$

$$\begin{cases} A+B+C=4 \\ 2B-2C=16 \\ -4A=-8 \end{cases} \quad \begin{cases} C=2-B \\ 2B-4+2B=16 \\ A=2 \end{cases} \quad \begin{cases} C=-3 \\ B=5 \\ A=2 \end{cases}$$

2)

$x=2 \Rightarrow 4\cdot 4+16\cdot 2-8 = 8B \Rightarrow 40=8B \Rightarrow B=5$

$x=-2 \Rightarrow 4\cdot 4-2\cdot 16-8 = 8C \Rightarrow -24=8C \Rightarrow C=-3$

Тогда $\frac{2}{x} + \frac{5}{x-2} - \frac{3}{x+2}$ $] =$

$$= 2\ln|x| + 5\ln|x-2| - 3\ln|x+2| + \frac{x^3}{3} + \frac{x^2}{2} + 4x + C$$

№ 8.3.17.

$$\int \frac{dx}{x^3-8} = \int \frac{dx}{(x-2)(x^2+2x+4)} = [\frac{A}{x-2} + \frac{Bx+C}{(x^2+2x+4)} =$$

$$\frac{A(x^2+2x+4)+(Bx+C)(x-2)}{(x-2)(x^2+2x+4)} \Rightarrow 1 = A(x^2+2x+4)+(Bx+C)(x-2)$$

1 способ:

$1 = Ax^2 + 2Ax + 4A + Bx^2 - 2Bx + Cx - 2C$

$1 = (A+B)x^2 + (2A - 2B + C)x + (4A - 2C)$

$$\begin{cases} A+B=0 \\ 2A-2B+C=0 \\ 4A-2C=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{12} \\ B=-\frac{1}{12} \\ C=-\frac{1}{3} \end{cases}$$

Тогда $\frac{1}{12(x-2)} - \frac{x+4}{12(x^2+2x+4)} \Big] =$

$= \frac{1}{12}\int\frac{dx}{x-2} - \frac{1}{12}\int\frac{x+4}{x^2+2x+4}dx = \Big[\int\frac{x+4}{x^2+2x+4}dx =$

$= \int\frac{\frac{1}{2}(2x+2)+3}{x^2+2x+4}dx = \frac{1}{2}\int\frac{(2x+2)dx}{x^2+2x+4} + 3\int\frac{dx}{x^2+2x+4} =$

$= \frac{1}{2}\ln|x^2+2x+4| + 3\int\frac{dx}{(x+1)^2+\sqrt{3}^2} =$

$= \frac{1}{2}\ln|x^2+2x+4| + \frac{3}{\sqrt{3}}\operatorname{arctg}\frac{x+1}{\sqrt{3}} + C \Big] =$

$= \frac{1}{12}\ln|x-2| - \frac{1}{24}\ln|x^2+2x+4| - \frac{3}{12\sqrt{3}}\operatorname{arctg}\frac{x+1}{\sqrt{3}} + C =$

$= \frac{1}{12}\ln|x-2| - \frac{1}{24}\ln|x^2+2x+4| - \frac{\sqrt{3}}{12}\operatorname{arctg}\frac{x+1}{\sqrt{3}} + C$

№8.3.18

$\int\frac{7x^3-10x^2+50x-77}{(x^2+9)(x^2+x-2)}dx = \int\frac{7x^3-10x^2+50x-77}{(x^2+9)(x-1)(x+2)}dx = \Big[$

$\frac{Ax+B}{x^2+9} + \frac{C}{x-1} + \frac{D}{x+2} =$

$= \frac{(Ax+B)(x-1)(x+2) + C(x^2+9)(x+2) + D(x-1)(x^2+9)}{(x^2+9)(x-1)(x+2)} =$

$= Ax^3 + Ax^2 - 2Ax + Bx^2 + Bx - 2B + Cx^3 + 9Cx + 2Cx^2 + 18C + Dx^3 + 9Dx -$

$- Dx^2 - 9D$

I способ $(A+C+D)x^3+(A+B+2C-D)x^2+(-2A+B+9C-9D)x+$
$+(-2B+18C-9D)$

$$\begin{cases} A+C+D=7 \\ A+B+2C-D=-10 \\ -2A+B+9C+9D=50 \\ -2B+18C-9D=-77 \end{cases} \qquad \begin{cases} A=7-C-D \\ B=-17-C+2D \\ C=\dfrac{81-13D}{10} \\ D=7 \end{cases} \qquad \begin{cases} A=1 \\ B=-2 \\ C=-1 \\ D=7 \end{cases}$$

Тогда $\dfrac{x-2}{x^2+9}-\dfrac{1}{x-1}+\dfrac{7}{x+2}\,\Big]=$

$$=\int\frac{x-2}{x^2+9}dx-\int\frac{dx}{x-1}+7\int\frac{dx}{x+2}=\int\frac{x\,dx}{x^2+9}-2\int\frac{dx}{x^2+9}-\int\frac{dx}{x-1}+$$

$$+7\int\frac{dx}{x+2}=\Big[\int\frac{x\,dx}{x^2+9}=\Big[\begin{matrix} x^2+9=t \\ 2x\,dx=dt \Rightarrow x\,dx=\frac{dt}{2}\end{matrix}\Big]=$$

$$=\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}\ln|t|+C=\frac{1}{2}\ln|x^2+9|+C\Big]=$$

$$=\frac{1}{2}\ln|x^2+9|-\frac{2}{3}\operatorname{arctg}\frac{x}{3}-\ln|x-1|+7\ln|x+2|+C$$