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Подгруппа №1

лекция (30.11.20)

№ 8.2 10

1) $\int \frac{x - \sin\frac{1}{x}}{x^2}\,dx = \int\left(\frac{x}{x^2} - \frac{\sin\frac{1}{x}}{x^2}\right)dx =$

$= \int\frac{dx}{x} - \int\frac{\sin\frac{1}{x}}{x^2}\,dx = \left[\begin{array}{l} 1)\ \int\frac{dx}{x} = \ln|x| + C \\ 2)\ t = \frac{1}{x} \Rightarrow dt = d\left(\frac{1}{x}\right) = \\ = \left(\frac{1}{x}\right)' dx = -\frac{dx}{x^2} \Rightarrow \\ \Rightarrow \frac{dx}{x^2} = -dt \end{array}\right] = \int\frac{dx}{x} - \int \sin t\cdot(-dt) =$

$= \int\frac{dx}{x} + \int \sin t\cdot dt = \ln|x| - \cos t + C =$

$= \ln|x| - \cos\frac{1}{x} + C$

2) $\int\frac{5x-1}{\sqrt{4-x^2}}\,dx = \int\left(\frac{5x}{\sqrt{4-x^2}} - \frac{1}{\sqrt{4-x^2}}\right)dx =$

$= 5\int\frac{x\,dx}{\sqrt{4-x^2}} - \int\frac{dx}{\sqrt{4-x^2}} = \left[\begin{array}{l} 1)\ t = 4-x^2 \Rightarrow dt = d(4-x^2) = \\ = (4-x^2)'dx = -2x\,dx \Rightarrow x\,dx = -\frac{1}{2}dt \end{array}\right] =$

$$= 5\int \frac{-\frac{1}{2}dt}{\sqrt{t}} - \int \frac{dx}{\sqrt{4-x^2}} = -\frac{5}{2}\int \frac{dt}{\sqrt{t}} - \int \frac{dx}{\sqrt{2^2-x^2}} =$$

$$= -\frac{5}{2}\cdot 2\cdot\sqrt{t} - \arcsin\frac{x}{2} + C = -5\sqrt{4-x^2} - \arcsin\frac{x}{2} + C$$

№ 2.15.

1) $\int \frac{\sqrt{1-x^2}}{x^2}dx = \left[\begin{array}{l} x = \psi(t),\ x = \sin t \\ 1-x^2 = 1-\sin^2 t = \cos^2 t \\ x = \sin t \Rightarrow dx = d(\sin t) = \\ = \cos t\,dt \end{array} \right] = \int \frac{\sqrt{\cos^2 t}\ \cos t\,dt}{\sin^2 t} =$

$$= \int \frac{\cos t\cdot\cos t}{\sin^2 t}dt = \int \frac{\cos^2 t}{\sin^2 t}dt =$$

$$= \int \frac{1-\sin^2 t}{\sin^2 t}dt = \int\left(\frac{1}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t}\right)dt =$$

$$= \int \frac{dt}{\sin^2 t} - \int 1\,dt = -\operatorname{ctg} t - t + C =$$

$$= -\operatorname{ctg}(\arcsin(x)) - \arcsin(x) + C =$$

$$= -\frac{\sqrt{1-x^2}}{x} - \arcsin x + C$$

2) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \left[\begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right] = \int \frac{2t\,dt}{\sqrt{t^2}(1+\sqrt{t^2})} =$

$$= \int \frac{2t\,dt}{t(1+t)} = 2\int \frac{dt}{1+t} = 2\int \frac{d(t+1)}{t+1} =$$

$$= 2\cdot\ln|t+1| + C = 2\ln|\sqrt{x}+1| + C =$$

$$= 2\ln(\sqrt{x}+1) + C$$

№ 8.2.20

1) $\int x\, e^x\, dx = \left[\begin{matrix} u = x \Rightarrow u' = 1 \\ v' = e^x \Rightarrow v = \int v'\, dx = \int e^x dx = e^x \end{matrix} \right] =$

$= x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + C$

$\int x \cdot e^x dx = \left[\begin{matrix} u = e^x \Rightarrow u' = e^x \\ v' = x \Rightarrow v = \frac{x^2}{2} \end{matrix} \right] =$

$= e^x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot e^x dx = \left[\begin{matrix} \text{т.к. для } x \text{ степень} \uparrow \\ \Rightarrow \text{усложнили} \end{matrix} \right]$

2) $\int \ln x\, dx = \left[\begin{matrix} u = \ln x \Rightarrow u' = \frac{1}{x} \\ v' = 1 \Rightarrow v = x \end{matrix} \right] =$

$= x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int dx =$

$= x \ln x - x + C$

3) $\int x^2 \cdot \cos x\, dx = \left[\begin{matrix} u = x^2 \Rightarrow u' = 2x \\ v' = \cos x \Rightarrow v = \sin x \end{matrix} \right] =$

$= x^2 \cdot \sin x - \int \sin x \cdot 2x\, dx =$

$= x^2 \sin x - 2 \int \sin x \cdot x\, dx = \left[\begin{matrix} u = x \Rightarrow u' = 1 \\ v = \sin x \Rightarrow v = -\cos x \end{matrix} \right.$

$= x^2 \sin x - 2(x \cdot (-\cos x) - \int (-\cos x) dx) =$

$= x^2 \cdot \sin x - 2(-x \cdot \cos x + \int \cos x\, dx) =$

$= x^2 \cdot \sin x - 2(-x \cdot \cos x + \sin x) + C =$

№8.2.27

$$\int e^x \cos x\,dx = \left[\begin{matrix} u = e^x \Rightarrow u' = e^x \\ v' = \cos x \Rightarrow v = \sin x \end{matrix}\right] =$$

$$= e^x \sin x - \int \sin x\, e^x = \left[\begin{matrix} u = e^x \Rightarrow u' = e^x \\ v' = \sin x \Rightarrow v = -\cos x \end{matrix}\right] =$$

$$= e^x \sin x - (e^x(-\cos x) - \int(-\cos x)\cdot e^x dx) =$$

$$= e^x \sin x - (-e^x \cos x + \int e^x \cos x\, dx) =$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x\, dx$$

То есть:

$$\int e^x \cos x\, dx = e^x \sin x + e^x \cos x - \int e^x \cos x\, dx + C$$

$$2\int e^x \cos x\, dx = e^x \sin x + e^x \cos x + C$$

$$\int e^x \cos x\, dx = \frac{e^x(\sin x + \cos x)}{2} + C$$

№8.2.30

1) $$\int \operatorname{arctg} x\, dx = \left[\begin{matrix} u = \operatorname{arctg} x \Rightarrow u' = \frac{1}{1+x^2} \\ v' = 1 \Rightarrow v = x \end{matrix}\right] =$$

$$= x \operatorname{arctg} x - \int \frac{x\,dx}{1+x^2} = \left[t = 1+x^2 \Rightarrow \right.$$

$$\left. \begin{matrix} \Rightarrow dt = d(1+x^2) = \\ = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dt \end{matrix} \right] = \int \frac{x\,dx}{1+x^2} = \int \frac{\frac{1}{2}dt}{t} =$$

$$= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln(1+x^2) + C$$

Отсюда

$$\int \operatorname{arctg} x\, dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$$

2) $$\int \sin\sqrt{x}\, dx = \left[\begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right] = \int \sin t \, 2t\, dt =$$

$$= \left[\begin{array}{l} v' = \sin t \Rightarrow v = -\cos t \\ u = 2t \Rightarrow u' = 2 \end{array} \right] =$$

$$= -2t\cos t - \int -2\cos t\, dt =$$

$$= -2t\cos t + 2\int \cos t\, dt =$$

$$= -2t\cos t + 2\sin t + C =$$

$$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C$$