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Подгруппа №1

№8.1.29

$$\int\frac{dx}{x^2\sqrt{x}} = \int x^{-2}\cdot x^{-\frac{1}{2}}dx = \int x^{-\frac{5}{2}}dx = \frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1}+C = \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}+C =$$

$$= -\frac{2}{3\sqrt{x^3}}+C = C-\frac{2}{3x\sqrt{x}}$$

№8.1.30

$$\int\frac{dx}{x^2+3} = \int\frac{dx}{x^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}+C$$

№8.1.31

$$\int\frac{1}{5^x}dx = \int\left(\frac{1}{5}\right)^x dx = \frac{\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}}+C = \frac{1}{5^x\ln 5^{-1}}+C =$$

$$= -\frac{1}{5^x\ln 5}+C$$

№8.1.32

$$\int\frac{dx}{\sqrt{4-x^2}} = \int\frac{dx}{\sqrt{2^2-x^2}} = \arcsin\frac{x}{2}+C$$

№8.1.33

$$\int\frac{dx}{\sqrt{x^2+(-1)}} = \ln|x+\sqrt{x^2+(-1)}|+C = \ln|x+\sqrt{x^2-1}|+C$$

№8.1.34

$$\int\frac{dx}{x^2-25} = \int\frac{dx}{x^2-5^2} = \frac{1}{2\cdot 5}\ln\left|\frac{x-5}{x+5}\right|+C = \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right|+C$$

№8.1.35

$$\int\left(x+\frac{2}{x}\right)^2dx = \int\left(x^2+\frac{4x}{x}+\frac{4}{x^2}\right)dx = \int x^2dx+4\int dx+4\int\frac{1}{x^2}dx =$$

$$= \frac{x^{2+1}}{2+1}+4x+4\frac{x^{-2+1}}{-2+1}+C = \frac{x^3}{3}+4x-\frac{4}{x}+C$$

№8.1.36

$$\int\frac{dx}{4x^2+1} = \frac{1}{4}\int\frac{dx}{x^2+\left(\frac{1}{2}\right)^2} = \frac{1}{4}\cdot\frac{1}{\frac{1}{2}}\operatorname{arctg}\frac{x}{\frac{1}{2}}+C = \frac{1}{2}\operatorname{arctg}(2x)+C$$

№8.1.37.

$$\int\left(7^x - \frac{8}{x} + 4\cos x\right)dx = \int 7^x dx - 8\int\frac{dx}{x} + 4\int \cos x\, dx =$$

$$= \frac{7^x}{\ln 7} - 8\ln|x| + 4\sin x + C$$

№8.1.38

$$\int\left(\frac{\sqrt{5}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right)dx = \sqrt{5}\int\frac{dx}{\cos^2 x} - \int x^{\frac{1}{3}}dx - 2\int\frac{dx}{x^4} =$$

$$= \sqrt{5}\,\mathrm{tg}\,x - \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - 2\,\frac{x^{-4+1}}{-4+1} + C = \sqrt{5}\,\mathrm{tg}\,x - \frac{3x\sqrt[3]{x}}{4} + \frac{2}{3x^3} + C$$

№8.1.39

$$\int\frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}}dx = \int\frac{\sqrt{x}}{\sqrt[4]{x}}dx - 3\int\frac{\sqrt[5]{x^2}}{\sqrt[4]{x}} + \int\frac{dx}{\sqrt[4]{x}} =$$

$$= \int x^{\frac{1}{2}-\frac{1}{4}}dx - 3\int x^{\frac{2}{5}-\frac{1}{4}}dx + \int x^{-\frac{1}{4}}dx =$$

$$= \int x^{\frac{1}{4}}dx - 3\int x^{\frac{3}{20}}dx + \int x^{-\frac{1}{4}}dx = \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} - 3\,\frac{x^{\frac{3}{20}+1}}{\frac{3}{20}+1} + \frac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} + C =$$

$$= \frac{4x\sqrt[4]{x}}{5} - \frac{60x\sqrt[20]{x^3}}{23} + \frac{4\sqrt[4]{x^3}}{3} + C$$

№8.1.40

$$\int\left(0{,}7x^{-0{,}1} + 0{,}2\cdot(0{,}5)^x\right)dx = \int\left(\frac{7x^{-\frac{1}{10}}}{10} + \frac{\left(\frac{1}{2}\right)^x}{5}\right)dx =$$

$$= \frac{7}{10}\int x^{-\frac{1}{10}}dx + \frac{1}{5}\int\left(\frac{1}{2}\right)^x dx = \frac{7}{10}\cdot\frac{x^{-\frac{1}{10}+1}}{-\frac{1}{10}+1} + \frac{1}{5}\cdot\frac{\left(\frac{1}{2}\right)^x}{\ln\frac{1}{2}} + C =$$

$$= \frac{7}{10}\cdot\frac{10\sqrt[10]{x^9}}{9} - \frac{1}{5\,2^x\ln 2} + C = \frac{7\sqrt[10]{x^9}}{9} - \frac{1}{2^x\,5\ln 2} + C$$

№8.1.41

$$\int(5\,\mathrm{sh}\,x - 7\,\mathrm{ch}\,x + 1)dx = 5\int \mathrm{sh}\,x\,dx - 7\int \mathrm{ch}\,x + \int dx =$$

$$= 5\,\mathrm{ch}\,x - 7\,\mathrm{sh}\,x + x + C$$

№8.1.42

$$\int(x^2-1)(\sqrt{x}+4)dx = \int(\sqrt{x^5}-4x^2-\sqrt{x}-4)dx =$$
$$= \int x^{\frac{5}{2}}dx + 4\int x^2dx - \int x^{\frac{1}{2}}dx - 4\int dx =$$
$$= \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + 4\frac{x^{2+1}}{2+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4x + C =$$
$$= \frac{2x^3\sqrt{x}}{7} + \frac{4x^3}{3} - \frac{2x\sqrt{x}}{3} - 4x + C$$

№8.1.43

$$\int\frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}dx = 7\int\frac{dx}{\sqrt{x^2+\pi}} - \int dx = 7\ln|x+\sqrt{x^2+\pi}| - x + C$$

№8.1.44

$$\int\left(\frac{\sqrt{x}-5}{x}\right)^3dx \quad \int\frac{(\sqrt{x}-5)^3}{x^3}dx = \int\frac{\sqrt{x^3}-3\sqrt{x^2}\cdot 5+3\sqrt{x}\,25-125}{x^3}dx =$$
$$= \int\frac{\sqrt{x^3}}{x^3}dx - 15\int\frac{\sqrt{x^2}}{x^3}dx + 75\int\frac{\sqrt{x}}{x^3}dx - 125\int\frac{dx}{x^3} =$$
$$= \int x^{\frac{3}{2}-3}dx - 15\int x^{1-3}dx + 75\int x^{\frac{1}{2}-3}dx - 125\int x^{-3}dx =$$
$$= \int x^{-\frac{3}{2}}dx - 15\int x^{-2}dx + 75\int x^{-\frac{5}{2}}dx - 125\int x^{-3}dx =$$
$$= \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} - 15\frac{x^{-2+1}}{-2+1} + 75\frac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} - 125\frac{x^{-3+1}}{-3+1} + C =$$
$$= -\frac{2}{\sqrt{x}} + \frac{15}{x} - \frac{150}{3x\sqrt{x}} + \frac{125}{2x^2} + C = \frac{15}{x} - \frac{2}{\sqrt{x}} - \frac{50}{x\sqrt{x}} + \frac{125}{2x^2} + C$$

№8.1.45

$$\int \sin 7x\,dx = -\frac{1}{7}\cos 7x + C$$

№8.1.46

$$\int\sqrt[5]{2x-8}\,dx = \left[2x-8 = ax+b\right]\frac{1}{2}\cdot\frac{(2x-8)^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C =$$
$$= \frac{1}{2}\cdot\frac{\sqrt[5]{(2x-8)^6}}{\frac{6}{5}} + C = \frac{5}{12}\cdot\sqrt[5]{(2x-8)^6}$$

№8.1.47

$$\int (1-4x)^{2001}dx = [-4x+1 = ax+b] = -\frac{1}{4}\frac{(1-4x)^{2001+1}}{2001+1} + C =$$
$$= -\frac{(1-4x)^{2002}}{8008} + C$$

№8.1.48

$$\int \frac{dx}{9x+7} = [9x+7 = ax+b] = \frac{1}{9}\ln|9x+7| + C$$

№8.1.49

$$\int\frac{dx}{(6x+11)^4} = [6x+11=ax+b] = \frac{1}{6} \frac{(6x+11)^{-4+1}}{-4+1} + C =$$
$$= -\frac{(6x+11)^{-3}}{18} + C = -\frac{1}{18(6x+11)^3} + C$$

№8.1.50

$$\int\frac{dx}{25x^2+1} = \frac{1}{25}\int\frac{dx}{x^2+\frac{1}{25}} = \frac{1}{25}\int\frac{dx}{x^2+(\frac{1}{5})^2} =$$
$$= \frac{1}{25}\cdot\frac{1}{\frac{1}{5}}\operatorname{arctg}\frac{x}{\frac{1}{5}} + C = \frac{1}{5} \operatorname{arctg}5x + C$$

№8.1.51

$$\int 3^{2-11x}dx = [-11x+2=ax+b] = -\frac{1}{11}\frac{3^{2-11x}}{\ln 3} + C = -\frac{3^{2-11x}}{11\ln 3}+C$$

№8.1.52

$$\int\frac{dx}{\sqrt{4x^2-1}} = \int\frac{dx}{\sqrt{4(x^2-\frac{1}{4})}} = \frac{1}{\sqrt{4}}\int\frac{dx}{\sqrt{x^2-\frac{1}{4}}} = \frac{1}{\sqrt{4}}\int\frac{dx}{\sqrt{x^2+(-\frac{1}{4})}} =$$
$$= \frac{1}{\sqrt{4}}\ln|x+\sqrt{x^2-\frac{1}{4}}| + C = \frac{1}{2}\ln|x+\sqrt{x^2-\frac{1}{4}}|+C$$

№8.1.53

$$\int \sin^2 3x\,dx = \int\frac{1-\cos 6x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int\cos 6x\,dx =$$
$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{1}{6}\sin 6x + C = \frac{x}{2} - \frac{\sin 6x}{12} + C$$

№8.1.54

$$\int \cos^2 8x\,dx = \int \frac{1+\cos 16x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 16x\,dx =$$

$$= \frac{x}{2} + \frac{1}{2}\cdot\frac{1}{16}\sin 16x + C = \frac{x}{2} + \frac{\sin 16x}{32} + C$$

№8.1.55

$$\int tg^2 x\,dx = \int \frac{1-\cos 2x}{1+\cos 2x}dx = :($$

$$\int tg^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx =$$

$$= \int \frac{dx}{\cos^2 x} - \int dx = tg x - x + C$$

№8.1.56

$$\int \frac{4x+1}{x-5}dx = \int \frac{4x+1-21+21}{x-5}dx = \int \frac{4x-20+21}{x-5}dx =$$

$$= 4\int \frac{x-5}{x-5}dx + 21\int \frac{dx}{x-5} = 4\int dx + 21\int \frac{dx}{x-5} =$$

$$= \left[\begin{array}{l} x-5=t \\ dt=d(x-5) \Rightarrow dt=dx \end{array}\right] = 4\int dx + 21\int \frac{dt}{t} =$$

$$= 4x + 21\ln|t| + C = 4x + 21\ln|x-5| + C$$

№8.1.57

$$\int (3tg x - 2ctg x)^2 dx = \int (9tg^2 x - 12 tg x\, ctg x + 4ctg^2 x)dx =$$

$$= 9\int tg^2 x\,dx - 12\int dx + 4\int ctg^2 x\,dx = 9\int \frac{\sin^2 x}{\cos^2 x}dx - 12\int dx +$$

$$+ 4\int \frac{\cos^2 x}{\sin^2 x}dx = 9\int \frac{1-\cos^2 x}{\cos^2 x}dx - 12\int dx + 4\int \frac{1-\sin^2 x}{\sin^2 x}dx =$$

$$= 9\int \frac{dx}{\cos^2 x} - 9\int dx - 12\int dx + 4\int \frac{dx}{\sin^2 x} - 4\int dx =$$

$$= 9tg x - 9x - 12x - 4ctg x - 4x + C =$$

$$= 9tg x - 4ctg x - 25x + C$$

№8.1.58

$$\int \frac{4\sqrt{1-x^2}+3x^2}{x^2-1}dx = \int \frac{4\sqrt{1-x^2}}{x^2-1}dx + \int \frac{3x^2}{x^2-1}dx =$$

$$= -4\int \frac{\sqrt{1-x^2}}{1-x^2}dx + 3\int \frac{x^2}{x^2-1}dx =$$

$$= -4\int (1-x^2)^{\frac{1}{2}-1}dx + 3\int \frac{x^2-1+1}{x^2-1}dx =$$

$$= -4\int (1-x^2)^{-\frac{1}{2}}dx + 3\int dx + 3\int \frac{dx}{x^2-1} =$$

$$= -4\int \frac{dx}{\sqrt{1-x^2}} + 3\int dx + 3\int \frac{dx}{x^2-1} =$$

$$= -4\arcsin x + 3x + \frac{3}{2}\ln\left|\frac{x-1}{x+1}\right| + C =$$

$$= 3x + \frac{3}{2}\ln\left|\frac{x-1}{x+1}\right| - 4\arcsin x + C$$

№8.1.59

$$\int \frac{\cos 2x\,dx}{\sin^2 x\cos^2 x} = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x\cos^2 x}dx = \int \frac{\cos^2 x\,dx}{\sin^2 x\cos^2 x} - \int \frac{\sin^2 x}{\sin^2 x\cos^2 x}$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} = -ctg\,x - tg\,x + C = C - ctg\,x - tg\,x$$

№8.1.60

$$\int \frac{\sin 2x}{\cos x}dx = \int \frac{2\sin x\cos x}{\cos x}dx = 2\int \sin x\,dx = -2\cos x + C$$

$$= C - 2\cos x$$