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Подгруппа №1

Интегралы и диф. уравнения (07.12.20)

№8.1.2

$$\int x^{10}dx = \frac{x^{10+1}}{10+1} + C = \frac{x^{11}}{11} + C$$

№8.1.3

$$\int \frac{dx}{x^7} = \int x^{-7}dx = \frac{x^{-7+1}}{-7+1} + C = -\frac{x^{-6}}{6} + C = -\frac{1}{6x^6} + C$$

№8.1.4

$$\int \sqrt[4]{x}\,dx = \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + C = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C = \frac{4\sqrt[4]{x^5}}{5} + C = \frac{4x\sqrt[4]{x}}{5} + C$$

№8.1.5

$$\int \frac{dx}{x^2+9} = \int \frac{dx}{x^2+3^2} = \frac{1}{3}\operatorname{arctg}\frac{x}{3} + C$$

№8.1.6

$$\int \frac{dx}{x^2-\frac{1}{2}} = \int \frac{dx}{x^2-\left(\sqrt{\frac{1}{2}}\right)^2} = \frac{1}{2\sqrt{\frac{1}{2}}}\ln\left|\frac{x-\sqrt{\frac{1}{2}}}{x+\sqrt{\frac{1}{2}}}\right| + C = \frac{1}{\sqrt{2}}\ln\left|\frac{x\sqrt{2}-1}{x\sqrt{2}+1}\right| + C$$

№3.1.7

$$\int \frac{dx}{\sqrt{x^2+3}} = \ln|x+\sqrt{x^2+3}|+C$$

№3.1.9

$$\int \frac{x^4+x^2-6x}{x^3}dx = \int\frac{x^4}{x^3}dx + \int\frac{x^2}{x^3}dx - \int\frac{6x}{x^3}dx =$$

$$= \int\left(x+\frac{1}{x}-\frac{6}{x^2}\right)dx = \int x\,dx + \int\frac{dx}{x} - 6\int\frac{dx}{x^2} =$$

$$= \frac{x^{1+1}}{1+1} + \ln|x| - 6\frac{x^{-2+1}}{-2+1} + C = \frac{x^2}{2} + \ln|x| + \frac{6}{x} + C$$

№3.1.10

$$\int\left(\frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7}\right)dx = 5\int\frac{dx}{x} - 10\int\frac{dx}{\sqrt[4]{x^3}} - 3\int\frac{dx}{x^2+7} =$$

$$= 5\int\frac{dx}{x} - 10\int\frac{dx}{\sqrt[4]{x^3}} - 3\int\frac{dx}{x^2+(\sqrt{7})^2} = \left[\frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} = \frac{x^{\frac{1}{4}}}{\frac{1}{4}} = 4\sqrt[4]{x}\right] =$$

$$= 5\ln|x| - 40\sqrt[4]{x} - \frac{3}{\sqrt{7}}\operatorname{arctg}\frac{x}{\sqrt{7}} + C$$

№3.1.11

$$\int\sqrt{x}(x^2+1)dx = \int\left(x^{2+\frac{1}{2}} + x^{\frac{1}{2}}\right)dx = \int(\sqrt{x^5}+\sqrt{x})dx =$$

$$= \int x^{\frac{5}{2}}dx + \int x^{\frac{1}{2}}dx = \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{x^7}}{7} + \frac{2\sqrt{x^3}}{3} + C$$

№3.1.12

$$\int\frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx = 3\int\frac{dx}{\sqrt{4-x^2}} + \int dx = 3\int\frac{dx}{\sqrt{2^2-x^2}} + \int dx =$$

$$= 3\arcsin\frac{x}{2} + x + C$$

№3.1.13

$$\int\frac{(x^3+2)^2}{\sqrt{x}}dx = \int\frac{x^6+4x^3+4}{\sqrt{x}}dx = \int\left(\frac{x^6}{\sqrt{x}} + \frac{4x^3}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx =$$

$$= \int x^{\frac{11}{2}}dx + 4\int x^{\frac{5}{2}}dx - 4\int\frac{dx}{\sqrt{x}} = \frac{2x^6\sqrt{x}}{13} + \frac{8x^3\sqrt{x}}{7} + 8\sqrt{x} + C$$

№8.1.14

$$\int\left(4\sin x+8x^3-\frac{11}{\cos^2 x}\right)dx=4\int\sin x\,dx+8\int x^3dx-11\int\frac{dx}{\cos^2 x}=$$

$$=-4\cos x+2x^4-11\,tg\,x+C$$

№8.1.16.

$$\int\cos(2x)dx=\left[\begin{matrix}dx=\frac{1}{2}\cdot 2dx=\frac{1}{2}d(2x)\\ \int f(ax+b)dx=\frac{1}{a}F(ax+b)+C,\ a\neq 0\end{matrix}\right]=$$

$$=\frac{1}{2}\sin(2x)+C$$

№8.1.17.

$$\int(9x+2)^{17}dx=\left[\ 9x+2=ax+b\ \right]=\frac{1}{9}\frac{(9x+2)^{17+1}}{17+1}+C=$$

$$=\frac{(9x+2)^{18}}{162}+C$$

№8.1.18.

$$\int\frac{dx}{3x-1}=\frac{1}{3}\ln|3x-1|+C$$

№8.1.19

$$\int4^{3-5x}dx=-\frac{1}{5}\,\frac{4^{3-5x}}{\ln4}+C=-\frac{4^{3-5x}}{5\ln4}+C$$

№8.1.20

$$\int\sqrt{3x+4}\,dx=\left[\begin{matrix}ax+b=3x+4\\ \int\sqrt{x}dx=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}=\frac{2\sqrt{x^3}}{3}\end{matrix}\right]=$$

$$=\frac{1}{3}\cdot\frac{2\sqrt{(3x+4)^3}}{3}+C=\frac{2\sqrt{(3x+4)^3}}{9}+C$$

№8.1.21.

$$\int\frac{dx}{3x^2-25}=\int\frac{dx}{3\left(x^2-\frac{25}{3}\right)}=\frac{1}{3}\int\frac{dx}{x^2-\left(\frac{5}{\sqrt3}\right)^2}=\frac{1}{3}\cdot\frac{1}{2\cdot\frac{5}{\sqrt3}}\cdot\ln\left|\frac{x-\frac{5}{\sqrt3}}{x+\frac{5}{\sqrt3}}\right|+C=$$

$$=\frac{\sqrt3}{30}\ln\left|\frac{x\sqrt3-5}{x\sqrt3+5}\right|+C$$

№8.1.23

$$\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2x)dx =$$
$$= \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C = \frac{x}{2} + \frac{\sin(2x)}{4} + C$$

№8.1.24

$$\int \frac{x-2}{x+3}dx = \int \frac{x+3-3-2}{x+3}dx = \int \frac{(x+3)-5}{x+3}dx =$$
$$= \int\left(\frac{x+3}{x+3}\right)dx - 5\int\frac{dx}{x+3} = \int dx - 5\int\frac{dx}{x+3} =$$
$$= x - 5\ln|x+3| + C$$

№8.1.25

$$\int \frac{x^2\,dx}{x^2-9} = \int\frac{x^2-9+9}{x^2-9}dx = \int dx + 9\int\frac{dx}{x^2-9} =$$
$$= x + \frac{9}{6}\ln\left|\frac{x-3}{x+3}\right| + C$$

№8.1.26

$$\int\frac{5+\sin^3 x}{\sin^2 x}dx = 5\int\frac{dx}{\sin^2 x} + \int\frac{\sin^3 x}{\sin^2 x}dx =$$
$$= 5\int\frac{dx}{\sin^2 x} + \int \sin x\,dx = -5\,\mathrm{ctg}\,x - \cos x + C$$

№8.2.2

$$\int\sqrt{4x-5}\,dx = \left[\begin{array}{l} t = 4x-5 \Rightarrow dt = d(4x-5) = (4x-5)'_x dx = 4dx \Rightarrow \\ \Rightarrow dx = \frac{1}{4}dt \end{array}\right] =$$
$$= \int\sqrt{t}\,\frac{1}{4}dt = \frac{1}{4}\int\sqrt{t}\,dt = \frac{1}{4}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2\sqrt{t^3}}{12} + C =$$
$$= \frac{\sqrt{t^3}}{6} + C = \frac{\sqrt{(4x-5)^3}}{6} + C$$

№8.2.3

$$\int\frac{dx}{(3x+2)^4} = \int(3x+2)^{-4}dx = \left[\begin{array}{l} t = 3x+2 \Rightarrow dt = d(3x+2) = (3x+2)'_x dx \Rightarrow \\ \Rightarrow 3dx = dt \Rightarrow dx = \frac{1}{3}dt \end{array}\right] =$$
$$= \int t^{-4}\,\frac{1}{3}dt = \frac{1}{3}\int t^{-4}dt = \frac{1}{3}\,\frac{t^{-3}}{-3} + C = -\frac{1}{9(3x+2)^3} + C$$

№8.2.4.

$$\int \sin^3 x \cdot \cos x \, dx = \left[\begin{array}{l} t = \sin x \Rightarrow \sin^3 x = t^3 \\ dt = d(\sin x) = \cos x \, dx \end{array} \right] =$$

$$= \int t^3 \, dt = \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$$

№8.2.5.

$$\int e^{x^3} \cdot x^2 \, dx = \left[\begin{array}{l} t = x^3 \Rightarrow dt = 3x^2 dx \Rightarrow \\ \Rightarrow \frac{1}{3} dt = x^2 dx \end{array} \right] =$$

$$= \int e^t \frac{1}{3} dt = \frac{1}{3} \int e^t \, dt = \frac{e^t}{3} + C = \frac{e^{x^3}}{3} + C$$