(20.05.20)

Обратная матрица.

№1.4.1

$A=\begin{pmatrix}1&2&3\\4&5&6\\7&8&0\end{pmatrix}$ 1) $\Delta A=\begin{vmatrix}1&2&3\\4&5&6\\7&8&0\end{vmatrix} = 7\begin{vmatrix}2&3\\5&6\end{vmatrix}-8\begin{vmatrix}1&3\\4&6\end{vmatrix}=-21-(-48)=27$

$27\neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11}=(-1)^{1+1}\cdot\begin{vmatrix}5&6\\8&0\end{vmatrix}=-48$ $A_{12}=-\begin{vmatrix}4&6\\7&0\end{vmatrix}=42$

$A_{13}=\begin{vmatrix}4&5\\7&8\end{vmatrix}=-3$ $A_{21}=-\begin{vmatrix}2&3\\8&0\end{vmatrix}=24$ $A_{22}=\begin{vmatrix}1&3\\7&0\end{vmatrix}=-21$

$A_{23}=-\begin{vmatrix}1&2\\7&8\end{vmatrix}=6$ $A_{31}=\begin{vmatrix}2&3\\5&6\end{vmatrix}=-3$ $A_{32}=-\begin{vmatrix}1&3\\4&6\end{vmatrix}=6$

$A_{33}=\begin{vmatrix}1&2\\4&5\end{vmatrix}=-3$

$A_{ij}=\begin{pmatrix}-48&42&-3\\24&-21&6\\-3&6&-3\end{pmatrix}$

3) $\tilde{A}=(A_{ij})^T$ $\tilde{A}=\begin{pmatrix}-48&24&-3\\42&-21&6\\-3&6&-3\end{pmatrix}$

4) $A^{-1}=\frac{1}{\det A}\cdot\tilde{A}$

$A^{-1}=\frac{1}{27}\cdot\begin{pmatrix}-48&24&-3\\42&-21&6\\-3&6&-3\end{pmatrix}=\begin{pmatrix}-48/27&24/27&-3/27\\42/27&-21/27&6/27\\-3/27&6/27&-3/27\end{pmatrix}=\begin{pmatrix}-16/9&8/9&-1/9\\14/9&-7/9&2/9\\-1/9&2/9&-1/9\end{pmatrix}$

№1.4.2

$A=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$ 1) $\Delta A=\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}=1\cdot\begin{vmatrix}1&0\\0&1\end{vmatrix}=1\neq 0\Rightarrow\exists A^{-1}$

2. $A_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad A_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \quad A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0 \quad A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0 \quad A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \quad A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$$A_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3) $\tilde{A} = (A_{ij})^T \Rightarrow \tilde{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

4) $A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = \frac{1}{1}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

N1.4.3.

$A = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{vmatrix} = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\left(-\frac{2}{3}\right)\cdot\frac{2}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\left(-\frac{2}{3}\right) - \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{2}{3} - \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} - \frac{1}{3}\cdot\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) =$

$= \frac{1}{27} - \frac{8}{27} - \frac{8}{27} - \frac{4}{27} - \frac{4}{27} - \frac{4}{27} = -\frac{27}{27} = -1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = \begin{vmatrix} 1/3 & -2/3 \\ -2/3 & 1/3 \end{vmatrix} = -\frac{1}{3} \quad A_{12} = -\begin{vmatrix} 2/3 & -2/3 \\ 2/3 & 1/3 \end{vmatrix} = -\frac{6}{9} = -\frac{2}{3} \quad A_{13} = \begin{vmatrix} 2/3 & 1/3 \\ 2/3 & -2/3 \end{vmatrix} = -\frac{6}{9} = -\frac{2}{3}$

$A_{21} = -\begin{vmatrix} 2/3 & 2/3 \\ -2/3 & 1/3 \end{vmatrix} = -\frac{6}{9} = -\frac{2}{3} \quad A_{22} = \begin{vmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{vmatrix} = -\frac{3}{9} = -\frac{1}{3} \quad A_{23} = -\begin{vmatrix} 1/3 & 2/3 \\ 2/3 & -2/3 \end{vmatrix} = \frac{6}{9} = \frac{2}{3}$

$A_{31} = \begin{vmatrix} 2/3 & 2/3 \\ 1/3 & -2/3 \end{vmatrix} = -\frac{6}{9} = -\frac{2}{3} \quad A_{32} = -\begin{vmatrix} 1/3 & 2/3 \\ 2/3 & -2/3 \end{vmatrix} = \frac{6}{9} = \frac{2}{3} \quad A_{33} = \begin{vmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{vmatrix} = -\frac{3}{9} = -\frac{1}{3}$

$$A_{ij} = \begin{pmatrix} -1/3 & -2/3 & -2/3 \\ -2/3 & -1/3 & 2/3 \\ -2/3 & 2/3 & -1/3 \end{pmatrix}$$

3) $\tilde{A} = (A_{ij})^T = \begin{pmatrix} -1/3 & -2/3 & -2/3 \\ -2/3 & -1/3 & 2/3 \\ -2/3 & 2/3 & -1/3 \end{pmatrix}$

4) $A^{-1} = \frac{1}{-1}\tilde{A} = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix}$

N1.4.4.

$A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{vmatrix} \leftarrow = 2\begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} =$

$= -4 + 5 = 1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = \begin{vmatrix} 2 & -4 \\ -1 & 0 \end{vmatrix} = -4$ $\quad A_{12} = -\begin{vmatrix} 3 & -4 \\ 2 & 0 \end{vmatrix} = -8$ $\quad A_{13} = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -7$

$A_{21} = -\begin{vmatrix} 2 & -3 \\ -1 & 0 \end{vmatrix} = 3$ $\quad A_{22} = \begin{vmatrix} 1 & -3 \\ 2 & 0 \end{vmatrix} = 6$ $\quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = 5$

$A_{31} = \begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix} = -2$ $\quad A_{32} = -\begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix} = -5$ $\quad A_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4$

3) $\tilde{A} = \begin{pmatrix} -4 & 3 & -2 \\ -8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}$

4) $A^{-1} = \begin{pmatrix} -4 & 3 & -2 \\ -8 & 6 & -5 \\ -7 & 5 & -4 \end{pmatrix}$