(08.06.20)

№6.4.31

$$\lim_{x\to\infty}\frac{x+5x^2-x^3}{2x^3-x^2+7x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{x(1+5x-x^2)}{x(2x^2-x+7)}=$$

$$=\lim_{x\to\infty}\frac{1+5x-x^2}{2x^2-x+7}=\lim_{x\to\infty}\frac{x^2\left(\frac{1}{x^2}+\frac{5}{x}-1\right)}{x^2\left(2-\frac{1}{x}+\frac{7}{x^2}\right)}=\lim_{x\to\infty}\frac{\frac{1}{x^2}+\frac{5}{x}-1}{2-\frac{1}{x}+\frac{7}{x^2}}=$$

$$=\frac{0+0-1}{2-0+0}=-\frac{1}{2}$$

№6.4.32

$$\lim_{x\to\infty}\frac{1-3x^2}{x^2-7x-2}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{x^2\left(\frac{1}{x^2}-3\right)}{x^2\left(1+\frac{7}{x}-\frac{2}{x^2}\right)}=$$

$$=\lim_{x\to\infty}\frac{\frac{1}{x^2}-3}{1+\frac{7}{x}-\frac{2}{x^2}}=\frac{0-3}{1+0-0}=-3$$

№6.4.33

$$\lim_{x\to\infty}\frac{x^3+x}{x^4-3x^2+1}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{1}{x^3}}{1-\frac{3}{x^2}+\frac{1}{x^4}}=\frac{0+0}{1-0+0}=\frac{0}{1}=0$$

№6.4.34

$$\lim_{x\to\infty}\frac{x^5-2x}{2x^3+x^2+1}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{1-\frac{2}{x^4}}{\frac{2}{x^2}+\frac{1}{x^3}+\frac{1}{x^5}}=\frac{1-0}{0+0+0}=\infty$$

№6.4.35

$$\lim_{x\to\infty}\left(\sqrt{x^2+4}-x\right)=[\infty-\infty]=\lim_{x\to\infty}\frac{(\sqrt{x^2+4}-x)(\sqrt{x^2+4}+x)}{(\sqrt{x^2+4}+x)}=$$

$$=\lim_{x\to\infty}\frac{x^2+4-x^2}{\sqrt{x^2+4}+x}=\lim_{x\to\infty}\frac{4}{\sqrt{x^2+4}+x}=\frac{4}{\infty}=0$$

№6.4.36

$$\lim_{x\to\infty}\left(\frac{x^3}{x^2-3}-x\right)=\left[\frac{\infty}{\infty}-\infty\right]=\lim_{x\to\infty}\frac{x^3-x^3+3x}{x^2-3}=$$

$$=\lim_{x\to\infty}\frac{3x}{x^2-3}=\lim_{x\to\infty}\frac{x^2\left(\frac{3}{x}\right)}{x^2\left(1-\frac{3}{x^2}\right)}=\lim_{x\to\infty}\frac{\frac{3}{x}}{1-\frac{3}{x^2}}=\frac{0}{1-0}=0$$

№6.4.37

1) $$\lim_{x\to0}\frac{\sin\alpha x}{x}=\left[\frac{0}{0}\right]=\lim_{y\to0}\frac{\sin y}{\frac{y}{\alpha}}=\lim_{y\to0}\frac{\alpha\sin y}{y}=$$

$$=\alpha\lim_{y\to0}\frac{\sin y}{y}=\alpha$$

2) $$\lim_{x\to0}\frac{\sin5x}{\sin3x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{\frac{\sin5x}{x}}{\frac{\sin3x}{x}}=\frac{\lim\limits_{x\to0}\frac{\sin5x}{x}}{\lim\limits_{x\to0}\frac{\sin3x}{x}}=\frac{5}{3}$$

3) $$\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{2x-\pi}=\left[\frac{0}{0}\right]=\lim_{y\to0}\frac{\cos\left(y+\frac{\pi}{2}\right)}{2\left(y+\frac{\pi}{2}\right)-\pi}=\lim_{y\to0}\frac{-\sin y}{2y}=$$

$$=-\frac{1}{2}\lim_{y\to0}\frac{\sin y}{y}=-\frac{1}{2}$$

4) $$\lim_{x\to0}\frac{\arcsin x}{x}=\lim_{t\to0}\frac{t}{\sin t}=\lim_{t\to0}\frac{1}{\frac{\sin t}{t}}=1.$$