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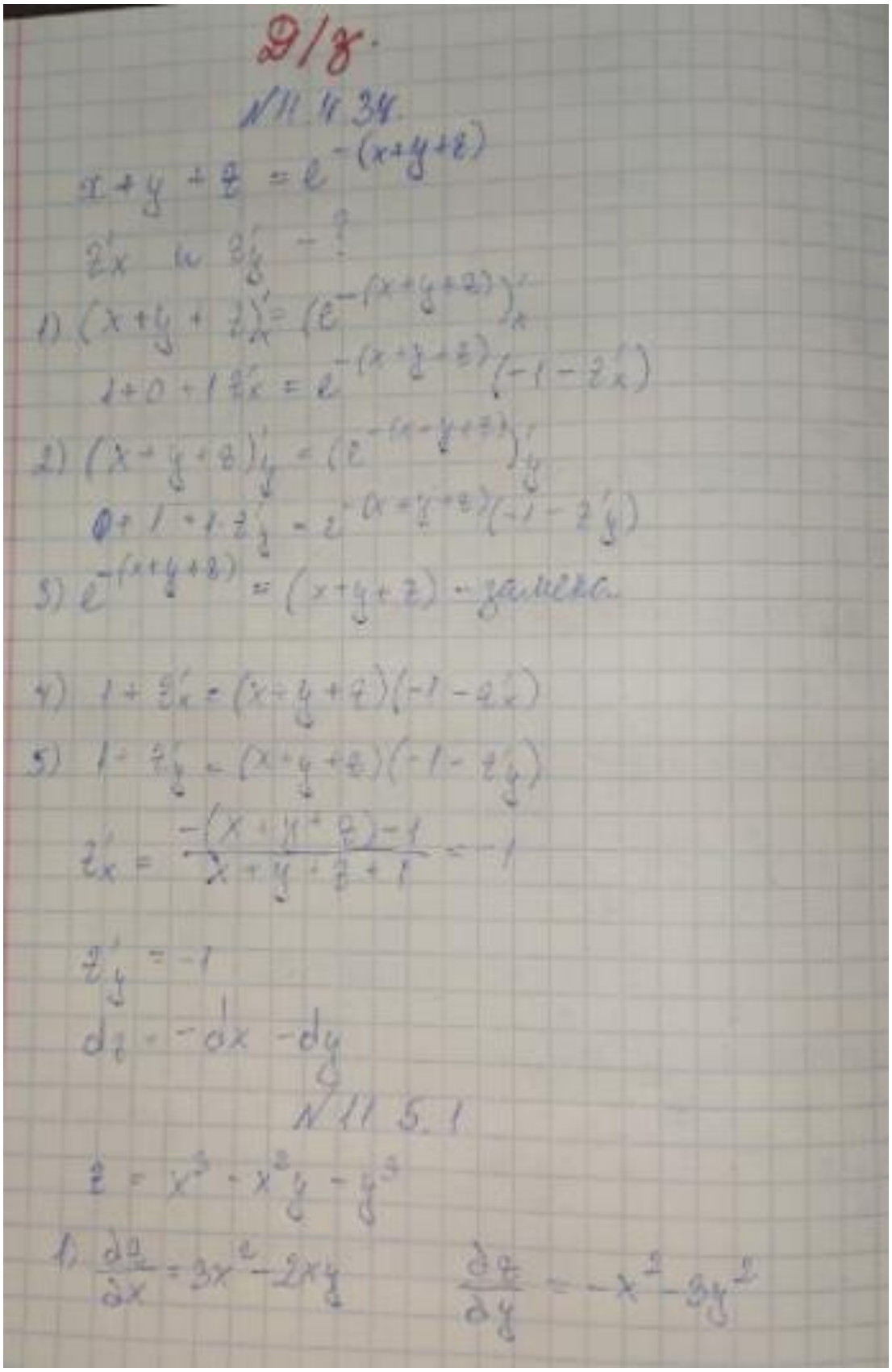
Подгруппа №1



2) $\frac{\partial^2 z}{\partial x^2} = 6x - 2y \qquad \frac{\partial^2 z}{\partial y^2} = -6y$

$\frac{\partial^2 z}{\partial x \partial y} = -2x \qquad \frac{\partial^2 z}{\partial y \partial x} = -2x$

3) $\frac{\partial^3 z}{\partial x^3} = 6 \qquad \frac{\partial^3 z}{\partial y^3} = -6$

$\frac{\partial^3 z}{\partial x^2 \partial y} = -2 \qquad \frac{\partial^3 z}{\partial x \partial y^2} = 0$

№ 11.5.2

$z = e^{xy^3} \qquad \frac{\partial^4 z}{\partial x^4} \quad \frac{\partial^4 z}{\partial x^3 \partial y} \qquad \frac{\partial^4 z}{\partial x^2 \partial y^2}$

1) $\frac{\partial z}{\partial x} = y^3 e^{xy^3} \qquad \frac{\partial^2 z}{\partial x^2} = y^6 e^{xy^3} \qquad \frac{\partial^3 z}{\partial x^3} = y^9 e^{xy^3}$

$\frac{\partial^4 z}{\partial x^4} = y^{12} e^{xy^3}$

2) $\frac{\partial^4 z}{\partial x^3 \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^3 z}{\partial x^3}\right) = \frac{\partial}{\partial y}\left(y^9 e^{xy^3}\right) =$

$= 9y^8 e^{xy^3} + 3y^{11} x e^{xy^3}$

3) $\frac{\partial^3 z}{\partial x^2 \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^2 z}{\partial x^2}\right) = \frac{\partial}{\partial y}\left(y^6 e^{xy^3}\right) =$

$= 6y^5 e^{xy^3} + 3y^8 x e^{xy^3} = 3y^5 e^{xy^3}(2 + y^3 x)$

4) $\frac{\partial^4 z}{\partial x^2 \partial y^2} = \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 z}{\partial x^2}\right) = \frac{\partial}{\partial y}\left(\frac{\partial^3 z}{\partial x^2 \partial y}\right) = \frac{\partial}{\partial y}\left[3y^5 e^{xy^3}(2 + y^3 x)\right] =$

$= 3\left[5y^4 e^{xy^3}(2 + y^3 x) + 3xy^7 e^{xy^3}(2 + y^3 x) + 3y^7 x e^{xy^3}\right] =$

$= 3y^4 e^{xy^3}\left[10 + 11xy^3 + 3x^2 y^6\right]$

№11.5.3

$d^2 z - ?$ $\quad z = \operatorname{arctg} \frac{y}{x}$

1) $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = -\frac{y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy$

2) $\frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2}$ $\quad \frac{\partial^2 z}{\partial x \partial y} = \frac{y^2 - x^2}{(x^2 + y^2)^2}$

$\frac{\partial^2 z}{\partial y^2} = -\frac{2xy}{(x^2 + y^2)^2}$

$d^2 z = \frac{2\left[xy\,dx^2 + (y^2 - x^2)\,dx\,dy - xy\,dy^2\right]}{(x^2 + y^2)^2}$

№11.5.4

$d^2z - ?$ $z = \frac{xy}{x-y}$ Инвариантность $z_{xx}'' dx^2 + 2z_{xy}'' dx dy + z_{yy}'' dy^2 = \frac{x}{x-y}$

1) $z_x' = -\frac{y^2}{(x-y)^2}$ $z_y' = \frac{x^2}{(x-y)^2}$

$z_{xx}'' = \frac{2y^2}{(x-y)^3}$ $z_{xy}'' = \frac{2xy}{(x-y)^3}$ $z_{yy}'' = -\frac{2x^2}{(x-y)^3}$

$d^2\left(\frac{xy}{x-y}\right) = \frac{2(y^2dx^2 - 2xy\,dx\,dy + x^2dy^2)}{(x-y)^3}$

2) $\frac{y^2}{x^2} + 2\frac{z}{x}\frac{z}{y} + \frac{z}{y^2} = \frac{2y^2 - 4xy - 2x^2}{(x-y)^2} = \frac{-2(x+y)^2}{(x-y)^2} = -\frac{2}{x-y}$

№11.5.5

$d^2z - ?$ $z = \frac{xy}{x-y}$

1) $z_x' = \frac{y^2}{(x-y)^2}$ $z_y' = \frac{x^2}{(x+y)^2}$

2) $z_{xx}'' = -\frac{2y^2}{(x+y)^3}$ $z_{xy}'' = \frac{2xy}{(x+y)^3}$ $z_{yy}'' = -\frac{2x^2}{(x+y)^3}$

3) $d^2z = \frac{2y^2}{(x+y)^3}dx^2 + 4\frac{xy}{(x+y)^3}dx\,dy - \frac{2x^2}{(x+y)^3}dy^2 =$

$= -\frac{2(y^2dx^2 - 2xy\,dx\,dy + x^2dy^2)}{(x+y)^3} = -\frac{2(ydx - xdy)^2}{(x+y)^3}$

4) $d^3z = z_{xxx}'''dx^3 + 3z_{xxy}'''dx^2dy + 3z_{xyy}'''dx\,dy^2 + z_{yyy}'''dy^3$

$d^3z = \frac{6}{(x+y)^4}[y^3dx^3 - (3xy - y^2)dx^2dy - (3xy - x^2)dx\,dy^2 + x^3dy^3]$

№11.5.6

d^2z - ? $z = \ln(x^2+y^2)$

$d^2z = d(dz)$

$$dz = 2\frac{x\,dx + y\,dy}{x^2+y^2}$$

$$d^2z = 2\frac{\partial}{\partial x}\left(\frac{x\,dx + y\,dy}{x^2+y^2}\right)dx + 2\frac{\partial}{\partial y}\left(\frac{x\,dx + y\,dy}{x^2+y^2}\right)dy =$$

$$= 2\frac{(x^2+y^2)dx - 2x(x\,dx + y\,dy)}{(x^2+y^2)^2}dx +$$

$$+ 2\frac{(x^2+y^2)dy - 2y(x\,dx + y\,dy)}{(x^2+y^2)^2}dy =$$

$$= 2\frac{(y^2-x^2)dx^2 - 4xy\,dx\,dy + (x^2-y^2)dy^2}{(x^2+y^2)^2}$$

№11.5.22

$y(x)$: $x^3y^2 - xy^5 + 5x - y = 0$ $y^{IV}(0)$ - ?

1) $3x^2y^2 + 2x^3yy' - y^5 - 5xy^4y' + 5 - y' = 0$

2) $6xy^2 + 6x^2yy' + 6x^2yy' + 2x^3(y')^2 + 2x^3yy'' - 5y^4y' -$
$- 5y^4y' - 20xy^3(y')^2 - 5xy^4y'' - y'' = 0$

3) $6xy^2 + 12x^2yy' + 2x^3(y')^2 + 2x^3yy'' - 10y^4y' - 20xy^3(y')^2 -$
$- 5xy^4y'' - y'' = 0$

4) $6y^2 + 12xyy' + 24xyy' + 12x^2(y')^2 + 12x^2yy'' + 6x^2(y')^2 +$
$+ 4x^3y'y'' + 6x^2yy'' + 2x^3y'y'' + 2x^3yy''' - 40y^3(y')^2 - 10y^4y'' -$

$-20y^4(y')^4 - 60xy^3(y')^3$ [illegible]

$-5xy^4y''' - y''' + 0$ [illegible]

Шаг 1: $x=0;\ y=0 \Rightarrow y'(0)=5$

Шаг 3: $x=0;\ y=0;\ y'(0)=5 \Rightarrow y''(0)=0$

Шаг 4: $x=0;\ y=0,\ y'(0)=5;\ y''(0)=0 \Rightarrow y'''(0)=0$

№ 11.5.27.

$y(x)$, $ye^x + e^y = 0$ $\quad y'' - ?$

1) $y'e^x + e^x y + e^y y' = 0$

2) $y''e^x + e^x y' + e^x y' + e^x y + e^y (y')^2 + e^y y'' = 0$

3) $y'' = -\dfrac{2e^x y' + e^x y + e^y (y')^2}{e^x + e^y}$

4) $y' = -\dfrac{ye^x}{e^x + e^y}$

5) $y'' = -\dfrac{-2e^x \frac{ye^x}{e^x+e^y} + e^x y + e^y\left(\frac{ye^x}{e^x+e^y}\right)^2}{e^x + e^y} =$

$= -\dfrac{-2e^{2x}y(e^x+e^y) + e^x y(e^x+e^y)^2 + y^2 e^y e^{2x}}{(e^x+e^y)^3}$

№ 11.5.31.

$y', y'' - ?$ $\quad y^4 - 3x^2 + 2x + 3y - 7 = 0$

1 способ

1) $y' = 2\frac{3x-1}{2y+3}$

2) $y'' = 2\frac{3(2y+3) - 2y'(3x-1)}{(2y+3)^2}$

3) $y'' = 2\frac{3(2y+3)^2 - 2(3x-1)^2}{(2y+3)^3}$

2 способ

1) $2yy' - 6x + 2 - 3y' = 0$

2) $2(y')^2 + 2yy'' + 6 - 3y'' = 0$

3) $4y'y'' + 2y'y'' + 2yy''' - 3y''' = 0$

№ 11.5.357

$\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2} - ?$

$x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0$

1) $2x + 6zz'_x + y - z'_x = 0$

2) $z'_x = \frac{2x + y}{1 - 6z}$

3) $2 + 6(z'_x)^2 + 6zz''_{xx} - z''_{xx} = 0$

4) $z''_{xx} = 2\frac{1 + 3(z'_x)^2}{1 - 3z}$

5) $4y + 12 z z'_y + x - z'_y = 0$

6) $z'_y = \frac{x - 4y}{1 - 6z}$

7) $4 + 6(z'_y)^2 + 6 z z''_{y^2} - z''_{y^2} = 0$

8) $z''_{y^2} = \frac{4 + 6(z'_y)^2}{1 - 6z}$

9) $z''_{x^2} = -2\frac{1 + 3\left(\frac{2x+y}{1-6z}\right)^2}{1 - 6z} = 2\frac{(1-6z)^2 + 3(2x+y)^2}{(1-6z)^3}$

10) $z''_{xy} = \frac{1 + 6\frac{(2x+y)(x+4y)}{(1-6z)^2}}{1 - 6z} = \frac{(1-6z)^2 + 6(2x+y)(x+4y)}{(1-6z)^3}$

11) $z''_{y^2} = \frac{4 + 6\left(\frac{x+4y}{1-6z}\right)^2}{1 - 6z} = \frac{4(1-6z)^2 + 6(x+4y)^2}{(1-6z)^3}$

№ 11.5.36

$\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$ при $x = 1,\ y = -2,\ z = 1$

если $x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0$

Из 11.5.35 =>

$z''_{x^2} = -\frac{2}{5},\quad z''_{xy} = -\frac{1}{5},\quad z''_{y^2} = -\frac{394}{125}$