(27.05.20)

Обратная матрица

№1.4.9

$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ 1) $\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\cdot a_{22} - a_{12}\cdot a_{21} \neq 0 \Rightarrow \exists A^{-1}$

2) $A_{11} = (-1)^{1+1}\cdot |a_{22}| = a_{22}$ $A_{12} = -a_{21}$

$A_{21} = -a_{12}$ $A_{22} = a_{11}$

3) $\tilde{A} = (A_{ij})^T = \begin{pmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{pmatrix}^T = \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$

4) $A^{-1} = \frac{1}{\det A}\cdot\tilde{A} = \frac{1}{a_{11}\cdot a_{22} - a_{12}\cdot a_{21}}\cdot\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$

№1.4.10

$A=\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3-2 = 1 \neq 0 \Rightarrow \exists A^{-1}$

2) $A^{-1} = \frac{1}{\Delta A}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix} = \frac{1}{1}\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$

№1.4.11

$A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ 1) $\Delta A = 4-6 = -2 \neq 0 \Rightarrow \exists A^{-1}$

2) $A^{-1} = -\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

№1.4.12

$A = \begin{pmatrix} x & 2 \\ y & -x \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} x & 2 \\ y & -x \end{vmatrix} = -x^2 - 2y \neq 0 \Rightarrow \exists A^{-1}$

2) $A^{-1} = \frac{1}{-x^2-2y} \begin{pmatrix} -x & -2 \\ -y & x \end{pmatrix} = \begin{pmatrix} \frac{-x}{-x^2-2y} & \frac{-2}{-x^2-2y} \\ \frac{-y}{-x^2-2y} & \frac{x}{-x^2-2y} \end{pmatrix} = \begin{pmatrix} \frac{x}{x^2+2y} & \frac{2}{x^2+2y} \\ \frac{y}{x^2+2y} & -\frac{x}{x^2+2y} \end{pmatrix}$

№1.4.13

$A = \begin{pmatrix} a & -b \\ -a & b \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} a & -b \\ -a & b \end{vmatrix} = ab - ab = 0 \Rightarrow \nexists A^{-1}$

№1.4.14

Метод Гаусса

$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 2 & 4 \end{pmatrix}$ 1) $\Delta A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 2 & 4 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 \\ 2 & 4 \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} =$

$= 10 - 6 - 2 = 2 \neq 0 \Rightarrow \exists A^{-1}$

2) $\Gamma = (A|E) \rightarrow$ преобразования $\rightarrow \Gamma_2 = (E|A^{-1})$

$\Gamma = \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ II - I \\ III - 2I \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 2 & -2 & 0 & 1 \end{array}\right) II = II + III \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 2 & -2 & 0 & 1 \end{array}\right) III \cdot \frac{1}{2} \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1/2 \end{array}\right) I - II - III \sim$

$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -1 & -3/2 \\ 0 & 1 & 0 & -3 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1/2 \end{array}\right) = \Gamma_2$

Тогда

$A^{-1} = \begin{pmatrix} 5 & -1 & -3/2 \\ -3 & 1 & 1 \\ -1 & 0 & 1/2 \end{pmatrix}$

№1.4.15

$A=\begin{pmatrix}1&1&1\\1&2&1\\1&1&2\end{pmatrix}$ 1) $\Delta A=\begin{vmatrix}1&1&1\\1&2&1\\1&1&2\end{vmatrix}=\begin{vmatrix}2&1\\1&2\end{vmatrix}-\begin{vmatrix}1&1\\1&2\end{vmatrix}+\begin{vmatrix}1&2\\1&1\end{vmatrix}=$

$=3-1-1=1\neq 0\Rightarrow \exists A^{-1}$

2) $\Gamma=\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\1&2&1&0&1&0\\1&1&2&0&0&1\end{array}\right)\begin{matrix}\\ II-I\\ III-I\end{matrix}\sim\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&0&-1&1&0\\0&0&1&-1&0&1\end{array}\right)^{I-II-III}$

$\sim\left(\begin{array}{ccc|ccc}1&0&0&3&-1&-1\\0&1&0&-1&1&0\\0&0&1&-1&0&1\end{array}\right)=\Gamma_2\Rightarrow A^{-1}=\begin{pmatrix}3&-1&-1\\-1&1&0\\-1&0&1\end{pmatrix}$

№1.4.16

$A=\begin{pmatrix}1&1&1\\-2&-1&-2\\2&3&3\end{pmatrix}$ 1) $\Delta A=\begin{vmatrix}1&1&1\\-2&-1&-2\\2&3&3\end{vmatrix}=\begin{vmatrix}-1&-2\\3&3\end{vmatrix}-\begin{vmatrix}-2&-2\\2&3\end{vmatrix}+\begin{vmatrix}-2&-1\\2&3\end{vmatrix}=$

$=3+2-4=1\neq 0\Rightarrow \exists A^{-1}$

2) $\Gamma=\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\-2&-1&-2&0&1&0\\2&3&3&0&0&1\end{array}\right)\begin{matrix}\\ II+2I\\ III-2I\end{matrix}\sim\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&0&2&1&0\\0&1&1&-2&0&1\end{array}\right)_{III-II}$

$\sim\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&0&2&1&0\\0&0&1&-4&-1&1\end{array}\right)^{I-II-III}\sim\left(\begin{array}{ccc|ccc}1&0&0&3&0&-1\\0&1&0&2&1&0\\0&0&1&-4&-1&1\end{array}\right)=\Gamma_2\Rightarrow$

$\Rightarrow A^{-1}=\begin{pmatrix}3&0&-1\\2&1&0\\-4&-1&1\end{pmatrix}$

№1.4.17

$A=\begin{pmatrix}1&2&3\\2&6&4\\3&10&8\end{pmatrix}$ 1) $\Delta A=\begin{vmatrix}1&2&3\\2&6&4\\3&10&8\end{vmatrix}=3\cdot\begin{vmatrix}2&6\\6&4\end{vmatrix}-10\begin{vmatrix}1&3\\2&4\end{vmatrix}+8\begin{vmatrix}1&2\\2&6\end{vmatrix}=$

$=-30+20+16-6\neq 0\Rightarrow \exists A^{-1}$

2) $\Gamma=\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\2&6&4&0&1&0\\3&10&8&0&0&1\end{array}\right)\begin{matrix}\\ II-2I\\ III-3I\end{matrix}\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&2&-2&-2&1&0\\0&4&-1&-3&0&1\end{array}\right)_{III-2II}$

$\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&2&-2&-2&1&0\\0&0&3&1&-2&1\end{array}\right)_{III\cdot\frac{1}{3}}\sim\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&2&-2&-2&1&0\\0&0&1&1/3&-2/3&1/3\end{array}\right)_{II+2III}\sim$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 0 & -4/3 & -1/3 & 2/3 \\ 0 & 0 & 1 & 1/3 & -2/3 & 1/3 \end{array}\right) \mathrm{II}:2 \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2/3 & -1/6 & 1/3 \\ 0 & 0 & 1 & 1/3 & -2/3 & 1/2 \end{array}\right)^{\mathrm{I}-2\mathrm{II}-3\mathrm{III}} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4/3 & 7/3 & -5/3 \\ 0 & 1 & 0 & -2/3 & -1/6 & 1/3 \\ 0 & 0 & 1 & 1/3 & -2/3 & 1/2 \end{array}\right) = \Gamma_2 \Rightarrow A^{-1} = \begin{pmatrix} 4/3 & 7/3 & -5/3 \\ -2/3 & -1/6 & 1/3 \\ 1/3 & -2/3 & 1/2 \end{pmatrix}$$