Rational Instance Checking in Defeasible $\mathcal{ELI}_\perp$

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TECHNISCHE UNIVERSITÄT DRESDEN

IME - Instituto de Matemática e Estatística

Challenge 1

$$\mathcal{A} = \{a : C, b : C, (a, b) : r\}$$
$$\mathcal{T} = \{D \sqcap \exists r.D \sqsubseteq \bot\}$$
$$\mathcal{D} = \{C \sqsubset\!\!\!\sim D\}$$

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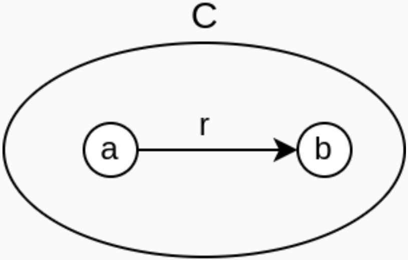

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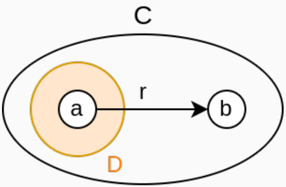
(**Solution no. 1** – Apply $C \precsim D$ to a)



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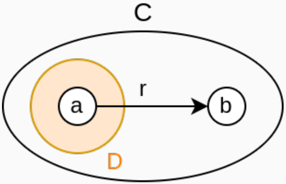
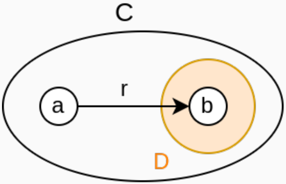
(**Solution no. 1** – Apply $C \mathrel{\underset{\sim}{\sqsubset}} D$ to a)



(**Solution no. 2** – Apply $C \mathrel{\underset{\sim}{\sqsubset}} D$ to b)



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$s = (a, b)$ characterizes the first solution, i.e.

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Adapting C&S method to typicality model semantics

Procedure defined in Pensel & Turhan (2018)

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4. Define the new minimal typicality model as $\mathcal{I}_{\mathcal{A}^*,\mathcal{T}} \cup \mathcal{I}_{\mathsf{min}}(\mathcal{T})$.

Adapting C&S method to typicality model semantics

Inducing an interpretration from $\mathcal{A}^*$ and $\mathcal{T}$

Let $\mathcal{K} = (\mathcal{A}, \mathcal{T})$. Then $\mathcal{I}_{\mathcal{A},\mathcal{T}} = (\Delta^{\mathcal{I}_{\mathcal{A},\mathcal{T}}}, \cdot^{\mathcal{I}_{\mathcal{A},\mathcal{T}}})$ s.t.:

$\Delta^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = \mathrm{sig}(\mathcal{A}) \cup Qc(\mathcal{K})$ where $Qc(\mathcal{K})$ denotes quantified concepts in $\mathcal{K}$.

$a^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = a$

$A^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = \{a \in \mathrm{sig}(\mathcal{A}) : \mathcal{K} \models A(a)\}$

$r^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = \{r(a, b) \in \mathcal{A}\} \cup \{(a, E_{\emptyset}) : \mathcal{K} \models (\exists r.E)(a)\}$

Adapting C&S method to typicality model semantics – Example

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$$\mathcal{I}_{\mathcal{A}^*, \mathcal{T}}$$

$$\Delta^{\mathcal{I}_{\mathcal{A}^*, \mathcal{T}}} = \{a, b\}$$

$$C^{\mathcal{I}_{\mathcal{A}^*, \mathcal{T}}} = \{a, b\}$$

$$D^{\mathcal{I}_{\mathcal{A}^*, \mathcal{T}}} = \{a\}$$

$$r^{\mathcal{I}_{\mathcal{A}^*, \mathcal{T}}} = \{(a, b)\}$$

Remarks

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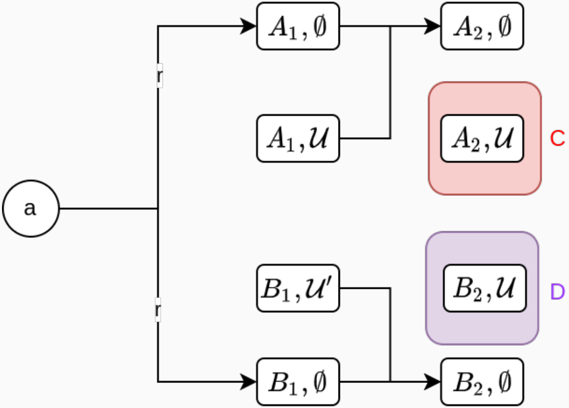

$C \sqsubseteq \forall r^-. E$

$\exists r. E \sqcap \exists r. F \sqsubseteq \bot$

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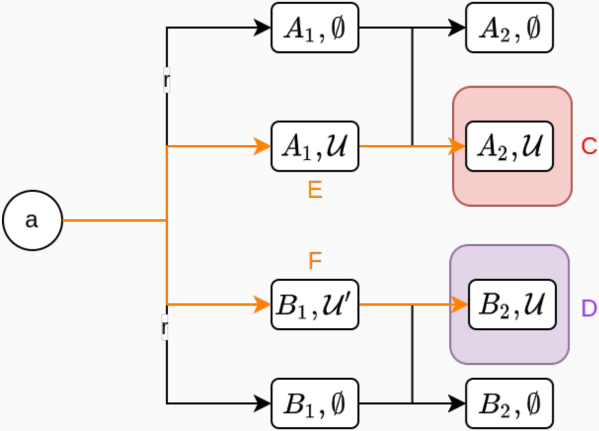

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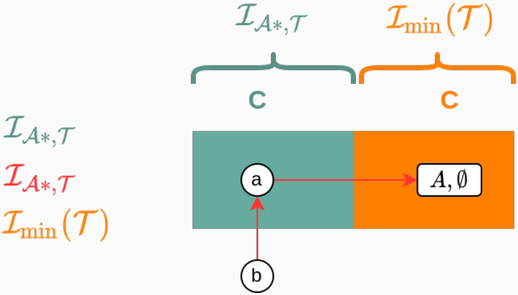
Quasi-disjointness in $\mathcal{EL}_\bot$

(Definition) $\mathcal{J}$ *is quasi-disjoint from* $\mathcal{I}$ *iff*

- $\forall A \in \mathsf{N_C}$, $A^{\mathcal{J}} \cap \Delta^{\mathcal{I}} = \emptyset$ *and*
- $\forall r \in \mathsf{N_R}$, $r^{\mathcal{J}} \cap (\Delta^{\mathcal{I}} \times (\Delta^{\mathcal{I}} \cup \Delta^{\mathcal{J}})) = \emptyset$



What happens with defeasible $\mathcal{ELI}_\bot$

The following property holds for $\mathcal{EL}_\bot$

For $\mathcal{I}, \mathcal{J}$ s.t. $\mathcal{J}$ is quasi-disjoint from $\mathcal{I}$ it holds that $C^{\mathcal{I} \cup \mathcal{J}} \cap \Delta^{\mathcal{I}} = C^{\mathcal{I}}$ For all $\mathcal{EL}_\bot$ concepts C

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... but not in $\mathcal{ELI}_\bot$

$\mathcal{J}$ s.t. $\Delta^{\mathcal{J}} = \{a, C\}$, $r^{\mathcal{J}} = \{(a, C)\}$ and $(\exists r^{-}.\top)^{\mathcal{J}} = \{C\}$

$\mathcal{I}$ s.t. $\Delta^{\mathcal{I}} = \{C\}$, $r^{\mathcal{I}} = \emptyset$ and $(\exists r^{-}.\top)^{\mathcal{J}} = \emptyset$.

Without quasi-disjointness...

Definition (New ABox Interpretation)

Let $\mathcal{K} = (\mathcal{A}, \mathcal{T})$. Then:

$$\Delta^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = \mathsf{sig}(\mathcal{A}) \cup \{M_\emptyset : M \in \mathcal{P}(\mathsf{sig}(\mathcal{T}))\}$$

$$a^{\mathcal{I}_{\mathcal{A},\mathcal{T}}} = a$$

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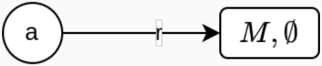
$$\cup \{(a, M_\emptyset) : \mathcal{K} \models (\exists r.\lceil M \rceil)(a) \text{ and } M \text{ is maximal for } \mathcal{K}, r \text{ and } a\}$$

$$\cup \{(M_\emptyset, a) : \mathcal{K} \models (\exists r^-.\lceil M \rceil)(a) \text{ and } M \text{ is maximal for } \mathcal{K}, r^- \text{ and } a\}$$

Why the new construction do NOT break the model property

$$A \sqsubseteq B \mid A \sqsubseteq \exists r.B \mid A \sqsubseteq \forall r.B$$



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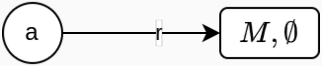

With the new definition

- $\mathcal{I}_{\mathcal{A}^*,\mathcal{T}} \cup \mathcal{I}_{\mathsf{min}}(\mathcal{T}) \models \mathcal{K}$.
- $\mathcal{I}_{\mathcal{A}^*,\mathcal{T}} \cup \mathcal{I}_{\mathsf{min}}(\mathcal{T})$ is *canonical* w.r.t. rational defeasible subsumption and defeasible instance checking (for named concepts).

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Upgrading the typicality of the edges

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2. Individuals own every edge to which they belong.

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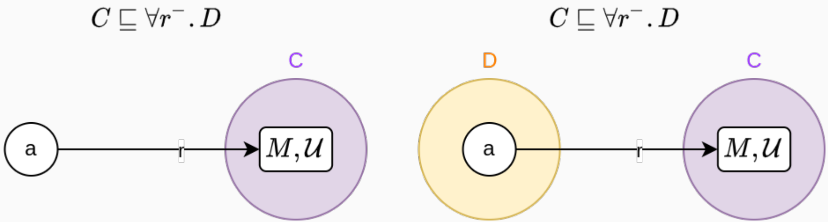
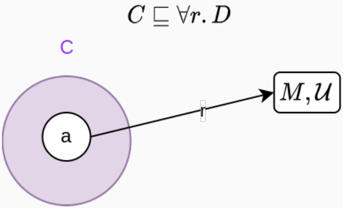
2. Individuals own every edge to which they belong.
3. There are three types of violations brought by upgrades on edges with individuals.

Violation #1



$C \sqsubseteq \forall r^-.D$
C
a
r
$M, \mathcal{U}$
$C \sqsubseteq \forall r^-.D$
D
C
a
r
$M, \mathcal{U}$

Violation #2



$C \sqsubseteq \forall r.\, D$
C
a
r
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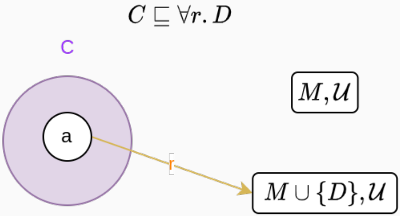

$C \sqsubseteq \forall r.\, D$
C
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r
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$M \cup \{D\}, \mathcal{U}$

Violation #3

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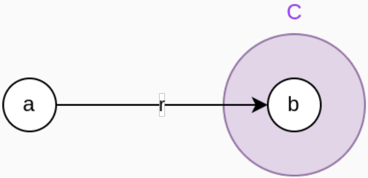
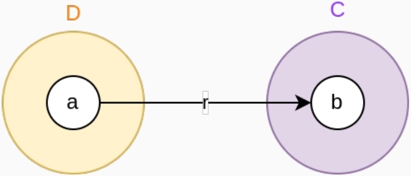
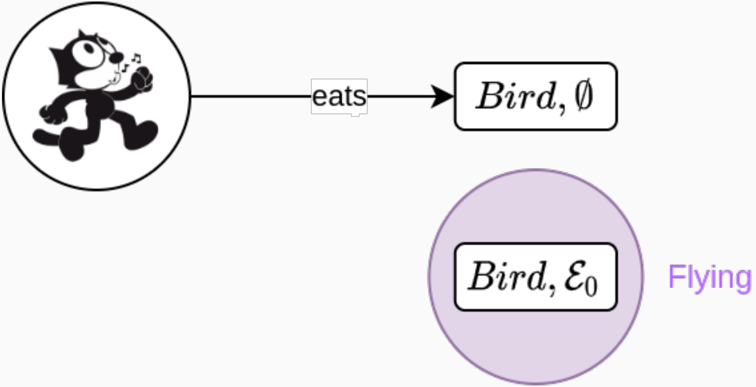




Quantification Neglect and Defeasible Instance Checking



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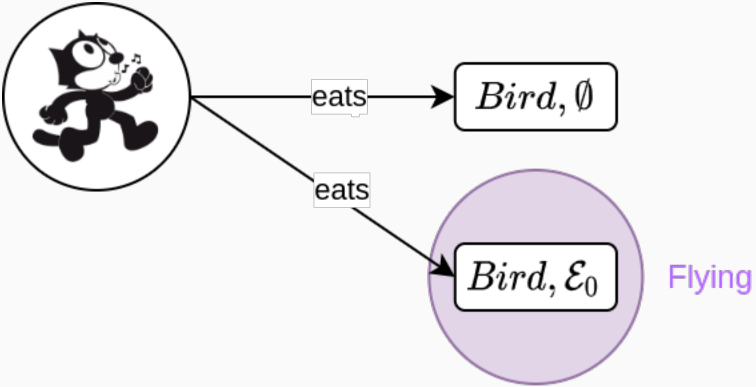

$$\mathcal{K} \models_{rat,\mathsf{nest}} (\exists \mathsf{eats}.\mathsf{Flying})(\mathsf{felix})$$

Next chapters

- Instance checking in relevant and lexicographic defeasible reasoning.
- Skeptical instance checking without order over individuals.
- Defeasible assertions?