Lexicographic Closure and Typicality Models

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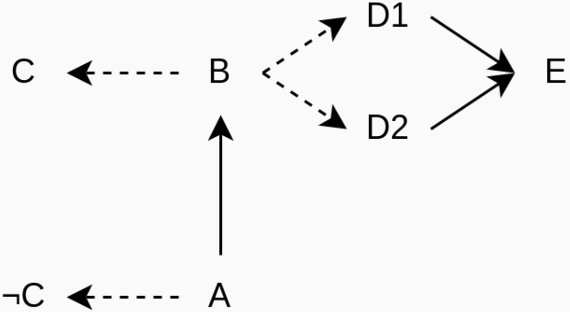
Motivation

"At this time, we conjecture that ***Lexicographic Closure*** *(...) can be similarly merged with our new semantics."* [1]

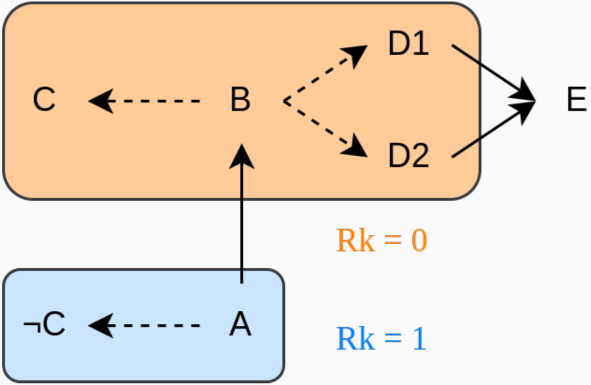
[1] PENSEL, M. A Lightweight Defeasible Description Logic in Depth, 2019.

A simple DKB

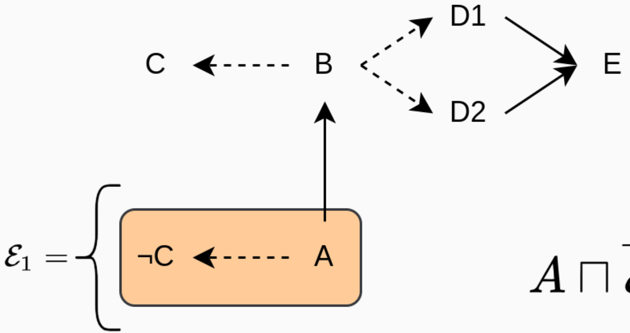


$$A \sqcap D1 \sqcap D2 \sqsubseteq \bot$$

Ranking the axioms and concepts



$$A \sqcap D1 \sqcap D2 \sqsubseteq \perp$$



$$A \sqcap \overline{\mathcal{E}_1}$$

$$A \sqcap D1 \sqcap D2 \sqsubseteq \bot$$

Lexicographic order

$$\langle n_0, \ldots, n_k \rangle <_{lex} \langle m_0, \ldots, m_k \rangle$$

iff

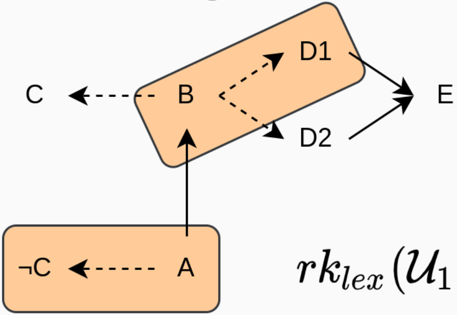
$$\exists i \in \{0, \ldots, k\}.(\forall j < i . n_j = m_j) \wedge n_i < m_i$$

Lexicographic order – Example

$$\langle 0, 0, 1 \rangle <_{lex} \langle 0, 1, 0 \rangle <_{lex} \langle 2, 0, 1 \rangle$$

Lexicographic rank #1

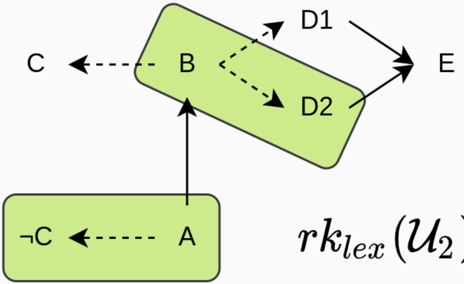
$$\mathcal{U}_1 = \text{orange}$$



$$rk_{lex}(\mathcal{U}_1) = \langle 1, 1 \rangle$$

$$A \sqcap D1 \sqcap D2 \sqsubseteq \bot$$

$$\mathcal{U}_2 = \text{green}$$



$$rk_{lex}(\mathcal{U}_2) = \langle 1, 1 \rangle$$

$$A \sqcap D1 \sqcap D2 \sqsubseteq \bot$$

Lexicographic preference

Lexicographic preference over $\mathcal{U} \subseteq \mathcal{D}$ consistent with A

$$\emptyset <_{lex} \mathcal{E}_0 <_{lex} \mathcal{U}_1 =_{lex} \mathcal{U}_2$$

i.e.

$$\langle 0, 0 \rangle <_{lex} \langle 1, 0 \rangle <_{lex} \langle 1, 1 \rangle =_{lex} \langle 1, 1 \rangle$$

Lexicographic Closure #1 (Casini & Straccia, 2012)

$\mathcal{K} \models_{lex} C \mathrel{\tilde{\sqsubseteq}} D$

iff

$\mathcal{K} \models_{lex} C \sqcap \overline{\mathcal{U}} \sqsubseteq D$,

for every $\mathcal{U} \subseteq \mathcal{D}$ that

1. Is not exceptional *w.r.t.* C and $\mathcal{K}$.
2. Is maximal according to the lexicographic order.

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Example

$\mathcal{K} \models_{lex} A \precsim E$

$\mathcal{K} \models A \sqcap \overline{\mathcal{U}_1} \sqsubseteq E$ and $\mathcal{K} \models A \sqcap \overline{\mathcal{U}_2} \sqsubseteq E$

Lexicographic Closure #2 (Pensel, 2019)

$\mathcal{K} \models_{lex} C \precsim D$

iff

$\mathcal{K} \models_{lex} C \sqcap \overline{\mathcal{U}} \sqsubseteq D$, where

$$\mathcal{U}_{lex} = \bigcap \{\mathcal{U} \subseteq \mathcal{D} : \mathcal{U} \text{ is maximal amongst non-exceptional subsets of } \mathcal{D} \text{ w.r.t. } \mathcal{K} \text{ and } C\}$$

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Example

$\mathcal{K} \not\models_{lex} A \precsim E$

$\mathcal{U}_{lex} = \bigcap\{\mathcal{U}_1, \mathcal{U}_2\} = \{A \precsim \neg C\}$ and $\mathcal{K} \not\models A \sqcap \overline{\{A \precsim \neg C\}} \sqsubseteq E$

Preliminary Considerations

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- It may be impossible to pinpoint a single $\mathcal{U} \subseteq \mathcal{D}$ that could serve as the consistent subset to be materialized alongside the antecedent of a DCI, defining lexicographic closure.
- The conjecture in Pensel (2019) remains open, but the solution – if there is any – is not straightforward.