



Algorithm utilizing GARCH Volatility Forecasting in Long-Short Equity

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I. INTRODUCTION

As of the first quarter of 2018, VIX has transformed from being stable and low to volatile and high, accompanying a transition from a bullish market to a bearish market.¹ With this transition, volatility forecasting is becoming more and more important. The algorithm allocates securities into either a long or short portfolio, and trades said securities throughout the day. Securities from the S&P 500 are fed into a linear univariate GARCH model which is parametrized through a maximum likelihood estimation (MLE). The GARCH model predicts volatility and price point of each security. These forecasts are inputted into a Monte Carlo simulation which simulates potential returns. Returns are then measured and ranked before being allocated into either a long or short portfolio.

II. GARCH FORECASTING

GARCH modeling is heavily used in time-series analyses for forecasting volatility. This model is used frequently because it takes into account multiple factors including financial data's heteroscedasticity, the principle that variances in error terms are not constant. The GARCH model used in this algorithm is a GARCH (1,1) model, signifying that there is only one autoregressive lag and one moving-average lag. Even though greater accuracy can be achieved with more terms, the risk of

overfitting increases as well as more uncertainty in the MLE.

Typically, a GARCH (p,q) model takes the form:²

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2$$

A GARCH (1,1) model can be simplified to

$$\begin{aligned} \sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ \varepsilon_t &= \sigma_t z_t \\ z_t &= N(0,1) \\ \omega > 0, \alpha_i, \beta_j &\geq 0 \end{aligned}$$

In the above simplification, ω and α represent the parameters for the ARCH component while β represents the parameter of the GARCH component. The ε_t component represents the standardized residuals. The GARCH model estimates returns as:³

$$r_t = \mu + \sigma_t z_t$$

Given the influence of VIX in the markets today, having a returns equation directly related to forecasted volatility takes the influence of volatility into account giving a more robust forecast in our current economy.

III. MLE FOR PARAMETRIZATION

The GARCH (1,1) model has three unknown parameters, ω, α , and β . A Maximum

Likelihood Estimation (MLE) determines how likely an outcome can be modeled as a function of multiple parameters, and maximizes this probability. In other words, the MLE determines which values for each parameter would most likely produce a distribution similar to that of the data given. This process is similar to the Least Square Estimation (LSE) or Ordinary Least Squares (OLS) in that they both attempt to reduce error between a model and observed data. However, LSE focuses on reducing the sum of the squared residuals and is more useful for analyzing linear regression models. The MLE improves on LSE by asymptotically lowering the variance of parameter estimates while also maintaining invariance in the parametrization to ensure the MLE solution runs independent of the chosen parametrization while performing extremely well on large datasets.⁴

Given a data set of previous price points, $\{y_1, y_2 \dots y_n\}$ for the given security, the MLE models the data *assuming* a parameter θ . This is denoted as follows:⁵

$$L(\theta) = f(y_1, y_2, y_3 \dots y_n | \theta) \\ = \prod_{i=1}^n f(y_i | \theta)$$

This denotes the probability of observing the given data with parameter θ . Obviously, our goal is to maximize this probability, we do so by maximizing the logarithm of the function as shown below:

$$l(\theta) = \sum_{i=1}^n \log(f(y_i | \theta))$$

We can maximize this function by equating:

$$\frac{dl(\theta)}{d\theta} = 0$$

The solution to the above differential solves for the parameter θ . This is done for each unknown parameter ω, α , and β .

IV. MONTE CARLO SIMULATION

When making a forecast for price points based on various uncertain variables, instead of taking the average of each variable and forecasting price, Monte Carlo simulations take into account the probability distributions of said variables into the price point forecast. This is especially important in today's market given the increased variance in volatility and price, leading to a smaller kurtosis in VIX's distribution. The simulation chooses values for each factor based on its probability distribution by utilizing Brownian motion, the movement of particles in a suspended liquid, in order to randomly generate a value based on each probability distribution.⁶ Each time the values are chosen, the simulation runs in order to determine a price point through the following equation similar to that of the Black-Scholes Model:⁷

$$S_t = S_0 e^{\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}z}$$

Where S_t represents the stock price at time t , S_0 represents the current price, α is the expected rate of return of the stock, σ^2 is the variance of returns, and z represents a standard normal random variable.

This process is repeated 100 times for each stock with differing values for the uncertain

variables. Once the simulation is complete, a distribution for the price point is determined and the mean is taken as the forecasted price point.

V. ALLOCATION INTO LONG/SHORT

After the Monte Carlo simulation runs, a forecasted price point for each security is returned. This price point is divided by its current price point to give an estimate of potential returns as shown:

$$r_t = \frac{y_t}{y_0}$$

The algorithm then ranks the top three positive returning stocks and allocates an order percentage based on the stock's returns compared to the total returns of the three stocks shown below:

$$\zeta_i = \frac{r_i}{\sum_{i=1}^n r_n}$$

Where ζ_i represents the percentage of capital in that specific trade allocated to stock i given n stocks. In the case of this algorithm, $n = 3$. Even though the algorithm trades three stocks at a given time, the top three returning stocks are dynamic and eventually the portfolio diversifies. The stocks are shorted if the price falls below 0.97 of its current price.

VI. PERFORMANCE ANALYSIS

The algorithm performed well against the market in a backtest from January 3, 2015 to November 11, 2016 with returns of 42.93% compared to the 9.23% Benchmark returns leading to a favorable alpha. The algorithm

was able to outperform the market while maintaining a Beta value below 1. This shows that the algorithm was not as susceptible to sudden changes compared to the market. This proves helpful during market downturns with increased VIX. The algorithm also produced both a Sharpe and Sortino ratio greater than 1. This shows the algorithm was able to utilize volatility measures in order to optimize the risk-adjusted performance.

VII. REFERENCES

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