Algebraic Path Problems and GraphBLAS: a Way to High-Performance Network Analysis

Semyon Grigorev

Saint Petersburg State University

April 28, 2022

Agenda

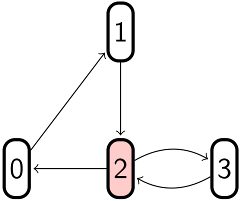
- Algebraic Path Problems
- GraphBLAS
- Our team

GraphBLAS API[1,2]

- Graph-matrix duality
- Operations over matrices and vectors
  - Parametrized by semiring-like structures
  - Sparse data
  - Parallel
- High-performance implementations
  - SuiteSparse:GraphBLAS
  - GraphBLAST
  - Huawei
  - ...

[1] https://graphblas.org/
[2] https://graphblas.org/GraphBLAS-Pointers/

BFS-like Skeleton



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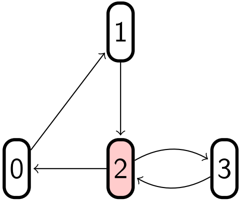

Adjacency matrix

Current front

New front

$$\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$$

Semiring

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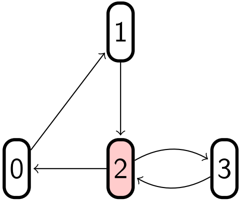

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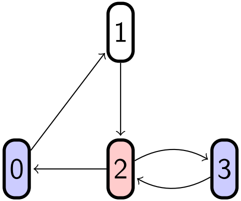

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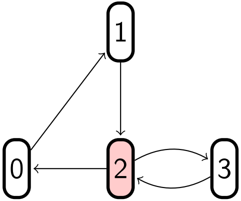

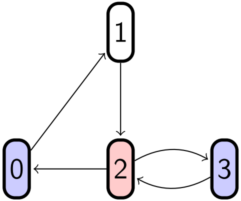

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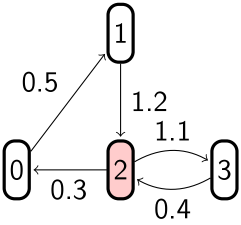
Shortest Paths



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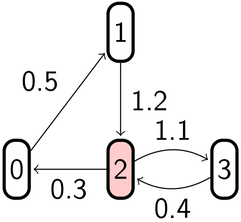
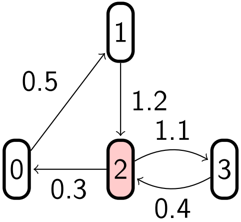


$$\begin{pmatrix} +\infty & +\infty & 0 & +\infty \end{pmatrix} \underset{\{min, +\}}{\times} \begin{pmatrix} 0 & 0.5 & +\infty & +\infty \\ +\infty & 0 & 1.2 & +\infty \\ 0.3 & +\infty & 0 & 1.1 \\ +\infty & +\infty & 0.4 & 0 \end{pmatrix} = \begin{pmatrix} 0.3 & +\infty & 0 & 1.1 \end{pmatrix}$$



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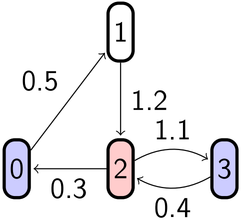

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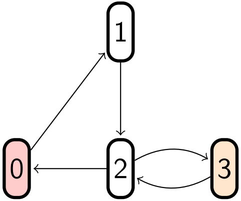
Multiple Sources Traversion Skeleton



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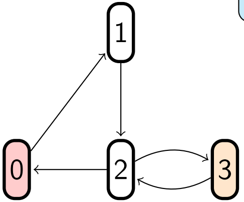

Current fronts for independent tracking

New independent fronts

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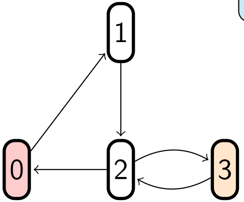

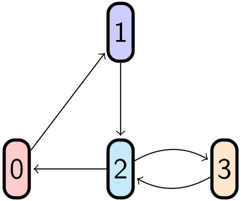

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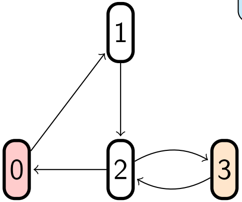

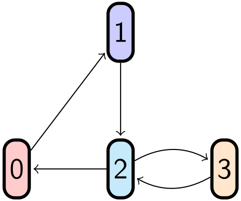

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Research areas

- High-performance graph analysis
- Path problems with constraints
- Graph databases

Team

- Semyon Grigorev (Lead)
  - PhD (2016)
  - Associate professor (2016, SPbSU)
  - s.v.grigoriev@spbu.ru
  - High-performance graph analysis
  - Graph databases
  - dblp: https://dblp.org/pid/181/9903.html
- Ekaterina Shemetova
  - PhD student
  - Path problems with constraints
  - Fine-grained complexity
  - Dynamic graph problems
- Rustam Azimov
  - PhD student
  - Linear algebra based graph analysis
  - GraphBLAS API
  - Algebraic path problem

Team: Master Students

- Alexandra Istomina
  - Master student
  - Fine-grained complexity
  - Path problems with constraints
  - Algebraic path problem
- Egor Orachev
  - Master student
  - Linear algebra based graph analysis
  - GraphBLAS API
  - GPGPU programming
- Vladimir Kutuev
  - Master student
  - Linear algebra based graph analysis
  - GraphBLAS API
  - Parallel programming
- Julia Susanina
  - Master student
  - Linear algebra based graph analysis
  - Probabilistic graph analysis
  - GPGPU programming

High-Performance Graph Analysis

- Linear algebra based algorithms for graph analysis
  - Parallel algorithms on CPU and GPGPU
  - Sparse linear algebra for graph analysis
  - GraphBLAS API[3]

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High-Performance Graph Analysis

- Linear algebra based algorithms for graph analysis
  - Parallel algorithms on CPU and GPGPU
  - Sparse linear algebra for graph analysis
  - GraphBLAS API[3]
- Research directions
  - GraphBLAS-based algorithms design, implementation and evaluation
  - Portable multi-GPGPU implementation of GraphBALS-like API
  - GraphBLAS API analysis

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Path Problems With Constraints: Algebraic Path Problems

- Semiring-like structures to specify constraints on paths
  - Reachability — boolean semiring
  - Shortest paths — tropical semiring
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- Compositionality
  - Having two semirings one can create a new one
  - Single solution for similar problems
    - Generic solution
    - Configurable solution

Formal Language Constraint Path Querying

- Particular case of algebraic path problem
  - Multiplication is not associative
  - Multiplication is not commutative
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- Research directions
  - New algorithms development
  - Complexity analysis
  - New classes of languages investigation
  - High performance algorithms implementation and evaluation

Our Results

- Tools
  - Spla: sparse linear algebra framework for multi-GPU computations based on OpenCL
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  - CFPQ_PyAlgo: set of GraphBLAS-based FLPQ algorithms
  - LDBC Graphalytics extension for evaluation of formal language constrained path querying
  - GLL4Graph: CFPQ for Neo4j
  - CFPQ for RedisGraph
- Papers (> 10)
  - SPbLA: The Library of GPGPU-Powered Sparse Boolean Linear Algebra Operations (GrAPL@IPDPS)
  - Evaluation of the context-free path querying algorithm based on matrix multiplication (GRADES-NDA@SIGMOD)
  - Multiple-Source Context-Free Path Querying in Terms of Linear Algebra (EDBT, Core A)
  - Context-free path querying by matrix multiplication (GRADES-NDA@SIGMOD)

Possible Ways for Collaboration

- Algebraic Path Problem framework applicability for network analysis
  - Which constraints can be specified in terms of semirings?
    - Length minimality
    - Nodes to visit
    - ...
  - Is it flexible enough?
- High-performance network analysis
  - GraphBLAS-based solution
  - Algorithms development and analysis
  - Algorithms implementation and evaluation

Possible Ways for Collaboration

- Algebraic Path Problem framework applicability for network analysis
  - Unstructured path problems and the making of semirings (https://link.springer.com/chapter/10.1007/BFb0028261)
  - Efficient Algorithms for Path Problems with Gernal Cost Citeria (https://www.semanticscholar.org/paper/Efficient-Algorithms-for-Path-Problems-with-Gernal-Lengauer-Theune/3fd320d97db0a581952d2919587b112f8df57c0b)
  - Path Problems in Networks https://www.morganclaypool.com/doi/abs/10.2200/S00245ED1V01Y201001CNT003
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