Fine-grained Reductions Around CFL Reachability

ALEKSANDRA ISTOMINA, Saint Petersburg State University, Russia and JetBrains Research, Russia
RESEARCH ADVISOR: SEMYON GRIGOREV, Saint Petersburg State University, Russia and JetBrains Research, Russia

1 INTRODUCTION

1.1 brief description of the problem, areas of usage, idea of solution

cfpq appears in static code analysis, graph databases, bioinformatics
2npda hard problem
finding valid paths between vertices

1.2 problems with current cfpq results

several cubic algorithms exist
can we do significantly better? no such algorithm had been found for several decades
maybe we can prove that no such algorithm exist under some hypothesis

1.3 main problem

fine-grained complexity has some results in the area
results are scattered, have no structure
maybe everything is already proven

1.4 main goals, overview

collect existing results into easy-to-read form
focus on static, dynamic problems are omitted from this research
state open problems

2 PRELIMINARIES

cfg, directed graph, cfl reachability and recognition, note on Dyck-1, fine-grained reduction

3 MAIN RESULTS

In [1]

3.1 existing problems and hypotheses

There are several problems that are connected with CFL Recognition and Reachability. *Boolean satisfiability problem (SAT, k-SAT)* determines if there exists an interpretation of variables that satisfies a given Boolean formula on n variables written in k-CNF, $k > 3$. The hypothesis about SAT, that we are interested about, is NSETH which proposes that there is no $\epsilon > 0$ such that k-SAT can be solved co-nondeterministically in time $2^{(1-\epsilon)n}$ for any k. In *Boolean Matrix Multiplication (BMM)* problem it is needed to calculate matrix product of the two given $n \times n$ matrices over (AND, OR). BMM hypothesis states that there is no $O(n^{3-\epsilon})$ combinatorial algorithm for that. *Orthogonal Vectors (OV)* problem decides whether the set of n boolean vectors contain two which dot product equals zero. Hypothesis states that OV problem can not be solved in $O(n^{2-\epsilon})$ time. Given a context free language $L(G)$ the *Language Edit Distance (LED)* problem seeks the minimum number of edits (insertions, deletions and substitutions) required to convert the given string s into a member of

Aleksandra Istomina: graduate student, email: aleksandra2999@mail.ru, ACM student member number: 4678238.

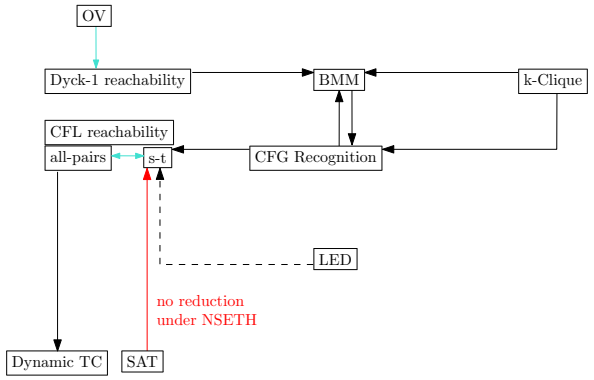

Fig. 1. Black arrow $a \longrightarrow b$ represent existing reduction from a to b, directions of future work are marked with dashed arrows. Red and turquoise arrows analogously represent non-existence of the reduction and open problems respectively.

$L(G)$. The fully *dynamic transitive closure (DTC)* problem asks to maintain reachability information in a directed graph between arbitrary pairs of vertices under insertions and deletions of edges.

3.2 existing reductions

CLFLR <-> SetConstr
dynamic TC to all-pairs
BMM to Dyck-1
s-t -> cfg -> bmm => no combinatorial algorithm
short s-t certificates => no reduction from SAT

3.3 open problems

This paper is a part of research dedicated to determination of existence or non-existence of a truly subcubic algorithm for CFL Reachability.

There are several reductions that seem promising: reduction from OV to s-t CFL Reachability that
global: subcubic cfpq
s-t vs all-pairs reachability: comparison with triangles detection problem
ov -> dyck-1

4 CONCLUSION AND FUTURE WORK

subcubic cfpq
reduction form ov, similar to OV -> APA
formalisation of naive reduction LED to s-t
possible reduction form APSP and reformulations

5 ACKNOWLEDGMENTS

REFERENCES

[1] Leslie G Valiant. 1975. General context-free recognition in less than cubic time. *Journal of computer and system sciences* 10, 2 (1975), 308–315.