

# Modeling Risk and Realities: Week 3

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Senthil Veeraraghavan  
Operations, Information and Decisions Department

# Week 3: Choosing Distributions that fit your Data

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- ◆ Data and visualization: Graphical representation
- ◆ Choosing among the family of distributions: Discrete and continuous distributions.
- ◆ How good does a certain distribution fit? Hypothesis testing and goodness of fit.

**Session 3**

# Fitting distributions to data

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- ◆ We made a case that it is important to fit a “right” distribution by visualizing the data.
- ◆ We generated histograms for our two datasets.
  - Dataset1\_histogram.xlsx
  - Dataset2\_histogram.xlsx
- ◆ Now, we can use those files to test goodness of fit.
- ◆ Before we do that, let us understand the concept behind testing the goodness of fit of a distribution.

# Goodness of Fit tests

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- ◆ After evaluating the histograms and summary statistics (mean, standard deviation, etc), we can explore distributions that can provide a good fit.
- ◆ Goodness of fit tests provide statistical evidence to test hypotheses about the nature of distribution that can fit the data.
- ◆ Two popular statistical goodness-of-fit tests are
  - Chi-Square test ( $\chi^2$  test)
  - Kolmogorov-Smirnov test.
- ◆ Anderson-Darling test is another test that is used less frequently.
- ◆ We will focus on the Chi-Square test.

# Chi-Square test

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- ◆ The Chi-Square tests the following null hypothesis against the alternate hypothesis.
  - Null Hypothesis: the studied data comes from a random variable following a specified distribution (e.g. uniform or normal).
  - Alternate Hypothesis: The sample data does not come from the specified distribution.
- ◆ Note: this is a one-sided test.
- ◆ In other words, you can disprove that data came from a specific distribution, but you cannot prove it came from that distribution.

# Running a Chi-Square Test on Your Data

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- ◆ We will run the Chi-Square tests on our datasets. However, we will first look at some thumb rules to run the test.
- ◆ Ideally, you should have at least 50 data points.
- ◆ Divide your data into  $n$  “buckets” with at least 5 observations in each bucket.
- ◆ Every Chi-Square test has “degrees of freedom” = number of buckets – parameters of specified distribution – 1.
- ◆ For example, if you have  $n=10$  buckets and try to fit a normal distribution with 2 parameters (mean and standard deviation),
  - Degrees of freedom =  $10 - 2 - 1 = 7$ .

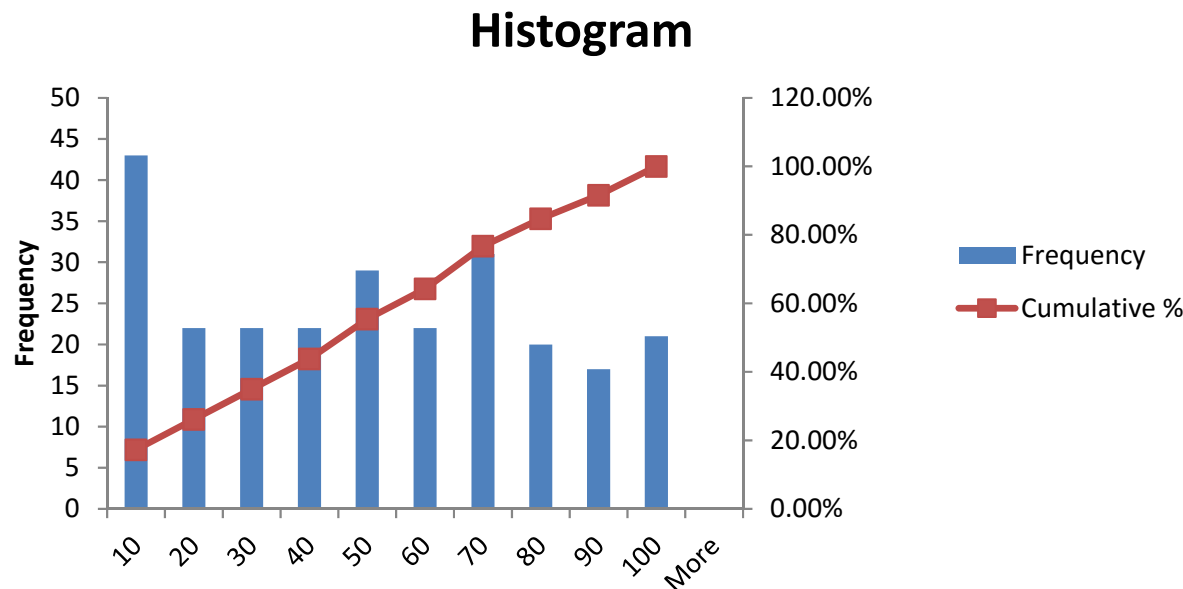
# Chi-Square Test

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- ◆ For each Chi-Square test with some degree of freedom, you can reject the null hypothesis with some confidence.
  - This could be set at 99%, 95%, etc.
- ◆ Chi-Square confidence tables are available at lots of sources.
  - For example, see the table at the following online link.  
<https://www.medcalc.org/manual/chi-square-table.php>
- ◆ We will explore Chi-Square test on our two-data sets
  - Dataset1\_histogram.xlsx
  - Dataset2\_histogram.xlsx

# Data Set 1

- ◆ The figure below gives the histogram
  - pdf in blue bars
  - CDF in red curve.
- ◆ Given the visualization of the pdf suggests a uniform distribution
  - We run a chi-square test for uniform distribution based on calculated min and max values from the data





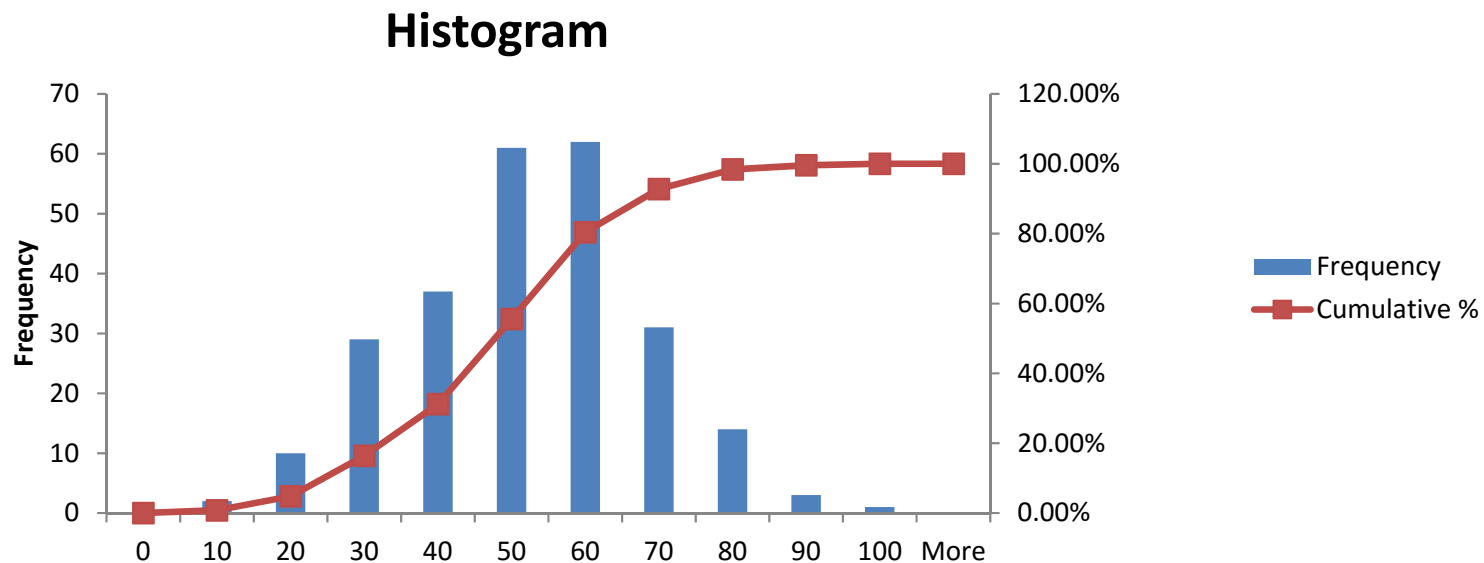
# Data Set 1

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- ◆ Descriptors for Uniform distribution (2 parameters).
  - MIN value = 0.09
  - MAX value = 99.87
- ◆ Recall that our Null Hypothesis is that data comes from a uniform distribution.
- ◆ Degrees of Freedom = Number of bins – Number of parameters -1  
=  $10 - 2 - 1 = 7$ .
- ◆ Chi-squared test gives a value of 0.013
- ◆ Looking at the tables (e.g. [link](#)), for degree of freedom 7.
  - We fail to reject the null hypothesis that data came from the uniform distribution (with confidence of 99.5%).

# Data Set 2

- ◆ The figure below gives the histogram
  - pdf in blue bars
  - CDF in red curve.
- ◆ Given the visualization of the pdf suggests a normal distribution
  - We run a chi-square test for normal distribution based on calculated average and standard deviation from the data



# Data Set 2

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- ◆ Descriptors for Normal distribution (2 parameters).
  - Sample average (sample mean) = 47.20
  - Standard deviation = 15.78
- ◆ Recall that our Null Hypothesis is that data comes from a normal distribution.
- ◆ Degrees of Freedom = Number of bins – Number of parameters -1  
=  $10 - 2 - 1 = 7$ .
- ◆ Chi-squared test gives a value of 0.8851
- ◆ Looking at the tables (e.g. [link](#)), for degree of freedom 7.
  - We fail to reject the null hypothesis that data came from normal distribution (with confidence of 99.5%).

# Goodness of Fit Files

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- ◆ The tabulated excel files are now reported in
  - Dataset1\_FIT.xlsx
  - Dataset2\_FIT.xlsx

# Kolmogorov-Smirnov test (K-S test)

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- ◆ For small samples, K-S test is more suitable.
- ◆ Basic Idea of K-S test:
  - Arrange the data values in ascending order
  - Arrange theoretical values similarly (from cumulative distribution function).
  - Find the maximal difference between the data value and its corresponding theoretical value.
  - If this maximal difference value is low, the fit is good.
- ◆ Typically, a value of 0.03-0.04 or lower is considered good.

# Modeling Using Continuous Distributions

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- ◆ Depending on size and nature of data, modeling reality using continuous distributions and choosing the correct distribution can be a challenging task.
- ◆ It is mathematically elegant to use a continuous distribution, but the approach creates complexities.
- ◆ Hence, often simulation is used.
- ◆ This will be our focus in Week 4. Congrats on ending Week 3.
- ◆ Best wishes for Week 4!

# Conclusion

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Senthil Veeraraghavan  
Operations, Information and Decisions Department  
@senthil\_veer