Interest Rates: APR and EAR

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Time Value of Money

- Intuition, tools and discounting
- Compounding
- Useful shortcuts
- Taxes
- Inflation

This Time

Interest Rates

- Interest rate quotes
- Non-annual cash flows and compounding

APR & EAR

Current 5-Year Jumbo CD Rates

Insititution	APY	Rate
CIT Bank FDIC ★★★★½	2.40% Tue Dec 16	2.37% Compounded daily
Maximize growth and savings. Member FDIC. Apply Now!		
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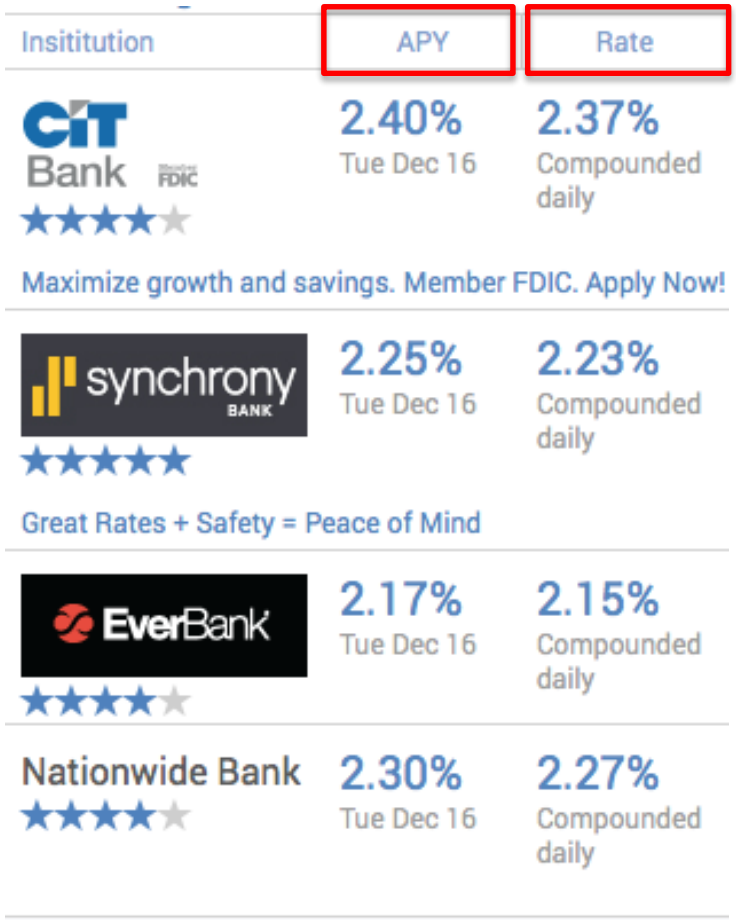

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Difference between "Rate" and "APY"?

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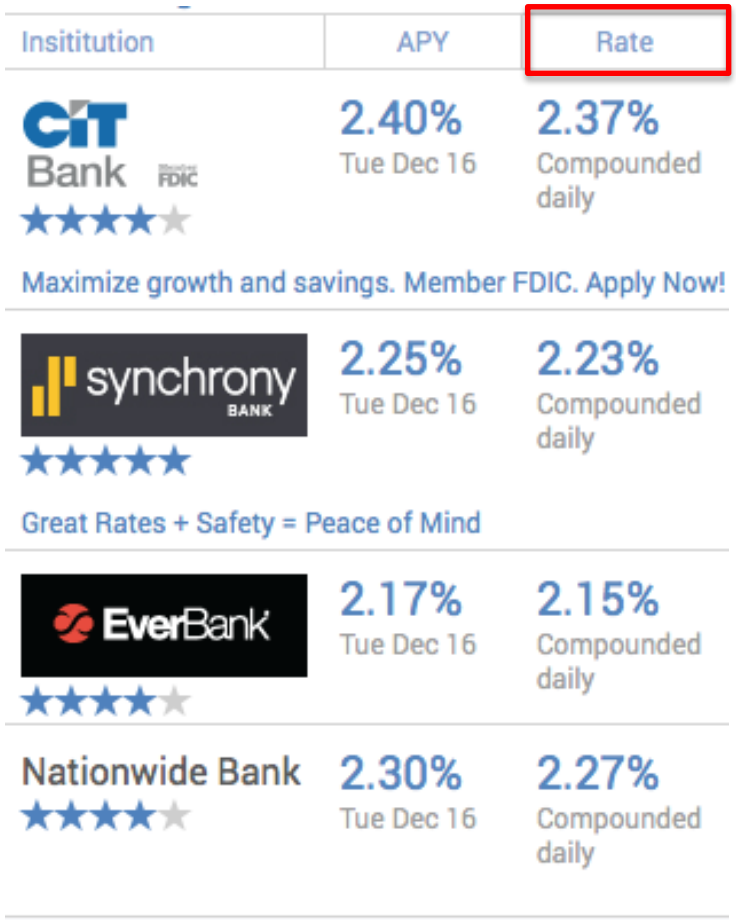

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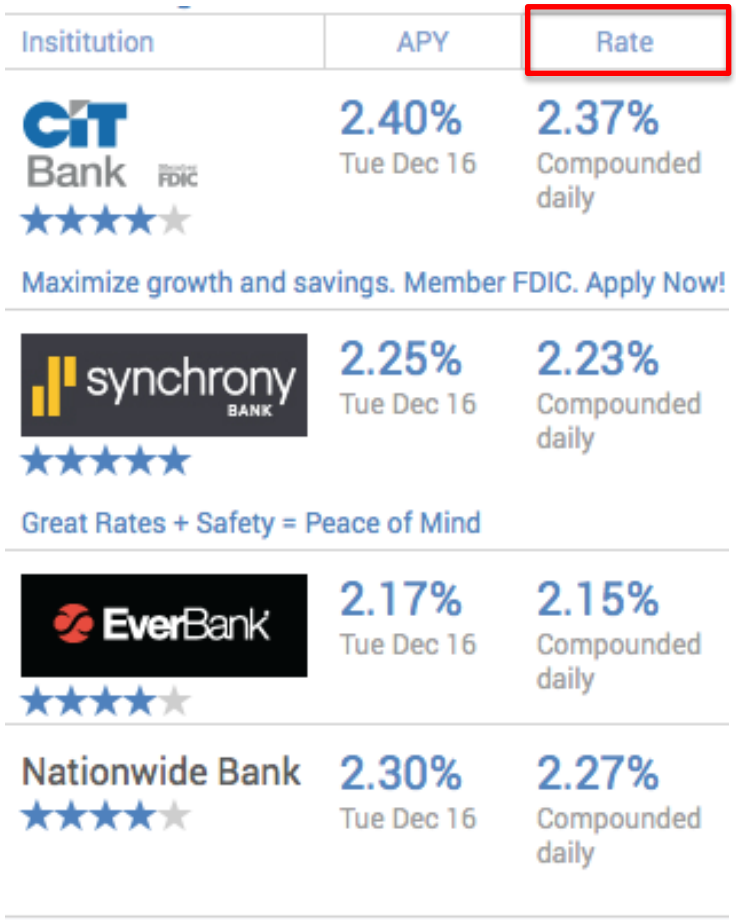

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Measures amount of simple interest earned in a year

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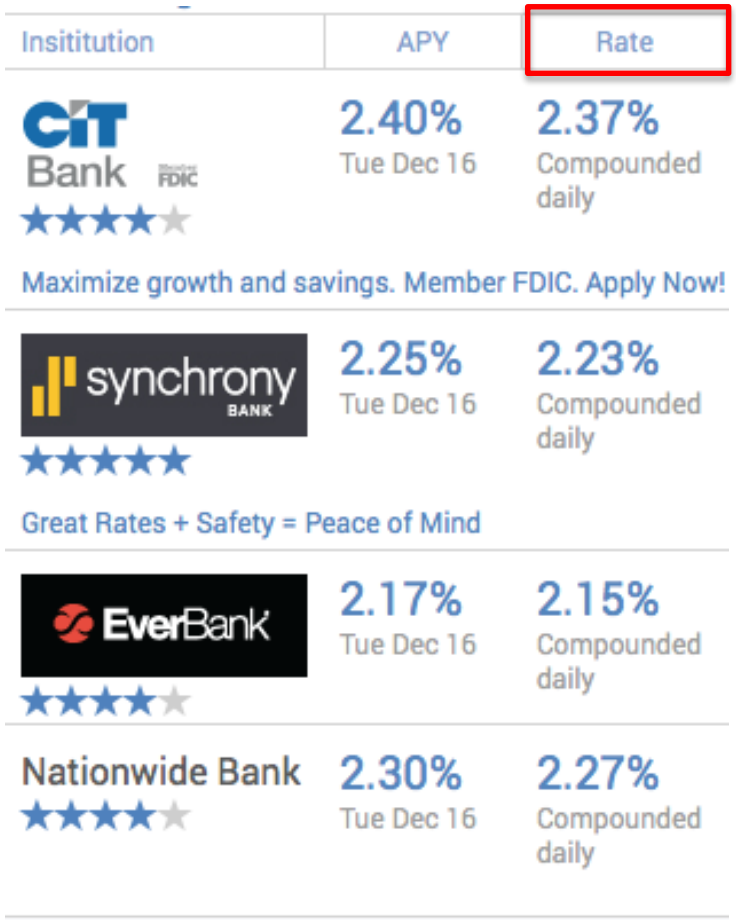

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Simple interest = interest earned *without* compounding

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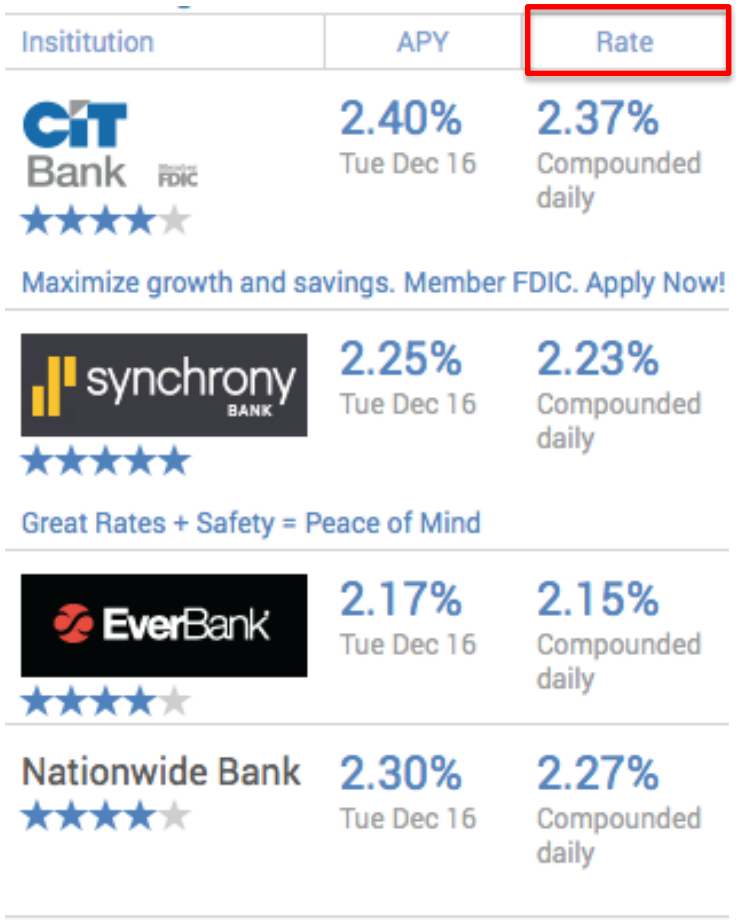

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Rate = APR or Annual Percentage Rate

Many bank quotes are in terms of APR

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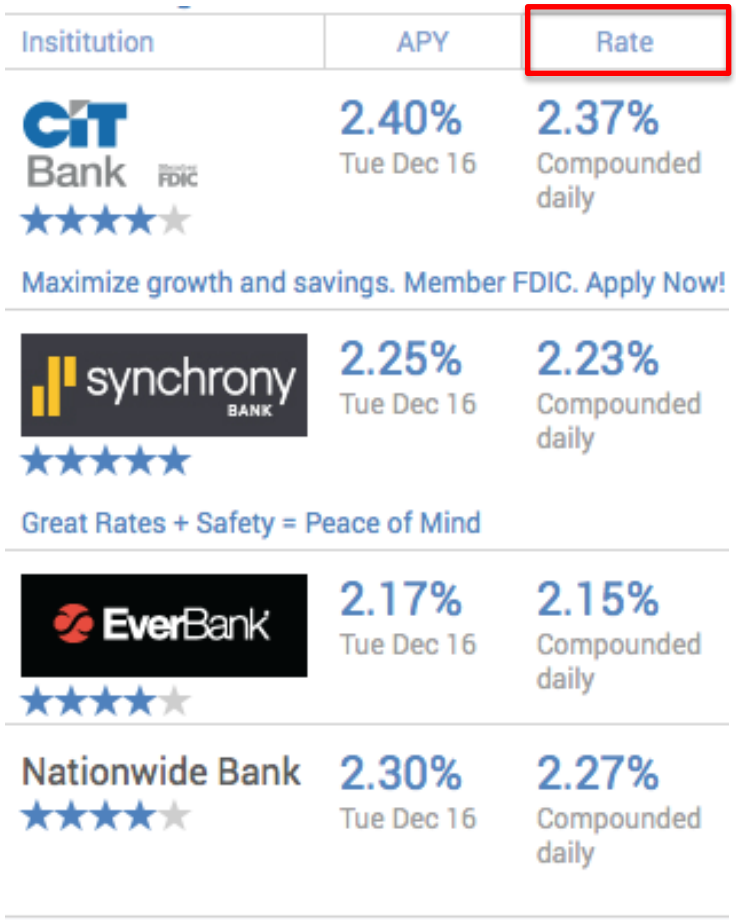

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Rate = APR or Annual Percentage Rate

APR typically *not* what we earn or pay

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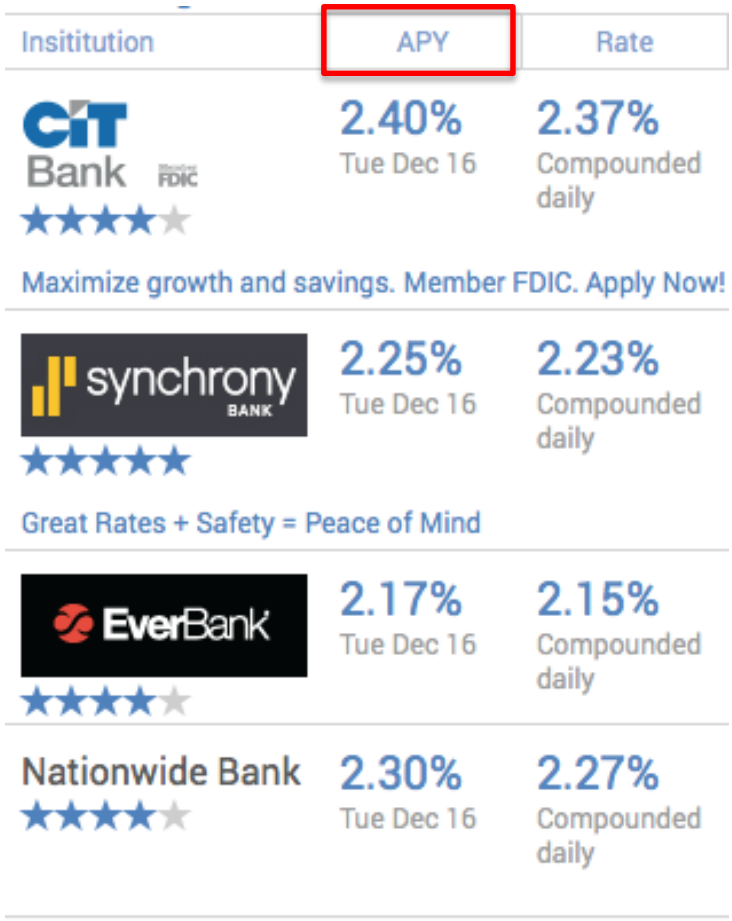

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APY = Annual Percentage Yield

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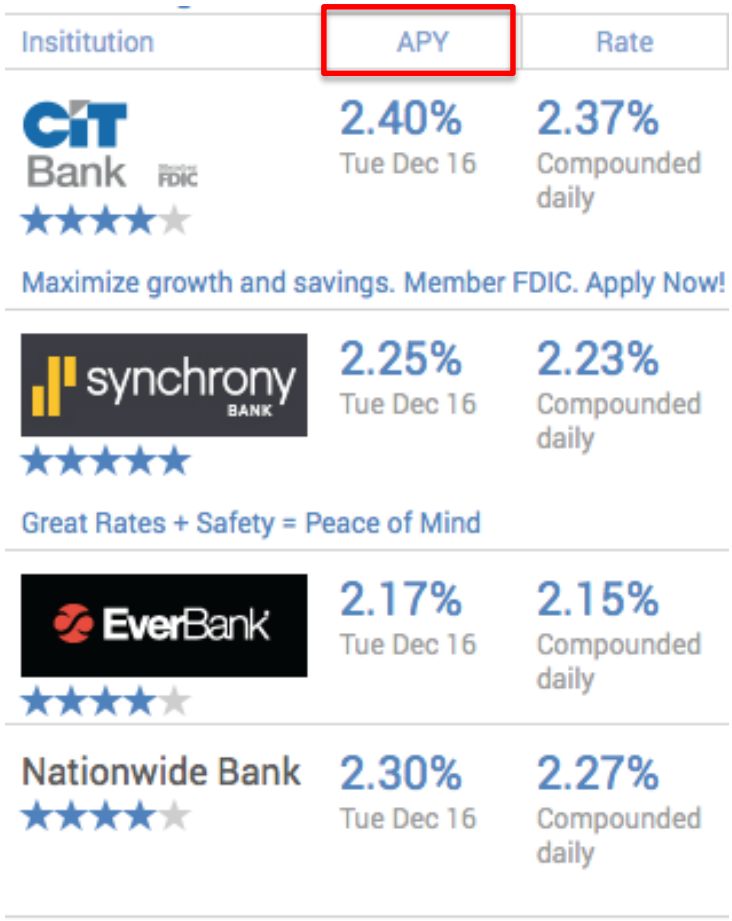

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APY = EAR or
Effective Annual Rate

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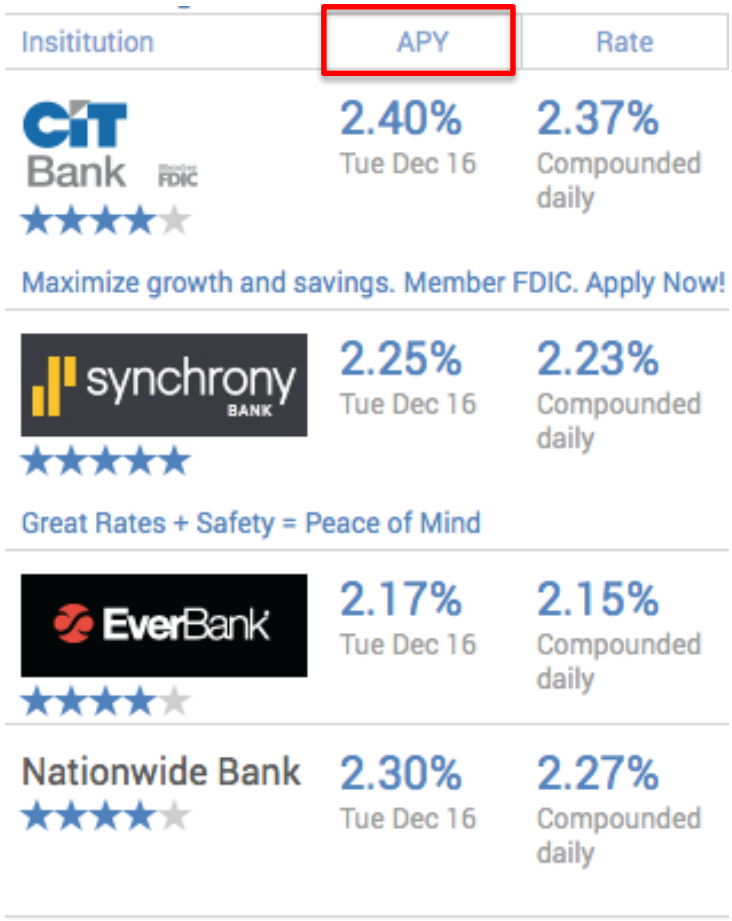

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APY = EAR or Effective Annual Rate

EAR measures actual amount of interest earned/paid in year

HOW ARE DIFFERENT RATES RELATED?

Lesson: EAR is a discount rate

EAR is what matters for computing interest and discounting cash flows

Lesson: APR is *not* a discount rate.

APR is a means to an end. We use it to get a discount rate (e.g., EAR)

How do we get from an APR to an EAR (and vice versa)?

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k}\right)^{k} - 1$$

$$= (1 + i)^{k} - 1$$

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k is the number of compounding periods per year

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k}\right)^{k} - 1$$

$$= (1 + i)^{k} - 1$$

i is the periodic interest rate, or periodic discount rate

Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

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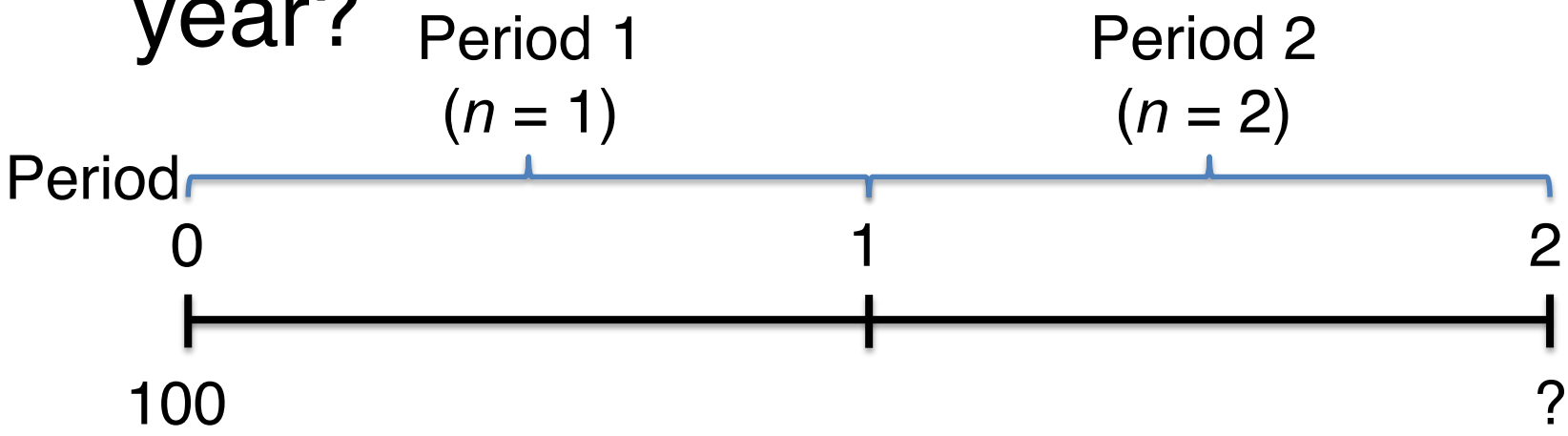

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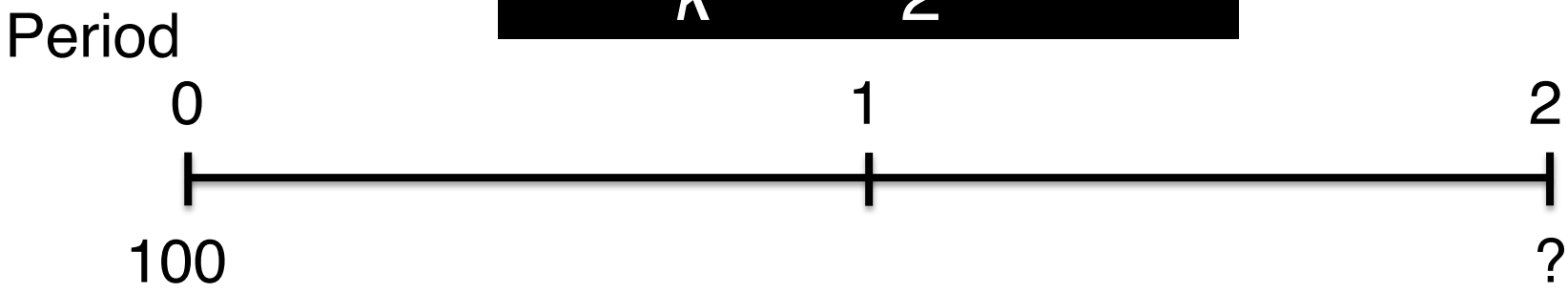

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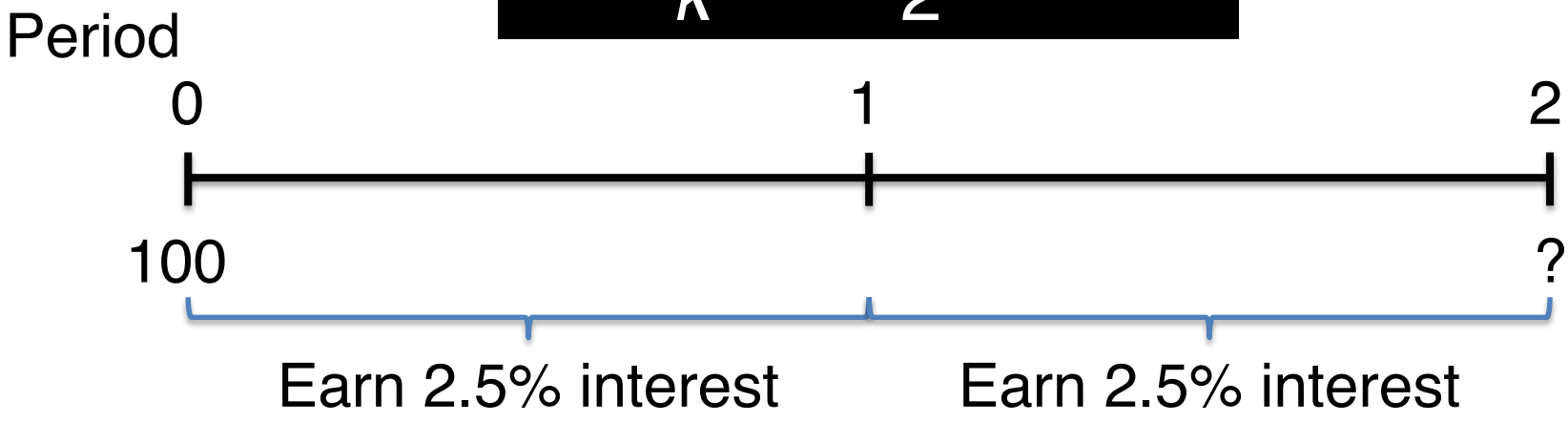

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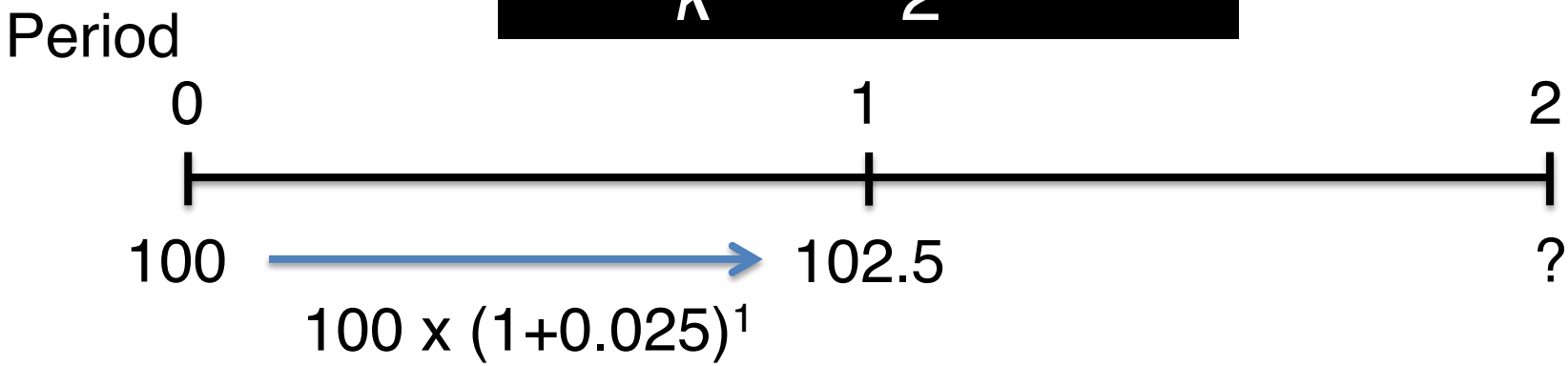

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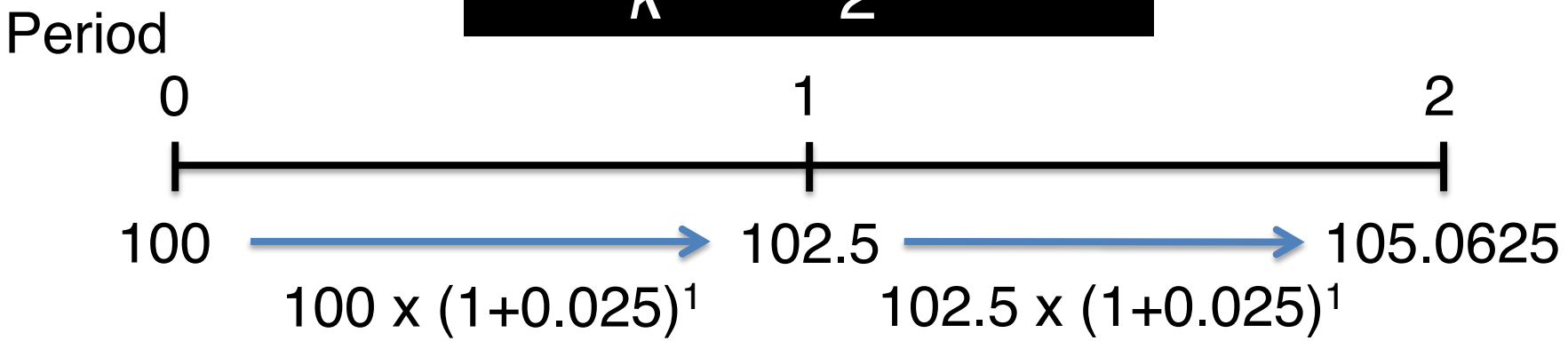
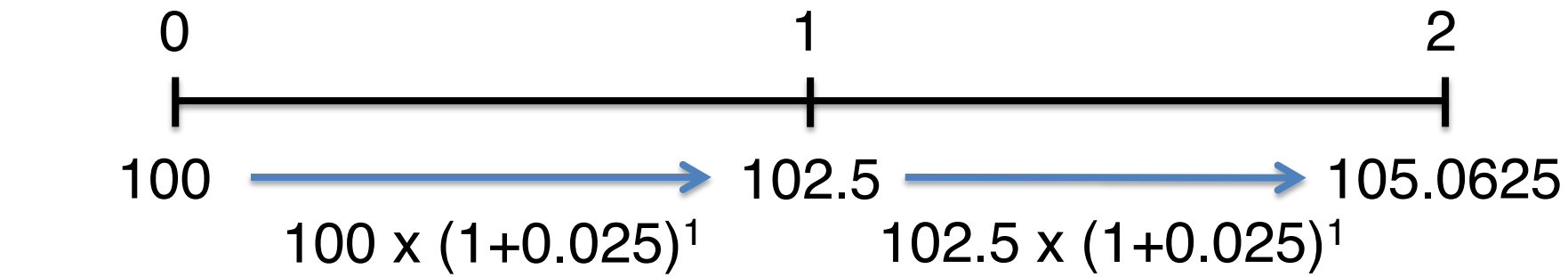


Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_2(100) = 100 \times (1 + 0.025)^2 = 105.0625$$



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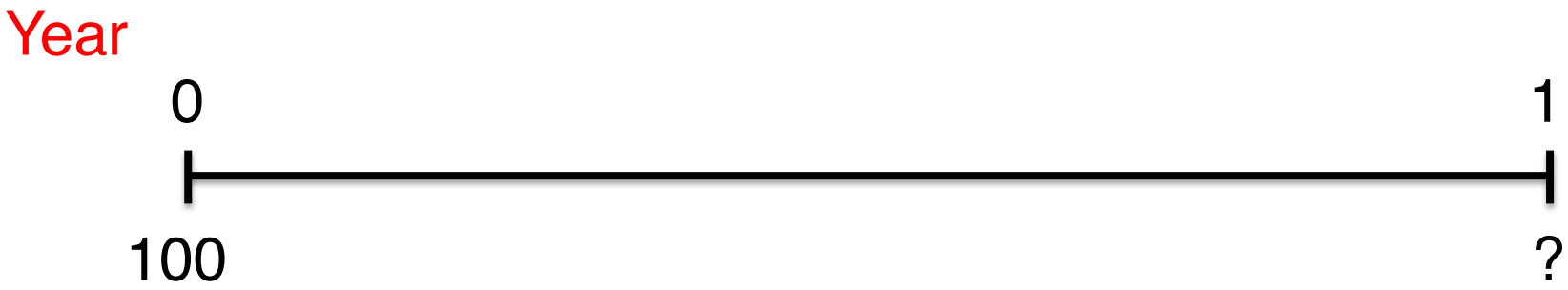

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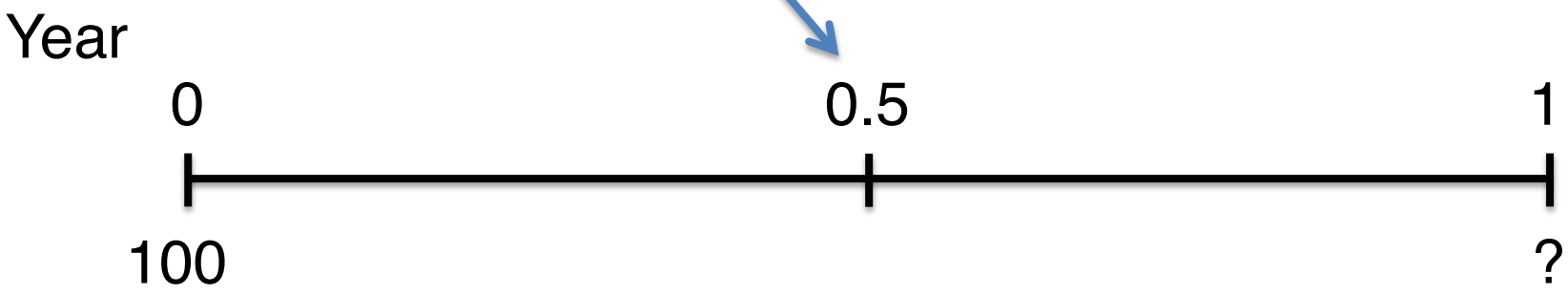

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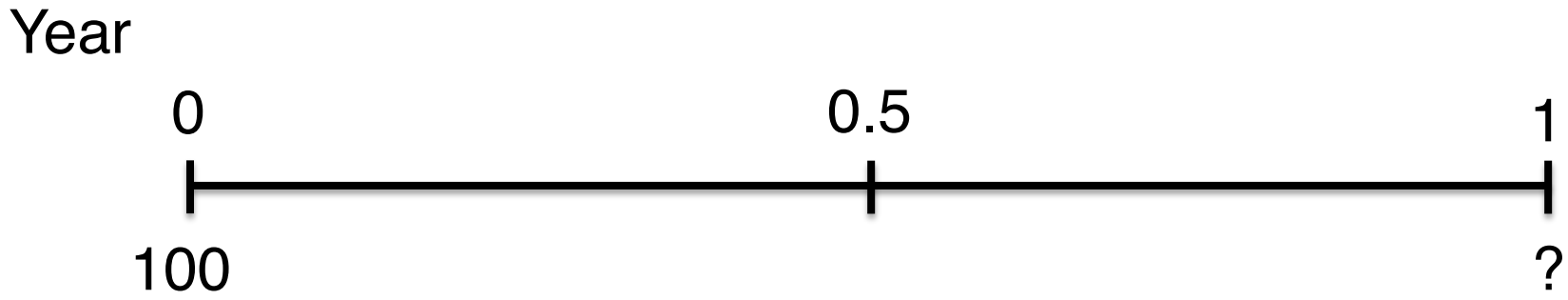
- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$EAR = (1+i)^k - 1 = (1+0.025)^2 - 1 = 5.0625\%$$



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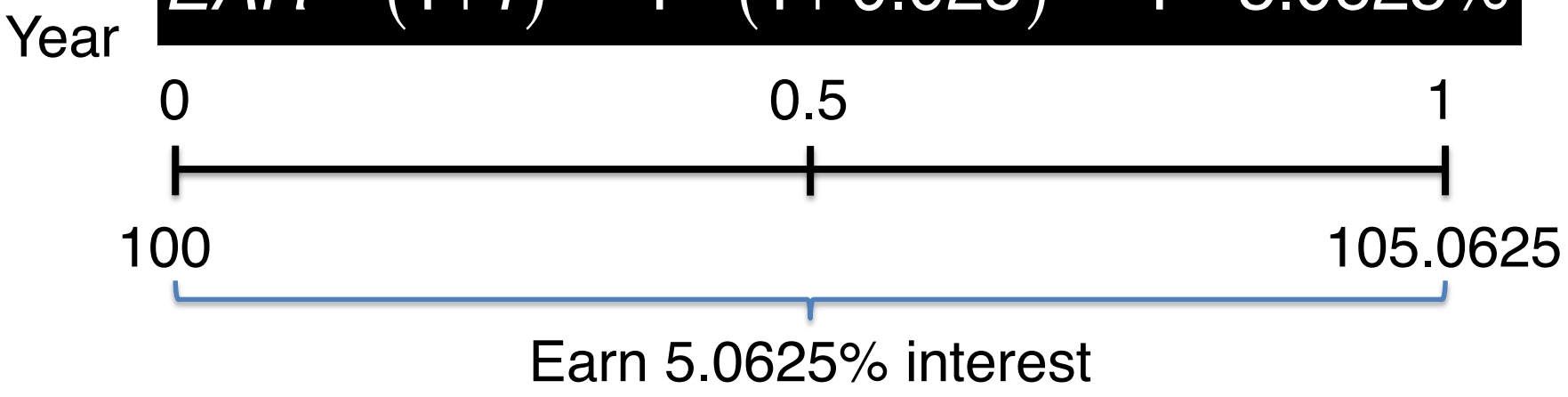

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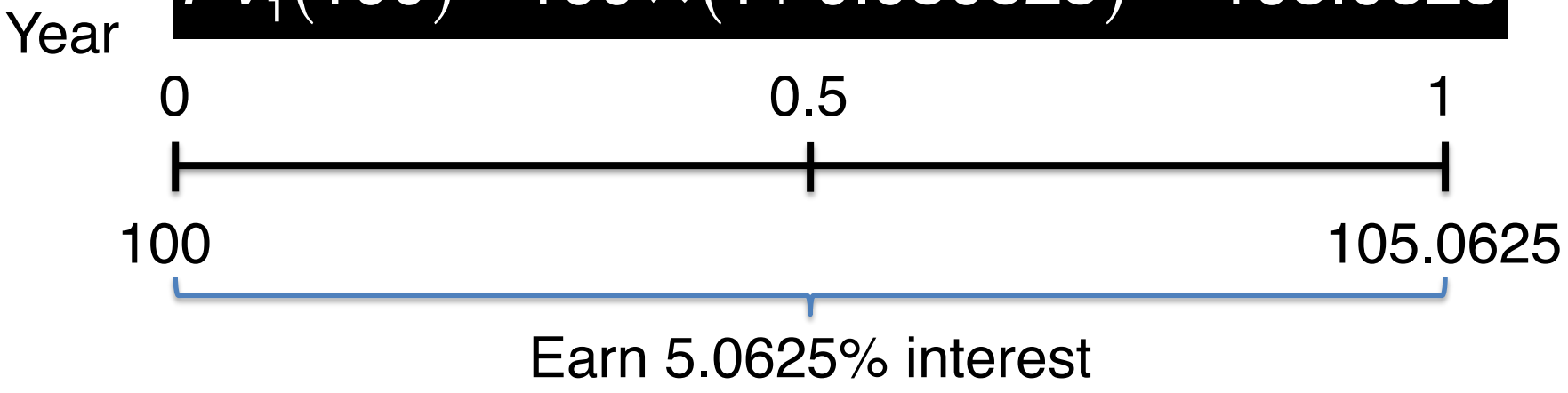
$$FV_1(100) = 100 \times (1 + 0.050625)^1 = 105.0625$$



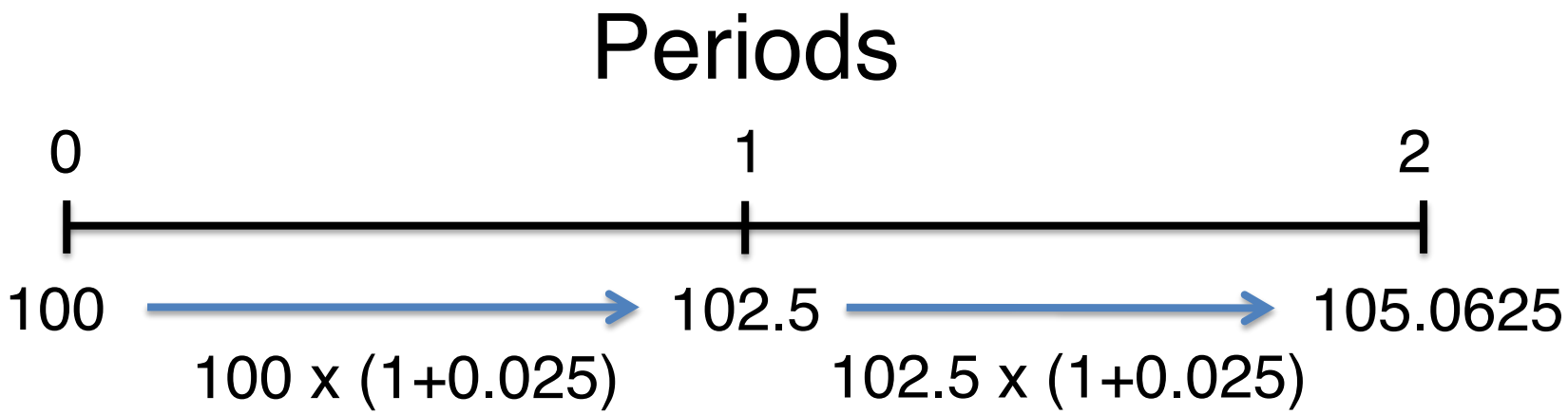
Lesson: If you discount cash flows using EAR, then measure time in years. If you discount cash flows using periodic interest rate, then measure time in periods.

Proof

$$
\begin{aligned}
(1+EAR)^T &= \left(1+(1+i)^k-1\right)^T \\
&= \left((1+i)^k\right)^T \\
&= (1+i)^{kT} \\
&= (1+i)^N
\end{aligned}
$$

where $N = kT$ = # of periods

Periods vs Years



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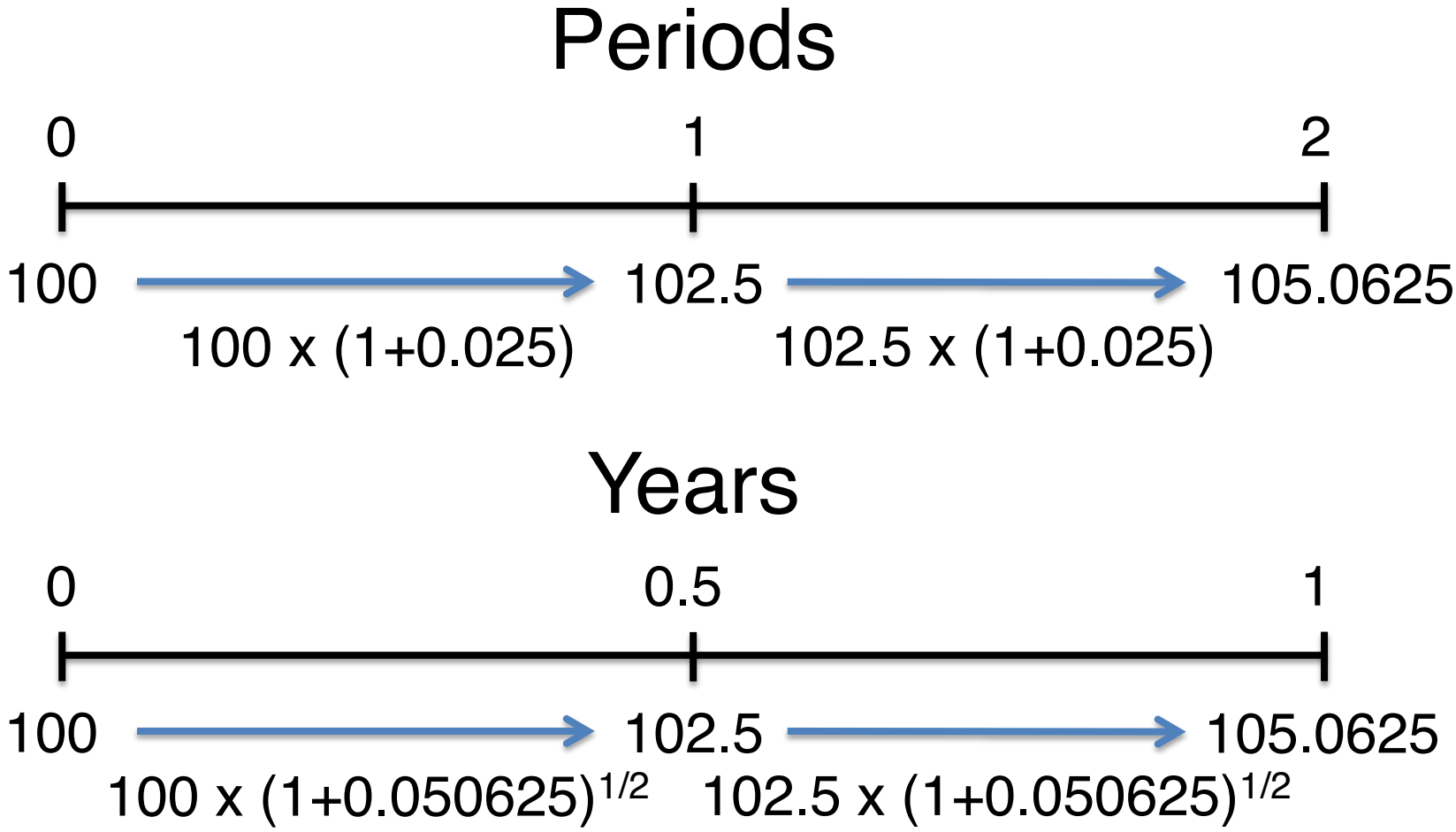

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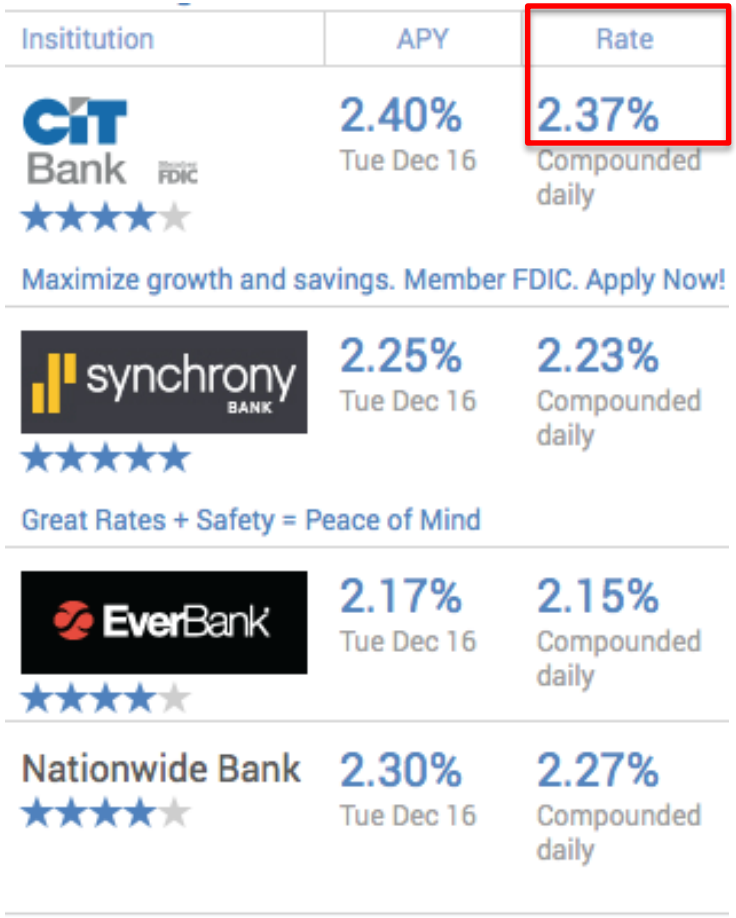

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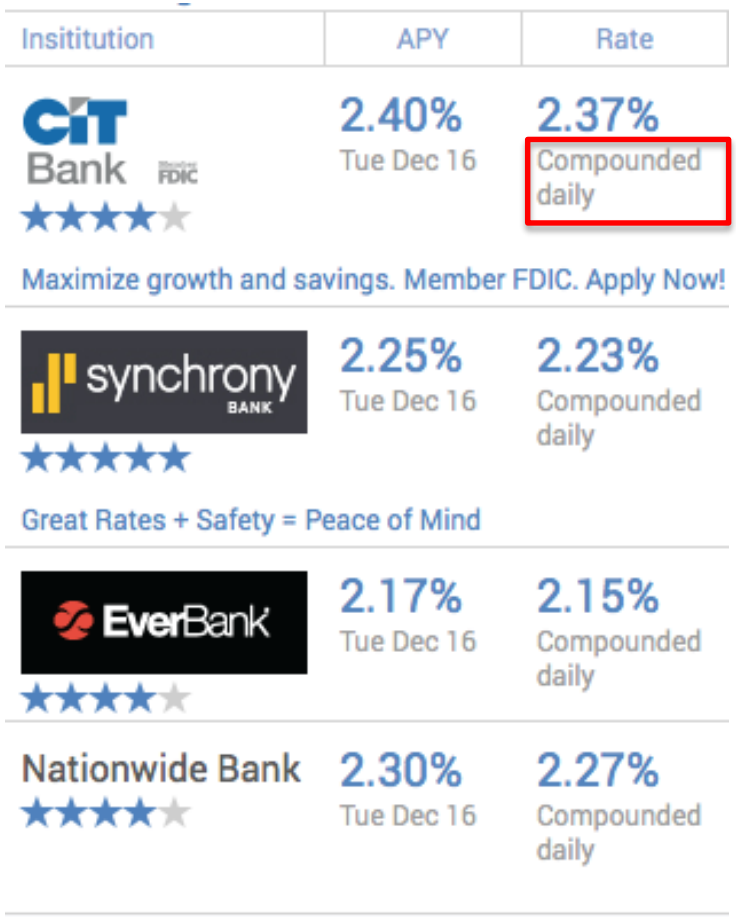

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APR = 2.37%

k = 365 (or 360, 252)

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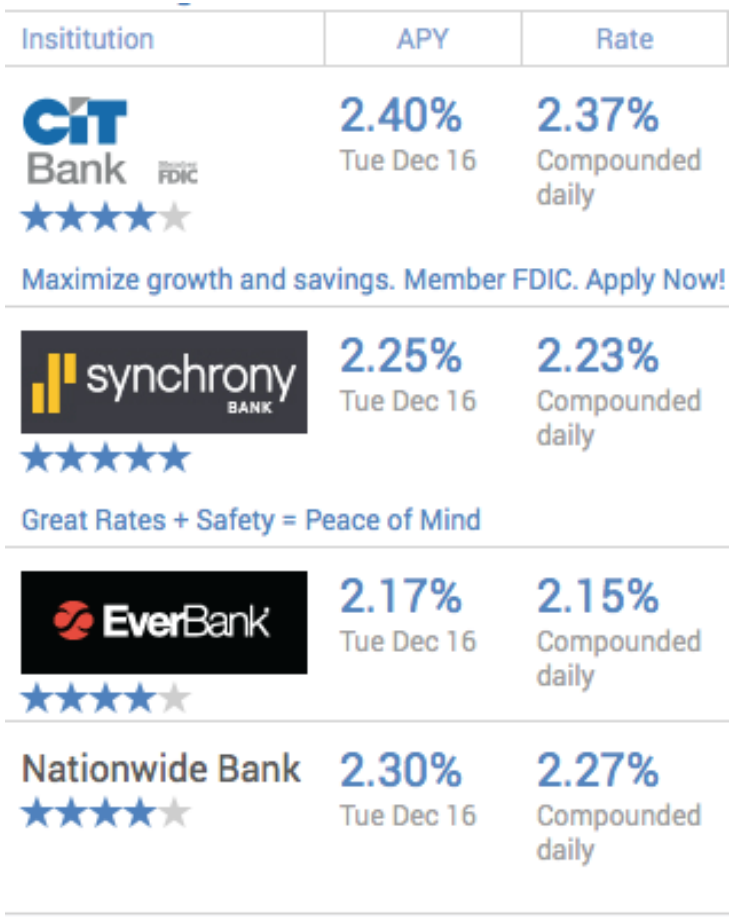

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→ i = 2.37% / 365

= 0.006714%

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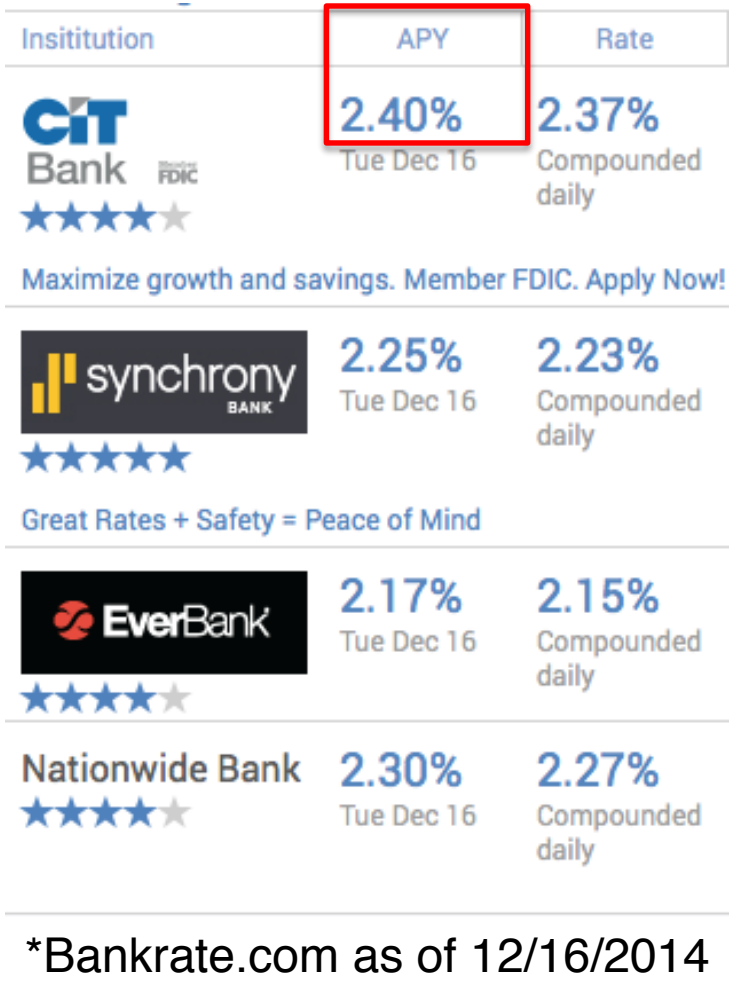

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k = 365 (or 360, 252)

→ i = 2.37% / 365

= 0.006714%

→ EAR = $(1+0.006714\%)^{365}-1$

= 2.398%

Summary

Lessons

- EAR is a discount rate
  - Measures cash flows in years
- Period interest rate, *i*, is a discount rate
  - Measures cash flows in periods
- APR is not a discount rate

Lessons

- Moving between EAR and APR

$$EAR = \left(1 + \frac{APR}{k}\right)^{k} - 1$$

$$= (1 + i)^{k} - 1$$

where

$i = APR / k$ and k = # of periods per year

Coming up next

- Interest Rates
  - Term Structure of interest rates and the yield curve