Week 1: Modeling Business Decisions in Low-Uncertainty Settings

- ◆ The First Example: Advertising Campaign at Hudson Readers Inc.

- ◆ How to Build an Optimization Model: Decisions Variables, Objective Function, Constraints

- ◆ Optimizing with Solver   **Session 1**

- ◆ Alternative Data Inputs

- ◆ Bringing in Risk Considerations: Managing Investments at Epsilon Delta Capital

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- Bringing in Risk Considerations: Managing Investments at Epsilon Delta Capital

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Session 3

Managing Investments at Epsilon Delta Capital

◆ The Epsilon Delta Capital (EDC) is a financial company that would like to determine its investment policy for the upcoming year

◆ In particular, EDC needs to determine how to allocate the total of $125 million at its disposal among the following categories of financial products: US government bonds, US municipal bonds, US corporate bonds, as well as financial products based on high-quality consumer loans

Managing Investments at Epsilon Delta Capital

◆ EDC estimates that the expected returns on each group of financial products in the upcoming year will be as follows:

Group of Financial Products	Expected Annual Return, %
Government Bonds	1.5
Municipal Bonds	3.0
Corporate Bonds	4.5
Consumer Loans	8

Managing Investments at Epsilon Delta Capital

◆ EDC uses an internal scale to assign a "quality score" to each group of products

◆ Such quality scores reflect the company's evaluation of the likelihood that the actual returns from a particular group of financial products will turn out to be below the expected values due to, for example, defaults of bond-issuing entities on their bond obligations or consumers' inability to repay their loans

Managing Investments at Epsilon Delta Capital

◆ The quality scores for each product group are shown in the following Table. Higher scores correspond to a lower likelihood of substandard returns

Group of Financial Products	Quality Score
Government Bonds	5
Municipal Bonds	3
Corporate Bonds	2
Consumer Loans	1

◆ The notion of "*risk*" can be associated with likelihood and/or magnitude of "undesirable" future events. Quality scores can serve as risk measures associated with investing in a particular group of financial products

Managing Investments at Epsilon Delta Capital

◆ EDC would like to determine the amounts to be invested in each group of financial products to maximize the expected annual $ return from its investments while ensuring that the following requirements are met:

- – The total amount of $125 million must be invested
- – The weighted quality score for the investment portfolio must not fall below 2.5 (the weight assigned to each product group is equal to the fraction of the total investment allocated to this product group)
- – No product group can be allocated less than $20 million
- – No product group can be allocated more than 50% of the entire investment amount
- – The amount allocated to government bonds must be at least 25% of the total amount allocated to three bond product groups

Epsilon Delta Capital: Building an Analytical Model

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Epsilon Delta Capital: Decision Variables

◆ We have four groups of financial products, and we have four decision variables:

G = the amount (in $ millions) to invest in government bonds

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C = the amount (in $ millions) to invest in corporate bonds

L = the amount (in $ millions) to invest in consumer loans

Epsilon Delta Capital: Objective

◆ EDC would like to determine the amounts to be invested in each group of financial products to **maximize the expected annual $ return** from its investments while ensuring that the following requirements are met:

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Epsilon Delta Capital: Objective

Group of Financial Products	Expected Annual Return, %
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Municipal Bonds	3.0
Corporate Bonds	4.5
Consumer Loans	8

- If EDC invests **G** (in $ millions) in government bonds, the expected annual return on this investment, in $ millions, will be 0.015***G**

- For an investment decision described by **G**, **M**, **C**, and **L**, **the expected annual return on this investment decision, in $ millions**, will be 0.015***G** + 0.03***M** + 0.045***C** + 0.08***L**

Epsilon Delta Capital: Constraints

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Epsilon Delta Capital: Budget Constraint

◆ The total amount of $125 million must be invested:

$$G + M + C + L = 125$$

Epsilon Delta Capital: Constraints

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Epsilon Delta Capital: Quality Score Constraint

Group of Financial Products	Quality Score
Government Bonds	5
Municipal Bonds	3
Corporate Bonds	2
Consumer Loans	1

◆ The weighted quality score of the entire investment must be 2.5 or higher:

$$5*(G)/125 + 3*(M)/125 + 2*(C)/125 + (1)*L/125 \geq 2.5$$

Epsilon Delta Capital: Constraints

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Epsilon Delta Capital: Constraint on Minimum and Maximum Amounts

◆ Since the total investment amount is fixed at $125 million, a 50% maximum investment requirement translates into the upper limit of 0.5*$125 million =$62.5 million on investment into any of the four product groups

Epsilon Delta Capital: Constraint on Minimum and Maximum Amounts

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◆ Minimum investment requirement:

   G, **M**, **C**, **L** ≥ 20

◆ Maximum investment requirement:

   G, **M**, **C**, **L** ≤ 62.5

Epsilon Delta Capital: Constraints

◆ EDC would like to determine the amounts to be invested in each group of financial products to maximize the expected annual $ return from its investments while ensuring that the following requirements are met:

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Epsilon Delta Capital: Constraint on Minimum Fraction of Government Bonds

◆ The amount allocated to government bonds (in $ millions) is **G**

◆ The amount allocated all three bond groups is **G** + **M** + **C**

◆ The fraction of investment into government bonds in all bond investments must be at least 0.25:

$$\mathbf{G}/(\mathbf{G} + \mathbf{M} + \mathbf{C}) \geq 0.25$$

Epsilon Delta Capital: Analytical Model for Investment Decision

Maximize 0.015*G + 0.03*M + 0.045*C + 0.08*L

subject to

G + M + C + L = 125 (investment amount)

5*G/125 + 3*M/125 + 2*C/125 + 1*L/125 ≥ 2.5

(minimum quality score)

G, M, C, L ≥ 20 (minimum investment amounts)

G, M, C, L ≤ 62.5 (maximum investment amounts)

G/(G + M + C) ≥ 0.25 (minimum fraction of government bonds)

G, M, C, L ≥ 0   (non-negative investment amounts)

Epsilon Delta Capital: Analytical Model for Investment Decision

Maximize 0.015*G + 0.03*M + 0.045*C + 0.08*L

subject to

        G + M + C + L = 125 (investment amount)

        5*G/125 + 3*M/125 + 2*C/125 + 1*L/125 ≥ 2.5

        (minimum quality score)

        G, M, C, L ≥ 20 (minimum investment amounts)

        G, M, C, L ≤ 62.5 (maximum investment amounts)

        G/(G + M + C) ≥ 0.25 (minimum fraction of government bonds)

        G, M, C, L ≥ 0   (non-negative investment amounts)

◆ See Epsilon Delta Capital_0.xlsx for the model set up

Epsilon Delta Capital: Optimal Investment Decision

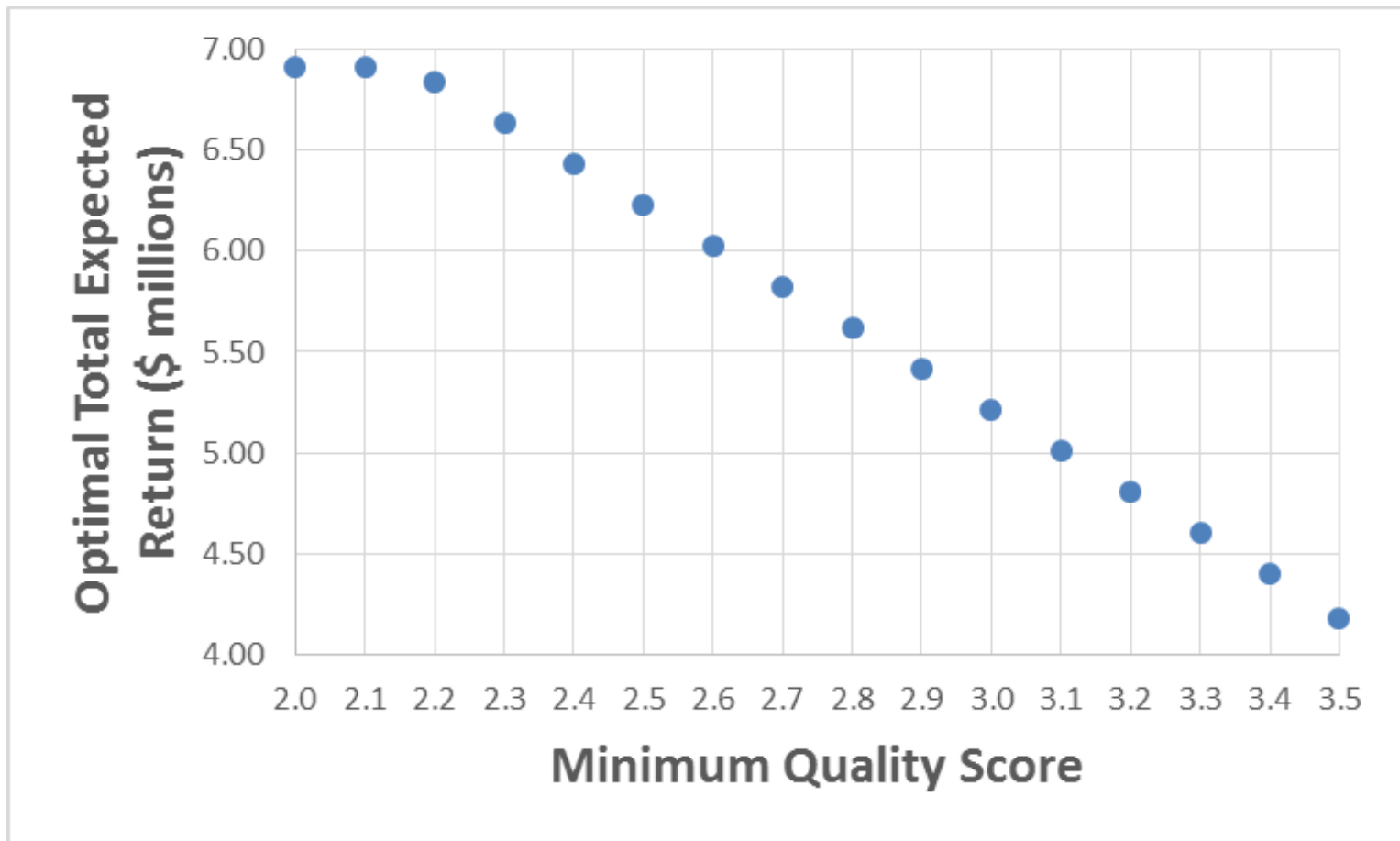|	A	B	C	D	E	F	G	H	I
1	**Epsilon Delta Capital**								
2	**Modeling Risk and Realities MOOC**					=SUMPRODUCT(B14:E14,B6:E6)/100			
3									
4		**Govt**	**Muni.**	**Corp.**	**Consumer**		**Total Expected**		
5	**Product Group**	**Bonds**	**Bonds**	**Bonds**	**Loans**		**Return (in $ millions)**		
6	**Expected Return, %**	1.5	3.0	4.5	8.0		**6.23**		
7	**Quality Score**	5.0	3.0	2.0	1.0				
8									
9	**Minimum Quality Score**	2.50				=SUM(B14:E14)			
10	**Minimum Investment Amount ($ millions)**	20.00							
11	**Maximum Investment Fraction**	0.50							
12	**Minimum Fraction of Government Bonds**	0.25							
13							**Total Invested**		
14	**Investment Amount ($ millions)**	31.88	20.00	20.00	53.12		125.00	=	125.00
15		=B14/I14							
16	**Investment Fractions, %**	0.26	0.16	0.16	0.42				
17			=B9						
18	**Risk Score Requirement**	2.50	=>	2.50					
19	**Fraction Allocated to Government Bonds**	0.44	=>	0.25					
20		=SUMPRODUCT(B16:E16,B7:E7)							
21			=B14/SUM(B14:D14)	=B12					
22									

◆ See Epsilon Delta Capital.xlsx

Epsilon Delta Capital: Optimal Expected Return Change vs. Quality Score Requirement

◆ How does the optimal expected return changes if we change the quality score requirement?

◆ How much more reward can EDC earn (in terms of expected return) if it is willing to tolerate higher risk (i.e., lower quality score)?
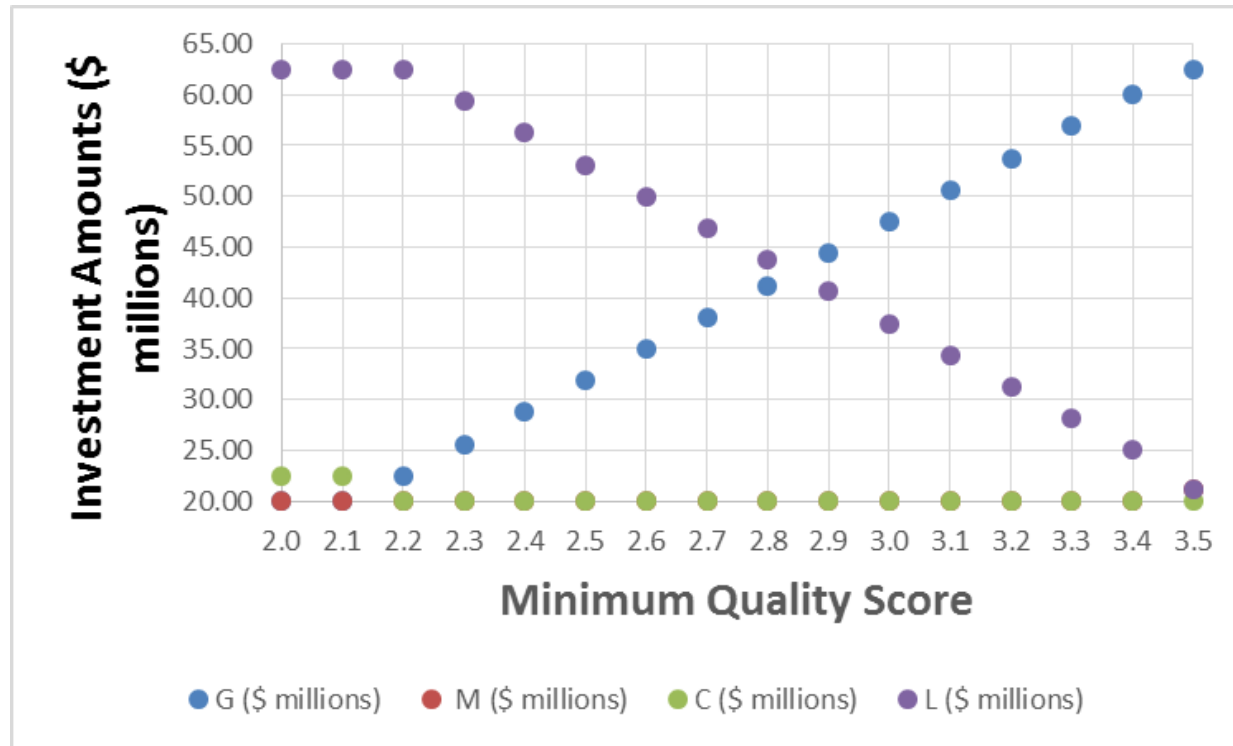
Epsilon Delta Capital: Optimal Expected Return Change vs. Quality Score Requirement

◆ See Epsilon Delta Capital.xlsx, Sheet "Analysis"

Epsilon Delta Capital: Optimal Expected Return Change vs. Quality Score Requirement

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Epsilon Delta Capital: Optimal Expected Return Change vs. Quality Score Requirement

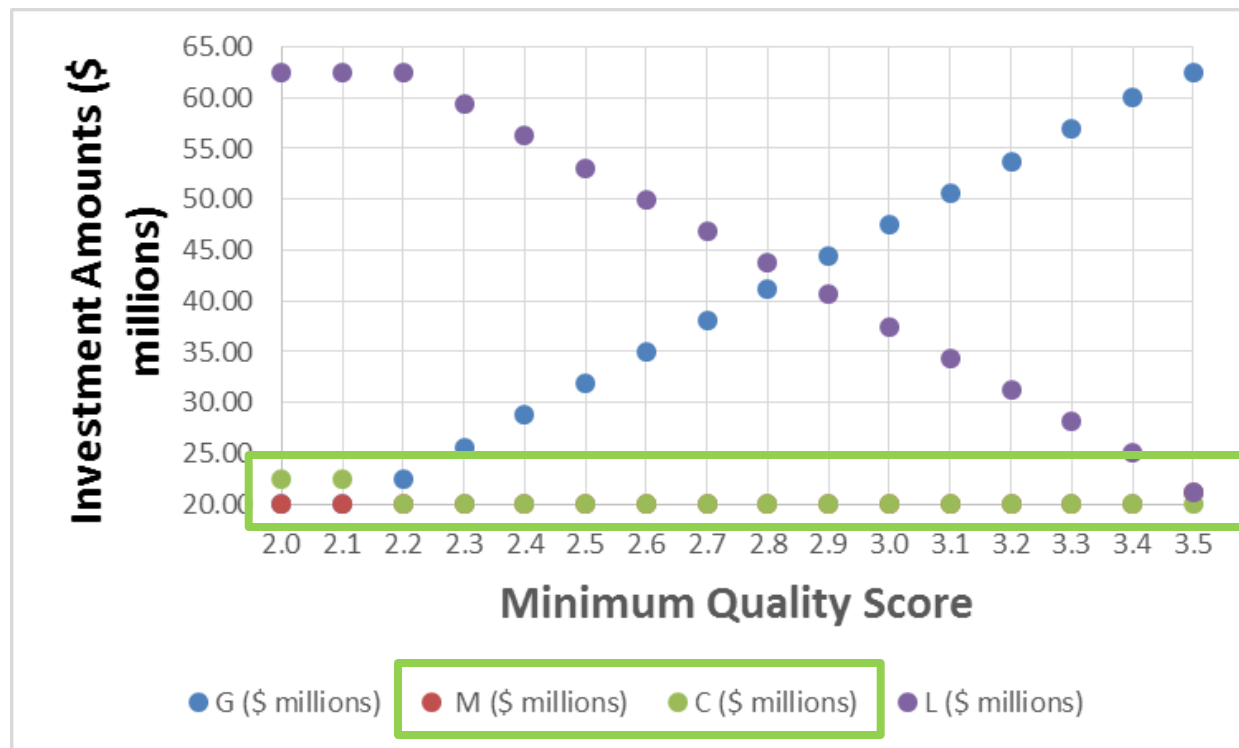◆ See Epsilon Delta Capital.xlsx, Sheet "Analysis"

Epsilon Delta Capital: Optimal Investment Amounts vs. Quality Score Requirement

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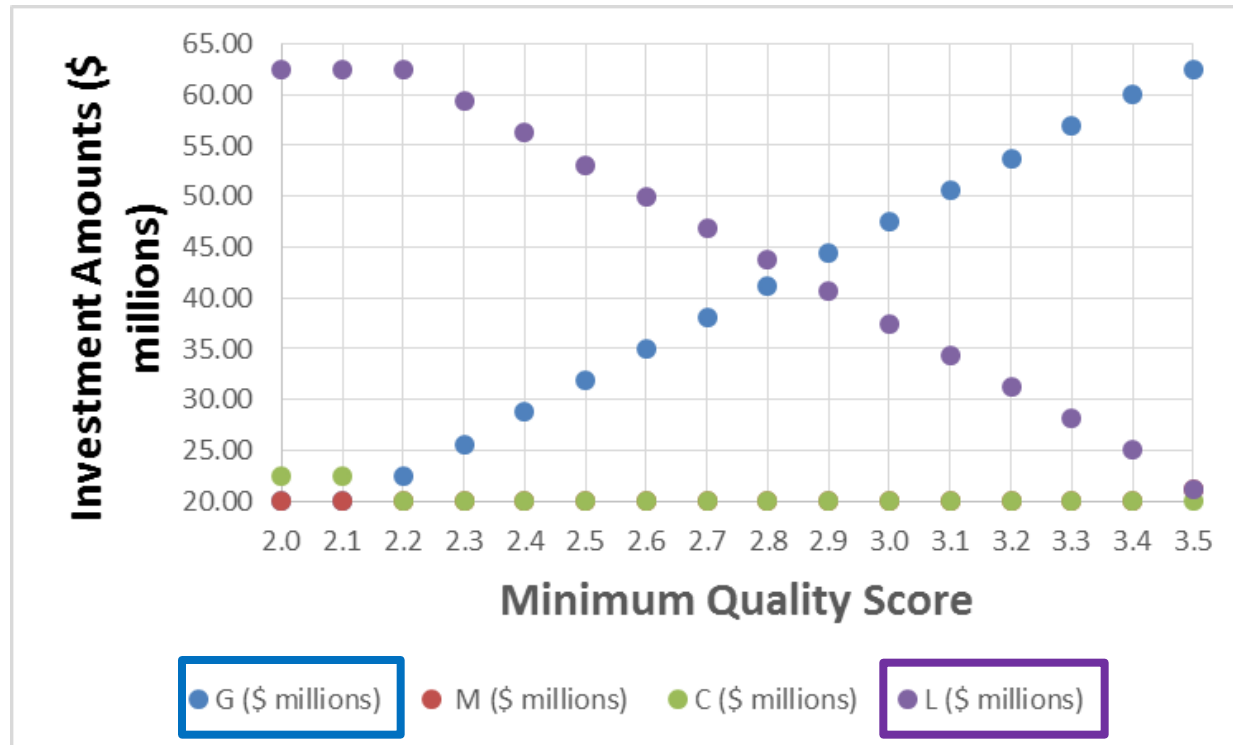# Epsilon Delta Capital: Optimal Investment Amounts vs. Quality Score Requirement

◆ See Epsilon Delta Capital.xlsx, Sheet "Analysis"

◆ Municipal and corporate bond product groups appear to have largely unattractive risk-reward profiles

Epsilon Delta Capital: Optimal Investment Amounts vs. Quality Score Requirement

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Epsilon Delta Capital: Optimal Investment Amounts vs. Quality Score Requirement

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Epsilon Delta Capital: Optimal Investment Amounts vs. Quality Score Requirement

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◆ As the degree of risk EDC is willing to take goes up, the investments shift towards consumer loans: assets with the highest risk but also the highest reward