Time Value of Money: Intuition and Discounting

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This Time

Time Value of Money

- Intuition, tools, and discounting

Intuition

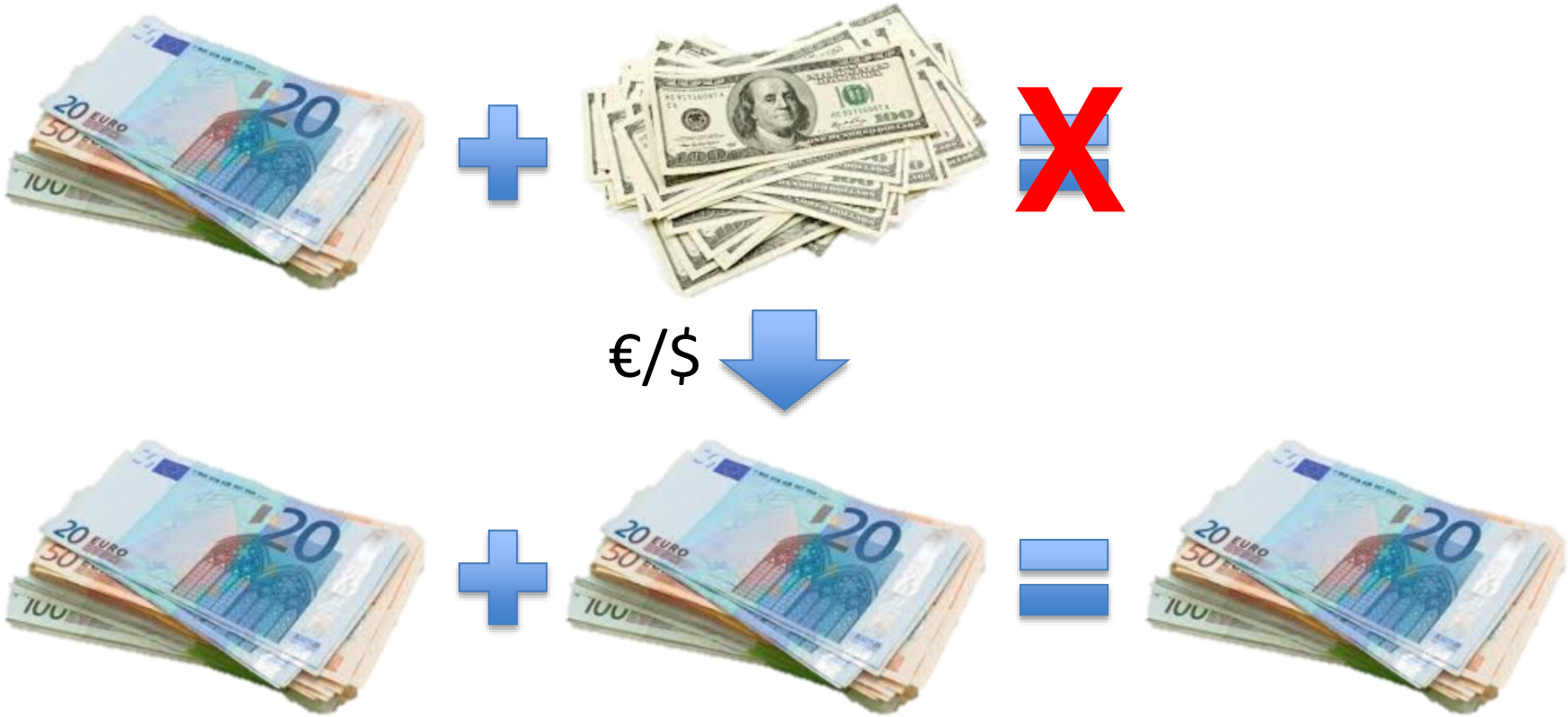
Currency

Currency

Currency

$/€

Currency



Currency

Currency

¥/€ ¥/$

Currency

$/€ $/¥

Currency

Messages (Look up)

1. Can't add/subtract different currencies

2. Must convert currencies to common (base) currency using exchange rate

Time Value of Money

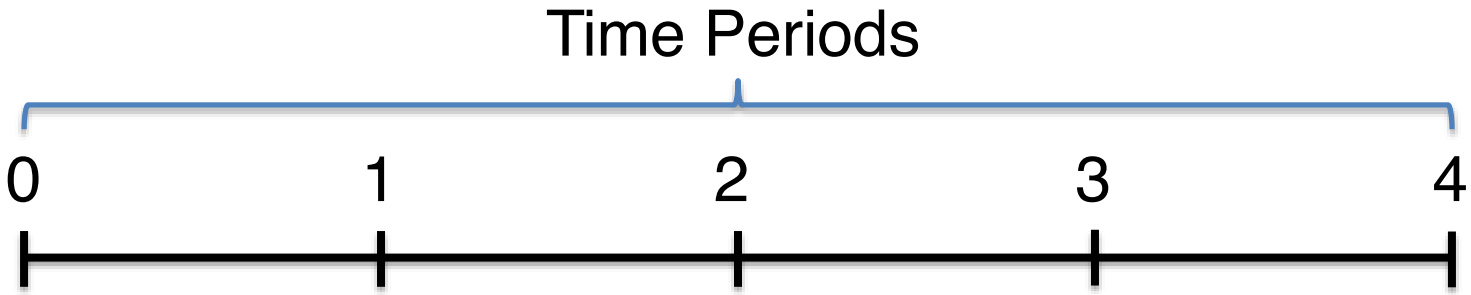
Time Value of Money

- Money received/paid at different times is like different currencies
  - Money has a time unit

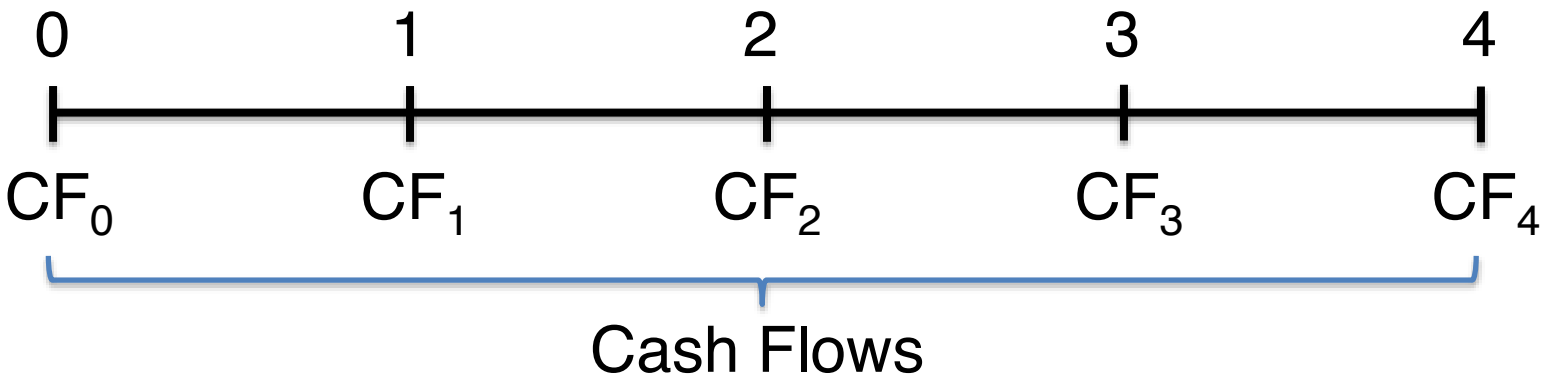
- Must convert to common/base unit to aggregate
  - Need exchange rate for time

THE TOOLS: TIME LINE & DISCOUNT FACTOR

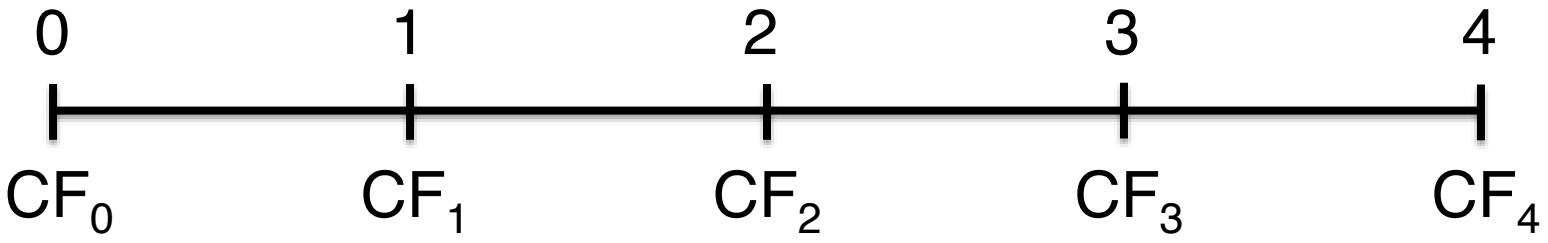
Time Line



Time Line

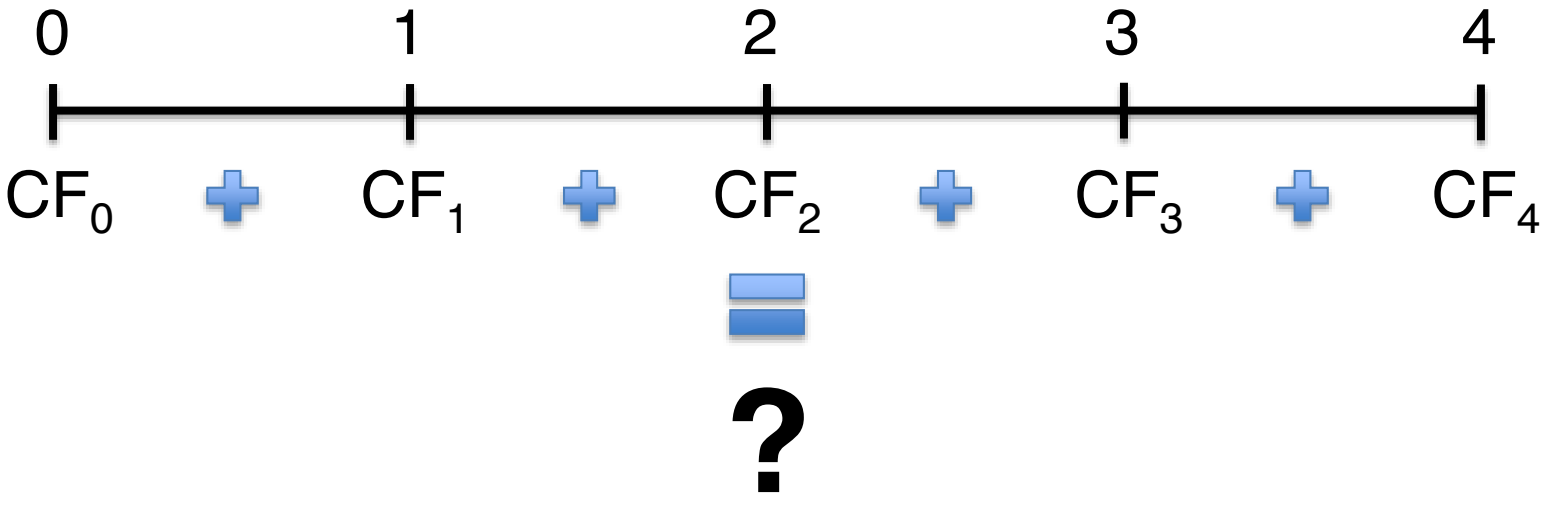


Time Line



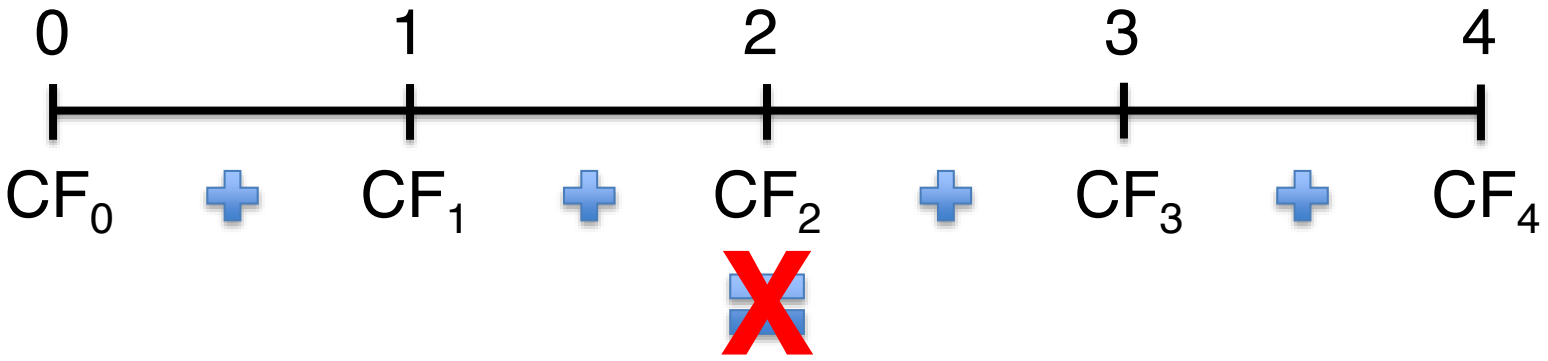
Lesson: Get in the habit of placing cash flows on a time line

Aggregating Cash Flows



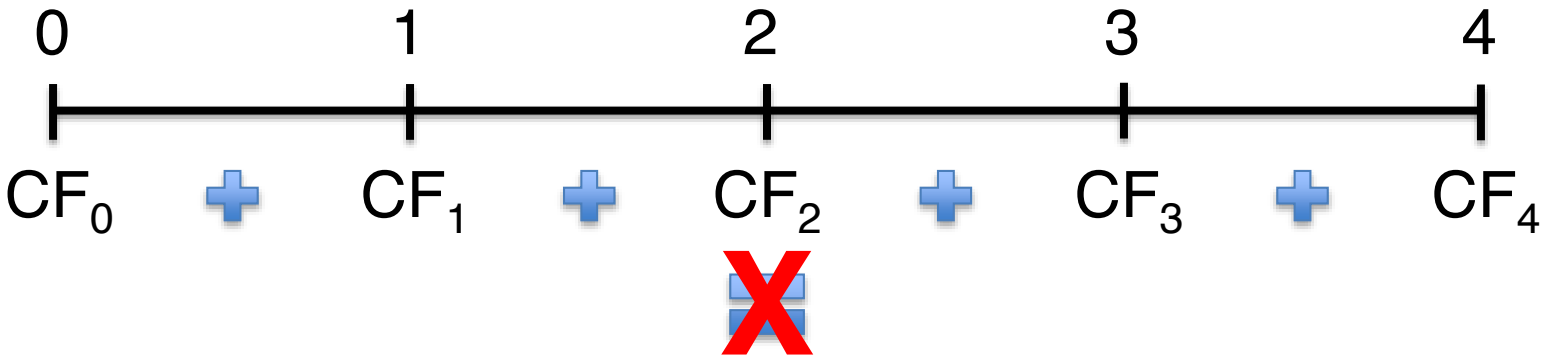
Can we add/subtract cash flows in different time periods

Aggregating Cash Flows



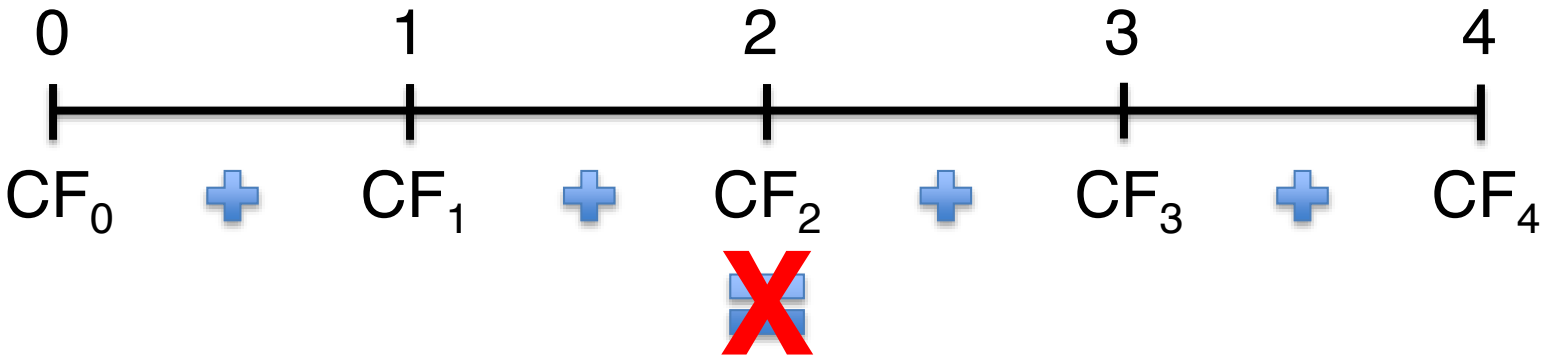
No!

Aggregating Cash Flows



Lesson: Never* add/subtract cash flows received at different times

Aggregating Cash Flows



Need exchange rate for time to convert to common time unit

Discount Factor

The discount factor is our exchange rate for time

$$(1+R)^t$$

t = time periods into future ($t > 0$) or past ($t < 0$) to move CFs

R = …

Definition: R is the rate of return offered by investment alternatives in the capital markets of equivalent risk.

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A.k.a., discount rate, hurdle rate, opportunity cost of capital

To determine *R*, consider the risk of the cash flows that you are discounting.

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Investment	Average Annual Return, *R*
Treasury-Bills (30-Day)	3.49%
Treasury-Notes (10-Year)	5.81%
Corporate Bonds (Investment Grade)	6.60%
Large-Cap Stocks	11.23%
Mid-Cap Stocks	15.15%
Small-Cap Stocks	25.32%

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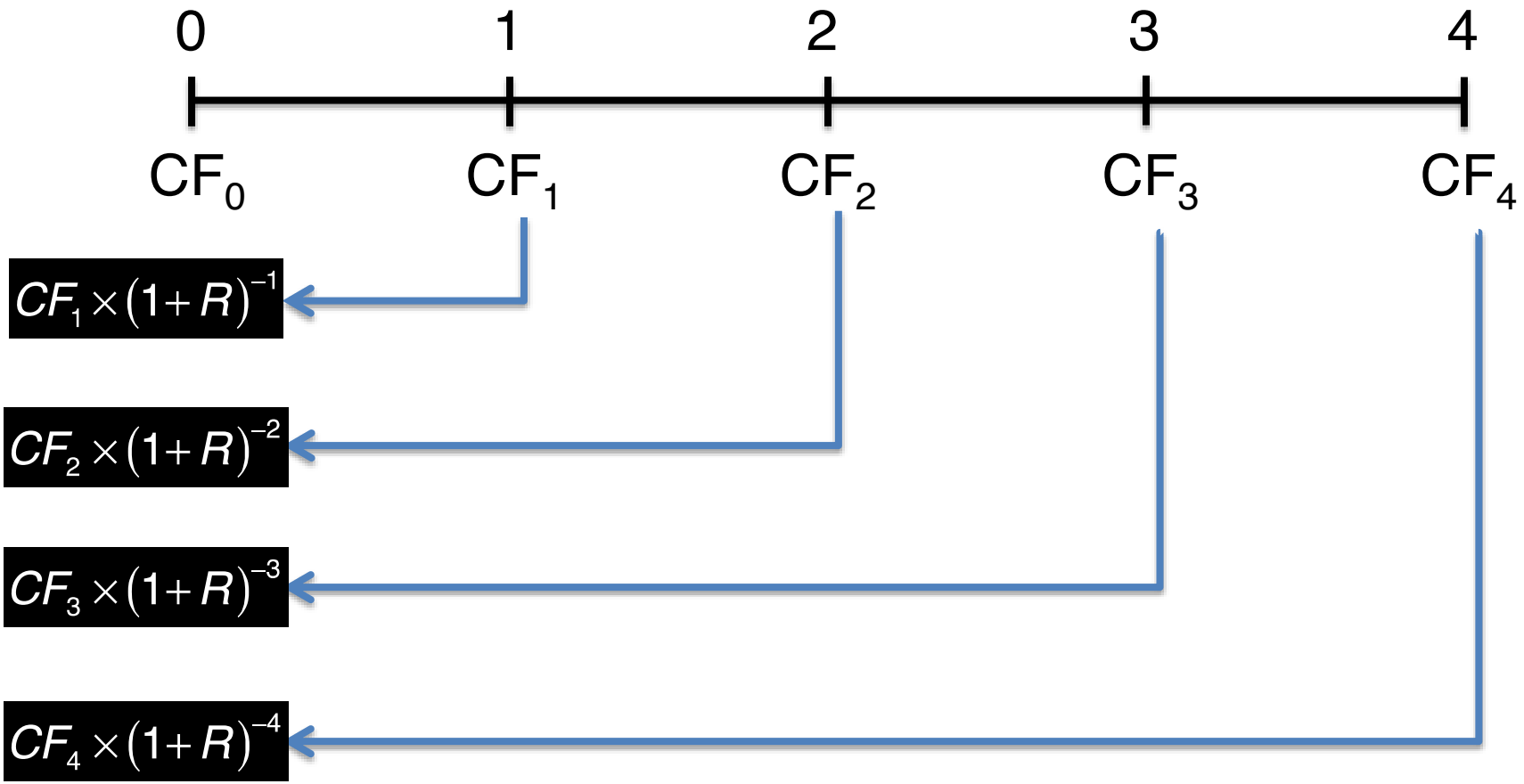
Riskier investment, higher return

USING THE TOOLS: DISCOUNTING

Discounting

Discounting CFs moves them back in time



Discounting

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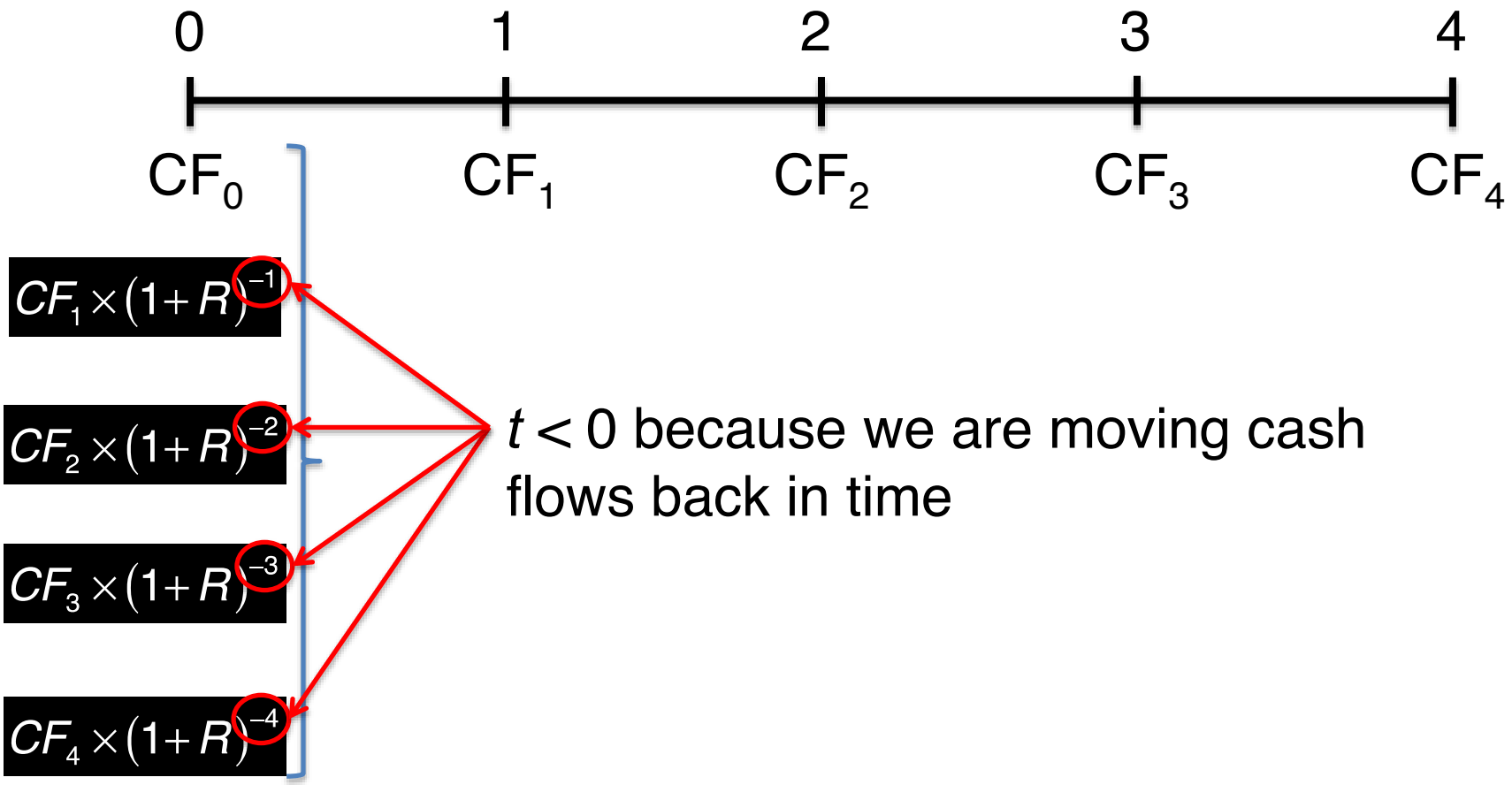

Discounting

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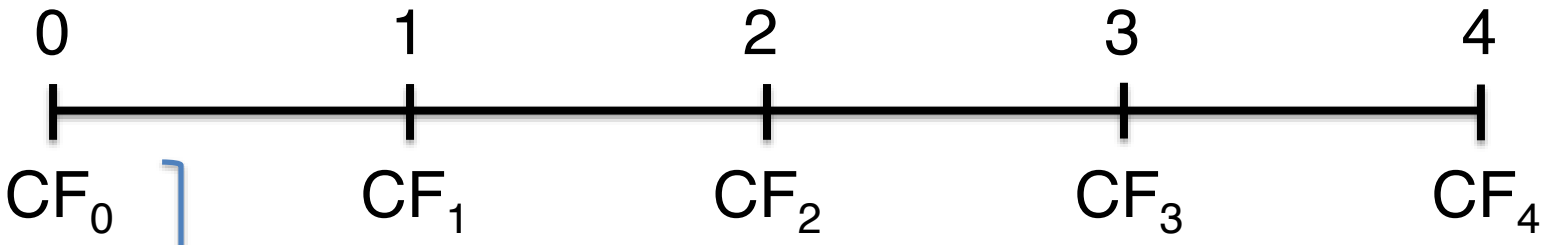


$CF_1 \times (1+R)^{-1}$

$CF_2 \times (1+R)^{-2}$

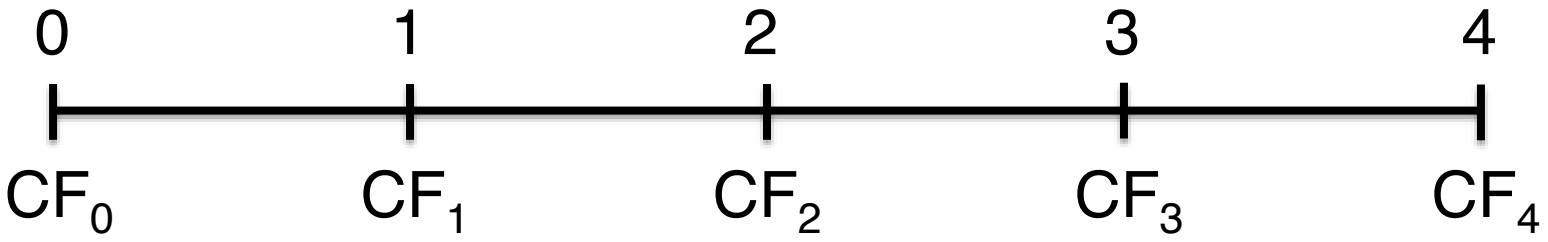
$CF_3 \times (1+R)^{-3}$

$CF_4 \times (1+R)^{-4}$

We can add/subtract these CFs because they are in the same time units (date 0)

Present Value

Present value, $PV_t(\bullet)$ of CFs is discounted value of CFs as of t



$$CF_1 \times (1+R)^{-1} = PV_0(CF_1)$$

$$CF_2 \times (1+R)^{-2} = PV_0(CF_2)$$

$$CF_3 \times (1+R)^{-3} = PV_0(CF_3)$$

$$CF_4 \times (1+R)^{-4} = PV_0(CF_4)$$

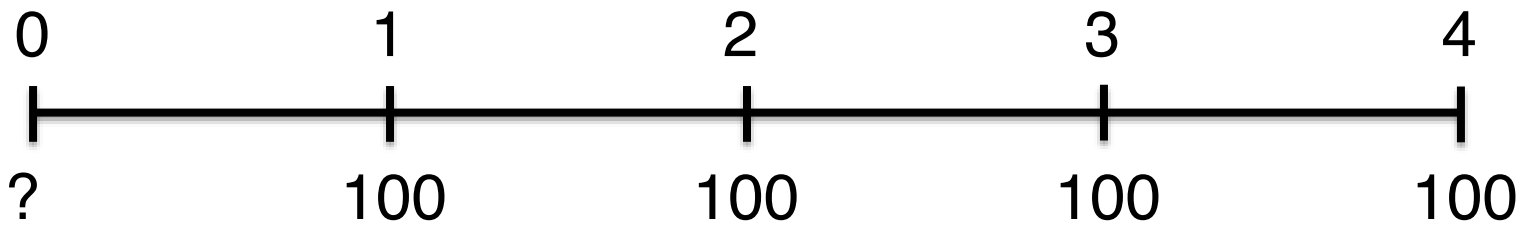
These are present values of future CFs as of today (period 0)

Example – Savings

How much do you have to save today to withdraw $100 at the end of each of the next four years if you can earn 5% per annum?

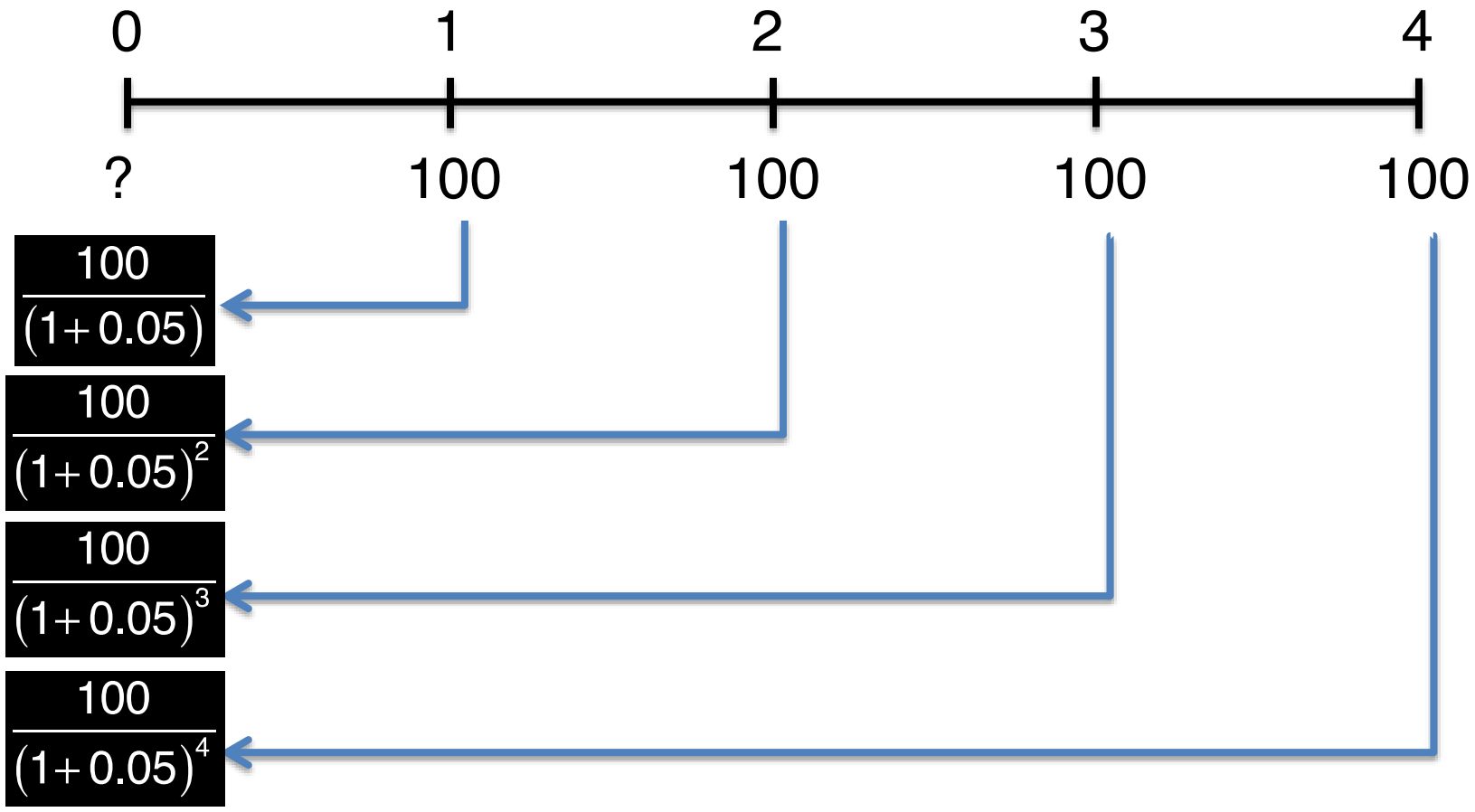
Example – Savings

Step 1: Put cash flows on a time line



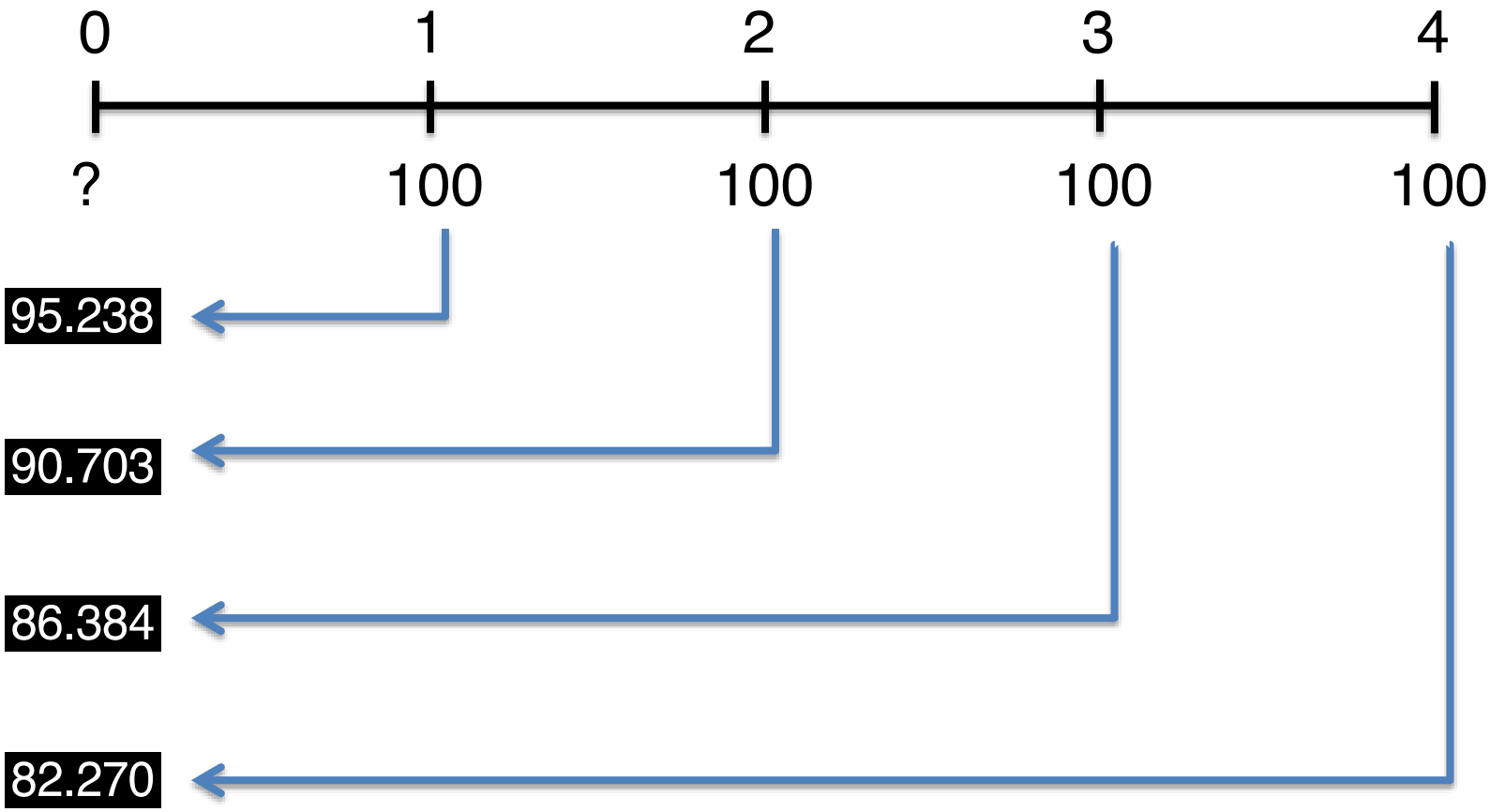
Example – Savings

Step 2: Move CFs back in time to today



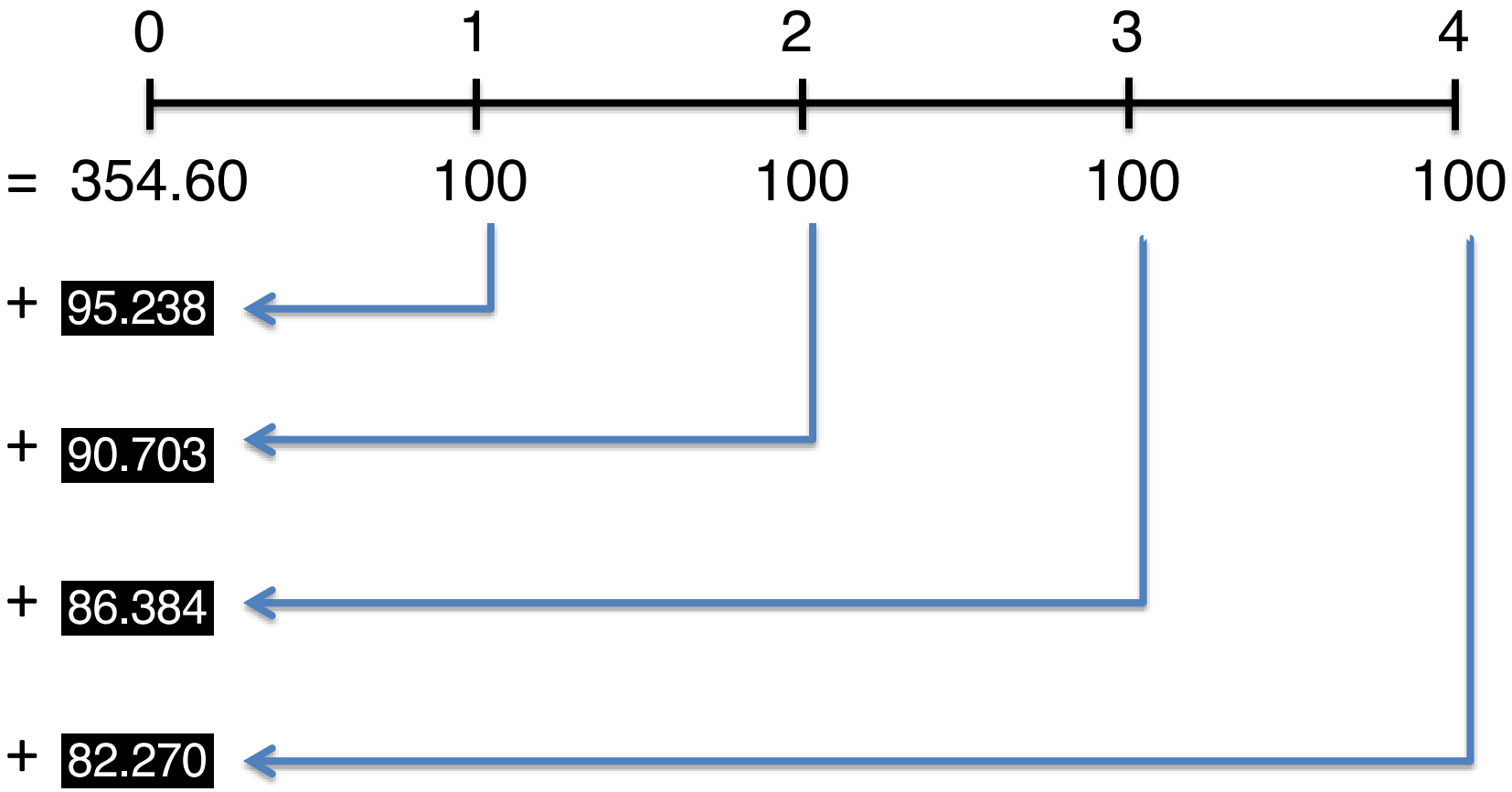
Example – Savings

Step 2: Move CFs back in time to today

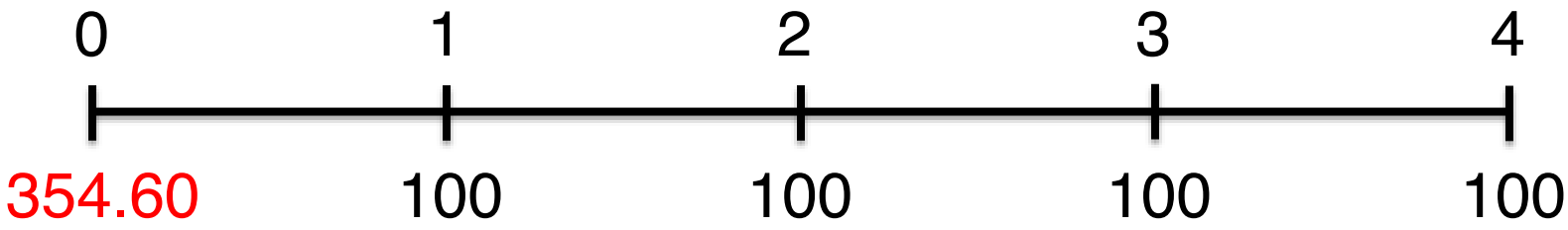


Example – Savings

Step 3: Add up CFs (all in time 0 units)

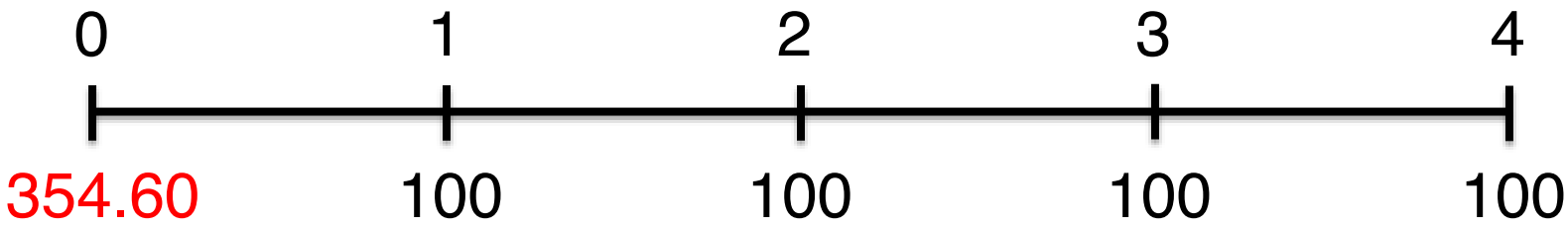


Example – Savings



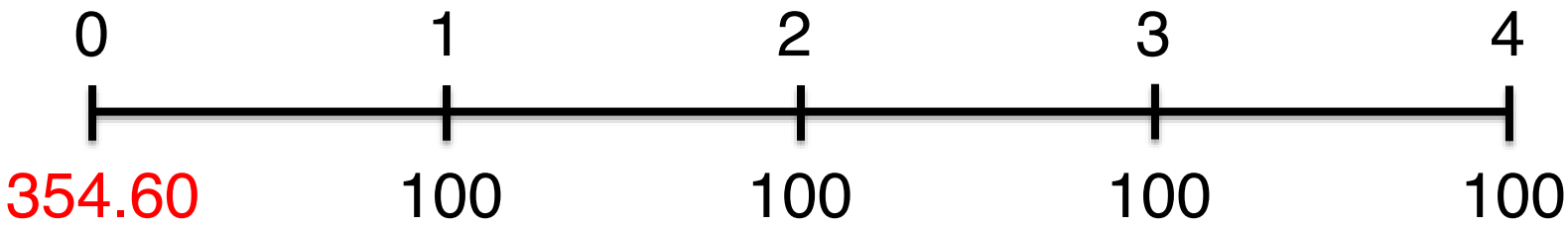
Interpretation 1: We need $354.60 today in an account earning 5% each year so that we can withdraw $100 at the end of each of the next four years

Example – Savings



Interpretation 2: The present value of $100 received at the end of each of the next four years is $354.60 when the discount rate is 5%.

Example – Savings



Interpretation 3: Today's price for a contract that pays $100 at the end of each of the next four years is $354.60 when the discount rate is 5%.

Comment: We are assuming that the discount rate, R, is constant over time.

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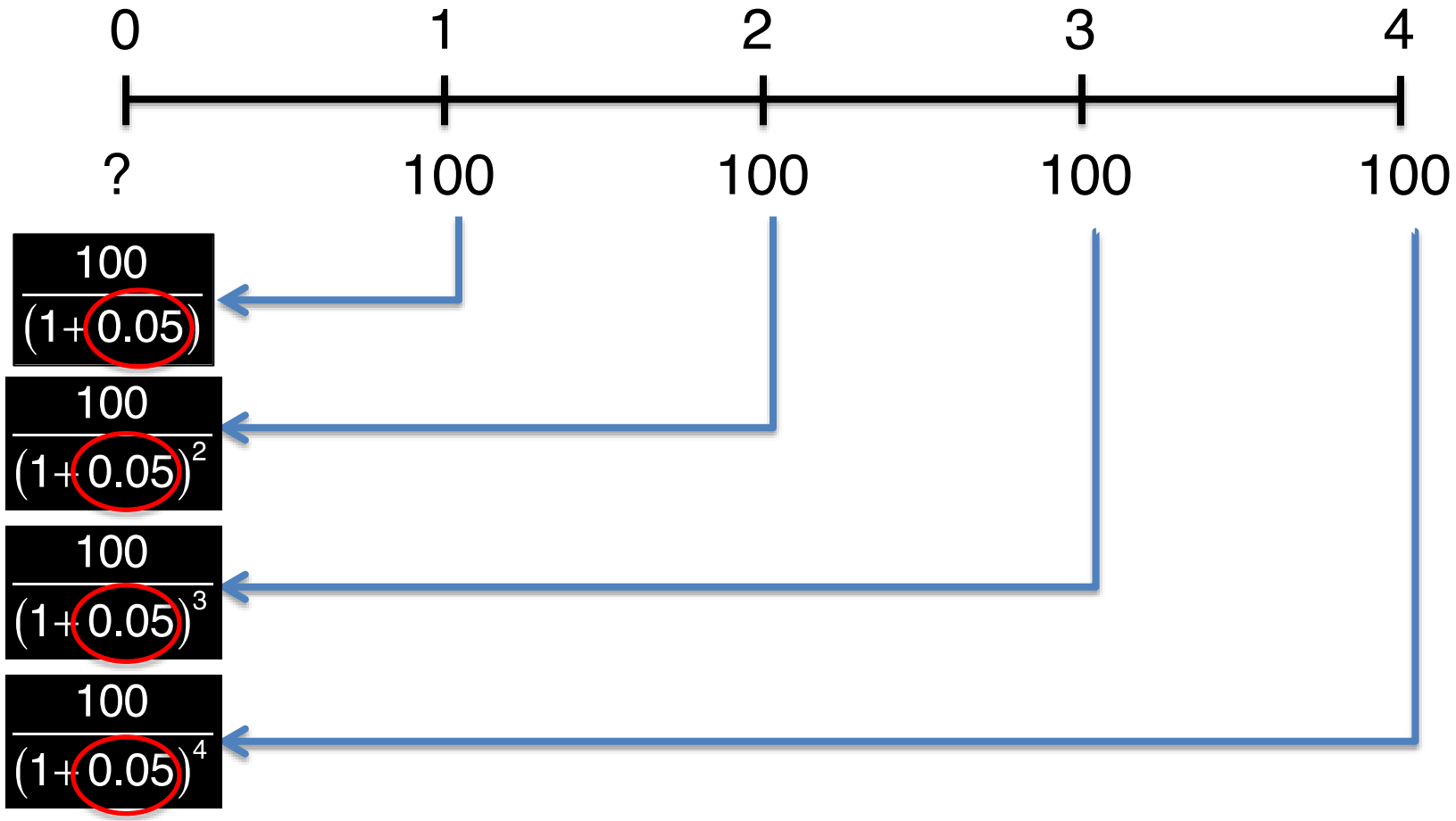

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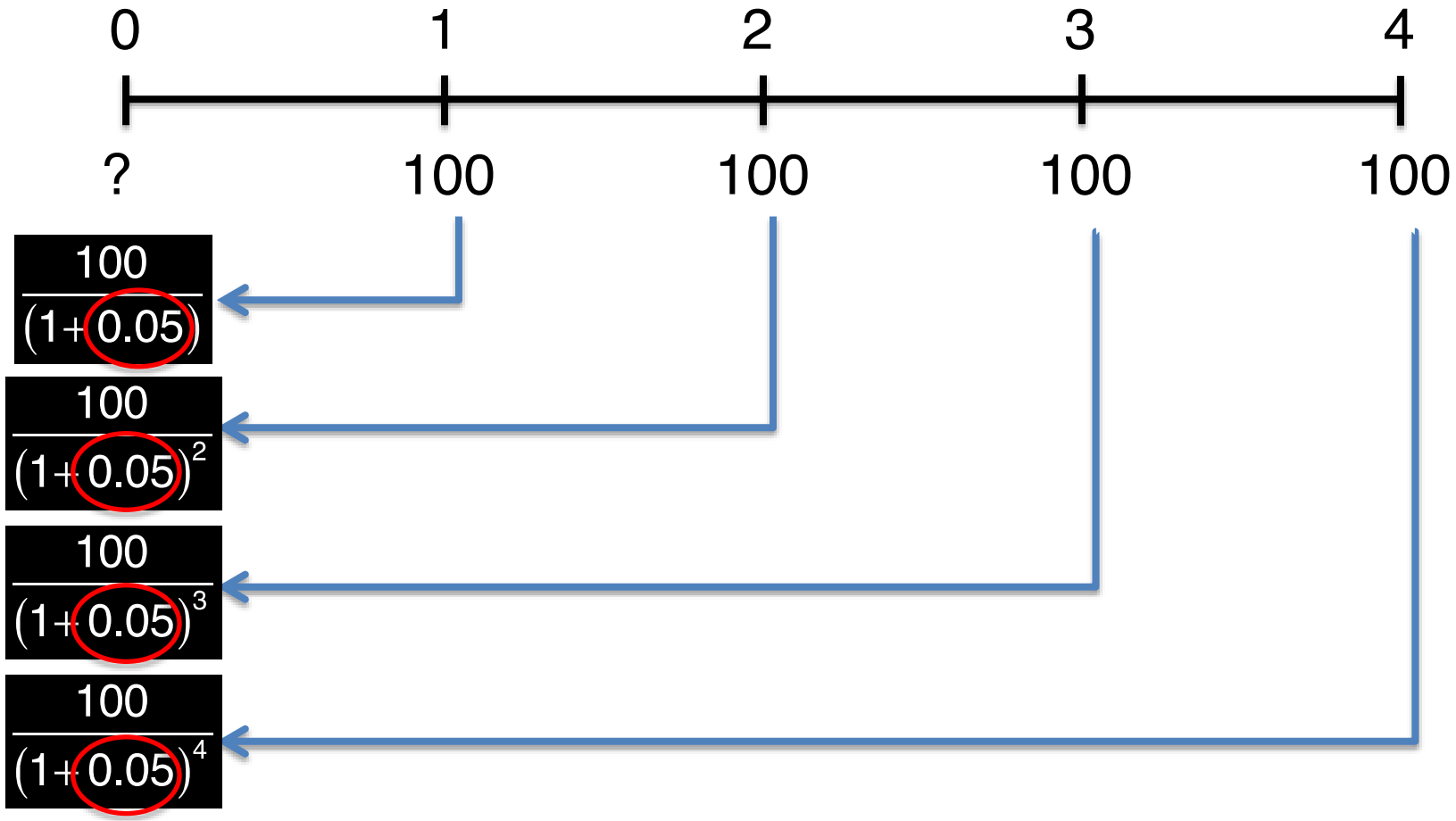

Common assumption but still an *assumption*

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawl Balance
0				$354.60

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Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73		354.60×0.05	

*Activity happens at end of the period

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32		

=

354.60 + 17.73

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32		

=

$$PV_0(\$372.32) = \$372.32 \times (1+0.05)^{-1} = \$354.60$$

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32	$100.00	

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawl Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32

=

372.32 – 100

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32
2	$13.62	$285.94	$100.00	$185.94
3	$9.30	$195.24	$100.00	$95.24
4	$4.76	$100.00	$100.00	$0.00

Summary

Lessons

- Never add/subtract cash flows from different time periods

- Use (i.e., multiply by) discount factor to change cash flows' time units

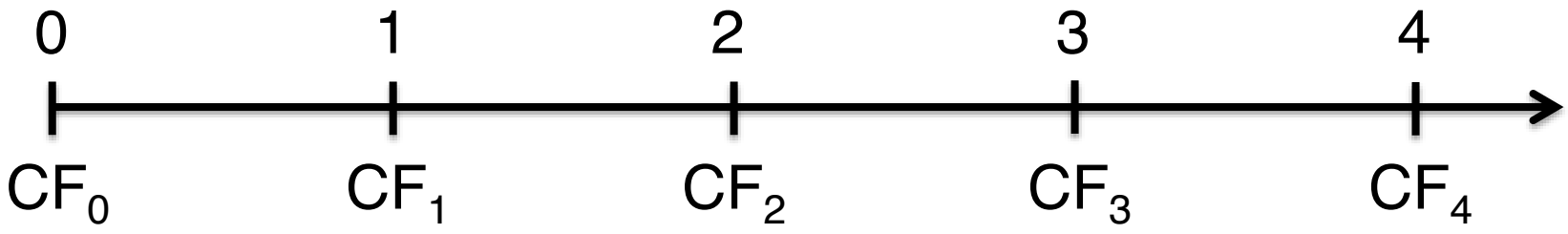
$$(1+R)^t$$

$t < 0$ moves CF back in time (discounting)

$t > 0$ moves CF forward in time (compounding)

Lessons

- Use a time line to help formulate problems



Lessons

- Present value as of time s of a cash flow at time $t > s$ is denoted, PV_s (CF_t)
  - Tells us the value future cash flows
  - Tells us the price of a claim to those cash flows

Coming up next

- Compounding