

Time Value of Money: Inflation

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Last Time Time Value of Money

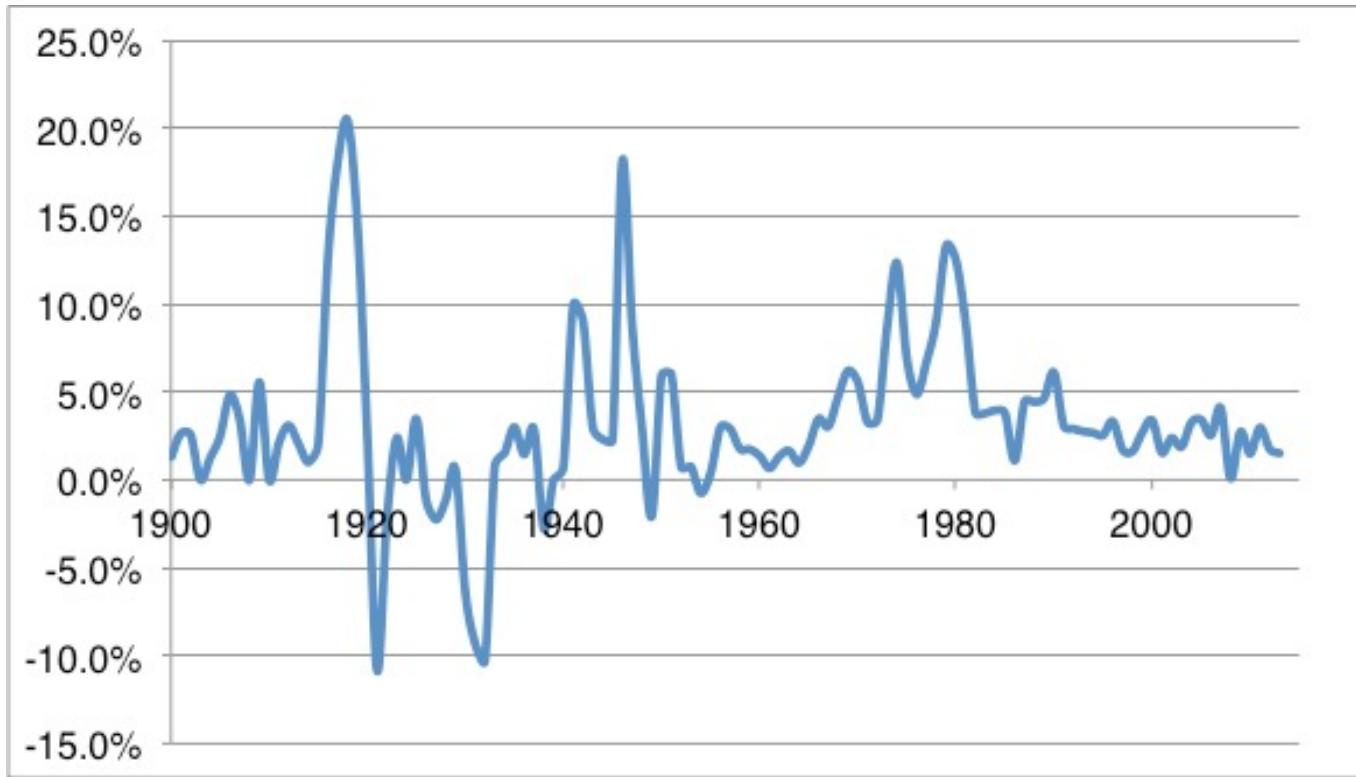
- Taxes

This Time Time Value of Money

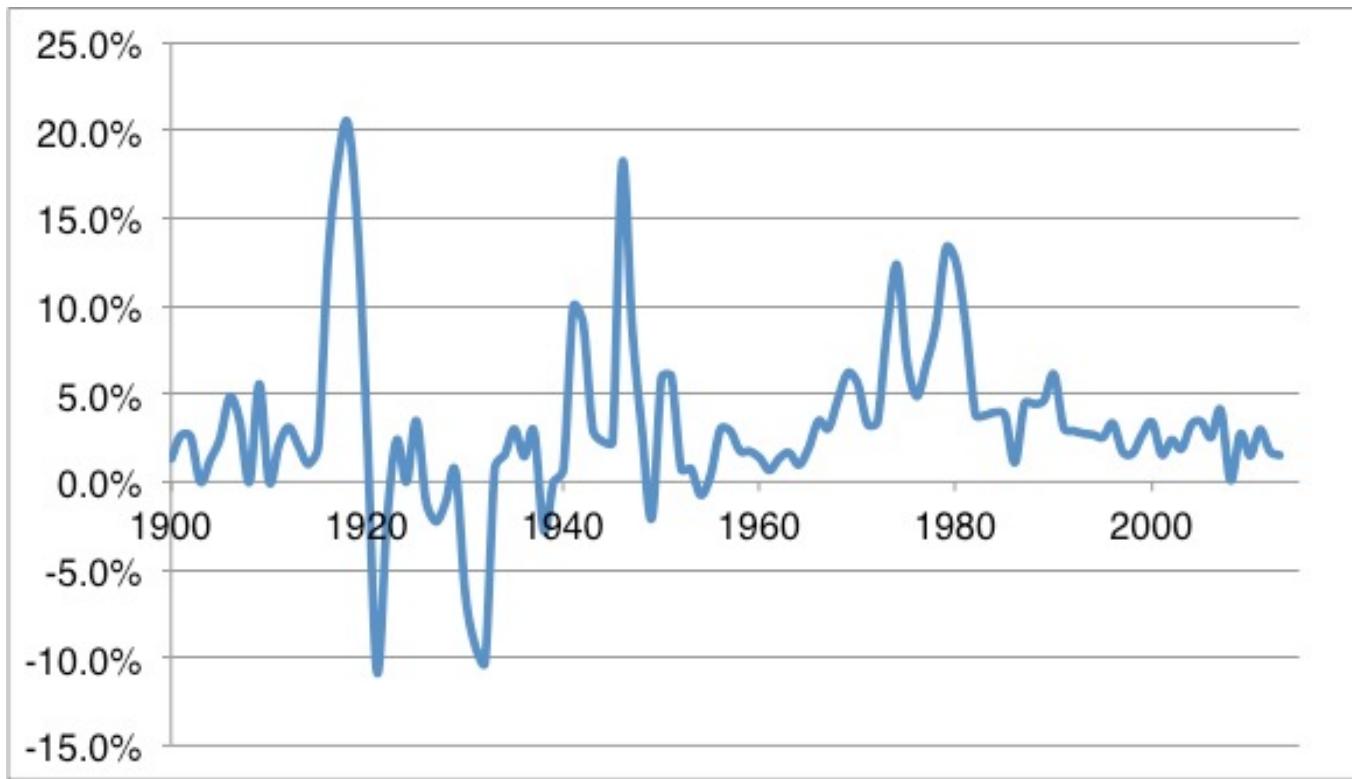
- Inflation

Inflation

Inflation



Inflation



How does inflation impact our returns?

Example – Savings (Account)

| Year | Interest | Pre-Withdrawal | | Post-Withdrawal | |
|------|----------|----------------|------------|-----------------|------------|
| | | Balance | Withdrawal | Balance | Withdrawal |
| 0 | | | | \$354.60 | |
| 1 | \$17.73 | \$372.32 | \$100.00 | \$272.32 | |
| 2 | \$13.62 | \$285.94 | \$100.00 | \$185.94 | |
| 3 | \$9.30 | \$195.24 | \$100.00 | \$95.24 | |
| 4 | \$4.76 | \$100.00 | \$100.00 | \$0.00 | |



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Lesson: Inflation won't affect the money we earn

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Lesson: Inflation will affect what we can buy with the money

Real Discount Rate

$$1 + RR = (1+R) / (1+\pi)$$

RR is the real discount rate

π is expected inflation

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- Commonly used approximation:

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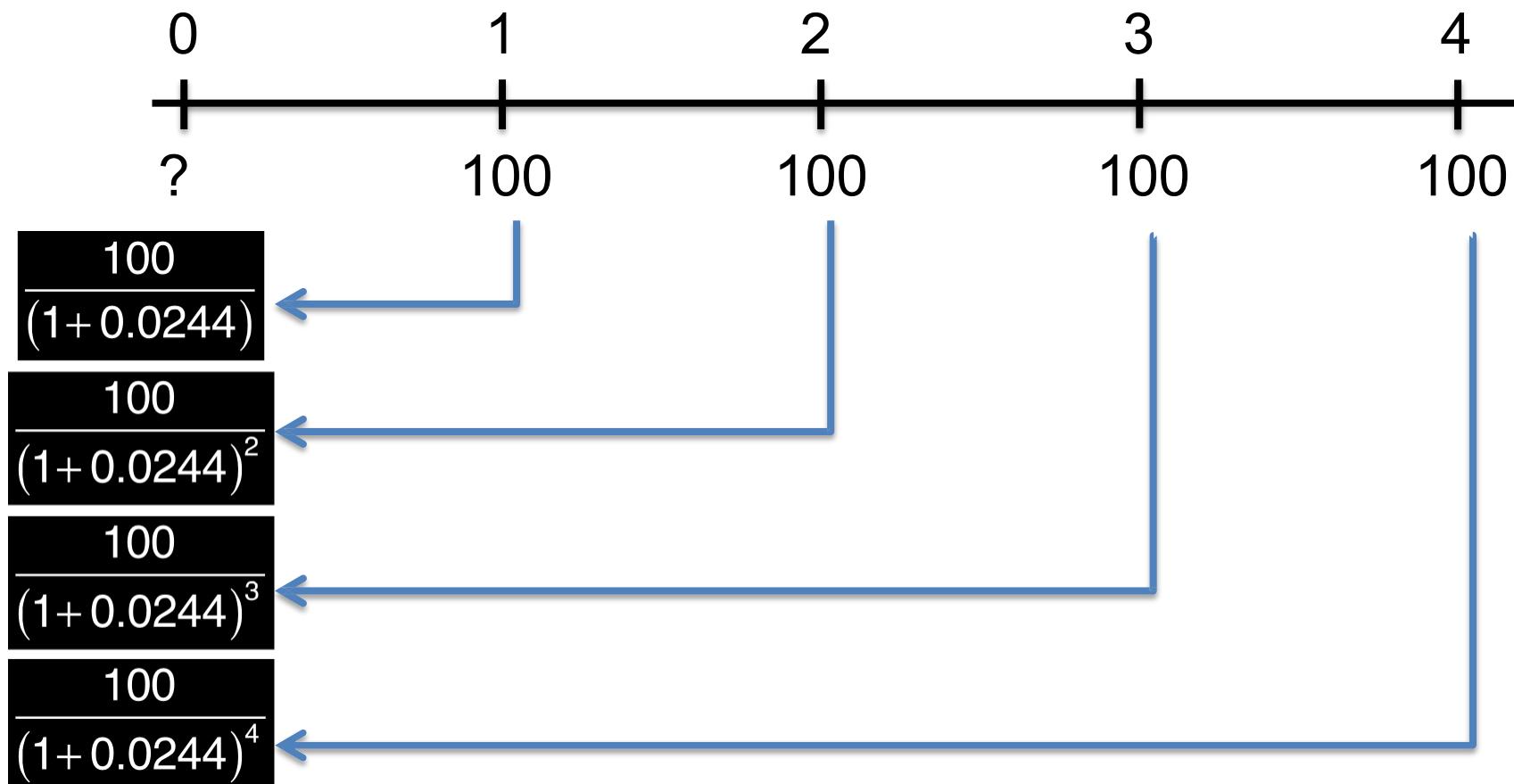
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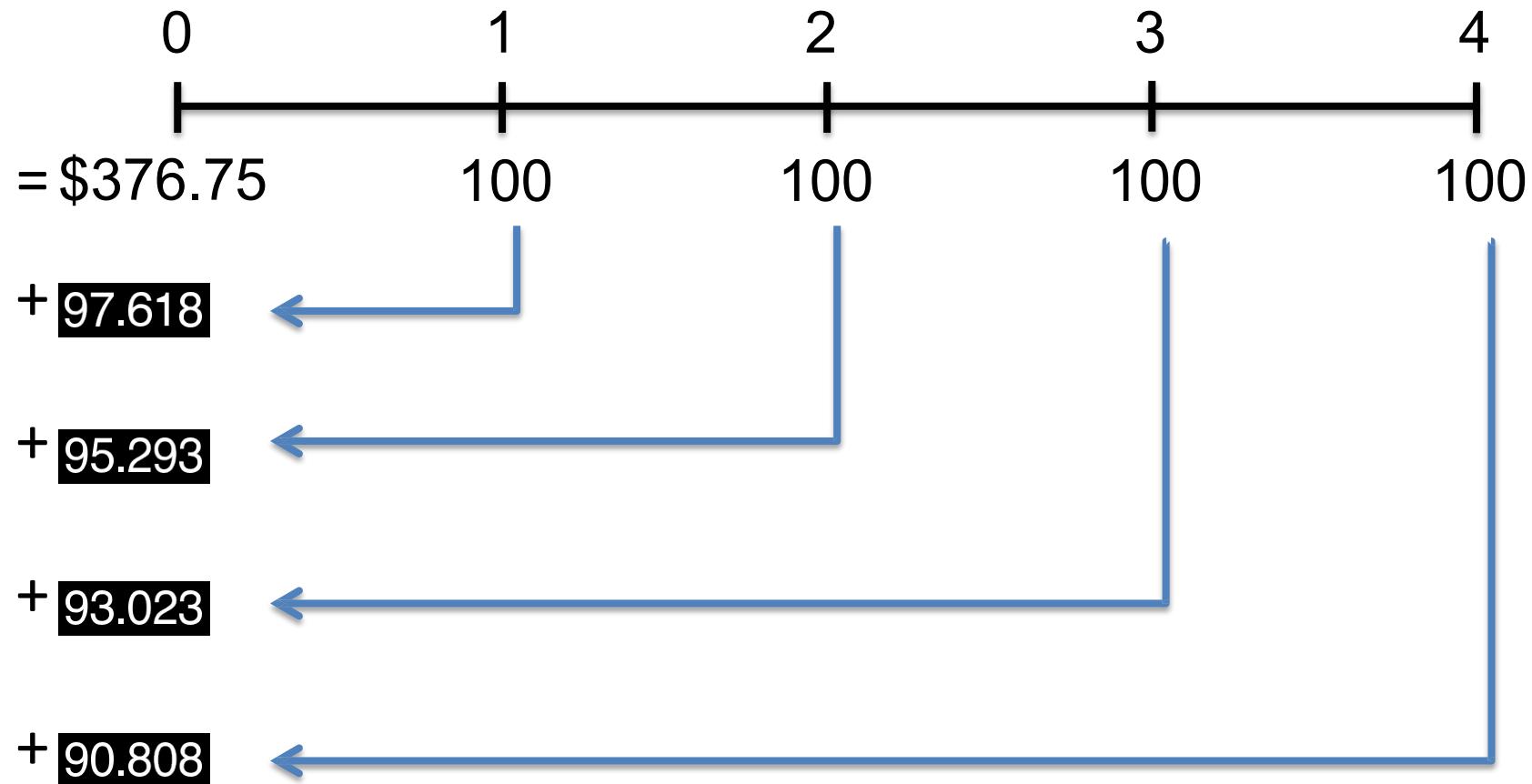
- For our example:

$$RR = (1+0.05) / (1+0.025) - 1 = 2.44\%$$

Savings with Inflation



Savings with Inflation



Savings with Inflation

- Difference:
 - taxes affect \$
 - Inflation affects consumption, not \$
 - Earn nominal return but can't buy as much

Savings with Inflation

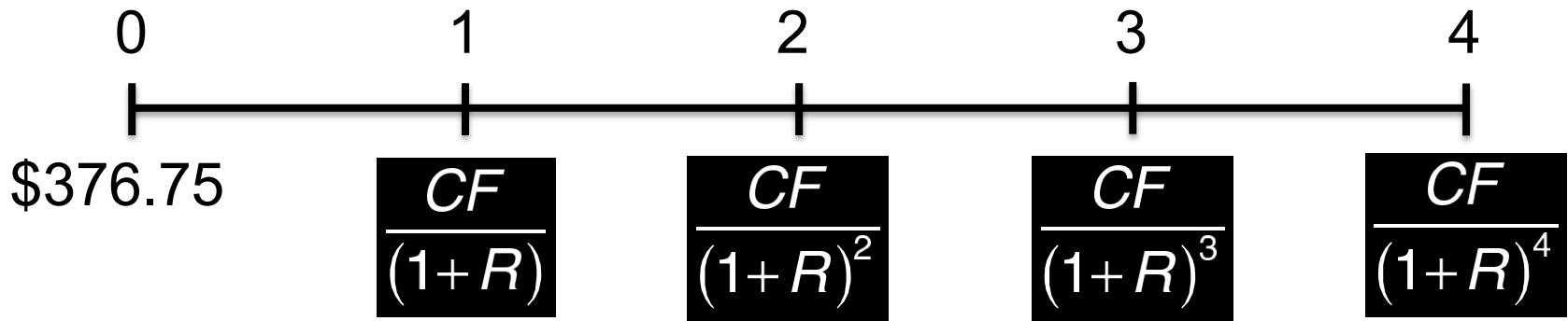
| Year | Pre-Withdrawal | | Post-Withdrawal | |
|------|----------------|----------|-----------------|----------|
| | Interest | Balance | Withdrawal | Balance |
| 0 | | | | \$376.75 |
| 1 | \$18.84 | \$395.59 | \$100.00 | \$295.59 |
| 2 | \$14.78 | \$310.37 | \$100.00 | \$210.37 |
| 3 | \$10.52 | \$220.89 | \$100.00 | \$120.89 |
| 4 | \$6.04 | \$126.93 | \$100.00 | \$26.93 |

Savings with Inflation

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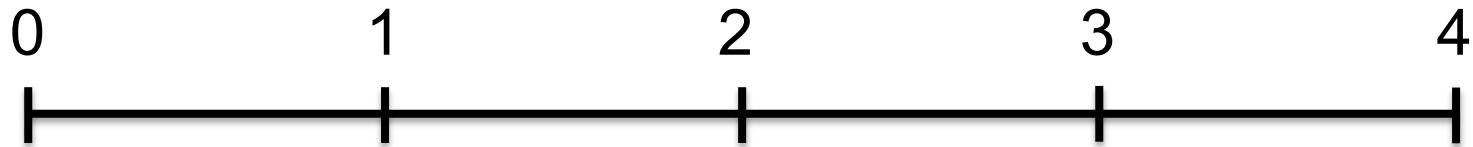
Implication: We have extra money(?).
We need to change withdrawal amount.
(Increase to buy costlier goods.)

Savings with Inflation



What is CF , the amount of money we can withdraw each year?

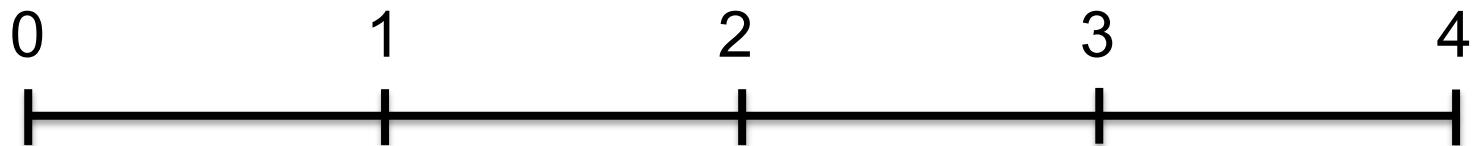
Savings with Inflation



$$\$376.75 = \frac{CF}{(1+0.05)} + \frac{CF}{(1+0.05)^2} + \frac{CF}{(1+0.05)^3} + \frac{CF}{(1+0.05)^4}$$

Use nominal rate since that reflects \$ we earn

Savings with Inflation



$$\$376.75 = \frac{CF}{(1+0.05)} + \frac{CF}{(1+0.05)^2} + \frac{CF}{(1+0.05)^3} + \frac{CF}{(1+0.05)^4}$$

$$\begin{aligned} CF &= \$376.75 \left(\frac{1}{(1+0.05)} + \frac{1}{(1+0.05)^2} + \frac{1}{(1+0.05)^3} + \frac{1}{(1+0.05)^4} \right)^{-1} \\ &= \$106.25 \end{aligned}$$

Savings with Inflation

| Year | Pre-Withdrawal | | Post-Withdrawal | |
|------|----------------|----------|-----------------|----------|
| | Interest | Balance | Withdrawal | Balance |
| 0 | | | | \$376.75 |
| 1 | \$18.84 | \$395.59 | \$106.25 | \$289.34 |
| 2 | \$14.47 | \$303.81 | \$106.25 | \$197.56 |
| 3 | \$9.88 | \$207.44 | \$106.25 | \$101.19 |
| 4 | \$5.06 | \$106.25 | \$106.25 | \$0.00 |

Savings with Inflation

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| 3 | \$9.88 | \$207.44 | \$106.25 | \$101.19 | |
| 4 | \$5.06 | \$106.25 | \$106.25 | \$0.00 | |

Ideally withdrawals grow each year to accommodate inflation

Savings with Inflation

| Year | Withdrawal |
|------|---------------------------------------|
| 0 | |
| 1 | $100 \times (1 + 0.025)^1 = \102.50 |
| 2 | $100 \times (1 + 0.025)^2 = \105.06 |
| 3 | $100 \times (1 + 0.025)^3 = \107.69 |
| 4 | $100 \times (1 + 0.025)^4 = \110.38 |

This sequence of withdrawals maintains purchasing power of \$100 in today's terms

Savings with Inflation

| Year | Withdrawal |
|------|---------------------------------------|
| 0 | |
| 1 | $100 \times (1 + 0.025)^1 = \102.50 |
| 2 | $100 \times (1 + 0.025)^2 = \105.06 |
| 3 | $100 \times (1 + 0.025)^3 = \107.69 |
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These are “nominal” values corresponding to the real \$100 purchasing power in year 0.

Savings with Inflation

| Year | Withdrawal |
|-----------------------------------|------------|
| 0 | |
| 1 | \$102.50 |
| 2 | \$105.06 |
| 3 | \$107.69 |
| 4 | \$110.38 |
| PV at 5% discount rate = \$376.75 | |

We discount nominal cash flows by the nominal rate to get the price.

Savings with Inflation

| Year | Withdrawal |
|-----------------------------------|------------|
| 0 | |
| 1 | \$102.50 |
| 2 | \$105.06 |
| 3 | \$107.69 |
| 4 | \$110.38 |
| PV at 5% discount rate = \$376.75 | |

Note: PV of nominal CFs at nominal discount rate = PV of real cash flows at real rate

Savings with Inflation

| Year | Withdrawal |
|-----------------------------------|------------|
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| 1 | \$102.50 |
| 2 | \$105.06 |
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| PV at 5% discount rate = \$376.75 | |

Intuition: The inflation term in the numerator and denominator cancel

Savings with Inflation

| Year | Pre-Withdrawal | | Post-Withdrawal | |
|------|----------------|----------|-----------------|----------|
| | Interest | Balance | Withdrawal | Balance |
| 0 | | | | \$376.75 |
| 1 | \$18.84 | \$395.59 | \$102.50 | \$293.09 |
| 2 | \$14.65 | \$307.74 | \$105.06 | \$202.68 |
| 3 | \$10.13 | \$212.81 | \$107.69 | \$105.13 |
| 4 | \$5.26 | \$110.38 | \$110.38 | \$0.00 |

Summary

Lessons

- Inflation does not affect \$ return
- Inflation does purchasing power of \$
- Real return, RR

$$RR = \frac{1+R}{1+\pi} - 1 \simeq R - \pi$$

where R is the nominal return and π is the rate of inflation

Lessons

- Discount real cash flows by the real rate of return, nominal cash flows by the nominal rate of return.

Coming up next

- Interest Rates
 - How do we value non-annual and irregular cash flows streams?
 - How do different compounding periods affect our valuations?