Time Value of Money: Intuition and Discounting

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This Time

Time Value of Money

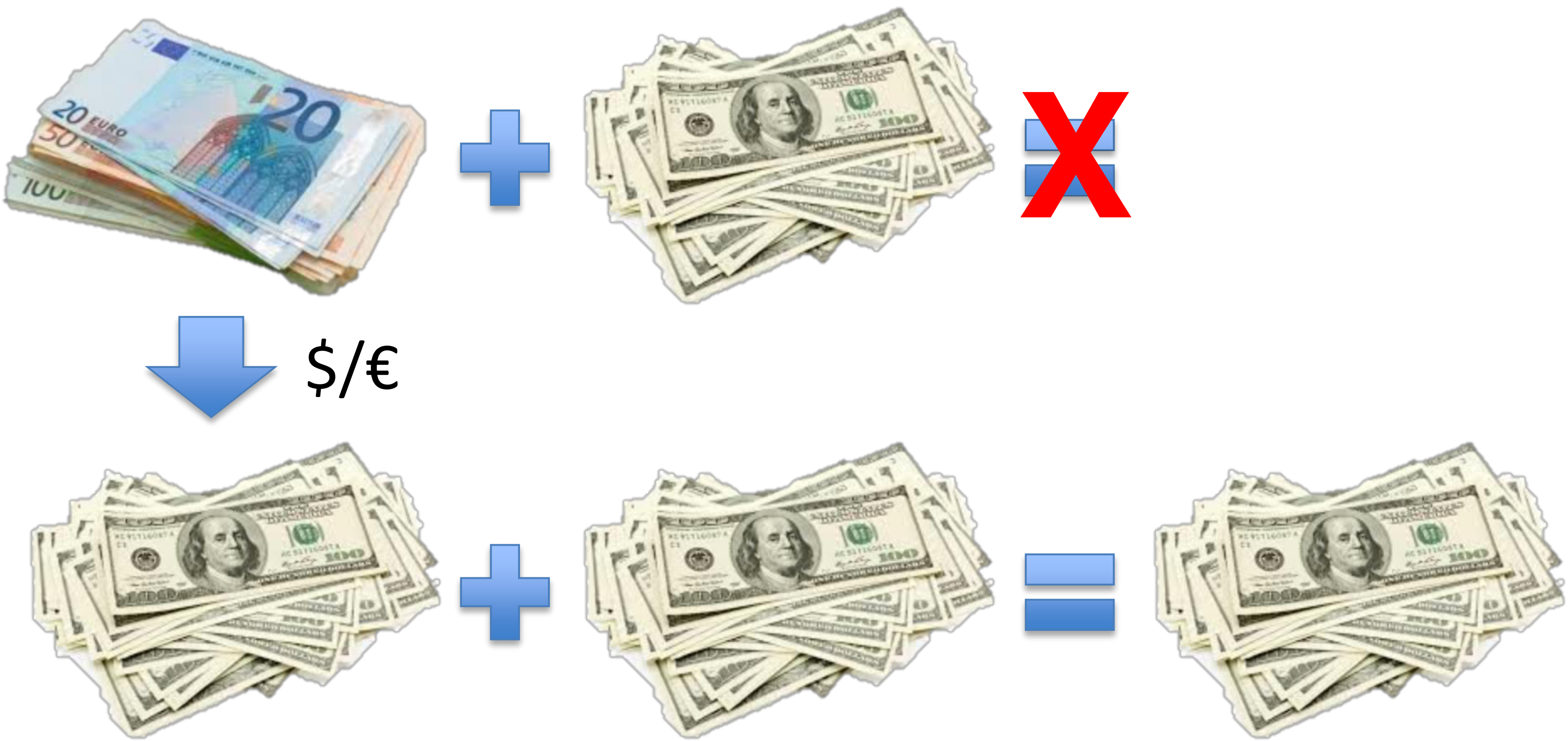
- Intuition, tools, and discounting

Intuition

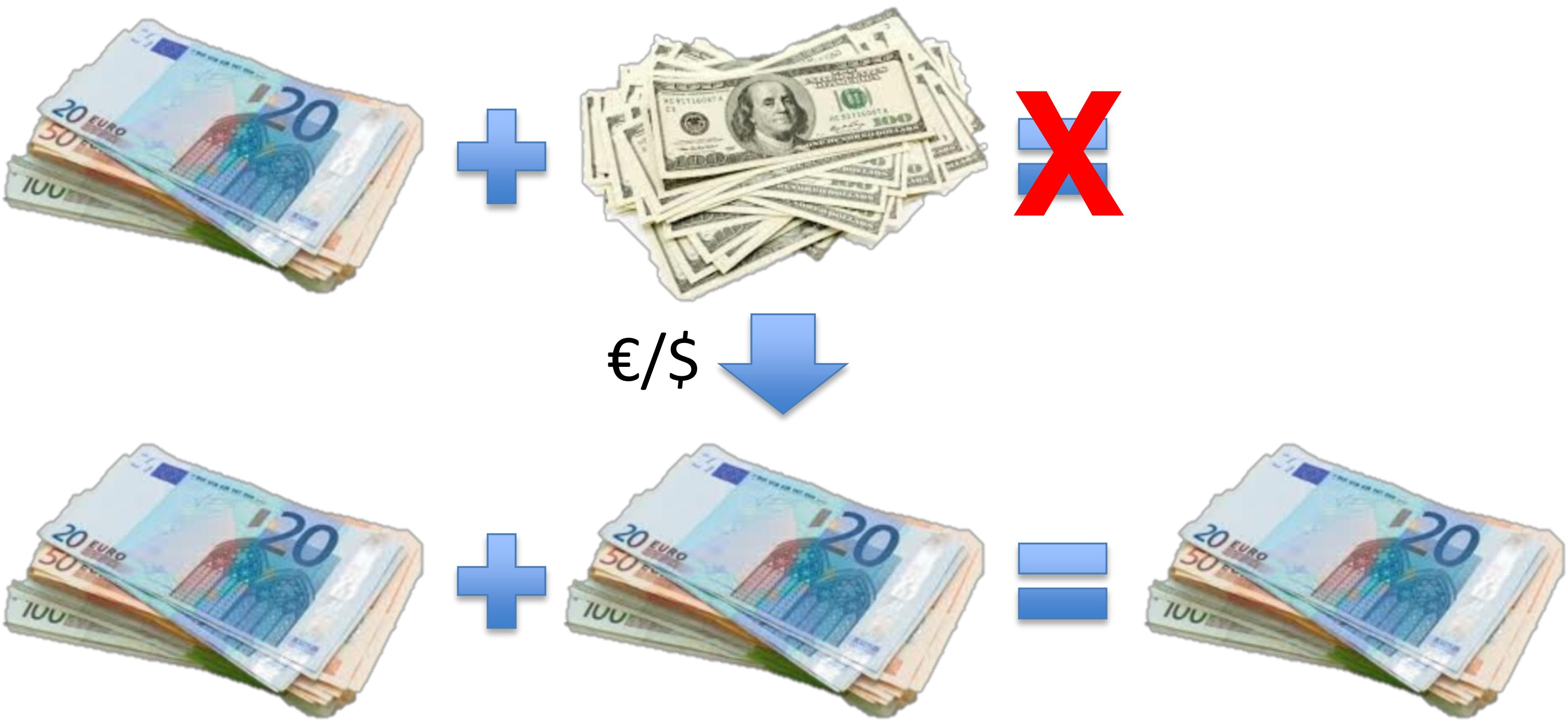
Currency

Currency

Currency



Currency



Currency

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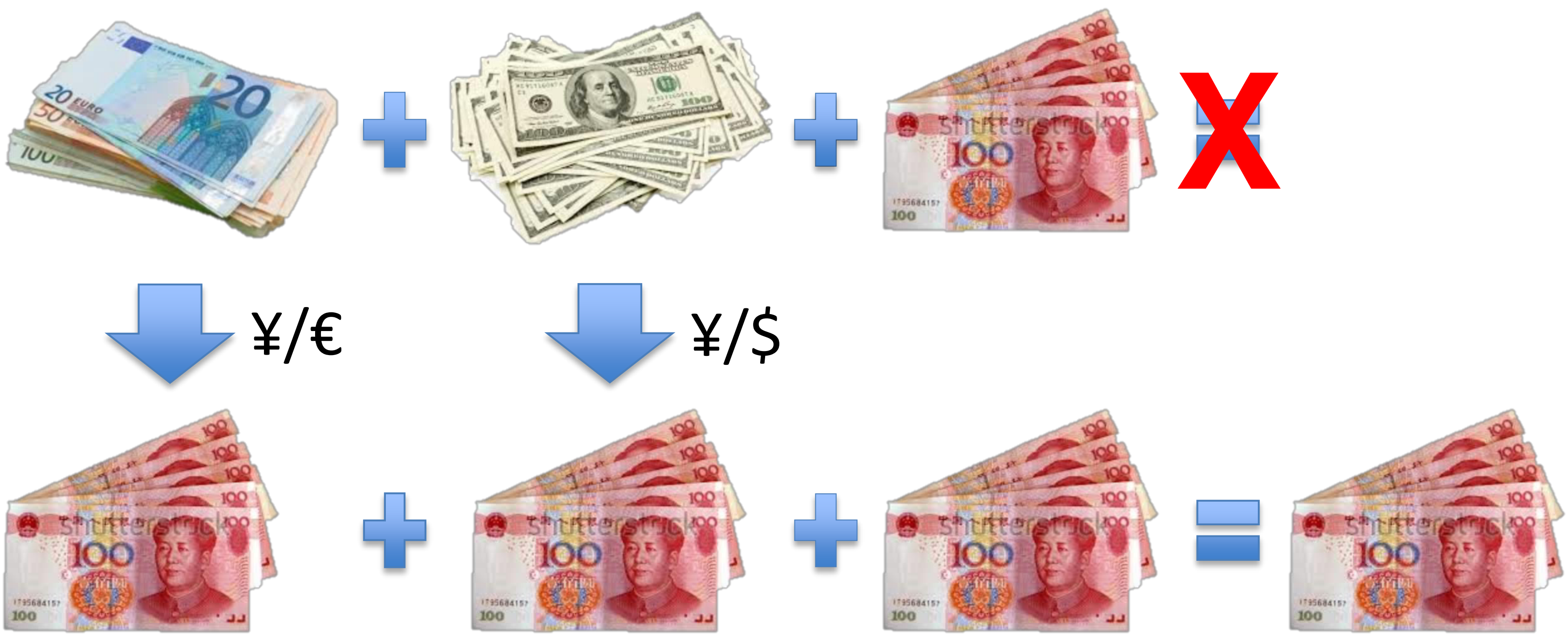

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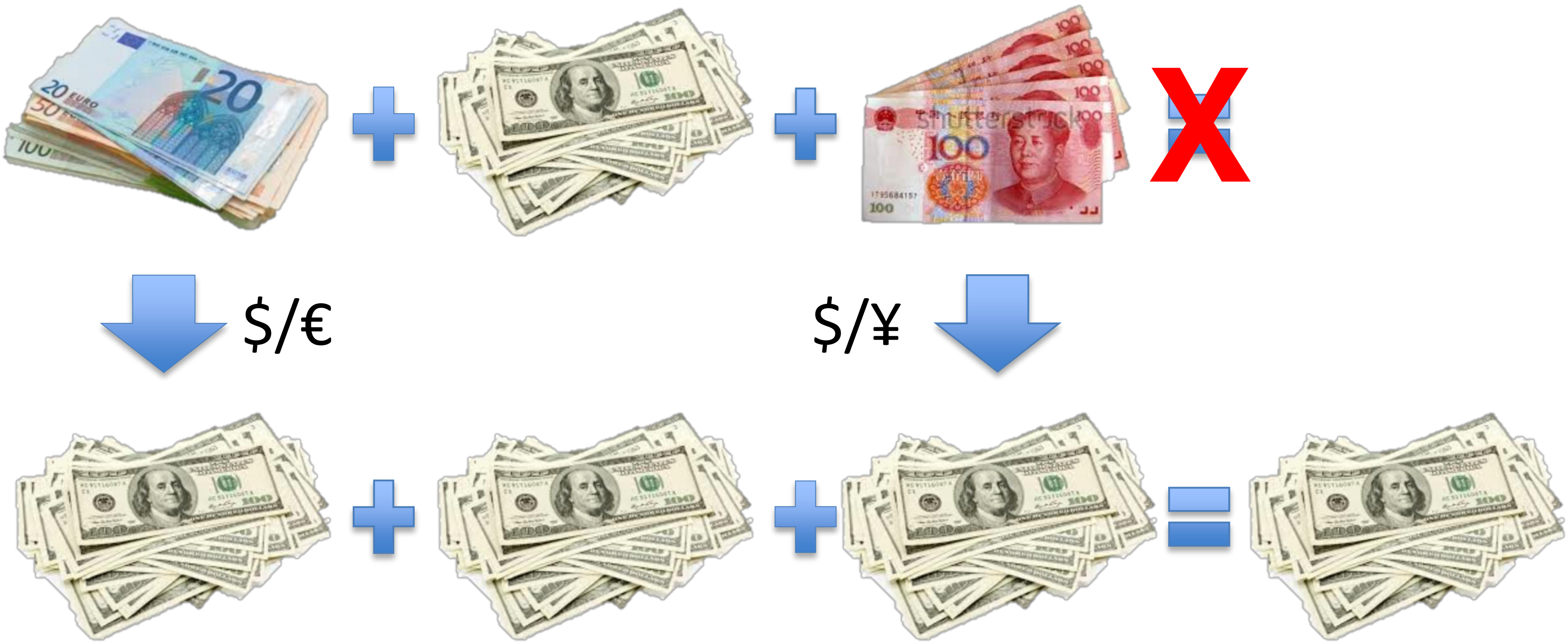

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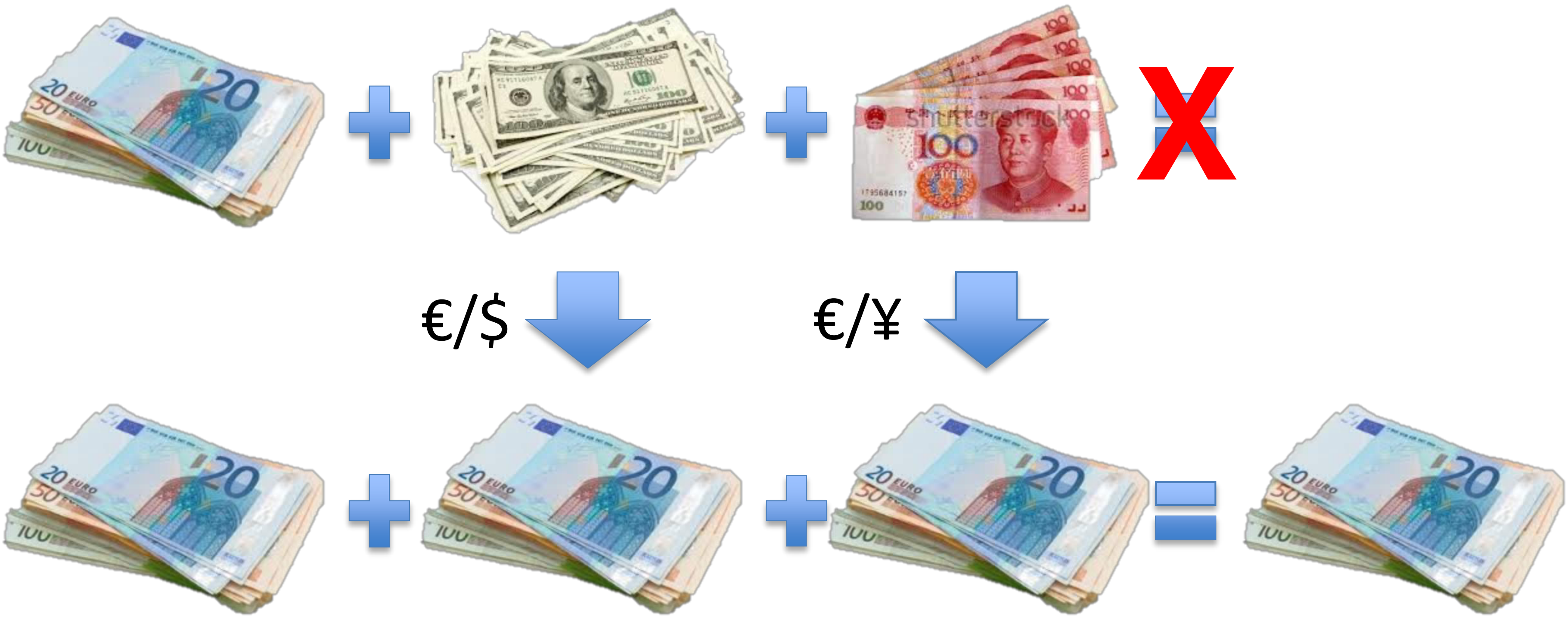


Messages (Look up)

1. Can’t add/subtract different currencies

2. Must convert currencies to common (base) currency using exchange rate

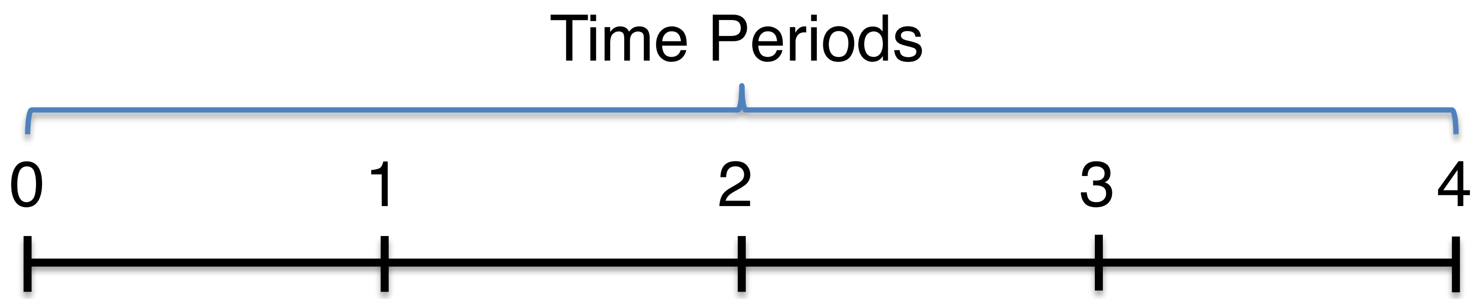
Time Value of Money

Time Value of Money

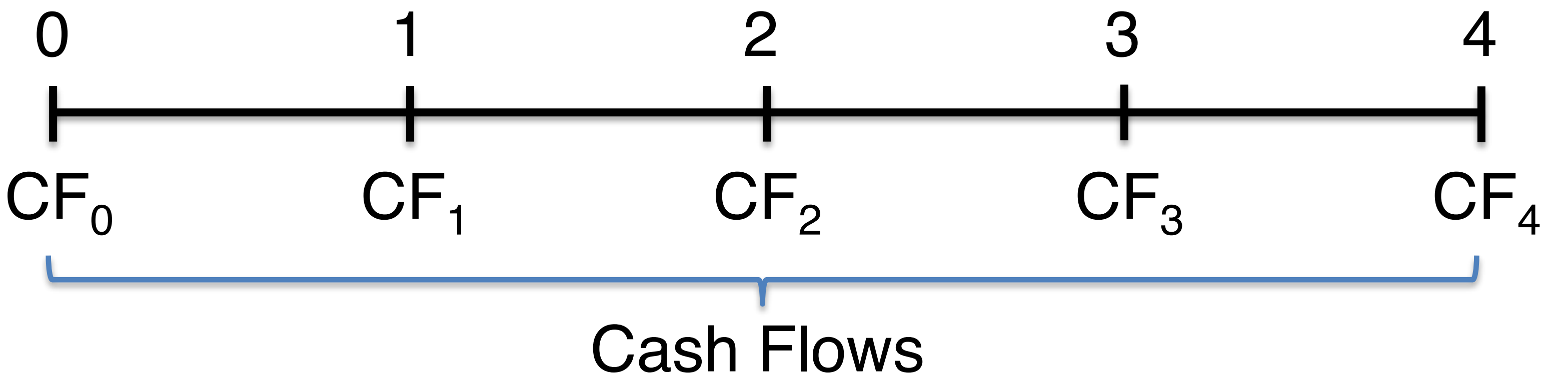
- Money received/paid at different times is like different currencies
  - Money has a time unit
- Must convert to common/base unit to aggregate
  - Need exchange rate for time

THE TOOLS: TIME LINE & DISCOUNT FACTOR

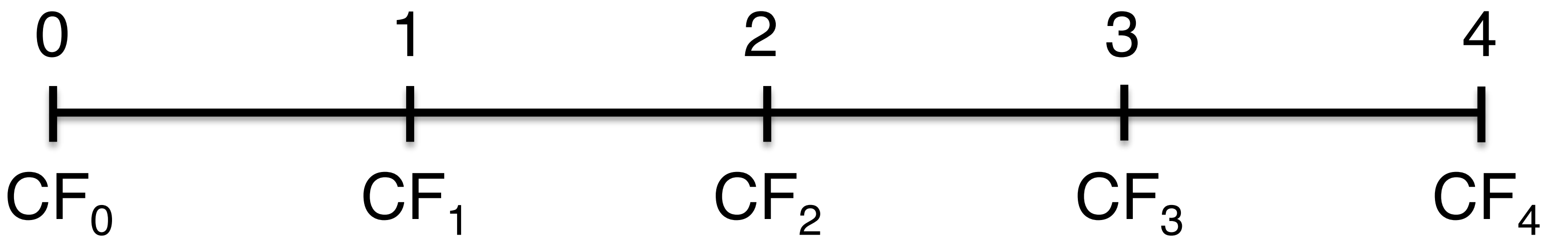
Time Line



Time Line

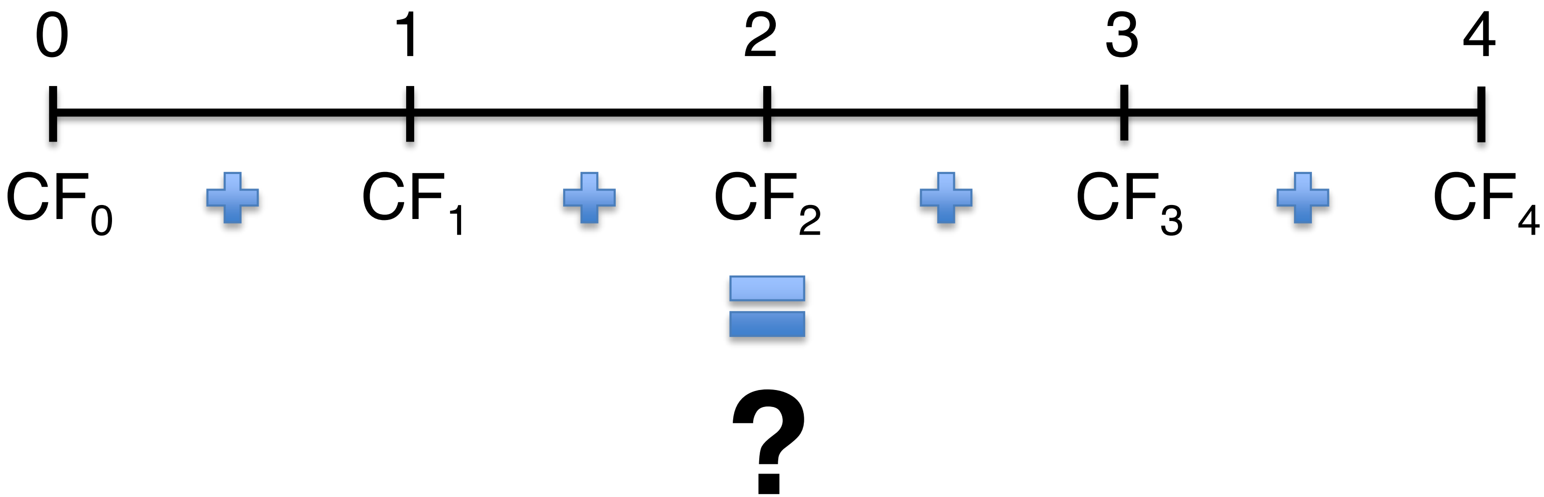


Time Line



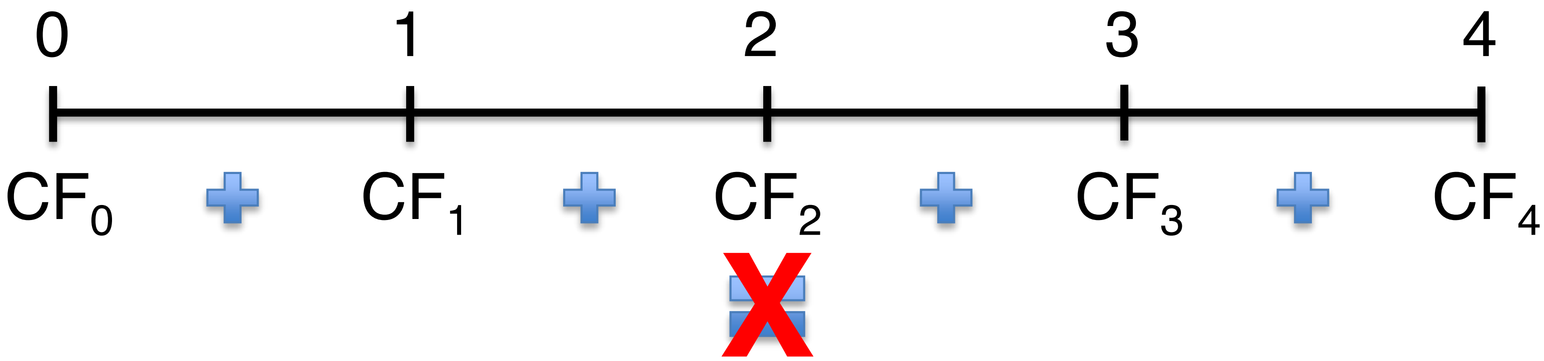
Lesson: Get in the habit of placing cash flows on a time line

Aggregating Cash Flows



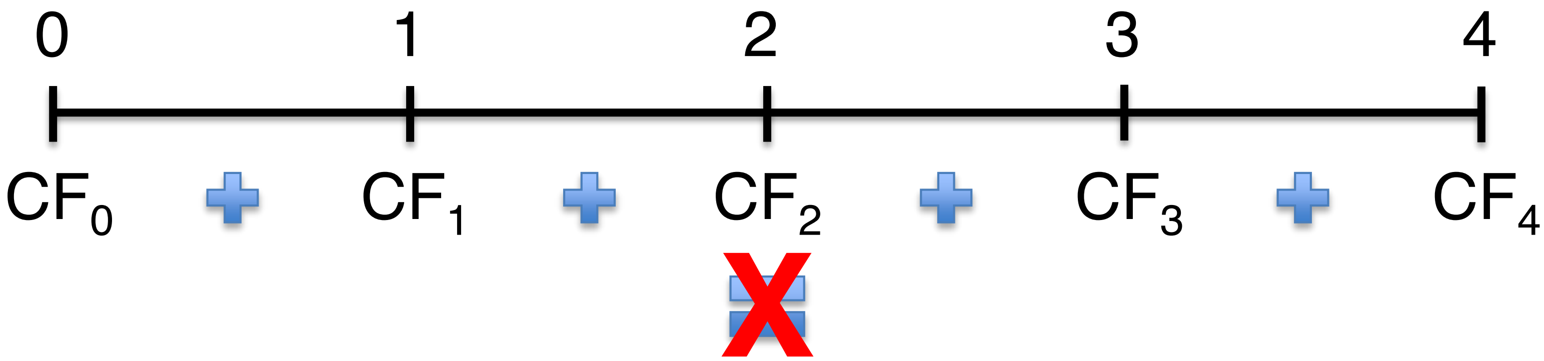
Can we add/subtract cash flows in different time periods

Aggregating Cash Flows



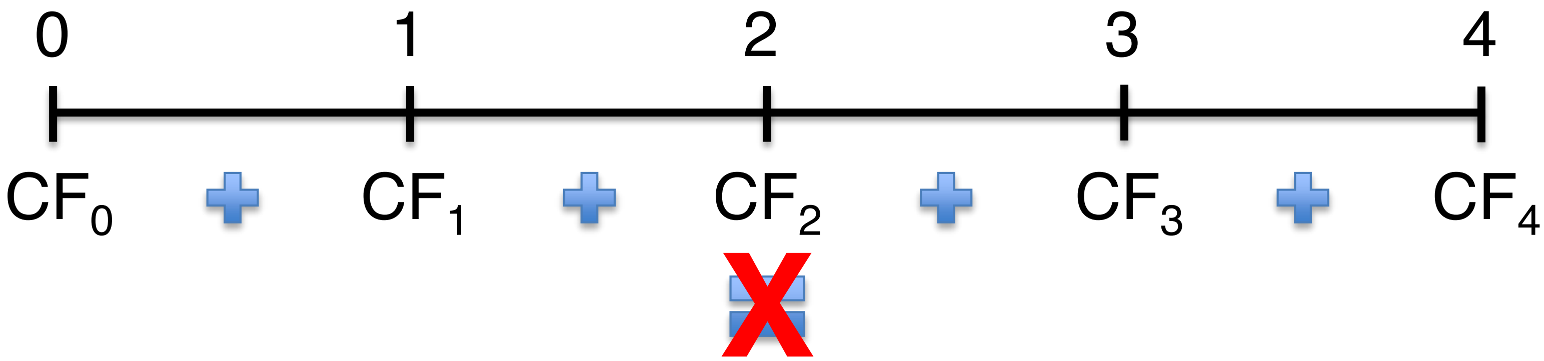
No!

Aggregating Cash Flows



Lesson: Never* add/subtract cash flows received at different times

Aggregating Cash Flows



Need exchange rate for time to convert to common time unit

Discount Factor

The discount factor is our exchange rate for time

$$\left(1+R\right)^{t}$$

t = time periods into future ($t > 0$) or past ($t < 0$) to move CFs

R = …

Definition: R is the rate of return offered by investment alternatives in the capital markets of equivalent risk.

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A.k.a., discount rate, hurdle rate, opportunity cost of capital

To determine *R*, consider the risk of the cash flows that you are discounting.

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Investment	**Average Annual Return, *R***
Treasury-Bills (30-Day)	3.49%
Treasury-Notes (10-Year)	5.81%
Corporate Bonds (Investment Grade)	6.60%
Large-Cap Stocks	11.23%
Mid-Cap Stocks	15.15%
Small-Cap Stocks	25.32%

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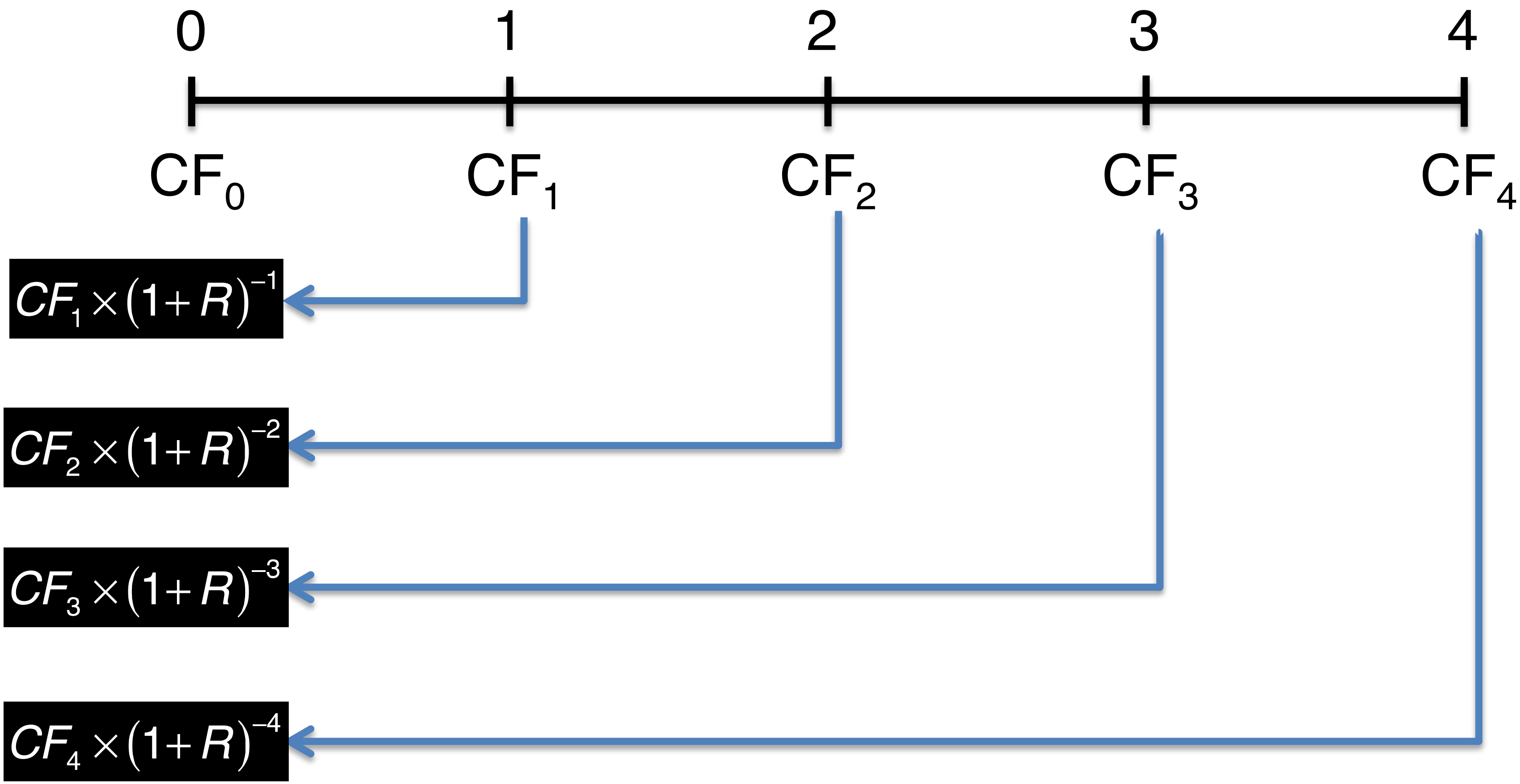
Riskier investment, higher return

USING THE TOOLS: DISCOUNTING

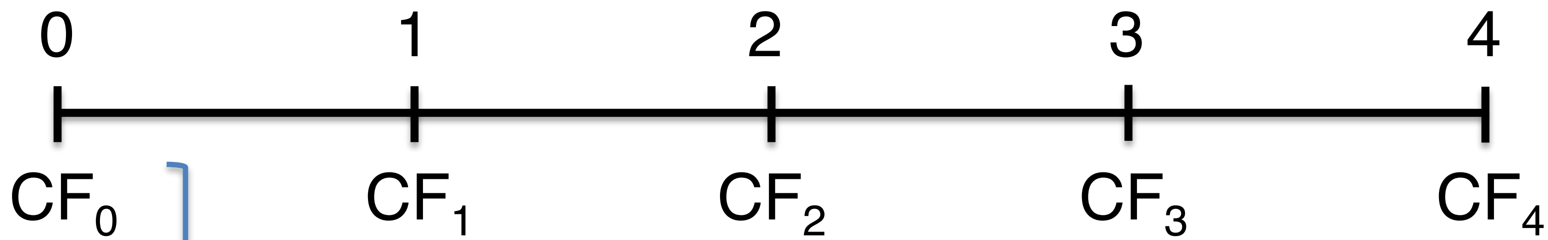
Discounting

Discounting CFs moves them back in time



Discounting

Discounting CFs moves them back in time



$$CF_1 \times (1+R)^{-1}$$

$$CF_2 \times (1+R)^{-2}$$

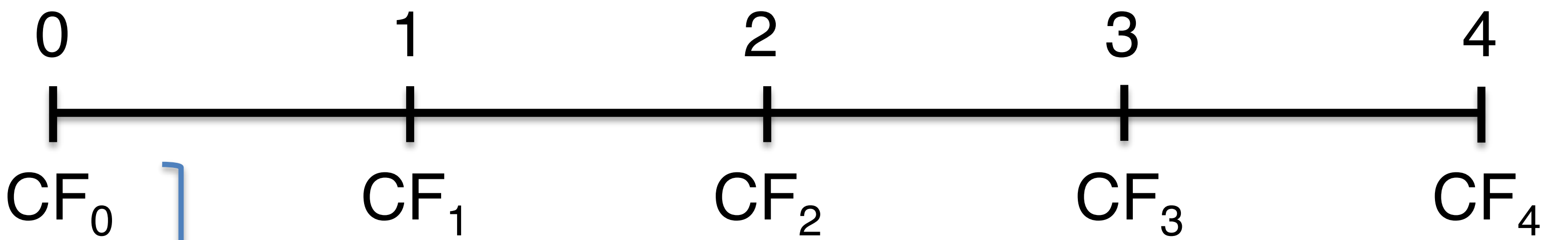
$$CF_3 \times (1+R)^{-3}$$

$$CF_4 \times (1+R)^{-4}$$

$t < 0$ because we are moving cash flows back in time

Discounting

Discounting CFs moves them back in time



$CF_1 \times (1+R)^{-1}$

$CF_2 \times (1+R)^{-2}$

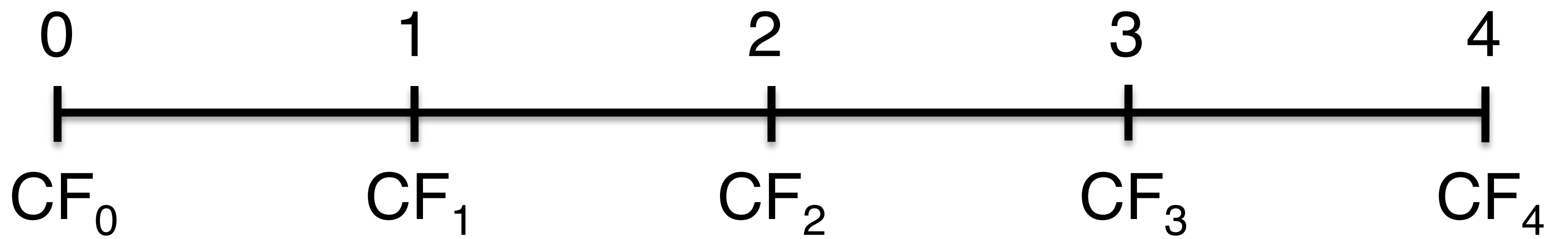
$CF_3 \times (1+R)^{-3}$

$CF_4 \times (1+R)^{-4}$

We can add/subtract these CFs because they are in the same time units (date 0)

Present Value

Present value, $PV_t(\bullet)$ of CFs is discounted value of CFs as of t



$$CF_1 \times (1+R)^{-1} = PV_0(CF_1)$$

$$CF_2 \times (1+R)^{-2} = PV_0(CF_2)$$

$$CF_3 \times (1+R)^{-3} = PV_0(CF_3)$$

$$CF_4 \times (1+R)^{-4} = PV_0(CF_4)$$

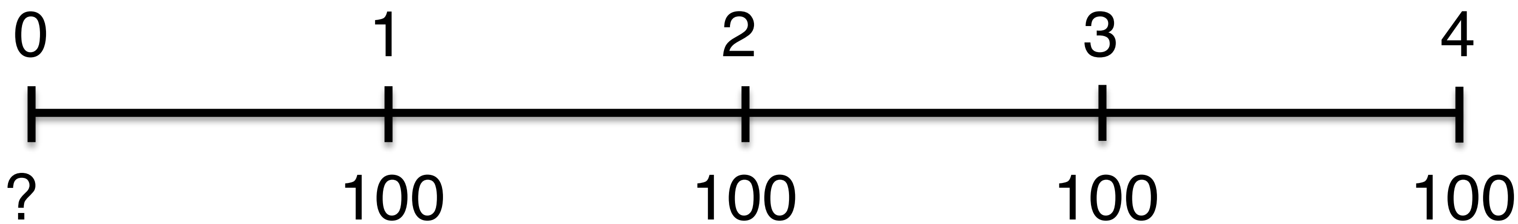
These are present values of future CFs as of today (period 0)

Example – Savings

How much do you have to save today to withdraw $100 at the end of each of the next four years if you can earn 5% per annum?

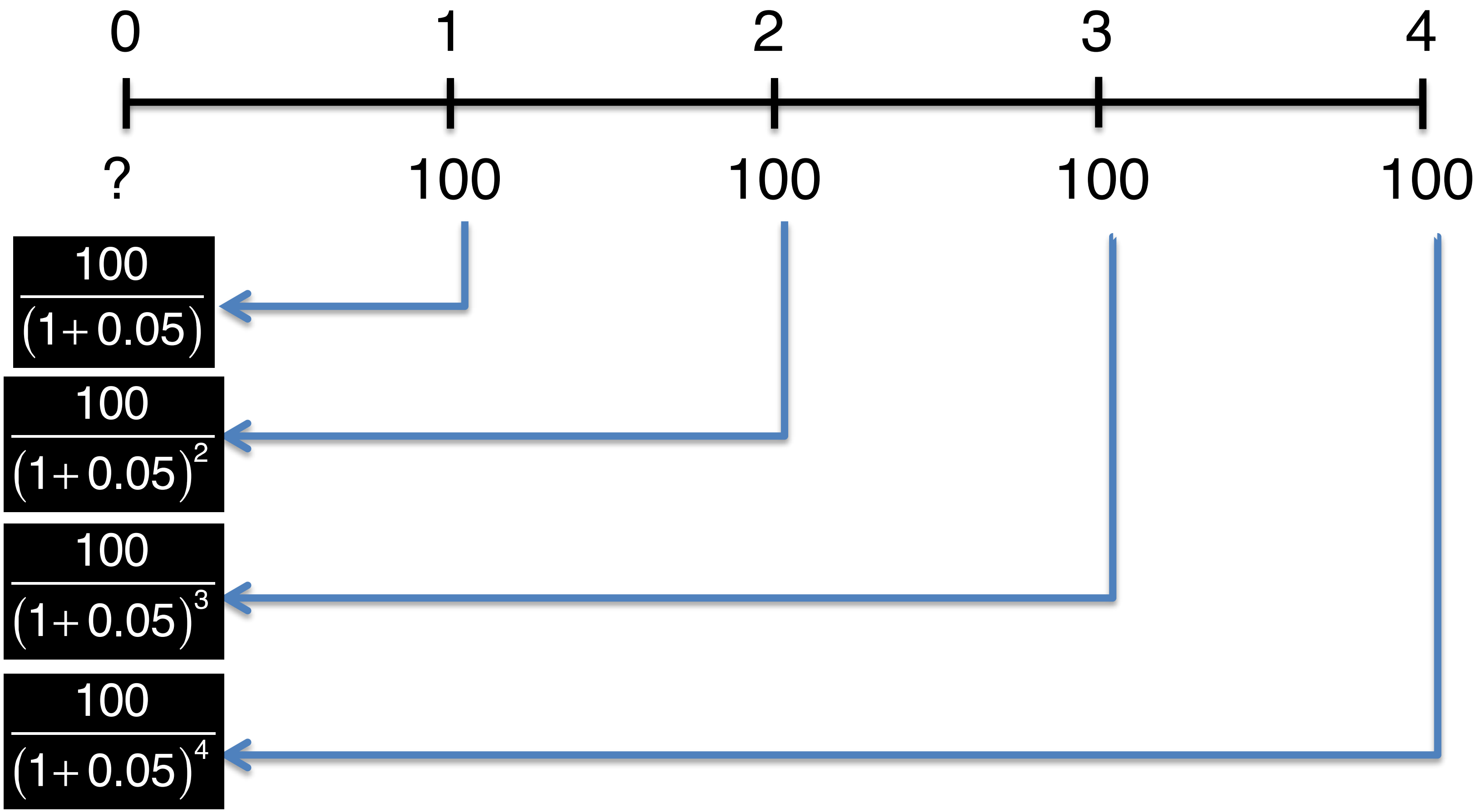
Example – Savings

Step 1: Put cash flows on a time line



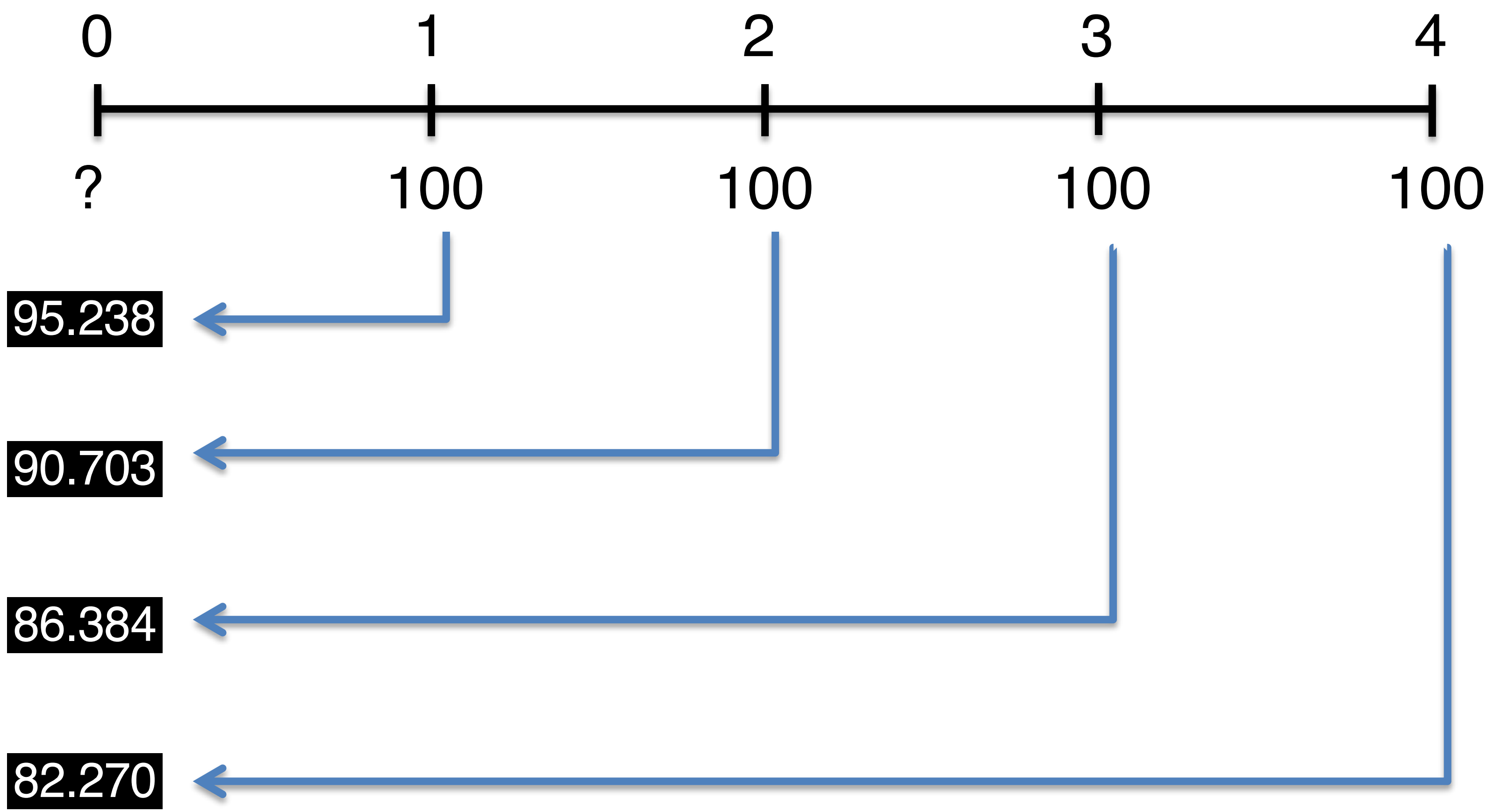
Example – Savings

Step 2: Move CFs back in time to today



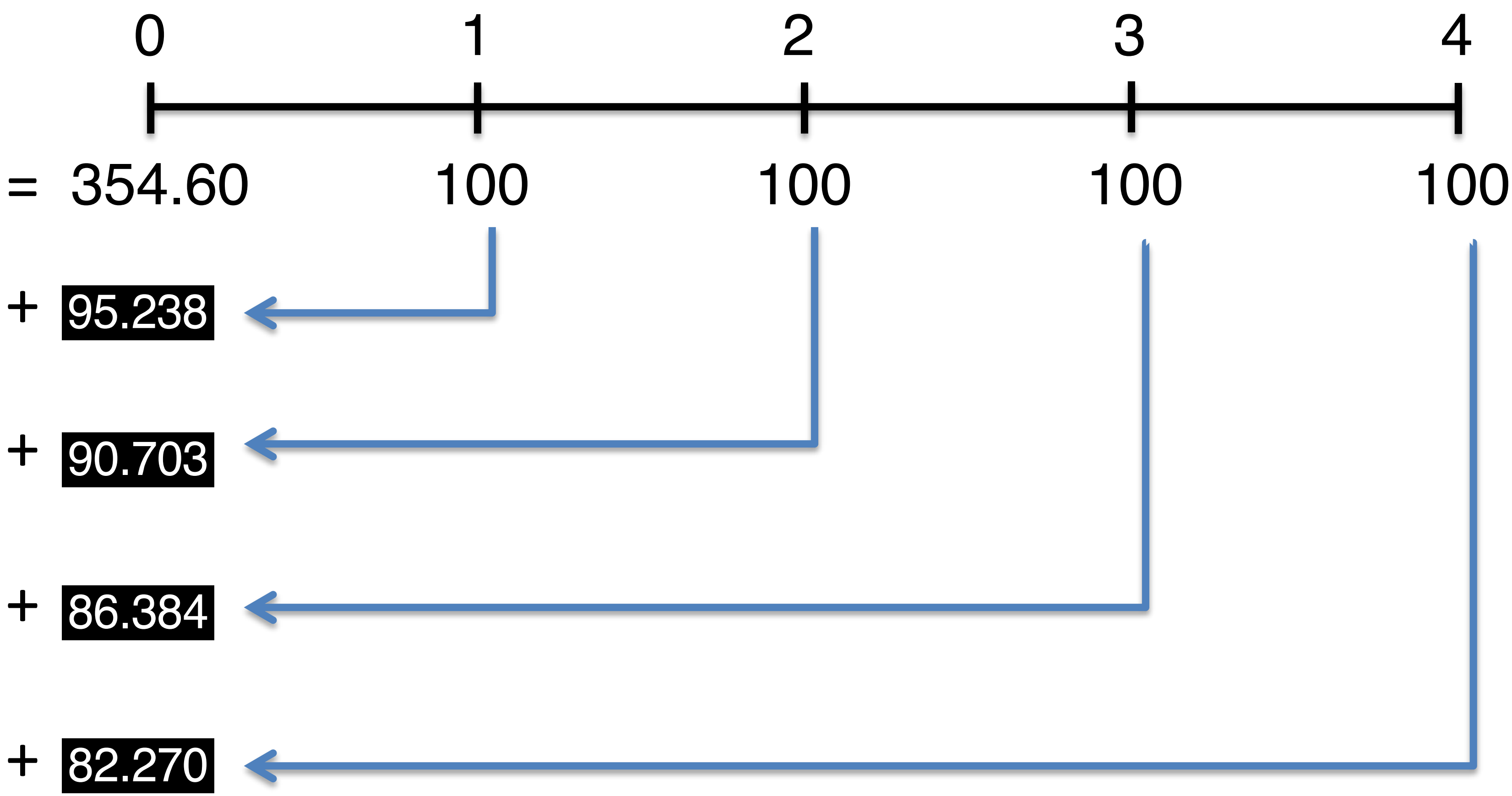
Example – Savings

Step 2: Move CFs back in time to today

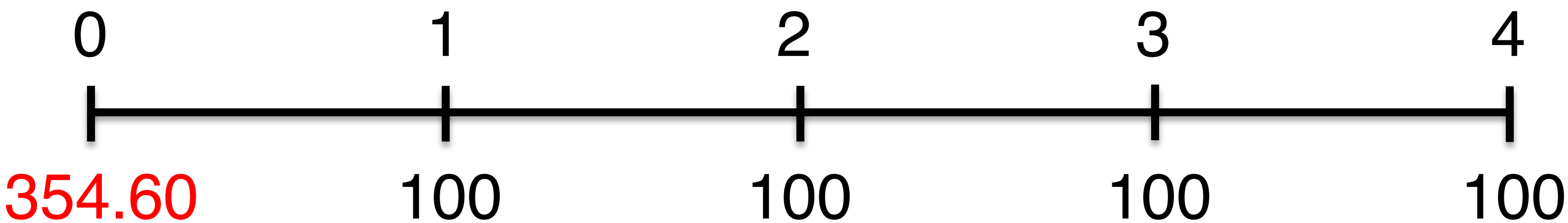


Example – Savings

Step 3: Add up CFs (all in time 0 units)

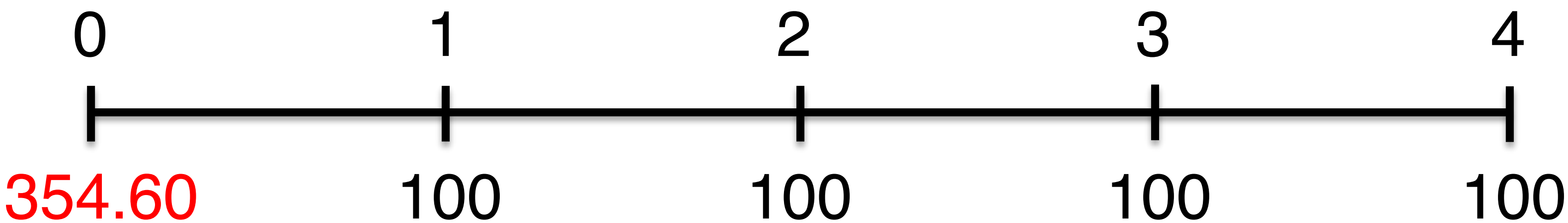


Example – Savings



Interpretation 1: We need $354.60 today in an account earning 5% each year so that we can withdraw $100 at the end of each of the next four years

Example – Savings



Interpretation 2: The present value of $100 received at the end of each of the next four years is $354.60 when the discount rate is 5%.

Example – Savings

0	1	2	3	4
354.60	100	100	100	100

Interpretation 3: Today's price for a contract that pays \$100 at the end of each of the next four years is \$354.60 when the discount rate is 5%.

Comment: We are assuming that the discount rate, R, is constant over time.

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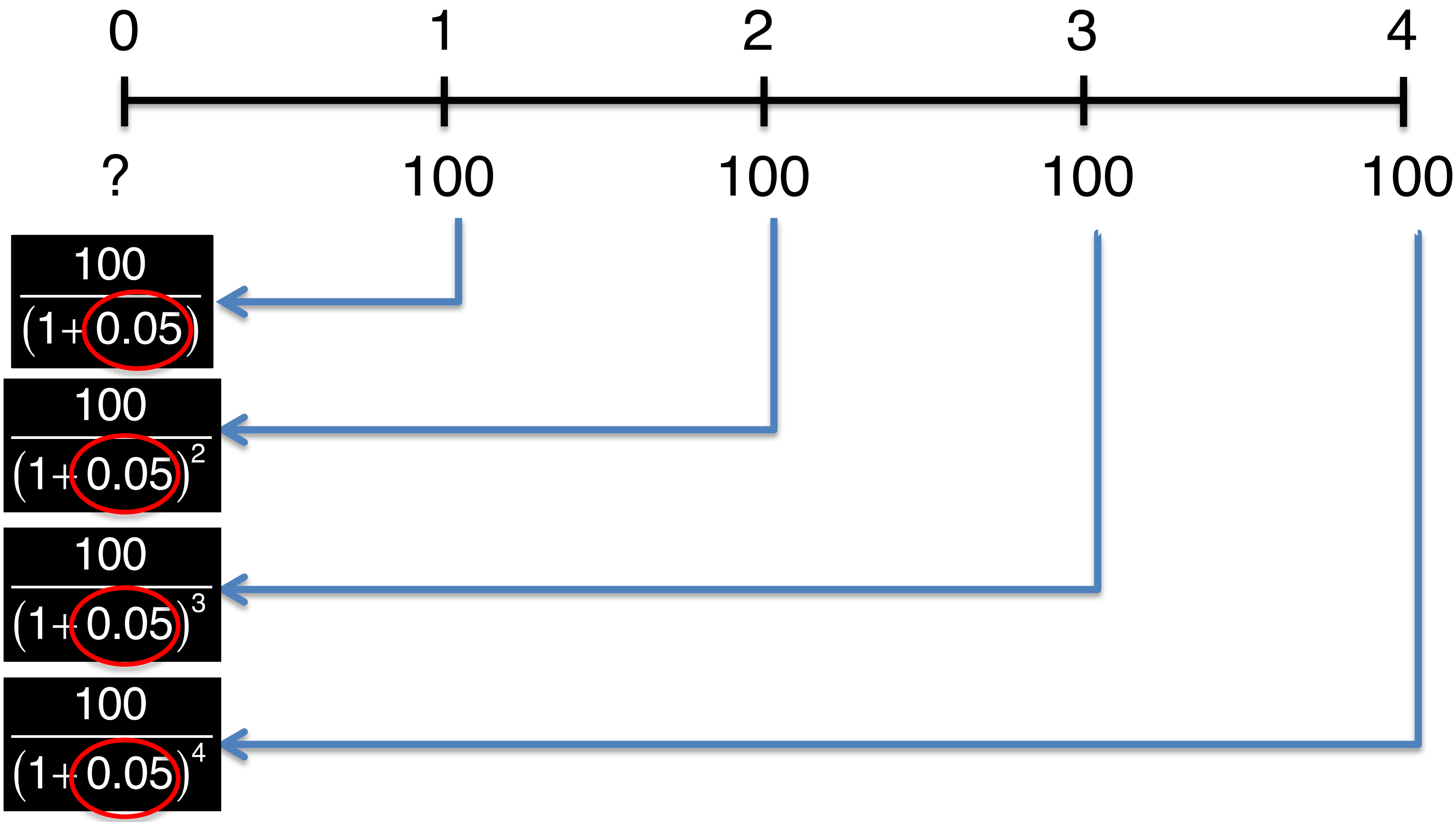

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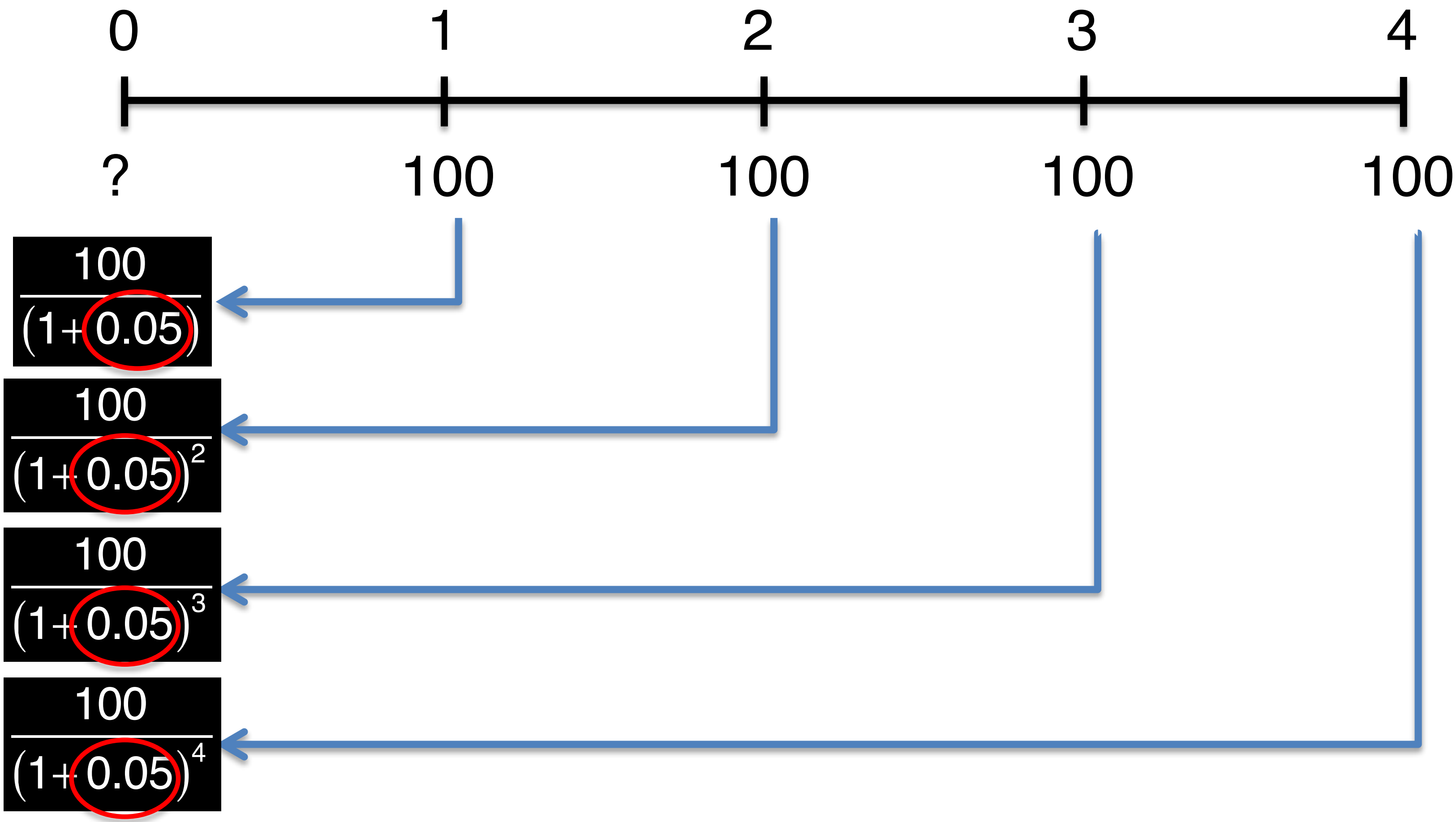


Common assumption but still an *assumption*

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawl Balance
0				$354.60

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				\$354.60
1	\$17.73			

354.60×0.05

*Activity happens at end of the period

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32		
		=		
		354.60 + 17.73		

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				\$354.60
1	\$17.73	\$372.32		

=

$$PV_0\left(\$372.32\right) = \$372.32 \times \left(1+0.05\right)^{-1} = \$354.60$$

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32	$100.00	

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawl Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32
				=
				372.32 – 100

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32
2	$13.62	$285.94	$100.00	$185.94
3	$9.30	$195.24	$100.00	$95.24
4	$4.76	$100.00	$100.00	$0.00

Summary

Lessons

- Never add/subtract cash flows from different time periods

- Use (i.e., multiply by) discount factor to change cash flows' time units

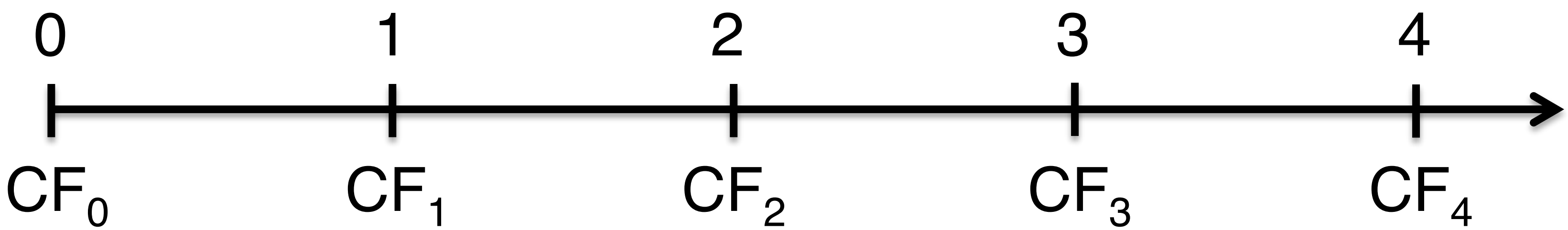
$$\boxed{(1+R)^t}$$

$t < 0$ moves CF back in time (discounting)

$t > 0$ moves CF forward in time (compounding)

Lessons

- Use a time line to help formulate problems



Lessons

- Present value as of time s of a cash flow at time $t > s$ is denoted, $PV_s(CF_t)$
  - Tells us the value future cash flows
  - Tells us the price of a claim to those cash flows

Coming up next

- Compounding

Time Value of Money: Compounding

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Last Time

Time Value of Money

- Intuition – time units like different currencies
- Tools – time line and discount factor
- Discounting – Moving CFs back in time
- Lesson: Don't add CFs with different time units…ever!

This Time

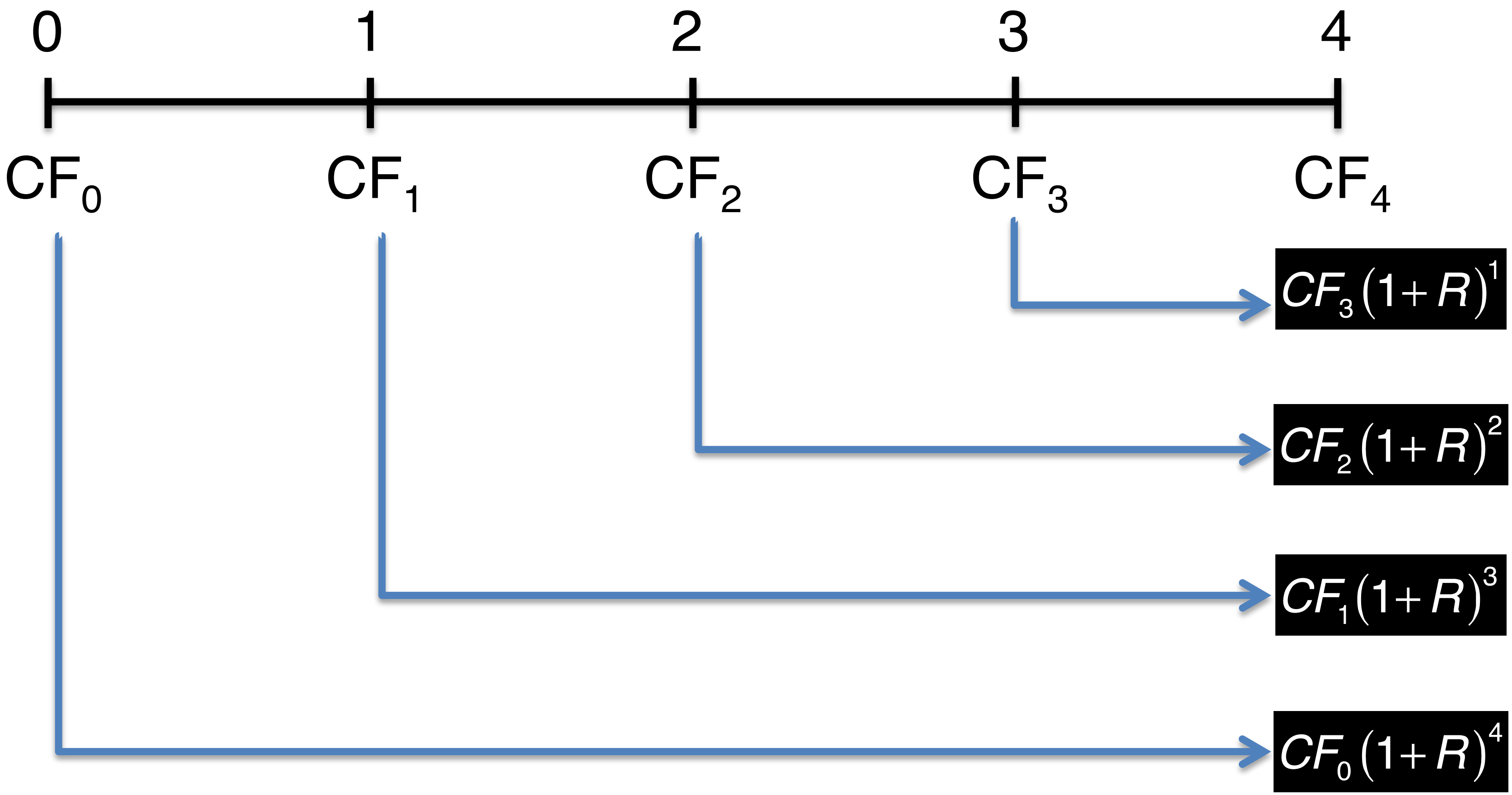
Time Value of Money

- Compounding

USING THE TOOLS: COMPOUNDING

Compounding

Compounding CFs moves them forward in time



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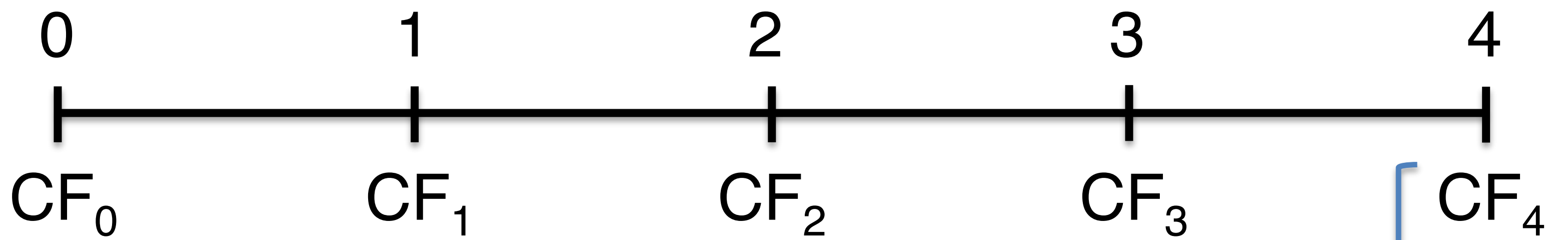


$CF_3(1+R)^1$

$CF_2(1+R)^2$

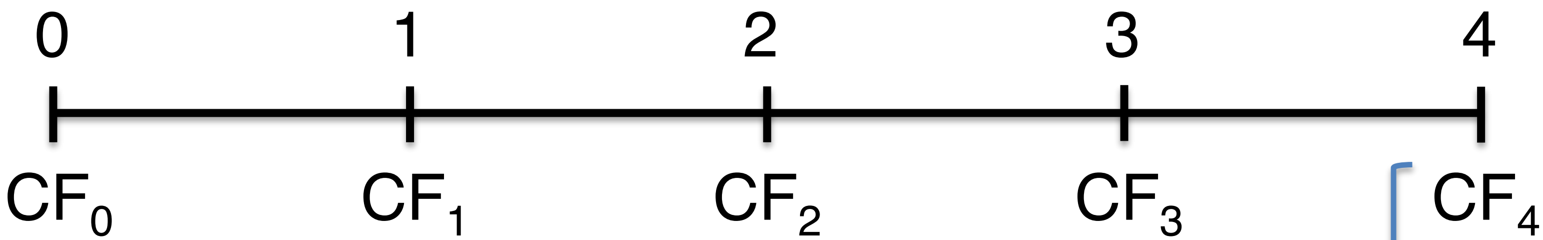
$CF_1(1+R)^3$

$CF_0(1+R)^4$

$t > 0$ because we are moving cash flows forward in time

Compounding

Compounding CFs moves them forward in time



$$CF_3(1+R)^1$$

$$CF_2(1+R)^2$$

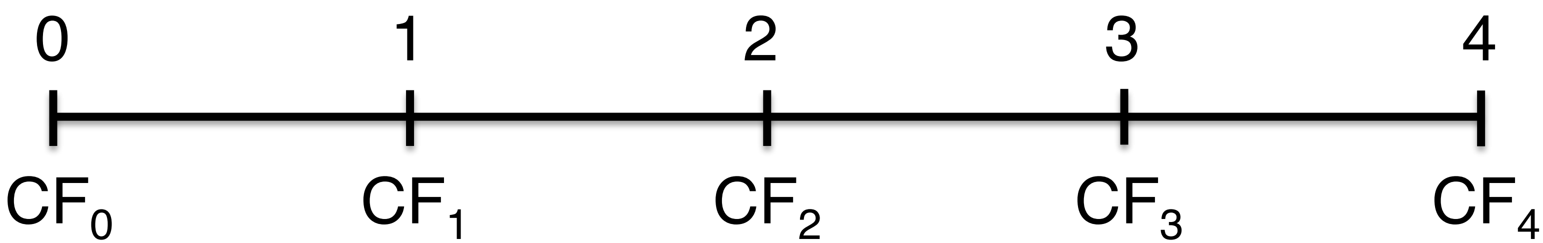
$$CF_1(1+R)^3$$

$$CF_0(1+R)^4$$

We can add/subtract these CFs because they are in the same time units (date 4)

Future Value

Future value, $FV_t(\bullet)$ of CFs is compounded value of CFs as of t



These are future values of CFs as of year 4

$$CF_3(1+R)^1 = FV_4(CF_3)$$

$$CF_2(1+R)^2 = FV_4(CF_2)$$

$$CF_1(1+R)^3 = FV_4(CF_1)$$

$$CF_0(1+R)^4 = FV_4(CF_0)$$

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

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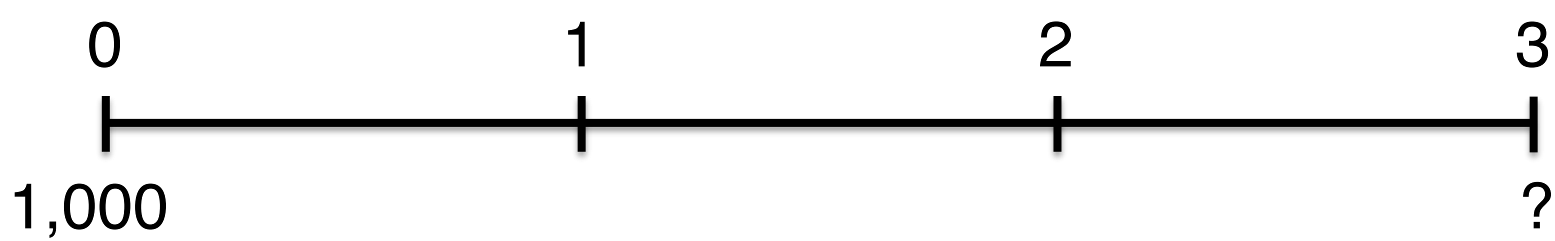
Step 1: Put cash flows on a time line



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Step 2: Move cash flow forward

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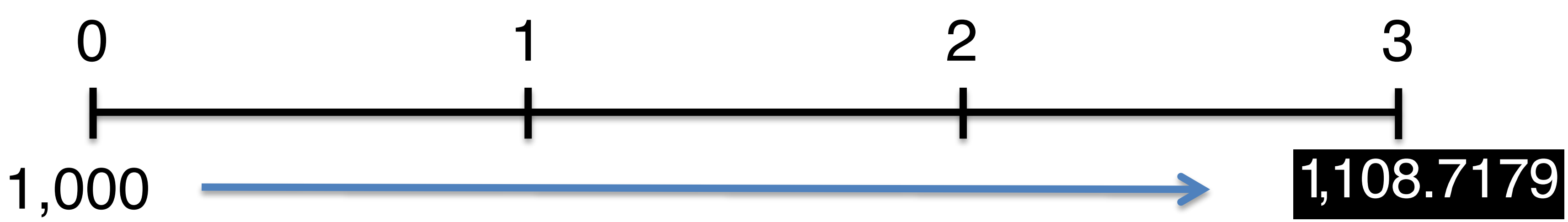

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

Step 2: Move cash flow forward

This is the future value of the 1,000

Example 2 – Savings

How much money will we have four years from today if we save $100 a year, beginning today, for the next three years, assuming we earn 5% per annum?

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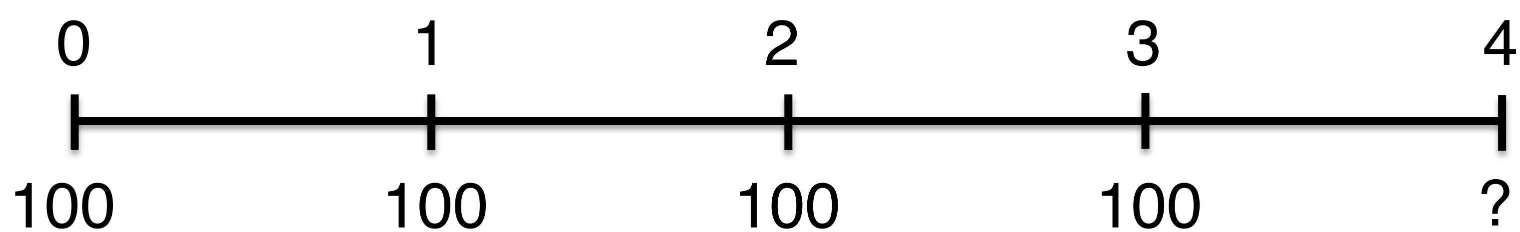
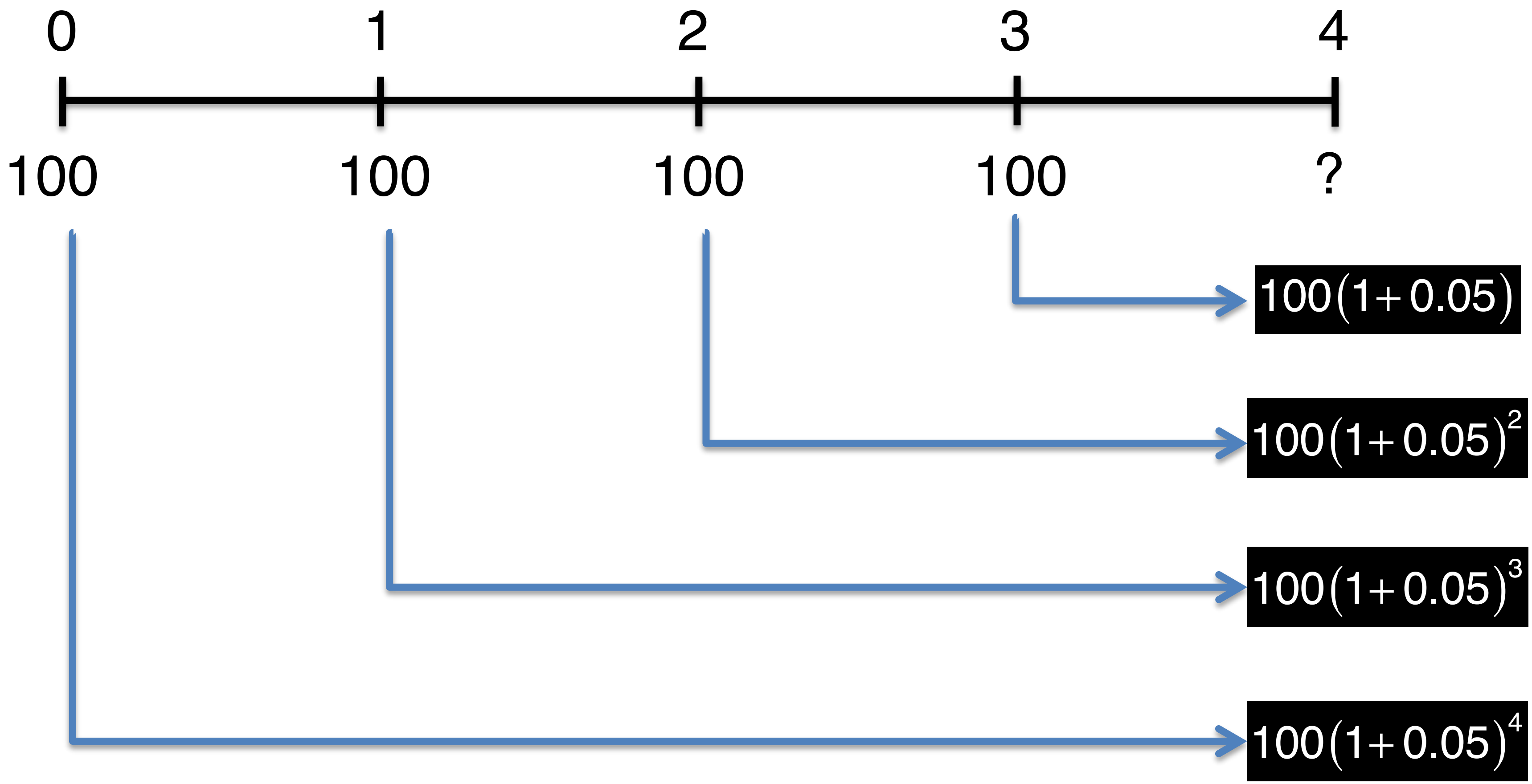
How much money will we have four years from today if we save $100 a year, beginning today, for the next three years, assuming we earn 5% per annum?

Step 1: Put cash flows on a time line



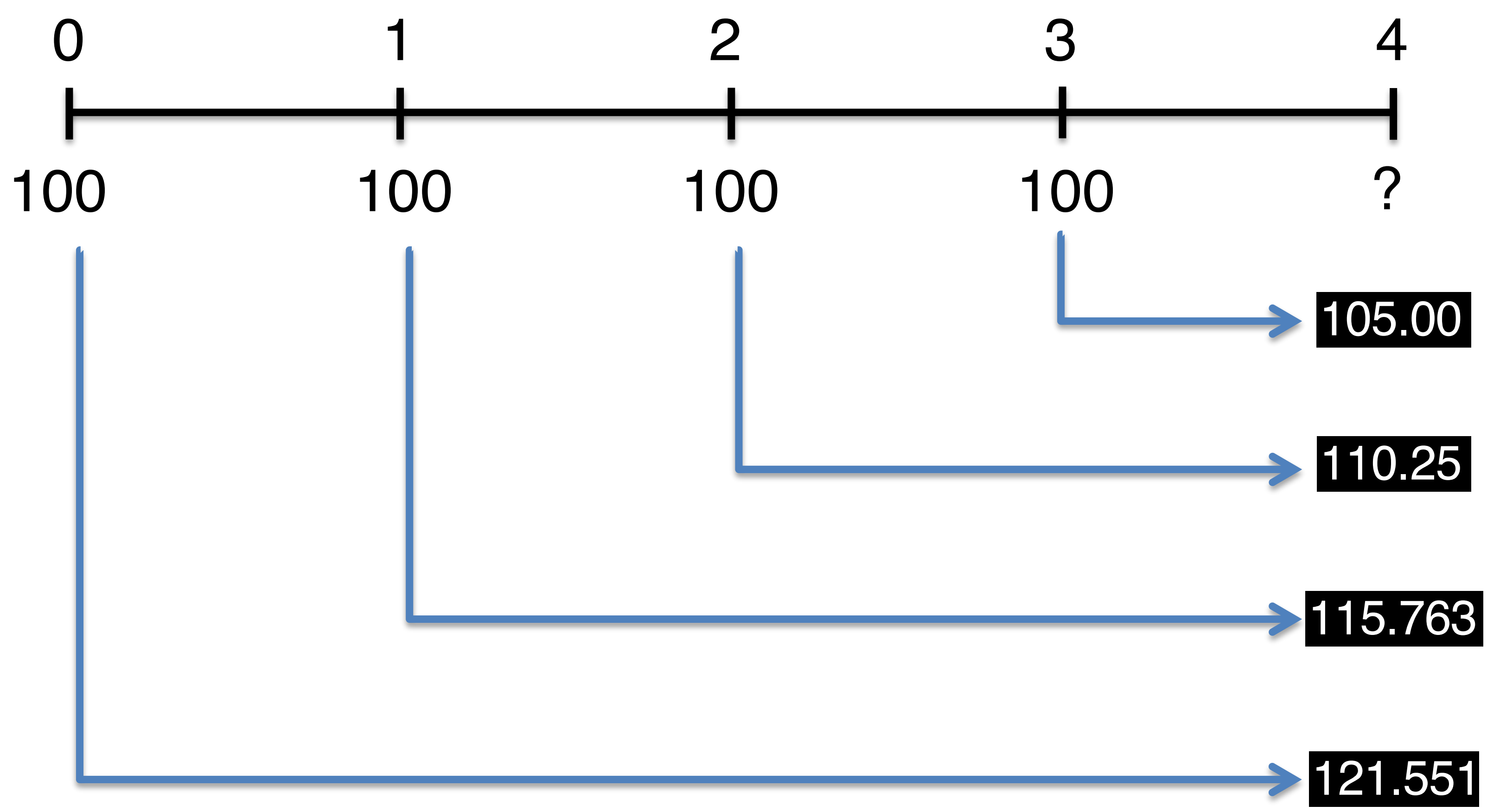
Example 2 – Savings

Step 2: Move CFs forward in time



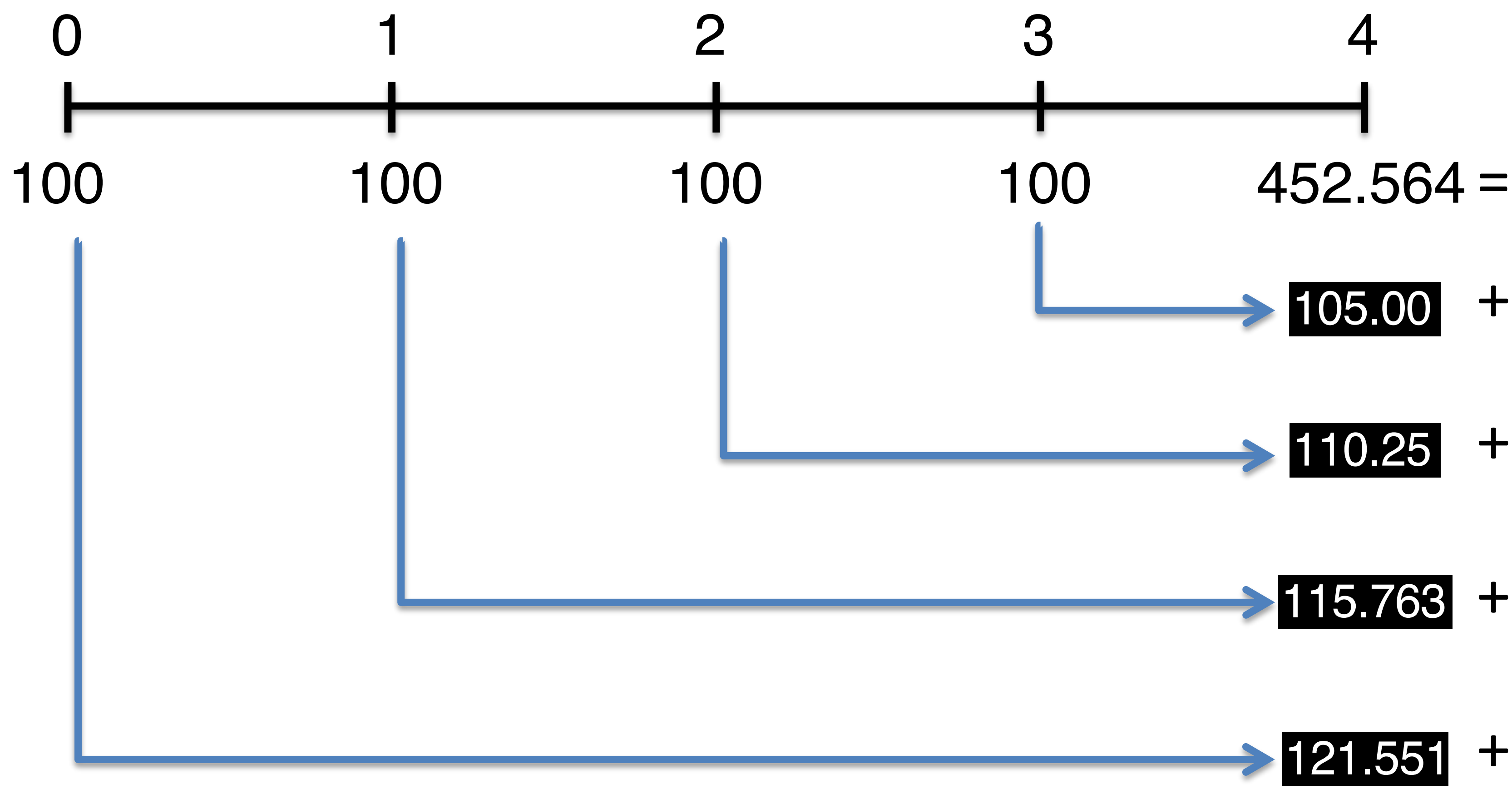
Example 2 – Savings

Step 2: Move CFs forward in time

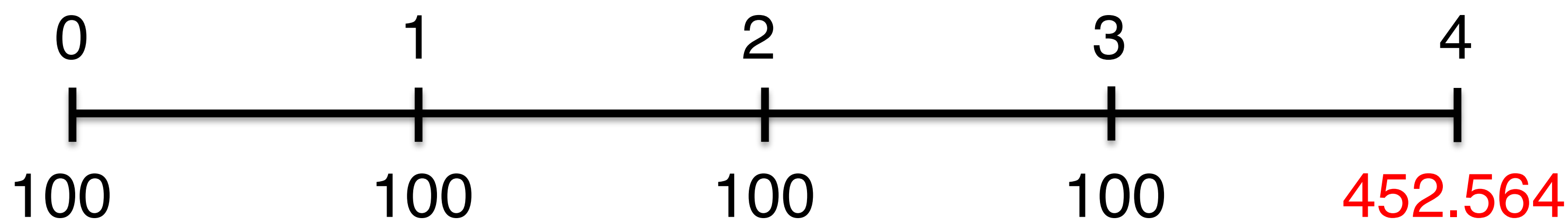


Example 2 – Savings

Step 3: Add up cash flows

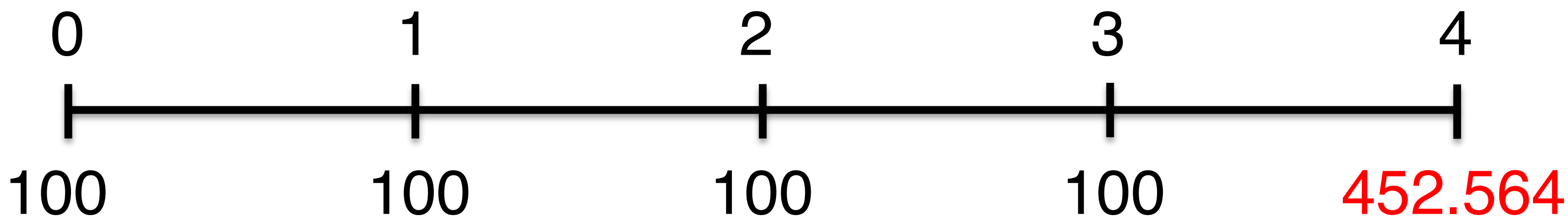


Example 2 – Savings



Interpretation 1: We will have $452.56 at the end of four years if we save $100 starting today for the next three years and our money earns 5% per annum.

Example 2 – Savings



Interpretation 2: The future value four years from today of saving $100 starting today for the next three years at 5% per annum is $452.56.

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			\$100.00	\$100.00

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			\$100.00	\$100.00
1	\$5.00	[redacted]		

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00		

$$=$$

$$100 + 5.00$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			\$100.00	\$100.00
1	\$5.00	\$105.00		

$$=$$

$$FV_1(100) = 100 \times (1 + 0.05)^1$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			\$100.00	\$100.00
1	\$5.00	\$105.00	\$100.00	

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00

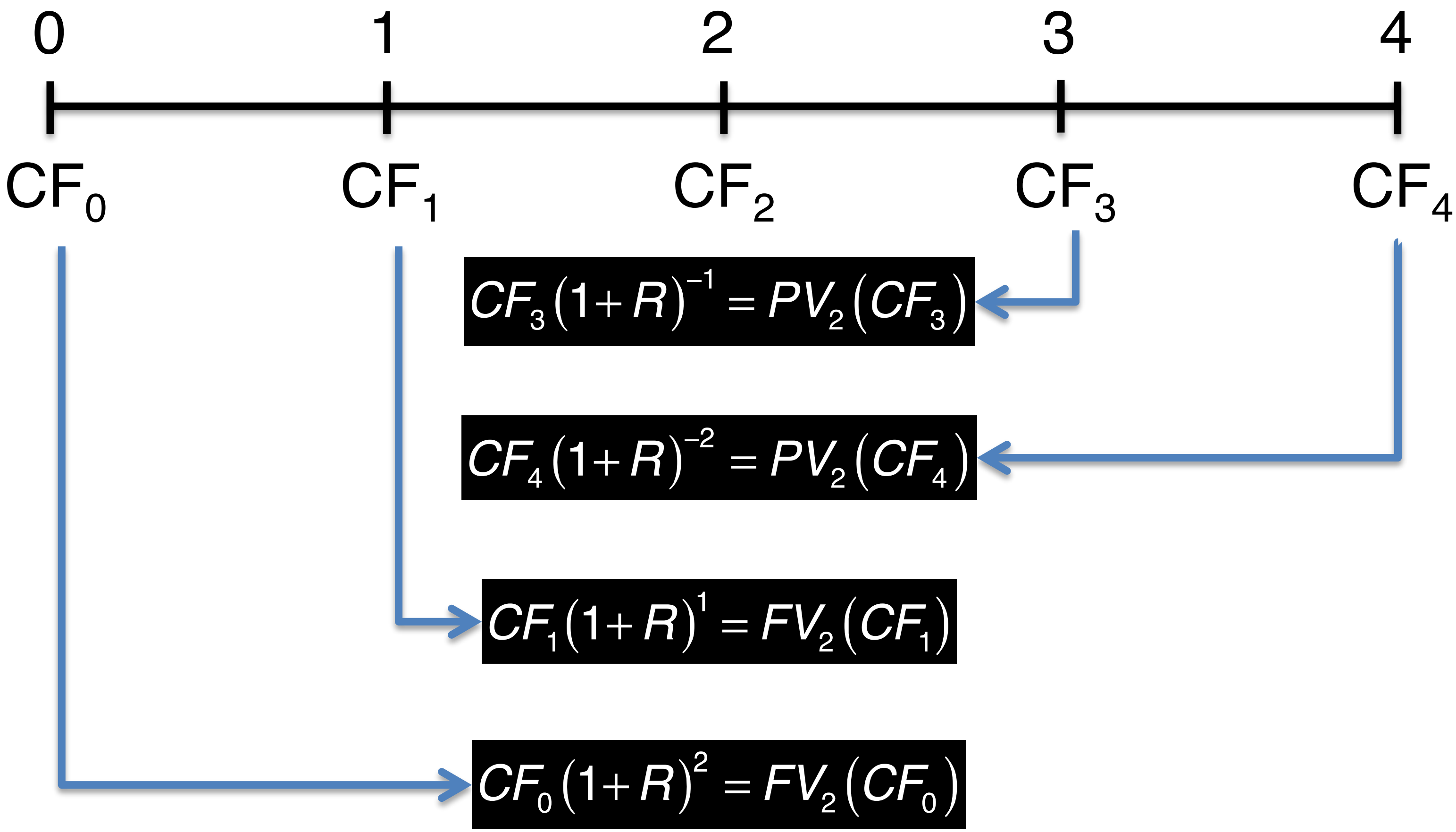
$$= 105 + 100$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00
2	$10.25	$215.25	$100.00	$315.25
3	$15.76	$331.01	$100.00	$431.01
4	$21.55	$452.56	$0.00	$452.56

More Generally

Can add CFs at any point in time if same units



$$CF_3(1+R)^{-1} = PV_2(CF_3)$$

$$CF_4(1+R)^{-2} = PV_2(CF_4)$$

$$CF_1(1+R)^{1} = FV_2(CF_1)$$

$$CF_0(1+R)^{2} = FV_2(CF_0)$$

Summary

Lessons

- We use compounding to move cash flows forward in time
- Denote the value of cash flows in the future as future value $FV_s\,(\mathrm{CF}_t)$

$$FV_s\left(CF_t\right) = CF_t\left(1+R\right)^{s-t} \quad \text{for } t < s$$

Coming up next

- Problem Set

- Useful shortcuts for *PV* and *FV* of common streams of cash flows

Time Value of Money: Useful Shortcuts

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Last Time

Time Value of Money

- Compounding

This Time

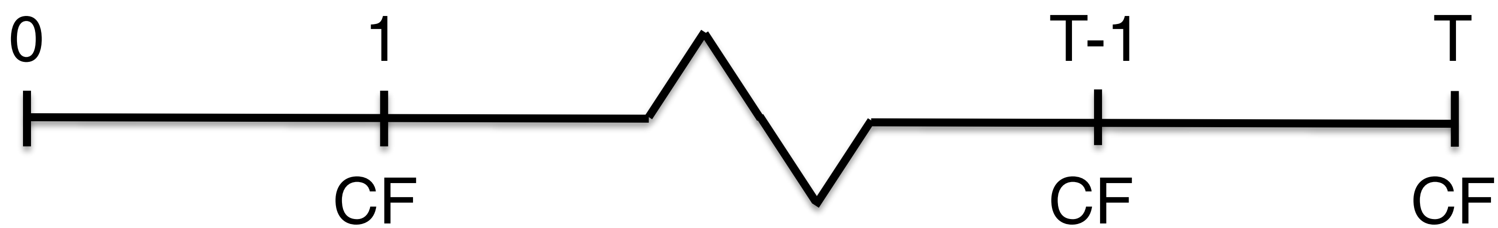
Time Value of Money

- Useful Shortcuts

ANNUITY

Annuity

An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time



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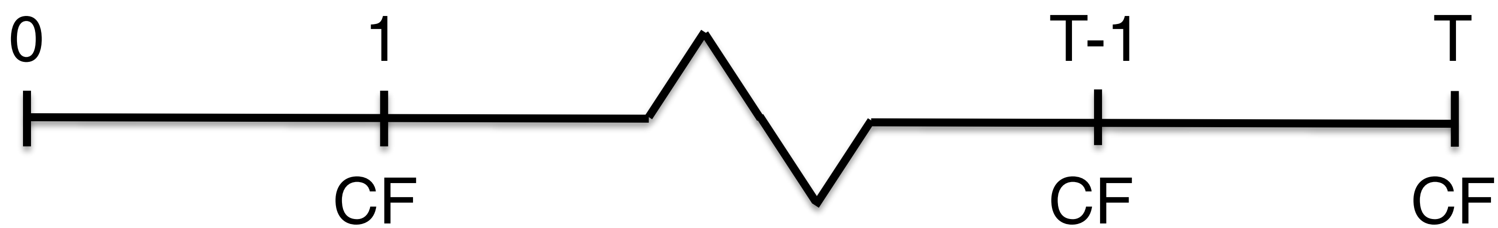
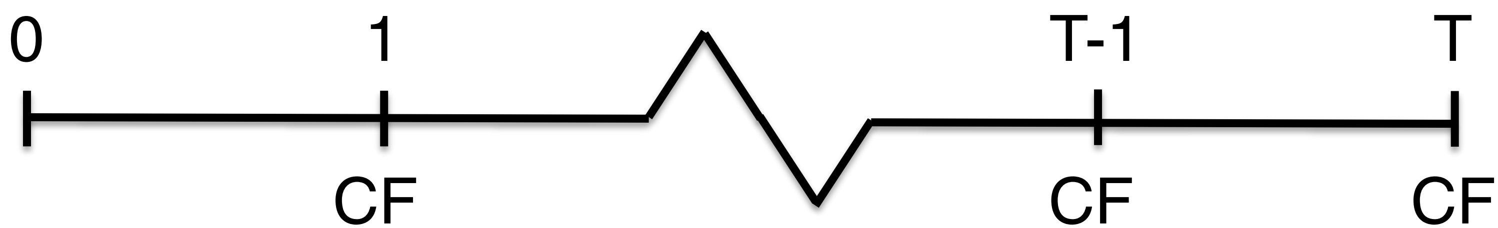


E.g., Savings, vehicle, home mortgage, auto lease, bond payments

Annuity

An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time



$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

$$= CF \times \underbrace{\frac{1-(1+R)^{-T}}{R}}_{\text{Annuity Factor}}$$

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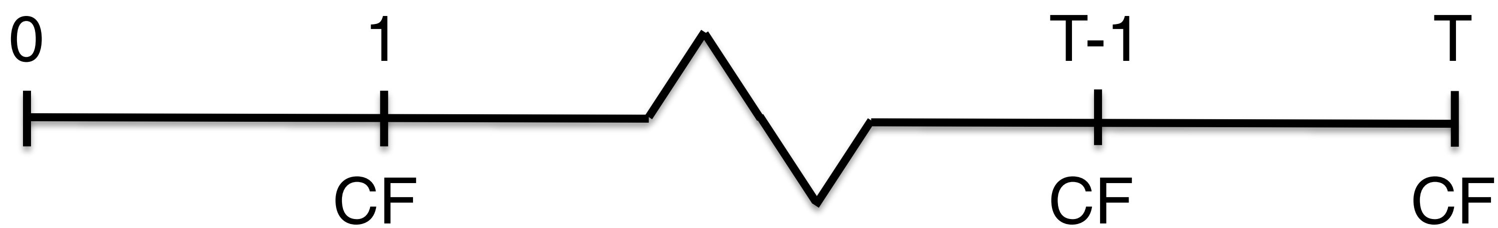

$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

*The first cash flow arrives one period from today

Example 1 – Savings

How much do you have to save today to withdraw $100 at the end of each of the next 20 years if you can earn 5% per annum?

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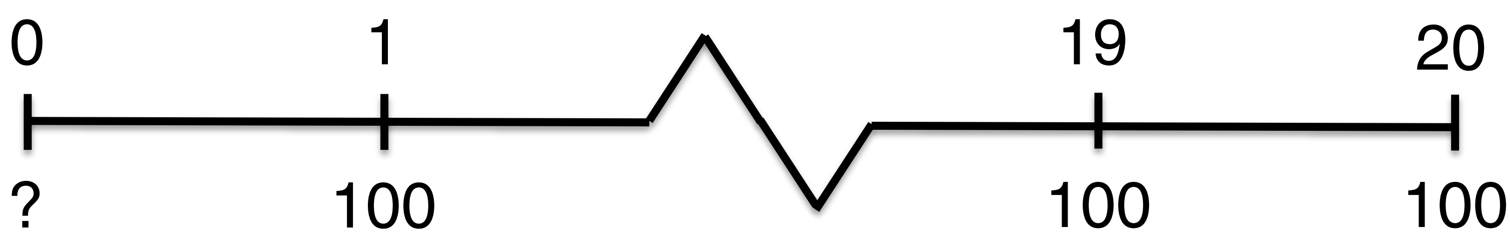

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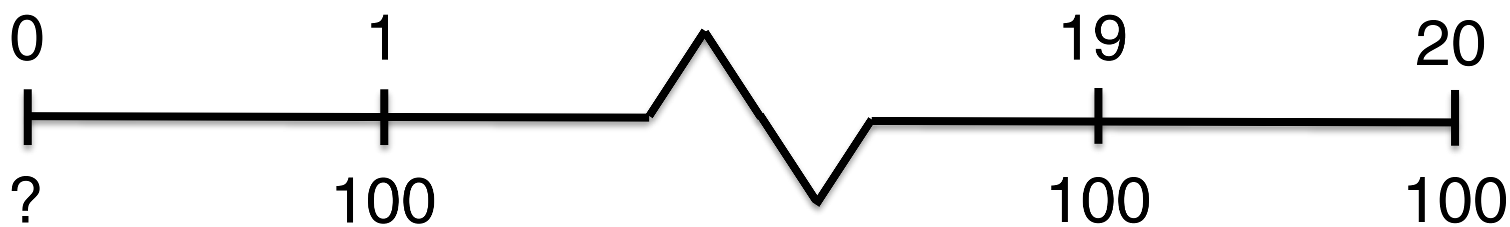
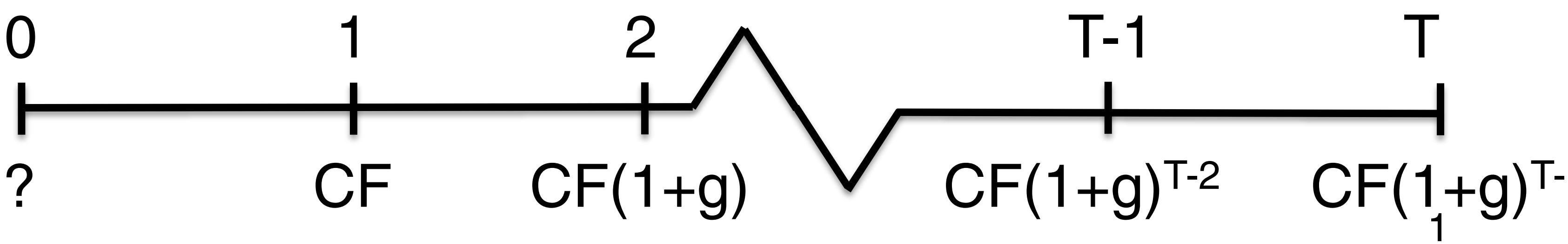


$$\text{PV of Annuity} = \frac{100}{0.05}\left(1-(1+0.05)^{-20}\right) = 1{,}246.22$$

GROWING ANNUITY

Growing Annuity

A growing annuity is a finite stream of cash flows that grow at a constant rate and that are evenly spaced through time



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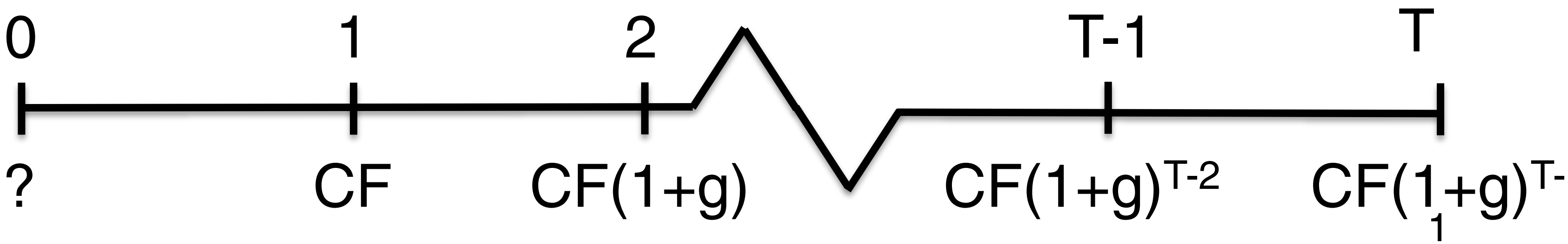

E.g., Income streams, savings strategies, project revenue/expense streams

Growing Annuity

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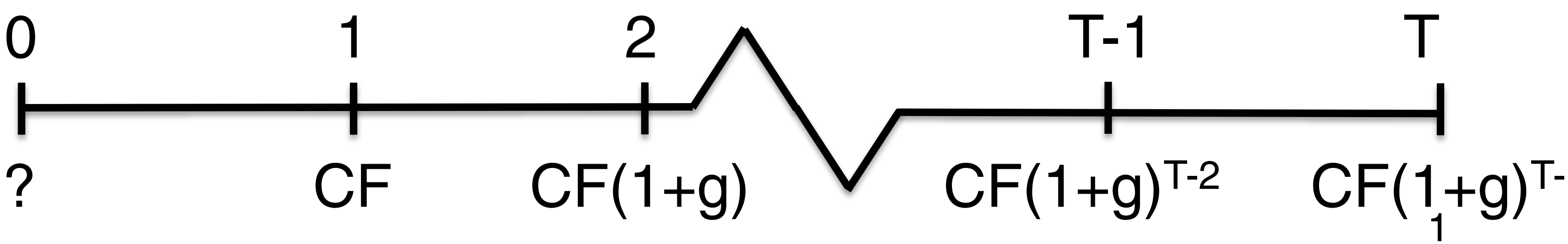

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

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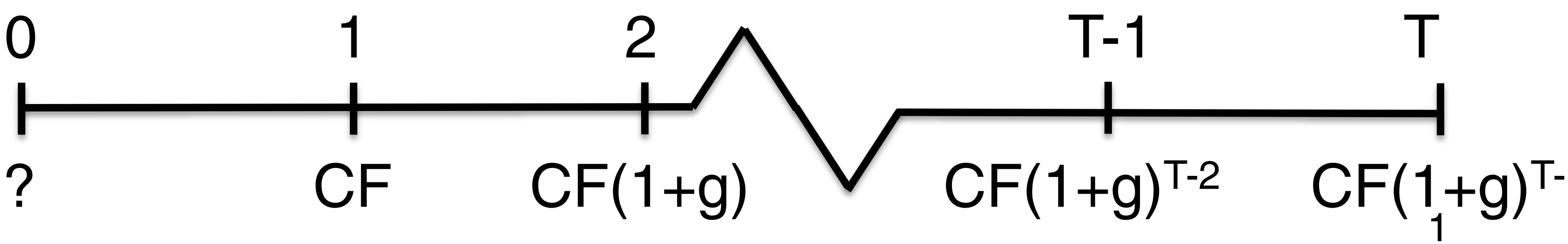

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

*The first cash flow arrives one period from today

Example 2 – Savings

How much do you have to save today to withdraw $100 at the end of this year, 102.5 next year, 105.06 the year after, and so on for the next 19 years if you can earn 5% per annum?

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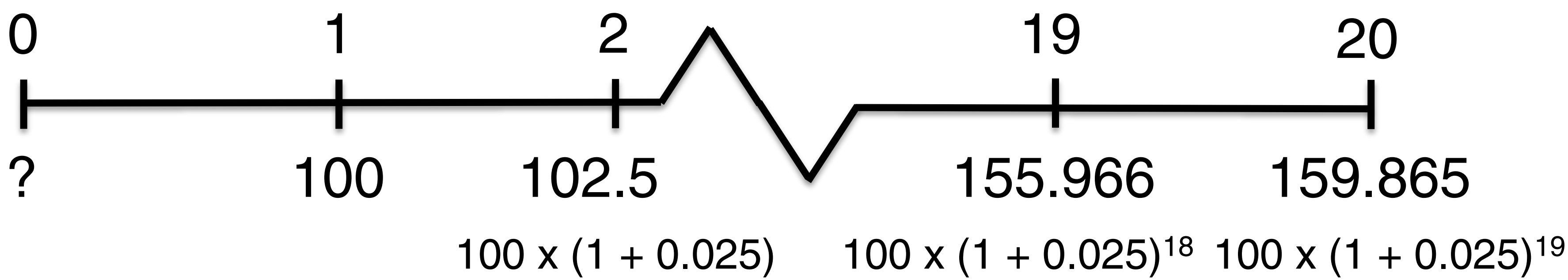

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How much do you have to save today to withdraw $100 at the end of this year, 102.5 next year, 105.06 the year after, and so on for the next 19 years if you can earn 5% per annum?

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

$$= \frac{100}{0.05-0.025}\left(1-\left(\frac{1+0.05}{1+0.025}\right)^{-20}\right) = 1{,}529.69$$

PERPETUITY

Perpetuity

An perpetuity is an infinite stream of cash flows of identical magnitude and equal spacing in time

Perpetuity

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E.g., Perpetuities, consol bonds

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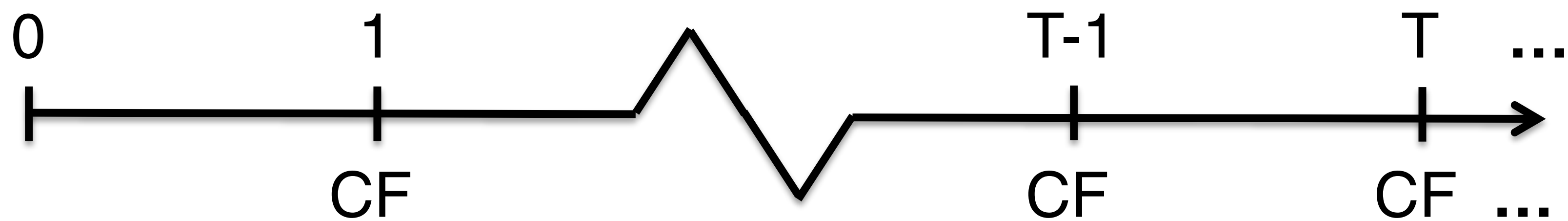

$$\text{PV of Perpetuity} = \frac{CF}{R}$$

Example 3 – Savings

How much do you have to save today to withdraw $100 at the end of each year forever if you can earn 5% per annum?

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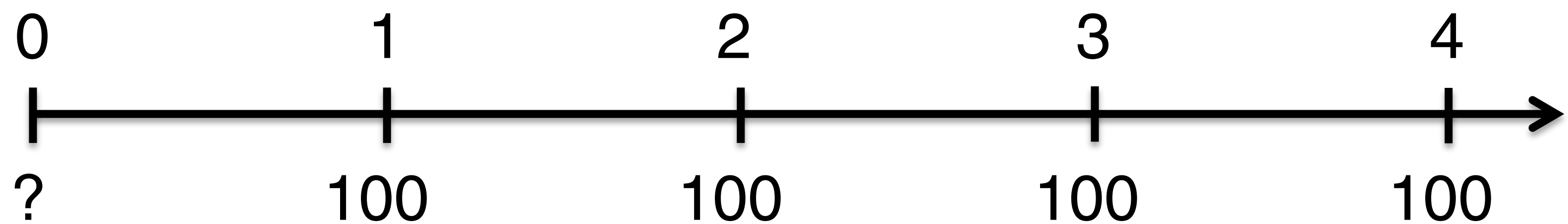
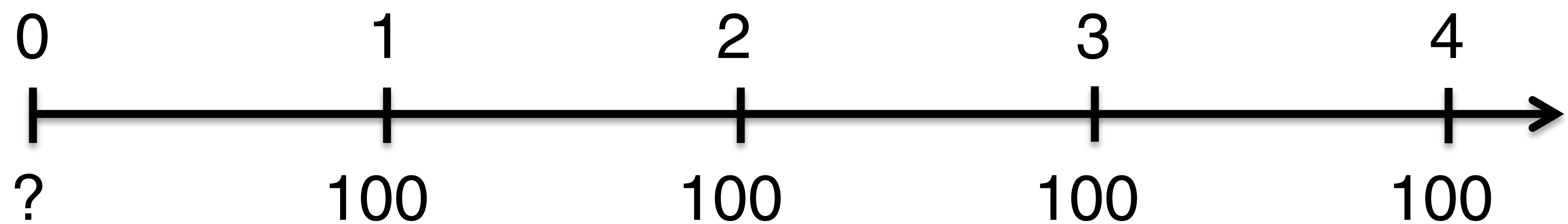


Discount CFs one at a time…impossible!

Example 3 – Savings

How much do you have to save today to withdraw $100 at the end of each year forever if you can earn 5% per annum?



$$\text{PV of Perpetuity} = \frac{100}{0.05} = 2{,}000$$

GROWING PERPETUITY

Growing Perpetuity

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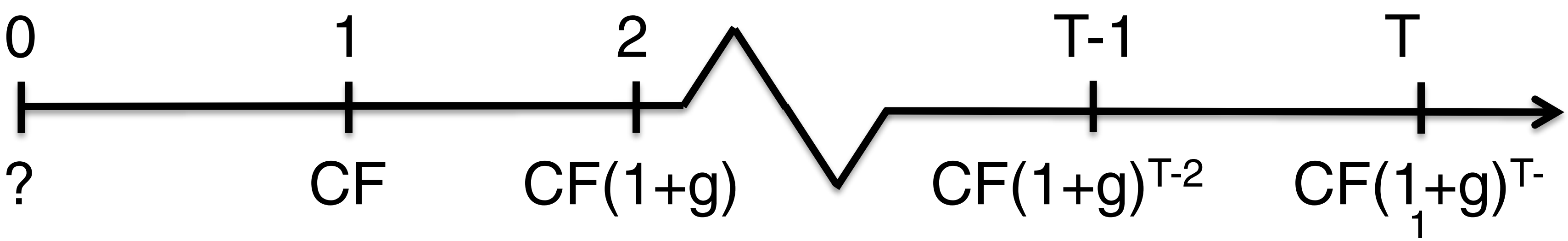

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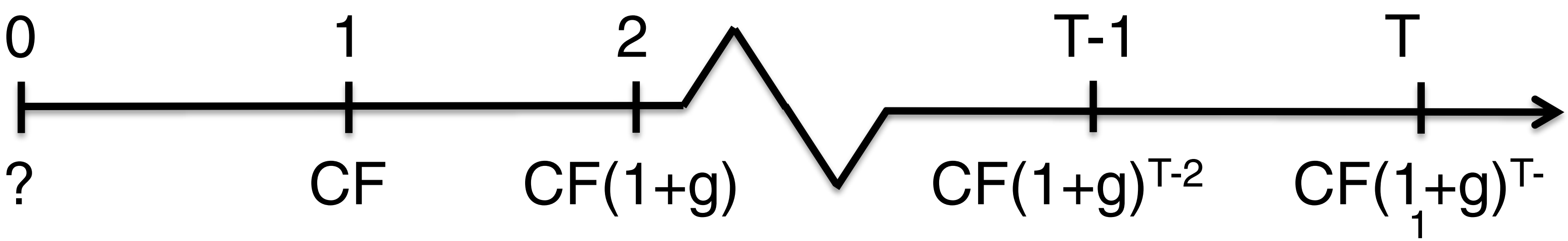

E.g., Dividend streams

Growing Perpetuity

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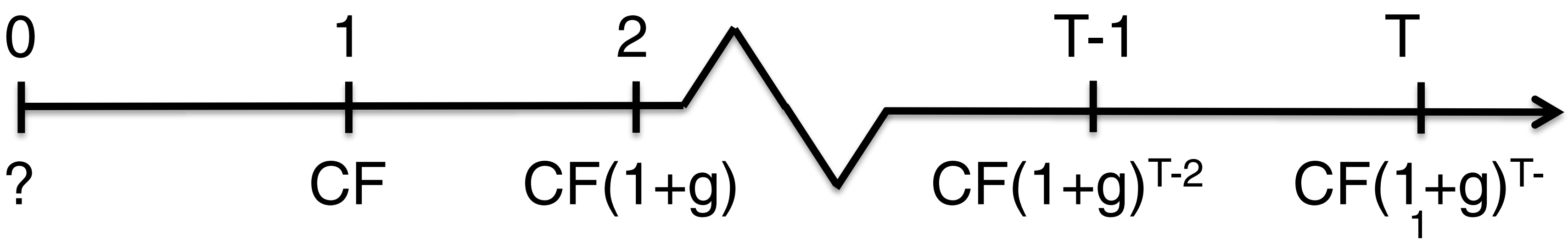

$$\text{PV of Growing Perpetuity} = \frac{CF}{R - g}$$

Growing Perpetuity

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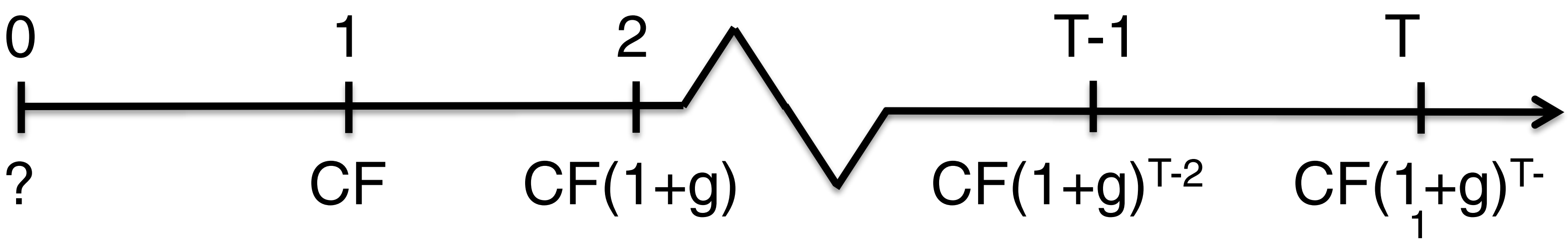

$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g}$$

*The first cash flow arrives one period from today

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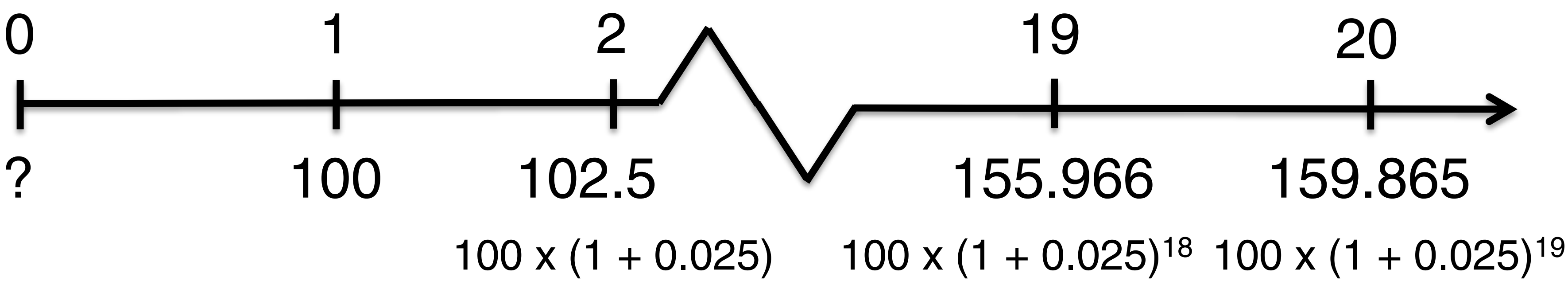

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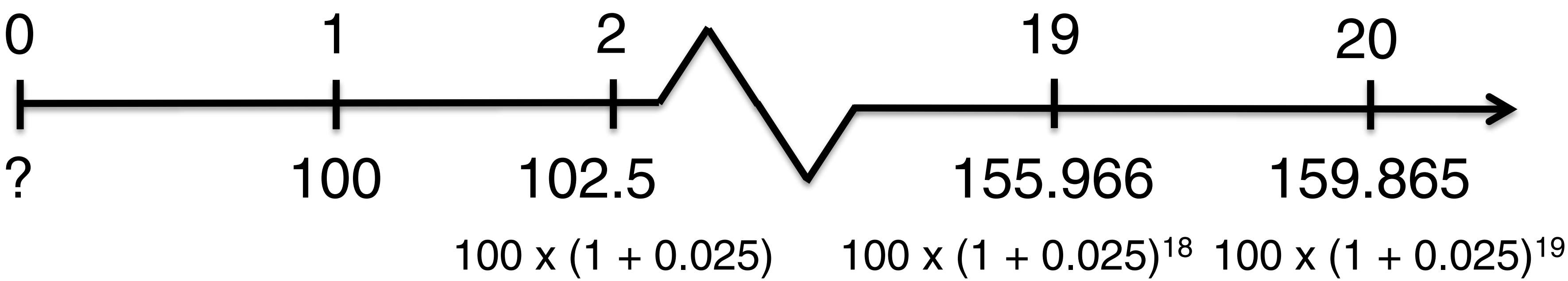


$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g} = \frac{100}{0.05-0.025} = 4{,}000$$

Summary

Lessons

- An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time

$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

- A perpetuity is an infinite stream of cash flows of identical magnitude and equal spacing in time

$$\text{PV of Perpetuity} = \frac{CF}{R}$$

Lessons

- A growing annuity is a finite stream of cash flows growing at a constant rate and equally spaced in time

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

- A growing perpetuity is an infinite stream of cash flows growing at a constant rate and equally spaced in time

$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g}$$

Caution

- Annuity and perpetuity formulas assume first cash flow occurs one period from today
- Growth rate, *g,* must be less than the discount rate, *R,* for PV formulas to make sense
- Understand excel functions assumptions

Coming up next

- Problem Set

- Taxes

Time Value of Money: Taxes

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Time Value of Money

- Useful shortcuts

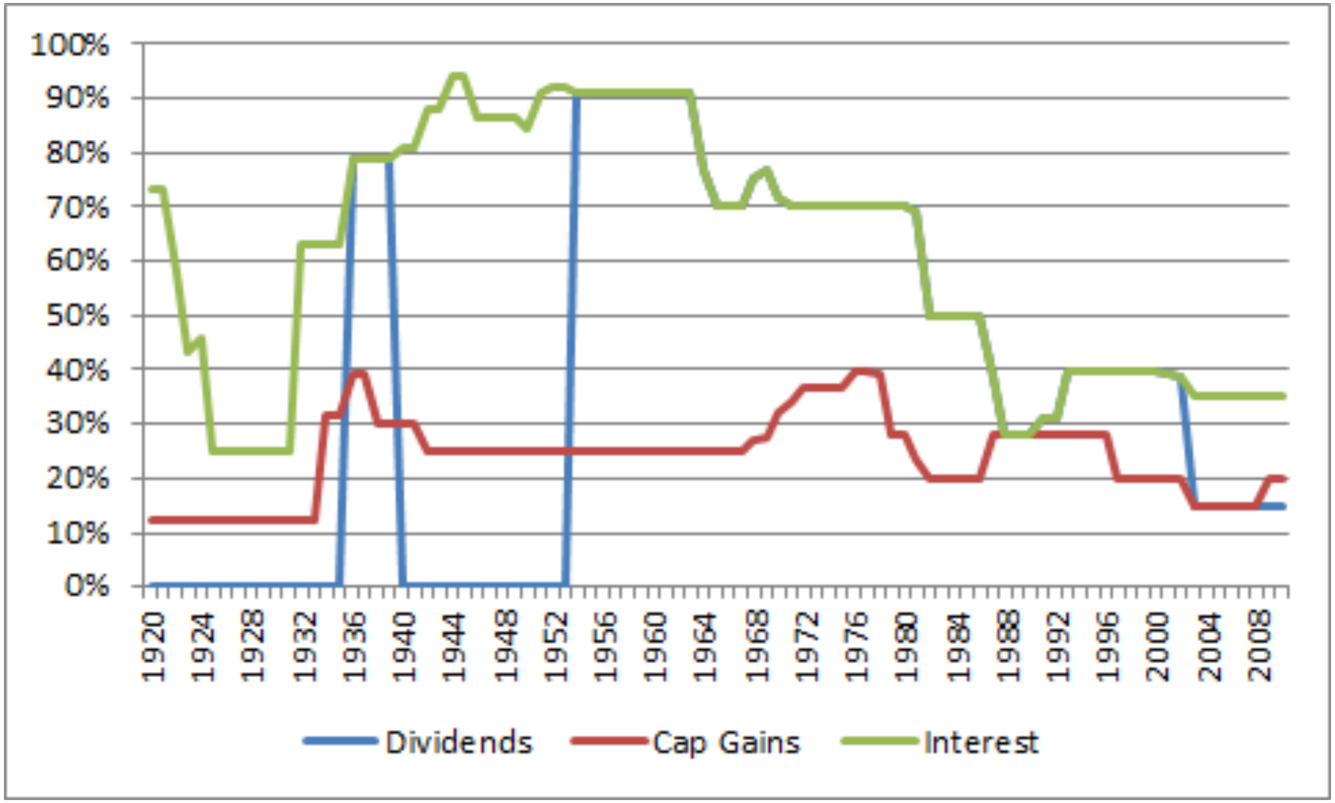
This Time

Time Value of Money

- Taxes

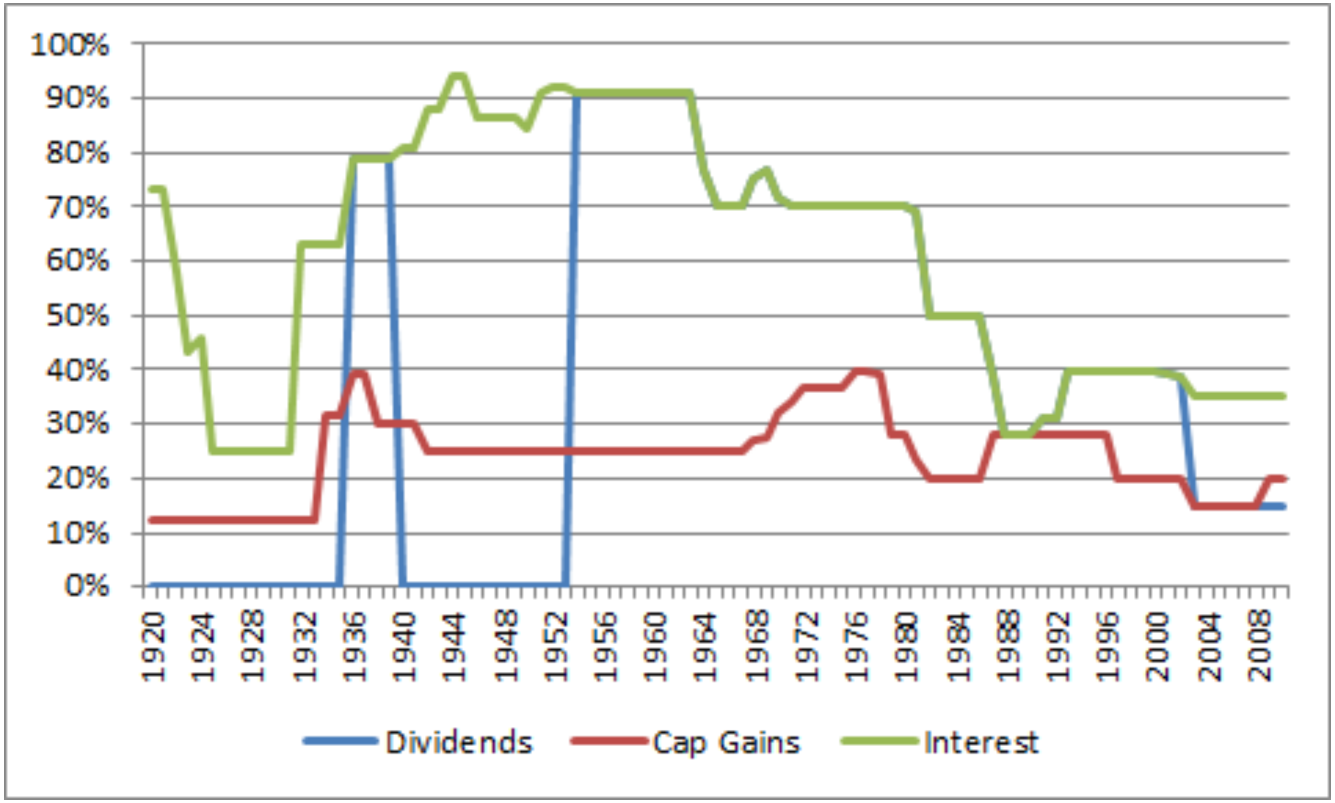
Taxes

Tax Rates



Source: Graham, John R., Mark T. Leary, and Michael R. Roberts, 2014, "A Century of Corporate Capital Structure: The Leverage of Corporate America," forthcoming *Journal of Financial Economics*

Tax Rates



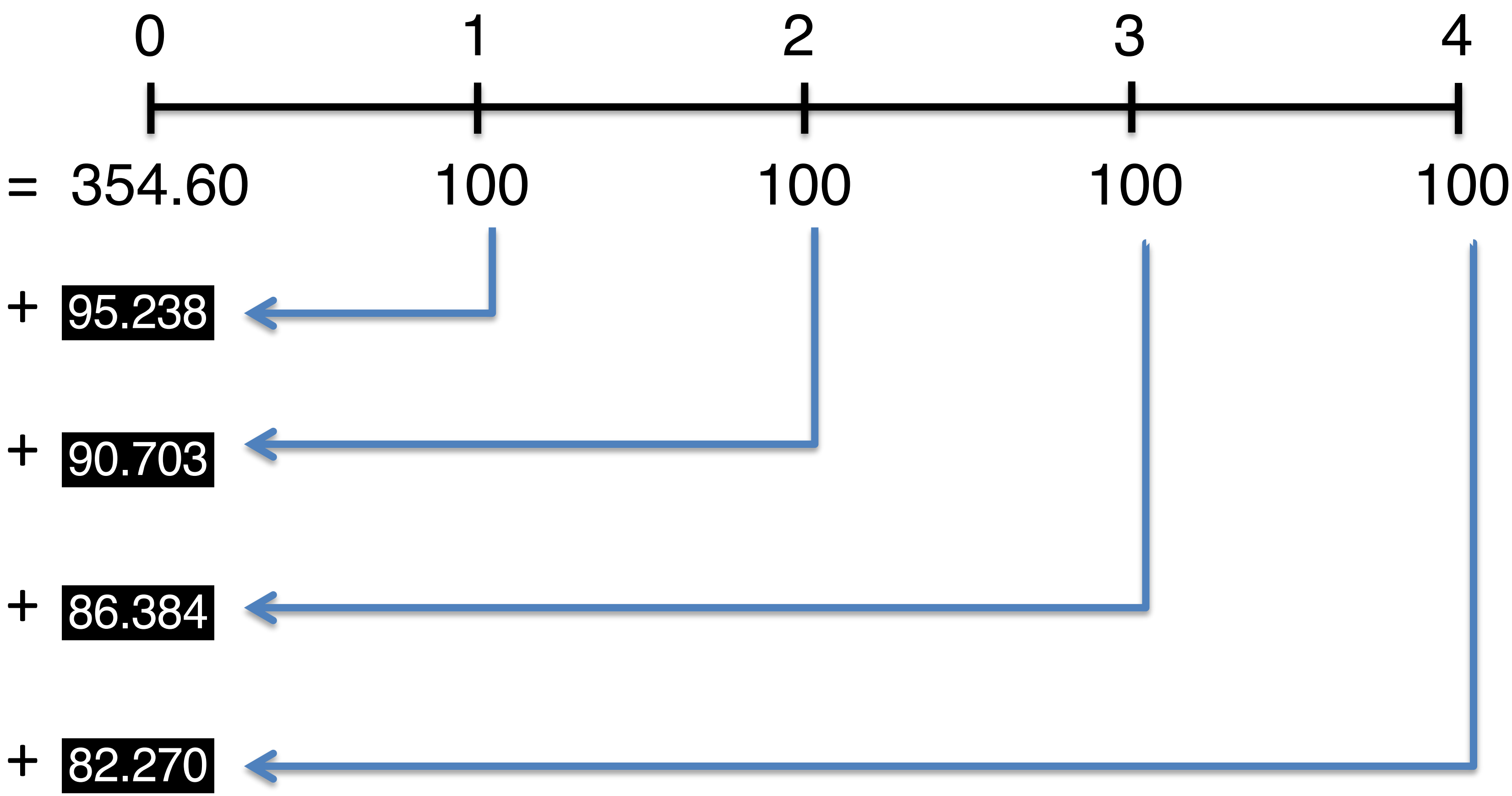
How do taxes impact our returns?

Example – Savings (Discounting)

How much do you have to save today to withdraw $100 at the end of each of the next four years if you can earn 5% per annum?

Example – Savings (Discounting)

Recall…



Example – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				\$354.60
1	\$17.73	\$372.32	\$100.00	\$272.32
2	\$13.62	\$285.94	\$100.00	\$185.94
3	\$9.30	\$195.24	\$100.00	\$95.24
4	\$4.76	\$100.00	\$100.00	\$0.00

Savings with Taxes (Account)

Year	Interest	Taxes (35%)	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0					$354.60
1	$17.73	-$6.21	$366.12	$100.00	$266.12
2	$13.31	-$4.66	$274.77	$100.00	$174.77
3	$8.74	-$3.06	$180.45	$100.00	$80.45
4	$4.02	-$1.41	$83.06	$83.06	$0.00

Savings with Taxes (Account)

Year	Interest	Taxes (35%)	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0					$354.60
1	$17.73	-$6.21	$366.12	$100.00	$266.12
2	$13.31	-$4.66	$274.77	$100.00	$174.77
3	$8.74	-$3.06	$180.45	$100.00	$80.45
4	$4.02	-$1.41	$83.06	$83.06	$0.00

We are $100 – $83.06 = $16.94 short. Taxes reduce funds available for withdrawal. We run out of money early

Lesson: Taxes reduce the return on our investment, *R*

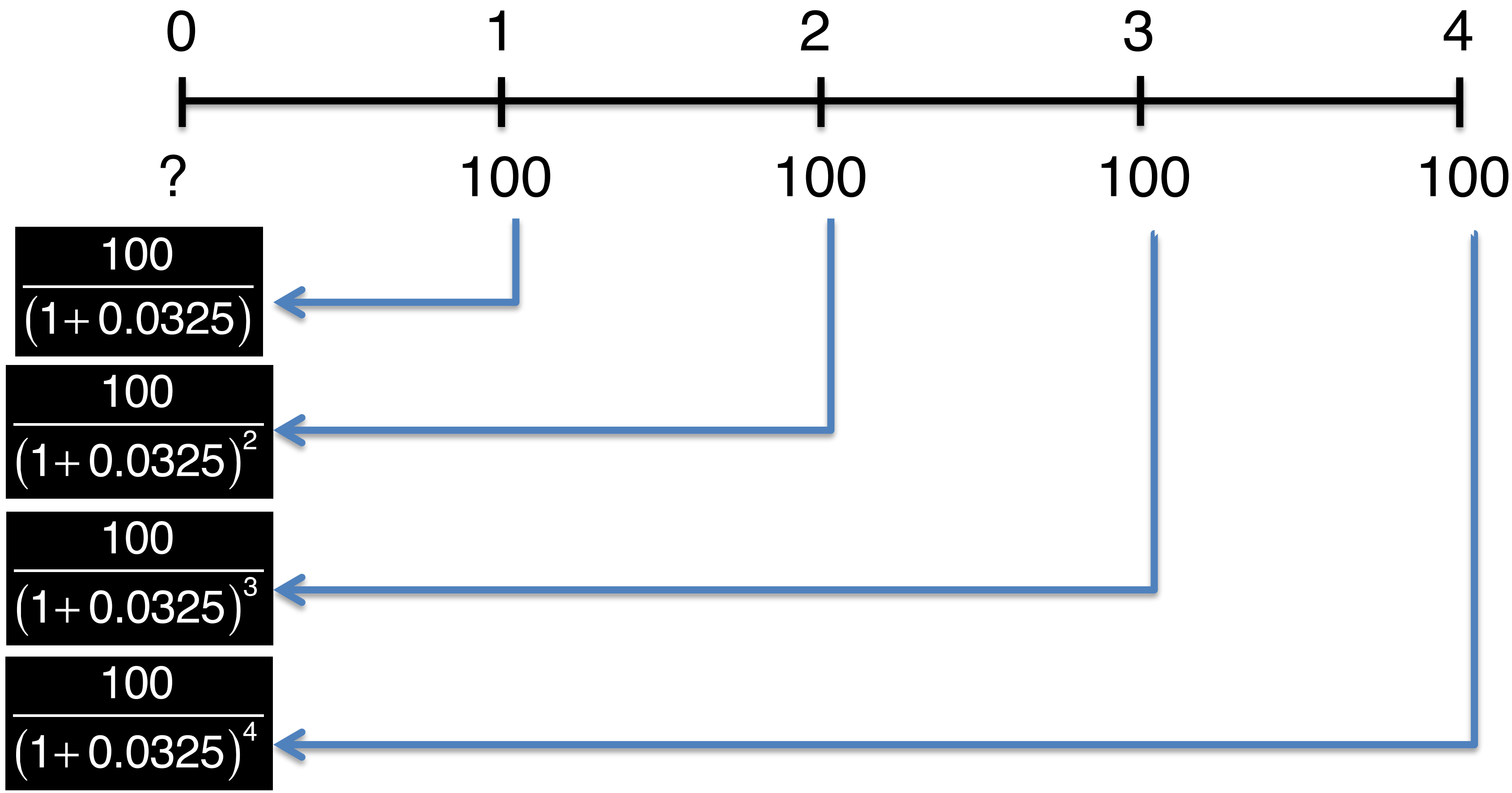
After-tax Discount Rate

$$Rt = R \times (1 - t)$$

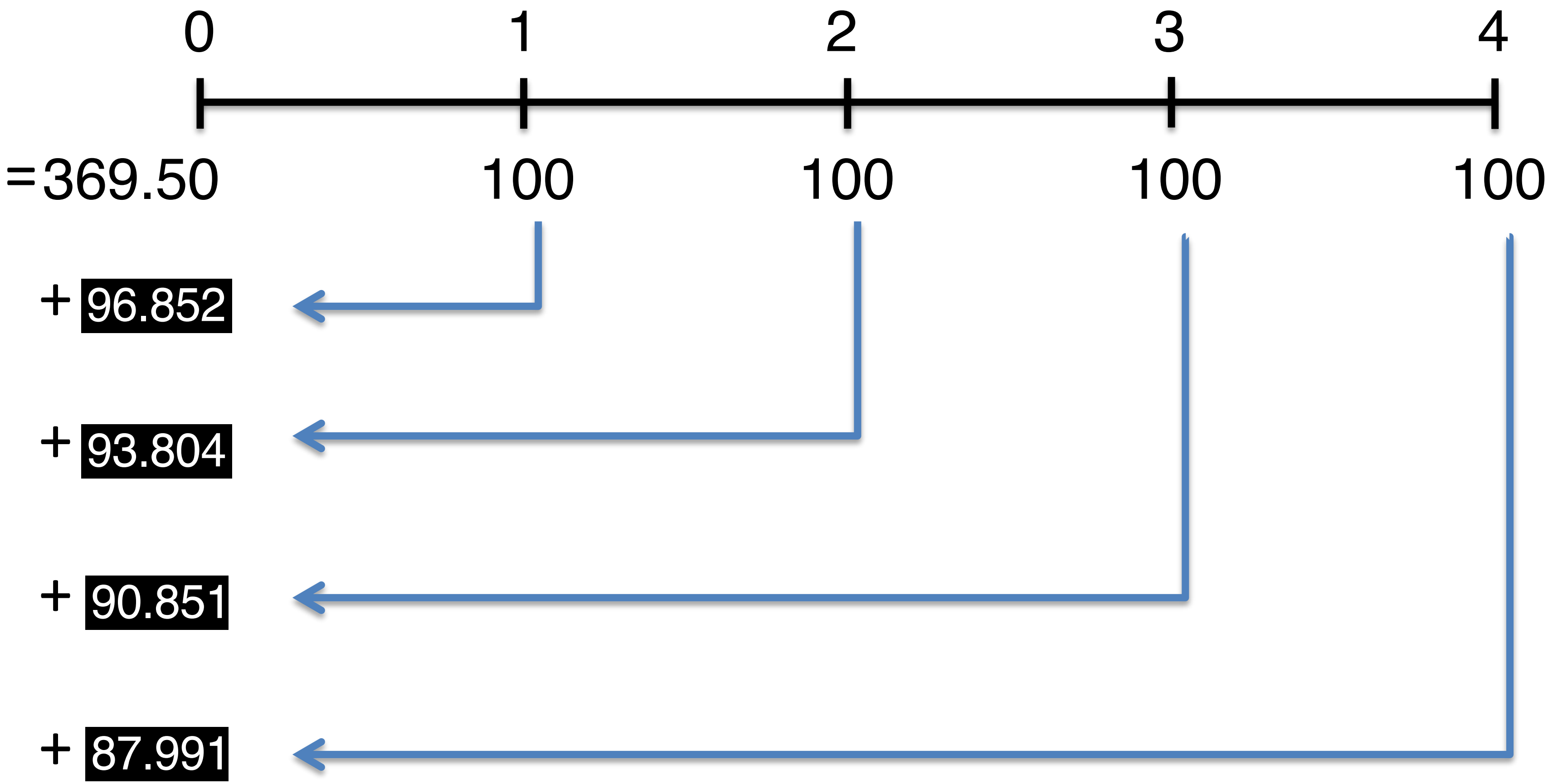
- For our example:

$$5\% \times (1 - 35\%) = 3.25\%$$

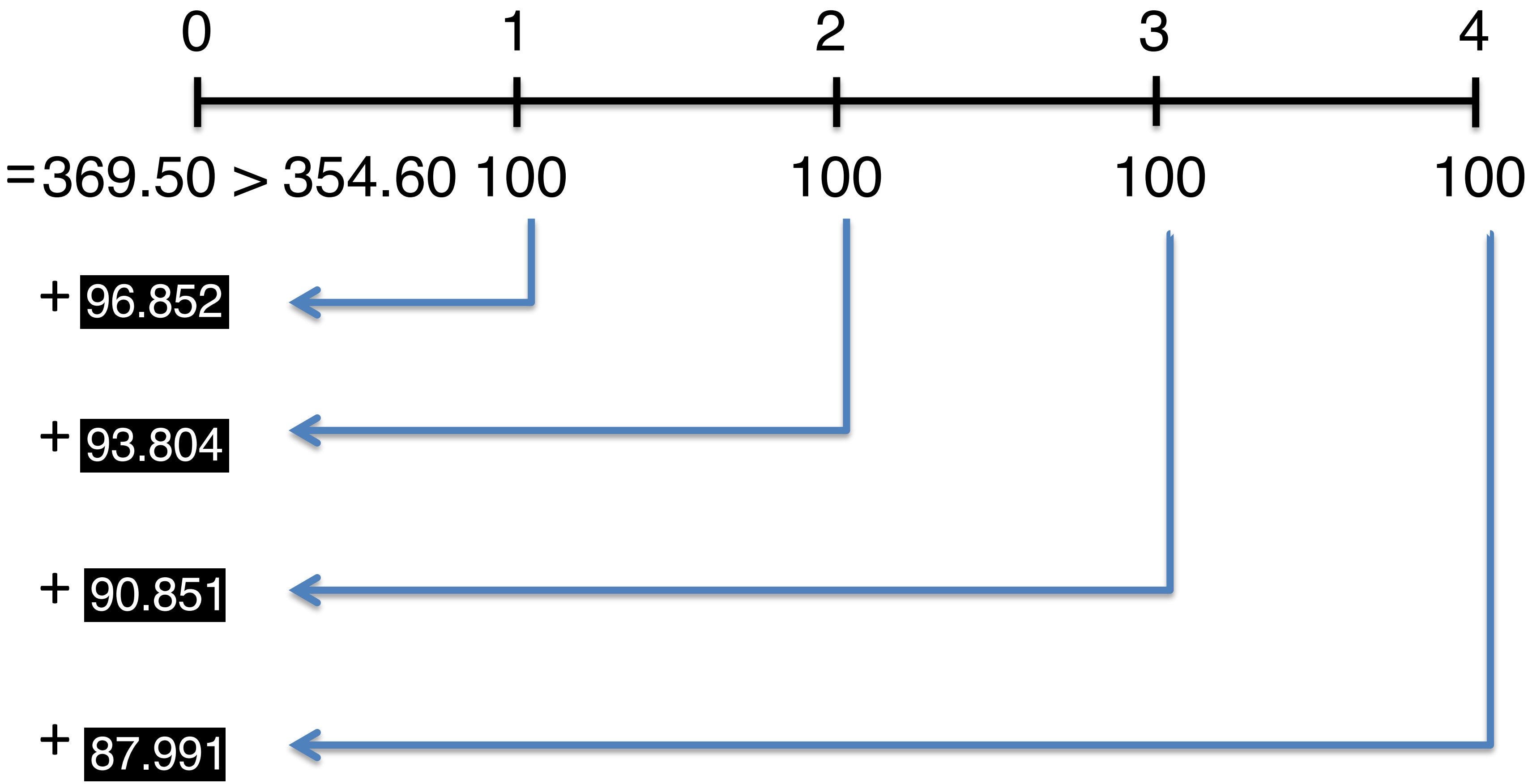
Savings with Taxes



Savings with Taxes



Savings with Taxes



Savings with Taxes

Year	Interest	Taxes	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0					$369.50
1	$18.47	-$6.47	$381.51	$100.00	$281.51
2	$14.08	-$4.93	$290.66	$100.00	$190.66
3	$9.53	-$3.34	$196.85	$100.00	$96.85
4	$4.84	-$1.69	$100.00	$100.00	$0.00

Savings with Taxes

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Implication: We need to save more to (\$369.50 > \$354.60) to withdraw \$100 each year *after taxes*

Savings with Taxes

Year	Interest	Taxes	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
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3	$9.53	-$3.34	$196.85	$100.00	$96.85
4	$4.84	-$1.69	$100.00	$100.00	$0.00

Note: $369.50 – $354.60 = $14.90 which also equals the present value of the taxes at 5%. (Check this!)

Summary

Lessons

- Taxes reduce our dollar return

- The after-tax return, *Rt*, on an investment is:

$$Rt = R \times (1 - t)$$

where R is the nominal return and t is the tax rate

Coming up next

- Time Value of Money
  - How does inflation affect our returns and value of money?

Time Value of Money: Inflation

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Time Value of Money

- Taxes

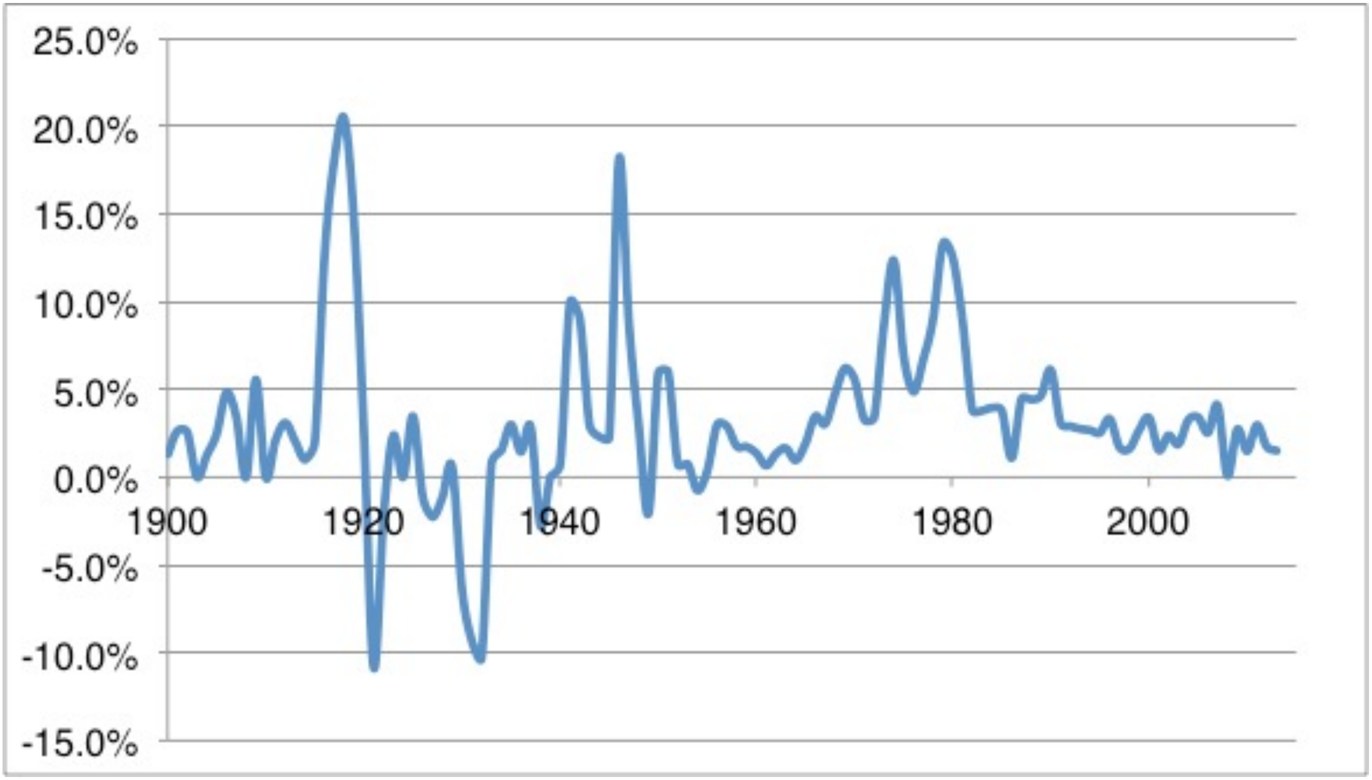
This Time

Time Value of Money

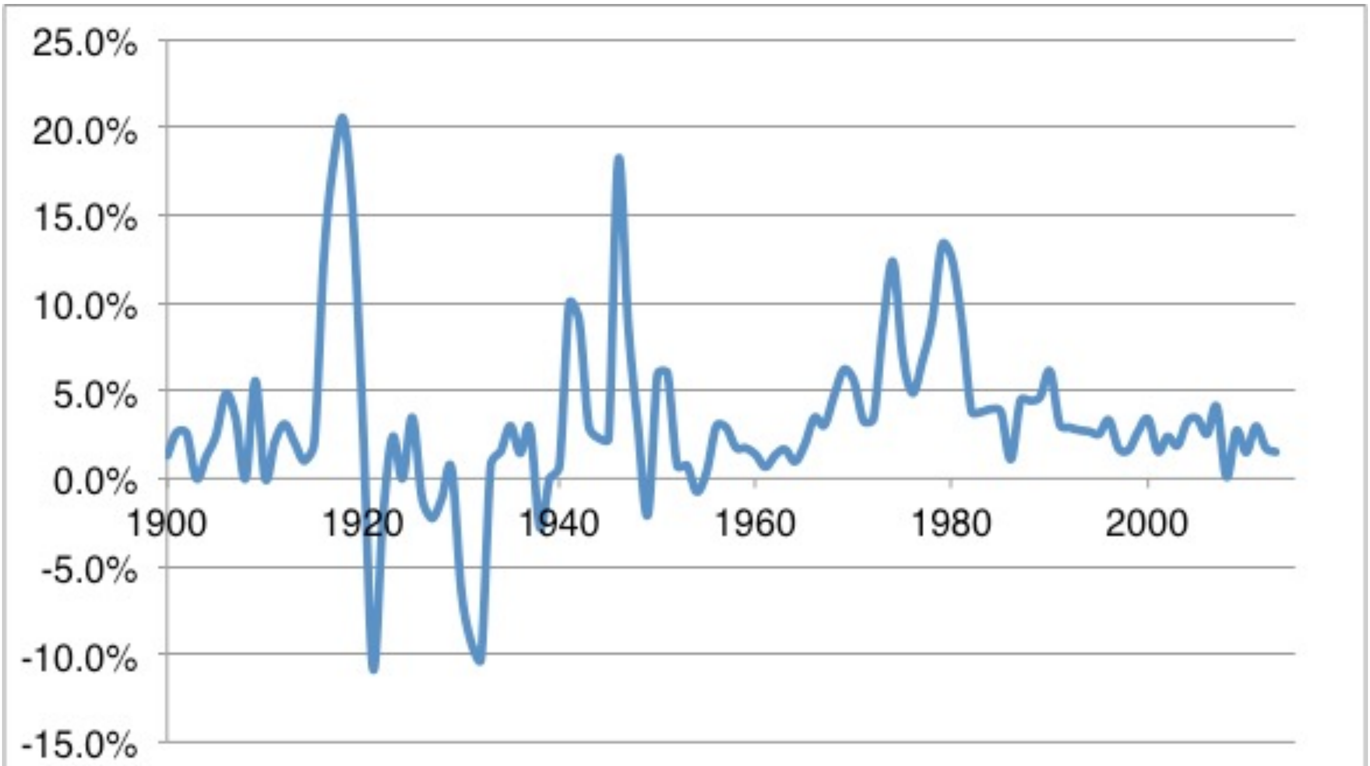
- Inflation

Inflation

Inflation



Inflation



How does inflation impact our returns?

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Lesson: Inflation won't affect the money we earn

Example – Savings (Account)

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32
2	$13.62	$285.94	$100.00	$185.94
3	$9.30	$195.24	$100.00	$95.24
4	$4.76	$100.00	$100.00	$0.00

Lesson: Inflation will affect what we can buy with the money

Real Discount Rate

$$1 + RR = (1+R) / (1+\pi)$$

RR is the real discount rate

π is expected inflation

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- Commonly used approximation:

$$RR = R - \pi$$

Real Discount Rate

$$1 + RR = (1+R) / (1+\pi)$$

RR is the real discount rate
π is expected inflation

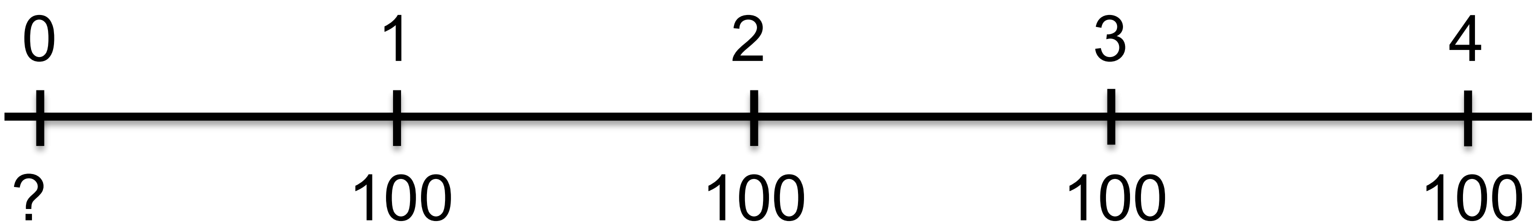
- Commonly used approximation:

$$RR = R - \pi$$

- For our example:

$$RR = (1+0.05) / (1+0.025) - 1 = 2.44\%$$

Savings with Inflation



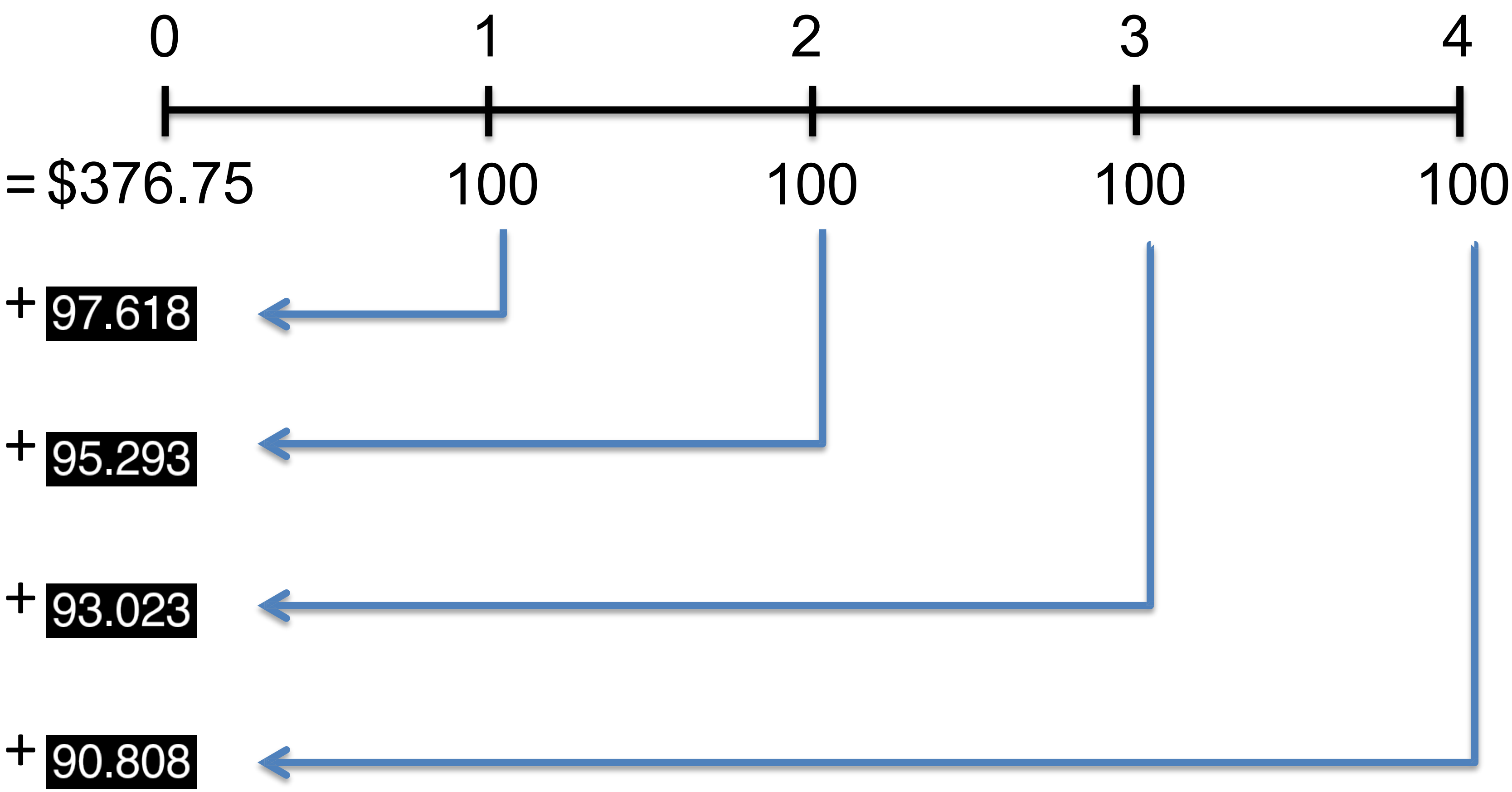
$$\frac{100}{(1+0.0244)}$$

$$\frac{100}{(1+0.0244)^2}$$

$$\frac{100}{(1+0.0244)^3}$$

$$\frac{100}{(1+0.0244)^4}$$

Savings with Inflation



Savings with Inflation

- Difference:
  - taxes affect $
  - Inflation affects consumption, not $
    - Earn nominal return but can't buy as much

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$376.75
1	$18.84	$395.59	$100.00	$295.59
2	$14.78	$310.37	$100.00	$210.37
3	$10.52	$220.89	$100.00	$120.89
4	$6.04	$126.93	$100.00	$26.93

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
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3	$10.52	$220.89	$100.00	$120.89
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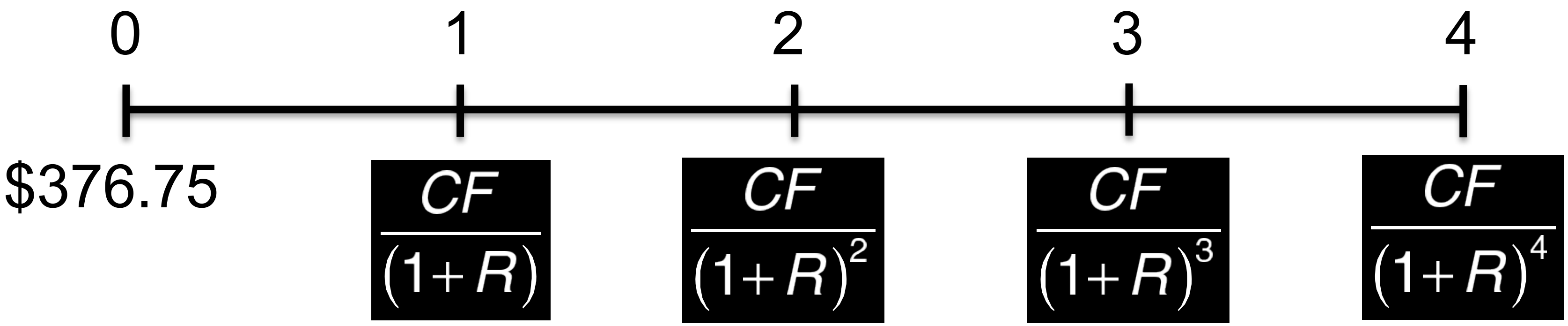
Implication: We have extra money(?). We need to change withdrawal amount. (Increase to buy costlier goods.)

Savings with Inflation



What is *CF*, the amount of money we can withdraw each year?

Savings with Inflation

0 1 2 3 4

$$\$376.75 = \frac{CF}{(1+0.05)} + \frac{CF}{(1+0.05)^2} + \frac{CF}{(1+0.05)^3} + \frac{CF}{(1+0.05)^4}$$

Use nominal rate since that reflects $ we earn

Savings with Inflation

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array}$$

$$\$376.75 = \frac{CF}{(1+0.05)} + \frac{CF}{(1+0.05)^2} + \frac{CF}{(1+0.05)^3} + \frac{CF}{(1+0.05)^4}$$

$$\begin{aligned} CF &= \$376.75\left(\frac{1}{(1+0.05)} + \frac{1}{(1+0.05)^2} + \frac{1}{(1+0.05)^3} + \frac{1}{(1+0.05)^4}\right)^{-1} \\ &= \$106.25 \end{aligned}$$

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$376.75
1	$18.84	$395.59	$106.25	$289.34
2	$14.47	$303.81	$106.25	$197.56
3	$9.88	$207.44	$106.25	$101.19
4	$5.06	$106.25	$106.25	$0.00

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$376.75
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3	$9.88	$207.44	$106.25	$101.19
4	$5.06	$106.25	$106.25	$0.00

Ideally withdrawals grow each year to accommodate inflation

Savings with Inflation

Year	Withdrawal
0	
1	$100 \times (1 + 0.025)^1 =$ \$102.50
2	$100 \times (1 + 0.025)^2 =$ \$105.06
3	$100 \times (1 + 0.025)^3 =$ \$107.69
4	$100 \times (1 + 0.025)^4 =$ \$110.38

This sequence of withdrawals maintains purchasing power of \$100 in today's terms

Savings with Inflation

Year		Withdrawal
0		
1	$100 \times (1 + 0.025)^1 =$	\$102.50
2	$100 \times (1 + 0.025)^2 =$	\$105.06
3	$100 \times (1 + 0.025)^3 =$	\$107.69
4	$100 \times (1 + 0.025)^4 =$	\$110.38

These are "nominal" values corresponding to the real \$100 purchasing power in year 0.

Savings with Inflation

Year	Withdrawal
0	
1	$102.50
2	$105.06
3	$107.69
4	$110.38

PV at 5% discount rate = $376.75

We discount nominal cash flows by the nominal rate to get the price.

Savings with Inflation

Year	Withdrawal
0	
1	$102.50
2	$105.06
3	$107.69
4	$110.38
PV at 5% discount rate =	$376.75

Note: PV of nominal CFs at nominal discount rate = PV of real cash flows at real rate

Savings with Inflation

Year	Withdrawal
0	
1	\$102.50
2	\$105.06
3	\$107.69
4	\$110.38
PV at 5% discount rate =	\$376.75

Intuition: The inflation term in the numerator and denominator cancel

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$376.75
1	$18.84	$395.59	$102.50	$293.09
2	$14.65	$307.74	$105.06	$202.68
3	$10.13	$212.81	$107.69	$105.13
4	$5.26	$110.38	$110.38	$0.00

Summary

Lessons

- Inflation does not affect $ return
- Inflation does purchasing power of $
- Real return, *RR*

$$RR = \frac{1+R}{1+\pi} - 1 \simeq R - \pi$$

where R is the nominal return and π is the rate of inflation

Lessons

- Discount real cash flows by the real rate of return, nominal cash flows by the nominal rate of return.

Coming up next

- Interest Rates
  - How do we value non-annual and irregular cash flows streams?
  - How do different compounding periods affect our valuations?

Interest Rates: APR and EAR

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Time Value of Money

- Intuition, tools and discounting
- Compounding
- Useful shortcuts
- Taxes
- Inflation

This Time

Interest Rates

- Interest rate quotes
- Non-annual cash flows and compounding

APR & EAR

Current 5-Year Jumbo CD Rates

Insititution	APY	Rate
CIT Bank ★★★★☆	2.40% Tue Dec 16	2.37% Compounded daily
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Difference between “Rate” and “APY”?

Current 5-Year Jumbo CD Rates

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Rate = APR or Annual Percentage Rate

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Rate = APR or Annual Percentage Rate

Measures amount of simple interest earned in a year

Current 5-Year Jumbo CD Rates

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*Bankrate.com as of 12/16/2014

Rate = APR or Annual Percentage Rate

Simple interest = interest earned *without* compounding

Current 5-Year Jumbo CD Rates

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Rate = APR or Annual Percentage Rate

Many bank quotes are in terms of APR

Current 5-Year Jumbo CD Rates

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*Bankrate.com as of 12/16/2014

Rate = APR or Annual Percentage Rate

APR typically *not* what we earn or pay

Current 5-Year Jumbo CD Rates

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APY = Annual Percentage Yield

Current 5-Year Jumbo CD Rates

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APY = EAR or

Effective Annual Rate

Current 5-Year Jumbo CD Rates

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*Bankrate.com as of 12/16/2014

APY = EAR or Effective Annual Rate

EAR measures actual amount of interest earned/paid in year

HOW ARE DIFFERENT RATES RELATED?

Lesson: EAR is a discount rate

EAR is what matters for computing interest and discounting cash flows

Lesson: APR is *not* a discount rate.

APR is a means to an end. We use it to get a discount rate (e.g., EAR)

How do we get from an APR to an EAR (and vice versa)?

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k}\right)^{k} - 1$$
$$= (1 + i)^{k} - 1$$

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k}\right)^k - 1$$
$$= (1+i)^k - 1$$

k is the number of compounding periods per year

Lesson: The relation between APR and EAR is:

$$\begin{aligned} EAR &= \left(1+\frac{APR}{k}\right)^k - 1 \\ &= (1+i)^k - 1 \end{aligned}$$

i is the periodic interest rate, or periodic discount rate

Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

Period

0 ?

100 ?

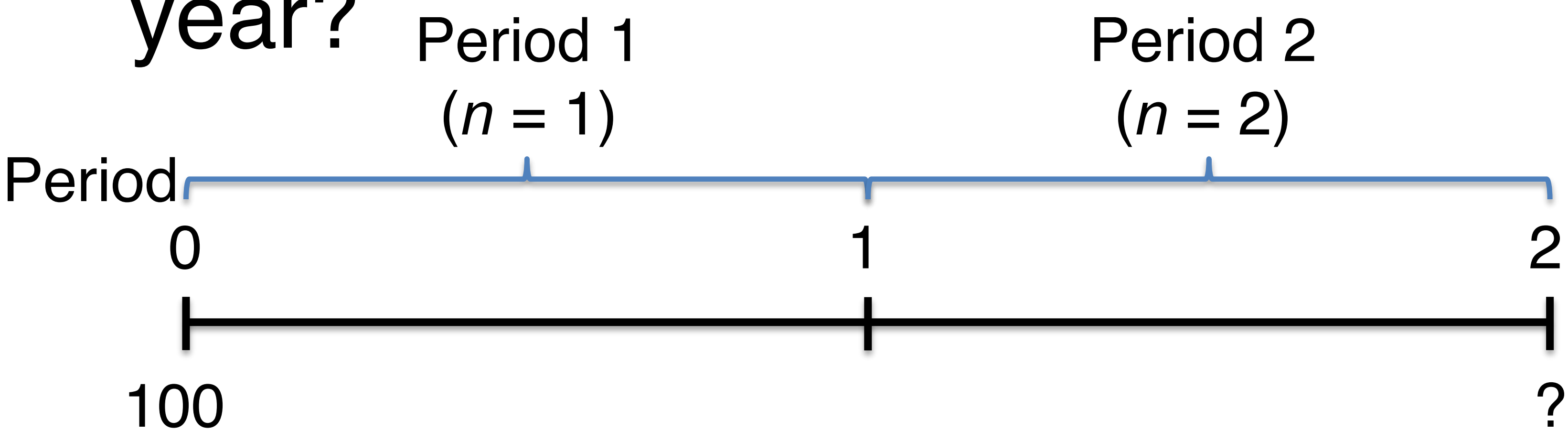
Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$k = 2$

Example

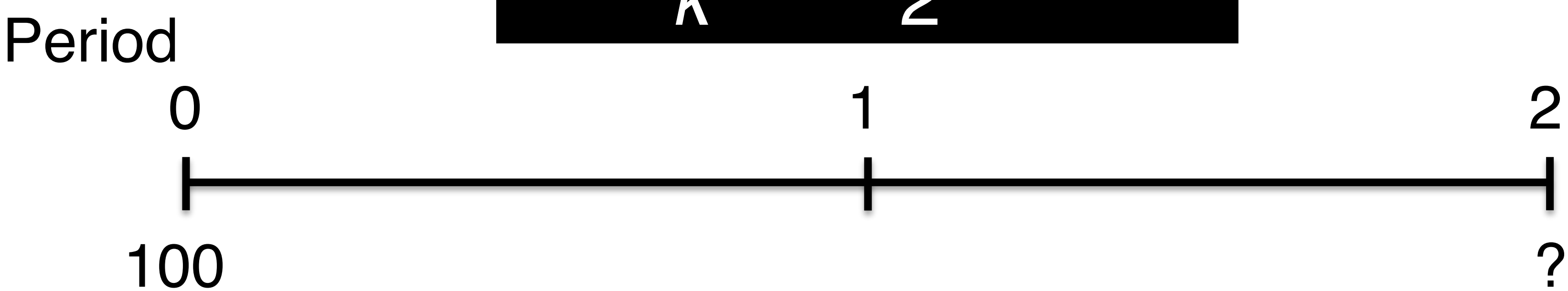
- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?



Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$i = \frac{APR}{k} = \frac{5\%}{2} = 2.5\%$$



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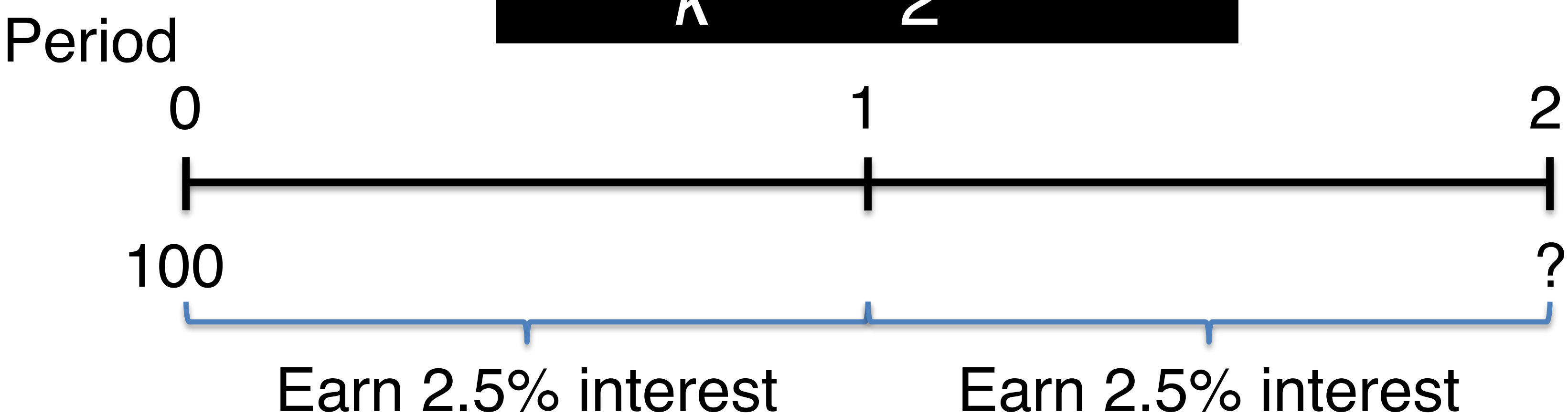

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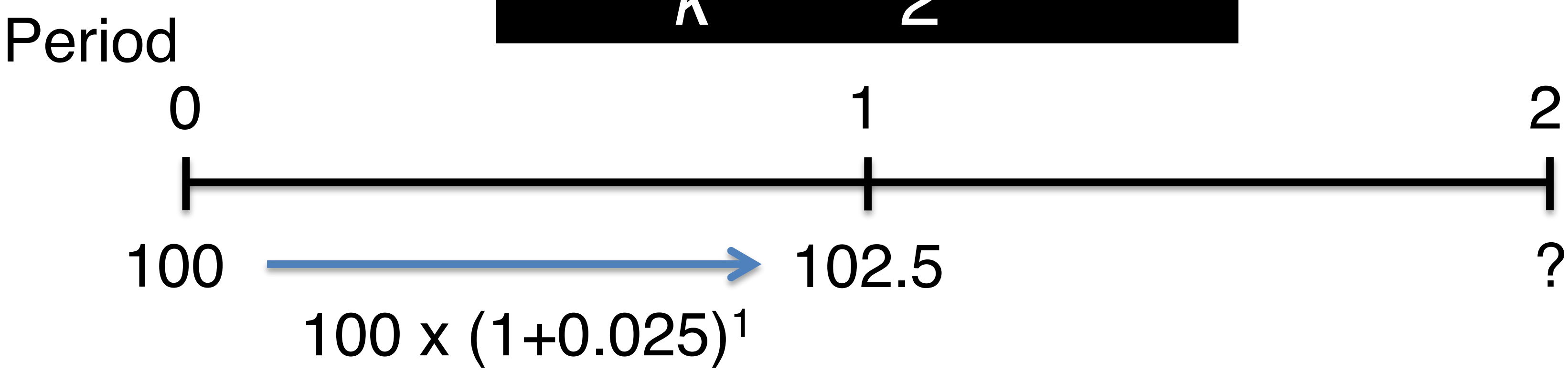


Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$i = \frac{APR}{k} = \frac{5\%}{2} = 2.5\%$$

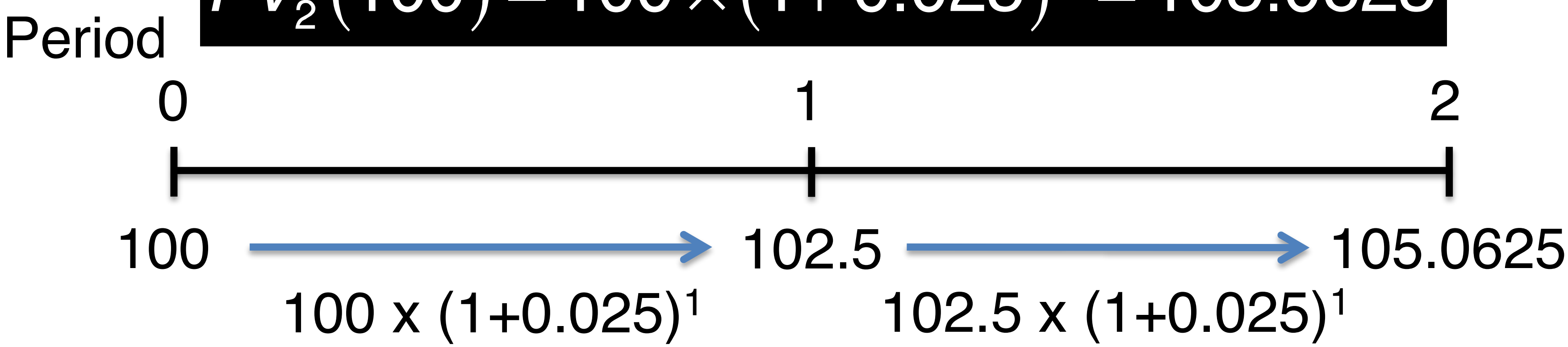
Period

0		1		2
100	$\rightarrow$	102.5	$\rightarrow$	105.0625
	$100 \times (1+0.025)^1$		$102.5 \times (1+0.025)^1$	

Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_2(100) = 100 \times (1 + 0.025)^2 = 105.0625$$



Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

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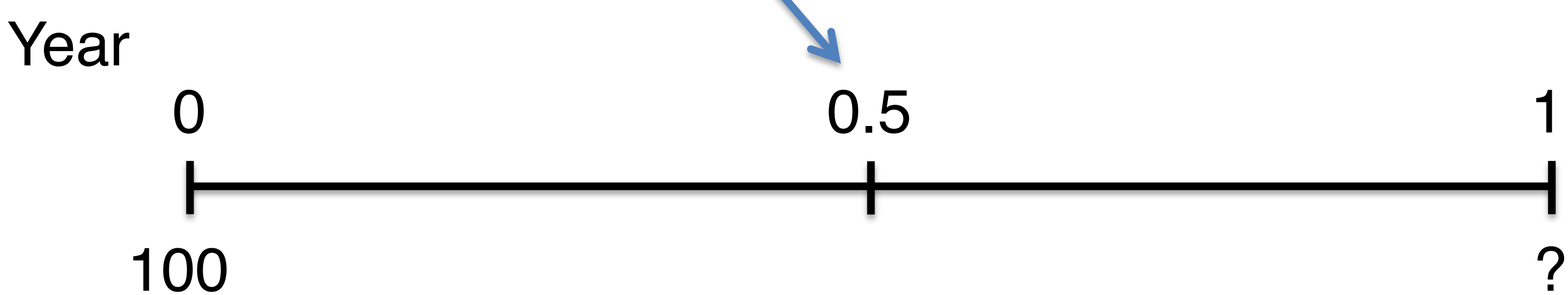
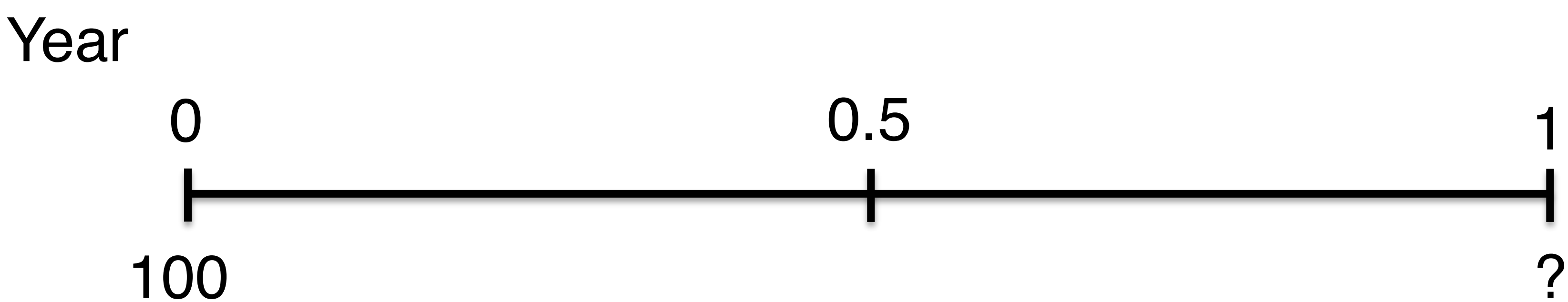


Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$EAR = (1+i)^k - 1 = (1+0.025)^2 - 1 = 5.0625\%$$



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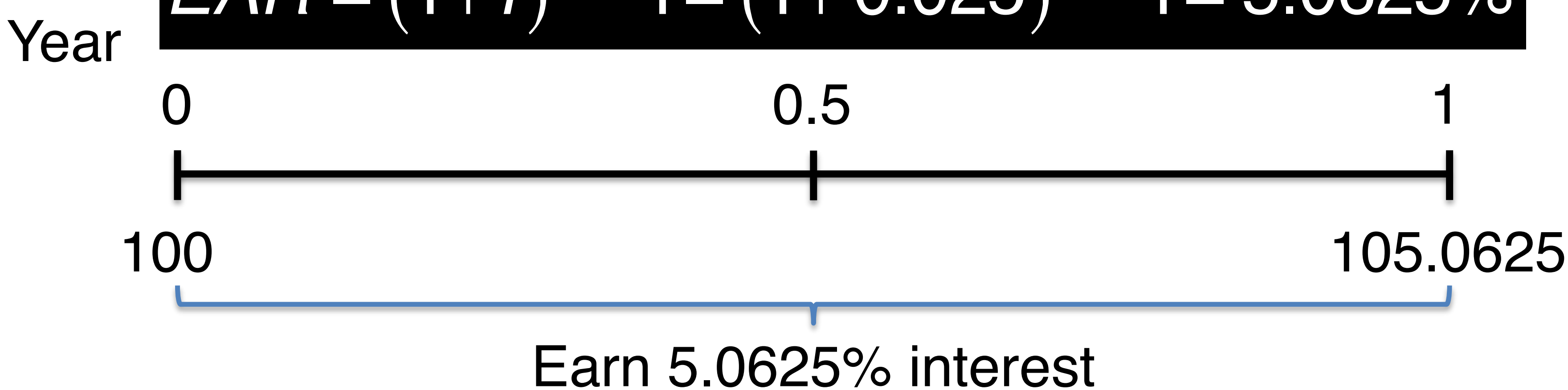
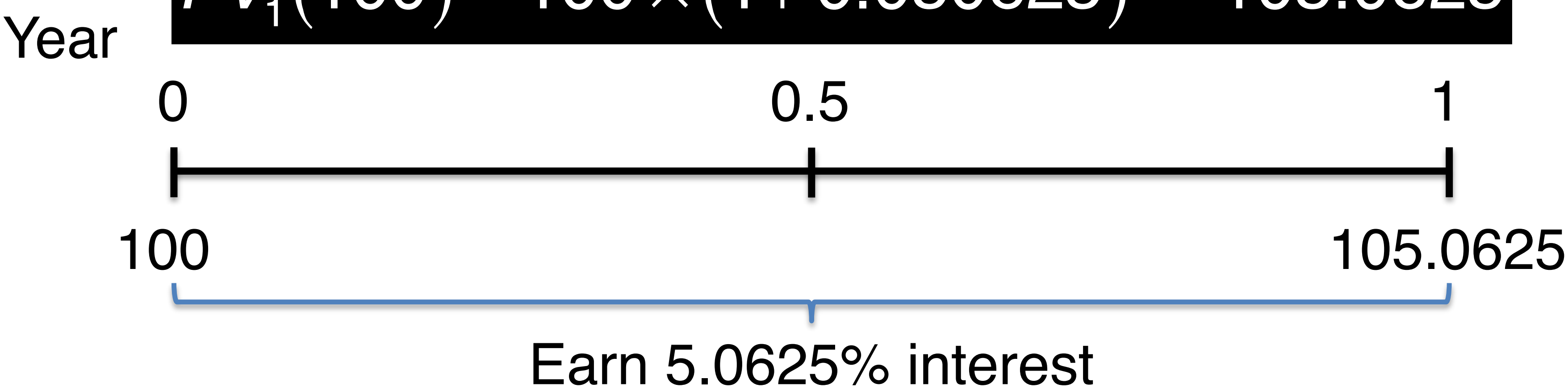
$$EAR = (1+i)^k - 1 = (1+0.025)^2 - 1 = 5.0625\%$$



Example

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_1(100) = 100 \times (1 + 0.050625)^1 = 105.0625$$



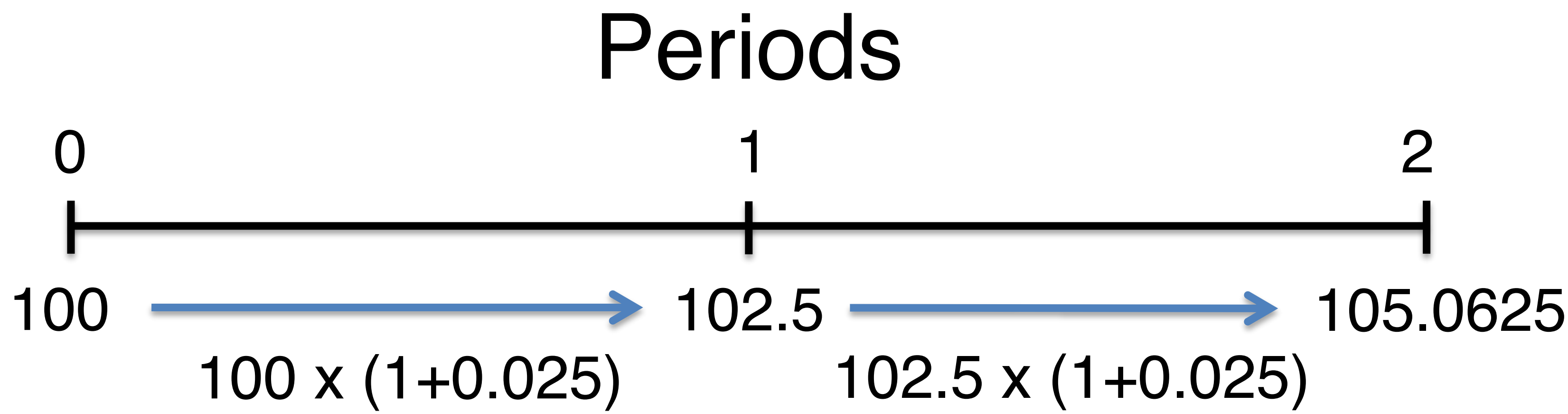
Lesson: If you discount cash flows using EAR, then measure time in years. If you discount cash flows using periodic interest rate, then measure time in periods.

Proof

$$
\begin{aligned}
(1+EAR)^T &= \left(1+(1+i)^k-1\right)^T \\
&= \left((1+i)^k\right)^T \\
&= (1+i)^{kT} \\
&= (1+i)^N
\end{aligned}
$$

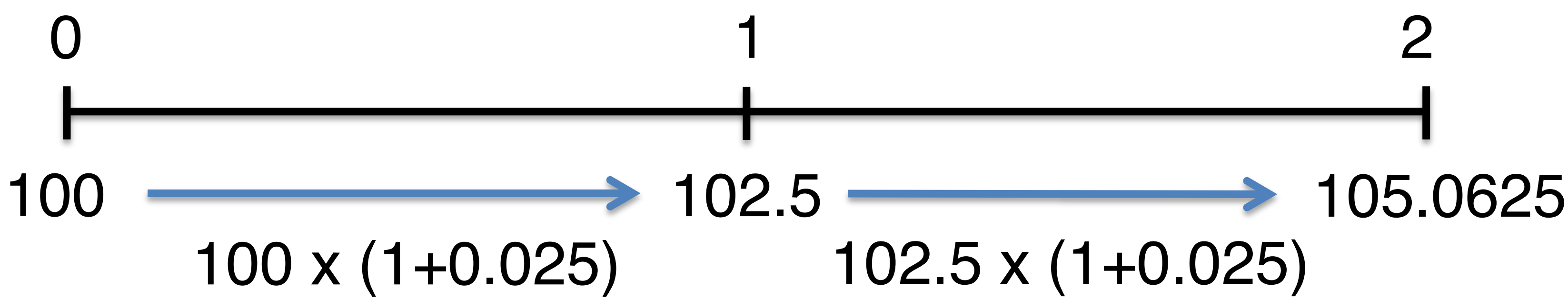
where $N = kT = \#$ of periods

Periods vs Years

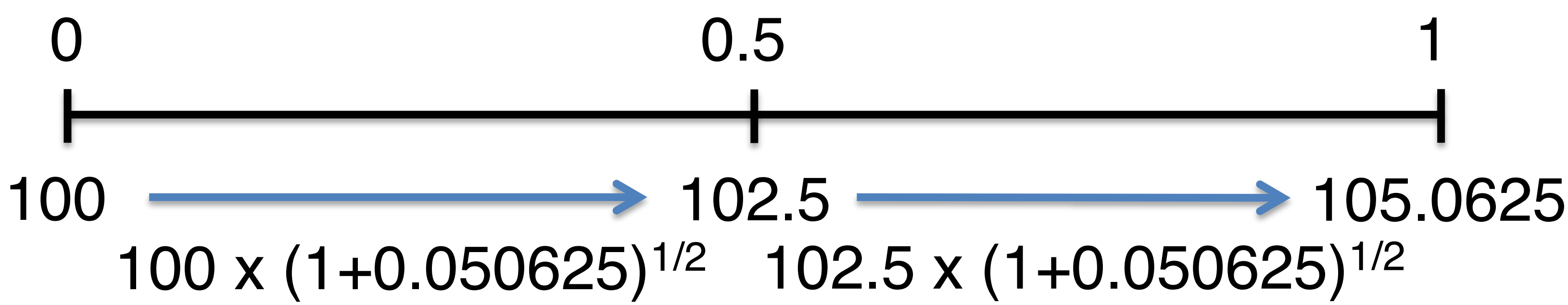


Periods vs Years

Periods



Years



Current 5-Year Jumbo CD Rates

Insitition	APY	Rate
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APR = 2.37%

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APR = 2.37%

k = 365 (or 360, 252)

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APR = 2.37%

k = 365 (or 360, 252)

→ i = 2.37% / 365

= 0.006714%

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APR = 2.37%

k = 365 (or 360, 252)

→ i = 2.37% / 365

= 0.006714%

→ EAR =

$(1+0.006714\%)^{365}-1$

= 2.398%

Summary

Lessons

- EAR is a discount rate
  – Measures cash flows in years

- Period interest rate, *i*, is a discount rate
  – Measures cash flows in periods

- APR is not a discount rate

Lessons

- Moving between EAR and APR

$$EAR = \left(1 + \frac{APR}{k}\right)^k - 1$$
$$= (1+i)^k - 1$$

where

$i = APR\,/\,k$ and k = # of periods per year

Coming up next

- Interest Rates
  - Term Structure of interest rates and the yield curve

Interest Rates: Term Structure

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Interest Rates

- Interest rate quotes
- Non-annual cash flows and compounding

This Time

Interest Rates

- Term Structure
- Yield Curve

Term Structure

Thus far we have assumed discount rates are constant through time

Thus far we have assumed discount rates are constant through time

$$PV = \frac{CF_1}{(1+R)} + \frac{CF_2}{(1+R)^2} + \frac{CF_3}{(1+R)^3} + \ldots$$

Same *R* …

Home Mortgage Refinancing Rates

REFINANCE RATES AVERAGES

Product	Rate	Change	Last week
30 year fixed refi	4.12%	-- 0.00	4.12%
15 year fixed refi	3.19%	▲ 0.03	3.16%
10 year fixed refi	3.23%	▲ 0.01	3.22%

Fixed Term CD Rates

Fixed Term CD - Time Deposits & IRA/CESA CDs†

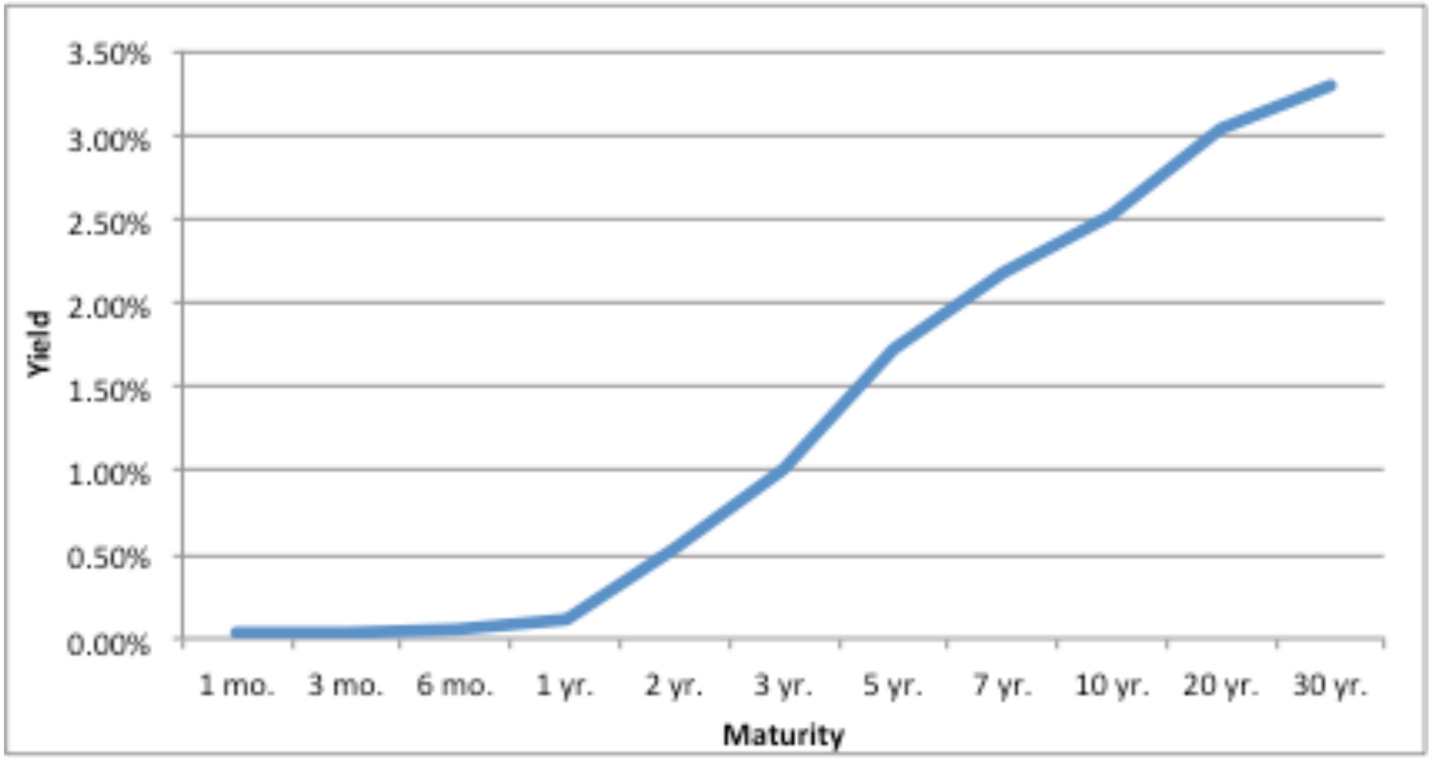
	Less than $10,000		$10,000-$99,999		$100,000 and over	
	Rate %	APY %†	Rate %	APY %†	Rate %	APY %†
28 - 179 Days*	0.03	0.03	0.03	0.03	0.03	0.03
06 - 11 Months	0.03	0.03	0.03	0.03	0.03	0.03
12 - 17 Months	0.05	0.05	0.05	0.05	0.05	0.05
18 - 23 Months	0.07	0.07	0.07	0.07	0.07	0.07
24 - 35 Months	0.10	0.10	0.10	0.10	0.10	0.10
36 - 47 Months	0.12	0.12	0.12	0.12	0.12	0.12
48 - 59 Months	0.15	0.15	0.15	0.15	0.15	0.15
60 - 119 Months	0.15	0.15	0.15	0.15	0.15	0.15
120 Months	0.15	0.15	0.15	0.15	0.15	0.15

CD - Time Deposit Minimum to open: $1,000
IRA/CESA CDs Minimum to open: $1,000 IRAs / $500 CESAs
*IRA/CESA CDs are not available for a term less than 6 Months

The Term Structure is the relation between the investment term and the interest rate

The Yield Curve is a graph of the relation between the investment term and the interest rate

Treasury Yield Curve – 7/24/2014



What is a yield?

A yield, y, is the one discount rate that when applied to the promised cash flows of the security recover the price of the security.

What is a yield?

A yield, *y*, is the one discount rate that when applied to the promised cash flows of the security recover the price of the security.

$$\text{Price} = \frac{CF_1}{(1+y)} + \frac{CF_2}{(1+y)^2} + \frac{CF_3}{(1+y)^3} + ... + \frac{CF_T}{(1+y)^T}$$

To build the yield curve simply compute the yield for securities of different maturities..

$$P_1 = \frac{CF_1}{(1+y_1)}$$

$$P_2 = \frac{CF_1}{(1+y_2)} + \frac{CF_2}{(1+y_2)^2}$$

$$\vdots$$

$$P_T = \frac{CF_1}{(1+y_T)} + \frac{CF_2}{(1+y_T)^2} + \frac{CF_3}{(1+y_T)^3} + \ldots + \frac{CF_T}{(1+y_T)^T}$$

Same as computing the discount rate for securities with different maturities

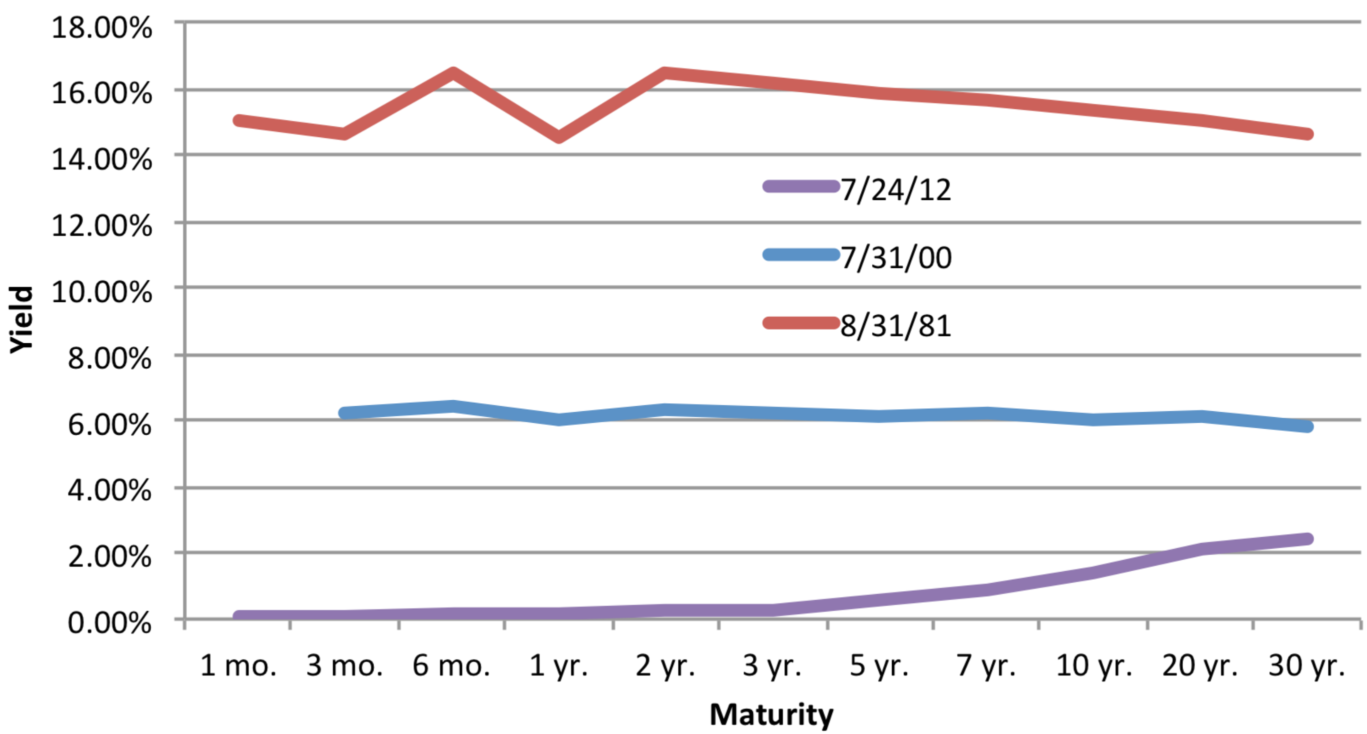
$$P_1 = \frac{CF_1}{(1+R_1)}$$

$$P_2 = \frac{CF_1}{(1+R_2)} + \frac{CF_2}{(1+R_2)^2}$$

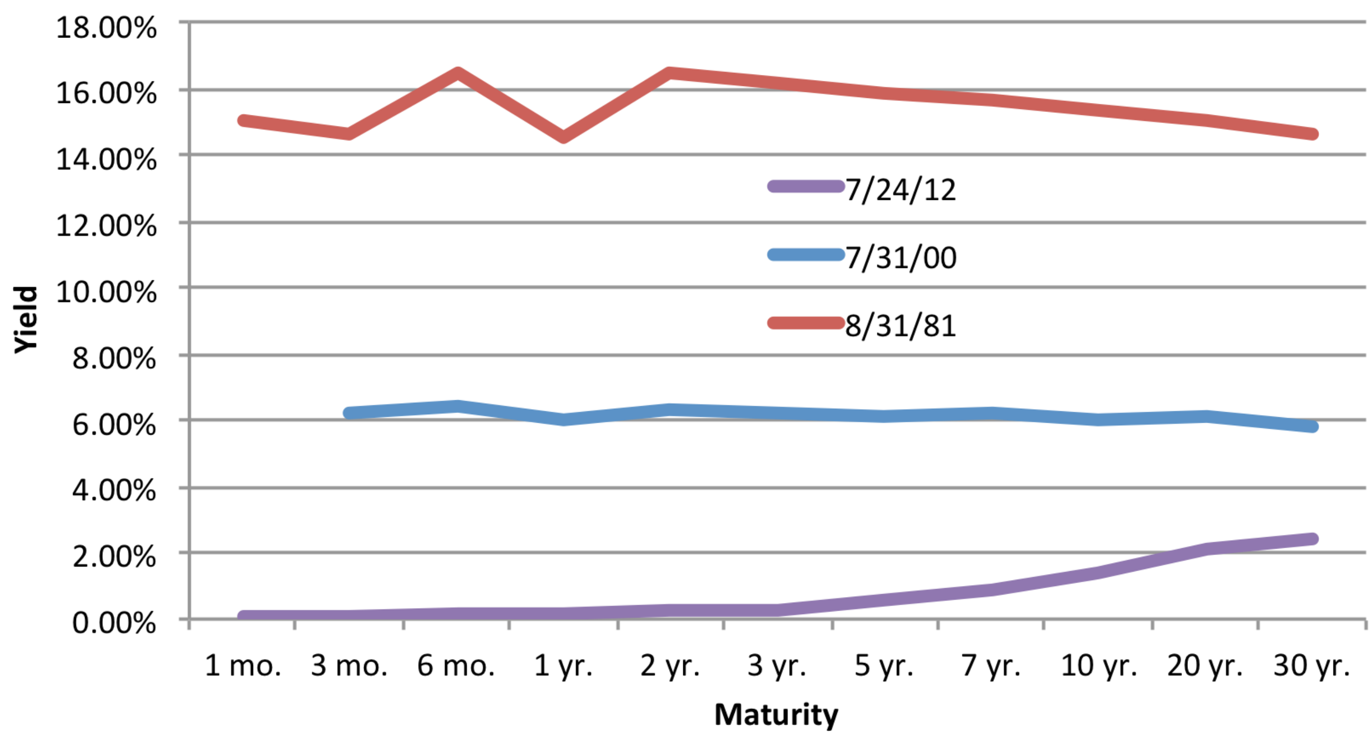
$$\vdots$$

$$P_T = \frac{CF_1}{(1+R_T)} + \frac{CF_2}{(1+R_T)^2} + \frac{CF_3}{(1+R_T)^3} + \ldots + \frac{CF_T}{(1+R_T)^T}$$

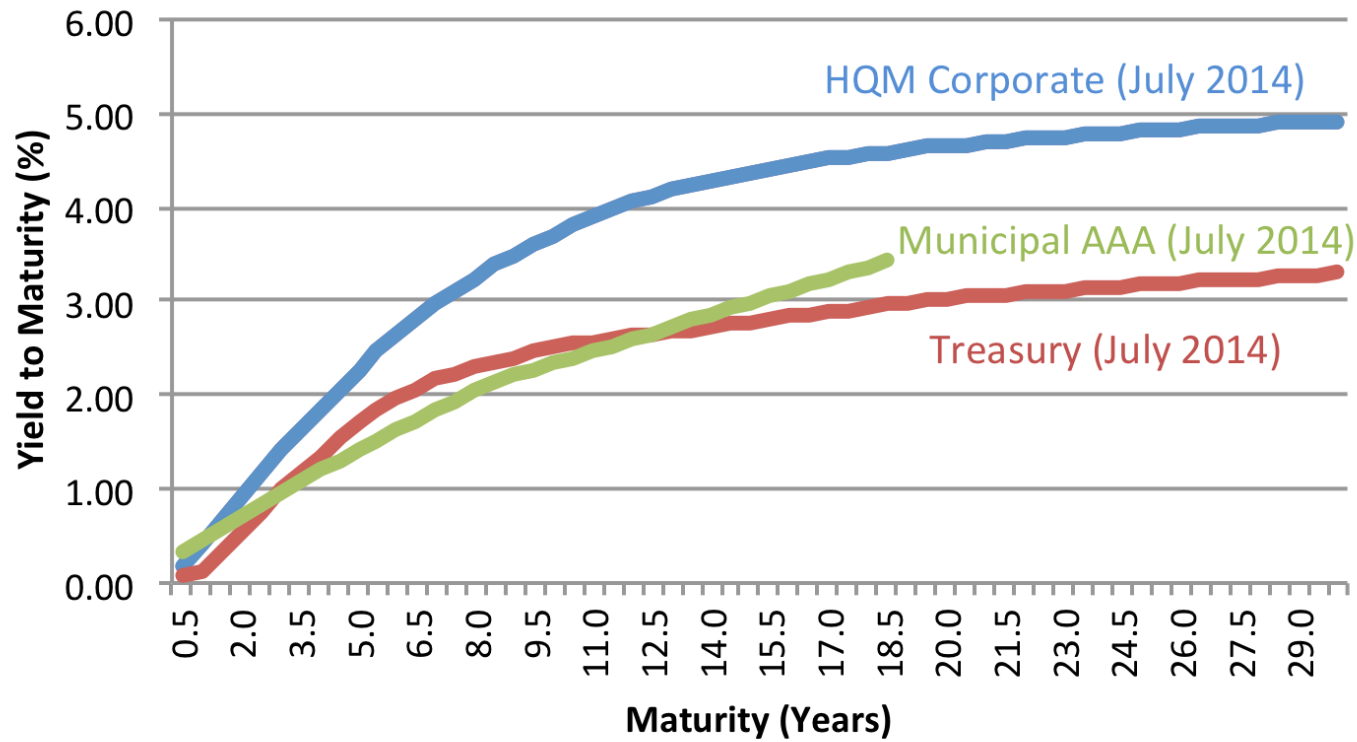
Yield Curves can move around a lot



Treasury Yield Curves graph the relation between interest rates on risk-free loans and loan maturity



Other yield curves graph the relation between interest rates on risky loans and loan maturity



Lesson: Yields vary by maturity and risk

ALL CORPORATES: YIELD BY CREDIT RATING AND MATURITY
(Median Yield For Previous Trading Day)

	1-2 Year	2-5 Year	5-10 Year	10 Year+
AAA	0.58	1.46	2.45	3.38
AA	0.47	1.54	2.70	3.89
A	0.84	1.82	3.01	3.97
BBB	1.23	2.33	3.82	5.51

All of these interest rates are referred to as spot rates

The spot rate is the interest rate for a loan made today

- Typically a different spot rate for loans of different maturities and risk

Punch line:

This is an approximation:

$$PV = \frac{CF_1}{(1+R)} + \frac{CF_2}{(1+R)^2} + \frac{CF_3}{(1+R)^3} + \ldots$$

for

$$PV = \frac{CF_1}{(1+R_1)} + \frac{CF_2}{(1+R_2)^2} + \frac{CF_3}{(1+R_3)^3} + \ldots$$

Summary

Lesson

- The term structure refers to the relation between interest rates and investment term
  - Loans (savings) of different maturities (terms) typically have different interest rates

Lesson

- The yield curve graphs the relation between interest rates and investment term

- Interest rates vary by the risk of the investment

Lessons

- The spot rate is the interest rate for a loan made today

- The spot rate comes from the yield curve and there is typically a different spot rate for loans of different maturities (and risk)

Coming up next

- Discounted Cash Flow (DCF)
  - Time value of money in a corporate setting
  - Figure out how to derive (free) cash flows
  - Capital budgeting is illustrative vehicle

Discounted Cash Flow: Decision Making

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Interest Rates

- APR
- EAR
- Non-annual and cash flows and compounding
- Term Structure

This Time

Discounted Cash Flow (DCF)

- Decision making

Decision Making

Q: How should we make financial decisions?

Q: How should we make financial decisions?

A: Undertake those actions that create value

Q: Which actions create value?

Q: Which actions create value?

A: Those in which the benefits exceed the costs…

Q: What if costs and benefits arrive at different times?

Q: What if costs and benefits arrive at different times?

A: Compare *PV*(Benefits) to *PV*(Costs)

Q: What if costs and benefits arrive at different times?

A: The discount rate, *R*, adjusts for timing *and* risk of cash flows

Lesson: The NPV Decision Rule says accept all projects with a positive NPV and reject all projects with a negative NPV

$$NPV = PV(\text{Benefits}) - PV(\text{Costs})$$

Lesson: The NPV Decision Rule says accept all projects with a positive NPV and reject all projects with a negative NPV

$$NPV = PV(\text{Benefits}) - PV(\text{Costs})$$

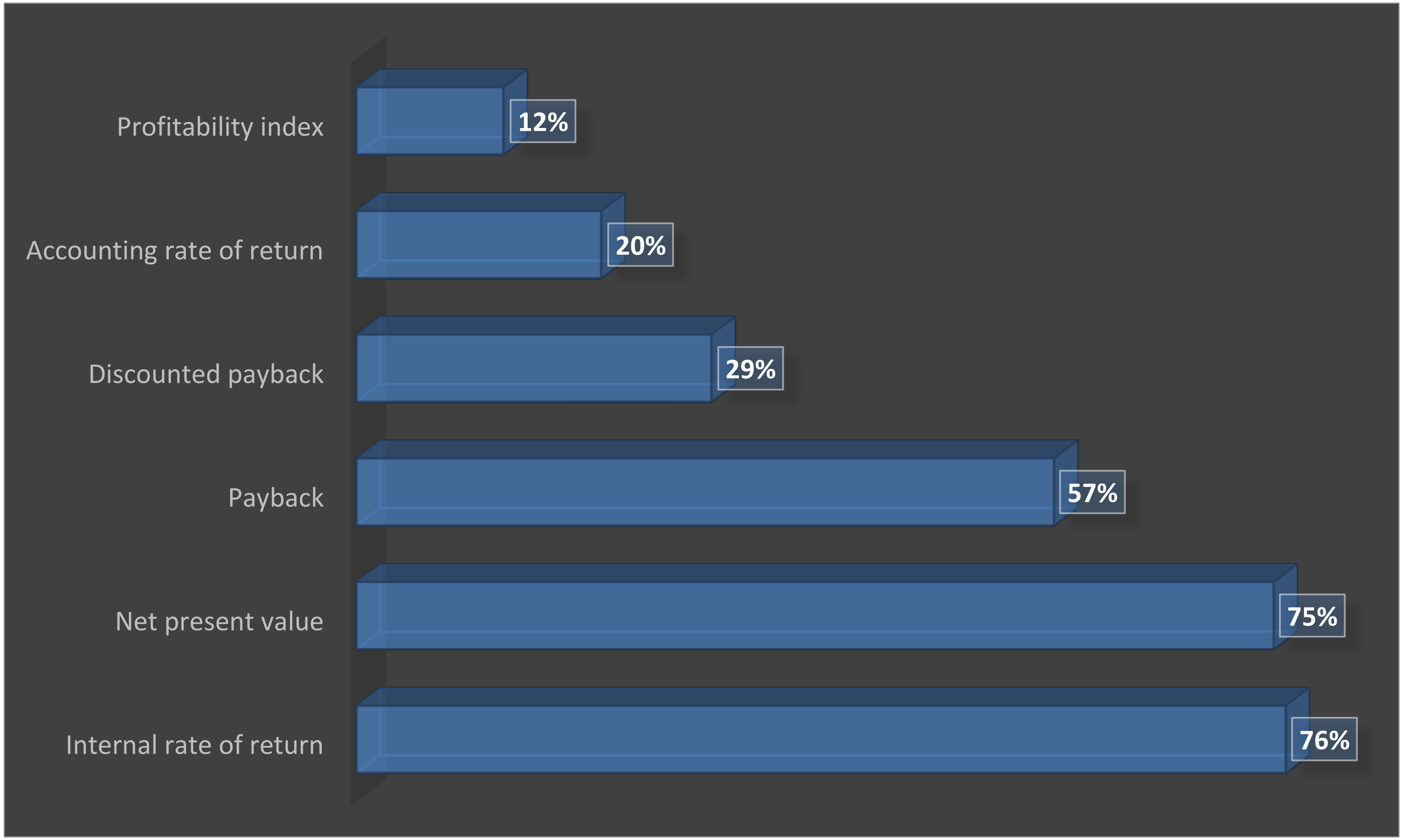
$$= FCF_0 + \frac{FCF_1}{(1+R)} + \frac{FCF_2}{(1+R)^2} + \ldots + \frac{FCF_T}{(1+R)^T}$$

Lesson: The NPV Decision Rule says accept all projects with a positive NPV and reject all projects with a negative NPV

$$NPV > 0 \Rightarrow \text{Accept}$$

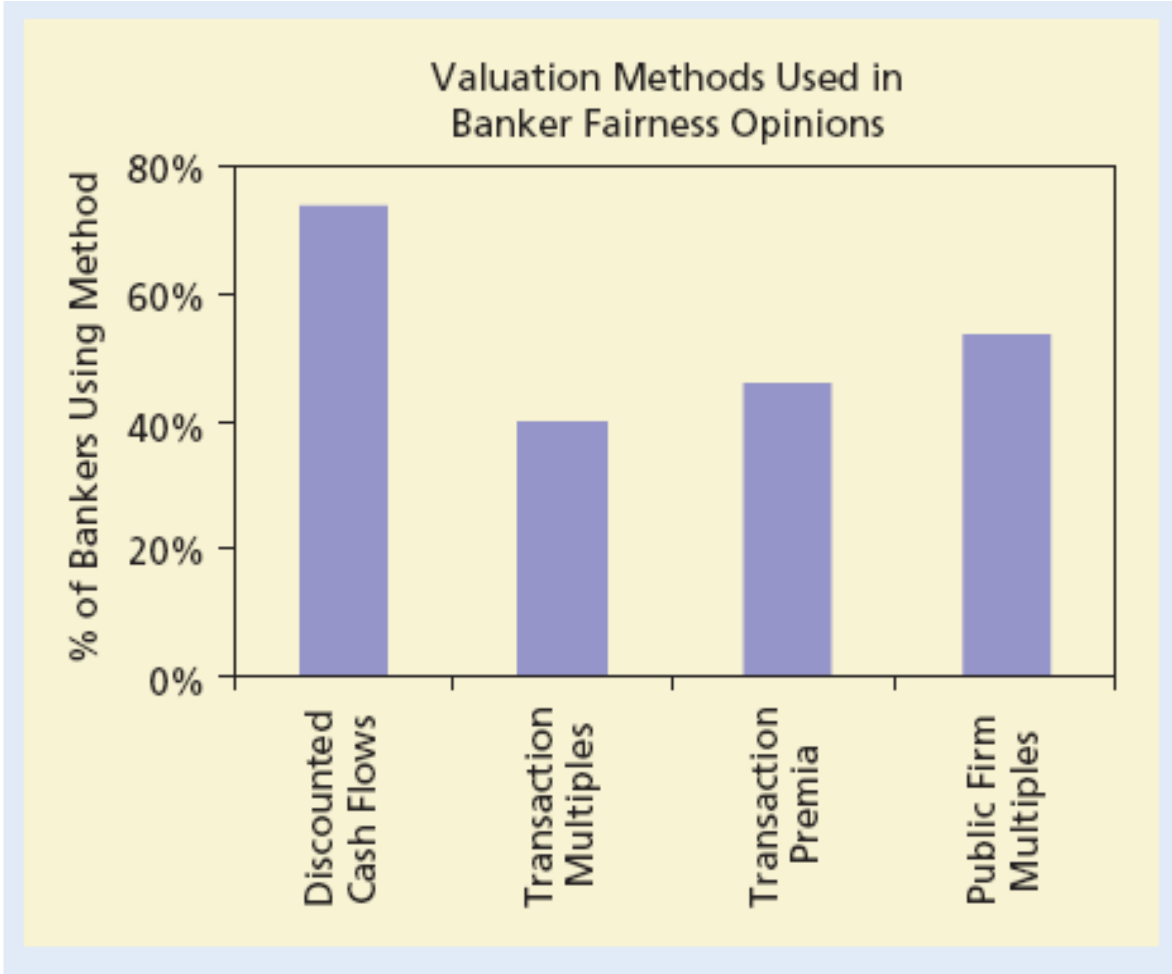
$$NPV < 0 \Rightarrow \text{Reject}$$

DECISION MAKING IN PRACTICE

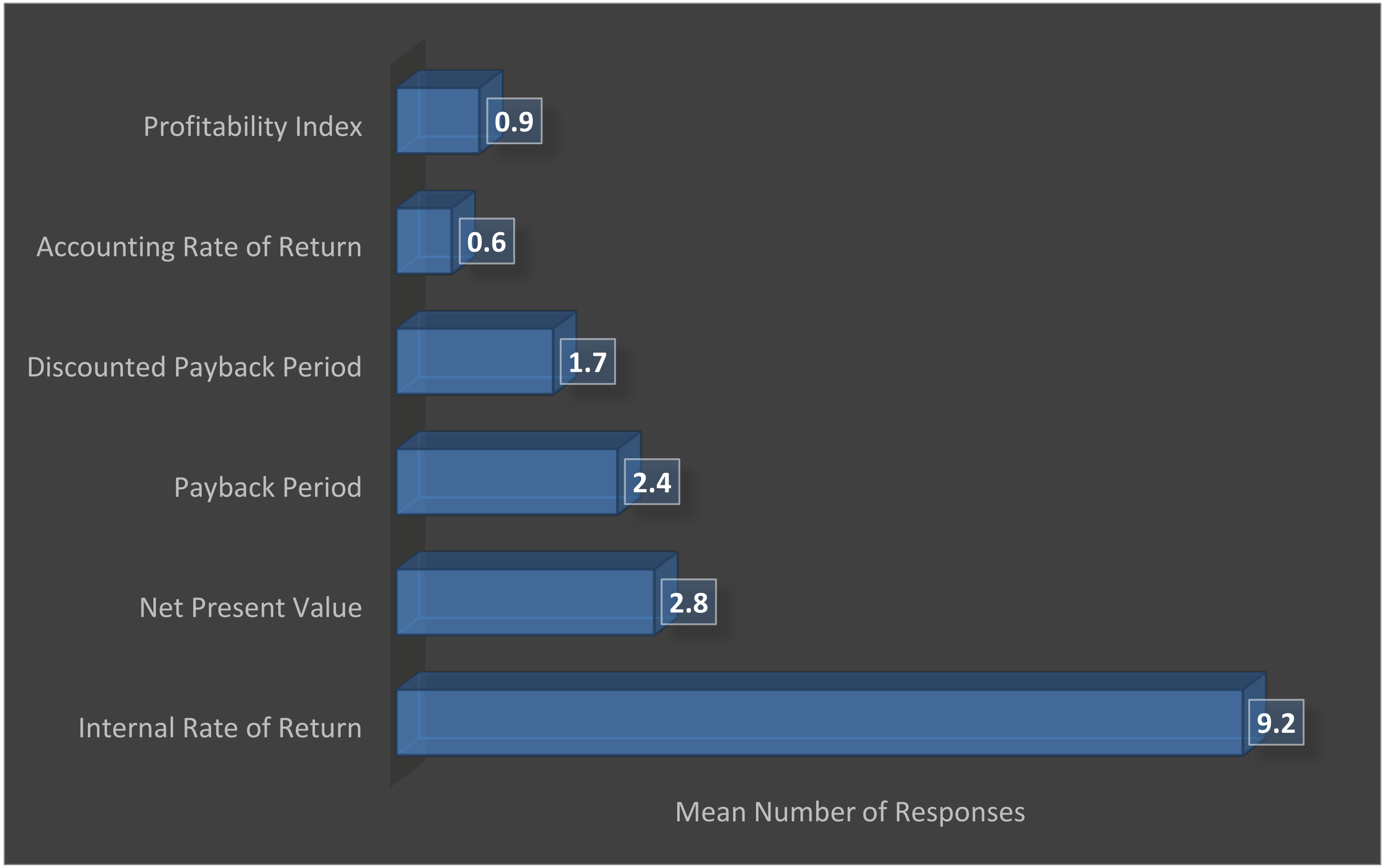


Survey of US CFOs: How frequently do you use capital budgeting techniques?

Graham and Harvey, 2001, The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics*



How Investment Bankers Value Companies (Source: page 145 in Graham, Smart, Megginson, 2010)



Which criterion do you use to evaluate an investment?

What do Private Equity Firms Say they Do? (Paul Gompers, Steve Kaplan, and Vladimir Mukharlyamov)

Summary

Lessons

- NPV is a decision rule that quantifies the value implications of decisions
  - Optimal rule
- Other rules still informative
  - Understand their weaknesses

Coming up next

- Discounted Cash Flow (DCF)
  - How to compute free Cash Flow

Discounted Cash Flow: Free Cash Flow

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Decision making
  - NPV rule
  - IRR
  - Payback
- Practical approach

This Time

Discounted Cash Flow (DCF)

- Free Cash Flow

Free Cash Flow

Recall: Two components to NPV

Recall: Two components to NPV

1. Free Cash Flows

Recall: Two components to NPV

1. Free Cash Flows
2. Discount Rate

Recall: Two components to NPV

1. Free Cash Flows
2. Discount Rate

FCF = (Revenue

FCF = (Revenue – Costs

$$FCF = (Revenue - Costs - Depreciation)$$

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$

$$\text{FCF} = \overbrace{(\text{Revenue} - \text{Costs} - \text{Depreciation}) \text{ x } (1 - t_C)}^{\substack{\text{Unlevered Net Income} \\ \text{Net Operating Profit After Taxes (NOPAT)} \\ \text{Earnings Before Interest After Taxes (EBIAT)}}}$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation}$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures}$$

$$\begin{aligned}\text{FCF} = &\ (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) \\ &+ \text{Depreciation} - \text{Capital Expenditures} \\ &- \text{Change in Net Working Capital}\end{aligned}$$

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

Lesson: FCF is the residual cash flow left over after **all** of the project's requirements have been satisfied and implications accounted for.

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Lesson: FCF is the cash flow that can be distributed to the financial claimants (e.g., debt and equity) of the project or company

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Lesson: FCF is **not** the same as accounting cash flow from the statement of cash flows (SCF) but we can derive FCF from the SCF.

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Lesson: FCF is more precisely unlevered free cash flow to distinguish it from free cash flow to equity (FCFE) or levered free cash flow.

$$
\begin{aligned}
\text{FCFE} = {} & (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) \\
& + \text{Depreciation} - \text{Capital Expenditures} \\
& - \text{Change in Net Working Capital} \\
& - \text{Interest} \times (1 - t_C) \\
& + \text{Net Borrowing}
\end{aligned}
$$

$$FCFE = FCF - Interest \times (1 - t_C) + Net\ Borrowing$$

$$\text{FCFE} = \text{FCF} - \text{Interest} \times (1 - t_C) + \text{Net Borrowing}$$

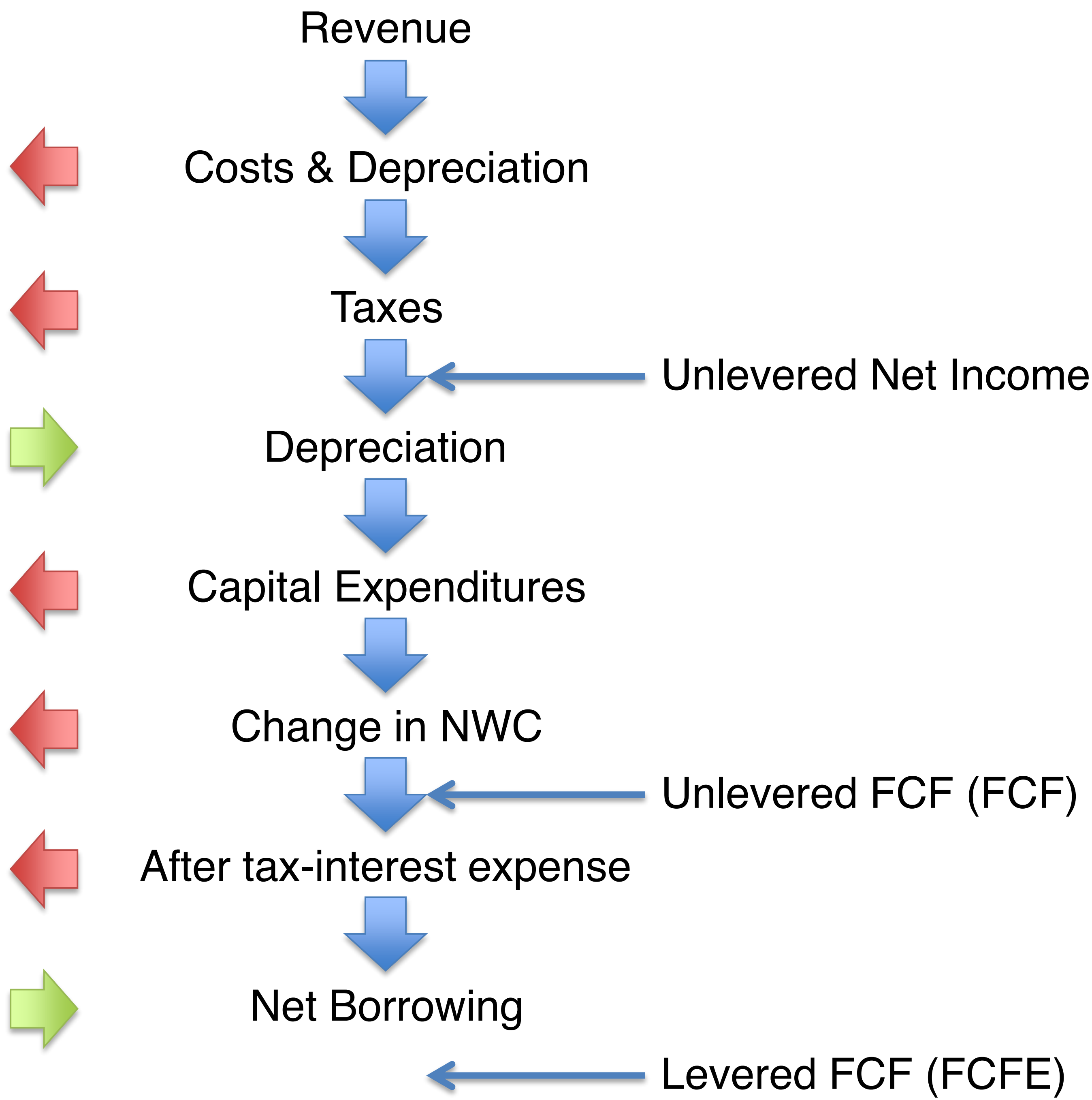
Lesson: FCFE is residual cash flow left over after **all** of the project's requirements have been satisfied, implications accounted for, *and* all debt financing has been satisfied

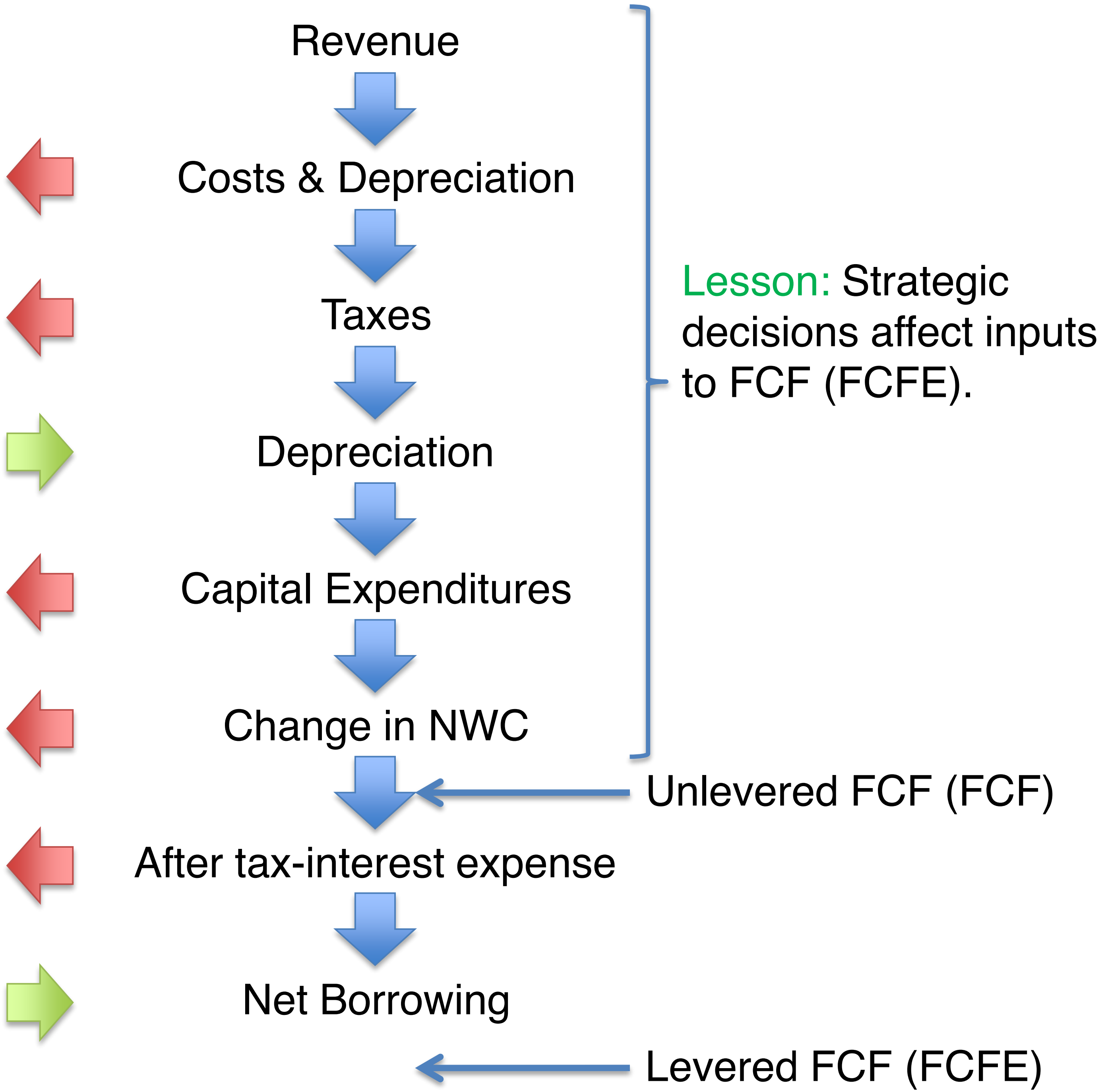
$$FCFE = FCF - Interest \times (1 - t_C) + Net\ Borrowing$$

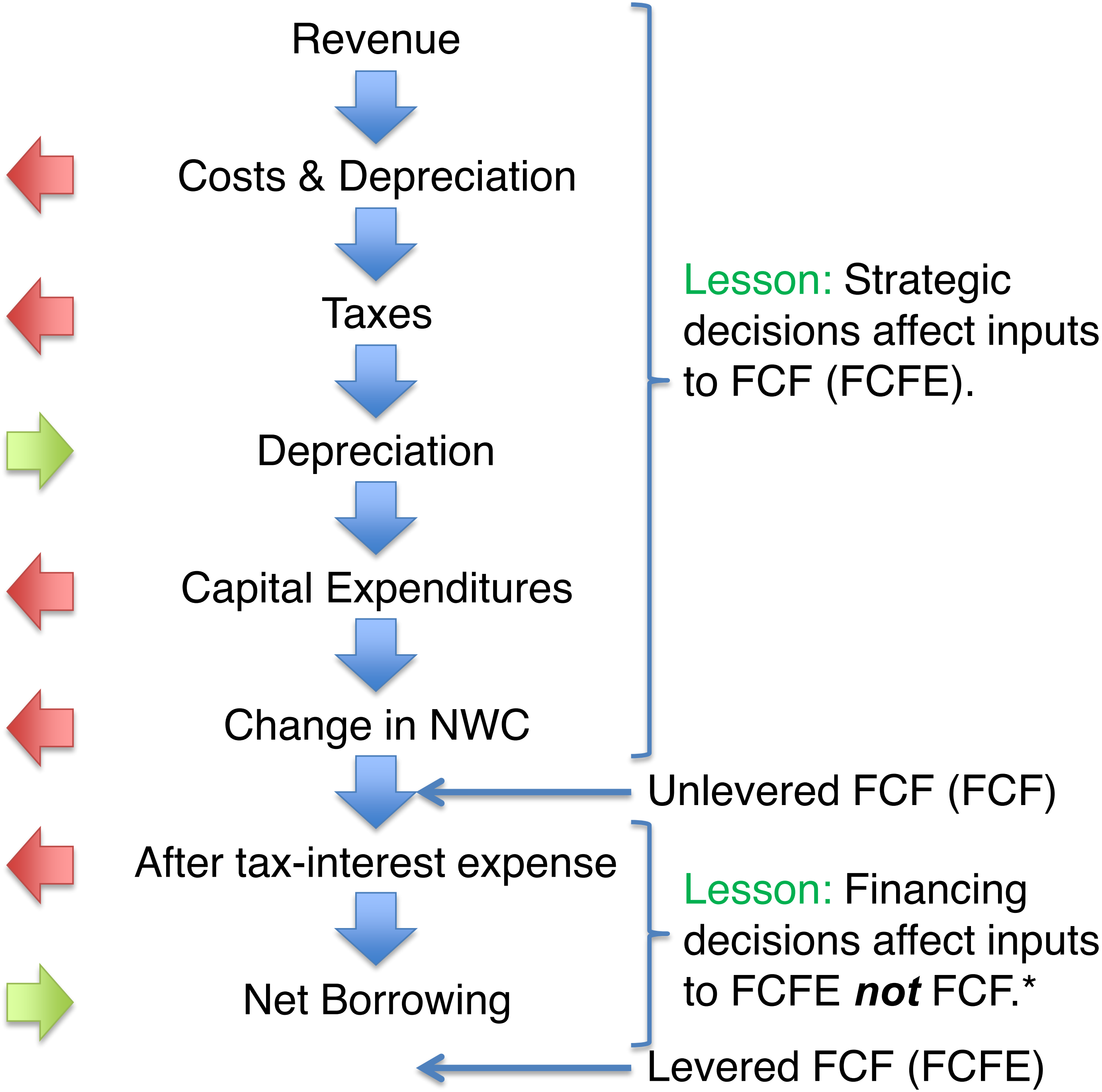
Lesson: FCFE is the cash flow that can be distributed to the shareholders (i.e., equity) of the project or company

$$FCFE = FCF - Interest \times (1 - t_C) + Net\ Borrowing$$

Lesson: FCF is more precisely levered free cash flow because it is affected by the choice of leverage (i.e., debt)


Revenue
Costs & Depreciation
Taxes
Unlevered Net Income
Depreciation
Capital Expenditures
Change in NWC
Unlevered FCF (FCF)
After tax-interest expense
Net Borrowing
Levered FCF (FCFE)


Revenue
Costs & Depreciation
Taxes
Depreciation
Capital Expenditures
Change in NWC
After tax-interest expense
Net Borrowing
Lesson: Strategic decisions affect inputs to FCF (FCFE).
Unlevered FCF (FCF)
Levered FCF (FCFE)


Revenue
Costs & Depreciation
Taxes
Depreciation
Capital Expenditures
Change in NWC
Unlevered FCF (FCF)
After tax-interest expense
Net Borrowing
Levered FCF (FCFE)
Lesson: Strategic decisions affect inputs to FCF (FCFE).
Lesson: Financing decisions affect inputs to FCFE *not* FCF.*

Summary

Lessons

- NPV is a decision rule that quantifies the value implications of decisions
  - Positive NPV implies value increasing
  - Negative NPV implies value decreasing

Coming up next

- Discounted Cash Flow (DCF)
  - Forecast Drivers

Discounted Cash Flow: Forecast Drivers

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Free Cash Flow

This Time

Discounted Cash Flow (DCF)

- Forecast Drivers

Forecast Drivers

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\begin{aligned} FCF = &(\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) \\ &+ \text{Depreciation} - \text{Capital Expenditures} \\ &- \text{Change in Net Working Capital} \end{aligned}$$

$$\text{Revenue} = \text{Market Size} \times \text{Market Share} \times \text{Price}$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{Revenue} = \text{Market Size} \times \text{Market Share} \times \text{Price}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Revenue Forecasts						
Market Forecasts						
Initial Market Size (Units, million)		1.00				
Market Growth Rate			2500.00%	128.0%	9.4%	3.5%
Market Size (Units, million)		1.0	26.0	59.3	64.9	67.1

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{Revenue} = \text{Market Size} \times \text{Market Share} \times \text{Price}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
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(Actual Market Size, Units Mil)		1.0	60.0	116.3	195.4	229.0

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

$$\text{Revenue} = \text{Market Size} \times \text{Market Share} \times \text{Price}$$

	Year					
	0 (F2008)	1	2	3	4	5
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(Actual Market Size, Units Mil)		1.0	60.0	116.3	195.4	229.0
Corp Market Share						
Initial Market Share		25.00%				
Market Share Annual Growth Rate			5.00%	5%	5%	5%
Market Share		25.0%	26.3%	27.6%	28.9%	30.4%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{Revenue} = \text{Market Size} \times \text{Market Share} \times \text{Price}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Revenue Forecasts						
Market Forecasts						
Initial Market Size (Units, million)		1.00				
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Initial Market Share		25.00%				
Market Share Annual Growth Rate			5.00%	5%	5%	5%
Market Share		25.0%	26.3%	27.6%	28.9%	30.4%
Pricing Strategy						
Initial Unit Price ($/unit)		200.00				
Bi-Annual Price Increases ($/unit)			-	49.99	-	49.99
Unit Price ($/unit)		200.00	200.00	249.99	249.99	299.98

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{Costs} = \text{Cost Margin} \times \text{Revenue}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Costs = R&D Expenditures

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

$$FCF = (Revenue - Costs - Depreciation) \times (1 - t_C) + Depreciation - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Capital Expenditures

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Capital Expenditures

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%
PP&E Liquidation Value	50.00%	50.0%	50.0%	50.0%	50.0%	50.0%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Depreciation

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%
PP&E Liquidation Value	50.00%	50.0%	50.0%	50.0%	50.0%	50.0%
PP&E life for depreciation (Years)	5.00	5	5	5	5	5

*Straight line depreciation

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

Net Working Capital = Cash + Inventory + AR – AP

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{Net Working Capital} = \text{Cash} + \text{Inventory} + \text{AR} - \text{AP}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Net Working Capital = Cash + Inventory + AR – AP

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Net Working Capital = Cash + Inventory + AR – AP

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
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$$\text{Net Working Capital} = \text{Cash} + \text{Inventory} + \text{AR} - \text{AP}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49
Accounts Payable						
Days Payable (365 x Accounts Payable / COGS)	61.54	61.54	61.54	61.54	61.54	61.54

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\Delta \text{ Net Working Capital} = \text{Net Working Capital (t)} - \text{Net Working Capital (t-1)}$$

where Δ = change over one period

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Taxes

We want the marginal tax rate (MTR)

=

Tax rate on additional $ of earnings

25.5%

This is Nonsense!

This is Nonsense!

Impossible to make accurate forecasts!

This is Nonsense!

Impossible to make accurate forecasts!

I agree, but that's not the point!!!!

Lesson: Point of DCF is to focus discussion and analysis on relevant issues

Lesson: Successful valuation (i.e., decision making) depends critically on input from non-finance personnel

Summary

Lessons

- Forecast Drivers are the assumptions used to populate our free cash flow forecasts

- Goal is to establish framework for discussion
  - Think about value drivers

Coming up next

- Discounted Cash Flow (DCF)
  - Forecasting free cash flow

Discounted Cash Flow: Forecasting Free Cash Flows

Michael R. Roberts

William H. Lawrence Professor of Finance

The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Forecast drivers

This Time

Discounted Cash Flow (DCF)

- Forecasting free cash flows

Forecasting Free Cash Flows

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Translate forecast drivers into $ forecasts

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
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	Year					
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Market Size (Units, million)		1.0	26.0	59.3	64.9	67.1
(Actual Market Size, Units Mil)		1.0	60.0	116.3	195.4	229.0
Dell's Market Share						
Initial Market Share		25.00%				
Market Share Annual Growth Rate			5.00%	5%	5%	5%
Market Share		25.0%	26.3%	27.6%	28.9%	30.4%
Pricing Strategy						
Initial Unit Price ($/unit)		200.00				
Bi-Annual Price Increases ($/unit)			-	49.99	-	49.99
Unit Price ($/unit)		200.00	200.00	249.99	249.99	299.98

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

	Year					
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Unit Price ($/unit)		200.00	200.00	249.99	249.99	299.98

Revenue = Market Size x Market Share x Price

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

	Year					
	0 (F2008)	1	2	3	4	5
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Unit Price ($/unit)		200.00	200.00	249.99	249.99	299.98

Revenue 1 = 1.0 x 0.25 x 200 = 50

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0	1	2	3	4	5
Incremental Earnings Forecasts						
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9

$$\text{COGS} = (\text{COGS} / \text{Sales}) \times \text{Sales}$$
$$\text{Year 1: } 0.8066 \times 50.0 = 40.33$$

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

Year 1: SG&A = 1% of '08 SG&A

= 0.01 x $6,959 = $69.6

Year 2 – 5: SG&A = 25% Annual Growth Rate

= $69.59 x (1+0.25) = $87.0

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) \\ + \text{Depreciation} - \text{Capital Expenditures} \\ - \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
SG&A		69.6	87.0	108.7	135.9	169.9

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Operating Expenses						
COGS						
COGS / Sales (% Sales)	80.66%	80.66%	80.66%	80.66%	80.66%	80.66%
SG&A						
1% of 2008 Company SG&A ($mil)		69.59				
SG&A Expense Growth Rate			25.00%	25.00%	25.00%	25.00%
R&D						
R&D Upfront ($mil)	200.00					
R&D for Versioning ($mil)		25.00	25.00	25.00	25.00	25.00

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
R&D	200.0	25.0	25.0	25.0	25.0	25.0

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9
Gross Profit = (28) - (29)	0.0	9.7	264.0	790.0	907.4	1,183.0
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0
EBITDA = (30) - (31) - (32)	-200.0	-84.9	152.0	656.2	746.5	988.1

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

Year 0: Initial investment = 227.7

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

Year 0: Initial investment = 227.7

Year 1: 10% of initial investment = 0.10 x $227.7 = $22.77

$$FCF = (Revenue - Costs - Depreciation) \times (1 - t_C)$$
$$+ \text{ Depreciation} - \text{Capital Expenditures}$$
$$- \text{ Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

Year 0: Initial investment = 227.7

Year 1: 10% of initial investment = 0.10 x $227.7 = $22.77

Year 2: 5% annual growth = $22.77 x (1+0.05) = $23.9

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

Year 0: Initial investment = 227.7

Year 1: 10% of initial investment = 0.10 x $227.7 = $22.77

Year 2: 5% annual growth = $22.77 x (1+0.05) = $23.9

Year 3-5: 1% annual growth = $23.9 x (1+0.01) = $24.1

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Capital Expenditures & PP&E Information						
Initial Investment (Fixed Cost, $mil)	227.70					
Future Investment (% of initial Investment)		10.0%				
Future Investment (Annual Growth)			5.0%	1.0%	1.0%	1.0%

	Year					
Capital Expenditure Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Capital Expenditure Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6

+

Straight line depreciation over 5 years

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0	1	2	3	4	5
Capital Expenditure Forecasts						
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6

Year 1: \$227.7 / 5 = \$45.5

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Capital Expenditure Forecasts	0	1	2	3	4	5
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6

Year 1: 227.7 / 5 = 45.5

Year 2: 250.5 / 5 = 50.1

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

	Year					
	0	1	2	3	4	5
Capital Expenditure Forecasts						
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6

What happens to all of that physical capital?

$$FCF = (Revenue - Costs - Depreciation) \times (1 - t_C) + Depreciation - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Capital Expenditure Forecasts	Year 0	1	2	3	4	5
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6
Book Value of CapEx						72.8
Liquidation Value (LV)						36.4
After tax proceeds from asset sale						45.6

Book Value = Accum CapEx – Accum Depreciation
= 347.5 – 274.8
= 72.8

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Capital Expenditure Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6
Book Value of CapEx						72.8
Liquidation Value (LV)						36.4
After tax proceeds from asset sale						45.6

$$LV = \text{Book Value} \times \text{Recovery Rate}$$
$$= 72.8 \times 0.50$$
$$= 36.4$$

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Capital Expenditure Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6
Book Value of CapEx						72.8
Liquidation Value (LV)						36.4
After tax proceeds from asset sale						45.6

$$\text{After-tax proceeds} = \text{LV} - (\text{LV} - \text{Book Value}) \times \text{Tax Rate}$$
$$= 36.4 - (36.4 - 72.8) \times 0.255$$
$$= 45.6$$

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)
+ Depreciation – Capital Expenditures
– Change in Net Working Capital

Capital Expenditure Forecasts	Year					
	0	1	2	3	4	5
Project CapEx	227.7	22.8	23.9	24.1	24.4	24.6
Accumulated CapEx	227.7	250.5	274.4	298.5	322.9	347.5
Depreciation		45.5	50.1	54.9	59.7	64.6
Book Value of CapEx						72.8
Liquidation Value (LV)						36.4
After tax proceeds from asset sale						45.6
Net Project CapEx	227.7	22.8	23.9	24.1	24.4	-21.0

Year 5: 24.6 – 45.6 = -21.0

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Net Working Capital = Cash + Inventory + AR – AP

$$FCF = (Revenue - Costs - Depreciation) \times (1 - t_C) + Depreciation - Capital\ Expenditures - Change\ in\ Net\ Working\ Capital$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49
Accounts Payable						
Days Payable (365 x Accounts Payable / COGS)	61.54	61.54	61.54	61.54	61.54	61.54

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0 (F2008)	1	2	3	4	5
Project Assumptions						
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%

	Year					
Incremental Earnings Forecasts	0	1	2	3	4	5
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0

Cash for SG&A: 50% of SG&A

Year 1: 0.50 x \$69.6 = \$34.8

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0

Cash for SG&A: 50% of SG&A

Year 1: 0.50 x \$69.6 = \$34.8

Cash for R&D: 100% of R&D

Year 1: 1.00 x \$25.0 = \$25.0

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Working Capital Assumptions						
Cash Requirements						
% of SG&A	50.00%	50%	50%	50%	50%	50%
% R&D Expenditures	100.00%	100%	100%	100%	100%	100%

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Project Assumptions	**0 (F2008)**	**1**	**2**	**3**	**4**	**5**
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Project Assumptions	Year					
	0 (F2008)	1	2	3	4	5
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%

Incremental Earnings Forecasts	Year					
	0	1	2	3	4	5
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

Inventory = Inventory Days x COGS / 365

Year 1: Inventory = 7.58 x \$40.3 / 365 = \$0.837

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Project Assumptions	Year					
	0 (F2008)	1	2	3	4	5
Inventory						
Inventory Days (365 x Inventory / COGS)	7.58	7.58	7.58	7.58	7.58	7.58
Excess Inventory liquidation value (% of Inventory Cost)						25.00%

Incremental Earnings Forecasts	Year					
	0	1	2	3	4	5
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

Working Capital Forecasts	Year					
	0	1	2	3	4	5
Inventory		0.8	22.9	68.4	78.6	102.5

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49

FCF = (Revenue – Costs – Depreciation) x (1 – t_C)

+ Depreciation – Capital Expenditures

– Change in Net Working Capital

	Year					
Working Capital Forecasts	0	1	2	3	4	5
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49

	Year					
Incremental Earnings Forecasts	0	1	2	3	4	5
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9

AR = AR Days x Sales / 365

Year 1: AR = 38.49 x $50.0 / 365 = $5.272

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Accounts Receivable						
Days Receivable (365 x Accounts Receivable / Sales)	38.49	38.49	38.49	38.49	38.49	38.49

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Accounts Receivable		5.3	143.9	430.7	494.8	645.0

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	0	1	2	3	4	5
Accounts Payable						
Days Payable (365 x Accounts Payable / COGS)	61.54	61.54	61.54	61.54	61.54	61.54

$$FCF = (Revenue - Costs - Depreciation) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
	0	1	2	3	4	5
Working Capital Forecasts						
Accounts Payable						
Days Payable (365 x Accounts Payable / COGS)	61.54	61.54	61.54	61.54	61.54	61.54

	Year					
	0	1	2	3	4	5
Incremental Earnings Forecasts						
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

AP = AP Days x COGS / 365

Year 1: AR = 61.54 x \$40.3 / 365 = \$6.794

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Accounts Payable						
Days Payable (365 x Accounts Payable / COGS)	61.54	61.54	61.54	61.54	61.54	61.54

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9

	Year					
Incremental Earnings Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Acounts Payable		6.8	185.6	555.5	638.1	831.9

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9
Inventory		0.8	22.9	68.4	78.6	102.5
Accounts Receivable		5.3	143.9	430.7	494.8	645.0
Acounts Payable		6.8	185.6	555.5	638.1	831.9
Net Working Capital	0	59.1	49.7	23.0	28.2	25.6

$$\text{Net Working Capital} = \text{Cash} + \text{Inventory} + \text{AR} - \text{AP}$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	0	1	2	3	4	5
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9
Inventory		0.8	22.9	68.4	78.6	102.5
Accounts Receivable		5.3	143.9	430.7	494.8	645.0
Acounts Payable		6.8	185.6	555.5	638.1	831.9
Net Working Capital	0	59.1	49.7	23.0	28.2	25.6

What happens to this working capital?

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9
Inventory		0.8	22.9	68.4	78.6	102.5
Accounts Receivable		5.3	143.9	430.7	494.8	645.0
Acounts Payable		6.8	185.6	555.5	638.1	831.9
Net Working Capital	0	59.1	49.7	23.0	28.2	25.6
Recovered NWC at end of Project						51.3

(Most of) it is recovered!

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

Working Capital Forecasts	Year 0	1	2	3	4	5
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9
Inventory		0.8	22.9	68.4	78.6	102.5
Accounts Receivable		5.3	143.9	430.7	494.8	645.0
Acounts Payable		6.8	185.6	555.5	638.1	831.9
Net Working Capital	0	59.1	49.7	23.0	28.2	25.6
Recovered NWC at end of Project						51.3

(Most of) it is recovered!

– Cash – Inventory x Recovery Rate – AR + AP

Year 5: – 109.90 – 102.5 x 0.25 – 645 + 831.9 = 51.375

$$FCF = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

	Year					
Working Capital Forecasts	**0**	**1**	**2**	**3**	**4**	**5**
Cash Requirements - SG&A Funding		34.8	43.5	54.4	68.0	84.9
Cash Requirements - R&D Funding		25.0	25.0	25.0	25.0	25.0
Cash		59.8	68.5	79.4	93.0	109.9
Inventory		0.8	22.9	68.4	78.6	102.5
Accounts Receivable		5.3	143.9	430.7	494.8	645.0
Acounts Payable		6.8	185.6	555.5	638.1	831.9
Net Working Capital	0	59.1	49.7	23.0	28.2	25.6
Recovered NWC at end of Project						51.3
Change in NWC		59.1	-9.4	-26.6	5.2	48.6

$$\text{Change in NWC} = \Delta NWC = NWC(t) - NWC(t-1)$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) \\ + \text{Depreciation} - \text{Capital Expenditures} \\ - \text{Change in Net Working Capital}$$

We have all the pieces. Organize into a useful (and familiar) format

(Quasi-) Income Statement

Incremental Earnings Forecasts	Year 0	1	2	3	4	5
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9
Gross Profit = (28) - (29)	0.0	9.7	264.0	790.0	907.4	1,183.0
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0
EBITDA = (30) - (31) - (32)	-200.0	-84.9	152.0	656.2	746.5	988.1
Depreciation		45.5	50.1	54.9	59.7	64.6
EBIT = (33) - (34)	-200.0	-130.5	101.9	601.4	686.8	923.5
Taxes	-50.9	-33.2	25.9	153.0	174.8	235.0
NOPAT (35) - (36) (a.k.a. EBIAT, Unlevered Net Income)	-149.1	-97.3	76.0	448.3	512.0	688.5

(Quasi-) Income Statement

Incremental Earnings Forecasts	Year 0	1	2	3	4	5
Sales		50.0	1,365.0	4,084.6	4,692.0	6,116.9
COGS		40.3	1,101.0	3,294.6	3,784.6	4,933.9
Gross Profit = (28) - (29)	0.0	9.7	264.0	790.0	907.4	1,183.0
SG&A		69.6	87.0	108.7	135.9	169.9
R&D	200.0	25.0	25.0	25.0	25.0	25.0
EBITDA = (30) - (31) - (32)	-200.0	-84.9	152.0	656.2	746.5	988.1
Depreciation		45.5	50.1	54.9	59.7	64.6
EBIT = (33) - (34)	-200.0	-130.5	101.9	601.4	686.8	923.5
Taxes	-50.9	-33.2	25.9	153.0	174.8	235.0
NOPAT (35) - (36) (a.k.a. EBIAT, Unlevered Net Income)	-149.1	-97.3	76.0	448.3	512.0	688.5

NOPAT

$$\text{FCF} = \underbrace{(\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C)}_{\text{NOPAT}}$$
$$+ \text{Depreciation} - \text{Capital Expenditures}$$
$$- \text{Change in Net Working Capital}$$

$$\text{FCF} = (\text{Revenue} - \text{Costs} - \text{Depreciation}) \times (1 - t_C) + \text{Depreciation} - \text{Capital Expenditures} - \text{Change in Net Working Capital}$$

Free Cash Flow Forecasts	**Year 0**	**1**	**2**	**3**	**4**	**5**
NOPAT (Unlevered Net Income, EBIAT)	-149.1	-97.3	76.0	448.3	512.0	688.5
Depreciation		45.5	50.1	54.9	59.7	64.6
Capital Expenditures	227.7	22.8	23.9	24.1	24.4	-21.0
Changes in NWC		59.1	-9.4	-26.6	5.2	48.6
Free Cash Flows = (38) +(39) - (40) - (41)	**-376.8**	**-133.6**	**111.6**	**505.7**	**542.1**	**725.5**

Other Free Cash Flow Considerations

- Opportunity Costs (Alternative uses of resources)
- Project Externalities (Cannibalization, spillovers,)
- Sunk Costs (Ignore)
- Other non-cash items (E.g., amortization)
- Salvage values (Assets do not disappear)
- Execution Risk (Idiosyncratic)
- Cash flow frequency (Project dependent)

Summary

Lessons

- Forecasting free cash flows is a matter of converting our forecast drivers into dollar forecasts

- One of the two basic inputs into a DCF

Coming up next

- Discounted Cash Flow (DCF)
  - Decision Criteria

Discounted Cash Flow: Decision Criteria

Michael R. Roberts

William H. Lawrence Professor of Finance

The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Forecasting free cash flows

This Time

Discounted Cash Flow (DCF)

- Decision Criteria

Decision Criteria

What do we do with cash flows?

1. Compute the NPV (assume discount rate of 12%)

1. Compute the NPV

$$NPV = \frac{-\$376.8}{(1+0.12)^0} + \frac{-\$133.6}{(1+0.12)^1} + \frac{\$111.6}{(1+0.12)^2} + \frac{\$505.7}{(1+0.12)^3} + \frac{\$542.1}{(1+0.12)^4} + \frac{\$725.5}{(1+0.12)^5}$$

$$= \$708.42$$

1. Compute the NPV

Firm value (i.e., debt plus equity) increases by \$708.42 million, in expectation, if the project is undertaken → undertake the project

Lesson: The NPV Rule says accept all projects with positive NPV, reject all projects with negative NPV

2. Compute internal rate of return

The internal rate of return (IRR) of a project is the one discount rate such that the net present value of the project's free cash flows equals zero.

2. Compute internal rate of return

$$NPV = \frac{-\$376.8}{(1+IRR)^0} + \frac{-\$133.6}{(1+IRR)^1} + \frac{\$111.6}{(1+IRR)^2} + \frac{\$505.7}{(1+IRR)^3} + \frac{\$542.1}{(1+IRR)^4} + \frac{\$725.5}{(1+IRR)^5}$$

$$\Rightarrow \quad IRR = 43.7\%$$

2. Compute internal rate of return

$$NPV = \frac{-\$376.8}{(1+IRR)^0} + \frac{-\$133.6}{(1+IRR)^1} + \frac{\$111.6}{(1+IRR)^2} + \frac{\$505.7}{(1+IRR)^3} + \frac{\$542.1}{(1+IRR)^4} + \frac{\$725.5}{(1+IRR)^5}$$

$$\Rightarrow \quad IRR = 43.7\%$$

Typically need to solve numerically (e.g., *IRR* function in Excel), or trial and error.

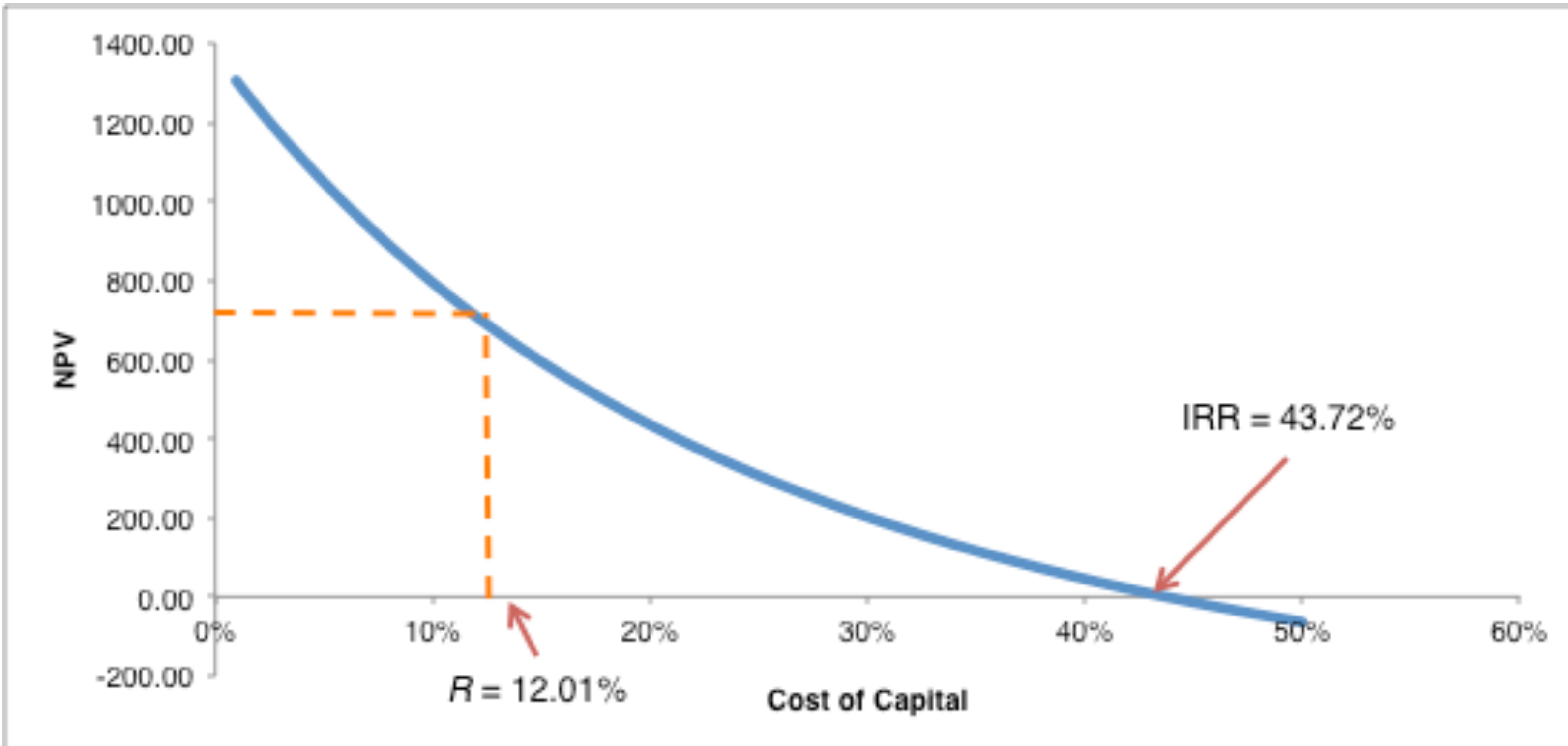
2. Compute internal rate of return

The promised return on investing in the project is 43.7% > 12% (hurdle rate) → undertake the project

Lesson: The IRR Rule says accept all projects whose $IRR > R$, reject all projects whose $IRR < R$

Lesson: The IRR Rule is informative but has several shortcomings that we explore in Topic 4 (Return on Investment)

NPV vs. IRR


1400.00
1200.00
1000.00
800.00
600.00
400.00
200.00
0.00
-200.00
NPV
0%
10%
20%
30%
40%
50%
60%
Cost of Capital
IRR = 43.72%
R = 12.01%

3. Compute payback period

The payback period, *pp*, of a project is the duration until the the cumulative free cash flows turn positive.

3. Compute payback period

Alternative Decision Criteria	Year 0	1	2	3	4	5
Free Cash Flows	-376.8	-133.6	111.6	505.7	542.1	725.5
Cumulative Free Cash Flows	**-376.8**	**-510.4**	**-398.8**	**106.9**	**649.0**	**1,374.5**
Payback Period	3					

3. Compute payback period

It takes 3 years to recover your investment. Good? Bad? Compare to some threshold payback period, *pp**

Lesson: The Payback Period Rule says accept all projects with $pp < pp^*$, reject all projects whose $pp > pp^*$

Lesson: The Payback Period Rule has several shortcomings…

Lesson: The Payback Period Rule has several shortcomings

#1: Ignores time value of money and risk of cash flows

3a. Compute discounted payback period

The discounted payback period, *dpp*, of a project is the duration until the the cumulative discounted free cash flows turn positive.

3a. Compute discounted payback period

	Year					
Alternative Decision Criteria	**0**	**1**	**2**	**3**	**4**	**5**
Discounted Free Cash Flows	-376.8	-119.3	88.9	359.8	344.4	411.4
Cumulative Discounted Free Cash Flows	**-376.8**	**-496.1**	**-407.1**	**-47.3**	**297.0**	**708.4**
Discounted Payback Period	4.0					

Lesson: The Discounted Payback Period Rule has several shortcomings

#1: Ignores cash flows after cutoff leading to myopic decision making

Lesson: The Discounted Payback Period Rule has several shortcomings

#2: Does not tell us value implications of our decision

Lesson: The Discounted Payback Period Rule has several shortcomings

#3: Does not help in choosing among projects with similar payback periods

Summary

Lessons

- Several decision criteria
  - NPV unambiguously the best but
  - Others are informative. Understand their shortcomings and use judiciously

Coming up next

- Discounted Cash Flow (DCF)
  - Sensitivity analysis

Discounted Cash Flow: Sensitivity Analysis

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Decision criteria

This Time

Discounted Cash Flow (DCF)

- Sensitivity analysis

Sensitivity Analysis

BREAK EVEN ANALYSIS

Break Even Analysis finds the parameter value that sets the NPV of the project equal to zero holding fixed all other parameters

	Base
Cost of Capital	12.01%
	708.42
Initial Investment ($mil)	227.7
	708.42

Base
Parameter Value

NP
V

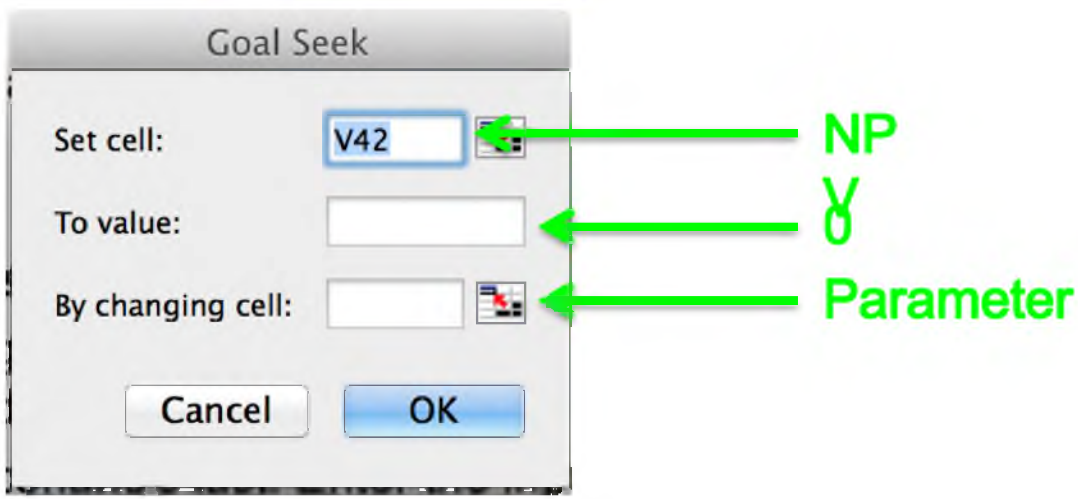
Break Even (BE)
Parameter Value

	Base	BE
Cost of Capital	12.01%	43.72%
	708.42	0.00
Initial Investment ($mil)	227.7	904.24
	708.42	0.00

NP
V

	Base	BE
Cost of Capital	12.01%	43.72%
	708.42	0.00
Initial Investment ($mil)	227.7	904.24
	708.42	0.00

Excel's Goal Seek



	Base	BE
Cost of Capital	12.01%	43.72%
	708.42	0.00
Initial Investment ($mil)	227.7	904.24
	708.42	0.00
PP&E Liquidation Value	50%	-2253%
	708.42	0.00
PP&E Life for Depreciation	5.00	0.32
	708.42	0.00
Initial Market Size (Units Mil)	1.00	0.52
	708.42	0.00
Market Growth Rate	2500%	1571%
	708.42	0.00
Initial Market Share	25.00%	13%
	708.42	0.00
Market Share Growth Rate	5.00%	-18%
	708.42	0.00
Initial Unit Price ($/unit)	200.00	77.36
	708.42	0.00
Bi-Annual Price Increases ($/unit)	49.99	(59.44)
	708.42	0.00
Tax Rate	25.45%	88%
	708.42	0.00
COGS / Sales (% Sales)	80.66%	90%
	708.42	0.00
SG&A First-Year ($mil)	69.59	229.81
	708.42	0.00
SG&A Expense Growth Rate	25%	98%
	708.42	0.00
R&D Upfront ($mil)	200.00	1,150.26
	708.42	0.00
R&D Versioning ($mil)	25.00	260.19
	708.42	0.00

Large
Margin for
Error

	Base	BE
Cost of Capital	12.01%	43.72%
	708.42	0.00
Initial Investment ($mil)	227.7	904.24
	708.42	0.00
PP&E Liquidation Value	50%	-2253%
	708.42	0.00
PP&E Life for Depreciation	5.00	0.32
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Initial Market Size (Units Mil)	1.00	0.52
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	708.42	0.00
R&D Upfront ($mil)	200.00	1,150.26
	708.42	0.00
R&D Versioning ($mil)	25.00	260.19
	708.42	0.00

Potentially Important

Lesson: Break Even Analysis is a partial equilibrium analyses that assume parameters are independent of one another.

COMPARATIVE STATICS

Comparative statics quantifies the sensitivity of the valuation to variation in a parameter holding fixed all other parameters

How does valuation change parameter variation from worst to best case?

		Worst	Base	Best
Cost of Capital		15.01%	12.01%	9.61%
	708.42	594.19	708.42	812.29
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	649.47	708.42	753.13

		Worst	Base	Best
Cost of Capital		15.01%	12.01%	9.61%
	708.42	**594.19**	**708.42**	**812.29**
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	**649.47**	**708.42**	**753.13**

Does the valuation vary sensibly with variation in the parameters?

Excel: Data Tables

1) Highlight matrix: NPV in bottom left, parameters in top ***row***

		Worst	Base	Best
Cost of Capital		15.01%	12.01%	9.61%
	708.42	594.19	708.42	812.29
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	649.47	708.42	753.13

Excel: Data Tables

1) Highlight matrix: NPV in bottom left, parameters in top ***row***

		Worst	Base	Best
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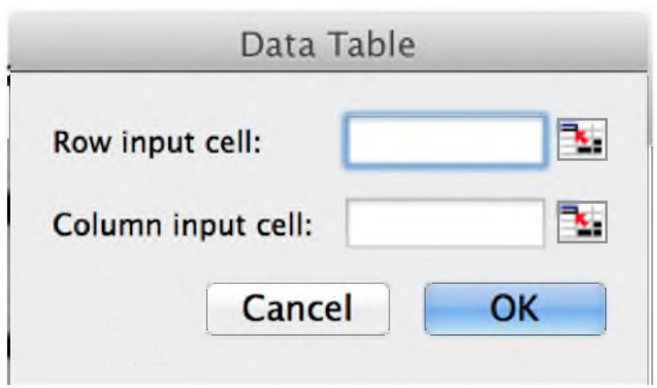
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	**649.47**	**708.42**	**753.13**

2) Data Table: Choose Row Input Cell and enter parameter cell



		Worst	Base	Best
Cost of Capital		15.01%	12.01%	9.61%
	708.42	**594.19**	**708.42**	**812.29**
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	**649.47**	**708.42**	**753.13**
PP&E Liquidation Value		30%	50%	75%
	708.42	**702.27**	**708.42**	**716.11**
PP&E Life for Depreciation		3.00	5.00	7.00
	708.42	**676.20**	**708.42**	**722.23**
Initial Market Size (Units Mil)		0.50	1.00	2.00
	708.42	**-31.05**	**708.42**	**2187.36**
Market Growth Rate		1000%	2500%	5000%
	708.42	**-332.83**	**708.42**	**3696.24**
Initial Market Share		15.00%	25.00%	35.00%
	708.42	**116.84**	**708.42**	**1300.00**
Market Share Growth Rate		1.00%	5.00%	8.00%
	708.42	**557.30**	**708.42**	**830.22**
Initial Unit Price ($/unit)		175.00	200.00	250.00
	708.42	**564.00**	**708.42**	**997.25**
Bi-Annual Price Increases ($/unit)		24.99	49.99	99.99
	708.42	**546.58**	**708.42**	**1032.10**
Tax Rate		35%	25.45%	12.50%
	708.42	**600.32**	**708.42**	**855.01**
COGS / Sales (% Sales)		84.30%	80.66%	74.25%
	708.42	**425.90**	**708.42**	**1205.93**
SG&A First-Year ($mil)		45.00	69.59	85.00
	708.4	**817.14**	**708.42**	**640.28**
SG&A Expense Growth Rate		30%	25%	15%
	708.4	**680.34**	**708.42**	**757.99**
R&D Upfront ($mil)		250.00	200.00	150.00
	708.4	**671.14**	**708.42**	**745.69**
R&D Versioning ($mil)		30.00	25.00	15.00
	708.4	**693.36**	**708.42**	**738.54**

What is the elasticity of the valuation with respect to each parameter?

What is the elasticity of the valuation with respect to each parameter?

$$\text{Elasticity} = \frac{\%\ \text{Change in NPV}}{\%\ \text{Change in Parameter}} = \frac{\Delta NPV}{\Delta P}\frac{P}{NPV}$$

		Base x (1-0.01)	Base	Base x (1+0.01)	Elasticity
Cost of Capital		11.89%	12.01%	12.13%	
	708.42	713.34	708.42	703.53	-0.69

$$\text{Elasticity} = \frac{(703.53 - 713.34)}{(0.1213 - 0.1189)} \times \frac{\frac{1}{2}(0.1189 + 0.1213)}{\frac{1}{2}(703.53 + 713.34)}$$

		Base x (1-0.01)	Base	Base x (1+0.01)	Elasticity
Cost of Capital		11.89%	12.01%	12.13%	
	708.42	713.34	708.42	703.53	-0.69
Initial Investment ($mil)		225.4	227.7	230.0	
	708.42	710.80	708.42	706.03	-0.34
PP&E Liquidation Value		50%	50%	51%	
	708.42	708.26	708.42	708.57	0.02
PP&E Life for Depreciation		4.95	5.00	5.05	
	708.42	707.93	708.42	708.90	0.07
Initial Market Size (Units Mil)		0.99	1.00	1.01	
	708.42	693.63	708.42	723.21	2.09
Market Growth Rate		2475%	2500%	2525%	
	708.42	686.61	708.42	730.38	3.09
Initial Market Share		24.75%	25.00%	25.25%	
	708.42	693.63	708.42	723.21	2.09
Market Share Growth Rate		4.95%	5.00%	5.05%	
	708.42	706.45	708.42	710.39	0.28
Initial Unit Price ($/unit)		198.00	200.00	202.00	
	708.42	696.87	708.42	719.97	1.63
Bi-Annual Price Increases ($/unit)		49.49	49.99	50.49	
	708.42	705.18	708.42	711.65	0.46
Tax Rate		25%	25.45%	25.70%	
	708.42	711.30	708.42	705.54	-0.41
COGS / Sales (% Sales)		79.85%	80.66%	81.47%	
	708.42	771.02	708.42	645.81	-8.84

Lesson: Comparative statics implicitly assumes parameters are independent of one another.

SCENARIO ANALYSIS

Scenario Analysis quantifies the sensitivity of the valuation to variation in multiple parameters

		Worst	Base	Best
Cost of Capital		15.01%	12.01%	9.61%
	708.42	594.19	708.42	812.29
Initial Investment ($mil)		284.0	227.7	185.0
	708.42	649.47	708.42	753.13
PP&E Liquidation Value		30%	50%	75%
	708.42	702.27	708.42	716.11
PP&E Life for Depreciation		3.00	5.00	7.00
	708.42	676.20	708.42	722.23
Initial Market Size (Units Mil)		0.50	1.00	2.00
	708.42	-31.05	708.42	2187.36
Market Growth Rate		1000%	2500%	5000%
	708.42	-332.83	708.42	3696.24
Initial Market Share		15.00%	25.00%	35.00%
	708.42	116.84	708.42	1300.00
Market Share Growth Rate		1.00%	5.00%	8.00%
	708.42	557.30	708.42	830.22
Initial Unit Price ($/unit)		175.00	200.00	250.00
	708.42	564.00	708.42	997.25
Bi-Annual Price Increases ($/unit)		24.99	49.99	99.99
	708.42	546.58	708.42	1032.10
Tax Rate		35%	25.45%	12.50%
	708.42	600.32	708.42	855.01
COGS / Sales (% Sales)		84.30%	80.66%	74.25%
	708.42	425.90	708.42	1205.93
SG&A First-Year ($mil)		45.00	69.59	85.00
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Scenario Analysis

Worst	Base	Best
(654.16)	708.42	33,112.71

Scenario Analysis

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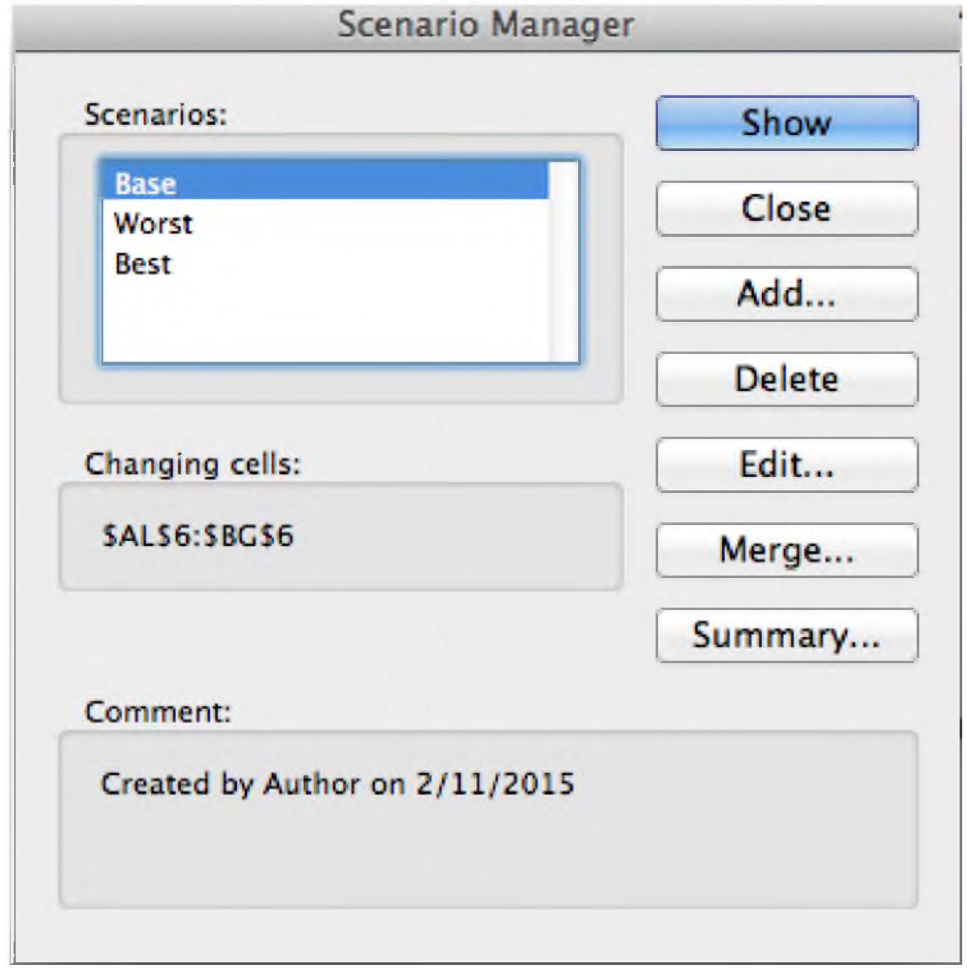
Excel: Scenarios



Scenario Analysis	Worst	Base	Best
	(654.16)	708.42	33,112.71

Excel: Lookup tables + a toggle are more efficient

Strategy wants to reduce the price by $30 in order to increase the initial market penetration from 25% to 30%

Strategy wants to reduce the price by \$30 in order to increase the initial market penetration from 25% to 30%

Does this make sense?

		Quantity (Initial Market Share)										
	708	20.92%	21.62%	22.38%	23.19%	24.06%	25.00%	26.02%	27.12%	28.32%	29.63%	31.07%
Price	250	708.42	758.38	811.83	869.16	930.80	997.25	1069.10	1147.05	1231.89	1324.58	1426.27
	240	660.09	708.42	760.13	815.58	875.20	939.48	1008.99	1084.39	1166.45	1256.12	1354.49
	230	611.76	658.46	708.42	762.00	819.61	881.72	948.88	1021.72	1101.02	1187.66	1282.70
	220	563.43	608.50	656.71	708.42	764.01	823.95	888.76	959.06	1035.59	1119.19	1210.92
	210	515.11	558.54	605.00	654.84	708.42	766.18	828.65	896.40	970.15	1050.73	1139.13
	200	466.78	508.58	553.30	601.26	652.82	708.42	768.53	833.74	904.72	982.27	1067.35
	190	418.45	458.62	501.59	547.68	597.23	650.65	708.42	771.08	839.29	913.81	995.56
	180	370.12	408.66	449.88	494.10	541.63	592.89	648.30	708.42	773.85	845.34	923.78
	170	321.79	358.69	398.17	440.52	486.04	535.12	588.19	645.76	708.42	776.88	851.99
	160	273.47	308.73	346.47	386.94	430.44	477.35	528.08	583.10	642.98	708.42	780.20
	150	225.14	258.77	294.76	333.35	374.85	419.59	467.96	520.43	577.55	639.96	708.42

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NPV will increase from 708.42 to something greater than 776.88

Marketing is concerned about the uncertainty surrounding the market for tablets

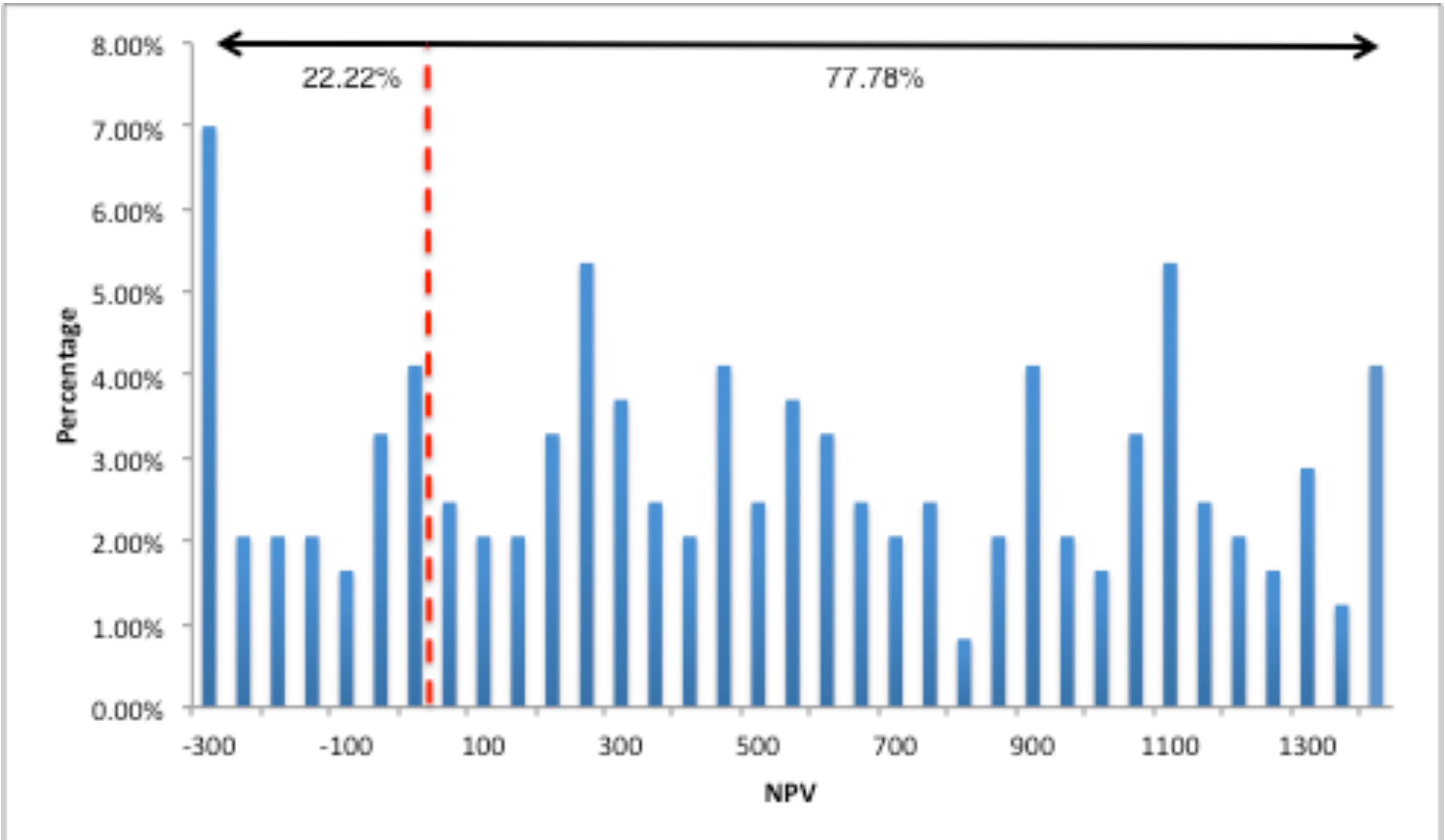
Marketing is concerned about the uncertainty surrounding the market for tablets

Can you provide some information?

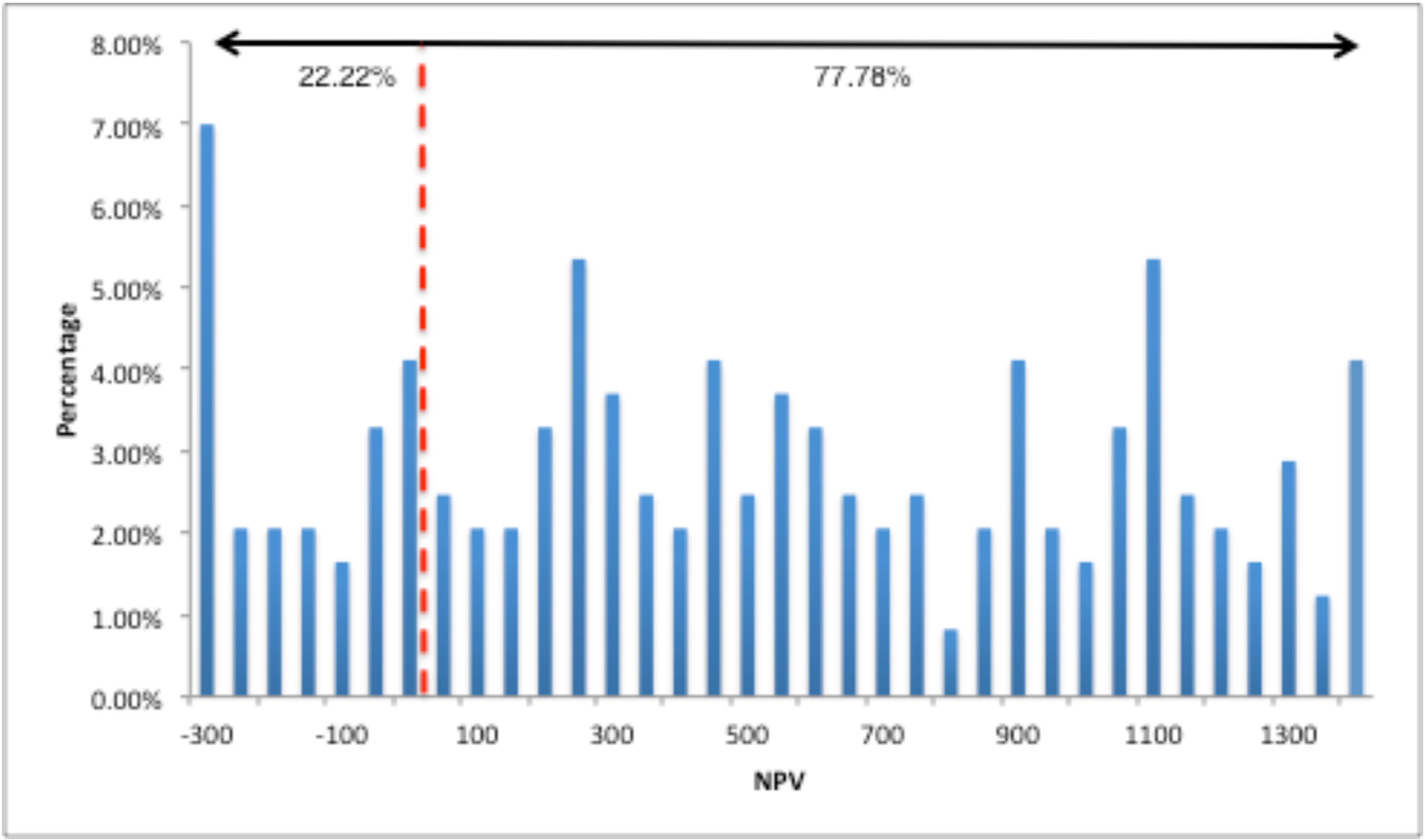
		Market Growth Rate										
	708	1390.55%	1522.34%	1682.86%	1883.75%	2144.35%	2500.00%	3024.12%	3905.32%	5879.44%	9680.69%	13770.16%
Market Size	2.25	708.42	893.29	1130.97	1447.89	1891.57	2557.10	3666.30	5884.72	12539.97	32505.73	65781.97
	2.00	544.09	708.42	919.70	1201.40	1595.78	2187.36	3173.32	5145.25	11061.03	28808.37	58387.24
	1.75	379.76	523.55	708.42	954.91	1300.00	1817.63	2680.34	4405.78	9582.08	25111.01	50992.52
	1.50	215.44	338.68	497.14	708.42	1004.21	1447.89	2187.36	3666.31	8103.14	21413.65	43597.80
	1.25	51.11	153.81	285.86	461.93	708.42	1078.15	1694.38	2926.83	6624.20	17716.29	36203.08
	1.00	-113.22	-31.05	74.59	215.44	412.63	708.42	1201.40	2187.36	5145.25	14018.92	28808.36
	0.75	-277.54	-215.92	-136.69	-31.05	116.84	338.68	708.42	1447.89	3666.31	10321.56	21413.64
	0.50	-441.87	-400.79	-347.97	-277.54	-178.95	-31.05	215.44	708.42	2187.36	6624.20	14018.92
	0.25	-606.20	-585.66	-559.25	-524.04	-474.74	-400.79	-277.54	-31.05	708.42	2926.84	6624.20
	0.10	-704.79	-696.58	-686.01	-671.93	-652.21	-622.63	-573.33	-474.74	-178.95	708.42	2187.36
	0.05	-737.66	-733.55	-728.27	-721.23	-711.37	-696.58	-671.93	-622.63	-474.74	-31.05	708.42

SIMULATION ANALYSIS

Simulation Analysis performs the valuation for a large number of simulated parameter values (i.e., scenarios)



500 simulated scenarios.
Parameters drawn independently of one another.
Parameters bounded by Best and Worst case scenario.



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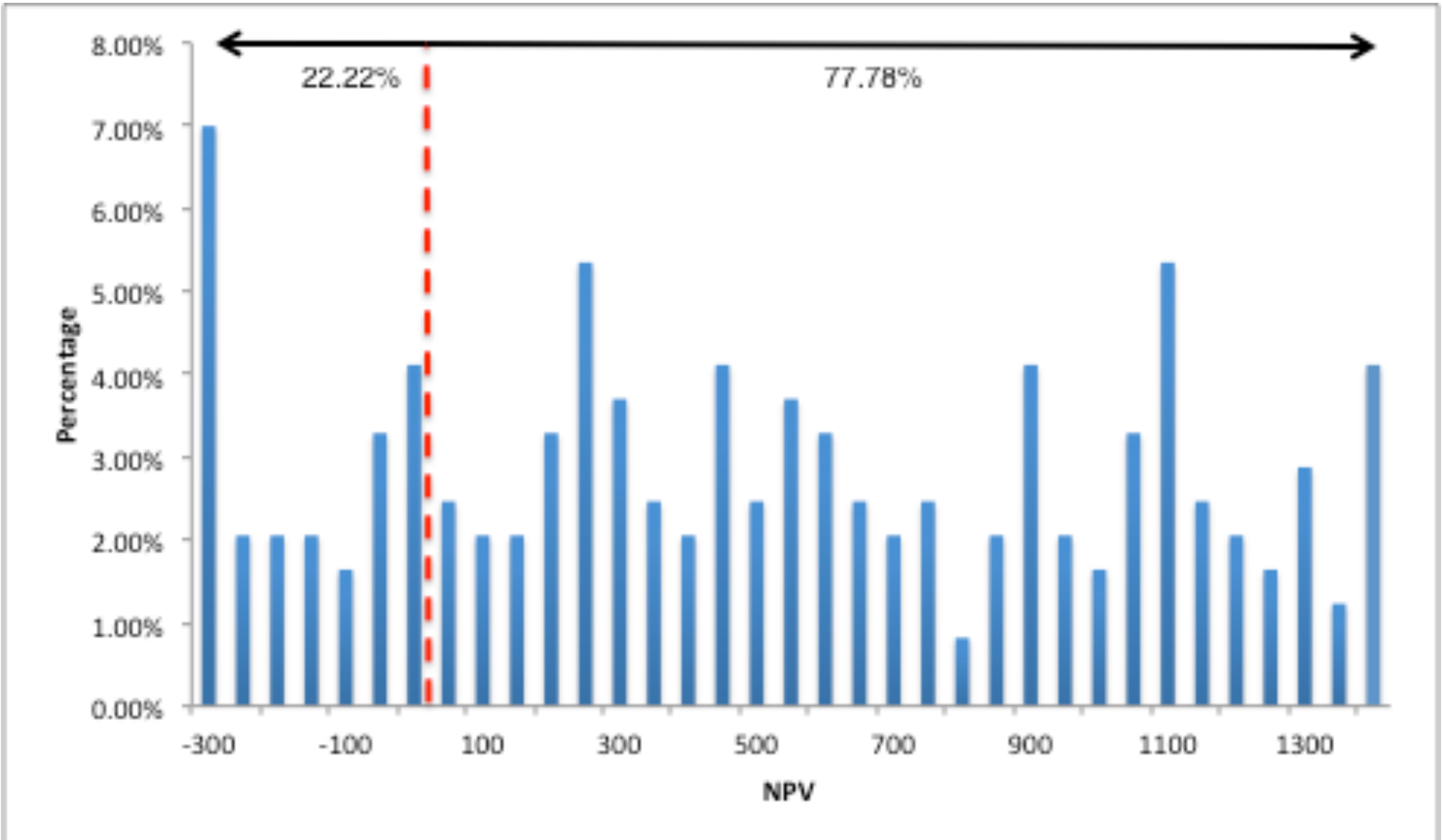

500 simulated scenarios.

Parameters drawn independently of one another.

Unreasonable assumption (e.g., price & quantity)

→ Can lead to implausible outcomes

Summary

Lessons

- Sensitivity analysis is an integral part of any valuation
  - Where value is created and destroyed?
  - What are the key value drivers?
  - What is the risk exposure?
  - How robust is the profitability of the project?

Coming up next

- Return on investment

Return on Investment

Michael R. Roberts
William H. Lawrence Professor of Finance
The Wharton School, University of Pennsylvania

Last Time

Discounted Cash Flow (DCF)

- Decision making
- Free cash flow
- Forecast drivers
- Forecasting free cash flow
- Sensitivity analysis
- Decision criteria

This Time

Return on investment

- IRR versus NPV

IRR

RECALL…

The internal rate of return of an asset is the one discount rate such that the NPV of the asset's free cash flows equals zero.

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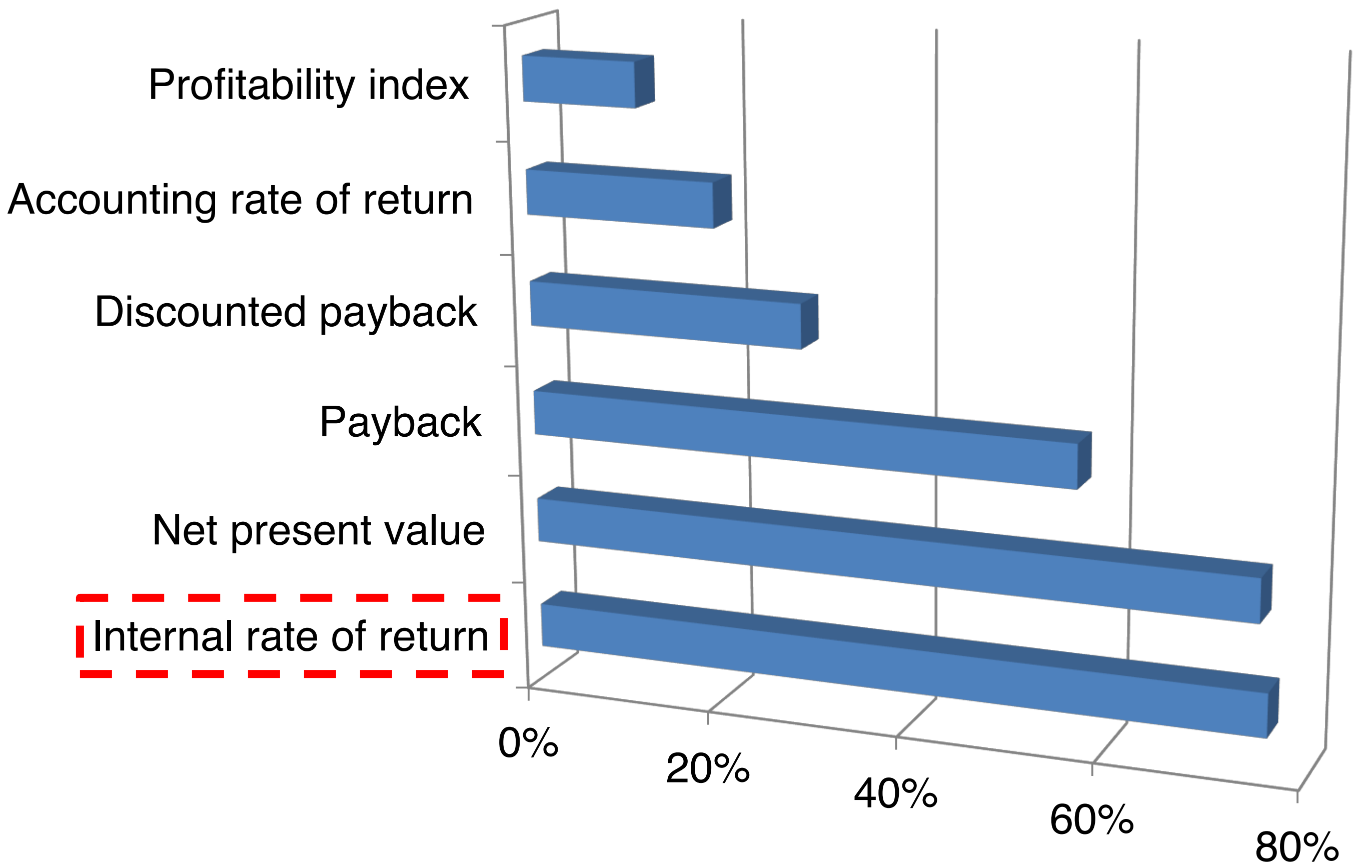
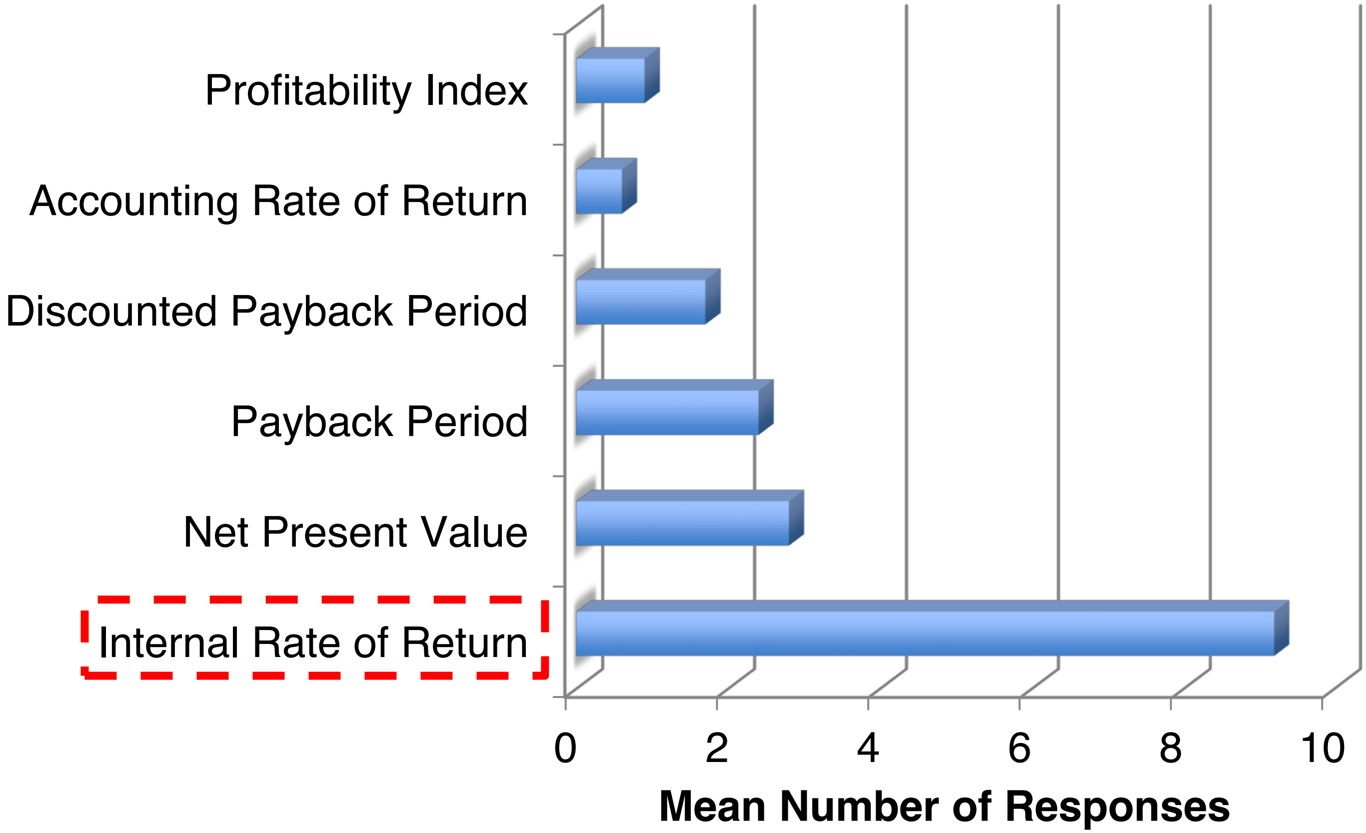
$$NPV = \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} + \ldots + \frac{CF_T}{(1+IRR)^T} = 0$$

The IRR Decision Rule says accept all projects whose $IRR > R$, reject all projects whose $IRR < R$ where R is the hurdle rate

Rates of return are popular measures used for making decisions



Graham and Harvey, 2001, The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics*



What do Private Equity Firms Say they Do? (Paul Gompers, Steve Kaplan, and Vladimir Mukharlyamov)

IRR V NPV

Lesson: The IRR rule leads to the same decisions – accept or reject – as the NPV rule if all negative cash flows precede all positive cash flows

Examples of CF sequences where IRR and NPV rules will coincide:

-, +, +, +, +

-, -, -, +, +, +, +, +, +

-, -, -, -, - , +

Examples of CF sequences where IRR and NPV rules may not coincide:

-, +, -, +, -, +

+, +, +, +, +, +, -, -, -, -

-, +, +, +, +, -

Can we compare projects using IRR?

Comparing Projects

Wharton wants to upgrade IT system and overhaul network infrastructure

Wharton wants to upgrade IT system and overhaul network infrastructure

Puts out request for proposals (RFP)

Bid #1 from Cisco

Generate $60 million in cost savings over three years for up front cost of $100 million

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Generate $60 million in cost savings over three years for up front cost of $100 million

If Wharton's cost of capital is 12%, what is your assessment of this bid?

Bid #1 from Cisco

Generate $60 million in cost savings over three years for up front cost of $100 million

Cash flows first:

	Year			
	0	1	2	3
Bid #1: Cisco	-100	60	60	60

Bid #1 from Cisco

Generate $60 million in cost savings over three years for up front cost of $100 million

	Year				IRR
	0	1	2	3	
Bid #1: Cisco	-100	60	60	60	36%

$$0 = -100 + \frac{60}{(1+IRR)} + \frac{60}{(1+IRR)^2} + \frac{60}{(1+IRR)^3}$$

Bid #1 from Cisco

Generate \$60 million in cost savings over three years for up front cost of \$100 million

	Year				
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IRR > *R* **and** CFs signs proper → Looks good!

Bid #1 from Cisco

Generate \$60 million in cost savings over three years for up front cost of \$100 million

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11

$$NPV = -100 + \frac{60}{(1+0.12)} + \frac{60}{(1+0.12)^2} + \frac{60}{(1+0.12)^3}$$

Bid #1 from Cisco

Generate $60 million in cost savings over three years for up front cost of $100 million

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11

NVP > 0 → Looks good!

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

Cash flows first:

	Year			
	0	1	2	3
Bid #1a: Cisco (Costs)	-20	-35	-35	-35

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

Cash flows first:

	Year			
	0	1	2	3
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Savings)	0	60	60	60

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

Cash flows first:

	Year			
	0	1	2	3
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Savings)	0	60	60	60
Bid #1a: Cisco (Net)	-20	25	25	25

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

	Year				
	0	1	2	3	IRR
Bid #1a: Cisco (Costs)	-20	-35	-35	-35	
Bid #1a: Cisco (Savings)	0	60	60	60	
Bid #1a: Cisco (Net)	-20	25	25	25	112%

$$0 = -20 + \frac{25}{(1+IRR)} + \frac{25}{(1+IRR)^2} + \frac{25}{(1+IRR)^3}$$

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

	Year				
	0	1	2	3	IRR
Bid #1a: Cisco (Costs)	-20	-35	-35	-35	
Bid #1a: Cisco (Savings)	0	60	60	60	
Bid #1a: Cisco (Net)	-20	25	25	25	112%

Bid #1a IRR (112%) > Bid #1 IRR (36%)

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

	Year					
	0	1	2	3	IRR	NPV
Bid #1a: Cisco (Costs)	-20	-35	-35	-35		
Bid #1a: Cisco (Savings)	0	60	60	60		
Bid #1a: Cisco (Net)	-20	25	25	25	112%	40.05

$$NPV = -20 + \frac{25}{(1+0.12)} + \frac{25}{(1+0.12)^2} + \frac{25}{(1+0.12)^3}$$

Bid #1a from Cisco

Same cost savings ($60 mil over three years) but costs spread over time: $20 mil today, $35 mil over three years

	Year					
	0	1	2	3	IRR	NPV
Bid #1a: Cisco (Costs)	-20	-35	-35	-35		
Bid #1a: Cisco (Savings)	0	60	60	60		
Bid #1a: Cisco (Net)	-20	25	25	25	112%	40.05

Bid #1 NPV ($44.11) > Bid #1a NPV ($40.05)

What is going on?

NPV → Bid #1 is better

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11
Bid #1a: Cisco (Costs)	-20	-35	-35	-35		
Bid #1a: Cisco (Savings)	0	60	60	60		
Bid #1a: Cisco (Net)	-20	25	25	25	112%	40.05

NPV → Bid #1 is better

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11
Bid #1a: Cisco (Costs)	-20	-35	-35	-35		
Bid #1a: Cisco (Savings)	0	60	60	60		
Bid #1a: Cisco (Net)	-20	25	25	25	112%	40.05

IRR → Bid #1a is better

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

Bid 1a incorporates a loan from Cisco

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

Bid 1a incorporates a loan from Cisco
What is the interest rate?

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

$$80 = \frac{35}{(1+R)} + \frac{35}{(1+R)^2} + \frac{35}{(1+R)^3} \Rightarrow R = 15\%$$

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

$$0 = -80 + \frac{35}{(1+IRR)} + \frac{35}{(1+IRR)^2} + \frac{35}{(1+IRR)^3} \Rightarrow IRR = 15\%$$

Note: This is also the IRR of the loan

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

$$80 = \frac{35}{(1+YTM)} + \frac{35}{(1+YTM)^2} + \frac{35}{(1+YTM)^3} \Rightarrow YTM = 15\%$$

Note: This is also the Yield-to-Maturity of the loan

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

$$80 = \frac{35}{(1+YTM)} + \frac{35}{(1+YTM)^2} + \frac{35}{(1+YTM)^3} \Rightarrow YTM = 15\%$$

Is this high or low?

A closer look

	Year			
	0	1	2	3
Bid #1: Cisco (Costs)	-100	0	0	0
Bid #1a: Cisco (Costs)	-20	-35	-35	-35
Bid #1a: Cisco (Implicit Loan)	80	-35	-35	-35

$$80 = \frac{35}{(1+YTM)} + \frac{35}{(1+YTM)^2} + \frac{35}{(1+YTM)^3} \Rightarrow YTM = 15\%$$

Loan interest rate (15%) > Cost of Capital (12%)

Lesson: IRR increased because initial investment fell more than future cash flows.

Lesson: IRR increased because initial investment fell more than future cash flows.

(Intuition: Small payoffs on a smaller investment can generate very large returns because of division by small numbers.)

Lesson: NPV fell because Cisco is lending you money at an interest rate that is greater than your cost of capital.

Lesson: IRR can mislead when deciding among projects.

Lesson: NPV will not mislead in comparisons. The larger the NPV, the greater the value

ADDITIONAL BIDS

	Year			
	0	1	2	3
Bid #1: Cisco	-100	60	60	60
Bid #2: Juniper	-100	90	70	5
Bid #3: Huawei	-20	20	20	20

	Year			
	0	1	2	3
Bid #1: Cisco	-100	60	60	60
Bid #2: Juniper	-100	90	70	5
Bid #3: Huawei	-20	20	20	20

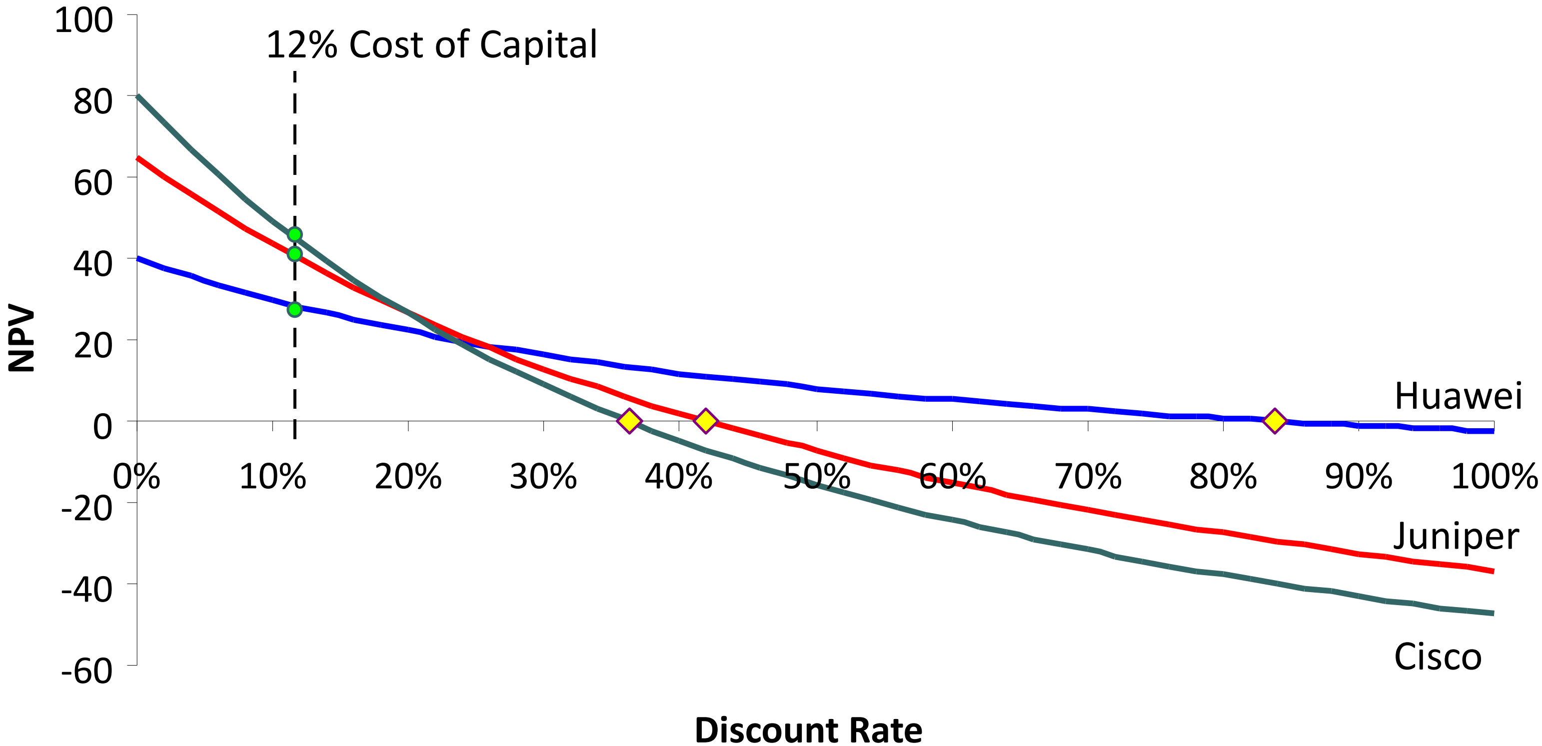
How would you rank the bids according to the IRR and the NPV criterion?

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11
Bid #2: Juniper	-100	90	70	5	42%	39.72
Bid #3: Huawei	-20	20	20	20	84%	28.04

	Year					
	0	1	2	3	IRR	NPV
Bid #1: Cisco	-100	60	60	60	36%	44.11
Bid #2: Juniper	-100	90	70	5	42%	39.72
Bid #3: Huawei	-20	20	20	20	84%	28.04

IRR: #3 > #2 > #1

NPV: #1 > #2 > #3



100
80
60
40
20
0
-20
-40
-60
NPV
12% Cost of Capital
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Huawei
Juniper
Cisco
Discount Rate

Intuition:

Huawei has small upfront cost → IRR ↑

Juniper has front-loaded CFs → IRR ↑

Lesson: IRR does not address differences in scale.

Lesson: IRR does not address differences in scale.

Would you rather earn 100% on a $1 investment or 10% on a $1,000,000 investment?

Intuition:
Juniper's bid is like Cisco's with an embedded loan…

	Year			
	0	1	2	3
Bid #1: Cisco	-100	60	60	60
Bid #3: Huawei	-20	20	20	20
Bid #3: Huawei (Implicit Loan)	80	-40	-40	-40

Intuition:
Juniper's bid is like Cisco's with an embedded loan…with a 23% interest rate!

	Year				
	0	1	2	3	IRR
Bid #1: Cisco	-100	60	60	60	
Bid #3: Huawei	-20	20	20	20	
Bid #3: Huawei (Implicit Loan)	80	-40	-40	-40	23%

Summary

Lessons

- The internal rate of return of an asset is the one discount rate such that the NPV of the asset's free cash flows equals zero.

$$NPV = \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3} + \ldots + \frac{CF_T}{(1+IRR)^T} = 0$$

- The IRR Decision Rule says accept all projects whose $IRR > R$, reject all projects whose $IRR < R$ where R is the hurdle rate

Lessons

- IRR Rule can mislead decision making when cash flow signs are anything other than all negatives before all positives

- IRR Rule can mislead decision making when comparing projects even when cash flow signs are proper.
  - IRR does not account for differences in scale

Lessons

- IRR should be used in conjunction with NPV analysis

Coming up next

- Fixed Income Securities
  - Institutional environment
  - Valuation
  - Risk analysis