Time Value of Money: Compounding

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Last Time

Time Value of Money

- Intuition – time units like different currencies
- Tools – time line and discount factor
- Discounting – Moving CFs back in time
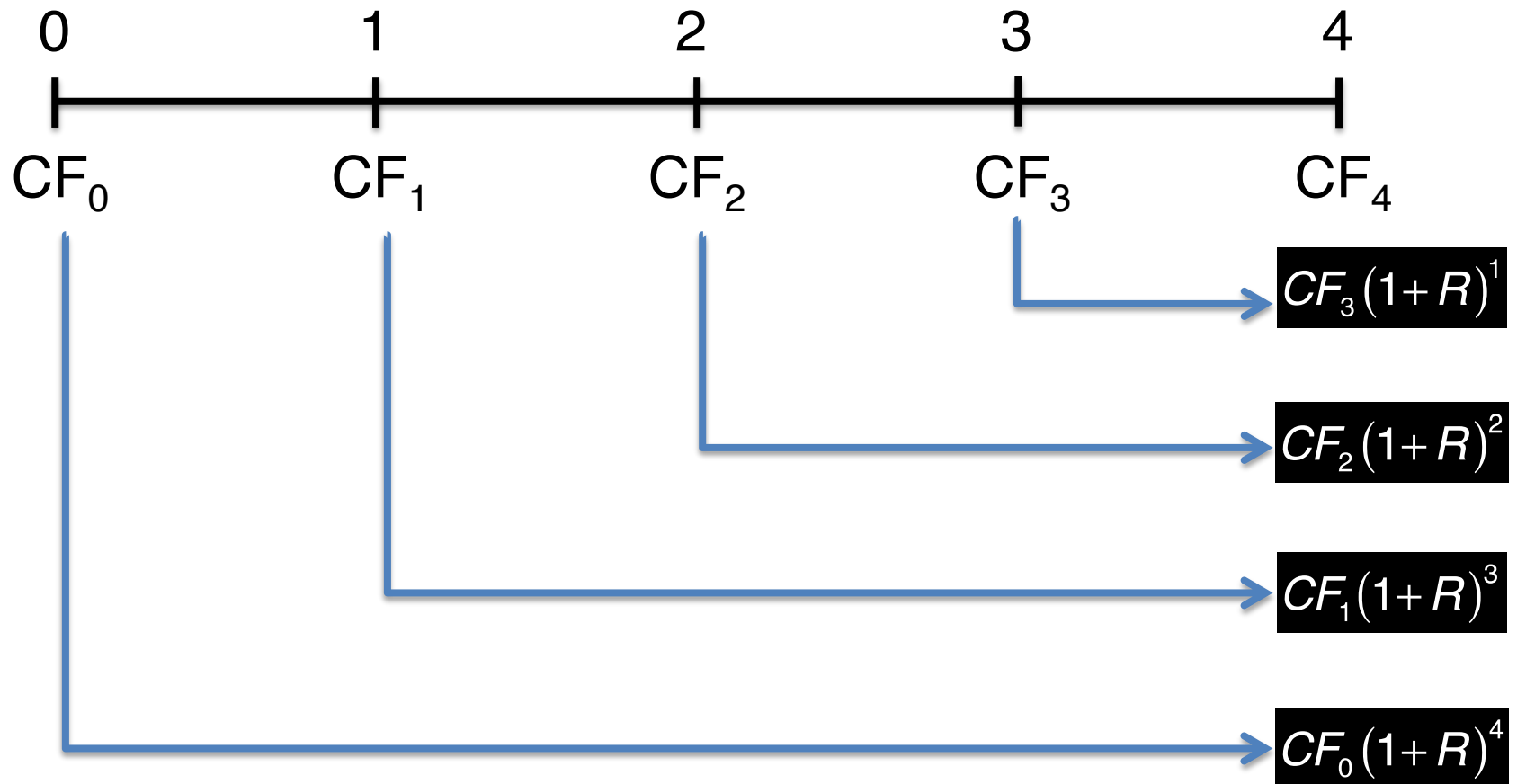- Lesson: Don't add CFs with different time units…ever!

This Time

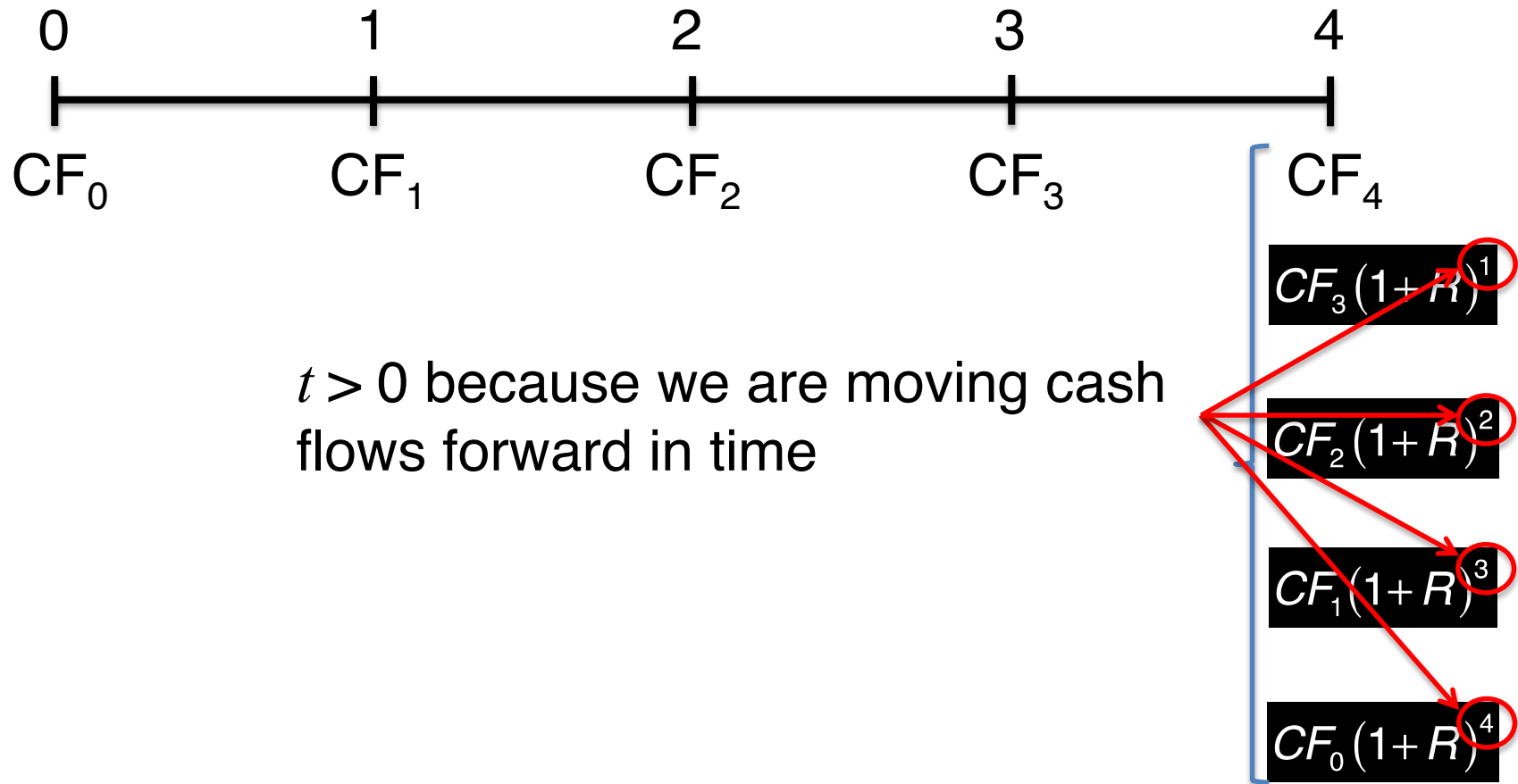## Time Value of Money
- Compounding

USING THE TOOLS: COMPOUNDING

Compounding

Compounding CFs moves them forward in time

Compounding

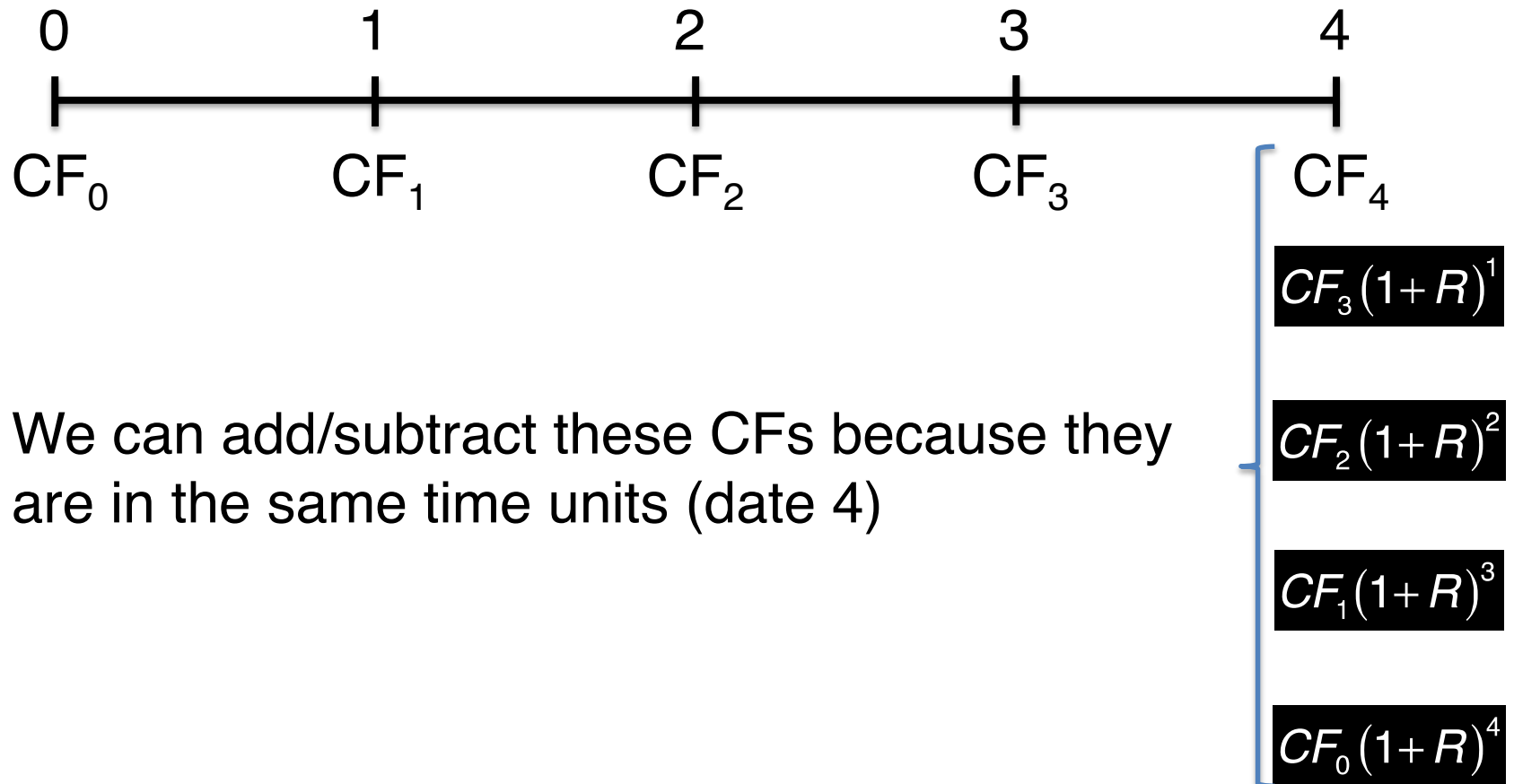Compounding CFs moves them forward in time
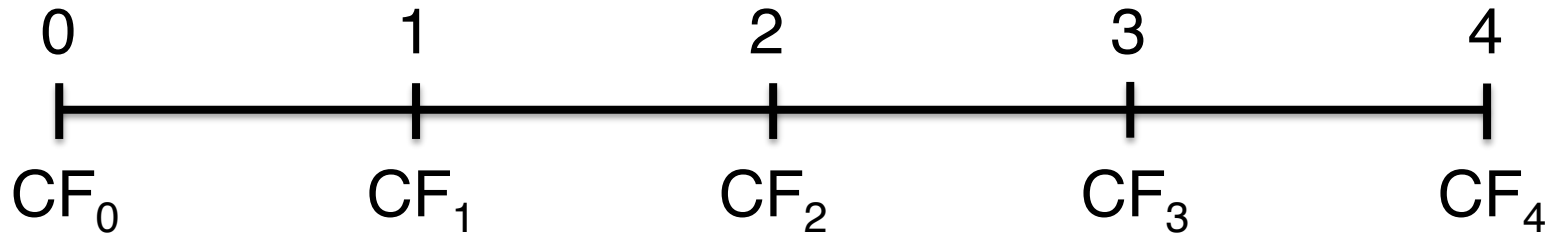
$$0 \qquad 1 \qquad 2 \qquad 3 \qquad 4$$

$CF_0 \qquad CF_1 \qquad CF_2 \qquad CF_3 \qquad CF_4$

$CF_3(1+R)^1$

$t > 0$ because we are moving cash flows forward in time

$CF_2(1+R)^2$

$CF_1(1+R)^3$

$CF_0(1+R)^4$

Compounding

Compounding CFs moves them forward in time

0	1	2	3	4
CF_0	CF_1	CF_2	CF_3	CF_4

$$CF_3(1+R)^1$$

We can add/subtract these CFs because they are in the same time units (date 4)

$$CF_2(1+R)^2$$

$$CF_1(1+R)^3$$

$$CF_0(1+R)^4$$

Future Value

Future value, $FV_t(\bullet)$ of CFs is compounded value of CFs as of t

```
  0        1        2        3        4
  |--------|--------|--------|--------|
 CF_0     CF_1     CF_2     CF_3     CF_4
```

$$CF_3(1+R)^1 = FV_4(CF_3)$$

$$CF_2(1+R)^2 = FV_4(CF_2)$$

These are future values of CFs as of year 4

$$CF_1(1+R)^3 = FV_4(CF_1)$$
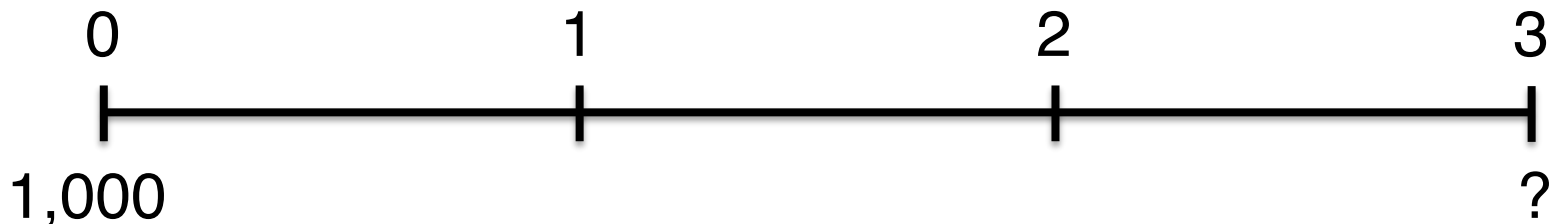
$$CF_0(1+R)^4 = FV_4(CF_0)$$

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

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Step 1: Put cash flows on a time line

0	1	2	3
1,000			?

Example 1 – Savings

How much money will I have after three years if I invest \$1,000 in a savings account paying 3.5% interest per annum?

Step 2: Move cash flow forward

0	1	2	3

1,000 $\longrightarrow$ $1,000(1+0.035)^3$

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

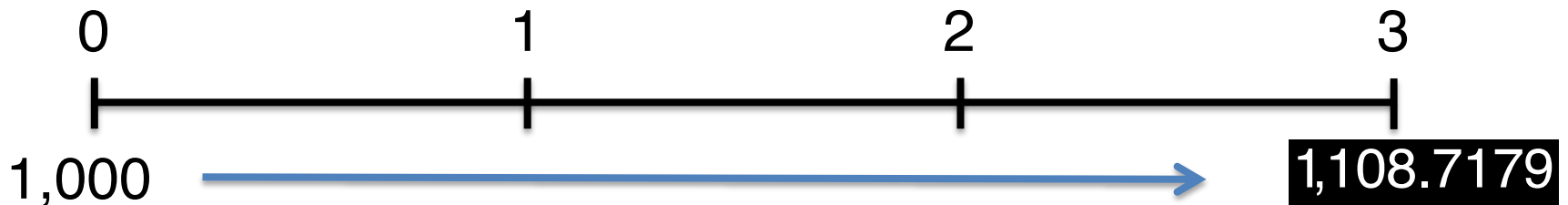Step 2: Move cash flow forward

```
0              1              2              3
|--------------|--------------|--------------|

1,000  ──────────────────────────▶   1,108.7179
```

Example 1 – Savings

How much money will I have after three years if I invest $1,000 in a savings account paying 3.5% interest per annum?

Step 2: Move cash flow forward

```
      0              1              2              3
      |              |              |              |
1,000  ──────────────────────────────────▶  1,108.7179
```
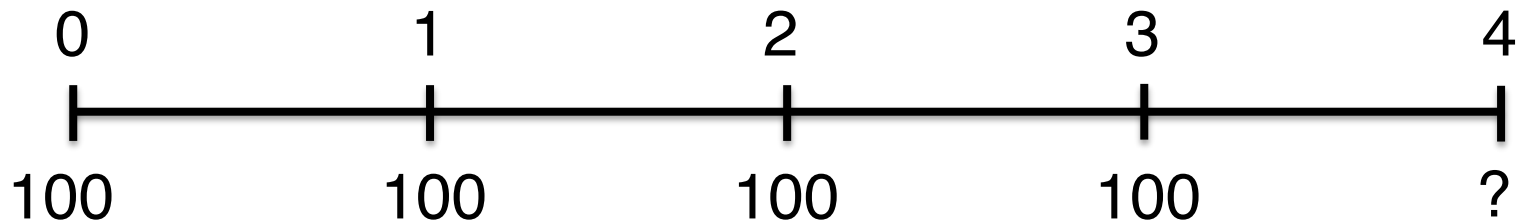
This is the future value of the 1,000

Example 2 – Savings

How much money will we have four years from today if we save $100 a year, beginning today, for the next three years, assuming we earn 5% per annum?
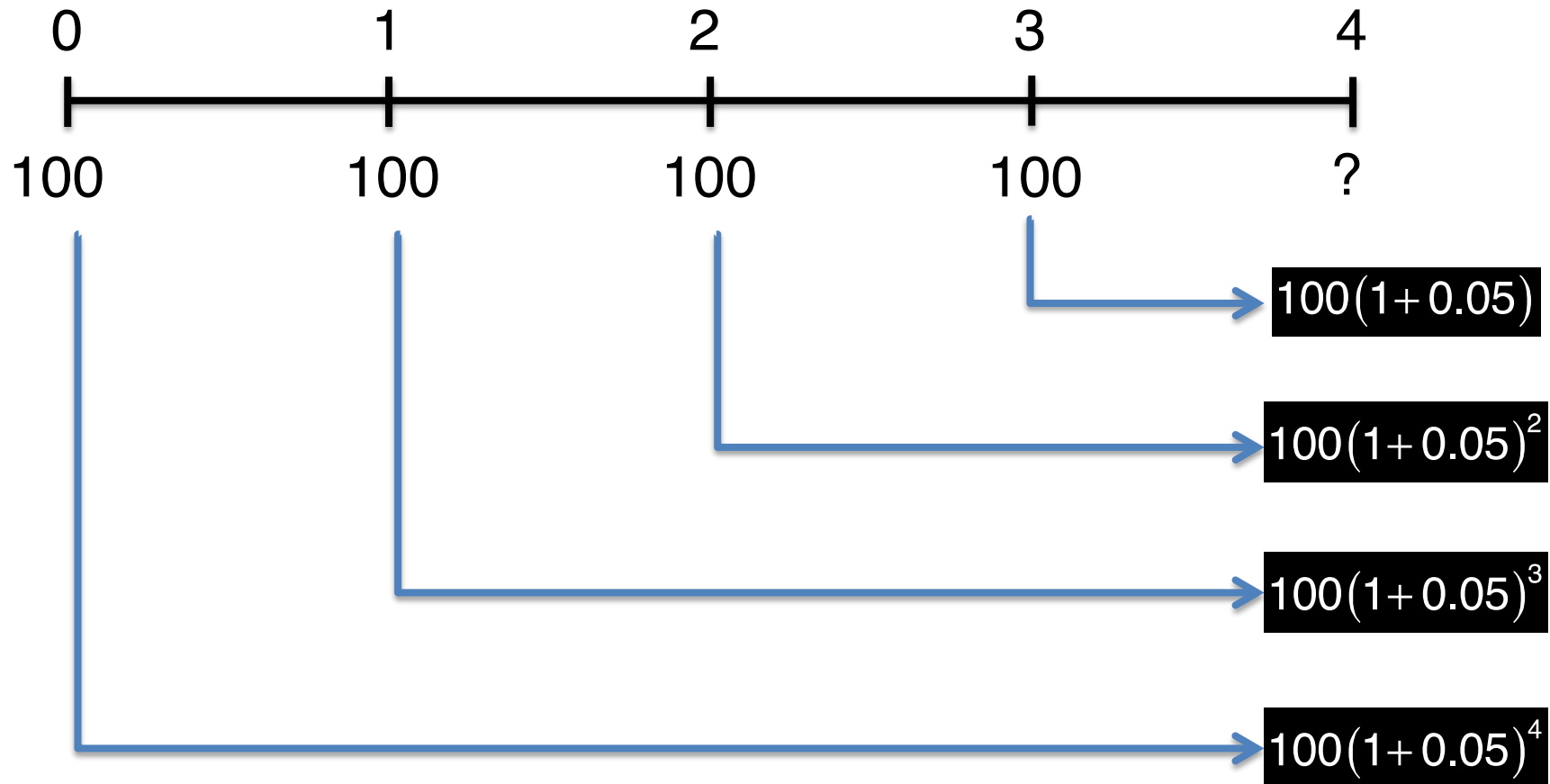
Example 2 – Savings

How much money will we have four years from today if we save $100 a year, beginning today, for the next three years, assuming we earn 5% per annum?

Step 1: Put cash flows on a time line

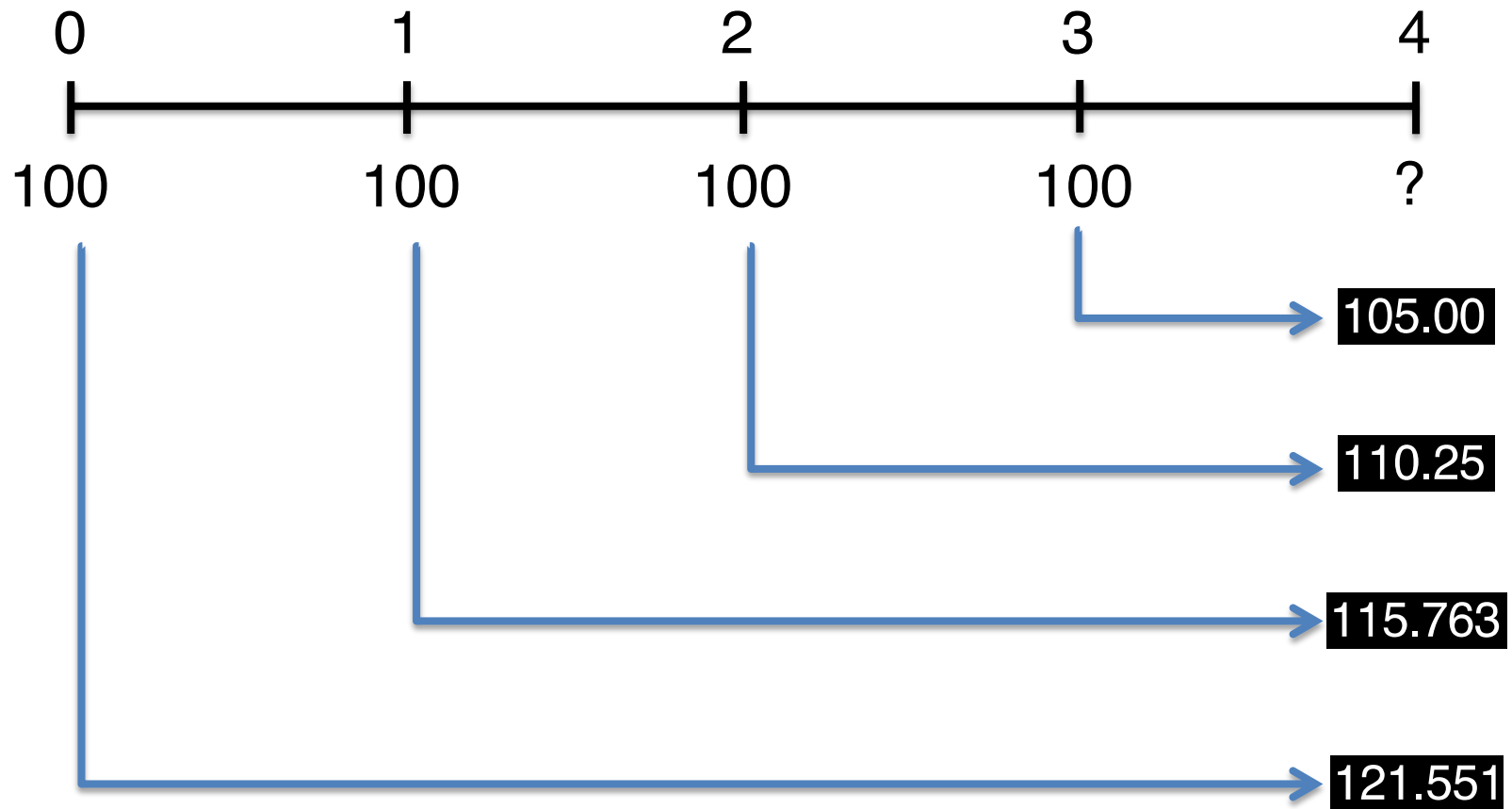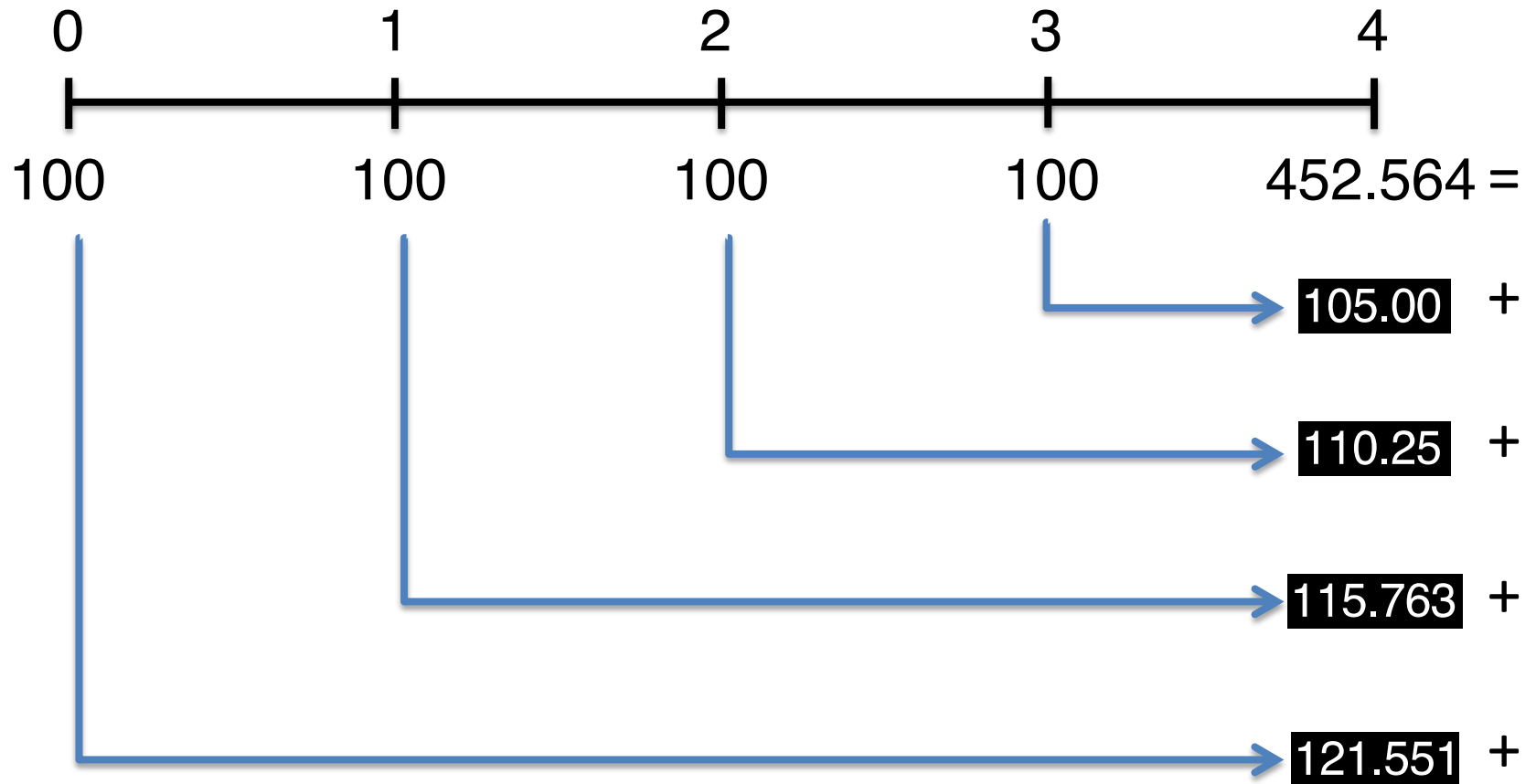| 0	1	2	3	4
100	100	100	100	?

Example 2 – Savings

Step 2: Move CFs forward in time

Example 2 – Savings

Step 2: Move CFs forward in time

Example 2 – Savings

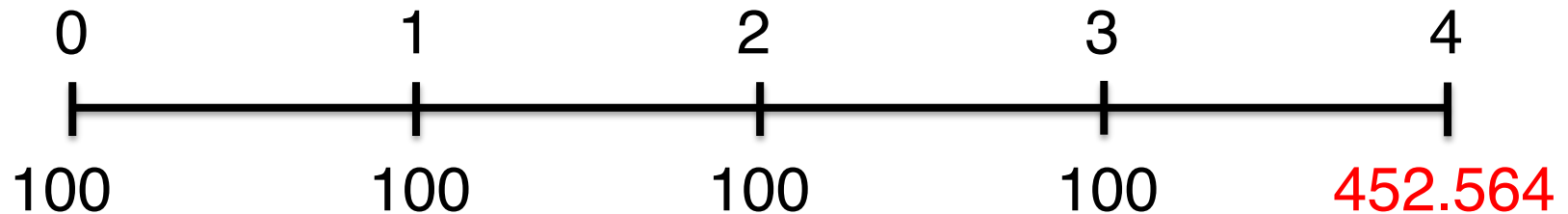Step 3: Add up cash flows

Example 2 – Savings

```
0           1           2           3           4
|-----------|-----------|-----------|-----------|
100         100         100         100      452.564
```
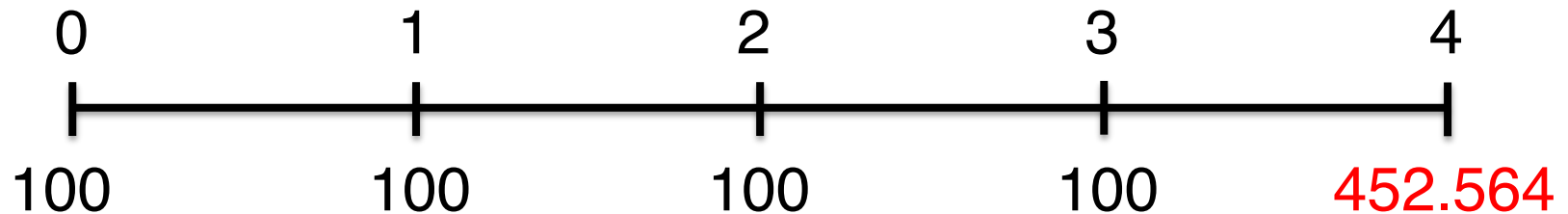
Interpretation 1: We will have $452.56 at the end of four years if we save $100 starting today for the next three years and our money earns 5% per annum.

Example 2 – Savings

Interpretation 2: The future value four years from today of saving $100 starting today for the next three years at 5% per annum is $452.56.

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00			

Example 2 – Savings (Account)

		Pre-Deposit		Post-Deposit
Year	Interest	Balance	Deposit	Balance
0			$100.00	$100.00
1	$5.00	$105.00		

$$= 100 + 5.00$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00		

$=$

$$FV_1(100) = 100 \times (1+0.05)^1$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00

$$= $$

$$105 + 100$$

Example 2 – Savings (Account)

Year	Interest	Pre-Deposit Balance	Deposit	Post-Deposit Balance
0			$100.00	$100.00
1	$5.00	$105.00	$100.00	$205.00
2	$10.25	$215.25	$100.00	$315.25
3	$15.76	$331.01	$100.00	$431.01
4	$21.55	$452.56	$0.00	$452.56

More Generally

Can add CFs at any point in time if same units

$$CF_3(1+R)^{-1} = PV_2(CF_3)$$

$$CF_4(1+R)^{-2} = PV_2(CF_4)$$

$$CF_1(1+R)^{1} = FV_2(CF_1)$$

$$CF_0(1+R)^{2} = FV_2(CF_0)$$

Summary

Lessons

- We use compounding to move cash flows forward in time

- Denote the value of cash flows in the future as future value $FV_s(CF_t)$

$$FV_s(CF_t) = CF_t(1+R)^{s-t} \quad \text{for } t < s$$

Coming up next

- Problem Set

- Useful shortcuts for *PV* and *FV* of common streams of cash flows