

Week 4: Balancing Risk and Reward Using Simulation

- ◆ Modeling Uncertainty: From Scenarios to Continuous Distributions
- ◆ Example: Designing a New Apartment Building
- ◆ Connecting Random Inputs and Random Outputs in a Simulation
- ◆ Setting up and Running a Simulation in Excel
- ◆ Analyzing and Interpreting Simulation Output
- ◆ Evaluating Alternative Decisions using Simulation Results

Week 4: Balancing Risk and Reward Using Simulation

- ◆ Modeling Uncertainty: From Scenarios to Continuous Distributions
- ◆ Example: Designing a New Apartment Building **Session 1**

- ◆ Connecting Random Inputs and Random Outputs in a Simulation
- ◆ Setting up and Running a Simulation in Excel
- ◆ Analyzing and Interpreting Simulation Output
- ◆ Evaluating Alternative Decisions using Simulation Results

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- ◆ Modeling Uncertainty: From Scenarios to Continuous Distributions
 - ◆ Example: Designing a New Apartment Building
- ◆ Connecting Random Inputs and Random Outputs in a Simulation
 - ◆ Setting up and Running a Simulation in Excel Session 2
- ◆ Analyzing and Interpreting Simulation Output
 - ◆ Evaluating Alternative Decisions using Simulation Results

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- ◆ Modeling Uncertainty: From Scenarios to Continuous Distributions
 - ◆ Example: Designing a New Apartment Building
 - ◆ Connecting Random Inputs and Random Outputs in a Simulation
 - ◆ Setting up and Running a Simulation in Excel
- ◆ Analyzing and Interpreting Simulation Output Session 3
 - ◆ Evaluating Alternative Decisions using Simulation Results

Simulation: Results for n=10 Simulation Runs

- ◆ The profit (in \$):

$$\Pi = 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))$$

- ◆ Stargrove.xlsx

Stargrove.xlsx									
Modeling Risk and Realities MOOC									
Apartments			Regular	Luxury	Reward	\$ 50,778,195.32	Risk	0.2	
Profit during the year	\$ 500,000.00	\$ 900,000.00							
Salvage Profit	\$ 100,000.00	\$ 150,000.00							
Profit Threshold	\$ 45,000,000.00								
Numbers of Apartments			Regular	Luxury					
96			96	12					
Demand Parameters									
Expected	90		10						
St. Dev.	25		3						
Demand Realizations	RV for Regular Demand	RV for Luxury Demand	DR	DL	Profit from Regular apartments	Profit from Luxury apartments	Total Profit	Profit below Threshold?	
1	34.65997588	6.536523668	34	6	\$ 23,463,990.35	\$ 6,702,392.75	\$ 30,166,383.10	1	
2	95.14134513	2.548728187	95	2	\$ 47,656,538.05	\$ 3,711,546.14	\$ 51,368,084.19	0	
3	103.4140237	9.157716957	103	9	\$ 48,000,000.00	\$ 8,668,287.72	\$ 56,668,287.72	0	
4	83.66079351	8.136827344	83	8	\$ 43,064,317.40	\$ 7,902,620.51	\$ 50,966,937.91	0	
5	146.1676643	11.60464879	146	11	\$ 48,000,000.00	\$ 10,503,486.59	\$ 58,503,486.59	0	
6	57.91029844	7.837553565	57	7	\$ 32,764,119.37	\$ 7,678,165.17	\$ 40,442,284.55	1	
7	128.539838	9.745067498	128	9	\$ 48,000,000.00	\$ 9,108,800.62	\$ 57,108,800.62	0	
8	159.5054041	12.00413297	159	12	\$ 48,000,000.00	\$ 10,800,000.00	\$ 58,800,000.00	0	
9	76.73592927	6.531155375	76	6	\$ 40,294,371.71	\$ 6,698,366.53	\$ 46,992,738.24	0	
10	116.6149982	9.286600314	116	9	\$ 48,000,000.00	\$ 8,764,950.24	\$ 56,764,950.24	0	

Simulation: Results for n=10 Simulation Runs

- ◆ Sample of random variables from the normal distribution with mean 90 and standard deviation of 25 and its descriptive statistics

Demand Realizations	RV for Regular Demand	RV for Regular Demand
1	34.65997588	Mean
2	95.14134513	Standard Error
3	103.4140237	Median
4	83.66079351	Mode
5	146.1676643	#N/A
6	57.91029844	Standard Deviation
7	128.539838	Sample Variance
8	159.5054041	Kurtosis
9	76.73592927	Skewness
10	116.6149982	Range
		Minimum
		Maximum
		Sum
		Count
		Confidence Level(95.0%)

- ◆ Sample mean is based on a small sample of n=10 instances of the underlying random variable. It is just an approximation to the true expected value of the random variable being simulated

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		Confidence Level(95.0%)

- ◆ **95% confidence level** identifies the “95% confidence interval” for the true expected value of the simulated random variable: based on the results of this simulation, we are 95% confident that the true expected value is in the interval = sample mean +/- 95% confidence level $\approx 110.24 \pm 27.86 = [82.38, 138.10]$

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RV for Regular Demand	
Mean	100.235027
Standard Error	12.31380659
Median	99.2776844
Mode	#N/A
Standard Deviation	38.9396755
Sample Variance	1516.298328
Kurtosis	-0.581372514
Skewness	-0.101693197
Range	124.8454282
Minimum	34.65997588
Maximum	159.5054041
Sum	1002.35027
Count	10
Confidence Level(95.0%)	27.85576579

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Simulation: Results for n=10 Simulation Runs

- ◆ With a simulation that samples the random input variables only 10 times, the reliability of the estimates for the mean and the standard deviation for any random quantity involved may be limited

Total Profit
\$ 30,166,383.10
\$ 51,368,084.19
\$ 56,668,287.72
\$ 50,966,937.91
\$ 58,503,486.59
\$ 40,442,284.55
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Reward \$	50,778,195.32
Risk	0.2

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Reward \$	50,778,195.32
Risk	0.2

Profit	
Mean	50778195.32
Standard Error	2945866.396
Median	54018185.95
Mode	#N/A
Standard Deviation	9315647.495
Sample Variance	8.67813E+13
Kurtosis	1.570184843
Skewness	-1.419981677
Range	28633616.9
Minimum	30166383.1
Maximum	58800000
Sum	507781953.2
Count	10
Confidence Level(95.0%)	6664012.769

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- Based on the results of this simulation, we can be 95% confident that the true expected profit under the decision we consider lies in the interval $\approx \$50,778,195 \pm \$6,664,013 = [\$44,114,182, \$57,442,208]$

Simulation: Results for n=1000 Simulation Runs

- ◆ Stargrove_1000.xlsx, seed 123 for the B column and seed 1234 for the C column

Total Profit
\$ 30,166,383.10
\$ 51,368,084.19
\$ 56,668,287.72
\$ 50,966,937.91
\$ 58,503,486.59
\$ 40,442,284.55
\$ 57,108,800.62
\$ 58,800,000.00
\$ 46,992,738.24
\$ 56,764,950.24
\$ 57,920,800.05
\$ 35,017,702.31
\$ 45,448,387.76
\$ 40,417,837.90
\$ 56,460,689.30
\$ 56,113,851.70
\$ 55,434,315.43
\$ 58,302,165.39
\$ 53,191,844.26
\$ 56,471,197.37



Reward \$	52,131,111.07
Risk	0.156
<hr/> <i>Profit</i> <hr/>	
Mean	52131111.07
Standard Error	205518.4113
Median	54261174.8
Mode	58800000
Standard Deviation	6499062.808
Sample Variance	4.22378E+13
Kurtosis	0.783172165
Skewness	-1.151004166
Range	32537281.28
Minimum	26262718.72
Maximum	58800000
Sum	52131111066
Count	1000
Confidence Level(95.0%)	403297.2995

- ◆ Based on the results of this longer simulation, we can now be 95% confident that the true expected profit under the decision we consider lies in the interval $\approx \$52,131,111 \pm \$403,297 = [\$51,727,814, \$52,534,408]$

Simulation: Results for n=1000 Simulation Runs

- ◆ Stargrove_1000.xlsx

Total Profit
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\$ 58,302,165.39
\$ 53,191,844.26
\$ 56,471,197.37



Reward \$	52,131,111.07
Risk	0.156
<hr/> <i>below Threshold?</i> <hr/>	
Mean	0.156
Standard Error	0.011480235
Median	0
Mode	0
Standard Deviation	0.363036907
Sample Variance	0.131795796
Kurtosis	1.609118878
Skewness	1.898921967
Range	1
Minimum	0
Maximum	1
Sum	156
Count	1000
Confidence Level(95.0%)	0.022528141

- ◆ Based on the results of this longer simulation, we can be 95% confident that the true value of the risk measure under the decision we consider lies in the interval $\approx 0.156 \pm 0.023 = [0.133, 0.179]$

Simulation: Comparing Two Alternatives

- ◆ Suppose that Stargrove would like to compare the decision of building 12 regular floors and 3 luxury floors ($R=96$ and $L=12$) with the decision of building 11 regular floors and 4 luxury floors ($R=88$ and $L=16$)
- ◆ We can use 1000 random values we have already generated for the demand for regular apartments and 1000 random values we have already generated for the demand for luxury apartments to estimate the reward and the risk associated with the decision of $R=88$ and $L=16$
- ◆ We can then compare reward and risk estimates for the two decisions

Simulation: Comparing Two Alternatives

◆ Stargrove_1000_TwoDecisions.xlsx

<i>Profit for R=96, L=12</i>	<i>Profit for R=96, L=12 below Threshold?</i>
Mean	52131111 Mean
Standard Error	205518.4 Standard Error
Median	54261175 Median
Mode	58800000 Mode
Standard Deviation	6499063 Standard Deviation
Sample Variance	4.22E+13 Sample Variance
Kurtosis	0.783172 Kurtosis
Skewness	-1.151 Skewness
Range	32537281 Range
Minimum	26262719 Minimum
Maximum	58800000 Maximum
Sum	5.21E+10 Sum
Count	1000 Count
Confidence Level(95.0%)	403297.3 Confidence Level(95.0%)
	0.022528141

<i>Profit for R=88, L=16</i>	<i>Profit for R=88, L=16 below Threshold?</i>
Mean	50660188.5 Mean
Standard Error	176801.7579 Standard Error
Median	52266311.62 Median
Mode	58400000 Mode
Standard Deviation	5590962.492 Standard Deviation
Sample Variance	3.12589E+13 Sample Variance
Kurtosis	1.760500397 Kurtosis
Skewness	-1.349726248 Skewness
Range	32337281.28 Range
Minimum	26062718.72 Minimum
Maximum	58400000 Maximum
Sum	50660188497 Sum
Count	1000 Count
Confidence Level(95.0%)	346945.4199 Confidence Level(95.0%)
	0.022703

Simulation: Comparing Two Alternatives

◆ Stargrove_1000_TwoDecisions.xlsx

	Profit for R=96, L=12	Profit for R=96, L=12 below Threshold?
Mean	52131111 Mean	0.156
Standard Error	205518.4 Standard Error	0.011480235
Median	54261175 Median	0
Mode	58800000 Mode	0
Standard Deviation	6499063 Standard Deviation	0.363036907
Sample Variance	4.22E+13 Sample Variance	0.131795796
Kurtosis	0.783172 Kurtosis	1.609118878
Skewness	-1.151 Skewness	1.898921967
Range	32537281 Range	1
Minimum	26262719 Minimum	0
Maximum	58800000 Maximum	1
Sum	5.21E+10 Sum	156
Count	1000 Count	1000
Confidence Level(95.0%)	403297.3 Confidence Level(95.0%)	0.022528141

	Profit for R=88, L=16	Profit for R=88, L=16 below Threshold?
Mean	50660188.5 Mean	0.159
Standard Error	176801.7579 Standard Error	0.011569
Median	52266311.62 Median	0
Mode	58400000 Mode	0
Standard Deviation	5590962.492 Standard Deviation	0.365859
Sample Variance	3.12589E+13 Sample Variance	0.133853
Kurtosis	1.760500397 Kurtosis	1.491811
Skewness	-1.349726248 Skewness	1.867841
Range	32337281.28 Range	1
Minimum	26062718.72 Minimum	0
Maximum	58400000 Maximum	1
Sum	50660188497 Sum	159
Count	1000 Count	1000
Confidence Level(95.0%)	346945.4199 Confidence Level(95.0%)	0.022703

Reward and risk measures for two policies

Simulation: Comparing Two Alternatives

- ◆ Stargrove_1000_TwoDecisions.xlsx

Decision	R=96, L=12	R=88, L=16
95% Confidence Interval for Reward, in \$ millions	[51.73, 52.53]	[50.31, 51.01]
95% Confidence Interval for Risk	[0.133, 0.179]	[0.136, 0.182]

- ◆ Based on the results of the simulation with n=1000 runs, we are 95% confident that the expected profit under the decision R=96, L=12 is higher than the expected profit under the decision R=88, L=16

Simulation: Comparing Two Alternatives

- ◆ Stargrove_1000_TwoPolicies.xlsx

Decision	R=96, L=12	R=88, L=16
95% Confidence Interval for Reward, in \$ millions	[51.73, 52.53]	[50.31, 51.01]
95% Confidence Interval for Risk	[0.133, 0.179]	[0.136, 0.182]

- ◆ Based on the results of the simulation with n=1000 runs, we are 95% confident that the expected profit under the decision R=96, L=12 is higher than the expected profit under the decision R=88, L=16
- ◆ The results of this simulation do not allow us to distinguish between the levels of risk associated with those two decisions at the same level of confidence

Simulation: Comparing Two Alternatives

- ◆ We can add other reasonable decisions to our comparison set
- ◆ If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures

Simulation: Comparing Two Alternatives

- ◆ We can add other reasonable decisions to our comparison set
- ◆ If two decisions cannot be distinguished on the basis of the results of a particular simulation, we can also run longer simulations to obtain more narrow confidence intervals for reward and risk measures
- ◆ Ultimately, the goals are to 1) limit the consideration set to decisions that result in risk measures limited by the tolerance level of a decision maker, and 2) among the decisions that satisfy constraint(s) on acceptable risk level(s), choose one that generates highest reward, at the selected confidence level
- ◆ Simulation provides “imperfect” estimates of reward and risk, but the notion of confidence intervals enables a decision maker to compare alternatives even using those imperfect estimates