Masinsko ucenje u R-u

Igor Hut

December 5, 2016

Contents

Uvodna analiza podataka

- `data.frame` - primarna forma za smestanje, analizu i manipulaciju podacima
- Uvoz podataka - licna preporuka `readr` (Hadley Wickham)
- Prvi korak - upoznavanje sa podacima
- Dimenzije - broj promenljivih (features) i opservacija (observations)
- Ciscenje i uredjivanje podataka - licna preporuka `dplyr` i `tidyr`(Hadley Wickham)
- Vizualizacija - `base` i `ggplot2` (Hadley Wickham)

Primer inicijalne provere i analize podataka

```r
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 3.3.2
```

```r
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library(tidyr)
# broj promenljivih i broj opservacija
str(iris)
```

```
## 'data.frame':    150 obs. of  5 variables:
##  $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
##  $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
##  $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
##  $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
##  $ Species     : Factor w/ 3 levels "setosa","versicolor",..: 1 1 1 1 1 1 1 1 1 1 ...
```

```r
dim(iris)
```

```
## [1] 150   5
```

```r
# Nekoliko prvih i poslednjih vrsta iz `iris` baze
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

```r
tail(iris)
```

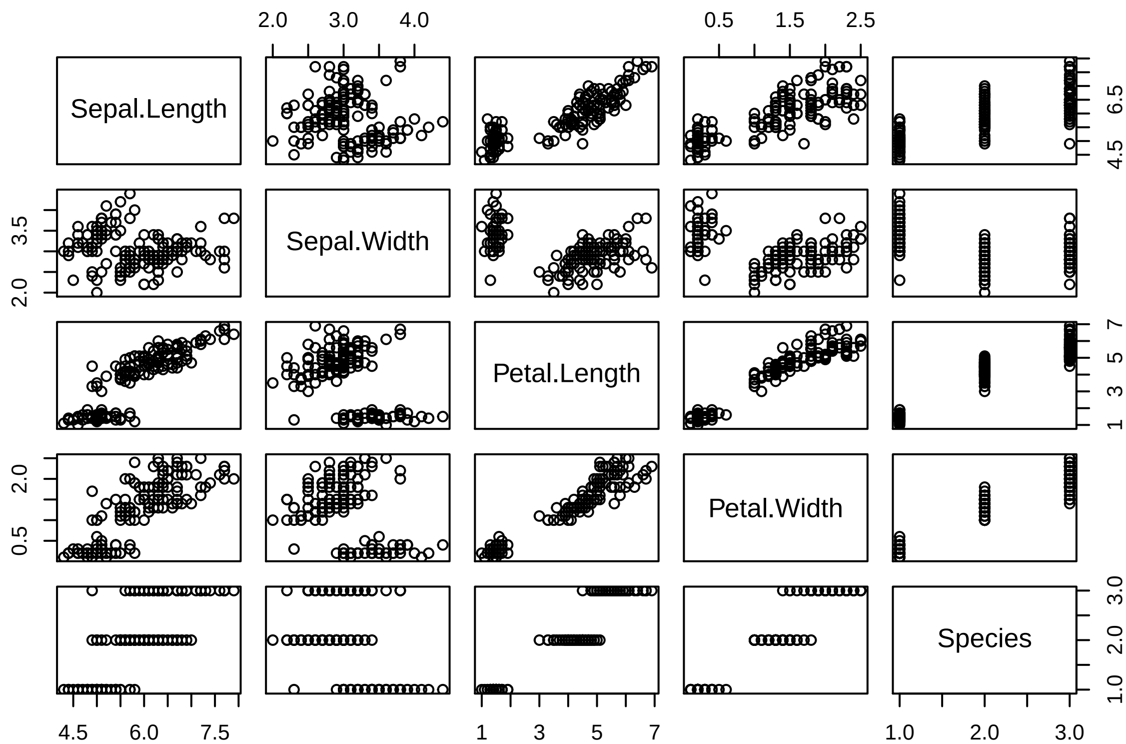
```
##     Sepal.Length Sepal.Width Petal.Length Petal.Width   Species
## 145          6.7         3.3          5.7         2.5 virginica
## 146          6.7         3.0          5.2         2.3 virginica
## 147          6.3         2.5          5.0         1.9 virginica
## 148          6.5         3.0          5.2         2.0 virginica
## 149          6.2         3.4          5.4         2.3 virginica
## 150          5.9         3.0          5.1         1.8 virginica
```

```r
# sumarna statistika za podatke u `iris` bazi
summary(iris)
```

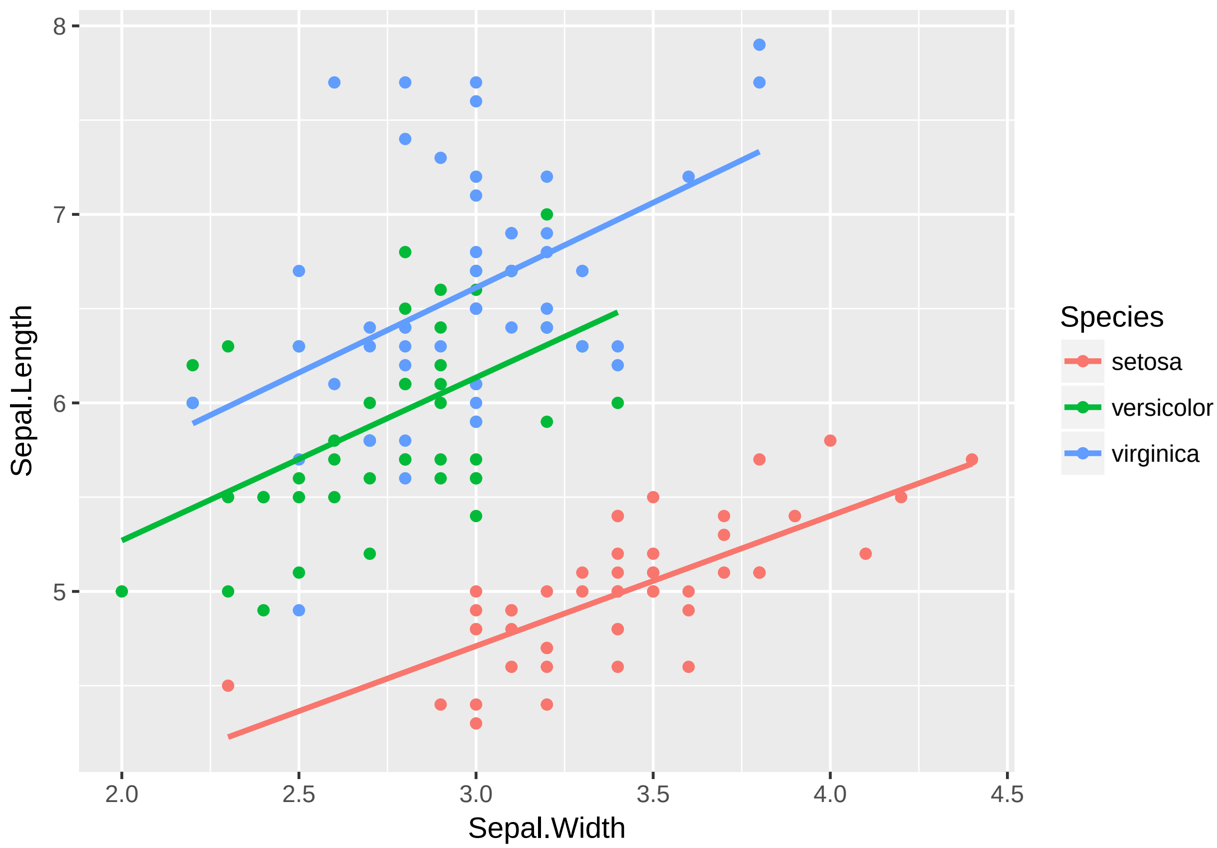
```
##   Sepal.Length    Sepal.Width     Petal.Length    Petal.Width
##  Min.   :4.300   Min.   :2.000   Min.   :1.000   Min.   :0.100
##  1st Qu.:5.100   1st Qu.:2.800   1st Qu.:1.600   1st Qu.:0.300
##  Median :5.800   Median :3.000   Median :4.350   Median :1.300
##  Mean   :5.843   Mean   :3.057   Mean   :3.758   Mean   :1.199
##  3rd Qu.:6.400   3rd Qu.:3.300   3rd Qu.:5.100   3rd Qu.:1.800
##  Max.   :7.900   Max.   :4.400   Max.   :6.900   Max.   :2.500
```

```
##         Species
##  setosa    :50
##  versicolor:50
##  virginica :50
##
##
##
```

```
plot(iris) #rezultat ce biti isti kao da smo upotrebili `pairs` funkciju iz `base` bilioteke
```




```
ggplot(iris, aes( x = Sepal.Width, y = Sepal.Length, col = Species)) +
  geom_point() +
  geom_smooth(method = "lm", se = FALSE)
```



Regresija, klasifikacija, klasterizacija - Uvod

Linearna regresija - uvodni primer 1

```
# Ucitavamo "Wage" bazu iz "ISLR" paketa koja sadrzi neke opste podatke o radnicima u srednje-Atlanstko
library(ISLR)
```

```
## Warning: package 'ISLR' was built under R version 3.3.2
```

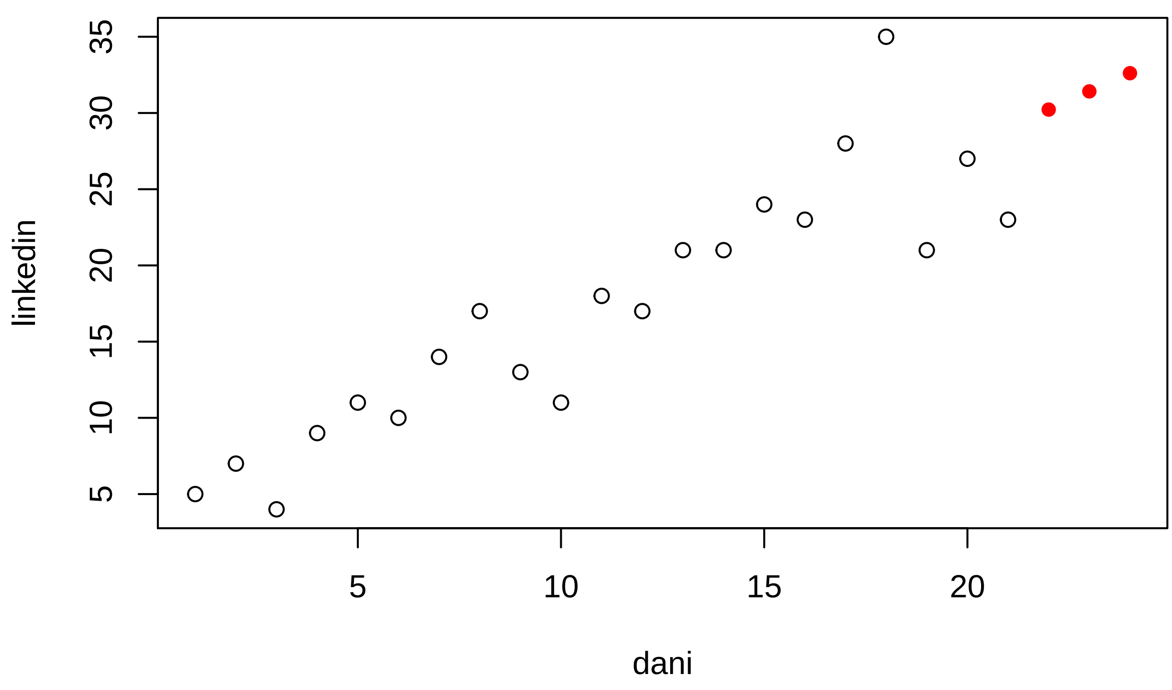
```
data("Wage")


# Generisemo linearni model "lm_wage"

lm_wage <- lm(wage ~ age, data = Wage)

# ?lm - kako se funkcija "lm()" koristi

# Definisemo data.frame sa novim vrednostima, koje nisu koriscenje za sintezu modela: "unseen"
unseen <- data.frame(age = 60)


# Na osnovu modela "lm_wage" predvidjamo koliko iznosi plata 60-togodisnjeg radnika
predict(lm_wage, unseen)
```

```
##        1
```

```
## 124.1413
```

Regresija - uvodni primer 2

```r
# Broj pregleda vased LinkedIn profila u periodu od tri nedelje
linkedin <- c(5, 7,  4,  9, 11, 10, 14, 17, 13, 11, 18, 17, 21, 21, 24, 23, 28, 35, 21, 27, 23)

# Vektor koji sadrzi korespodentne dane: "dani"
dani <- 1:21

# Linearni model - broj pregleda po danima: linkedin_lm

linkedin_lm <- lm(linkedin ~ dani)

# Predvidjamo broj pregleda u sledeca tri dana: linkedin_pred
buduci_dani <- data.frame(dani = 22:24)
linkedin_pred <- predict(linkedin_lm, buduci_dani)

# Plotujemo "istorijske" podatke i predvidjanje
plot(linkedin ~ dani, xlim = c(1, 24))
points(22:24, linkedin_pred, col = "red", pch = 16)
```



Klasifikacija - uvodni primer

Primer neadekvatnog klasifikatora - krajnje overfitovanje podataka iz trening seta!

```r
library(readr)
```

```
## Warning: package 'readr' was built under R version 3.3.2
```

```r
if (!"emails" %in% ls()) {
    emails <- read_csv("data/emails_small.csv")
}
```

```
## Parsed with column specification:
## cols(
##   avg_capital_seq = col_double(),
##   spam = col_integer()
## )
```

```r
# Proveravamo strukturu seta podataka
str(emails)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    13 obs. of  2 variables:
##  $ avg_capital_seq: num  1 2.11 4.12 1.86 2.97 ...
##  $ spam           : int  0 0 1 0 1 0 1 0 0 1 ...
##  - attr(*, "spec")=List of 2
##   ..$ cols   :List of 2
##   .. ..$ avg_capital_seq: list()
##   .. .. ..- attr(*, "class")= chr  "collector_double" "collector"
##   .. ..$ spam           : list()
##   .. .. ..- attr(*, "class")= chr  "collector_integer" "collector"
##   ..$ default: list()
##   .. ..- attr(*, "class")= chr  "collector_guess" "collector"
##   ..- attr(*, "class")= chr "col_spec"
```

```r
# Definisemo funkciju spam_classifier()
# 1 - spam, 0 - ham
spam_classifier <- function(x){
  prediction <- rep(NA,length(x))
  prediction[x > 4] <- 1
  prediction[x >= 3 & x <= 4] <- 0
  prediction[x >= 2.2 & x < 3] <- 1
  prediction[x >= 1.4 & x < 2.2] <- 0
  prediction[x > 1.25 & x < 1.4] <- 1
  prediction[x <= 1.25] <- 0
  return(prediction)
}

# Primenimo nas klasifikator na kolonu "avg_capital_seq": "spam_pred"
spam_pred <- spam_classifier(emails$avg_capital_seq)

# Uporedimo "spam_pred" i  "emails$spam"
spam_pred == emails$spam
```

```
##  [1] TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE TRUE
```

```r
identical(spam_pred, as.numeric(emails$spam))
```

```
## [1] TRUE
```

Klasterovanje - uvodni primer

```r
# Da bi smo obezbedili reproduktibilnost
set.seed(1)

# Proveravamo strukturu podataka
str(iris)
```

```
## 'data.frame':    150 obs. of  5 variables:
##  $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
##  $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
##  $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
##  $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
##  $ Species     : Factor w/ 3 levels "setosa","versicolor",..: 1 1 1 1 1 1 1 1 1 1 ...
```

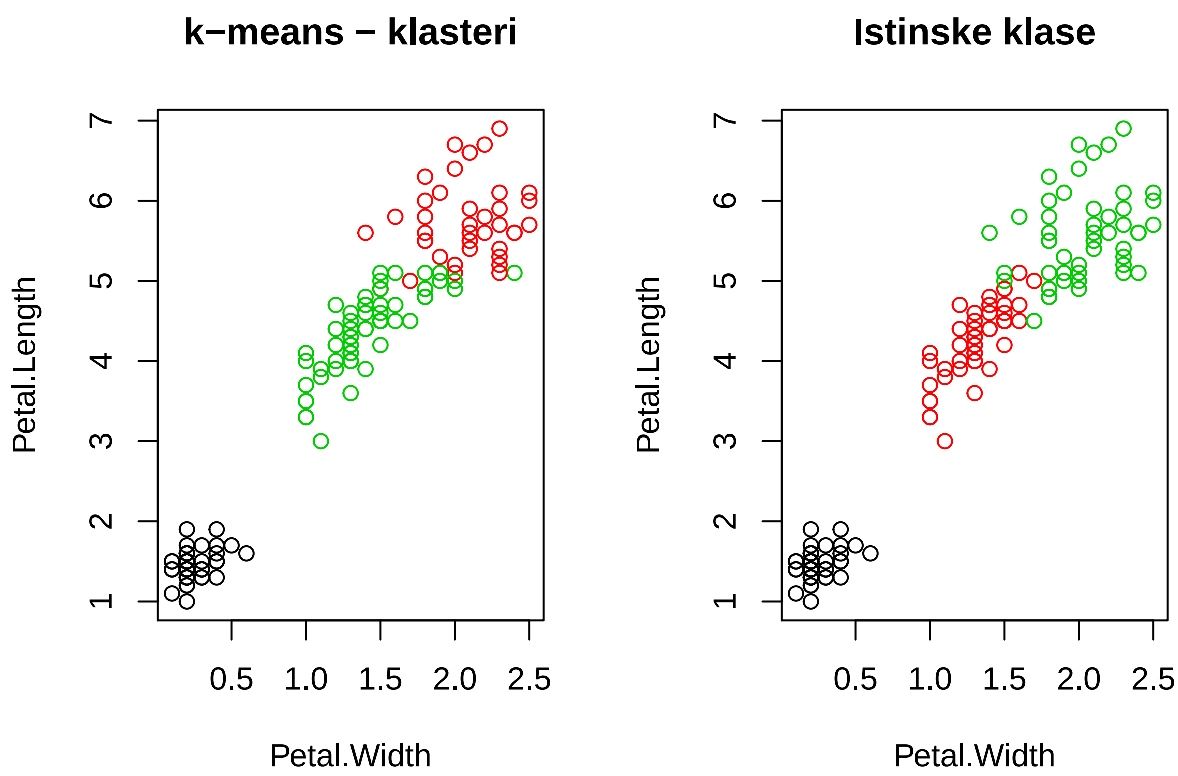
```r
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

```r
# Delimo "iris" na dva seta: "my_iris" i "species""
my_iris <- iris[-5]
species <- iris$Species

# Vrsimo k-means klasterizaciju za "my_iris", pretpostavljamo da postoje tri klase: "kmeans_iris"
kmeans_iris <- kmeans(my_iris,3)

# Poredimo dobijene klastere sa istinskim klasama (kategorijama)
table(species, kmeans_iris$cluster)
```

```
##             
## species       1  2  3
##   setosa     50  0  0
##   versicolor  0  2 48
##   virginica   0 36 14
```

```r
# Plotujemo "Petal.Width" vs "Petal.Length", bojimo po klasterima odn. postojecim kategorijama
par(mfrow = c(1,2))
plot(Petal.Length ~ Petal.Width, data = my_iris, col = kmeans_iris$cluster)
title("k-means - klasteri")
plot(Petal.Length ~ Petal.Width, data = my_iris, col = iris$Species)
title("Istinske klase")
```



Ocena modela

Konfuziona matrica - Primeri

Primer 1:

```
library(rpart)
library(readr)
library(purrr)
```

```
##
## Attaching package: 'purrr'

## The following objects are masked from 'package:dplyr':
##
##     contains, order_by
```

```
# Import podataka
if (!"titanic" %in% ls()) {
    titanic <- read_csv("data/train.csv")
}
```

```
## Parsed with column specification:
## cols(
##   survived = col_integer(),
##   pclass = col_integer(),
##   name = col_character(),
```

```
##   sex = col_character(),
##   age = col_double(),
##   sibsp = col_integer(),
##   parch = col_integer(),
##   ticket = col_character(),
##   fare = col_double(),
##   cabin = col_character(),
##   embarked = col_character()
## )
```

```r
# Da obezbedimo reproduktibilnost
set.seed(33)

# Proveravamo strukturu data seta
str(titanic)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    891 obs. of  11 variables:
##  $ survived: int  0 1 1 1 0 0 0 0 1 1 ...
##  $ pclass  : int  3 1 3 1 3 3 1 3 3 2 ...
##  $ name    : chr  "Braund, Mr. Owen Harris" "Cumings, Mrs. John Bradley (Florence Briggs Thayer)" "He
##  $ sex     : chr  "male" "female" "female" "female" ...
##  $ age     : num  22 38 26 35 35 NA 54 2 27 14 ...
##  $ sibsp   : int  1 1 0 1 0 0 0 3 0 1 ...
##  $ parch   : int  0 0 0 0 0 0 0 1 2 0 ...
##  $ ticket  : chr  "A/5 21171" "PC 17599" "STON/O2. 3101282" "113803" ...
##  $ fare    : num  7.25 71.28 7.92 53.1 8.05 ...
##  $ cabin   : chr  NA "C85" NA "C123" ...
##  $ embarked: chr  "S" "C" "S" "S" ...
##  - attr(*, "spec")=List of 2
##   ..$ cols   :List of 11
##   .. ..$ survived: list()
##   .. .. ..- attr(*, "class")= chr  "collector_integer" "collector"
##   .. ..$ pclass  : list()
##   .. .. ..- attr(*, "class")= chr  "collector_integer" "collector"
##   .. ..$ name    : list()
##   .. .. ..- attr(*, "class")= chr  "collector_character" "collector"
##   .. ..$ sex     : list()
##   .. .. ..- attr(*, "class")= chr  "collector_character" "collector"
##   .. ..$ age     : list()
##   .. .. ..- attr(*, "class")= chr  "collector_double" "collector"
##   .. ..$ sibsp   : list()
##   .. .. ..- attr(*, "class")= chr  "collector_integer" "collector"
##   .. ..$ parch   : list()
##   .. .. ..- attr(*, "class")= chr  "collector_integer" "collector"
##   .. ..$ ticket  : list()
##   .. .. ..- attr(*, "class")= chr  "collector_character" "collector"
##   .. ..$ fare    : list()
##   .. .. ..- attr(*, "class")= chr  "collector_double" "collector"
##   .. ..$ cabin   : list()
##   .. .. ..- attr(*, "class")= chr  "collector_character" "collector"
##   .. ..$ embarked: list()
##   .. .. ..- attr(*, "class")= chr  "collector_character" "collector"
##   ..$ default: list()
##   .. ..- attr(*, "class")= chr  "collector_guess" "collector"
##   ..- attr(*, "class")= chr "col_spec"
```

```r
# Koristicemo samo kolone 'survived', 'pclass', 'sex' i 'age'
titanic <- titanic[, c(1, 2, 4, 5)]
str(titanic)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    891 obs. of  4 variables:
##  $ survived: int  0 1 1 1 0 0 0 0 1 1 ...
##  $ pclass  : int  3 1 3 1 3 3 1 3 3 2 ...
##  $ sex     : chr  "male" "female" "female" "female" ...
##  $ age     : num  22 38 26 35 35 NA 54 2 27 14 ...
```

```r
# Prve tri promenlive bi evidentno trebalo da budu tretirane kao kategoricke promenljive - faktori
titanic[-4] <- map(titanic[-4], as.factor)

str(titanic)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    891 obs. of  4 variables:
##  $ survived: Factor w/ 2 levels "0","1": 1 2 2 2 1 1 1 1 2 2 ...
##  $ pclass  : Factor w/ 3 levels "1","2","3": 3 1 3 1 3 3 1 3 3 2 ...
##  $ sex     : Factor w/ 2 levels "female","male": 2 1 1 1 2 2 2 2 1 1 ...
##  $ age     : num  22 38 26 35 35 NA 54 2 27 14 ...
```

```r
table(titanic$survived)
```

```
##
##   0   1
## 549 342
```

```r
# Odnos prezivelih i poginulih
prop.table(table(titanic$survived))
```

```
##
##         0         1
## 0.6161616 0.3838384
```

```r
# Generisemo klasifikacioni model (drvo odlucivanja - decision tree) na osnovu datih podataka:
tree <- rpart(survived ~ ., data = titanic, method = "class")

# Koristimo predict() funkciju da predvidimo klase
pred <- predict(tree, newdata = titanic, type = "class")


# Konstruisemo konfuzionu matricu koristeci "table()":
conf_t <- table(titanic$survived, pred)
conf_t
```

```
##    pred
##       0   1
##   0 479  70
##   1  94 248
```

Primer 2:

```r
#Isto to sa "pima" bazom podataka
library(faraway)
```

```
## Warning: package 'faraway' was built under R version 3.3.2
```

```
##
```

```
## Attaching package: 'faraway'

## The following object is masked from 'package:rpart':
##
##     solder
```

```r
data(pima)

head(pima)
```

```
##   pregnant glucose diastolic triceps insulin  bmi diabetes age test
## 1        6     148        72      35       0 33.6    0.627  50    1
## 2        1      85        66      29       0 26.6    0.351  31    0
## 3        8     183        64       0       0 23.3    0.672  32    1
## 4        1      89        66      23      94 28.1    0.167  21    0
## 5        0     137        40      35     168 43.1    2.288  33    1
## 6        5     116        74       0       0 25.6    0.201  30    0
```

```r
str(pima)
```

```
## 'data.frame':    768 obs. of  9 variables:
##  $ pregnant : int  6 1 8 1 0 5 3 10 2 8 ...
##  $ glucose  : int  148 85 183 89 137 116 78 115 197 125 ...
##  $ diastolic: int  72 66 64 66 40 74 50 0 70 96 ...
##  $ triceps  : int  35 29 0 23 35 0 32 0 45 0 ...
##  $ insulin  : int  0 0 0 94 168 0 88 0 543 0 ...
##  $ bmi      : num  33.6 26.6 23.3 28.1 43.1 25.6 31 35.3 30.5 0 ...
##  $ diabetes : num  0.627 0.351 0.672 0.167 2.288 ...
##  $ age      : int  50 31 32 21 33 30 26 29 53 54 ...
##  $ test     : int  1 0 1 0 1 0 1 0 1 1 ...
```

```r
# Da bismo obezbedili reproduktibilnost
set.seed(33)

# Generisemo klasifikacioni model (drvo odlucivanja - decision tree) na osnovu datih podataka:
tree <- rpart(test ~ ., data = pima, method = "class")

# Koristimo predict() funkciju da predvidimo klase
pred <- predict(tree, newdata = pima, type = "class")


# Konstruisemo konfuzionu matricu koristeci "table()":
conf_p <- table(pima$test, pred)
conf_p
```

```
##    pred
##       0   1
##   0 449  51
##   1  72 196
```

Tacnost, preciznost, senzitivnost (recall), specificnost - Primer

```r
# Izracunajmo parametre za ocenu valjanosti modela "tree" za "titanic" skup podataka

# Formiramo TP, FN, FP i TN na osnovu "conf_t"

TP <- conf_t[2,2]
```

```r
FP <- conf_t[1,2]

FN <- conf_t[2,1]

TN <- conf_t[1,1]


# Tacnost (Accuracy)
acc <- (TP + TN)/sum(conf_t)
acc
```

```
## [1] 0.8159371
```

```r
# Preciznost (Precision)
prec <- TP/(TP + FP)
prec
```

```
## [1] 0.7798742
```

```r
# Senzitivnost (Sensitivity, Recall)
sens <- TP/(TP + FN)
sens
```

```
## [1] 0.7251462
```

```r
# Specificnost (Specificity)
spec <- TN/(TN + FP)
spec
```

```
## [1] 0.8724954
```

Zadatak za vezbanje na casu:

Izracunajte ove vrednosti za "tree" model generisan na osnovu "pima" seta podataka.

Kvalitet regresije

- Srednja kvadratna greska
- U nasem slucaju mozemo smatrati da se poklapa sa standardnom devijacijom
- `sqrt((1/nrow(truth)) * sum( (truth$col - pred)^2))`

Primer:

```r
# Koristicemo "pima" bazu

# Struktura seta podataka
str(pima)
```

```
## 'data.frame':    768 obs. of  9 variables:
##  $ pregnant : int  6 1 8 1 0 5 3 10 2 8 ...
##  $ glucose  : int  148 85 183 89 137 116 78 115 197 125 ...
##  $ diastolic: int  72 66 64 66 40 74 50 0 70 96 ...
##  $ triceps  : int  35 29 0 23 35 0 32 0 45 0 ...
##  $ insulin  : int  0 0 0 94 168 0 88 0 543 0 ...
##  $ bmi      : num  33.6 26.6 23.3 28.1 43.1 25.6 31 35.3 30.5 0 ...
```

```
##  $ diabetes : num  0.627 0.351 0.672 0.167 2.288 ...
##  $ age      : int  50 31 32 21 33 30 26 29 53 54 ...
##  $ test     : int  1 0 1 0 1 0 1 0 1 1 ...
```

```r
# Multivarijabilna linearna regresija - prostiji model (ukljucen manji broj promenljivih)
fit_1 <- lm(diabetes ~ bmi + triceps + age + glucose, data = pima)

# Predvidjanje na osnovu modela: pred_1
pred_1 <- predict(fit_1)

# RMSE na osnovu "pima$diabetes" (tacne vrednosti) i "pred_1" (vrednosti na osnovu modela fit_1)
rmse_1 <- sqrt(1/nrow(pima)*sum((pima$diabetes - pred_1) ^ 2))

rmse_1
```

```
## [1] 0.3222776
```

```r
# Multivarijabilna linearna regresija - kompleksniji model (ukljucen veci broj promenljivih)
fit_2 <- lm(diabetes ~ bmi + triceps + age + glucose + diastolic + insulin + pregnant, data = pima)

# Predvidjanje na osnovu modela: pred_1
pred_2 <- predict(fit_2)

# RMSE na osnovu "pima$diabetes" (tacne vrednosti) i "pred_1" (vrednosti na osnovu modela fit_1)
rmse_2 <- sqrt(1/nrow(pima)*sum((pima$diabetes - pred_2) ^ 2))

rmse_2
```

```
## [1] 0.3205351
```

Procena valjanosti klasterizacije: WSS vs BSS

```r
# Da bi smo obezbedili reproduktibilnost
set.seed(33)

# Proveravamo strukturu podataka
str(iris)
```

```
## 'data.frame':    150 obs. of  5 variables:
##  $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
##  $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
##  $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
##  $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
##  $ Species     : Factor w/ 3 levels "setosa","versicolor",..: 1 1 1 1 1 1 1 1 1 1 ...
```

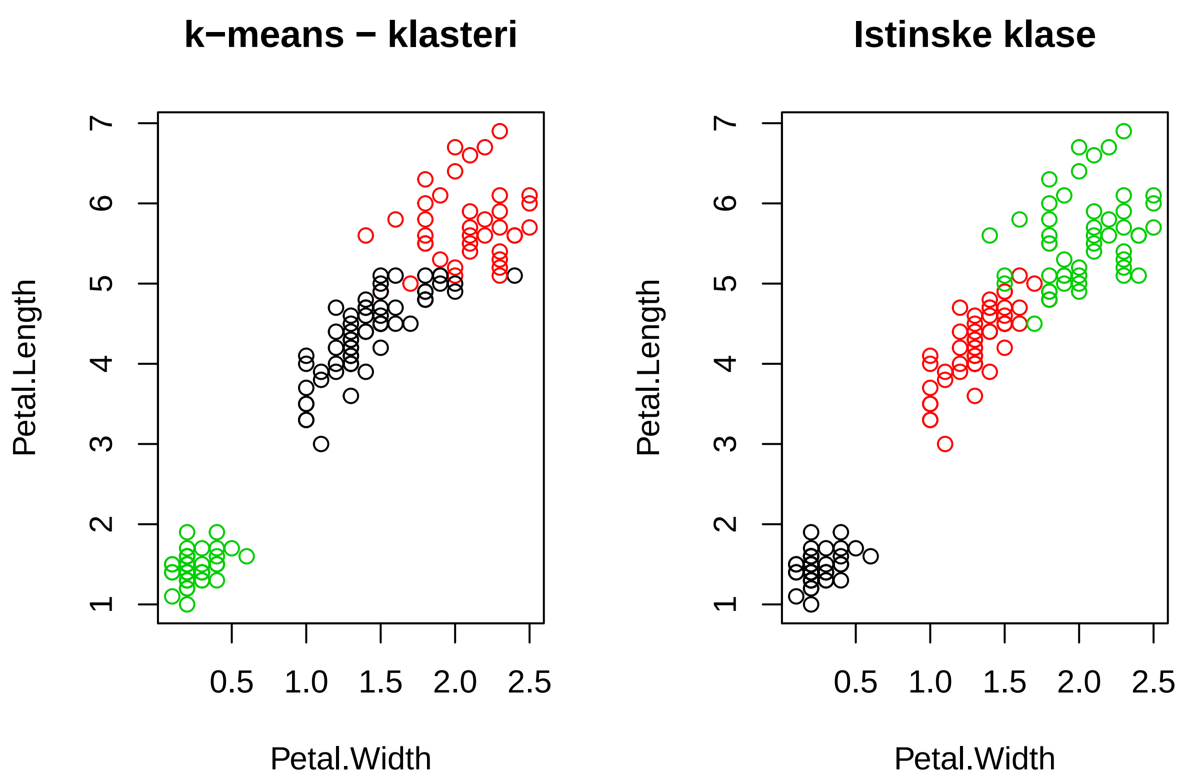
```r
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
## 6          5.4         3.9          1.7         0.4  setosa
```

```r
# Delimo "iris" na dva seta: "my_iris" i "species""
my_iris <- iris[-5]
species <- iris$Species

# Vrsimo k-means klasterizaciju za "my_iris" uz pretpostavku da postoje tri klase: "kmeans_iris"
kmeans_iris <- kmeans(my_iris,3)

# Poredimo dobijene klastere sa istinskim klasama (kategorijama)
table(species, kmeans_iris$cluster)
```

```
##
## species       1  2  3
##   setosa      0  0 50
##   versicolor 48  2  0
##   virginica  14 36  0
```

```r
# Plotujemo "Petal.Width" vs "Petal.Length", bojimo po klasterima odn. postojecim kategorijama
par(mfrow = c(1,2))
plot(Petal.Length ~ Petal.Width, data = my_iris, col = kmeans_iris$cluster)
title("k-means - klasteri")
plot(Petal.Length ~ Petal.Width, data = my_iris, col = iris$Species)
title("Istinske klase")
```




```r
kmeans_iris$tot.withinss/kmeans_iris$betweenss
```

```
## [1] 0.1308696
```

Trening set i test set

- Cilj implementacije algoritma **nadgledanog** ucenja jeste dobijanje "dovoljno" dobrog prediktivnog modela na osnovu raspolozivog seta podataka.

- Set podataka koji se koristi za formiranje modela - **trening set**
- Set podatak koji se koristi za procenu valjanosti modela - **test set**
- Trening set i test set ne smeju imati/deliti zajednicke elemente tj. opservacije
- Samo testiranjem modela na podacima koji nisu korisceni za ucenje mozemo izvesti adekvatnu estimaciju ocena valjanosti modela - generalizacija.
- Opste prihvacena praksa je da se rasploziv skup podataka podeli na sledeci nacin:
  - Trening set 70% ili 75%
  - Test set 30% ili 35%
- Prilikom podele raspolozivog skupa podataka treba strogo voditi racuna da zastupljenost, odn. distribucija, klasa (ovo se odnosi na algoritme za klasifikaciju) bude slicna u trening i test setu
  - ne bi smelo da se dogodi da jedan ili drugi set uopste ne sadrze ni jednu opservacuju koja pripada odredjenoj klasi
- Dobra praksa je da se poredak opservacija randomizuje (slucajno odabrana permutacija) pre deljenja skupa podataka na trening i test set
  - Ovo vazi i za klasifikaciju i za regresiju
- Odabiranje (semplovanje) opservacija za trening i test set moze ponekad i znacajno uticati na procenjene vrednosti ocena valjanosti datog modela
  - Da bi se ovaj efekat minimizovao koristi se **unakrsna validacija** (cross-validation)

Primer

```
# Koristicemo "titanic" set podataka formiran u jednom od prethodnih primera
str(titanic)
```

```
## Classes 'tbl_df', 'tbl' and 'data.frame':    891 obs. of  4 variables:
##  $ survived: Factor w/ 2 levels "0","1": 1 2 2 2 1 1 1 1 2 2 ...
##  $ pclass  : Factor w/ 3 levels "1","2","3": 3 1 3 1 3 3 1 3 3 2 ...
##  $ sex     : Factor w/ 2 levels "female","male": 2 1 1 1 2 2 2 2 1 1 ...
##  $ age     : num  22 38 26 35 35 NA 54 2 27 14 ...
```

```
table(titanic$survived)
```

```
##
##   0   1
## 549 342
```

```
# Odnos prezivelih i poginulih
prop.table(table(titanic$survived))
```

```
##
##         0         1
## 0.6161616 0.3838384
```

```
# Da bismo omogucili reproduktibilnost
set.seed(33)

# Prvo napravimo jednu slucajno odabranu permutaciju celog skupa podataka (dataset shuffle)
n <- nrow(titanic)
shuffled <- titanic[sample(n),] #f-a 'sample' vrsi slucajno odabiranje elemenata zadatog vektora

# Delimo skup podataka na trening i test set (70% i 30%)
```

```r
train_indicies <- 1:round(0.7 * n)
train <- shuffled[train_indicies, ]
test <- shuffled[-train_indicies, ]

# Generisemo klasifikacioni model (drvo odlucivanja - decision tree) na osnovu trening seta:
tree <- rpart(survived ~ ., data = train, method = "class")

# Koristeci dobijeni model "tree" vrsimo klasifikaciju podataka iz test seta:
pred <- predict(tree, newdata = test, type = "class")

# Racunamo matricu konfuzije
conf_t <- table(test$survived, pred)

# Prikaz matrice konfuzije
conf_t
```

```
##    pred
##       0   1
##   0 128  28
##   1  36  75
```

```r
# Formiramo TP, FN, FP i TN na osnovu "conf_t"

TP <- conf_t[2,2]

FP <- conf_t[1,2]

FN <- conf_t[2,1]

TN <- conf_t[1,1]


# Tacnost (Accuracy)
acc <- (TP + TN)/sum(conf_t)
acc
```

```
## [1] 0.7602996
```

```r
# Preciznost (Precision)
prec <- TP/(TP + FP)
prec
```

```
## [1] 0.7281553
```

```r
# Senzitivnost (Sensitivity, Recall)
sens <- TP/(TP + FN)
sens
```

```
## [1] 0.6756757
```

```r
# Specificnost (Specificity)
spec <- TN/(TN + FP)
spec
```

```
## [1] 0.8205128
```

Zadatak za vezbanje na casu:

Ponovite pokazanu proceduru koristeci "pima" skup podataka.

Upotreba unakrsne validacije (cross-validation)

Radi demonstracije cemo rucno formirati algroritam koji koristi unakrsnu validaciju za procenu tacnosti modela:

```
# Da bismo obezbedili reproduktibilnost
set.seed(33)

# Koristicemo prethodno formirani "shuffled" skup podataka

# Inicijalizujemo vektor accs - popunjavamo nulama
accs <- rep(0,9)


# Treniramo model koristeci kros-validacione intervale vrednosti i vrsimo estimaciju tacnosti modela ka
for (i in 1:9) {
  # Ovi indeksi ukazuju na trenutni interval test seta koji koristimo za treniranje modela
    indices <- (((i - 1) * round((1/9)*nrow(shuffled))) + 1):((i*round((1/9) * nrow(shuffled))))

   # Iskljucujemo ove intervale iz trening seta
   train <- shuffled[-indices,]

  # Ukljucimo ih u test set
  test <- shuffled[indices,]

  # Treniramo model sa svakim od dobijenih trening setova po iteracijama
  tree <- rpart(survived ~ ., train, method = "class")

  # Predvidjamo klase za tekuci test set u svakoj od iteracija
  pred <- predict(tree, test, type = "class")

  # Formiramo odgovarajucu konfuzionu matricu
  conf <- table(test$survived, pred)

  # Dodeljujemo vrednost za tacnost tekuceg modela i-tom indeksu u vektoru accs
  accs[i] <- sum(diag(conf))/sum(conf)
}

# Srednja vrednost za accs
mean(accs)
```

```
## [1] 0.7833895
```

Pitanje: Recimo da primenjujemo unakrsnu validaciju na skupu podataka koji sadrzi 22680 opservacija. Zelite da vas trening set sadrzi 21420 unosa (opservacija). Koliko iteracija moze da sadrzi kros-validacioni algoritam?

Bajas i varijansa (Bias and Variance)

Primer

Koristicemo *Spambase Data Set* koji mozete naci na https://archive.ics.uci.edu/ml/datasets/Spambase

```
if (!"emails_full" %in% ls()) {
    emails_full <- read.csv("data/spambase.data", header = FALSE)
}

# Proveravamo strukturu seta podataka
str(emails_full)
```

```
## 'data.frame':    4601 obs. of  58 variables:
##  $ V1 : num  0 0.21 0.06 0 0 0 0 0 0.15 0.06 ...
##  $ V2 : num  0.64 0.28 0 0 0 0 0 0 0 0.12 ...
##  $ V3 : num  0.64 0.5 0.71 0 0 0 0 0 0.46 0.77 ...
##  $ V4 : num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V5 : num  0.32 0.14 1.23 0.63 0.63 1.85 1.92 1.88 0.61 0.19 ...
##  $ V6 : num  0 0.28 0.19 0 0 0 0 0 0 0.32 ...
##  $ V7 : num  0 0.21 0.19 0.31 0.31 0 0 0 0.3 0.38 ...
##  $ V8 : num  0 0.07 0.12 0.63 0.63 1.85 0 1.88 0 0 ...
##  $ V9 : num  0 0 0.64 0.31 0.31 0 0 0 0.92 0.06 ...
##  $ V10: num  0 0.94 0.25 0.63 0.63 0 0.64 0 0.76 0 ...
##  $ V11: num  0 0.21 0.38 0.31 0.31 0 0.96 0 0.76 0 ...
##  $ V12: num  0.64 0.79 0.45 0.31 0.31 0 1.28 0 0.92 0.64 ...
##  $ V13: num  0 0.65 0.12 0.31 0.31 0 0 0 0 0.25 ...
##  $ V14: num  0 0.21 0 0 0 0 0 0 0 0 ...
##  $ V15: num  0 0.14 1.75 0 0 0 0 0 0 0.12 ...
##  $ V16: num  0.32 0.14 0.06 0.31 0.31 0 0.96 0 0 0 ...
##  $ V17: num  0 0.07 0.06 0 0 0 0 0 0 0 ...
##  $ V18: num  1.29 0.28 1.03 0 0 0 0.32 0 0.15 0.12 ...
##  $ V19: num  1.93 3.47 1.36 3.18 3.18 0 3.85 0 1.23 1.67 ...
##  $ V20: num  0 0 0.32 0 0 0 0 0 3.53 0.06 ...
##  $ V21: num  0.96 1.59 0.51 0.31 0.31 0 0.64 0 2 0.71 ...
##  $ V22: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V23: num  0 0.43 1.16 0 0 0 0 0 0 0.19 ...
##  $ V24: num  0 0.43 0.06 0 0 0 0 0 0.15 0 ...
##  $ V25: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V26: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V27: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V28: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V29: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V30: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V31: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V32: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V33: num  0 0 0 0 0 0 0 0 0.15 0 ...
##  $ V34: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V35: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V36: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V37: num  0 0.07 0 0 0 0 0 0 0 0 ...
##  $ V38: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V39: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V40: num  0 0 0.06 0 0 0 0 0 0 0 ...
##  $ V41: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V42: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V43: num  0 0 0.12 0 0 0 0 0 0.3 0 ...
##  $ V44: num  0 0 0 0 0 0 0 0 0 0.06 ...
##  $ V45: num  0 0 0.06 0 0 0 0 0 0 0 ...
```

```
##  $ V46: num  0 0 0.06 0 0 0 0 0 0 0 ...
##  $ V47: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V48: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V49: num  0 0 0.01 0 0 0 0 0 0 0.04 ...
##  $ V50: num  0 0.132 0.143 0.137 0.135 0.223 0.054 0.206 0.271 0.03 ...
##  $ V51: num  0 0 0 0 0 0 0 0 0 0 ...
##  $ V52: num  0.778 0.372 0.276 0.137 0.135 0 0.164 0 0.181 0.244 ...
##  $ V53: num  0 0.18 0.184 0 0 0 0.054 0 0.203 0.081 ...
##  $ V54: num  0 0.048 0.01 0 0 0 0 0 0.022 0 ...
##  $ V55: num  3.76 5.11 9.82 3.54 3.54 ...
##  $ V56: int  61 101 485 40 40 15 4 11 445 43 ...
##  $ V57: int  278 1028 2259 191 191 54 112 49 1257 749 ...
##  $ V58: int  1 1 1 1 1 1 1 1 1 1 ...
```

```r
# Na osnovu dokumentacije...

emails_full <- emails_full[, c(55, 58)]

str(emails_full)
```

```
## 'data.frame':    4601 obs. of  2 variables:
##  $ V55: num  3.76 5.11 9.82 3.54 3.54 ...
##  $ V58: int  1 1 1 1 1 1 1 1 1 1 ...
```

```r
colnames(emails_full) <-  c("avg_capital_seq", "spam")

str(emails_full)
```

```
## 'data.frame':    4601 obs. of  2 variables:
##  $ avg_capital_seq: num  3.76 5.11 9.82 3.54 3.54 ...
##  $ spam           : int  1 1 1 1 1 1 1 1 1 1 ...
```

```r
emails_full$spam <- as.factor(emails_full$spam)

str(emails_full)
```

```
## 'data.frame':    4601 obs. of  2 variables:
##  $ avg_capital_seq: num  3.76 5.11 9.82 3.54 3.54 ...
##  $ spam           : Factor w/ 2 levels "0","1": 2 2 2 2 2 2 2 2 2 2 ...
```

```r
# Definisemo funkciju spam_classifier()
# 1 - spam, 0 - ham
spam_classifier <- function(x){
  prediction <- rep(NA,length(x))
  prediction[x > 4] <- 1
  prediction[x >= 3 & x <= 4] <- 0
  prediction[x >= 2.2 & x < 3] <- 1
  prediction[x >= 1.4 & x < 2.2] <- 0
  prediction[x > 1.25 & x < 1.4] <- 1
  prediction[x <= 1.25] <- 0
  return(factor(prediction, levels = c("0","1")))
}

# Primenimo spam_classifier na emails_full: pred_full
pred_full <- spam_classifier(emails_full$avg_capital_seq)

# Konfuziona matrica za emails_full: conf_full
```

```r
conf_full <- table(emails_full$spam, pred_full)

# Racunamo tacnost na osnovu conf_full: acc_full
acc_full <- sum(diag(conf_full))/sum(conf_full)
acc_full
```

```
## [1] 0.6561617
```

```r
# Uproscen model za klasifikaciju
spam_classifier <- function(x){
  prediction <- rep(NA,length(x))
  prediction[x > 4] <- 1
  prediction[x <= 4] <- 0
  return(factor(prediction, levels = c("0","1")))
}

# Tacnost predikcije sa uproscenim modelom za emails data set
conf_small <- table(emails$spam, spam_classifier(emails$avg_capital_seq))
acc_small <- sum(diag(conf_small)) / sum(conf_small)
acc_small
```

```
## [1] 0.7692308
```

```r
# Primenimo uprosceni model i na "emails_full" i sracunamo konfuzionu matricu
conf_full <- table(emails_full$spam, spam_classifier(emails_full$avg_capital_seq))

# Izracunamo tacnost
acc_full <- sum(diag(conf_full)) / sum(conf_full)
acc_full
```

```
## [1] 0.7259291
```

Regresija

Prosta linearna regresija

Koriscenje funkcije `lm()` - Primeri

Primer 1:

U ovom primeru cemo koristiti set podataka “Boston” iz paketa `MASS`. Ovaj set podataka sadrzi podatke o trzisnoj vrednosti nekretnina u predgradjima Bostona, SAD, zajedno sa razlicitim parametrima koji uticu na formiranje ove vrednosti.

```r
library(MASS)
```

```
##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select
```

```r
library(ISLR)
library(ggplot2)

?Boston
```

```
## starting httpd help server ...
## done
```

```r
str(Boston)
```

```
## 'data.frame':    506 obs. of  14 variables:
##  $ crim   : num  0.00632 0.02731 0.02729 0.03237 0.06905 ...
##  $ zn     : num  18 0 0 0 0 0 12.5 12.5 12.5 12.5 ...
##  $ indus  : num  2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 7.87 ...
##  $ chas   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ nox    : num  0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 0.524 ...
##  $ rm     : num  6.58 6.42 7.18 7 7.15 ...
##  $ age    : num  65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
##  $ dis    : num  4.09 4.97 4.97 6.06 6.06 ...
##  $ rad    : int  1 2 2 3 3 3 5 5 5 5 ...
##  $ tax    : num  296 242 242 222 222 222 311 311 311 311 ...
##  $ ptratio: num  15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
##  $ black  : num  397 397 393 395 397 ...
##  $ lstat  : num  4.98 9.14 4.03 2.94 5.33 ...
##  $ medv   : num  24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...
```

```r
head(Boston)
```

```
##      crim zn indus chas   nox    rm  age    dis rad tax ptratio  black
## 1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3 396.90
## 2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8 396.90
## 3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8 392.83
## 4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7 394.63
## 5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7 396.90
## 6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7 394.12
##   lstat medv
## 1  4.98 24.0
## 2  9.14 21.6
## 3  4.03 34.7
## 4  2.94 33.4
## 5  5.33 36.2
## 6  5.21 28.7
```

```r
# Proverimo kako izgleda promena "medv" (median value of owner-occupied homes in \$1000s) sa
# "lstat" (lower status of the population (percent)
plot(medv~lstat,Boston)

# Kao sto vidimo postoji jasan trend opadanja vrednosti nekretnina sa porastom procenta
# siromasnijih stanovnika (ovakva korelacija je naravno i ocekivana). Ovakvi slucajevi su dobri
# kandidati za modelovanje prostom linerarnom regresijom.

fit_1 = lm(medv ~ lstat, data = Boston)

# Hajde da vidimo kako izgleda nas model
fit_1
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Coefficients:
```

```
## (Intercept)        lstat
##       34.55        -0.95
```

```r
# Detaljniji uvid
summary(fit_1)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

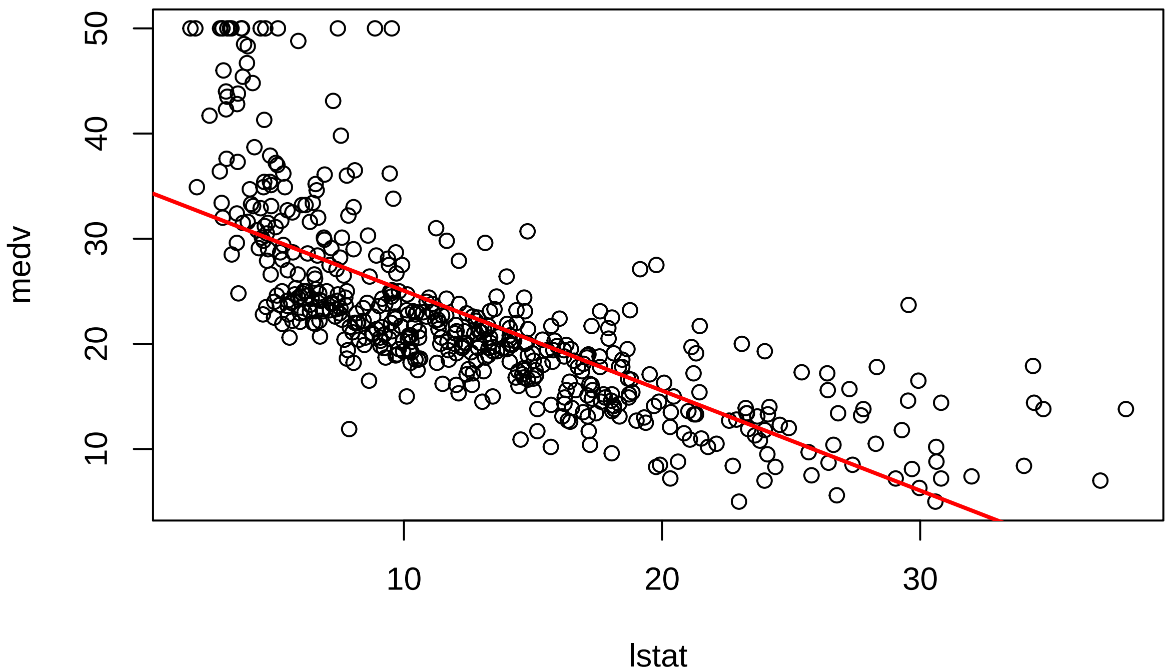
```r
# Sta sve model sadrzi
names(fit_1)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

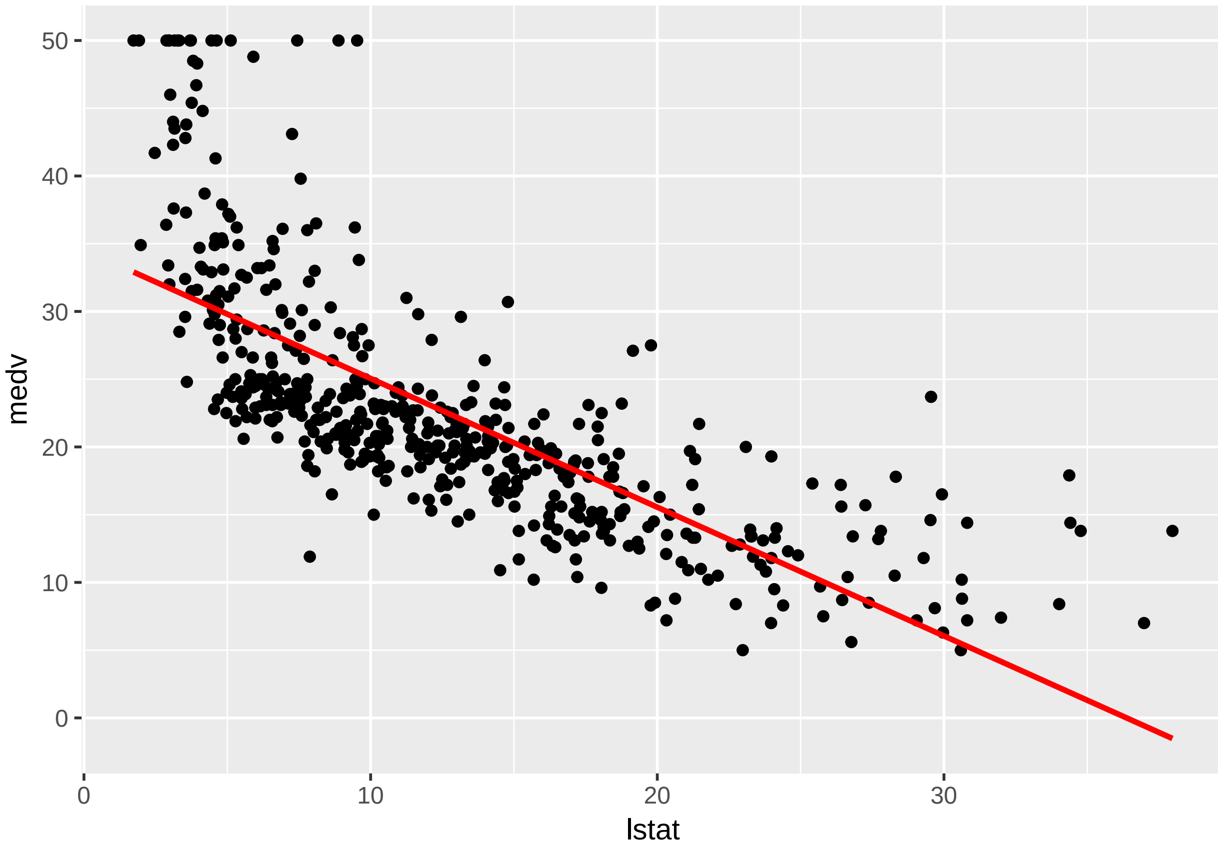
```r
# Samo koeficijenti
fit_1$coefficients
```

```
## (Intercept)       lstat
##  34.5538409  -0.9500494
```

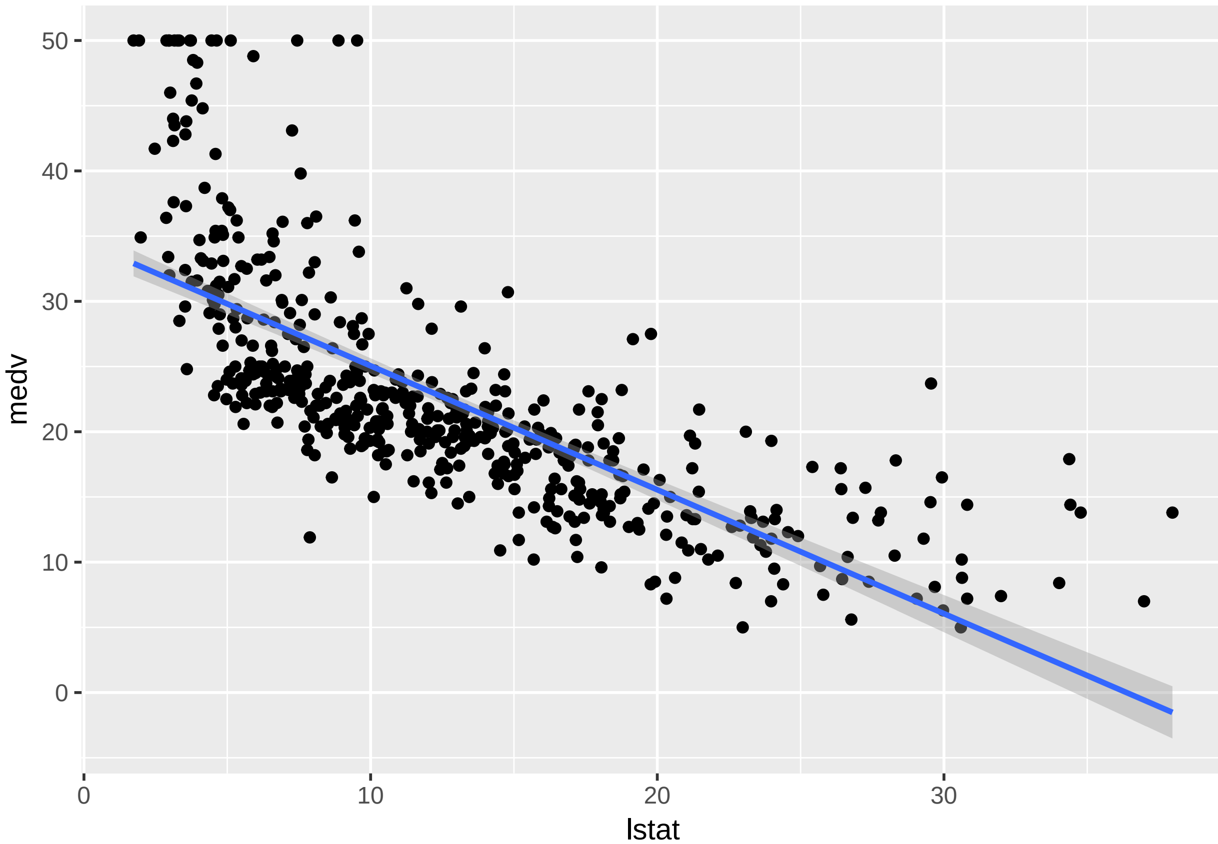
```r
# Ucrtajmo regresionu pravu na pocetni scatter plot
abline(fit_1$coefficients, col = "red", lwd = 2)
```




```r
# Ili sve zajedno koristerci "ggplot2"
ggplot(Boston, aes( x = lstat, y = medv)) +
    geom_point() +
    geom_smooth(method = "lm", se = FALSE, colour = "red")
```




```
# Ako zelimo i interval pouzdanosti sam izostavimo parametar "se" (podrazumevano se = TRUE)
ggplot(Boston, aes( x = lstat, y = medv)) +
    geom_point() +
    geom_smooth(method = "lm")
```




```
# Interval pouzdanosti
confint(fit_1)
```

```
##                  2.5 %     97.5 %
## (Intercept) 33.448457 35.6592247
## lstat       -1.026148 -0.8739505
```

```
# Da predvidimo vrednosti "medv" za dati vektor vrednosti "lstat" promenljive, uz
# proracun intervala pouzdanosti
predict(fit_1,data.frame(lstat = c(5,10,15)),interval = "confidence")
```

```
##        fit      lwr      upr
## 1 29.80359 29.00741 30.59978
## 2 25.05335 24.47413 25.63256
## 3 20.30310 19.73159 20.87461
```

Multivarijabilna linearna regresija

Primer 1

```
library(readr)
library(tidyr)
library(purrr)
library(ggpubr)
```

```
## Warning: package 'ggpubr' was built under R version 3.3.2
```

```r
# Uvoz i sredjivanje podataka
shop_data <- read_csv("data/shop_data.csv")
```

```
## Parsed with column specification:
## cols(
##   `"sales","sq_ft","inv","ads","size_dist","comp"` = col_character()
## )
```

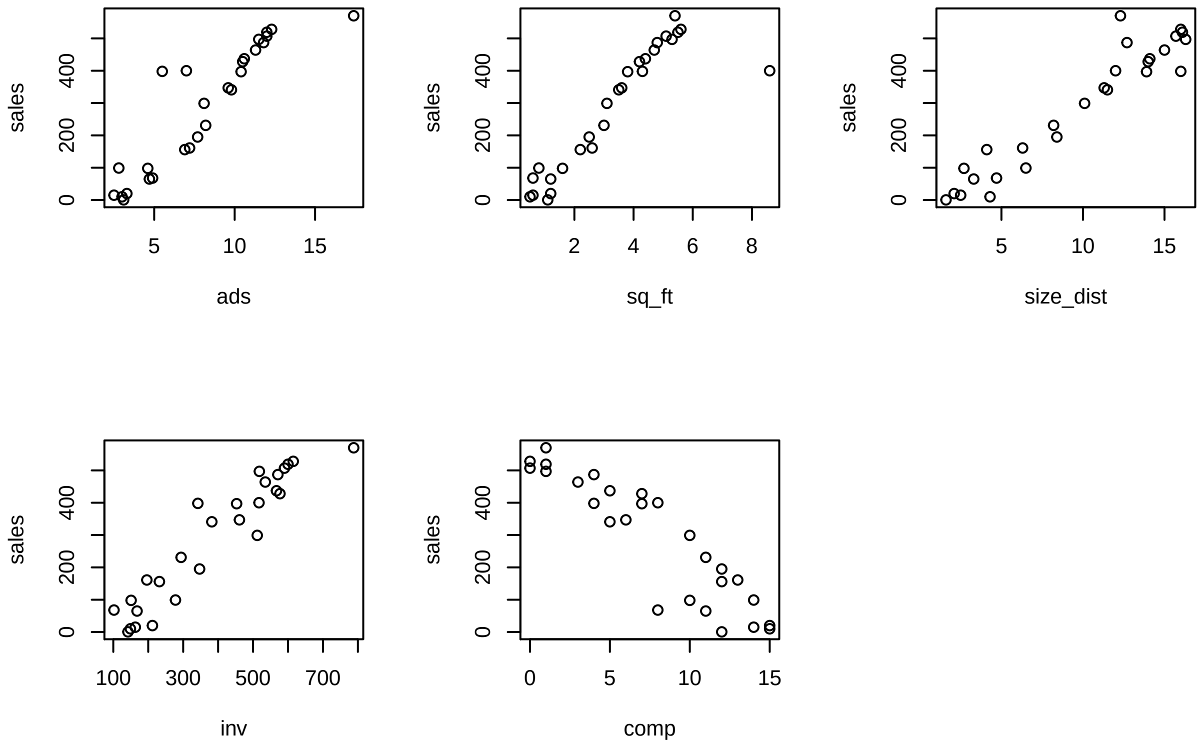
```r
shop_data <- separate(shop_data, '"sales","sq_ft","inv","ads","size_dist","comp"',
                      c("sales","sq_ft","inv","ads","size_dist","comp"), sep = ",")
shop_data <- as.data.frame(map(shop_data, as.numeric))

str(shop_data)
```

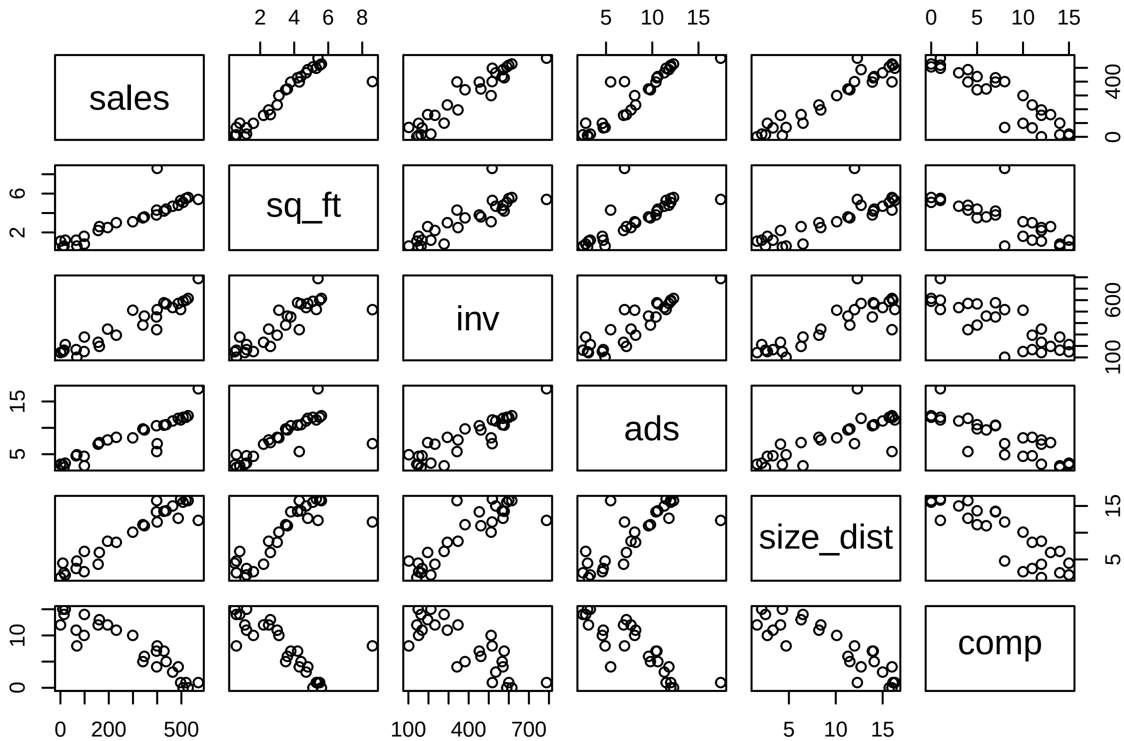
```
## 'data.frame':    27 obs. of  6 variables:
##  $ sales    : num  231 156 10 519 437 487 299 195 20 68 ...
##  $ sq_ft    : num  3 2.2 0.5 5.5 4.4 ...
##  $ inv      : num  294 232 149 600 567 571 512 347 212 102 ...
##  $ ads      : num  8.2 6.9 3 12 10.6 ...
##  $ size_dist: num  8.2 4.1 4.3 16.1 14.1 ...
##  $ comp     : num  11 12 15 1 5 4 10 12 15 8 ...
```

```r
head(shop_data)
```

```
##   sales sq_ft inv  ads size_dist comp
## 1   231   3.0 294  8.2       8.2   11
## 2   156   2.2 232  6.9       4.1   12
## 3    10   0.5 149  3.0       4.3   15
## 4   519   5.5 600 12.0      16.1    1
## 5   437   4.4 567 10.6      14.1    5
## 6   487   4.8 571 11.8      12.7    4
```

```r
# Hajde da proverimo kako se podaci ponasaju i mogu li se uociti relacije
# izmedju distribucija promenljivih koje bi ukazivale na opravdanost uvodjenja
# linearnog modela:

par(mfrow = c(2,3))
plot(sales ~ ads, shop_data)
plot(sales ~ sq_ft, shop_data)
plot(sales ~ size_dist, shop_data)
plot(sales ~ inv, shop_data)
plot(sales ~ comp, shop_data)

#Ili:
pairs(shop_data)
```


sales
0
200
400
5
10
15
ads
sales
0
200
400
2
4
6
8
sq_ft
sales
0
200
400
5
10
15
size_dist
sales
0
200
400
100
300
500
700
inv
sales
0
200
400
0
5
10
15
comp

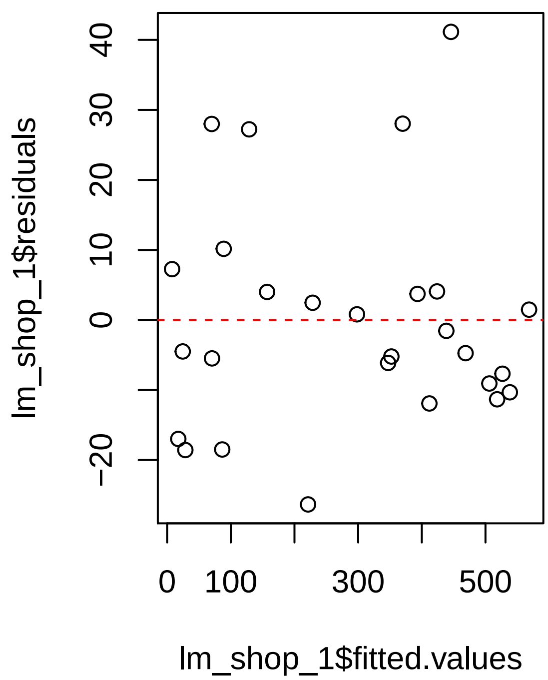


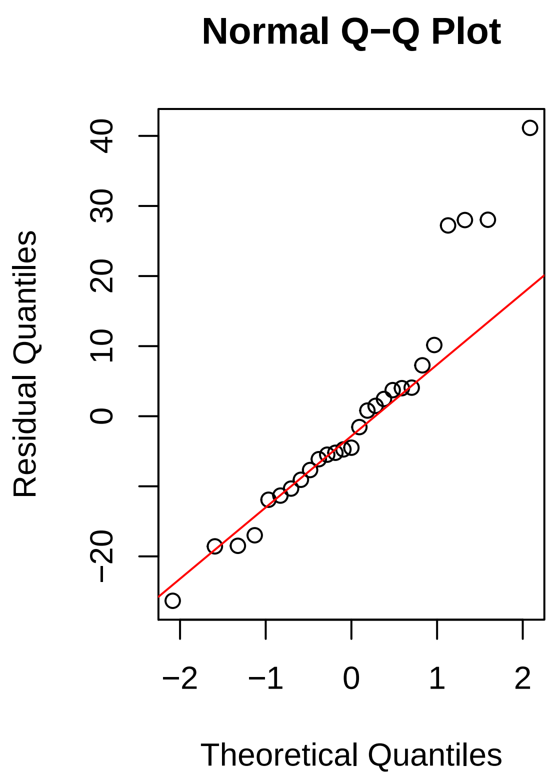

```r
# Linearni model za "sales" koji ukjucuje sve prediktore (sve preostale promenljive)
lm_shop_1 <- lm( sales ~., data = shop_data)

# Proverimo parametre valjanosti modela i koliko su pojedini prediktori znacajni u modelu:
summary(lm_shop_1)
```

```
##
## Call:
## lm(formula = sales ~ ., data = shop_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -26.338  -9.699  -4.496   4.040  41.139
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -18.85941   30.15023  -0.626 0.538372
## sq_ft        16.20157    3.54444   4.571 0.000166 ***
## inv           0.17464    0.05761   3.032 0.006347 **
## ads          11.52627    2.53210   4.552 0.000174 ***
## size_dist    13.58031    1.77046   7.671 1.61e-07 ***
## comp         -5.31097    1.70543  -3.114 0.005249 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.65 on 21 degrees of freedom
```

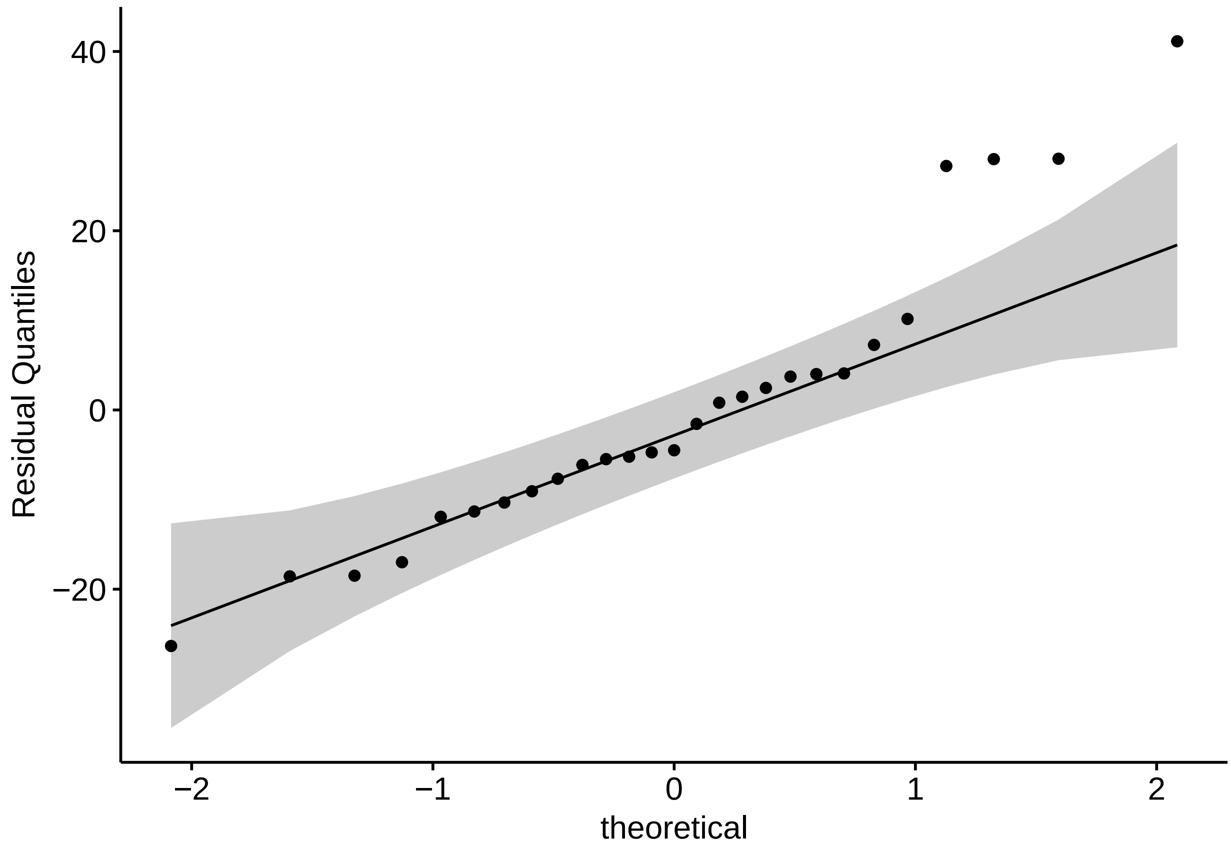
```
## Multiple R-squared:  0.9932, Adjusted R-squared:  0.9916
## F-statistic: 611.6 on 5 and 21 DF,  p-value: < 2.2e-16
```

```r
# Da bismo uopste mogli da koristimo p-vrednosti u ovom kontekstu treba prvo da proverimo
# da li je zadovoljena pretpostavka o normalnoj distribuciji reziduala!

par(mfrow = c(1,2))
# Plotujemo reziduale u funkciji fitovanih vrednosti za pojedinacje opservacije
plot(lm_shop_1$fitted.values, lm_shop_1$residuals)
abline(0,0, col = "red", lty = 2)

# Napravimo  Q-Q plot kvantila reziduala
qqnorm(lm_shop_1$residuals, ylab = "Residual Quantiles")
qqline(lm_shop_1$residuals, col = "red")
```






```r
par(mfrow = c(1,1))

# Mozemo i da upotrebimo f-ju "ggqqplot" iz paketa "ggpubr" koji sadrzi funkcije za
# plotovanje "lepih" grafika:

ggqqplot(lm_shop_1$residuals, ylab = "Residual Quantiles")
```




```r
# Me moze se uociti nikakav jasan "pattern" u distribucij reziduala, sta vise kvantili
# reziduala su uglavnom na liniji koja odgovara teorijskoj - normalnoj distribuciji

# Proverimo ponovo summary
summary(lm_shop_1)
```

```
##
## Call:
## lm(formula = sales ~ ., data = shop_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -26.338  -9.699  -4.496   4.040  41.139
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -18.85941   30.15023  -0.626 0.538372
## sq_ft        16.20157    3.54444   4.571 0.000166 ***
## inv           0.17464    0.05761   3.032 0.006347 **
## ads          11.52627    2.53210   4.552 0.000174 ***
## size_dist    13.58031    1.77046   7.671 1.61e-07 ***
## comp         -5.31097    1.70543  -3.114 0.005249 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17.65 on 21 degrees of freedom
```

```
## Multiple R-squared:  0.9932, Adjusted R-squared:  0.9916
## F-statistic: 611.6 on 5 and 21 DF,  p-value: < 2.2e-16
```

```r
# Iskoristimo sada dobijeni model da predvidimo neto prodajnu vrednost na osnovu novog
# skupa prediktora:
shop_new = data.frame("sq_ft" = 2.3, "inv" = 420, "ads" = 8.7,
                      "size_dist" = 9.1, "comp" = 10)
predict(lm_shop_1, shop_new)
```

```
##        1
## 262.5006
```

Primer 2

Za ovaj primer cemo ponovo koristiti set podataka "Boston" iz paketa `MASS`.

```r
# Linearni model za "medv" na osnovu dva prediktora: "lstat" i "age"
fit2 = lm(medv ~ lstat + age, data = Boston)
summary(fit2)
```

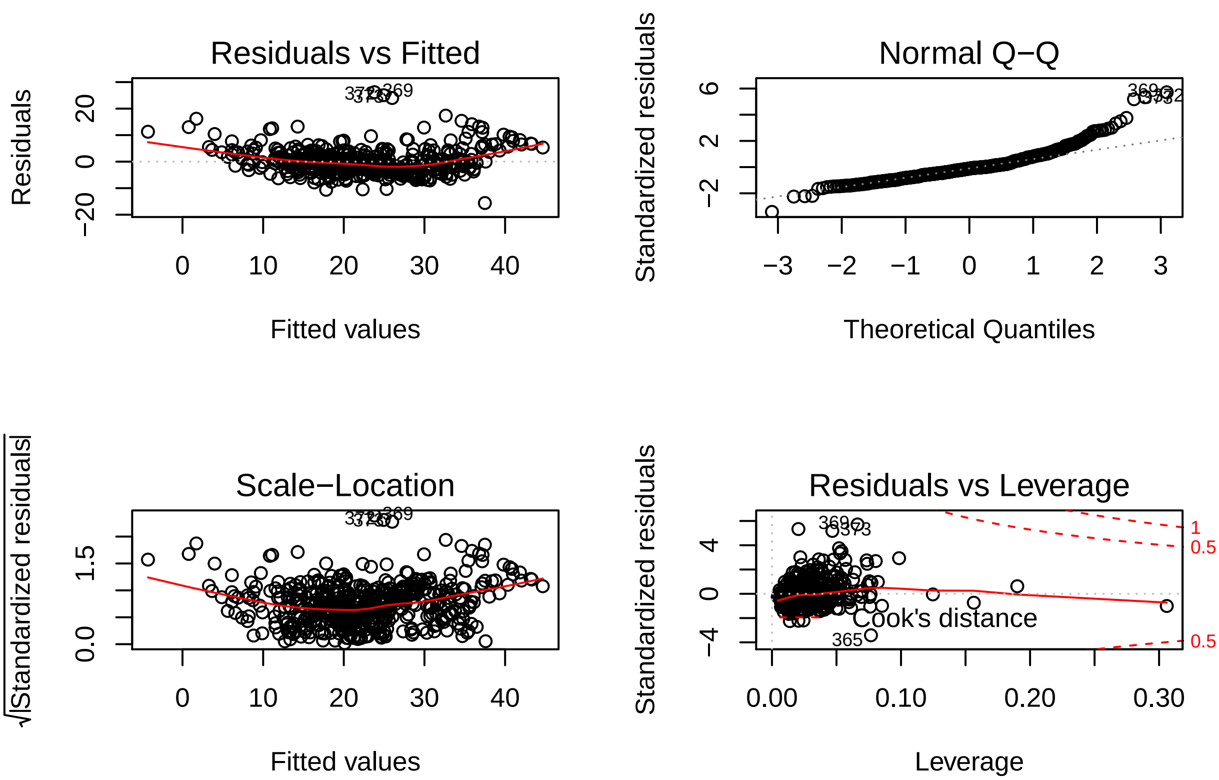
```
##
## Call:
## lm(formula = medv ~ lstat + age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
## lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
## age          0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

```r
#Linearni model za "medv" na osnovu svih raspolozivih prediktora
fit3 = lm(medv ~.,Boston)
summary(fit3)
```

```
##
## Call:
## lm(formula = medv ~ ., data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.595  -2.730  -0.518   1.777  26.199
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
## crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
```

```
## zn           4.642e-02  1.373e-02   3.382 0.000778 ***
## indus        2.056e-02  6.150e-02   0.334 0.738288
## chas         2.687e+00  8.616e-01   3.118 0.001925 **
## nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
## rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
## age          6.922e-04  1.321e-02   0.052 0.958229
## dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
## rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
## tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
## ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
## black        9.312e-03  2.686e-03   3.467 0.000573 ***
## lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.745 on 492 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7338
## F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

```r
# Jos jedan nacin da se iscraju grafici koji se koriste za procenu valjanosti i opravdanosti linearnog 
par(mfrow = c(2,2))
plot(fit3)
```




```r
# Na osnovu "summary" za model fit3 videli smo da promenljive "indus" i "age" ne igraju bitnu ulogu, te
fit4 = update(fit3,~.-age-indus)
summary(fit4)
```

```
##
## Call:
## lm(formula = medv ~ crim + zn + chas + nox + rm + dis + rad +
##     tax + ptratio + black + lstat, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.5984  -2.7386  -0.5046   1.7273  26.2373
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.341145   5.067492   7.171 2.73e-12 ***
## crim         -0.108413   0.032779  -3.307 0.001010 **
## zn            0.045845   0.013523   3.390 0.000754 ***
## chas          2.718716   0.854240   3.183 0.001551 **
## nox         -17.376023   3.535243  -4.915 1.21e-06 ***
## rm            3.801579   0.406316   9.356  < 2e-16 ***
## dis          -1.492711   0.185731  -8.037 6.84e-15 ***
## rad           0.299608   0.063402   4.726 3.00e-06 ***
## tax          -0.011778   0.003372  -3.493 0.000521 ***
## ptratio      -0.946525   0.129066  -7.334 9.24e-13 ***
## black         0.009291   0.002674   3.475 0.000557 ***
## lstat        -0.522553   0.047424 -11.019  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.736 on 494 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7348
## F-statistic: 128.2 on 11 and 494 DF,  p-value: < 2.2e-16
```