1. **This problem requires a tiny bit of programming :)** Consider a system that has two independent controls, A and B that can prevent the system from being destroyed. The system is activated at discrete time points $t_1, t_2, \ldots$, and the system is considered to be controlled if either control A or control B holds at the time of activation. The system is destroyed if both A and B fail simultaneously. If one control fails but the other control holds, the the defective control is replaced before the next activation. If a control holds at time $t = t_k$, then it is considered 90% reliable at $t = t_{k+1}$. If a control fails at time $t = t_k$, then its untested replacement is considered to be only 60% reliable at $t = t_{k+1}$.

   Can the system be expected to run indefinitely without ever being destroyed? If not, how long is the system expected to run before destruction occurs? Compare this result with a system that has a single control and a system that has three independent controls.

1 Solution problem 1

As stated in the problem if a control holds at time $t = t_k$, then it is considered 90% reliable at $t = t_{k+1}$. If a control fails at time $t = t_k$, then its untested replacement is considered to be only 60% reliable at $t = t_{k+1}$.

Let us call this conditions conditions A and B:

(a) $\rightarrow$ if a control holds at time $t = t_k$, then it is considered 90% reliable at $t = t_{k+1}$.

(b) $\rightarrow$ If a control fails at time $t = t_k$, then its untested replacement is considered to be only 60% reliable at $t = t_{k+1}$.

Taking this into account if one control fails at time $t = t_k$ the reliability drops to 60% in $t = t_{k+1}$ based on condition b, but in $t = t_{k+2}$ it raises to 90% based on condition a. Since in all cases the reliability of the components is above 60% in an ideal system the system would stay stable indefinitely. Since the conditions a and b state that the components will be in worst case 60% stable and if we are looking from the perspective of an ideal system, there should not even be a case when one component stops to work.

2 Questions related to interpretation

Based on my interpretation of the goal of the assignment, should the statement

"if a control holds at time** $t = t_k$**, then it is considered** 90% **reliable at** $t = t_{k+1}$**."

be interpreted as

"if a control holds at time** $t = t_k$**, then it is considered** 10% **less reliable at** $t = t_{k+1}$ **than at** $t = t_k$**."

and the statement

"If a control fails at time** $t = t_k$**, then its untested replacement is considered to be only** 60% **reliable at** $t = t_{k+1}$**."

as

" If a control fails at time** $t = t_k$**, then its untested replacement is considered to be** 40% **less reliable at** $t = t_{k+1}$ **then at** $t = t_k$**. "

2. Demonstrate that for node similarities σ_{ij} defined according to $\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1}$ the sum $\sum_j \sigma_{ij}$ gives the PageRank of node i divided by the degree k_i.

3 Question

Based on the presentation Measuring Network Properties Slide 97/118[1] the end formula for regular node similarity is denoted as:

$\sigma = (\mathbf{D} - \alpha\mathbf{A})^{-1}\mathbf{D}$

Is the difference intentional?

[1] https://courses.isds.tugraz.at/dhelic/netsci/measures.pdf