IE 5331-003 Homework 1

Spring 2025
Due Date: Sunday, Feb-09-2025, 11:59 PM

Problem 1

Implement the maxflow problem in class and compare with its dual formulation. You need to finish the following steps.

1. Provide the primal formulation and derive the dual (show all steps).
2. Implement a network generator with two inputs, number of vertices n and number of edges m, and one output, a connected network with (non-negative) flow capacities assigned on each edge. [You can use the python package: networkx]
3. Generate three network instances with at most 5 nodes and 10 arcs and plot them (in a readable manner) with flow capacities shown on each edge.
4. Implement the primal formulation in Python using Gurobi.
5. Implement the dual formulation in Python using Gurobi.
6. Report your results.

Problem 2 (Shortest Path)

Given a connected network $G = (V, E)$ with a source node s, a terminal node t, and a length $d_e \geq 0$ assigned on each edge $e \in E$. Finish the following tasks.

1. Develop a linear programming formulation to find the shortest path from s to t.
2. Derive the dual formulation (show all steps).
3. Try to interpret the dual formulation.

Problem 3 (Cargo Transportation)

The Houston Steamship Company operates ships between Ports X and Y and Ports A, B, and C. Table 1 lists the cargoes to be delivered for the next 15 days. An entire ship is required to carry each cargo. After a ship

Cargo ID	Departure Date	Port of Origin	Port of Destination
1	3	X	A
2	4	Y	A
3	6	X	C
4	6	Y	A
5	9	X	B
6	9	Y	A
7	10	X	A
8	10	X	C
9	10	Y	B
10	13	Y	B
11	15	Y	B
12	15	Y	C

Table 1: Cargo listing for the next 15 days.

reaches a destination, it may return to either port of origin (X or Y). The total travel time (in days) between between these ports (in either direction) is shown in Table 2. Finish the following questions.

Time	A	B	C
X	2	3	2
Y	1	2	1

Table 2: Sailing time (either direction) in days.

1. Propose a formulation for the above problem to minimize the number of ships required to deliver all cargoes.
2. Refine the formulation to minimize the total travel time (in days).
3. Implement two formulations in Python.
4. Report your results.