IE 5331-003 Homework 2

Spring 2025

Due Date: Sunday, Feb-23-2025, 11:59 PM

Problem 1

Consider the following quadratic optimization (with positive definite matrix) problem, finish the following questions.

$$\min_x \frac{1}{2}\langle x, Qx\rangle + \langle c, x\rangle$$
$$\text{s.t. } Ax = b$$

1. Provide the dual problem in the maximin form.
2. Reformulate the dual problem by solving an optimal x exactly. Simplify the resulting optimization problem (merge terms if possible).
3. Determine the optimal u of the dual problem by direct calculation (analytical solution).
4. Randomly generate five problem instances with $x \in \mathbb{R}^{10}$ (so each Q is a 10×10 positive definite matrix and each $c \in \mathbb{R}^{10}$). Solve the associated dual problem in Python using following methods:
   (a) Use the analytical solution.
   (b) Use the gradient decent (first-order method). [hint: numpy or scipy package]
   (c) Use the Newton's method (second-order method). [hint: numpy or scipy package]
   (d) Solve the linear system obtained by the KKT conditions.
5. Report your results in a table, including optimal values and running times.

Problem 2

A company has two distribution centers (DCs) that require purchasing a certain type of product from n potential suppliers. Ordering from each suppler $i \in [n]$ induces a fix cost of $\bar{c}_i$ dollars. Once an order is placed for the supplier i, parameter $\bar{a}_{ij}$ is the amount of products will be sent from supplier i to DC j. Each DC needs at least d_j amount of products. The details can be found in the following table. Finish the following questions.

Center ID	Supplier ID									Demands d
	1	2	3	4	5	6	7	8	9	
1	10	23	13	4	9	20	15	6	12	46
2	3	8	30	11	6	33	17	18	9	54
Cost $\bar{c}$	9	11	7	8	8	13	21	16	11	—

Table 1: Information Table. Center IDs are indexed by $[2] := \{1, 2\}$, Supplier IDs are indexed by $[9] := \{1, 2, \ldots, 9\}$. Number $\bar{a}_{ij}$ in each cell denotes the amount of products from supplier i to center j.

1. Set up a formulation assuming all parameters are deterministic as shown in the table.
2. Suppose each supply vector $a_j = (a_{ij})_{i\in[9]}$ is uncertain and can take any possible realization of $a_j = \bar{a}_j + \xi$ for some $\xi \in \Xi := \{\xi \mid \langle \xi, \xi\rangle \leq 1\}$, where ξ can be considered as some perturbation of the parameters.

(a) Set up the associated semi-infinite optimization to ensure solution robustness, i.e., the solution is feasible for the worst-case scenario.

(b) Solve this semi-infinite optimization using master-subproblem method. [hint: Gurobi callback + lazy cuts. Run with one-hour time limit if necessary.]

(c) Reformulate the problem using (convex) dualization.

(d) Solve the reformulated problem in Gurobi [one-hour time limit].

(e) Compare two methods in terms of optimal values, running times, and optimality gap.

3. Suppose the cost c is also uncertain with the representation $c := \bar{c} + \gamma$ for some $\gamma \in \Gamma := \{\gamma \mid B\gamma \geq b\}$.

   (a) Adjust the objective function of the previous semi-infinite optimization problem to ensure solution robustness. [hint: consider the worst-case in the objective function]

   (b) Reformulate the problem to have a one-level optimization problem.