IE 5331-003 Homework 3

Spring 2025

Due Date: Sunday, Mar-16-2025, 11:59 PM

Problem 1

The n-queens problem consist of placing n chess queens on an $n \times n$ chessboard so that no two queens have the chance to attack each other. Clearly, a solution requires that no two queens share the same row, column, or diagonal. It has been proven that there is a solution for $n \geq 4$. The most recognized case is when $n = 8$, simply because it can be solved directly on a regular chessboard (see Figure 1).

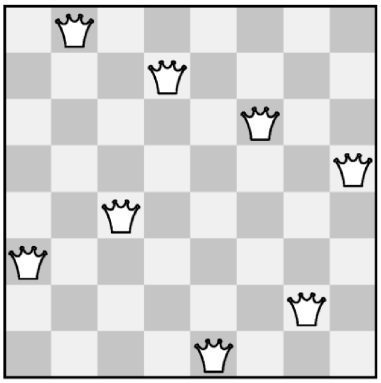

Figure 1: A Solution for $n = 8$

There are several ways to solve the n-queens problem. For this homework, you are required to solve it using a linear optimization model with binary variables.

1. Provide a generic optimization formulation (n is a parameter).
2. Code your model using Python and Gurobi.
3. Run for cases $n = 6, 7, 8, 9, 10$. Show the solutions and their computational time.

Problem 2

Given the following disjunctive programming

$$
\begin{aligned}
&\min\ 4x_1 + 2x_2 + 5x_3 \\
&\text{s.t. } 3x_2 - 2x_3 \geq 5 \\
&\quad \begin{bmatrix} 2x_1 + 3x_3 \geq 2 \\ x_1 - 2x_2 \leq 10 \end{bmatrix} \vee \begin{bmatrix} 4x_1 - x_2 \leq 8 \\ x_2 + 5x_3 \geq 10 \end{bmatrix} \vee \begin{bmatrix} 5x_1 - 3x_2 + 7x_3 \geq 4 \end{bmatrix} \\
&\quad x_1, x_2, x_3 \geq 0
\end{aligned}
$$

1. Reformulate and simplify the problem using the big-M method.
2. Reformulate and simplify the problem using the convex hull method.

Problem 3

Given n items, the classic knapsack problem aims to maximize the total item value in a knapsack with limited space. Let v_i and s_i be the value and size of item i, respectively, and use W to denote the size capacity of the knapsack. Finish the following problems.

1. Set up the formulation for the knapsack problem.
2. Reformulate it using the cut-cocut reforumation and explain the index set.
3. Describe how to generate the proposed constraints in the subproblem.