IE 5331-003 Homework 5

Spring 2025
Due Date: Sunday, May-04-2025, 11:59 PM

Problem 1

The company NexBuy has three potential suppliers, denoted by A, B, and C, with supply capacities $s_A = 100$, $s_B = 80$, and $s_C = 120$, respectively. Suppose the shipping cost per unit of product is random, where $c_A \sim \text{Uniform}(5, 8)$, $c_B \sim (5 + 3 \times \text{Beta}(2, 2))$, and $c_C \sim \text{Triangular}(4, 9, 11)$, the company plans to purchase 150 items in total. Finish the following questions.

(1). Set up a formulation to minimize the expected total cost with the variance bounded by some upper bound (you can try several different levels).

(2). Solve the problem with 10, 100, and 1000 sampling scenarios. Report your results.

(3). Now, the objective of the problem is to minimize the tail risk $\text{CVaR}_{0.05}$. Derive the associated CVaR formulation, then repeat the steps in (2).

(4). Design a new objective function to balance between the expectation cost and CVaR using one balancing parameter. [Hint: which type of combination could help to balance between two values?]

Problem 2 [Adaptation of Problem 2 in HW2]

A company has two distribution centers (DCs) that require purchasing a certain type of product from n potential suppliers. The parameter $\bar{a}_{ij}$ is the capacity of available products that can be sending from supplier i to DC j. The demand of each DC is random. So, the company needs to decide how many to order before and after demand realizations to minimize the total cost. Ordering from each suppler $i \in [n]$ **in advance** induces a fix cost of $\bar{c}_i$ dollars **per item**. Backlog orders (order after the demand realization) requires an additional $\$2$ dollars per item. The detailed parameters can be found in the following table. Finish the

Center ID	Supplier ID									Demands d
	1	2	3	4	5	6	7	8	9	
1	10	23	13	4	9	20	15	6	12	Uniform(48, 60)
2	3	8	30	11	6	33	17	18	9	Triangle(35, 67, 70)
Cost $\bar{c}$	9	11	7	8	8	13	21	16	11	—

Table 1: Center IDs are indexed by $[2] := \{1, 2\}$, Supplier IDs are indexed by $[9] := \{1, 2, \ldots, 9\}$. Number $\bar{a}_{ij}$ in each cell denotes the capacity of products can be sent from supplier i to center j.

following questions.

1. Set up the associated two-stage stochastic programming formulation.
2. With 50 samples, solving this problem as a one-level optimization, report your results.
3. Define the associated master and subproblems, explain how to solve this problem using the L-shaped method. (No need to solve it)

Problem 3

Consider a portfolio optimization problem involving n assets, where the (random) return rates are represented by a random vector $r = (r_i)_{i \in [n]}$. We are given m independent samples of r, denoted by r^j for $j \in [m]$, where each $r^j \in \mathbb{R}^n$. Answer the following questions:

1. Formulate the classical portfolio optimization problem that maximizes the expected return.

2. Formulate the problem in part (1) using a moment-based distributionally robust optimization (DRO) model. Reformulate the entire problem as a single-level optimization problem.

3. Formulate the problem in part (1) using a Wasserstein-distance-based DRO model. Reformulate the resulting problem as a single-level optimization problem.

4. Suppose the portfolio optimization problem also includes a chance constraint of the form $\mathbb{P}(\langle r, x \rangle \geq \gamma) \geq 1 - \epsilon$, for some confidence level γ. Reformulate the resulting problem using Wasserstein DRO as a single-level optimization problem.