Table A–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \qquad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \qquad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1}e^{-at} \qquad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \qquad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

(*continues on next page*)

Table A–1 (*continued*)

18	$\frac{1}{a^2}\left(1 - e^{-at} - ate^{-at}\right)$	$\frac{1}{s(s + a)^2}$
19	$\frac{1}{a^2}\left(at - 1 + e^{-at}\right)$	$\frac{1}{s^2(s + a)}$
20	$e^{-at}\sin\omega t$	$\frac{\omega}{(s + a)^2 + \omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s + a}{(s + a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1 - \zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1 - \zeta^2}\,t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1 - \zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1 - \zeta^2}\,t - \phi\right)$ $\phi = \tan^{-1}\frac{\sqrt{1 - \zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1 - \zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1 - \zeta^2}\,t + \phi\right)$ $\phi = \tan^{-1}\frac{\sqrt{1 - \zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$
25	$1 - \cos\omega t$	$\frac{\omega^2}{s\left(s^2 + \omega^2\right)}$
26	$\omega t - \sin\omega t$	$\frac{\omega^3}{s^2\left(s^2 + \omega^2\right)}$
27	$\sin\omega t - \omega t\cos\omega t$	$\frac{2\omega^3}{\left(s^2 + \omega^2\right)^2}$
28	$\frac{1}{2\omega}t\sin\omega t$	$\frac{s}{\left(s^2 + \omega^2\right)^2}$
29	$t\cos\omega t$	$\frac{s^2 - \omega^2}{\left(s^2 + \omega^2\right)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2}\left(\cos\omega_1 t - \cos\omega_2 t\right) \qquad \left(\omega_1^2 \neq \omega_2^2\right)$	$\frac{s}{\left(s^2 + \omega_1^2\right)\left(s^2 + \omega_2^2\right)}$
31	$\frac{1}{2\omega}\left(\sin\omega t + \omega t\cos\omega t\right)$	$\frac{s^2}{\left(s^2 + \omega^2\right)^2}$

Table A–2 Properties of Laplace Transforms

1	$\mathscr{L}[Af(t)] = AF(s)$
2	$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathscr{L}_{\pm}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathscr{L}_{\pm}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathscr{L}_{\pm}\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\overset{(k-1)}{f}(0\pm)$ where $\overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathscr{L}_{\pm}\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\,dt\right]_{t=0\pm}$
7	$\mathscr{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$
8	$\mathscr{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t)\,dt = \lim_{s \to 0} F(s)$ if $\int_0^\infty f(t)\,dt$ exists
10	$\mathscr{L}[e^{-\alpha t}f(t)] = F(s + a)$
11	$\mathscr{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$
12	$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathscr{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
14	$\mathscr{L}[t^nf(t)] = (-1)^n\frac{d^n}{ds^n}F(s) \qquad (n = 1, 2, 3, \dots)$
15	$\mathscr{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds$ if $\lim_{t \to 0}\frac{1}{t}f(t)$ exists
16	$\mathscr{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathscr{L}\left[\int_0^t f_1(t - \tau)f_2(\tau)\,d\tau\right] = F_1(s)F_2(s)$
18	$\mathscr{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(p)G(s - p)\,dp$

Finally, we present two frequently used theorems, together with Laplace transforms of the pulse function and impulse function.

Initial value theorem	$f(0+) = \lim_{t \to 0+} f(t) = \lim_{s \to \infty} sF(s)$
Final value theorem	$f(\infty) = \lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Pulse function $f(t) = \dfrac{A}{t_0}1(t) - \dfrac{A}{t_0}1(t - t_0)$	$\mathcal{L}[f(t)] = \dfrac{A}{t_0 s} - \dfrac{A}{t_0 s} e^{-st_0}$
Impulse function $g(t) = \lim_{t_0 \to 0} \dfrac{A}{t_0}, \quad \text{for } 0 < t < t_0$ $= 0, \quad \text{for } t < 0, t_0 < t$	$\mathcal{L}[g(t)] = \lim_{t_0 \to 0} \left[\dfrac{A}{t_0 s}(1 - e^{-st_0}) \right]$ $= \lim_{t_0 \to 0} \dfrac{\dfrac{d}{dt_0}[A(1 - e^{-st_0})]}{\dfrac{d}{dt_0}(t_0 s)}$ $= \dfrac{As}{s} = A$