Recap and exercises

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3 GOOD HEALTH AND WELL-BEING

SUSTAINABLE DEVELOPMENT GOAL 3:
Ensure healthy lives and promote well-being for all at all ages

https://sdgs.un.org/goals/goal3

QUESTIONS & ANSWERS
"Any time" on TELEGRAM

Friday with YAGO
@2pm !!

Exercises

Exercise L4E2: Determine the inverse Laplace transform of the following:

a. $\dfrac{7s-6}{s^2-s-6} = F(s)$

```
>> syms s
>> F=(7*s-6)/(s^2-s-6)
F =
-(7*s - 6)/(- s^2 + s + 6)
>> f=ilaplace(F)
f =
4*exp(-2*t) + 3*exp(3*t)
```

```
>>> import sympy as sym
>>> from sympy.abc import s,t
>>> from sympy.integrals import inverse_laplace_transform
>>> F =(7*s - 6)/(s**2 - s - 6)
>>> F
(7*s - 6)/(s**2 - s - 6)
>>> f=sym.inverse_laplace_transform(F,s,t)
>>> f
(3*exp(5*t) + 4)*exp(-2*t)*Heaviside(t)
```

$$F(s) = \frac{7s-6}{s^2-s-6} = \frac{7s-6}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2}$$

$$A = (s-3)F(s)\Big|_{s\to 3} = \frac{7s-6}{s+2}\Big|_{s\to 3} = 3$$

$$B = (s+2)F(s)\Big|_{s\to -2} = \frac{7s-6}{s-3}\Big|_{s\to -2} = 4$$

$$F(s) = \frac{3}{s-3} + \frac{4}{s+2}$$

$$f(t) = \left(3e^{3t} + 4e^{-2t}\right)\delta_{-1}(t)$$

Exercises

Exercise L4E2: Determine the inverse Laplace transform of the following:

b. $\dfrac{2s^2+5}{s^2+3s+2} = F(s)$

```
>> F=(2*s^2+5)/(s^2+3*s+2)
F =
(2*s^2 + 5)/(s^2 + 3*s + 2)
>> f=ilaplace(F)
f =
7*exp(-t) - 13*exp(-2*t) + 2*dirac(t)
```

```
>>> import sympy as sym
>>> from sympy.abc import s,t
>>> from sympy.integrals import inverse_laplace_transform
>>> F=(2*s**2+5)/(s**2+3*s+2)
>>> F
(2*s**2 + 5)/(s**2 + 3*s + 2)
>>>
>>> f=sym.inverse_laplace_transform(F,s,t)
>>> f
2*DiracDelta(t) + 7*exp(-t)*Heaviside(t) - 13*exp(-2*t)*Heaviside(t)
```

$$F(s) = \frac{2s^2+5}{s^2+3s+2} = 2 + \frac{1-6s}{s^2+3s+2}\,{}^{*} = 2 + \underbrace{\frac{1-6s}{(s+2)(s+1)}}_{F'(s)}$$

$$= 2 + \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = (s+2)F'(s)\Big|_{s\to -2} = \frac{1-6s}{s+1}\Big|_{s\to -2} = -13$$

$$)F'(s)\Big|_{s\to -1} = \frac{1-6s}{s+2}\Big|_{s\to} = 7$$

* Note that $n = m = 2 \Rightarrow$

2	0	5	1 3 2
-2	-6	-4	2
0	-6	1	

$(1-6s) \rightsquigarrow$ 0 -6 1

Exercises

Exercise L4E2: Determine the inverse Laplace transform of the following:

c. $\frac{6(s+34)}{s(s^2+10s+34)} = F(s)$

$$f(t) = 6 + 10e^{-5t}(\cos(3t+2.21))$$

or

$$f(t) = 6 - 6e^{-5t}\left(\cos(3t) + \frac{4}{3}\sin(3t)\right)$$

① $\mathcal{L}^{-1}\left[\frac{R}{s-p} + \frac{R'}{s-p'}\right] = Me^{\alpha t}\cos(\omega t + \phi)$

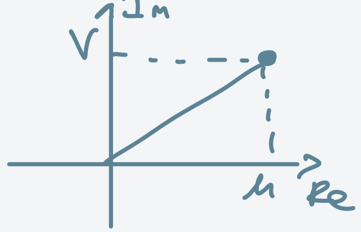
Where $p, p' = \alpha + j\omega$ $\quad R, R' = u + jv$

$M = 2|R| = 2\sqrt{u^2+v^2}$

$\phi = \arg(R) = \arctan\left(\frac{v}{u}\right)$

② $\mathcal{L}^{-1}\left[\frac{R}{s-p} + \frac{R'}{s-p'}\right] = Be^{\alpha t}\cos\omega t + Ce^{\alpha t}\sin\omega t$

where $B = 2u$ $\quad C = -2v$



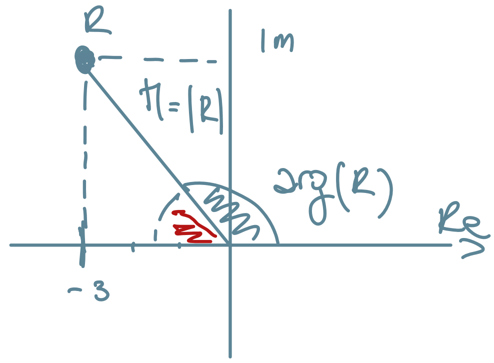
$$F(s) = \frac{6(s+34)}{s(s^2+10s+34)} = \frac{6(s+34)}{s(s+5-j3)(s+5+j3)} = \frac{R_1}{s} + \frac{R}{s+5-j3} + \frac{R'}{s+5+j3}$$

$$R_1 = s.F(s)\Big|_{s\to 0} = \frac{6(s+34)}{s^2+10s+34}\Big|_{s\to 0} = 6$$

$$R = (s+5-j3).F(s)\Big|_{s\to -5+j3} = \frac{6(s+34)}{s(s+5+j3)}\Big|_{s\to -5+j3} =$$

$$= \frac{6(-5+j3+34)}{(-5+j3)(-\cancel{5}+j3+\cancel{5}+j3)} = \frac{6(29+j3)}{-18-j30} = \frac{\cancel{6}(29+j3)(-3+j5)}{\cancel{6}(-3-j5)(-3+j5)} =$$

$$= \frac{-102+j136}{34} = -3+j4$$



$$M = 2|R| = 2\sqrt{3^2+4^2} = 2\times 5$$

$$\phi = \arg(R) = \operatorname{atan}\left(\frac{4}{-3}\right) + 3.14 =$$
$$= 2.2127$$

$$B = 2\times(-3) = -6$$

$$C = -2\times(4) = -8$$

```
% TI0118 - L04E2c
clear; close all; clc

num=[6 6*34];
den=[1 10 34 0];
[r,p,~]=residue(num,den);
sys=tf(num,den);

t = linspace(0,5,200);
y = impulse(sys,t);

alpha=real(p(1));
omega=imag(p(1));

M=2*abs(r(1));
phi=atan(imag(r(1))/real(r(1)))+pi; % phi=angle(r(1)); % built-in fuction

f1=(M*exp(alpha*t).*cos(omega.*t+phi)+real(r(3))).*heaviside(t);

B=2*real(r(1)); C=-2*imag(r(1));
f2=(B*exp(alpha*t).*cos(omega*t)+C*exp(alpha*t).*sin(omega*t)+real(r(3))).*heaviside(t);

plot(t,y,t,f1,t,f2); legend('built-in','algebric','polar')
```

→ impulse response - built-in function

Check and compare the solution in time domain