



Modelling dynamic systems in the Laplace domain

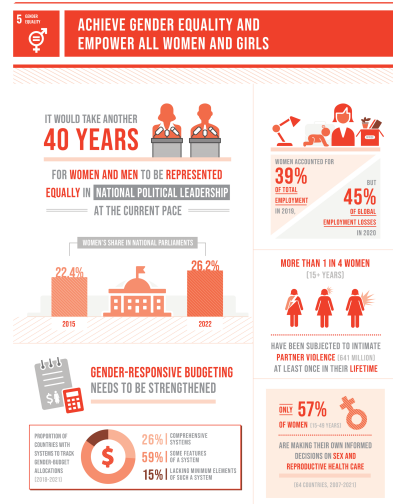
Transfer function examples

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EXTRA CLASS
TOMORROW!!!

Topic: Simulation of
dynamical systems



THE SUSTAINABLE DEVELOPMENT GOALS REPORT 2022: [UNSTATS.UN.ORG/SDGS/REPORT/2022/](https://unstats.un.org/sdgs/report/2022/)

SALA 11
@ 14:00

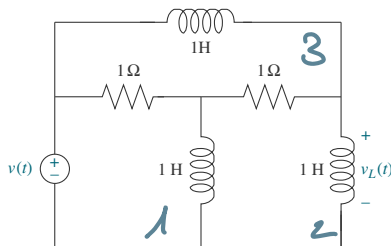


Electrical network transfer functions

Exercise L5E2: For the given circuit, find the transfer function:

$$G(s) = V_L(s)/V(s)$$

1. Solve the problem two ways: mesh analysis and nodal analysis.
2. Show that the two methods yield the same result.



Test ANALYSIS

$$\text{in } \mathcal{M}_1 \leadsto (\mathcal{S}+1) \bar{I}_1(s) - \mathcal{S} \bar{I}_2 - \bar{I}_3 = V(s)$$

$$\text{in } \mathcal{M}_2 \leadsto -\mathcal{S} \bar{I}_1(s) + (2\mathcal{S}+1) \bar{I}_2(s) - \bar{I}_3 = 0$$

$$\text{in } \mathcal{M}_3 \leadsto -\bar{I}_1(s) - \bar{I}_2(s) + (\mathcal{S}+2) \bar{I}_3 = 0$$

Solving for \bar{I}_2 (with Cramer)

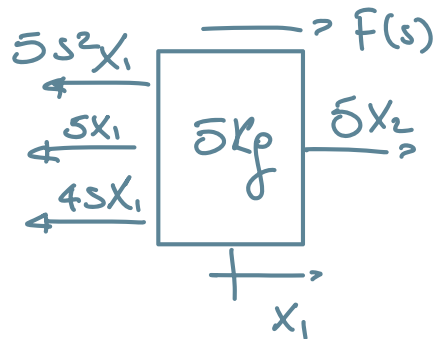
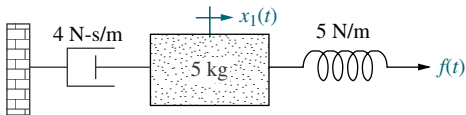
$$\bar{I}_2 = \frac{\begin{vmatrix} \mathcal{S}+1 & V(s) & -1 \\ -\mathcal{S} & 0 & -1 \\ -1 & 0 & \mathcal{S}+2 \end{vmatrix}}{\begin{vmatrix} \mathcal{S}+1 & -\mathcal{S} & -1 \\ -\mathcal{S} & 2\mathcal{S}+1 & -1 \\ -1 & -1 & \mathcal{S}+2 \end{vmatrix}} = \frac{(\mathcal{S}^2+2\mathcal{S}+1)V(s)}{\mathcal{S}(\mathcal{S}^2+\mathcal{S}+2)} \Rightarrow V(s) = \frac{\mathcal{S}(\mathcal{S}^2+\mathcal{S}+2)}{\mathcal{S}^2+2\mathcal{S}+1} \bar{I}_2$$

$$V_L(s) = \mathcal{S} \bar{I}_2 \leadsto \frac{V_L(s)}{V(s)} = \frac{\mathcal{S}^2+2\mathcal{S}+1}{\mathcal{S}^2+\mathcal{S}+2}$$



Translational mechanical system transfer functions

Exercise L5E3: Find the transfer function, $G(s) = X_1(s)/F(s)$, for the system below:



$X_1(s) \equiv$ mass displ.

$X_2(s) \equiv$ spring displ.

$$(5s^2 + 4s + 5) X_1(s) - 5 X_2(s) = 0$$

$$-5 X_1(s) + 5 X_2(s) = F(s)$$

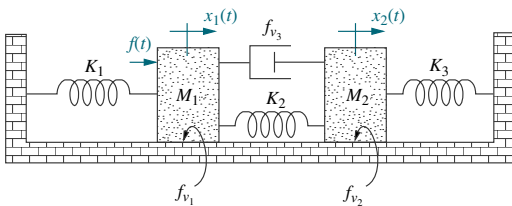
$$\Rightarrow (5s^2 + 4s) X_1(s) = F(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{1}{s(5s + 4)}$$



Translational mechanical system transfer functions

Exercise L5E4: Find the transfer function, $X_2(s)/F(s)$, for the system below:



SOLVED in NISE (6th edition)
EXAMPLE 2.17