Modelling dynamic systems

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2 ZERO HUNGER

End hunger, achieve food security and improved nutrition and promote sustainable agriculture

SUSTAINABLE DEVELOPMENT GOALS

Partial fraction expansion

Case 1. Roots of the denominator of $F(s)$ are real and distinct

Exercise L2E3: Using the Laplace transform, solve the following differential equation

- $y(t)$ if all initial conditions are zero
- $u(t)$ is a step function

$$\frac{d^2y(t)}{dt^2} + 12\frac{dy(t)}{dt} + 32y(t) = 32u(t)$$

NISE - EXAMPLE 2.3

$$s^2 Y(s) + 12s Y(s) + 32 Y(s) = \frac{32}{s}$$

Unit step
Zero initial conditions

$$Y(s)\left(s^2 + 12s + 32\right) = \frac{32}{s}$$

$$Y(s) = \frac{32}{s(s^2+12s+32)} = \frac{32}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

$$A = \left.\frac{32}{(s+4)(s+8)}\right|_{s\to 0} = 1$$

$$B = \left.\frac{32}{s(s+8)}\right|_{s\to -4} = 2$$

$$C = \left.\frac{32}{s(s+4)}\right|_{s\to -8} = 1$$

$$Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8} \Rightarrow y(t) = \left(1 - 2e^{-4t} + e^{-8t}\right)u(t)$$

Laplace transform
Exercises

Exercise L2E4: Find the Laplace transform of $f(t) = te^{-5t}$.

Exercise L2E5: Find the inverse Laplace transform of

$$F(s) = \frac{10}{s(s+2)(s+3)^2}$$

$$F(s) = \frac{s-10}{(s+2)(s+5)}$$

$$F(s) = \frac{100}{(s+1)(s^2+4s+13)}$$

$$F(s) = \frac{s+18}{s(s+3)^2}$$

$$F(s) = \frac{1}{(s+5)^2}$$

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt = \int_0^\infty t e^{-5t} e^{-st}\, dt =$$

$$= \int_0^\infty t\, e^{-(s+5)t}\, dt =$$

$$= \left[-\frac{t}{s+5} e^{-(s+5)t} - \frac{1}{(s+5)^2} e^{-(s+5)t} \right]_0^\infty =$$

$$= 0 - \frac{1}{(s+5)^2}(0-1) = \frac{1}{(s+5)^2}$$

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Exercise L2E5: Find the inverse Laplace transform of

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$$F(s) = \frac{s-10}{(s+2)(s+5)}$$

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$$F(s) = \frac{s+18}{s(s+3)^2}$$

$$F(s) = \frac{10}{S(S+2)(S+3)^2} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+3} + \frac{D}{(S+3)^2}$$

$$A = \left.\frac{10}{(S+2)(S+3)^2}\right|_{S\to 0} = \frac{5}{9} \qquad B = \left.\frac{10}{S(S+3)^2}\right|_{S\to -2} = -5$$

$$C = \left.\frac{10}{S(S+2)}\right|_{S\to -3} = \frac{10}{3}$$

$$D = \left.\frac{d}{ds}\left[\frac{10}{S(S+2)}\right]\right|_{S\to -3} = \left.\frac{-10(2S+2)}{S^2(S+2)^2}\right|_{S\to -3} = -\frac{40}{9}$$

$$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} - \frac{40}{9}e^{-3t}$$

Laplace transform
Exercises

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Exercise L2E5: Find the inverse Laplace transform of

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$$F(s) = \frac{s-10}{(s+2)(s+5)}$$

$$F(s) = \frac{100}{(s+1)(s^2+4s+13)}$$

$$F(s) = \frac{s+18}{s(s+3)^2}$$

- $F(s) = \dfrac{s-10}{(s+2)(s+5)} = \dfrac{A}{s+2} + \dfrac{B}{s+5} = -\dfrac{4}{s+2} + \dfrac{5}{s+5}$

  $f(t) = -4e^{-2t} + 5e^{-5t}$

- $F(s) = \dfrac{100}{(s+1)(s^2+4s+13)}$

  $f(t) = 10e^{-t} - \dfrac{10}{3}\sqrt{10}\, e^{-2t} \cos(3t + 2.81)$

  or

  $f(t) = 10e^{-t} - 10e^{-2t}\left(\cos 3t + \dfrac{1}{3}\sin 3t\right)$

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$$F(s) = \frac{s-10}{(s+2)(s+5)}$$

$$F(s) = \frac{100}{(s+1)(s^2+4s+13)}$$

$$F(s) = \frac{s+18}{s(s+3)^2}$$

- $F(s) = \frac{s+18}{s(s+3)^2}$ $\quad f(t) = 2 - 2e^{-3t} - 5te^{-3t}$

$F(s) = \frac{A}{s} + \frac{B}{(s+3)^2} + \frac{C}{s+3}$

$A = s \cdot F(s)\Big|_{s\to 0} = \frac{s+18}{(s+3)^2}\Big|_{s\to 0} = 2$

$B = \frac{d}{ds}\left[(s+3)^2 \cdot F(s)\right]_{s\to -3} = \frac{d}{ds}\left[\frac{s+18}{s}\right]_{s\to -3} = -\frac{18}{9} = -2$

$C = (s+3)^2 \cdot F(s)\Big|_{s\to -3} = \frac{s+18}{s}\Big|_{s\to -3} = \frac{15}{-3} = -5$