NOTE: we suppose without loss of generality that all elements of a are distinct and that a is sorted non-decreasing
(if not, just remove duplicates and sort it)

Lemma 1

The answer is 1 step **iff**

$$\gcd(\{a[i+1]-a[i] : 0 \le i < n-1\}) > 1.$$

Proof:
Let d be this gcd.

- ($\Rightarrow$) If the answer is 1, there exists $m > 1$ and some x such that

  $$a[i] \equiv x \pmod{m} \quad \text{for all } i,$$

  so $a[i+1]-a[i]$ is divisible by m for all i.
- ($\Leftarrow$) If $d > 1$, then

  $$a[n-1] \equiv a[n-2] \equiv \cdots \equiv a[0] \pmod{d}.$$

Lemma 2

The answer is always at most 2.

Proof:
Trivial (as $M = 2$ is a candidate to solve the problem in at most two steps).

We can now analyze remainders:

If $N \ge 3$, then the sequence starts out as one of the following: - $x, x, \ldots$ - $x, y, x, \ldots$ - $x, y, y, \ldots$

Therefore, m is a divisor of either:

$$a[1]-a[0], \quad a[2]-a[0], \quad \text{or} \quad a[2]-a[1].$$

We can collect these divisors in

$$O\big(3 \cdot (\max_{i=1..M} \operatorname{div}(i) + \sqrt{M})\big) = O\big(M^{1/3} + \sqrt{M}\big) = O\big(\sqrt{M}\big)$$

where $\operatorname{div}(x)$ is the number of divisors of x and then merge them into a set (to avoid double counting) in

$$O(M^{1/3} \log M).$$

Checking if a divisor is valid is $O(N)$.

Thus the final complexity is:

$$O\big(N \log N \ + \ \sqrt{M} + M^{1/3} \log M \ + \ N M^{1/3}\big).$$

If $N = 2$, and there is no solution in 1 step,
then any $1 < m \leq M$ is a solution.