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1) a) Podemos verificar através do critério das linhas

$a_{11}=6$, $a_{12}=3$, $a_{13}=0 \Rightarrow \left|\frac{3}{6}\right| + \left|\frac{0}{6}\right| = \left|\frac{1}{2}\right| < 1$ ✓

$a_{21}=2$, $a_{22}=5$, $a_{23}=-2 \Rightarrow \left|\frac{2}{5}\right| + \left|\frac{-2}{5}\right| = \left|\frac{4}{5}\right| < 1$ ✓

$a_{31}=3$, $a_{32}=-2$, $a_{33}=6 \Rightarrow \left|\frac{3}{6}\right| + \left|\frac{-2}{6}\right| = \left|\frac{5}{6}\right| < 1$ ✓

∴ podemos afirmar, através do critério das linhas, que a sequência gerada pelo método Gauss-Jacobi e Gauss-Seidel converge para x^*, independente do ponto inicial $x^{(0)}$

b) $x^{(k+1)} = Px^{(k)} + C$, $\quad P = -D^{-1}(L+U)$, $C = D^{-1}b$

$$A = \begin{bmatrix} 6 & 3 & 0 \\ 2 & 5 & -2 \\ 3 & -2 & 6 \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & -2 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}, \quad D^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{6} \end{bmatrix}$$

$$P = \begin{bmatrix} -\frac{1}{6} & 0 & 0 \\ 0 & -\frac{1}{5} & 0 \\ 0 & 0 & -\frac{1}{6} \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & -2 \\ 3 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & 0 \\ -\frac{2}{5} & 0 & \frac{2}{5} \\ -\frac{1}{2} & \frac{1}{3} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 \\ -\frac{6}{5} \\ 0 \end{bmatrix}$$

c) $$\|P\|_1 = \left|-\frac{1}{6}\right| + \left|-\frac{1}{5}\right| + \left|\frac{-1}{6}\right| = \frac{8}{15}$$

$$\|P\|_\infty = \max\left\{\left|-\frac{1}{6}\right|, \left|-\frac{1}{5}\right|, \left|-\frac{1}{6}\right|\right\} = \frac{1}{5}$$

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(d) $x^{(0)} = \begin{bmatrix} -1 \\ 0.5 \\ 0.2 \end{bmatrix}$,

$$x_1^{(k+1)} = \frac{-12 - 3x_2^{(k)}}{6}, \quad x_2^{(k+1)} = \frac{-6 + 2x_3^{(k)} - 2x_1^{(k)}}{5}, \quad x_3^{(k+1)} = \frac{-3x_1^{(k)} + 2x_2^{(k)}}{6}$$

$x_1^{(1)} = \frac{-12 - 3 \cdot 0,5}{6} = -2,25$

$x_2^{(1)} = \frac{-6 + 2 \cdot 0,2 - 2(-1)}{5} = -0,72 \Rightarrow$

$x_3^{(1)} = \frac{-3(-1) + 2(0,5)}{6} = 0,6$

$x_1^{(2)} = \frac{-12 - 3 \cdot (-0,72)}{6} = -1,64$

$x_2^{(2)} = \frac{-6 + 2 \cdot 0,6 - 2(-2,25)}{5} = -0,06$

$x_3^{(2)} = \frac{-3(-2,25) + 2(-0,72)}{6} = 0,885$

Resíduo

$\|b - Ax^{(1)}\|_\infty = 9,3$

$\|b - Ax^{(2)}\| = 2,02$

$\epsilon_r = \frac{\|x^{(2)} - x^{(1)}\|_\infty}{\|x^{2}\|_\infty} = \frac{0,85}{1,64} = 0,5183$

$x^{(0)} = \begin{bmatrix} -1 \\ 0.5 \\ 0.2 \end{bmatrix}$,

$$x_1^{(k+1)} = \frac{-12 - 3x_2^{(k)}}{6}, \quad x_2^{(k+1)} = \frac{-6 + 2x_3^{(k)} - 2x_1^{(k+1)}}{5}, \quad x_3^{(k+1)} = \frac{-3x_1^{(k+1)} + 2x_2^{(k+1)}}{6}$$

$x_1^{(1)} = \frac{-12 - 3 \cdot 0,5}{6} = -2,25$

$x_2^{(1)} = \frac{-6 + 2(0,2) - 2 \cdot (-2,25)}{5} = -2,02$

$x_3^{(1)} = \frac{-3(-2,25) + 2(-2,02)}{6} = 0,4516$

$\Rightarrow$

$x_1^{(2)} = \frac{-12 - 3(-2,02)}{6} = -0,99$

$x_2^{(2)} = \frac{-6 + 2(0,4516) - 2 \cdot (-0,99)}{5} = -0,6233$

$x_3^{(2)} = \frac{-3(-0,99) + 2(-0,6233)}{6} = 0,28$

$\|b - Ax^{(1)}\|_\infty = 9,5$

$\|b - Ax^{(1)}\|_\infty = 4,2$

$\epsilon_r = \frac{1,39}{0,99} = 1,4$

2) O método Gauss-Seidel seria o mais recomendado nesta situação, pois a fatoração LU fará as matrizes L e U serem densas, aumentando a quantidade de operações.