Igor Ribeira 140492

1- Tomando $f(x) = \frac{1}{6\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{x-\mu}{6}\right)^2\right]$, Com $\mu = 0$ e $6 = 1$,

temos que: $f(x) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-x^2}{2}\right]$

a) $V_R = \pi \int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \left(\frac{1}{\sqrt{2\pi}} \exp\left[\frac{-x^2}{2}\right]\right)^2 dx$, $g(x) = \frac{\pi}{2\pi} \cdot e^{-x^2} \Rightarrow g(x) = \frac{1}{2} e^{-x^2}$

(I) $a = \frac{1}{\sqrt{2}}$ e $b = \frac{\sqrt{3}}{\sqrt{2}}$, Com $m = 6$

$h = \frac{b-a}{h} \Rightarrow h = \frac{\frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{6} = \frac{\sqrt{3}-1}{6\sqrt{2}} = 0{,}0863$

(II) $x_0 = a = \frac{1}{\sqrt{2}} = 0{,}7071$

$x_1 = x_0 + m = 0{,}7934$

$x_2 = x_1 + m = 0{,}8797$

$x_3 = x_2 + m = 0{,}9660$

$x_4 = x_3 + m = 1{,}0523$

$x_5 = x_4 + m = 1{,}1386$

$x_6 = b = \frac{\sqrt{3}}{\sqrt{2}} = 1{,}2247$

(III) $g(x) = \frac{1}{2} e^{-x^2}$

$g(x_0) = 0{,}3033$

$g(x_1) = 0{,}2664$

$g(x_2) = 0{,}2306$

$g(x_3) = 0{,}1967$

$g(x_4) = 0{,}1652$

$g(x_5) = 0{,}1368$

$g(x_6) = 0{,}1158$

(IV) $\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} g(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2f(x_5) + f(x_6)\right]$

$\approx \frac{0{,}0863}{2}\left[0{,}3033 + 2(0{,}2664) + 2(0{,}2306) + 2(0{,}1967) + 2(0{,}1652) + 2(0{,}1368) + 0{,}1158\right.$

$\approx 0{,}1040$

b) $|E_{TR}| = \frac{m}{12} h^3 M_2$, $M_2 = \max_{x \in [a,b]} |f''(x)|$

$g(x) = \frac{1}{2} e^{-x^2} \Rightarrow g'(x) = -x e^{-x^2} \Rightarrow g''(x) = -e^{-x^2} + 2x^2 e^{-x^2} \geq 0 \quad \forall x \in \left[\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right]$

(II) $h(x) = g''(x)$

$h'(x) = \left[2x e^{-x^2} + (4x e^{-x^2} + 2x^2(-2x) e^{-x^2})\right]$

$\Rightarrow h'(x) = 6x e^{-x^2} - 4x^3 e^{-x^2}$

$M_2 = h(b) = h\left(\frac{\sqrt{3}}{\sqrt{2}}\right) = -\exp\left[-\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2\right] + 2\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2 \exp\left[-\left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2\right]$

$M_2 = 0{,}4463$

$\therefore$

$|E_{TR}| = 1{,}4343 \cdot 10^{-4}$, $|E_{TR}| < 10^{-6}$

$\frac{m h^3 M_2}{12} \leq 10^{-6} \Rightarrow m^2 \geq 10^6 \cdot \frac{0{,}0619}{12}$

$m^2 \geq 5158{,}5105 \Rightarrow m \geq 71{,}8228 \Rightarrow m \geq 72$

2. a) $f(x) = x^2 \ln(x)$ e $R = \{(x,y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2,\ 0 \leq y \leq f(x)\}$

$a = 1$, $b = 2$

$\int_1^2 x^2 \ln(x)\,dx$

(I) $h = \frac{b-a}{m} \Rightarrow h = 0{,}1667$

(II) x_m

$x_0 = a = 1$

$x_1 = 1{,}1667$

$x_2 = 1{,}3334$

$x_3 = 1{,}5001$

$x_4 = 1{,}6668$

$x_5 = 1{,}8335$

$x_6 = 2$

Igor Ribeira 140492

III
$f(x_0) = 0$
$f(x_1) = 0{,}2099$
$f(x_2) = 0{,}5158$
$f(x_3) = 0{,}4126$
$f(x_4) = 0{,}4194$
$f(x_5) = 2{,}0380$
$f(x_6) = 2{,}7726$

IV
$$\int_1^2 x^2 \ln(x)\,dx \approx \boxed{1{,}0716}$$

b) $f'''(x) = \frac{2}{x} \Rightarrow f^{(4)}(x) = \frac{-2}{x^2}$, $M_4 = h(1) = \frac{2}{1^2} = 2$

$|ESR| = 8{,}5820 \cdot 10^{-6}$
$|ESR| \leq 10^{-6}$, temos
$\frac{m}{180}\left(\frac{2-1}{m}\right)^5 \cdot 2 < 10^{6} \Longrightarrow m > 12$