Igor Ribeiro - 140492 Cn - Atividade 6

1) a) $h = \frac{b-a}{m} = \frac{\frac{\pi}{2} - 0}{6} = \frac{\pi}{12}$.

$x_0 = 0$; $x_1 = \frac{\pi}{12}$; $x_2 = \frac{\pi}{6}$; $x_3 = \frac{\pi}{4}$; $x_4 = \frac{\pi}{3}$; $x_5 = \frac{5\pi}{2}$; $x_6 = \frac{\pi}{2}$

O volume do sólido é dado pela integral:

$$V = \int_0^{\pi/2} \pi [2\cos(\tfrac{x}{2})]^2 dx = \int_0^{\frac{\pi}{2}} 4\pi . \cos^2(\tfrac{x}{2})\, dx \; ; \quad g(x) = \underbrace{4\pi}_{12.5663} . \cos^2(\tfrac{x}{2})$$

x_k	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$\frac{5\pi}{2}$	$\pi/2$
$g(x_k)$	12.5664	12.3523	11.7246	10.7261	9.4248	7.9094	6.2832

$$V = \int_0^{\pi/2} g(x) dx = \frac{\frac{\pi}{12}}{2} [g(x_0) + 2g(x_1) + 2g(x_2) + 2g(x_3) + 2g(x_4) + 2g(x_5) + g(x_6)] = 0.1309 . (123,124) = 16.1169$$

b) $k = 2$

$$P = \begin{cases} 2k+4, & \text{se } 0 \le k \le 4 \\ 15-k, & \text{se } 5 < k \le 9 \end{cases} \Rightarrow P = 2.2+4 = 8$$

$g(x) = 4\pi \cos^2(\frac{x}{2})$; $g'(x) = -2\pi . \sin(x)$; $g''(x) = -2\pi . \cos(x)$; $g'''(x) = 2\pi . \sin(x)$

$g''(x) \le 0$ para $x \in [0, \pi/2]$, $|g''(x)| = -g''(x)$

Como $g''' \ge 0$ para $x \in [0, \pi/2]$, g'' é crescente em $x \in [0, \pi/2]$, $-g''$ é decrescente e tem máximo em $x = 0$

$g''(0) = 2\pi . \cos(0) = 6.2832$

$|E_{TR}| \le 5^{-8}$

$\frac{m . h^3}{12} M_2 \le 5^{-8}$

$\frac{m . (\frac{\pi/2 - 0}{m})^3}{12} . 6,2832 \le 5^{-8}$

$\frac{\pi^3}{96 n^2} . 6.2832 \le 5^{-8}$

$n = 890,3448$

Seria preciso $m = 891$ para alcançar um erro $\le 5^{-8}$

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2) a) Seja $f(x) = \sqrt{x}$, $f'(x) = \frac{1}{2\sqrt{x}}$

$$S = \int_1^5 2\pi . (\sqrt{x}) . \sqrt{1 + \left[\frac{1}{2\sqrt{x}}\right]^2}\, dx = \int_1^5 2\pi \sqrt{x . (1 + \tfrac{1}{4x})}\, dx = \int_1^5 2\pi \sqrt{x . \left(\frac{4x+1}{4x}\right)} = \int_1^5 2\pi \sqrt{\frac{4x+1}{4}}$$

$$S = \int_1^5 \pi \sqrt{4x+1}\, dx$$

$h = \frac{b-a}{m} = \frac{5-1}{6} = \frac{4}{6} = 0.6667$

$x_0 = 1$ $x_1 = 1.6667$ $x_2 = 2.3334$ $x_3 = 3.0001$ $x_4 = 3.6668$ $x_5 = 4.3335$ $x_6 = 5.0002$

Seja $g(x) = \pi\sqrt{4x+1}$

x_k	1.0000	1.6667	2.3334	3.0001	3.6668	4.3335	5.0002
$f(x)$	7.0248	8.6988	10.0990	11.3273	12.4350	13.4517	14.3969

$$S = \int_1^5 \pi\sqrt{4x+1}\, dx \approx \frac{h}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4(f(x_3)) + 2f(x_4) + 4f(x_5) + f(x_6)]$$

$$\approx \frac{0.6667}{3} [200.4007] = 44.5357$$

b) $k = 9 \Rightarrow p = 12$

$f(x) = \pi\sqrt{4x+1}$

$f'(x) = \frac{2\pi}{\sqrt{4x+1}}$

$f''(x) = -\frac{4\pi}{\sqrt{4x+1}(4x+1)}$

$f'''(x) = \frac{24\pi}{\sqrt{4x+1}(4x+1)^2}$

$f^{(4)}(x) = \frac{-240\pi}{\sqrt{4x+1}(4x+1)^3}$

$g(x) = f^{4}(x)$

$|g(x)| = -f^{(4)}(x) \to x \in (1,5)$

$g'(x) \ge 0$, $g(x)$ é crescente

Porém $|g(x)|$ é decrescente em $(1,5)$

Valor máx global ocorre em $x = 1$

$|g(1)| = \left|\frac{-240\pi}{\sqrt{4+1}(5)^3}\right| = 2.6975$

$|E_{SR}| \le \frac{m . h^5}{180} . M_4 \le 10^{-12}$

$\frac{m . (\frac{5-1}{m})^5}{180} . 2.6975 \le 10^{-12} \Rightarrow \frac{\frac{1024}{m^4}}{180} . 0.198 \le 10^{-2}$

$\frac{256}{45 m^4} . 2.6975 \le 10^{-12} \Rightarrow \frac{690.56}{45 m^4} \le 10^{-12} \Rightarrow \frac{690,56 . 10^{12}}{45} \le m^4$

$m \le \sqrt[4]{15.3458 . 10^{12}} \le 1,9792 . 10^3$

$m \le 1979,2000$

$\therefore$ Será necessário um $m = 1980$ para atingir um erro $\le 10^{-12}$