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CG - Atividade Teórica 02

1) Tomando $x_0 = 3$, $y_0 = 2$; basta aplicar a seguinte sequência de transformações:

$M_{xy,x'y'} = R(-\theta)\,T(-x_0,-y_0)$, para $\theta = 45°$

$$\begin{bmatrix}\cos(-\theta) & -\sin(-\theta) & 0\\ \sin(-\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -x_0\\ 0 & 1 & -y_0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} \Rightarrow \begin{bmatrix}\cos(-45) & -\sin(-45) & 0\\ \sin(-45) & \cos(-45) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}$$

$$M_{xy,x'y'} = \begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -\frac{5\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} \Rightarrow \begin{bmatrix}\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{5\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y - \frac{\sqrt{2}}{2}\\ 1\end{bmatrix}$$

Para $P=(4,5)$, temos:

$$\begin{bmatrix}\frac{\sqrt{2}}{2}\cdot 4 + \frac{\sqrt{2}}{2}\cdot 5 - \frac{5\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}\cdot 4 + \frac{\sqrt{2}}{2}\cdot 5 - \frac{\sqrt{2}}{2}\\ 1\end{bmatrix}$$

Então $P'(x',y')$:

$$\begin{bmatrix}x'\\ y'\\ 1\end{bmatrix} = \begin{bmatrix}2\sqrt{2}\\ \sqrt{2}\\ 1\end{bmatrix}$$

2) A matriz de transformação inversa vem das seguintes transformações:

$M_{x'y',x,y} = T(x_0,y_0)\,R(\theta)$

Primeiro desfazemos a rotação e depois transladamos de volta.

$$M_{x'y',x,y} = \begin{bmatrix}1 & 0 & x_0\\ 0 & 1 & y_0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos(45) & -\sin(45) & 0\\ \sin(45) & \cos(45) & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2\\ 0 & 0 & 1\end{bmatrix}$$

Desfazendo a transformação em P':

$P = M_{x'y',x,y} \cdot P'$

$$\begin{bmatrix}x\\ y\\ 1\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3\\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}2\sqrt{2}\\ \sqrt{2}\\ 1\end{bmatrix} = \begin{bmatrix}4\\ 5\\ 1\end{bmatrix} \Rightarrow x = 4,\quad y = 5$$

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3. a) $P_1 = (2,2,2)$, $P_2 = (6,6,6)$

Primeiro, centramos $u = \frac{V}{|V|}$, $V = (4,4,4)$ e transladamos para origem

$$|V| = \sqrt{4^2+4^2+4^2} = \sqrt{48} = 4\sqrt{3}$$

$$u = \frac{(4,4,4)}{4\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \quad T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Então calculamos a projeção de u no plano yz

$$\text{Proj}_{\langle 0,1,1\rangle} u = \frac{\langle 0,1,1\rangle \cdot u}{|\langle 0,1,1\rangle|^2} \cdot \langle 0,1,1\rangle = \left(0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$u' = \left(0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \quad |u'| = \frac{\sqrt{2}}{\sqrt{3}}$$

$$u_z = (0,0,1), \quad |u_z| = 1$$

$$\cos\alpha = \frac{u' \cdot u_z}{|u'| \cdot |u_z|} = \frac{0+0+\frac{1}{\sqrt{3}}}{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\cdot(1)} = \frac{\overbrace{\frac{1}{\sqrt{3}}}^{c}}{\underbrace{\frac{\sqrt{2}}{\sqrt{3}}}_{d}} = \frac{\sqrt{2}}{2}, \quad \underline{\alpha = 45^\circ}$$

$$\sin\alpha = \frac{b}{d} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{2}}{\sqrt{3}}} = \frac{\sqrt{2}}{2}, \quad \underline{\alpha = 45^\circ}$$

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(\beta) = \begin{vmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_z(\theta) = \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$u'' = \left(\overset{a}{\frac{1}{\sqrt{3}}}, 0, \overset{d}{\frac{\sqrt{2}}{\sqrt{3}}}\right)$$

$$\cos\beta = d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin\beta = -a = -\frac{1}{\sqrt{3}}$$

$$R(\theta) = T^{-1} R_x^{-1}(\alpha) R_y^{-1}(\beta) R_z(\theta) R_y(\beta) R_x(\alpha) T$$

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{vmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \left(\frac{\sqrt{2}}{\sqrt{3}} + \cos\theta + \frac{\sqrt{2}}{\sqrt{3}}\right) & -\sin(\theta) & \left(-\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) & 0 \\ \sin\theta & \left(\frac{\sqrt{2}}{2} + \cos\theta + \frac{\sqrt{2}}{2}\right) & \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) & 0 \\ \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) & \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) & \left(2\left(\frac{\sqrt{2}}{2}\right) + 2\left(\frac{\sqrt{3}}{2}\right)\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \left(\frac{\sqrt{2}}{3} + \cos\theta + \frac{\sqrt{2}}{3}\right) & -\sin\theta & 0 & 0 \\ \sin\theta & \left(\cos\theta + 2\left(\frac{\sqrt{2}}{2}\right)\right) & 0 & 0 \\ 0 & 0 & \left(2\frac{\sqrt{2}}{2} + 2\frac{\sqrt{3}}{2}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $P_1 = (3,3,1)$, $P_2 = (6,8,6)$, $V = (3,5,5)$

$$u = \left(\frac{3}{\sqrt{59}}, \frac{5}{\sqrt{59}}, \frac{5}{\sqrt{59}}\right), \quad a = \frac{3}{\sqrt{59}}, \; b = \frac{5}{\sqrt{59}}, \; c = \frac{5}{\sqrt{59}}$$

$$T(-3,-3,-1) = \begin{vmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad u' = (0,b,c) = \left(0, \frac{5}{\sqrt{59}}, \frac{5}{\sqrt{59}}\right), \quad d = \sqrt{b^2+c^2} = \frac{5\sqrt{2}}{\sqrt{59}}$$

$$\sin\alpha = \frac{b}{d} = \frac{\frac{5}{\sqrt{59}}}{\frac{5\sqrt{2}}{\sqrt{59}}} = \frac{\sqrt{2}}{2} \quad ; \quad \cos\alpha = \frac{c}{d} = \frac{\sqrt{2}}{2} \quad \Longrightarrow \alpha = 45^\circ$$

$$R_x(\alpha) = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$u'' = (a, 0, d) = \left(\frac{3}{\sqrt{59}}, 0, \frac{5\sqrt{2}}{\sqrt{59}}\right), \quad u_z = (0,0,1)$$

$$\cos\beta = d = \frac{5\sqrt{2}}{\sqrt{59}}, \quad \sin\beta = -a = -\frac{3}{\sqrt{59}} \Rightarrow \beta = -0{,}401247$$

$$R_y(\beta) = \begin{vmatrix} \frac{5\sqrt{2}}{\sqrt{59}} & 0 & -\frac{3}{\sqrt{59}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{3}{\sqrt{59}} & 0 & \frac{5\sqrt{2}}{\sqrt{59}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}, \quad R_z(\theta) = \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$R(\theta) = T^{-1} R_x^{-1}(\alpha) R_y^{-1}(\beta) R_z(\theta) R_y(\beta) R_x(\alpha) T$$

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{vmatrix} \frac{5\sqrt{2}}{\sqrt{59}} & 0 & \frac{3}{\sqrt{59}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{\sqrt{59}} & 0 & \frac{5\sqrt{2}}{\sqrt{59}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \frac{5\sqrt{2}}{\sqrt{59}} & 0 & \frac{3}{\sqrt{59}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{\sqrt{59}} & 0 & \frac{5\sqrt{2}}{\sqrt{59}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \left(\frac{5\sqrt{2}}{\sqrt{59}} \cdot \cos(\theta) \cdot \frac{5\sqrt{2}}{\sqrt{59}}\right) & -\sin(\theta) & \left(\frac{3}{\sqrt{59}} + \frac{3}{\sqrt{59}}\right) & 0 \\ \sin\theta & \left(\frac{\sqrt{2}}{2} + \cos\theta + \frac{\sqrt{2}}{2}\right) & \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) & 0 \\ \left(\frac{3}{\sqrt{59}} - \frac{3}{\sqrt{59}}\right) & \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) & \left(2\left(\frac{\sqrt{2}}{2}\right) + 2\left(\frac{5\sqrt{2}}{\sqrt{59}}\right)\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \left(\frac{5\sqrt{2}}{\sqrt{59}} + \cos\theta + \frac{5\sqrt{2}}{\sqrt{59}}\right) & -\sin\theta & 0 & 0 \\ \sin\theta & \left(\cos\theta + 2\left(\frac{\sqrt{2}}{2}\right)\right) & 0 & 0 \\ 0 & 0 & \left(2\frac{\sqrt{2}}{2} + 2\frac{5\sqrt{2}}{\sqrt{59}}\right) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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4) $P_1 = (2,2,0)$, $P_2 = (6,2,0)$, $P_3 = (5,6,0)$, $P_4 = (4,2,4)$

$V = \overrightarrow{P_1P_3} = (3,4,0)$

$u = \left(\frac{3}{5}, \frac{4}{5}, 0\right)$ $a = 3/5$, $b = 4/5$, $c = 0$

$u' = \left(0, \frac{4}{5}, 0\right)$

$d = \frac{4}{5}$

$u'' = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$

$$T = \begin{vmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_x(\alpha) = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_y(\beta) = \begin{vmatrix} 4/5 & 0 & -\frac{3}{5} & 0 \\ 0 & 1 & 0 & 0 \\ 3/5 & 0 & 4/5 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$P' = R_y(\beta) R_x(\alpha)\, T \cdot P$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{vmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 4/5 & 0 & -\frac{3}{5} & 0 \\ 0 & 1 & 0 & 0 \\ 3/5 & 0 & 4/5 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{vmatrix} 4/5 & 0 & -3/5 & -2 \\ -3/5 & 0 & -4/5 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \\ 1 \end{vmatrix}$$

Para $P_2 = (6,2,0)$, temos:

$$\begin{vmatrix} x_2' \\ y_2' \\ z_2' \\ 1 \end{vmatrix} = \begin{vmatrix} 4/5 & 0 & -3/5 & -2 \\ -3/5 & 0 & -4/5 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 6 \\ 2 \\ 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 34/5 \\ -28/5 \\ 2 \\ 1 \end{vmatrix} \Longrightarrow P_2' = \left(\frac{34}{5}, -\frac{28}{5}, 2\right)$$

Para $P_4 = (4,2,4)$, temos

$$\begin{vmatrix} x_4' \\ y_4' \\ z_4' \\ 1 \end{vmatrix} = \begin{vmatrix} 4/5 & 0 & -3/5 & -2 \\ -3/5 & 0 & -4/5 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 4 \\ 2 \\ 4 \\ 1 \end{vmatrix} = \begin{vmatrix} 14/5 \\ -38/5 \\ 2 \\ 1 \end{vmatrix} \Longrightarrow P_4' = \left(\frac{14}{5}, \frac{-38}{5}, 2\right)$$

5) Dados $P_1 = (x_1, y_1, z_1)$ e $P_2 = (x_2, y_2, z_2)$

$V = (x_2 - x_1, y_2 - y_1, z_2 - z_1$

tomando $x_v = (x_2 - x_1)$, $y_v = (y_2 - y_1)$ e $z_v = (y_2 - y_1)$

$V = (x_v, y_v, z_v)$

$$u = \frac{(x_v, y_v, z_v)}{\sqrt{(x_v)^2 + y_v^2 + z_v^2}} = \Bigg(\underbrace{\frac{x_v}{\sqrt{(x_v)^2 + (y_v)^2 + z_v^2}}}_{a}, \underbrace{\frac{y_v}{\sqrt{(x_v)^2 + (y_v)^2 + z_v^2}}}_{b}, \underbrace{\frac{z_v}{\sqrt{x_v^2 + y_v^2 + z_v^2}}}_{c}\Bigg)$$

$$d = \frac{\sqrt{y_v^2 + z_v^2}}{\sqrt{x_v^2 + y_v^2 + z_v^2}} \qquad \frac{c}{d} = \frac{z_v}{\sqrt{y_v^2 + z_v^2}} \qquad \frac{b}{d} = \frac{y_v}{\sqrt{y_v^2 + z_v^2}}$$

$P' = R_y R_x T$

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_x = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_v}{\sqrt{y_v^2 + z_v^2}} & \frac{y_v}{\sqrt{y_v^2 + z_v^2}} & 0 \\ 0 & \frac{y_v}{\sqrt{y_v^2 + z_v^2}} & \frac{z_v}{\sqrt{y_v^2 + z_v^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_y = \begin{vmatrix} \frac{\sqrt{y_v^2 + z_v^2}}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 & \frac{-x_v}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_v}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 & \frac{\sqrt{y_v^2 + z_v^2}}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{z_v}{\sqrt{y_v^2 + z_v^2}} & \frac{y_v}{\sqrt{y_v^2 + z_v^2}} & 0 \\ 0 & \frac{y_v}{\sqrt{y_v^2 + z_v^2}} & \frac{z_v}{\sqrt{y_v^2 + z_v^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \frac{\sqrt{y_v^2 + z_v^2}}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 & \frac{-x_v}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{x_v}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 & \frac{\sqrt{y_v^2 + z_v^2}}{\sqrt{x_v^2 + y_v^2 + z_v^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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6) Considerando $A = (x_a, y_b, z_b)$ e $B = (x_b, y_b, z_b)$

Precisamos primeiro colocar $\vec{AB}$ no eixo z:

$V = B - A = (x_b - x_a,\ y_b - y_a,\ z_b - z_a)$

$|V| = 1$, pois é um cubo unitário

$U = V$, $a = (x_b - x_a)$, $b = (y_b - y_a)$, $c = (z_b - z_a)$

$U' = (0,\ y_b - y_a,\ z_b - z_a)$, $d = \sqrt{(y_b - y_a)^2 + (z_b - z_a)^2}$

$$T = \begin{vmatrix} 1 & 0 & 0 & -x_a \\ 0 & 1 & 0 & -y_a \\ 0 & 0 & 1 & -z_a \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_x(a) = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{-(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & \frac{(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_y = \begin{vmatrix} \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 & -(x_b - x_a) & 0 \\ 0 & 1 & 0 & 0 \\ (x_b - x_a) & 0 & \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Agora, com $\overline{BA}$ no eixo z, aplicamos a escala

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

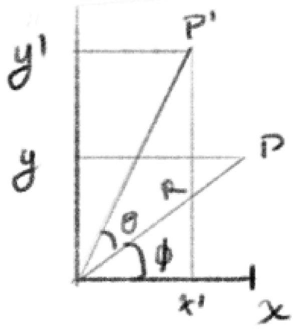
Então, desfazemos as rotações e levamos de volta o objeto ao ponto inicial

$$R_y^{-1} = \begin{vmatrix} \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 & (x_b - x_a) & 0 \\ 0 & 1 & 0 & 0 \\ -(x_b - x_a) & 0 & \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad R_x^{-1} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & \frac{-(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad T^{-1} = \begin{vmatrix} 1 & 0 & 0 & x_a \\ 0 & 1 & 0 & y_a \\ 0 & 0 & 1 & z_a \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Resultando em $P' = T^{-1} R_x^{-1} R_y^{-1} S R_y R_x T$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{vmatrix} 1 & 0 & 0 & -x_a \\ 0 & 1 & 0 & -y_a \\ 0 & 0 & 1 & -z_a \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{-(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & \frac{(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 & -(x_b - x_a) & 0 \\ 0 & 1 & 0 & 0 \\ (x_b - x_a) & 0 & \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 & (x_b - x_a) & 0 \\ 0 & 1 & 0 & 0 \\ -(x_b - x_a) & 0 & \sqrt{(y_b-y_a)^2+(z_b-z_a)^2} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & \frac{-(y_b - y_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & \frac{(z_b - z_a)}{\sqrt{(y_b-y_a)^2+(z_b-z_a)^2}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & x_a \\ 0 & 1 & 0 & y_a \\ 0 & 0 & 1 & z_a \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

7) Rotação em torno de z:



$\operatorname{Sen}\phi = \frac{y}{R}$

$y = R \operatorname{Sen}\phi \quad (1)$

$\operatorname{Sen}(\theta + \phi) = \frac{y'}{R}$

$y' = R \cdot \operatorname{Sen}(\theta + \phi)$

$y' = R\cos\phi \operatorname{Sen}\theta + R\operatorname{Sen}\phi \cos\theta \quad (1)$

$y' = x \operatorname{Sen}\theta + y\cos\theta$

$\cos\phi = \frac{x}{R}$

$x = R\cos\phi \quad (2)$

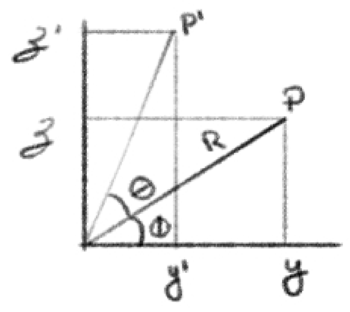
$\cos(\theta + \phi) = \frac{x'}{R}$

$x' = R\cos(\theta + \phi)$

$x' = R\cos\phi\cos\theta - R\operatorname{Sen}\phi\operatorname{Sen}\theta \quad (2)$

$x' = x\cos\theta - y\operatorname{Sen}\theta$

Rotação em torno de x:



$\operatorname{Sen}\phi = \frac{z}{R}$

$z = R\operatorname{Sen}\phi \quad (1)$

$\operatorname{Sen}(\theta + \phi) = \frac{z'}{R}$

$z' = R \cdot \operatorname{Sen}(\theta + \phi)$

$z' = R\cos\phi\operatorname{Sen}\theta + R\operatorname{Sen}\phi\cos\theta \quad (1)$

$z' = y\operatorname{Sen}\theta + z\cos\theta$

$\cos\phi = \frac{y}{R}$

$y = R\cos\phi \quad (2)$

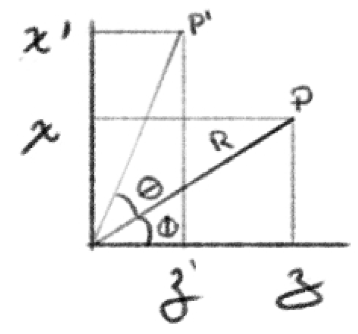
$\cos(\theta + \phi) = \frac{y'}{R}$

$y' = R\cos(\theta + \phi)$

$y' = R\cos\phi\cos\theta - R\operatorname{Sen}\phi\operatorname{Sen}\theta \quad (2)$

$y' = y\cos\theta - z\operatorname{Sen}\theta$

Rotação em torno de y:



$\operatorname{Sen}\phi = \frac{x}{R}$

$x = R\operatorname{Sen}\phi \quad (1)$

$\operatorname{Sen}(\theta + \phi) = \frac{x'}{R}$

$x' = R \cdot \operatorname{Sen}(\theta + \phi)$

$x' = R\cos\phi\operatorname{Sen}\theta + R\operatorname{Sen}\phi\cos\theta \quad (1)$

$x' = z\operatorname{Sen}\theta + x\cos\theta$

$\cos\phi = \frac{z}{R}$

$z = R\cos\phi \quad (2)$

$\cos(\theta + \phi) = \frac{z'}{R}$

$z' = R\cos(\theta + \phi)$

$z' = R\cos\phi\cos\theta - R\operatorname{Sen}\phi\operatorname{Sen}\theta \quad (2)$

$z' = z\cos\theta - x\operatorname{Sen}\theta$