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CN - Atividade 3

1) a) $A = \left[\begin{array}{cccc|c} 1 & -1 & 3 & 0 & 1 \\ 0 & -1 & 2 & 4 & 2 \\ -2 & 1 & 3 & 2 & 3 \\ -3 & 1 & 0 & -1 & 4 \end{array}\right]$ Max = L_4, $L_1 \leftarrow L_4$, $L_4 \leftarrow L_1$

$= \left[\begin{array}{cccc|c} -3 & 1 & 0 & -1 & 4 \\ 0 & -1 & 2 & 4 & 2 \\ -2 & 1 & 3 & 2 & 3 \\ 1 & -1 & 3 & 0 & 1 \end{array}\right]$ $u_{21} = 0$, $u_{31} = -2/3$, $u_{41} = 1/3$

$= \left[\begin{array}{cccc|c} -3 & 1 & 0 & -1 & 4 \\ 0 & -1 & 2 & 4 & 2 \\ 2/3 & 1/3 & 3 & 8/3 & 3 \\ -1/3 & -2/3 & 3 & -1/3 & 1 \end{array}\right]$ Max = L_2, $u_{32} = 1/3$, $u_{42} = -2/3$

$= \left[\begin{array}{cccc|c} -3 & 1 & 0 & -1 & 4 \\ 0 & -1 & 2 & 4 & 2 \\ 2/3 & -1/3 & 11/3 & 4 & 3 \\ -1/3 & 2/3 & 5/3 & -3 & 1 \end{array}\right]$ Max = L_3, $u_{43} = -\frac{5}{11}$

$= \left[\begin{array}{cccc|c} -3 & 1 & 0 & -1 & 4 \\ 0 & -1 & 2 & 4 & 2 \\ 2/3 & -1/3 & 11/3 & 4 & 3 \\ -1/3 & 2/3 & \frac{5}{11} & -\frac{53}{11} & 1 \end{array}\right]$

$$\underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & -1 & 2 & 4 \\ -2 & 1 & 3 & 2 \\ -3 & 1 & 0 & -1 \end{bmatrix}}_{A} \underset{=}{=} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2/3 & -1/3 & 1 & 0 \\ -1/3 & 2/3 & 5/11 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} -3 & 1 & 0 & -1 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 11/3 & 4 \\ 0 & 0 & 0 & -\frac{53}{11} \end{bmatrix}}_{U}$$

b) $\begin{cases} Ly = Pb \\ Ux = y \end{cases}$

$$Pb = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ 9 \\ 2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2/3 & -1/3 & 1 & 0 \\ -1/3 & 2/3 & 5/11 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 \\ 10 \\ 9 \\ 2 \end{bmatrix}}_{= Pb} \qquad \begin{cases} y_1 = 1 \\ y_2 = 10 \\ y_3 = 9 - \frac{2}{3} + \frac{10}{3} = 35/3 \\ y_4 = 2 + \frac{1}{3} - \frac{20}{3} - \frac{5\left(\frac{35}{3}\right)}{11} = -\frac{106}{11} \end{cases}$$

$$\begin{bmatrix} -3 & 1 & 0 & -1 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 11/3 & 4 \\ 0 & 0 & 0 & -\frac{53}{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ 35/3 \\ -\frac{106}{11} \end{bmatrix}$$

$$\begin{cases} x_4 = 2 \\ x_3 = 1 \\ x_2 = 0 \\ x_1 = -1 \end{cases}$$

2) Não, pois durante o método de pivoteamento parcial, temos a <u>maior</u> entrada como pivô. Portanto, não haverá nenhuma outra entrada $a_{ij} >$ pivô, logo o multiplicador sempre será ≤ 1 para cancelar o elemento.

3) Podemos encontrar a inversa de A solucionando $AX = I$.
Se A é uma matriz $N \times N$, X e I também será.
Temos a seguinte situação: $A = \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \cdots & \cdots & \cdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix}$ e $X = \begin{bmatrix} x_{11} & \cdots & x_{1N} \\ \cdots & \cdots & \cdots \\ x_{N1} & \cdots & x_{NN} \end{bmatrix}$
Portanto, podemos dividir X em N colunas $\{x_1, x_2, \ldots, x_n\}$, e I em $\{I_1, I_2 \ldots I_n\}$
Desta forma $\begin{cases} Ax_1 = e_1 \\ \cdots \\ Ax_N = e_N \end{cases}$, mas com a fatoração de Cholesky, sabemos que $A = G^T G$

então $Ax_1 = e_1 \Rightarrow G^T G x_1 = e_1 = \begin{cases} G^T y_1 = e_1 \\ G x_1 = y_1 \end{cases}$; Do qual resolveremos os N sistemas.

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4) a)

$$A = \begin{bmatrix} 1 & -3 & -1 & 2 \\ -3 & 13 & 7 & -12 \\ -1 & 7 & 6 & -7 \\ 2 & -12 & -7 & 18 \end{bmatrix}$$

Supondo que tal fatoração exista, então:

$$\underbrace{\begin{bmatrix} 1 & -3 & -1 & 2 \\ -3 & 13 & 7 & -12 \\ -1 & 7 & 6 & -7 \\ 2 & -12 & -7 & 18 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} g_{11} & 0 & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ g_{31} & g_{32} & g_{33} & 0 \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}}_{G^T} \underbrace{\begin{bmatrix} g_{11} & g_{21} & g_{31} & g_{41} \\ 0 & g_{22} & g_{32} & g_{42} \\ 0 & 0 & g_{33} & g_{43} \\ 0 & 0 & 0 & g_{44} \end{bmatrix}}_{G}$$

Primeira Coluna

$$\begin{bmatrix} 1 \\ -3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 & 0 \\ g_{21} & g_{22} & 0 & 0 \\ g_{31} & g_{32} & g_{33} & 0 \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} g_{11} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} g_{11}^2 \\ g_{21} g_{11} \\ g_{31} g_{11} \\ g_{41} g_{11} \end{bmatrix}$$

$g_{11}^2 = 1 \Rightarrow g_{11} = 1$

$g_{21} \cdot 1 = -3 \Rightarrow g_{21} = -3$

$g_{31} \cdot 1 = -1 \Rightarrow g_{21} = -1$

$g_{41} \cdot 1 = 2 \Rightarrow g_{41} = 2$

Segunda Coluna

$$\begin{bmatrix} -3 \\ 13 \\ 7 \\ -12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & g_{22} & 0 & 0 \\ -1 & g_{32} & g_{33} & 0 \\ 2 & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} -3 \\ g_{22} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 9 + g_{22}^2 \\ 3 + g_{32} g_{22} \\ -6 + g_{42} g_{22} \end{bmatrix}$$

$g_{22}^2 = 13 - 9 \Rightarrow g_{22} = 2$

$3 + 2g_{32} = 7 \Rightarrow g_{32} = 2$

$-6 + 2g_{42} = -12 \Rightarrow g_{42} = -3$

terceira Coluna

$$\begin{bmatrix} -1 \\ 7 \\ 6 \\ -7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ -1 & 2 & g_{33} & 0 \\ 2 & -3 & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ g_{33} \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 1 + 4 + g_{33}^2 \\ -2 - 6 + g_{33} g_{43} \end{bmatrix}$$

$5 + g_{33}^2 = 6 \Rightarrow g_{33} = 1$

$g_{43} - 8 = -7 \Rightarrow g_{43} = 1$

quarta Coluna

$$\begin{bmatrix} 2 \\ -12 \\ -7 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & -3 & 1 & g_{44} \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \\ g_{44} \end{bmatrix} = \begin{bmatrix} 2 \\ -12 \\ -7 \\ 4 + 9 + 1 + g_{44}^2 \end{bmatrix}$$

$14 + g_{44}^2 = 18$

$g_{44} = 2$

$$\underbrace{\begin{bmatrix} 1 & -3 & -1 & 2 \\ -3 & 13 & 7 & -12 \\ -1 & 7 & 6 & -7 \\ 2 & -12 & -7 & 18 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & -3 & 1 & 2 \end{bmatrix}}_{G^T} \cdot \underbrace{\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 2 & 2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}}_{G}$$

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4) b) para chegar no sistema proposto no ex 3, podemos primeiro fazer a fatoração de Choleski

$$\underbrace{\begin{bmatrix} 1 & -3 & -1 & 2 \\ -3 & 13 & 7 & -12 \\ -1 & 7 & 6 & -7 \\ 2 & -12 & -7 & 18 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & -3 & 1 & 2 \end{bmatrix}}_{G^T} \cdot \underbrace{\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 2 & 2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}}_{G} \Rightarrow A = G^T G$$

$$I = \begin{bmatrix} 1 & 0 & 0 & \underbrace{0}_{I_3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Então, aplicando a idea

$$Ax = I$$

e Dividindo X em colunas X_1, X_2 e X_3, temos que e I em colunas I_1, I_2 e I_3

$$Ax_3 = I_3$$

Como $A = G^T G$, temos: $G^T \underbrace{G X_3}_{Y_3} = I_3$

Então, podemos resolver

$$\begin{cases} G^T y_3 = I_3 \\ G x_3 = y_3 \end{cases}$$

- $G^T y_3 = I_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} y_{13} \\ y_{23} \\ y_{33} \\ y_{43} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$y_{13} = \frac{0}{1} = 0 \Rightarrow y_{13} = 0$

$-3y_{13} + 2y_{23} = 0 \Rightarrow 2y_{23} = 0 = y_{23} = 0$

$-y_{13} + 2y_{23} + y_{33} = 1 \Rightarrow y_{33} = 1$

$2y_{13} - 3y_{23} + y_{33} + 2y_{44} = 0 \Rightarrow y_{44} = -1/2$

$$Y_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1/2 \end{bmatrix}$$

- $G X_3 = Y_3$

$$\begin{bmatrix} 1 & -3 & -1 & 2 \\ 0 & 2 & 2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \\ x_{43} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1/2 \end{bmatrix}$$

$2x_{43} = \frac{1}{2} \Rightarrow x_{43} = -1/4$

$x_{33} + x_{43} = 1 \Rightarrow x_{33} = 1 - \frac{1}{4} = 5/4$

$2x_{23} + 2\cancel{x_{33}} - 3x_{43} = 0 \Rightarrow x_{23} = -13/8$

$x_{13} - 3\left(-\frac{13}{8}\right) - \frac{5}{4} + 2\left(\frac{-1}{4}\right) = 0 \Rightarrow x_{13} = -\frac{25}{8}$

$$X_3 = \begin{bmatrix} -\frac{25}{8} \\ -\frac{13}{8} \\ 5/4 \\ -1/4 \end{bmatrix}$$