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1) $x_0 = \frac{\pi}{6}$, $x_1 = \frac{\pi}{4}$, $x_2 = \frac{\pi}{3}$

a) $P_2(\pi/6) = \sqrt{3}/2$; $P_2(\frac{\pi}{4}) = 1$; $P_2(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ $\therefore$

	x_0	x_1	x_2
$f(x)$	$\pi/6$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$
	y_0	y_1	y_2

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-\frac{\pi}{4})(x-\frac{\pi}{3})}{(\frac{\pi}{6}-\frac{\pi}{4})(\frac{\pi}{6}-\frac{\pi}{3})} = \frac{48x-12\pi}{\pi}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-\pi/6)(x-\pi/3)}{(\pi/4-\pi/6)(\frac{\pi}{4}-\frac{\pi}{3})} = \frac{16\pi-96x}{\pi}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-\frac{\pi}{6})(x-\frac{\pi}{4})}{(\frac{\pi}{3}-\frac{\pi}{6})(\frac{\pi}{3}-\frac{\pi}{4})} = \frac{54x-9\pi}{\pi}$$

$$P_2(x) = y_0L_0(x) + y_1L_1(x) + y_2L_2(x)$$

$$= \frac{\sqrt{3}}{2}\left(\frac{48x-12\pi}{\pi}\right) + \left(\frac{16\pi-96x}{\pi}\right) + \frac{\sqrt{3}}{2}\left(\frac{54x-9\pi}{\pi}\right)$$

$$= \frac{102\sqrt{3}x - 21\sqrt{3}\pi + 32\pi - 192x}{2\pi}$$

$$P_2(0{,}6) \cong -19{,}4851$$

b) $|f(x) - P_2(x)| \leq E_{sup}(x)$,

$E_{sup}(x) = \frac{M_3}{3!}|(x-x_0)(x-x_1)(x-x_2)|$, onde $M_3 = \max_{x\in[x_0,x_2]} |f'''(x)|$

$f(x) = \operatorname{Sen}(2x)$, $f'(x) = 2\cos(2x)$, $f''(x) = -4\operatorname{Sen}(2x)$, $f'''(x) = -8\cos(2x)$

temos mudança de sinal em $\frac{\pi}{4}$, sendo a função crescente o intervalo todo, mas $|f'''(x_2)| = |f'''(x_0)|$

$$|f'''(x_0)| = |f'''(x_2)| = |-8\cos(2\tfrac{\pi}{3})| = 4$$

Para $x = 0.6$, temos

$$E_{sup} = \frac{4}{6} \cdot (0{,}6-\tfrac{\pi}{3})(0{,}6-\tfrac{\pi}{4})(0{,}6-\tfrac{\pi}{6}) = 4{,}2229 \cdot 10^{-3}$$

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2)

	0	1	2	3
x	-1	0	1	3
$f(x)$	a	1	0	1

$P_0(x) = a_0 \Rightarrow a_0 = a$

$P_1(x) = P_0(x) + a_1(x - x_0) \Rightarrow a + a_1(x+1) = P_1(x)$, $\quad P_1(\overset{0}{x}) = 1 \Rightarrow a + a_1 = 1$
$a_1 = 1 - a$

$P_1(x) = a + (1-a)(x-1)$

$P_2(x) = P_1(x) + a_2(x-x_0)(x-x_1) \Rightarrow a + (1-a)(x-1) + a_2(x+1)(x)$

$P_2(\overset{1}{x}) = 0 \Rightarrow a + \cancel{(1-a)}(0) + a_2(2)(1) = a + 2a_2 = 0 \Rightarrow a_2 = \frac{-a}{2}$

$P_2(x) = P_1(x) + \left(-\frac{a}{2}\right)(x+1)(x)$

$P_3(x) = P_2(x) + a_3(x-x_0)(x-x_1)(x-x_2) = a + (1-a)(x-1) + \left(-\frac{a}{2}\right)(x+1)(x) + a_3(x+1)(x)(x-1)$

$P_3(3) = 1 \Rightarrow a + (1-a)(3-1) + \left(-\frac{a}{2}\right)(3+1)(3) + a_3(3+1)(3)(3-1) =$
$= a + 2 - 2a + \left(-\frac{a}{2}\right)(4)(3) + a_3(4)(3)(2) = -7a + 2 + 24a_3$
$a_3 = \frac{7a-2}{24}$

$P_3(x) = P_2(x) + \left(\frac{7a-2}{24}\right)(x+1)\underbrace{(x)(x-1)}$

$\Rightarrow P_2(x) = P_1(x) + \left(\frac{a}{2}\right)\underbrace{(x+1)(x)}_{x^2+x}$, $\therefore P_2(x)$ já é de grau 2

$\Rightarrow$ mas $P_3(x)$ é de grau 3

$\therefore$ basta zerar $\left(\frac{7a-2}{24}\right)$, para dependermos apenas do Polinomio de segundo grau

$\frac{7a-2}{24} = 0 \Rightarrow \boxed{a = \frac{2}{7}}$

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3) Como $p(x)$ interpola nos 4 primeiros pontos, podemos tomá-lo como sendo um P_3 e partirmos para acharmos nosso $q(x)$, que seria o P_4.
Fazemos isto pois a forma de Newton nos permite reaproveitar os cálculos anteriores

$$P_3(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1)$$

$$q(x) = P_4(x) = P_3 + a_4 (x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

$$P_4(3) = 10 \Rightarrow P_3(3) + a_4(3+1)(3-0)(3-1)(3-2) = 10$$
$$\Rightarrow -38 + 24a_4 = 10$$
$$\Rightarrow a_4 = \frac{48}{24} = 2$$

$$q(x) = P_3(x) + 2(x+1)(x)(x-1)(x-2)$$
$$q(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1) + 2(x+1)(x)(x-1)(x+2)$$