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1) $L \, . \, I'(t) + R \, . \, I(t) = \mathcal{E}(t)$

a) $I'(x_0) = \dfrac{I(x_0+h) - I(x_0)}{h} = \dfrac{I(1.00+0.01) - I(1.00)}{0.01} = \dfrac{3.12 - 3.10}{0.01} = 2A$

$I'(1.00) = 2A \Rightarrow L \, . \, I'(1.00) + R \, . \, I(1.00) = 0{,}98H \, . \, 2 + 0{,}142\Omega \, . \, 3.10 = 2{,}4002 \, v$

b) $I'(x_0) = \dfrac{I(x_0+h) - I(x_0-h)}{2h} = \dfrac{I(1.01+0.01) - I(1.01-0.01)}{2.0{,}01} = \dfrac{3.14 - 3.10}{0{,}02} = 2A$

$I'(x_0) = \dfrac{I(x_0+h) - I(x_0-h)}{2h} = \dfrac{I(1.02+0.01) - I(1.02-0.01)}{2.0{,}01} = \dfrac{3.18 - 3.12}{0{,}02} = 3A$

$I'(x_0) = \dfrac{I(x_0+h) - I(x_0-h)}{2h} = \dfrac{I(1.03+0.01) - I(1.03-0.01)}{2.0{,}01} = \dfrac{3.24 - 3.14}{0{,}02} = 5A$

$\mathcal{E}(1.01) = L \, . \, I'(1.01) + R \, . \, I(1.01) = 0{,}98 . 2 + 0{,}142 . 3{,}12 = 2{,}4030 \, v$ ✓
$\mathcal{E}(1.02) = L \, . \, I'(1.02) + R \, . \, I(1.02) = 0{,}98 . 3 + 0{,}142 . 3{,}14 = 3{,}3858 \, v$
$\mathcal{E}(1.03) = L \, . \, I'(1.03) + R \, . \, I(1.03) = 0{,}98 . 5 + 0{,}142 . 3{,}18 = 5{,}3515 \, v$

c) $I'(x_0) = \dfrac{I(x_0) - I(x_0-h)}{h} \Rightarrow \dfrac{I(1{,}04) - I(1{,}04-0{,}01)}{0{,}01} = \dfrac{3{,}24 - 3{,}18}{0{,}01} = 6 \, v$

$\mathcal{E}(1.04) = L \, . \, I'(1.04) + R(1.04) = 0{,}98 . 6 + 0{,}142 . 3{,}24 = 6{,}3400 \, v$

d) $I''(x_0) = \dfrac{I(x_0+h) - 2I(x_0) + I(x_0-h)}{h^2} = \dfrac{I(1.02+0.01) - 2.I(1.02) + I(1.02-0.01)}{0{,}01^2} = \dfrac{I(1.03) - 2.I(1.02) + I(1.01)}{0{,}0001} =$

$= \dfrac{3{,}18 - 2(3{,}14) + 3{,}12}{0{,}0001} = 200$

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2) $f \in C^3[a,b]$, $x_0 \in (a,b)$

a) $D_1^c[f(x_0)] = \dfrac{f(x_0+h) - f(x_0-h)}{2h}$,

$\tilde{f}(x) = f(x) + \mathcal{E}(x)$, $|\mathcal{E}(x)| \leq \mathcal{E}_f$

$$\tilde{f}'(x_0) = \frac{f(x_0+h) + \mathcal{E}(x_0+h) - (f(x_0-h) - \mathcal{E}(x_0-h))}{2h} = \underbrace{\frac{f(x_0+h) - f(x_0-h)}{2h}}_{= f'(x_0) - \frac{h^2}{3} f'''(c)} - \frac{\mathcal{E}(x_0+h) + \mathcal{E}(x_0-h)}{2h}$$

$$\tilde{f}'(x_0) = f'(x_0) - \frac{h^2}{3} f'''(c) - \frac{\mathcal{E}(x_0+h) + \mathcal{E}(x_0-h)}{2h}$$

$$\mathcal{E}(h) = \tilde{f}'(x_0) - D[f(x_0) + \mathcal{E}(x_0)] = \frac{h^2}{3} f'''(c) - \frac{\mathcal{E}(x_0+h) + \mathcal{E}(x_0-h)}{2h}$$

Como $\| x+y \| \leq \|x\| + \|y\|$, por desigualdade triangular, então:

$$\left\| \frac{\mathcal{E}(x_0+h) + \mathcal{E}(x_0-h)}{2h} \right\| \leq \left| \frac{\mathcal{E}(x_0+h)}{2h} \right| + \left| \frac{\mathcal{E}(x_0-h)}{2h} \right| \leq \frac{\mathcal{E}_f}{2h} + \frac{\mathcal{E}_f}{2h} = \frac{2\mathcal{E}_f}{2h} = \frac{\mathcal{E}_f}{h}$$

$\therefore$

$$\left| \frac{h^2 f'''(c)}{3} - \frac{\mathcal{E}(x_0+h) + \mathcal{E}(x_0-h)}{2h} \right| \leq \left| \frac{h^2 f'''(c)}{3} \right| + \left| \frac{\mathcal{E}_f}{h} \right| \leq \left| \frac{h^2}{3} M_3 \right| + \left| \frac{\mathcal{E}_f}{h} \right|$$

Isolando h:

$$\left| \frac{h^2}{3} M_3 \right| = \left| \frac{\mathcal{E}_f}{h} \right| \Rightarrow \frac{h^3}{3} M_3 = \mathcal{E}_f \Rightarrow h^3 = \frac{\mathcal{E}_f . 3}{M_3} \Rightarrow h^* = \sqrt[3]{\frac{3\mathcal{E}_f}{M_3}}$$

onde

$$M_3 = \max_{x \in [a,b]} |f'''(x)|$$

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2. b) $h^* = \sqrt[3]{\dfrac{3 \cdot 10^{-16}}{M_3}}$

$M_3 = \max\limits_{x \in [1,2]} |f'''(x)|$, $f(x) = x \cdot e^{-x}$, $f'(x) = \dfrac{1-x}{e^x}$, $f''(x) = \dfrac{-2+x}{e^x}$

$f'''(x) = \dfrac{3-x}{e^x}$

$M_3 = \max\limits_{x \in [1,2]} \left[\dfrac{3-x}{e^x}\right] = f'''(1) = \dfrac{3-1}{e} = \dfrac{2}{e}$

$\therefore$

$h^* = \sqrt[3]{\dfrac{3 \cdot 10^{-16}}{\frac{2}{e}}} = 7{,}4153 \cdot 10^{-6}$