

Atividade 01

📅 Entrega	@October 29, 2021
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▼ 1.

• a)

$$fl(\frac{1}{60}) = \frac{0.100 \cdot 10^1}{0.600 \cdot 10^2} = 0.166 \cdot 10^{-1}$$

$$fl((\frac{1}{60})^2) = (0.166 \cdot 10^{-1})^2 = 0.02755600 \cdot 10^{-2} = 0.276 \cdot 10^{-3}$$

$$fl((\frac{1}{60})^3) = (0.166 \cdot 10^{-1})^3 = 0.00457429 \cdot 10^{-3} = 0.457 \cdot 10^{-5}$$

$$fl((\frac{1}{60})^4) = (0.166 \cdot 10^{-1})^4 = 0.00075933 \cdot 10^{-4} = 0.759 \cdot 10^{-7}$$

$$fl((\frac{1}{60})^5) = (0.166 \cdot 10^{-1})^5 = 0.00012604 \cdot 10^{-5} = 0.123 \cdot 10^{-8}$$

$$fl((\frac{1}{60})^6) = (0.166 \cdot 10^{-1})^6 = 0.00002092 \cdot 10^{-6} = 0.209 \cdot 10^{-10}$$

$$fl(22 \cdot (\frac{1}{60})) = 0.220 \cdot 10^2 \cdot 0.166 \cdot 10^{-1} = 0.03652 \cdot 10^1 = 0.365$$

$$fl(7 \cdot (\frac{1}{60})^2) = (0.700 \cdot 10^1)(0.276 \cdot 10^{-3}) = 0.1932 \cdot 10^{-2} = 0.193 \cdot 10^{-2}$$

$$fl(42 \cdot (\frac{1}{60})^3) = (0.420 \cdot 10^2)(0.457 \cdot 10^{-5}) = 0.19194 \cdot 10^{-3} = 0.192 \cdot 10^{-3}$$

$$fl(33 \cdot (\frac{1}{60})^4) = (0.330 \cdot 10^2)(0.759 \cdot 10^{-7}) = 0.25047 \cdot 10^{-5} = 0.250 \cdot 10^{-5}$$

$$fl(4 \cdot (\frac{1}{60})^5) = (0.400 \cdot 10^1)(0.123 \cdot 10^{-8}) = 0.0492 \cdot 10^{-7} = 0.492 \cdot 10^{-8}$$

$$fl(40 \cdot (\frac{1}{60})^6) = (0.400 \cdot 10^2)(0.209 \cdot 10^{-10}) = 0.0836 \cdot 10^{-8} = 0.836 \cdot 10^{-9}$$

$$fl(1 + (22 \cdot (\frac{1}{60}))) = (0.100 \cdot 10^1) + (0.0365 \cdot 10^1) = 0.137 \cdot 10^1$$

$$fl(1 + (22 \cdot (\frac{1}{60})) + (7 \cdot (\frac{1}{60})^2)) = (0.137 + 0.000193) \cdot 10^1 = 0.137 \cdot 10^1$$

$$fl(1 + (22 \cdot (\frac{1}{60})) + (7 \cdot (\frac{1}{60})^2) + (42 \cdot (\frac{1}{60})^3)) = (0.137 + 0.000193) \cdot 10^1 = 0.137 \cdot 10^1$$

$$fl(1 + (22 \cdot (\frac{1}{60})) + (7 \cdot (\frac{1}{60})^2) + (42 \cdot (\frac{1}{60})^3) + (33 \cdot (\frac{1}{60})^4)) = (0.137 + 0.00000250) \cdot 10^1 = 0.137 \cdot 10^1$$

$$fl(1 + (22 \cdot (\frac{1}{60})) + (7 \cdot (\frac{1}{60})^2) + (42 \cdot (\frac{1}{60})^3) + (33 \cdot (\frac{1}{60})^4) + (4 \cdot (\frac{1}{60})^5)) = (0.137 + 0.00000000492) \cdot 10^1 = 0.137 \cdot 10^1$$

$$fl(1 + (22 \cdot (\frac{1}{60})) + (7 \cdot (\frac{1}{60})^2) + (42 \cdot (\frac{1}{60})^3) + (33 \cdot (\frac{1}{60})^4) + (4 \cdot (\frac{1}{60})^5) + 40 \cdot (\frac{1}{60})^6) = (0.137 + 0.000000000836) \cdot 10^1 = 0.137 \cdot 10^1$$

$$fl(\alpha^*) = 0.137 \cdot 10^1$$

• b)

tomando $\alpha \approx 1,36881$

$$E_R(\alpha^*) = \left| \frac{1.36881 - 1.37}{1.36881} \right| = \left| \frac{-0.00119}{1.36881} \right| = 0.00086936828 \approx 0.08\%$$

▼ 2.

• a)

Calcule o valor aproximado de x_1 (x_1^*) utilizando um sistema de ponto flutuante com base 10, 4 dígitos na mantissa e arredondamento:

$$a = 1 = 0.1000 \times 10^1$$

$$b = 62.10 = 0.6210 \times 10^2$$

$$c = 1 = 0.1000 \times 10^1$$

$$fl(-b) = -0.6210 \times 10^2$$

$$fl(b^2) = (0.6210 \times 10^2)^2 = 0.385641 \times 10^4 = 0.3856 \times 10^4$$

$$fl(4c) = (0.4000 \times 10^1) \times (0.1000 \times 10^1) = (0.4000 \times 0.1000) \times 10^2 = 0.0400 \times 10^2 = 0.4000 \times 10^1$$

$$fl(b^2 - 4c) = 0.3856 \times 10^4 - 0.4000 \times 10^1 = (0.3856 - 0.0004) \times 10^4 = 0.3852 \times 10^4$$

$$fl(\sqrt{b^2 - 4c}) = \sqrt{0.3852 \times 10^4} = 0.620645 \times 10^2 \Rightarrow 0.6206 \times 10^2$$

$$fl(-b + \sqrt{b^2 - 4c}) = -0.6210 \times 10^2 + 0.6206 \times 10^2 = (-0.6210 + 0.6206) \times 10^2 = -0.0004 \times 10^2 \Rightarrow -0.4000 \times 10^{-1}$$

$$fl\left(\frac{-b + \sqrt{b^2 - 4c}}{2}\right) = \frac{-0.4000}{0.2000} \times 10^{1-1} = -2.0 \Rightarrow -0.2000 \times 10^{-1}$$

$$x_1^* = fl\left(\frac{b + \sqrt{b^2 - 4c}}{0.2000 \times 10^1}\right) = -0.2000 \times 10^{-1}$$

• b)

Erro relativo entre x_1 e x_1^* :

$$E_R(x) = \left| \frac{-0.016107235 - (-0.2000 \times 10^{-1})}{-0.016107235} \right| = 0.241678 \approx 24.17\%$$

• c)

Este erro consideravelmente alto se dá ao fato da perda de duas casas decimais em $fl(\sqrt{b^2 - 4c})$. e por causa da alta proximidade entre b e $\sqrt{b^2 - 4c}$.

• d)

Se fizéssemos uma simplificação do radical na parte superior da fórmula, poderíamos garantir que a raiz quadrada tivesse menos influência. Então, a partir da manipulação algébrica, teríamos:

$$fl(-b + \sqrt{b^2 - 4c}) = fl(\sqrt{b^2 - 4c} - b) = fl([\sqrt{b^2 - 4c} - b] \times \frac{\sqrt{b^2 - 4c} + b}{\sqrt{b^2 - 4c} + b}), \text{ que por sua vez é:}$$

$$fl\left(\frac{-4c}{\sqrt{b^2 - 4c} + b}\right), \text{ e depois aplicar a divisão por 2 da fórmula de bhaskara.}$$

• e)

$$c = 0.1000 \times 10^1$$

$$fl(-4c) = -(0.4000 \times 0.1000) \times 10^1 = -0.0400 \times 10^2 = -0.4000 \times 10^1$$

$$b = 62.10 = 0.6210 \times 10^2$$

$$fl(b^2) = (0.6210 \times 10^2)^2 = 0.385641 \times 10^4 = 0.3856 \times 10^4$$

$$fl(b^2 - 4c) = 0.3856 \times 10^4 - (0.4000 \times 10^1) = (0.3856 + 0.0004) \times 10^4 = 0.3852 \times 10^4$$

$$fl(\sqrt{b^2 - 4c}) = \sqrt{0.3852 \times 10^4} = 0.620645 \times 10^2 = 0.6206 \times 10^2$$

$$fl(\sqrt{b^2 - 4c} + b) = 0.6206 \times 10^2 + 0.6210 \times 10^2 = (0.6206 + 0.6210) \times 10^2 = 1.2416 \times 10^2 = 0.1242 \times 10^3$$

$$fl\left(\frac{-4c}{\sqrt{b^2 - 4c} + b}\right) = \frac{-0.4000}{0.1242} \times 10^{1-3} = -3.22061 \times 10^{-2} = -0.3221 \times 10^{-1}$$

tomando $z = fl\left(\frac{-4c}{\sqrt{b^2 - 4c} + b}\right)$, agora calculamos

$$fl\left(\frac{z}{2}\right) = \frac{-0.3221 \times 10^{-1}}{0.200 \times 10^1} = -1.6105 \times 10^{-1-1} = -0.1611 \times 10^{-1}$$

• f)

O novo erro relativo é:

$$E_R(x) = \left| \frac{-0.016107235 - (-0.1611 \cdot 10^{-1})}{-0.016107235} \right| = 0.000171662 \approx 0.017\%$$