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1.

a). Utilizando a técnica de diferenças finitas avançadas, temos que

- $Q'(t) = \frac{Q(t+h)-Q(t)}{h}$ (I)

Enquanto no atrasado temos:

- $Q'(t) = \frac{Q(t)-Q(t-h)}{h}$ (II)

tomado $h=2$, nosso menor intervalo conhecido, podemos descobrir

(I): $Q'(0) = \frac{Q(0+2)-Q(0)}{2} = \frac{21,3000-20,0000}{2,0000} = 0,6500$

(II): $Q''(10) = \frac{Q(10)-Q(10-2)}{2} = \frac{26,3000-25,1000}{2,0000} = 0,6000$

b) $Q'(t) = \frac{Q(t+h)-Q(t-h)}{2h}$, tomando $h=2$

- para $t=2 \Rightarrow Q'(2) = \frac{Q(2+2)-Q(2-2)}{2.2} = \frac{22,6-20}{4} = \frac{2,6}{4} = 0,65$
- para $t=4 \Rightarrow Q'(4) = \frac{Q(4+2)-Q(4-2)}{2.2} = \frac{23,9-21,3}{4} = \frac{2,6}{4} = 0,65$
- para $t=6 \Rightarrow Q'(6) = \frac{Q(6+2)-Q(6-2)}{2.2} = \frac{25,1-22,6}{4} = \frac{2,5}{4} = 0,625$
- para $t=8 \Rightarrow Q'(8) = \frac{Q(8+2)-Q(8-2)}{2.2} = \frac{26,3-23,9}{4} = \frac{2,4}{4} = 0,6$

c) $t=6$

$$Q''(t) = \frac{Q(t+h)-2Q(t)+Q(t-h)}{h^2} = \frac{Q(6+2)-2Q(6)+Q(6-2)}{2^2} = \frac{25.1-47.8+22.6}{4} = -0.025$$

$$D_2^C[f(x_0)] = \frac{f(x_0-h)-2f(x_0)+f(x_0+h)}{h^2}.$$

2) $\tilde{f}(x) = f(x)+\epsilon(x)$

a) $$D_2^C[\tilde{f}(x)] = D_2^C[f(x)+\epsilon(x)] = \frac{[f(x-h)+\epsilon(x-h)] - 2[f(x)+\epsilon(x)] + [f(x+h)+\epsilon(x+h)]}{h^2}$$

$$= \frac{f(x-h)-2f(x)+f(x+h)}{h^2} + \frac{\epsilon(x-h)-2\epsilon(x)+\epsilon(x+h)}{h^2}$$

Pelo Teor. Taylor temos que

$$\frac{f(x+h)+f(x-h)-2f(x)}{h^2} = f''(x)+\frac{h^2}{12}f^{(4)}(c),$$

Portanto:

$$D_2^C[f(x)+\epsilon(x)] = f''(x)+\frac{h^2}{12}f^{(4)}(c)+\frac{\epsilon(x-h)-2\epsilon(x)+\epsilon(x+h)}{h^2}$$

Como $|\epsilon(x)| \le \epsilon_s \ \forall x \in f$, pela desigualdade triangular temos:

$$\left|\frac{\epsilon(x+h)-2\epsilon(x)+\epsilon(x+h)}{h^2}\right| \le \left|\frac{\epsilon(x+h)}{h^2}\right|+\left|\frac{2\epsilon(x)}{h^2}\right|+\left|\frac{\epsilon(x-h)}{h^2}\right| \le \frac{\epsilon_s}{h^2}+\frac{2\epsilon_s}{h^2}+\frac{\epsilon_s}{h^2} = \frac{4\epsilon_s}{h^2}$$

$$f''(x)-D_2^C[f(x)+\epsilon(x)] = -\frac{h^2}{12}f^{(4)}(c)-\left[\frac{\epsilon(x-h)-2\epsilon(x)+\epsilon(x+h)}{h^2}\right]$$

$$E(h) = |f''(x)-D_2^C| = \left|-\frac{h^2}{12}f^{(4)}(c)-\left[\frac{\epsilon(x-h)-2\epsilon(x)+\epsilon(x+h)}{h^2}\right]\right| \le \frac{h^2}{12}M_4+\frac{4\epsilon_s}{h^2} = L(h)$$

$L'(h) = \frac{h}{6}M_4-\frac{8\epsilon_s}{h^3}$, $L'(h)=0 \Rightarrow \frac{h}{6}M_4-\frac{8\epsilon_s}{h^3}=0$

$\frac{h}{6}M_4 = \frac{8\epsilon_s}{h^3} \Rightarrow \frac{h^4}{6}M_4 = 8\epsilon_s \Rightarrow h^4 = \frac{48\epsilon_s}{M_4} \Rightarrow h^* = \sqrt[4]{\frac{48\epsilon_s}{M_4}} \Rightarrow h^* = 2\sqrt[4]{\frac{3\epsilon_s}{M_4}}$

b) para $f(x) = x^3\ln(x)$, temos $f'(x) = 3x^2.\ln(x)+x^2$, $f''(x) = 6x\ln(x)+5x$, $f'''(x) = 6\ln(x)+11$, $f^{(4)}(x) = 6/x$

tomando $g(x) = |f^{(4)}(x)| = f^{(4)}(x)$ no intervalo $[1,2]$. $g'(x) = \frac{-6}{x^2}$, ou seja, $g(x)$ é positiva e decrescente em $[1,2]$

$M_4 = \max_{x\in[1,2]}|f^{(4)}(x)| = g(1) = \frac{6}{1} = 6$

$h^* = 2\sqrt[4]{\frac{3\epsilon_s}{6}} \Rightarrow h^* = 2\sqrt[4]{\frac{3.10^{-16}}{6}} = 1,6818.10^{-4}$