Avaliação Matemática Discreta

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1- 2011100038 | 4

$$
\begin{array}{l|l}
2011100038 & 4 \\
\underline{-200} & 502.775.009 \\
00111 & \\
\quad \underline{-108} & \\
\quad 003000 & \\
\qquad \underline{-3000} & \\
\qquad 000038 & \\
\qquad\quad \underline{-36} & \\
\qquad\quad 02 &
\end{array}
$$

$\boxed{R = 2}$ $\boxed{n = 2011100038}$

2- Encontre o 83º termo da sequência dadas por

$$\begin{cases} a_0 = -1, \quad a_1 = 1, \quad a_2 = 0 \\ a_n = -k \cdot a_{n-1} + k^2 \cdot a_{n-2} + k^3 \cdot a_{n-3} + (n^2 + 2n + 1) \cdot k^n \end{cases}$$

onde $k = \begin{cases} -2 & \text{se } R = 0 \\ -1 & \text{se } R = 1 \\ 1 & \text{se } R = 2 \\ 2 & \text{se } R = 3 \end{cases} \longrightarrow R = 2 \rightarrow \boxed{1k}$

$a_n = -1 a_{n-1} + 1^2 \cdot a_{n-2} + 1^3 \cdot a_{n-3} + (n^2 + 2n + 1) \cdot 1^n$

$a_n = -1 a_{n-1} + 1 \cdot a_{n-2} + 1 \cdot a_{n-3} + (n^2 + 2n + 1) \cdot 1^n$

Solução geral da relação a_n Grau = 3

$r^3 - (-k \cdot r^2) + (k^2 \cdot r) + (k^3) \longrightarrow r^3 = r^2 + r - 1$

$r^3 - r^2 - r + 1$

Raiz 2 $= r:1 = 1 - 1 - 1 + 1 = 0$ $a_n = \alpha(1)^n + B(-1)^n$

$r = -1: -1^3 - (-1)^2 + 1 = 0$

$a_0 = 1 = \alpha(1)^1 + B(-1)^1 = \alpha - B = 1$

$a_1 = -1 = \alpha(1)^0 + B(-1)^0 = \alpha + B = -1$

$\alpha - B = 1$

$\alpha + B = -1$

$B = -1$

$\alpha - 1 = -1 \longrightarrow \alpha = 0$

$a_{82} = 0 \cdot 1^{82} + (-1)(-1)^{82} = 23°$ termo $= -1$

3- Mostre por indução que, $\forall n \in \mathbb{N}$

$$1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{n}} > 2.\sqrt{n+1-2} \qquad n=3$$

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}} > 2.\sqrt{3+1}-2$$

$$1+\frac{1}{1{,}41}+\frac{1}{1{,}73} > 2.\sqrt{4}-2$$

$$1+0{,}71+0{,}58 > 2.2-2 \rightarrow 2{,}29 > 4-2 \rightarrow 2{,}29 > 2$$

sabe-se então que

$$I = 1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{K}} > 2.\sqrt{K+1-2} \qquad \frac{1}{\sqrt{K}} > 2.\sqrt{K+1}-2$$

Para $K+1$

$$1+\frac{1}{\sqrt{2}}+\dots+\frac{1}{\sqrt{K+1}} > 2.\sqrt{K+2}-2 \qquad \frac{K^{\frac{1}{2}}.(K+1)^{\frac{1}{2}}}{1\ (K+1)} \Big| \frac{K^{\frac{1}{2}}}{(K+1)}$$

$$\frac{1}{\sqrt{K}}$$

$$\frac{1}{\sqrt{K}}+\frac{1}{\sqrt{K+1}} > 2.\sqrt{K+2}-2 = \frac{\sqrt{K}.\sqrt{K+1}+K+1}{\sqrt{K}.(K+1)}$$

$$\frac{K^{\frac{1}{2}}.(K+1)^{\frac{1}{2}}+K+1}{K^{\frac{1}{2}}.(K+1)} > 2(K+2)^{\frac{1}{2}}-2$$

5 - Mostre que se $\begin{cases} F_1 = 1, \ F_2 = 2 \\ F_n = F_{n-1} + F_{n-2}, \ \forall n \geq 3 \end{cases}$

Então:

$$F_n < \left(\frac{7}{4}\right)^n \quad \forall n \geq 1$$

Base: $P(n) = F_n < \left(\frac{7}{4}\right)^n \quad \forall n \quad n = 1$

$F_1 = 1 < \frac{7}{4}^{= 1,75}$ $\quad P(1) \to$ Verdadeiro

Seja $P(1), P(2), \ldots, P(n), \forall n \geq 2$ verdadeiras hipóteses de indução

$$F_{n+1} < \left(\frac{7}{4}\right)^{n+1} \longrightarrow F_{n+1} = F_n + F_{n-1} < \left(\frac{7}{4}\right)^{1} + \left(\frac{7}{4}\right)^{n-1}$$

$$\frac{7}{4}\left(\frac{7}{4}\right)^{n-1} + \left(\frac{7}{4}\right)^{n-1}$$

$$\underbrace{\left(1 + \frac{7}{4}\right)}\left(\frac{1}{4}\right)^{n-1} \qquad 1 + \frac{7}{4} = 2,75$$

é necessário

$$F_{n+1} < \left(\frac{7}{4}\right)^{2}\left(\frac{7}{4}\right)^{n-1} \qquad \underbrace{\left(\frac{7}{4}\right)^{2}}_{3,06} \text{ seja } > 1 + \frac{7}{4}$$

$$F_{n+1} < \left(\frac{7}{4}\right)^{n+1} \quad \forall n \geq 1$$