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Igor Raddtke - Geometria - Lista 04

1- a) $2\vec{u}.(-\vec{v}) = (4,-6,-2).(-1,1,-4) =$
$(-4)+(-6)+8 = -2$

b) $(\vec{u}+\vec{v}).(\vec{u}-\vec{v})$ $(3,-4,3).(1,-2,-5) =$
$3+8+(-15) = -4$

2- Determine o vetor $\vec{v}$, paralelo ao vetor $\vec{v} = (2,-1,3)$ tal que $\vec{u}.\vec{v} = -42$

$\vec{u} = K\vec{v} = K(2,-1,3) = (2K,-K,3K)$

$\vec{u}.\vec{v} = -42$

$(2K,-K,3K).(2,-1,3) = -42$

V é multiplo de u $\quad 4K + K + 9K = -42$

$V = x.u \quad 14K = -42$

$K = \frac{-42}{14} = -3$

$V = (2x,-x,3x)$

$\boxed{V = (-6,3,-9)}$

3- a) $(\vec{u}-3\vec{v}).\vec{u} = (u-3v).u = u^2 - 3v.u$
$u^2 - 3v.u = (2)^2 - 3(-1) = 7$

3- b) $(2\vec{v}-\vec{u}).(2\vec{v})$
$(2\vec{v}).(2\vec{v}) - \vec{u}.2\vec{v}$
$(2.2)\vec{v}.\vec{v} - 2\vec{u}.\vec{v}$
$4\|\vec{v}\|^2 - 2.(-1)$
$4.3^2 - 2.(-1) = 4.9+2 = 36+2 = 38$

3- c) $(\vec{u}+\vec{v}).(\vec{v}-4\vec{u})$

$-1.4.2.2+3.3-4(-1)$

$-1-16+9+4$

$-17+13=-4$

4- Qual o valor de a para que os vetores $\vec{V_1}=\vec{a}i+\vec{2}j-\vec{4}K$ e $\vec{V_2}=\vec{2}i+(1-2a)\vec{j}+\vec{3}K$ sejam ortogonais?

$$\begin{cases} a = ai+2j-4K \\ b=2i+(1-2a)+3K \end{cases}$$

$(a+2-4).(2+(1-2a)+3)=0$

$2.a+2(1-2.a)-4.3=0$

$2a+2-4a-12=0$

$2a-4a+2-12=0$

$-2a-10=0$

$a=\frac{10}{-2}=-5$

5- Dados os pontos A(m,1,0), B(m-1,2m,2) e C(1,3,-1), determinar m de modo que o triângulo seja retângulo em A. Calcule a área do triângulo.

$\vec{AB}$ e $\vec{AC}$

$AB=(xb-xa, yb-ya, zb-za)=((m-1)-m, 2m-1, 2-0)=$

$(-1, 2m-1, 2)$

$AC=(xc-xa, yc-ya, zc-za)=(1-m, 3-1, (-1)-0)=$

$(1-m, 2, -1)$

$A.B.AC=0$ $\quad (-1, 2m-1, 2).(1-m, 2, -1)=0$

$(-1).(1-m)+(2m-1).(2)+2.(-1)=0$

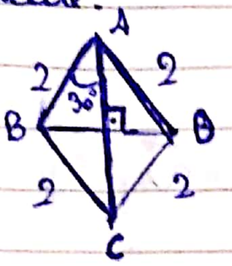
$-1+m+4m-2-2=0$

$5m-5=0$

$5m=5$

$m=\frac{5}{5}=1$

6- O quadrilátero ABCD é um losângulo de lado 2. Calcule.



a) $\vec{AC} \cdot \vec{BD} = |\vec{AC}| \cdot |\vec{BD}| \cdot \cos \theta$

$|\vec{AC}| \cdot |\vec{BD}| \cos 90° = 0$

b) $\vec{AB} \cdot \vec{AD} = |\vec{AB}| \cdot |\vec{AD}| \cos \theta = 2 \cdot 2 \cdot \cos 60°$

$4 \cdot \frac{1}{2} = 2$

c) $|\vec{BA}| \cdot \vec{BC} = |\vec{BA}| \cdot |\vec{BC}| \cdot \cos -60°$

$2 \cdot 2 \cdot (1/2) = -2$

d) $\vec{AB} \cdot \vec{BC} = |\vec{AB}| \cdot |\vec{BC}| \cdot \cos \theta = 2 \cdot 2 \cdot \cos 60°$

$4 \cdot 1/2 = 2$

7- $\vec{u} \perp \vec{v}$ $(\theta = 90°$ $\vec{u} \cdot \vec{v} = 0$

$|\vec{u}| = 6$ $\quad |\vec{v}| = 8$ $\quad |\vec{u} + \vec{v}|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) =$

$|\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |v|^2$

$6 + 2 \cdot 0 + 8^2 = 36 + 64 = 100$

$|\vec{u} \cdot \vec{v}| = \sqrt{100} = 10$

8- a) $\vec{u} = (2, -1, -1)$ e $\vec{v} = (-1, -1, 2)$ $\quad \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = -2, 1, -2$

$|\vec{u}| = \sqrt{4+1+1} = \sqrt{6}$

$|\vec{v}| = \sqrt{1+1+4} = \sqrt{6}$

$\cos \theta = \frac{-3}{\sqrt{6} \cdot \sqrt{6}} = \frac{-3^{\times 3}}{6} = \frac{-1}{2} = 120°$

b) $\vec{u} = (1, -2, 1)$ e $\vec{v}(-1, 1, 0)$ $\cos \theta$ $\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = 1 - 2 + 0 = -1$

$|\vec{u}| = \sqrt{1+4+1} = \sqrt{6}$ $\quad \sqrt{6} \cdot \sqrt{2}$ $\qquad \frac{-1}{\sqrt{6} \cdot \sqrt{2}} = 150°$

$|\vec{v}| = \sqrt{1+1+0} = \sqrt{2}$

9- $A(3,4,4)$, $B(2,-3,4)$ e $C(6,0,4)$.

$AB = (-1,7,0) \rightarrow |\vec{AB}| = \sqrt{1+49} = \sqrt{50}$

$AC = (3,-4,0) \rightarrow |\vec{AC}| = \sqrt{9+16} = \sqrt{25} = 5$ $\quad \cos K = \dfrac{\text{Prod. Esc}}{\text{Prod dos Mód}}$

$BC = (4,3,0) \rightarrow |\vec{BC}| = \sqrt{16+9} = \sqrt{25} = 5$

$AB \cdot AC = (-1,-7,0) \cdot (3,-4,0) = 25$

$AC \cdot BC = (4,3,0) \cdot (3,-4,0 = 0$

Prod. dos modulos:

$|\vec{AB}| \cdot |\vec{AC}| = ((50)^{1/2}) \cdot 5 = 45°$

$|\vec{BC}| \cdot |\vec{AC}| = 25 = \dfrac{0}{25} = 90°$

O ângulo B é igual a 45° internos

10- $|\vec{u}+\vec{v}|^2 = (\vec{u}+\vec{v}) \cdot (\vec{u}+\vec{v}) = |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 =$

$4^2 + 2|\vec{u}| \cdot |\vec{v}| \cos 60° + 3^2 = 16 + 2 \cdot 4 \cdot 3 \cdot \frac{1}{2} + 9 =$

$16 + 12 + 9 = 37$

$|\vec{u}+\vec{v}| = \sqrt{37}$:

$(\vec{u}+\vec{v}) \cdot (\vec{u}-\vec{v}) =$

$\vec{u} \cdot \vec{u} - \cancel{\vec{u} \cdot \vec{v}} + \cancel{\vec{v} \cdot \vec{u}} - \vec{v} \cdot \vec{v}$

$|\vec{u}|^2 - |\vec{v}|^2 = 16 - 9 = 7$

11- $\vec{u} = ?$ $|\vec{u}| = 2$, θ entre $\vec{u}$ e $\vec{v} = 45°$ $\quad \vec{v} = (1,-1,0)$ $\quad \vec{u} = (1,-1,z)$

$(x,y,z) \cdot (1,1,0) = 0$

$x + y + 0 = 0$

$x = -y$ $\qquad \vec{u} = (-y, y, z)$ $\qquad |\vec{u}| = 2$

$\sqrt{1+1+z^2} = 2^2$

$1+1+z^2 = 4$

$\cos 45° = \dfrac{(-y, y, z) \cdot (1,-1,0)}{2 \cdot \sqrt{1^2+1+0^2}} =$

$\dfrac{\sqrt{2}}{2} = \dfrac{-y-y+0}{2\sqrt{2}}$

$\dfrac{\sqrt{2}}{2} = \dfrac{2y}{2\sqrt{2}}$

$\sqrt{2} \cdot \sqrt{2} = -2y$ $\qquad 2 = -2y$ $\qquad y = -1$