2º Lista - Geometria Analítica - Semester 2020.2

Ciência da Computação

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1- Dados os vetores $\vec{u} = 2\vec{i} - 3\vec{j}$, $\vec{v} = \vec{i} - \vec{j}$ e $\vec{w} = -2\vec{i} + \vec{j}$, determinar:

a) $2\vec{u} - \vec{v} = 2(2,-3) - (1,-1) = (4,-6) - (1,-1) = (3,-5)$

b) $\vec{v} - \vec{u} + 2\vec{w} = (1,-1) - (2,-3) + 2(-2,1) =$
$(-1,2) + (-4,2) = (-5,4)$

c) $\frac{1}{2}\vec{u} - 2\vec{v} - \vec{w} = \frac{1}{2}(2,-3) - 2(1,-1) - (-2,1) =$

$\left(1, \frac{-3}{2}\right) - (2,-2) - (-2,1) = \left(1, -\frac{1}{2}\right)$

2-a) $4(\vec{u} - \vec{v}) + \frac{1}{3}\vec{x} = 2\vec{u} - \vec{x}$

$4.[(3,-1) - (-1,2)] + \frac{1}{3}\vec{x} = 2(3,-1) - \vec{x}$

$4(4,-3) + x \cdot \frac{1}{3} = (6,-2) - x$

$(4.4, 4(-3)) + x \cdot \frac{1}{3} = (6,-2) - x$

$(16-12) + x\frac{1}{3} = (6,-2) - x$

$x \cdot \frac{1}{3} + x = (6,-2) - (16,12)$

$\frac{4x}{3} = (-10,10)$

$x = \frac{3}{4}(-10,10)$

$x = \left(\frac{-15}{2}, \frac{15}{2}\right)$

2-b) $3\vec{x} - (2\vec{v} - u) = 2(4\vec{x} - 3\vec{u})$

$3x - (2(-1,2) - (3,-1)) = 2(4x - 3(3,-1))$

$3x - (-5,5) = 8x - 2(9,-3)$

$3x - (-5,5) = 8x - (18,-6)$

$8x - 3x = -(-5,5) + (18,-6)$

$5x = (23,-11)$ $\quad x = \left(\frac{23}{5}, \frac{-11}{5}\right)$

3- Dados os vetores $\vec{u} = (2,-4)$, $\vec{v} = (-5,1)$ e $\vec{w} = (-12,6)$, determinar a_1 e a_2 tais que $\vec{w} = a_1\vec{u} + a_2\vec{v}$.

$(-12,6) = a_1(2,-4) + a_2(-5,1)$

$(-12,6) = (2a' - 5a'', -4a' + a'')$

① $-12 = 2a' - 5a''$

② $6 = -4a' + a'' \rightarrow a'' = 6 + 4a'$

$a'' = 6 + 4a'$

$a'' = 6 + 4.(-1)$

$a'' = 2$

$-12 = 2a' - 5(6 + 4a')$

$-12 = 2a' - 30 - 20a'$

$-12 = -18a' \quad a' = \frac{-12}{12} \quad a' = -1$

4a) $B = A + 2\vec{v}$

$(1,3) = (-5,1) + 2.(a,b)$

$(1,3) = (-5,1) + (2a, 2b)$

$(1,3) = (-5+2a, 1+2b)$

$1 = -5 + 2a \Rightarrow a = \frac{(1+5)}{2} = 3$

$3 = 1 + 2b \Rightarrow b = \frac{(3-1)}{2} = \frac{2}{2} = 1$

$V = (3,1)$

b) $A = B + 3\vec{u}$

$(-5,1) = (1,3) + 3(a,b)$

$(-5,1) = 1,3 + (3a, 3b)$

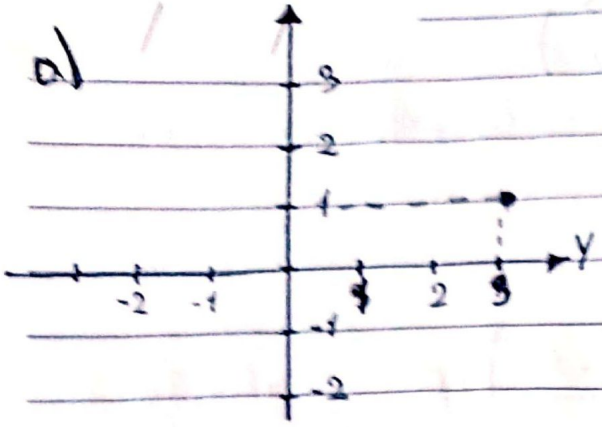
$(-5,1) = (1+3a, 3+3b)$

$-5 = 1 + 3a \quad a = (-5-2) = -2$

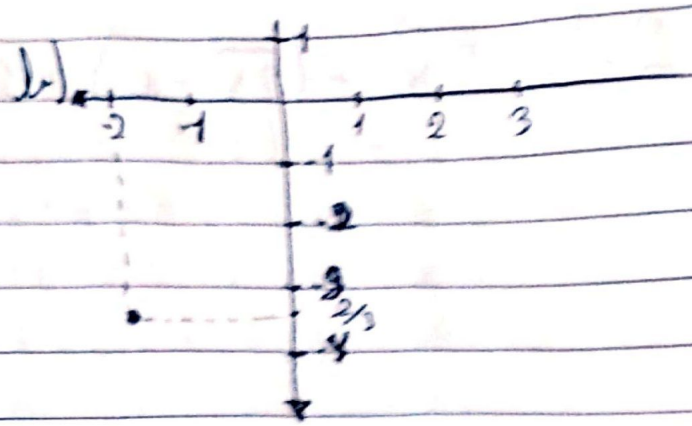
$1 = 3 + 3b \quad b = \frac{1-3}{3} = \frac{-2}{3}$

$\vec{V} = (-2, \frac{-2}{V})$

a)



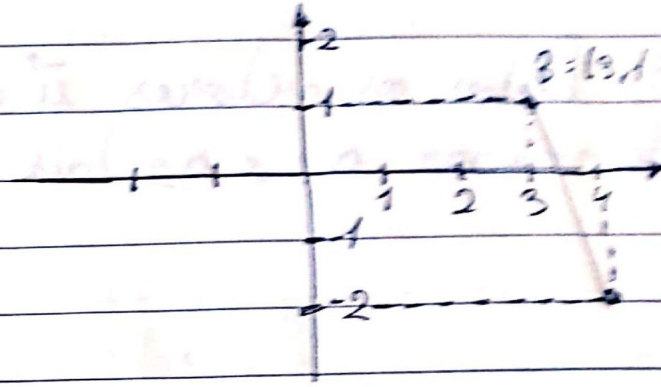
b)



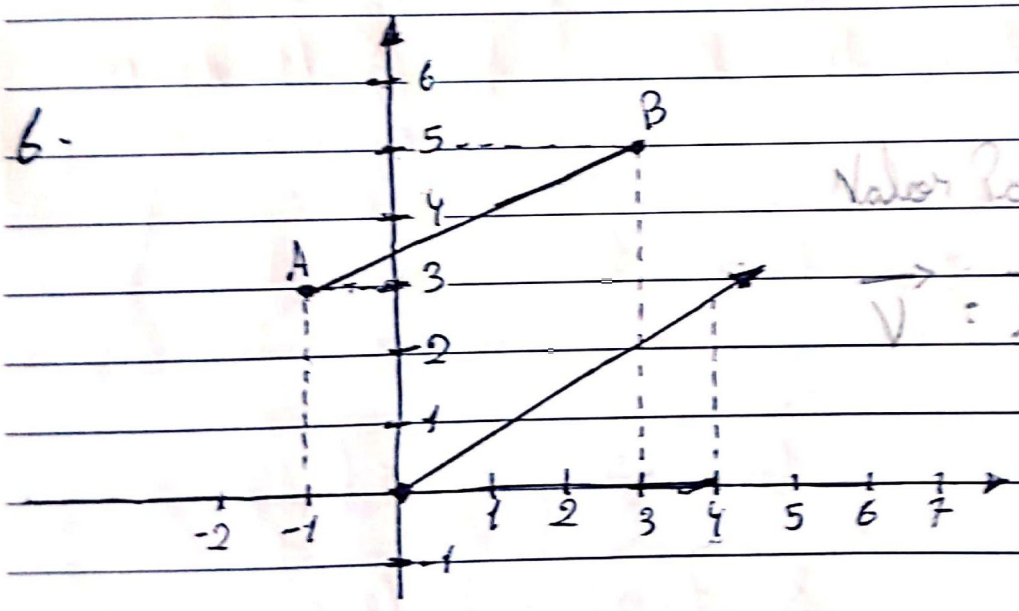
5- (3,1) - (-1,3) = (2 -0)

(4,-2) = (2-0)

(4,-2) ponto inicial



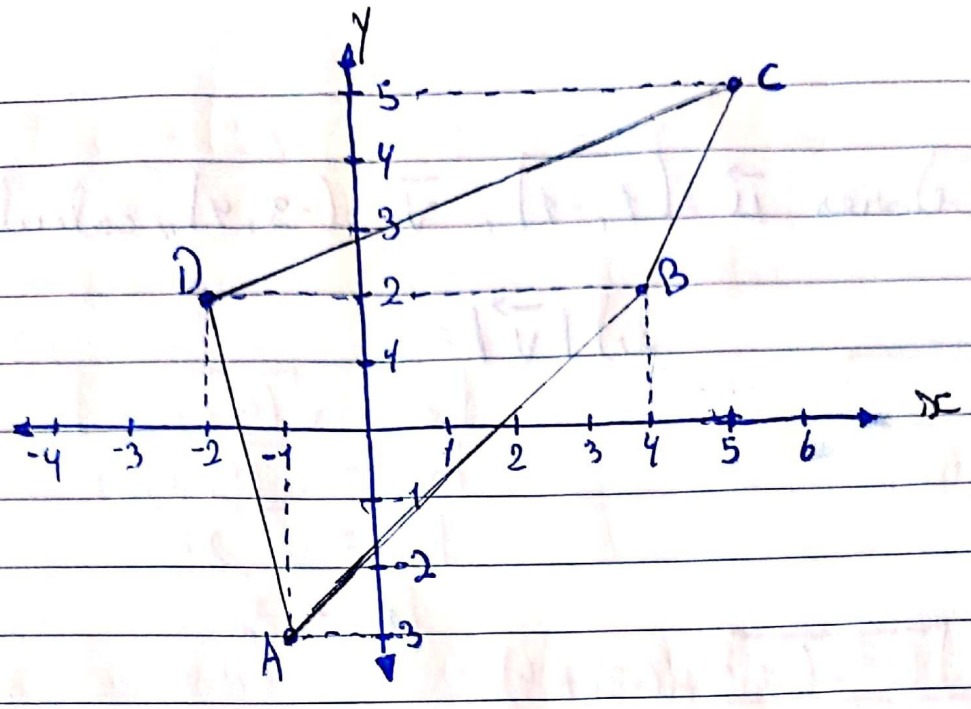
6-



Valor Posição

$\vec{v} = \overrightarrow{AB} = (4\ 2)$

7.



a) $AB = B - A = (5-2) - (-3,2) \rightarrow AB = (8,-4)$

$Am = (8,-4) = (4,-2)$

$m = (x,y)$

$m - A = (x,y) - (-3,2) \rightarrow Am = (x+3, y-2)$

$Am = \frac{1}{2} AB \rightarrow (x+3, y-0) = (4,-2)$

$x+3 = 4 \rightarrow x = 1$

$y - 2 = -2 \rightarrow y = 0$

$m = (1,0)$

Logo: $AN = (x+3, y-2)$

$AN = \frac{2}{3}(AB) = \frac{2}{3}(8,-4) =$

$AN = \left(\frac{16}{8}, \frac{-8}{3}\right)$

$AN = \frac{2}{3} AB$

$(x+3, y-2) = \left(\frac{16}{3}, \frac{-8}{3}\right) \rightarrow x+3 = \frac{16}{3} \rightarrow x = \frac{7}{3}$

$y - 2 = -\frac{8}{3} \rightarrow y = \frac{-2}{3}$

$N = \left(\frac{7}{3}, \frac{-2}{3}\right)$

9- Dados os vetores $\vec{u} = (1, -1)$, $\vec{v} = (-3, 4)$, calcule:

a) $|\vec{u}| = (1, -1)$

$|\vec{u}| = \sqrt{1^2 + (-1)^2}$

$|\vec{u}| = \sqrt{1 + 1}$

$|\vec{u}| = \sqrt{2}$

b) $|\vec{v}| = \sqrt{|-3|^2 + 4^2}$

$|v| = \sqrt{(9) + 16}$

$|v| = \sqrt{(9 + 16)}$

$|v| = \sqrt{25}$

$|v| = 5$

c) $|\vec{u} + \vec{v}| = \sqrt{(1^2 + (-1)^2 + (-3) + 4)^2}$

$\sqrt{((1 + 1) + (3 + 4))}$

$\sqrt{(2) + 25}$

$\sqrt{27}$

10- $||u||^2 = x^2 + y^2$

$4^2 = a^2 + (-2)^2$

$16 = a^2 + 4$

$a^2 = 16 - 4$

$a^2 = 12$

$a = \pm\sqrt{12}$ $\quad a = \pm 2\sqrt{3}$

11- a) $-\vec{v} + 2\vec{v} = -2\vec{v}$

$-2\vec{v} \cdot (1, -3) = (-2, 6$

b) $\frac{2 \cdot (1 \cdot 3)}{\sqrt{((1^2 + (-3)^2)}} = \frac{2 \cdot (1 - 3)}{\sqrt{10}} = \frac{2, 6}{\sqrt{10}} = \left(\frac{2}{\sqrt{10}} + \frac{6}{\sqrt{10}}\right)$

11 c) $\frac{-4(1,-3)}{\sqrt{(1^2+(-3)^2)}} = \frac{-4(1,-3)}{\sqrt{10}} \cdot \frac{4,12}{\sqrt{10}} \cdot \left(\frac{4}{\sqrt{10}}, \frac{12}{10}\right)$ /