

Lista 7 - Geometria Analítica

1ºº Rodilla

1- $A(2, -3, 4)$ $B(1, -1, 2)$

$AB = (-1, 2, -2)$

$(x, y, z) = (2, -3, 4) + k \cdot (-1, 2, -2)$
 $k \in \mathbb{R}$

$C = \frac{3}{2} = 2 + \frac{(-k+1)}{5} = -4 =$

C e d não pertence à reta.

2- $A = (1, 2, 3)$

$S = (1, 2, 3) + k(0, 0, 1)$

$r = (1, 4, 3) + k(0, 0, 1)$

$\begin{cases} x = 1 \\ y = 2 \\ z = 3 + k \end{cases}$

3- $r = \begin{cases} x = 2 + k \\ y = 3 - k \\ z = -4 + 2k \end{cases}$ a) $y = 3 - k = 0 \Rightarrow k = 3$
 $k = \boxed{-3}$

ponha $r = (-1, 6, -10)$

$x = 2 + k$

$z = -4 + 2k$

$x = 2 + (-3)$

$z = -4 + 2(-3)$

$x = \boxed{-1}$

$z = -10$

b) $x = y$

$2 + k = 3 - k$

$2k = 1 \Rightarrow k = \frac{1}{2}$

$x = 2 + \frac{1}{2} = \frac{5}{2}$

$z = -4 + 2 \cdot \left(\frac{1}{2}\right) = \left(\frac{5}{2}, \frac{5}{2}, -3\right)$

$y = \frac{5}{2}$

$-4 + 1 =$
 3

$$\begin{aligned}
 c) \quad z &= 4x & y &= 3x & x &= 2+x \\
 -4+2x &= 4(2+x) & y &= 3+6 & x &= 2-6 = -4 \\
 -4+2x &= 8+4x & y &= 9 \\
 -12 &= 2x \\
 x &= \frac{-12}{2} & z &= -4+2x & r &= (-4, 9, -16) \\
 x &= 6 & z &= -4+2(-6) \\
 & & z &= -4-12 = -16
 \end{aligned}$$

$$\begin{aligned}
 4- \quad A &= (4, 3, 2) \quad P(m, m, 5) \quad \vec{3i} - 4j - k \\
 r &= \begin{cases} x = 1+3x \\ y = 2-4x \\ z = 3-x \end{cases} & \begin{cases} m = 4+3 \\ m = 4+3 \cdot 3 \\ m = 13 \end{cases} & r = \begin{cases} x = 4+3x \\ y = -3-4x \\ z = -2-x \end{cases} \\
 & & & \begin{cases} -5 = -2-x \\ x = 3 \end{cases} \\
 m &= -3-4x \\
 m &= -3-4 \cdot 3 \\
 m &= -3-12 \\
 m &= -15
 \end{aligned}$$

$$5- \quad A(-1, 4, -2) \quad B(3, -3, 6) \quad C(2, -1, 4)$$

ponto médio de AB

$$\frac{A+B}{2} = \frac{-1+3}{2}; \frac{4+(-3)}{2}; \frac{-2+6}{2} \quad (-1, -\frac{3}{2}, 2)$$

$$\frac{A+B}{2} = (4; \frac{1}{2}; 2) =$$

$$(2, 1); (-1, \frac{1}{2}); (4, 2)$$

$$x = (2, -1, 4) + \frac{1}{2}(1, -3, 2)$$

$$x = 2 + \frac{1}{2}$$

$$y = -1 - \frac{3}{2}$$

$$z = 4 + 2$$

7. $A(5, -5, 6)$ e $B(4, -1, 12)$

A) $x = 5$ $y = -5$ $z = 6$
 $\frac{5-3}{-1} = k$ $\frac{-5+1}{2} = k$ $\frac{6-2}{-2} = k$
 $k = -2$ $k = -2$ $k = -2$

B) $z = 12$ $x = 4$ $y = -1$
 $\frac{12-2}{-2} = k$ $\frac{4-3}{2} = k$ $\frac{-1+1}{2} = k$
 $k = -5$ $k = \frac{1}{2}$ $k = 0$

8. $x = 1 + 2k$ $y = -3 \cdot k$ $z = 4k$

a) Alucina

$x = 5$ $y = -3 \cdot k$ $z = 4k$ $P = (5, -5, 2)$
 $5 = 1 + 2k$ $y = -3 - 2$ $z = 2$
 $4 = 2k$ $y = -5$
 $k = 2$

b) Cordovada

$x = 1 + 2k$ $y = 2$
 $x = 1 - 10$ $z = -3 - k$
 $x = -9$ $k = -5$

$z = 4k$ $P = (-5, -9, -20)$
 $z = -20$

$$9. a) \quad n = \begin{cases} x = 2 - t & x = 2 - 1 \\ y = 3t & t = -x + 2 \\ z = 4t - 5 & \end{cases} \quad \begin{cases} y = 3(-x+2) \\ y = -3x + 6 \end{cases}$$

$$x = 2 - (-x + 2)$$

$$x = 2 + x + 2$$

$$2x = 4$$

$$x = \frac{4}{2}$$

$$x = 2$$

$$z = 4(-x + 2) - 5$$

$$z = -4 + 8 - 5$$

$$z = -4x + 3$$

$$b) \quad A(4, 0, -3) \quad \vec{v}(2, 5, 5)$$

$$\begin{cases} x = 4 + 2t \end{cases}$$

$$\begin{cases} y = 4t \end{cases}$$

$$\begin{cases} z = -3 + 5t \end{cases}$$

$$y = x - 4$$

$$y = 2x - 8$$

$$t = y - 4$$

$$x = 4 + \frac{2y}{2}$$

$$x = 4 + y$$

$$y = 2x - 8$$

$$z = -3 + 5 \left(\frac{y}{2} \right) = -3 + 5 \left(\frac{2x - 8}{2} \right)$$

$$z = 5x - 13$$

$$z = 5x - 13$$

$$10. \quad A(-1, 6, 3)$$

$$B(2, 2, 1)$$

$$AB = (3, -4, -2)$$

$$n = \begin{cases} x = 2 + 3t \\ y = 2 - 4t \\ z = \end{cases}$$

$$t = \frac{x-2}{3}$$

$$t = \frac{y-2}{-4}$$

$$t = \frac{z-1}{-2}$$

$$\frac{x-2}{3} = \frac{z-1}{-2}$$

$$\frac{x-2}{3} = \frac{y-2}{-4} = \frac{z-1}{-2}$$

$$x = \frac{7-3z}{2}$$

$$\frac{y-2}{-4} = \frac{z-1}{-2} \Rightarrow y-2 = 2z-2 \Rightarrow y = 2z$$

$$\text{Resposta} = z$$

$$\text{em } z$$

$$\begin{cases} x = \frac{7-3z}{2} \\ y = 2z \end{cases}$$

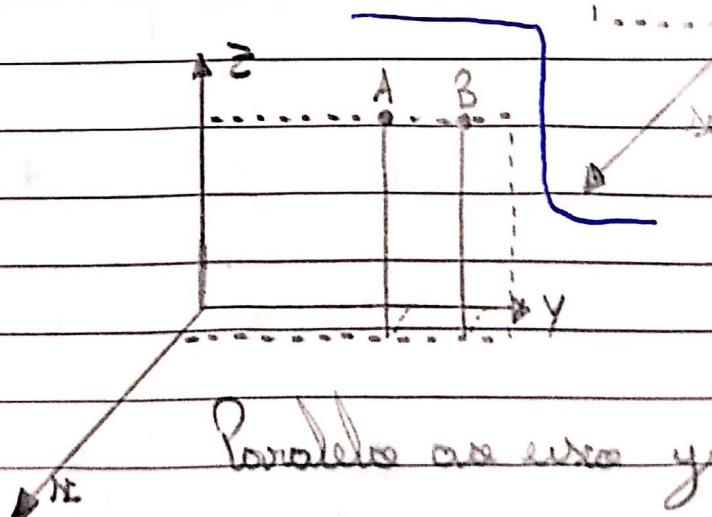
11. a) $y = 2x + 3$ $y = 9$ $x = 3$
 $z = y - 1$ $8 = 2x + 3$ $z = 3 - 1$ $P(3, 9, 2)$
 $2x = 6$ $z = 2$
 $x = 3$

b) $x = 27$ $z = x + 1$ $y = 2x + 3$
 $z = 2z - 1$ $1 = x - 2$ $y = 4 + 3$ $M(2, 7, 1)$
 $z = 1$ $x = 2$ $y = 7$

c) Ordenada igual ao triplo da reta $y = 3z$
 $y = 2x + 3$
 $z = x - 1$ $3z = 2x + 3$

13. a) $A(3, -2, 4)$ $\vec{v} = \vec{i} = (1, 0, 0)$
 $x = 3 + k$
 $y = -2 + 0k$ $y = -2$
 $z = 4 + 0k$ $z = 4$

b) $A(2, 2, 4)$
 $\vec{v} = \vec{j} = (0, 1, 0)$
 $x = 2$
 $z = 4$

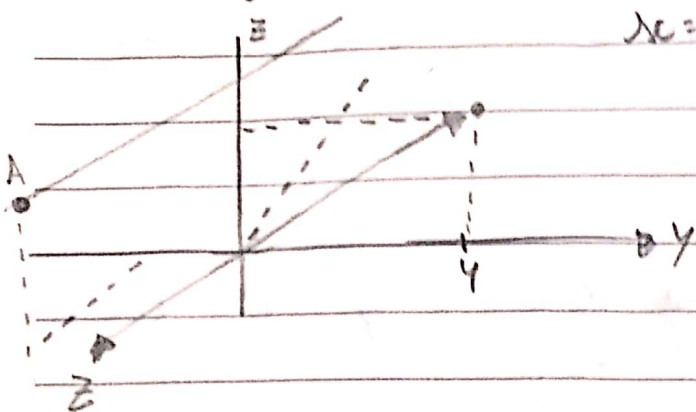


$$c) \vec{v} = (0, 0, 1) \quad (x, y, z) = (-2, 3, 4) \quad / \quad /$$

$$(-2, 3, 4)$$

$$+ t(0, 0, 1)$$

$$x = -2 \quad y = 3 \quad z = 4 + t$$



14. $A(4, -5, 3)$ Paralela $OZ(0, 0, 1)$ $(x, y, z) =$
 $x = 4$ $(4, -5, 3) + t(0, 0, 1)$
 $y = -5$ e igual mas o $\vec{v} = (1, 0, 0)$
 $z = 3 + t$

Paralela ao eixo $Ox =$

Paralela ao eixo Oy e igual a $\vec{v} = (0, 1, 0)$

16. $R_1 \begin{cases} x = 2m - 3 \\ y = 1 + 3k \\ z = -4k \end{cases} \quad R_2 \begin{cases} x = 2y - 1 \\ z = y + 4 \end{cases}$

$$(x, y, z) = P_0 + k(a, b, c) \quad k \in \mathbb{R}$$

$$r_1 = (x, y, z) = (-3, 1, 0) + k(2m, 3, -4)$$

$$r_2 = (x, y, z) = (-1, 0, 4) + k(2, 1, -1)$$

$$(2m, 3, -4) \cdot (2, 1, -1) = 0$$

$$4m + 3 + 4 = 0$$

$$4m = -7$$

$$m = -\frac{7}{4}$$

$$17 = 0 \quad (3, 2, -1)$$

$$s: \begin{cases} x = 3 \\ y = -1 \end{cases}$$

$$n: \begin{cases} y = x - 3 \\ z = -2x + 3 \end{cases}$$

$$(x, y, z) = (3, 2, -1) + (-1, 1, 0) \cdot t$$

Paramétrico

$$q: \begin{cases} x = 3 - t \\ y = 2 + t \\ z = -1 \end{cases}$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i} - \vec{j} = (1, -1, 0)$$

$$R_0: \begin{cases} x = t + 3 \\ y = 2 - t \\ z = -1 \end{cases}$$

$$18 = 0 \quad A(0, 0, 0), \quad r_1: \frac{x}{2} = y = \frac{z-3}{2} \quad \text{e} \quad r_2: \begin{cases} x = 3t \\ y = -t + 1 \\ z = 2 \end{cases}$$

$$r: (x, y, z) =$$

$$(2t, 6t, -5t)$$

$$18 = 0 \quad r_1: \begin{cases} y = 2x - 3 \\ z = -x + 5 \end{cases}$$

$$r_2: \begin{cases} y = 3x + 7 \\ z = x + 1 \end{cases}$$

$$R_1: \begin{cases} x = k \\ y = -3 + 2k \\ z = 5 - k \end{cases}$$

$$R_2: \begin{cases} x = k \\ y = 7 - 3k \\ z = 1 + k \end{cases}$$

$$-3 + 2k = 7 - 3k$$

$$5k = 10$$

$$k = 2$$

$$5 - k = k + 1$$

$$2k = 4$$

$$k = 2$$

$$k = k$$

$$2 = 2$$

$$I(2, 1, 3)$$

$$18) \begin{cases} x_1 \begin{cases} 3+2k \\ -1-3k \\ 2+4k \end{cases} & x_2 \begin{cases} -1+k_2 \\ 4-k_2 \\ -3+3+k_2 \end{cases} \end{cases}$$

$$\text{I) } 3+2k = -1+k_2 \\ k_2 = 2k+4 \\ k_2 = 2$$

$$\text{II) } -1-3k = 4-k_2-4 \\ -1 = 1$$

$$\text{I} (1, 2, -2)$$

$$19) \begin{cases} x_1 \begin{cases} x = 1+2k \\ y = -k \\ z = 3 \end{cases} & x_2 \begin{cases} x = k_2 \\ y = -1+k_2 \\ z = 2+k_2 \end{cases} \end{cases}$$

$$\text{I) } -k_1 = -1+k_2 \\ k_1 = 1-k_2$$

$$\text{II) } 3 = 2+k_2 \\ k_2 = 1$$

$$\text{III) } k_1 = 0 \quad \text{I} (1, 0, 3) \\ A (0, 1, 0)$$

$$\text{IV) } P = A + k(1-A) \quad (x, y, z) = (0, 1, 0) + k(1, -1, 3)$$

$$\begin{cases} x = 7 \\ y = 1-k \\ z = 3k \end{cases} \quad \begin{cases} y = -x+1 \\ z = 3x \end{cases}$$

$$20) \begin{cases} x = 2+k \\ y = k \\ z = -1+2k \end{cases}$$

$$\text{I) } \vec{AC} = C - A = (k, k+1, -1+1) \\ \vec{BC} = C - B = (1+k, k, 2k)$$

$$\text{II) } |\vec{AC}| = |\vec{BC}|$$

$$\text{III) } x = 2 + (-\frac{1}{4}) = \frac{3}{4} \\ y = \frac{1}{4} \\ z = -1 - \frac{2}{4} = -\frac{3}{2}$$

$$\sqrt{k^2 + (k+1)^2 + (2k+1)^2} = \sqrt{(k+1)^2 + k^2 + (2k)^2} \\ k^2 + k^2 + 2k + 1 + 4k^2 + 4k + 1 = k^2 + 2k + 1 + 4k^2 + 4k \\ 4k = -1 \\ k = -\frac{1}{4}$$

$$C (\frac{3}{4}, \frac{1}{4}, -\frac{3}{2})$$

$$k = -\frac{1}{4}$$

21. $A = (3, 4, -2)$
 $\vec{v} = \vec{AI}$, $I(x, y, z)$ $\vec{AI} = (x-3, y-4, z+2)$

$\vec{v}_n = (1, -1, 2)$ $(x-3, y-4, z+2) \cdot (1, -1, 2) = 0$

$I \in R$, $x = 1+t$, $y = 2-k$, $z = 4+2k$

$(1, -1, 2) = 0$

$(1-2, -2-2, 2k+6) \cdot (1, -1, 2) = 0$

$1 \cdot (1-2) + (-1) \cdot (-2-2) + 2(2k+6) = 0$

$k - 2 - k + 2 + 4k + 12 = 0$

$6k = -12 \rightarrow k = -2$

$k = -2 \rightarrow I(-1, 4, 0)$

$\vec{v} = \vec{AI} = I - A = -4, 0, 2$

$\vec{v} = \vec{AI}$ $I \in r$

$A = (3, 4, -2)$

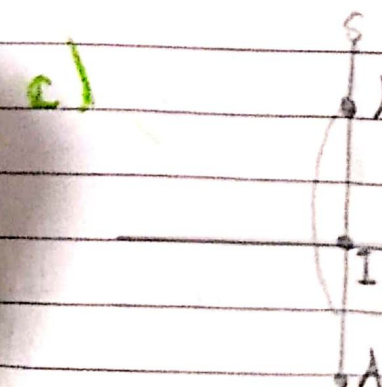
$\vec{v} = (-4, 0, 2)$

$S = \begin{cases} x = 3 - 4t \\ y = 4 + 0t \\ z = -2 + 2t \end{cases}$

b) $d(A, r) = d(A, I)$
 $Z = (-1, 4, 0)$

c) $\vec{AI} = \vec{IA}$ ou $A' = I + \vec{AI}$

r (Determinar A')



(A simétrica)
 de A em relação a r .