/ /   Ciência da Computação - Calculo I

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1- a) $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$ = $x = \frac{-1 \pm \sqrt{25}}{2.1} = \frac{-1 \pm 5}{2}$

$x' = \frac{-1+5}{2} = \frac{4}{2} = 2$

$x^2+x-6 = (x-2).(x-[-3]) =$

$x'' = \frac{-1-5}{2} = \frac{-6}{2} = -3$

$(x-2).(x+3$

$\lim_{x \to 2} \frac{(x-2).(x+3)}{x-2} = \lim_{x \to 2} (x+3) = 2+3 = 5$

1-b) $\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4}$ = $x^2+5+4 = (x-x').(x-x'')$

$S = -4 + -1 = -5$

$P = -4 + -1 = 4$

$(x-(-4)).(x-(-1))$

$(x+4).(x+1$

x^2+3x-4

$S = -4 + 1 = -3$ $(x-(-4)).(x-1)$

$P = -4 + 1 = -4$ $(x+4).(x-1)$

$\lim_{x \to -4} \frac{\cancel{(x+4)}.(x+1)}{\cancel{(x+4)}.(x-1)}$ $\lim_{x \to -4} \frac{(x+1)}{x-1} = \frac{-4+1}{-4-1} = \frac{-3}{-5} =$

$\frac{3}{5}$

1- c) $\lim_{x \to 2} \frac{x^2 - x + 6}{x - 2}$ Não existe

$\frac{x^2 - x + 6}{x - 2} = -\infty$ $\frac{x^2 - x + 6}{x - 2} = +\infty$

$$\frac{\left(x - \frac{1}{2}\right)^2 + \frac{25}{4}}{x - 2}$$

1- d) $\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4} =$

$x = \frac{(-b)^2 \pm \sqrt{-3^2 - 4.1.-4}}{2.a}$

$\lim_{x \to 4} \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$

$x = \frac{--3 \pm \sqrt{25}}{2.1}$ $x = \frac{(--3 + \sqrt{25})}{2.1}$

$x'' = \frac{(--3 - \sqrt{25})}{2.1}$ $x' = \frac{8}{2} = 4$

$\lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5}$

$x'' = \frac{-2}{2}$ $x'' = -1$

1- e) $\lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \left(\frac{(t-3)x(t+3)}{2t^2 + 6t + t + 3}\right)$

$\left(\frac{t-3}{2t+1}\right) = \lim_{t \to -3} \frac{3-3}{2.(-3)+1}$

$\frac{6}{5}$

1- f) $\lim_{x \to -1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$ $\begin{matrix} +\infty \\ -\infty \end{matrix}$ Não existe

2- a) $\lim_{x \to 3} \dfrac{x^4 - 8x^3 + 18x^2 - 27}{x^4 - 10x^3 + 36x^2 - 54x + 27} = \lim_{x \to 3} \dfrac{(x+1)}{(x-1)}$

$\lim_{x \to 3} \dfrac{(x+1)}{(x-1)} \to \dfrac{3+1}{3-1} = \dfrac{4}{2} = \boxed{2}$

2- b) $\lim_{x \to 2} \dfrac{x-2}{\sqrt{2x-4}} = \dfrac{(x-2)}{\sqrt{2x-4}} \cdot \dfrac{(\sqrt{2x-4})}{(\sqrt{2x-4})} \to \dfrac{(x-2) \cdot (\sqrt{2x-4})}{2x-4}$

$\lim_{x \to 2} \dfrac{\sqrt{2x-4}}{2} \to \dfrac{\sqrt{4-4}}{2} = \boxed{0}$

2- c) $\lim_{x \to 4} \dfrac{x-4}{\sqrt{x}-2} = \dfrac{(x-4) \cdot (\sqrt{2}+2)}{(\sqrt{x}-2) \cdot (\sqrt{2}+2)} \to \dfrac{(x-4)(\sqrt{x}+2)}{x-4}$

$\sqrt{x} + 2 = \sqrt{4} + 2 = \boxed{4}$

2 d) $\lim_{x \to 0} \dfrac{x}{2 - \sqrt{4-x}} = \dfrac{x(2 - \sqrt{4-x})}{(2 - \sqrt{4-x}) \cdot (2 - \sqrt{4-x})} = \dfrac{x(2 - \sqrt{4-x}}{x}$

$2 + \sqrt{-x+4} = 2 + \sqrt{4} = \boxed{4}$

3 - a) $\lim_{x \to \infty} \frac{1}{2x+3}$ $\frac{\lim x \to \infty (1)}{\lim x \to \infty (2x+3)}$

$\frac{\lim x \to \infty (1) = 1}{\lim x \to \infty (2x+3) = \infty}$ $\boxed{1/\infty}$

b) $\lim_{x \to \infty} \frac{3x+5}{x-4} = \left(\frac{3+5/x}{1-4/x}\right) = \frac{\lim x \to \infty (3+5/x)}{\lim x \to \infty (1-4/x)}$

$\lim_{x \to \infty} (3+5/x) = 3$

$\to \frac{3}{1} = \boxed{3}$

$\lim_{x \to \infty} (1-4/x) = 1$

c) $\lim_{x \to -\infty} \frac{1-x-x^2}{2x^2-7} = \lim_{x \to -\infty} \left(\frac{1}{x^2} - \frac{1}{x} - 1\right) = -1$

$\lim_{x \to -\infty} (2-7/x^2) = 2$ $\boxed{\frac{-1}{2}}$

d) $\lim_{x \to \infty} \frac{2-3x^2}{5x^2+4x} = \lim_{x \to \infty} \left(\frac{2}{x^2} - 3\right) = -3$

$\lim_{x \to \infty} \left(5 + \frac{4}{x}\right) = 5$ $\boxed{\frac{-3}{5}}$

e) $\lim_{x \to \infty} \frac{x^3+5x}{2x^3-x^2+4} = \frac{\lim x \to \infty (1+5/x^2)}{\lim x \to \infty (2-1/x+4/x^3)} \begin{matrix} \to 1 \\ \to 2 \end{matrix}$ $\boxed{\frac{1}{2}}$

f) $\lim_{x \to -\infty} \frac{x^2+2}{x^3+x^2-1}$ $\frac{\lim_{x \to -\infty} (1+2/x^3) = 1}{\lim_{x \to -\infty} \left(1 + \frac{1}{x} - \frac{1}{x}^3\right) = 1}$ $\to \frac{1}{1} = \boxed{1}$

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4 - a) $\lim_{x \to 5^+} \frac{6}{x-5} = \lim_{x \to 5^+} \frac{6}{x-5} \to x > 5 \to x - 5 > 0$

valor positivo

$+\infty$ para 0.

4 - b) $\lim_{x \to 5^-} \frac{6}{x-5} = \lim_{x \to 5^-} \frac{6}{x-5} \to x < 5 , x - 5 < 0 = -\infty$

Valor negativo que se aproxima de 0

4 - c) $\lim_{x \to 3} \frac{1}{(x-3)^8} = \lim_{x \to a} - f(x) = \lim_{x \to a} + f(x) = L$

$\to$ então $\lim_{x \to a} f(x) = 1$

$\lim_{x \to 3} 3 - \left(\frac{1}{(x-3)^8}\right) = \infty$

$\lim_{x \to 3} 3 + \left(\frac{1}{(x-3)^8}\right) = \infty$

$\boxed{\infty}$

4 - d) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$ $\lim_{x \to a} f(x) = L \to \lim_{x \to 0^-} \left(\frac{x-1}{x^2(x+2)}\right) = -\infty$

$\lim_{x \to 0^+} \left(\frac{x-1}{x^2(x+2)}\right) = -\infty$

$\boxed{-\infty}$

4 - e) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} \to (x-1) \cdot \frac{1}{x^2(x+2)}$

$\lim_{x \to -2^+} \left((x-1) \cdot \frac{1}{x^2(x+2)}\right) \to \lim_{x \to -2^+} (x-1) \cdot \lim_{x \to -2^+} \left(\frac{1}{x^2(x+2)}\right)$

$\lim_{x \to -2^+} (x-1) = -3$ $\lim_{x \to -2^+} \left(\frac{1}{x^2(x+2)}\right) = \infty$

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4- f) $\lim_{x \to 5^+} \ln(x+5) = \lim_{x \to 5^+} (\ln(x-5)) =$

$\ln(5-5) = \boxed{0}$

6- a) $f(x) = \frac{3x}{x-1}$ $\lim_{x \to +\infty} f(x) = \lim_{x \to \infty} \frac{3x}{x-1} = \frac{1}{x}\left(\frac{3x}{x-1}\right)$

$\lim_{x \to 1} \rightarrow \left(\frac{3x}{x-1}\right) \rightarrow \lim_{x \to 1} \rightarrow \left(\frac{3x}{x-1}\right)$ Vertical: 1
horizontal: 3

6- b) $f(x) = \frac{2x}{\sqrt{x^2+4}} = \lim_{x \to +\infty} \left(\frac{2x}{\sqrt{x^2+4}}\right) - \lim_{x \to -\infty} \left(\frac{2x}{\sqrt{x^2+4}}\right) \cdot \left(\frac{\frac{2x}{\sqrt{x^2+4}}}{x}\right)$

Assíntotas: $y = 2$
horizontal: $y = -2$

6- c) $f(x) = \frac{2x^2+1}{2x^2-3x} = f(x) = \frac{2x^2+1}{2x^2-3x} \rightarrow \lim_{x \to 0} \left(\frac{2x^2+1}{2x^2-3x}\right)$

$\lim_{x \to 3/2} \left(\frac{2x^2+1}{2x^2-3x}\right)$

Vertical $\begin{cases} x = 0 \\ x = 3/2 \end{cases}$

$\lim_{x \to +\infty} \left(\frac{2x^2+1}{2x^2-3x}\right) \rightarrow$ Horizontal $\rightarrow \{ y = 1$

6- d) $f(x) = \frac{x}{\sqrt{x^2-4}}$ $f(x) = \frac{x}{\sqrt{x^2-4}}$, $x \in (-\infty, -2) \cup (2, +\infty)$

$\lim_{x \to -2} \left(\frac{x}{\sqrt{x^2-4}}\right) -\infty$ $\lim_{x \to 2} \left(\frac{x}{\sqrt{x^2-4}}\right) \rightarrow +\infty$

Vertical $= x = -2,\ x = 2$

$\lim_{x \to +\infty} \left(\frac{x}{\sqrt{x^2-4}}\right) \rightarrow 1$ $\lim_{x \to -\infty} \left(\frac{x}{\sqrt{x^2-4}}\right) \rightarrow -1$

horizontal $= y = 1,\ y = -1$

6-e) $f(x) \frac{x^3+1}{x^2+4}$ $\lim_{x \to +\infty} \left(\frac{x^3+1}{x^2+4}\right) \to +\infty$

$\lim_{x \to +\infty} \left(\frac{x^3+1}{x^2+4}\right) \to -\infty$

não tem assíntotas

6-f) $f(x) = \frac{x}{\sqrt[4]{x^4+1}}$ $\lim_{x \to +\infty} \left(\frac{x}{\sqrt[4]{x^4+1}}\right) \to 1$ $\lim_{x \to -\infty} \left(\frac{x}{\sqrt[4]{x^4+1}}\right)$

verticais: $y = 1, y = -1$

7-a) $f(-2) = 1 \to f(-2)$ $\frac{x^2-4}{x+2}$ $\lim_{x \to -2} \frac{x^2-4}{x+2} \to \frac{(x-2)(x+2)}{x+2}$

$x - 2 \to -2-2 \to -4$

não é contínua

7-b) $f(1) = 1^3 - 2.1 + 3$

$\hookrightarrow$ $f(x)$ é contínua se $x = 1$

7-c) $f(x) = \frac{x}{x^2-1} \to \frac{-2}{-2^2-1} \to \frac{-2}{5}$ não é contínua, o resultado é dif. do nº inserido

8- $f(x) = \begin{cases} 1 + ax, & \text{se } x \leq 0 \\ x^4 + 2a, & \text{se } x > 0 \end{cases}$

$\lim_{x \to p^-} f(x) = f(p)$ $\lim_{x \to 0^-} 1 + ax = f(0) = f(0) = 1$

$\lim_{x \to p^+} f(x) = f(p)$ $\lim_{x \to 0^+} x^4 + 2a = f(0) \to 2a = 1$

$a = \frac{1}{2}$

$f(x) = \begin{cases} 1 + \frac{x}{2}, & \text{se } x \leq 0 \\ x^4 + 1, & \text{se } x > 0 \end{cases}$

9 - a) F é contínua (função polinomial) no intervalo [0, 1]

$f(0) = -1$

$f(1) = 1$ 0 está entre $f(0)$ e $f(1)$: $f(x_0) = 0$

9 - b) F é contínua (função polinomial) no intervalo [1, 2]

$f(1) = 1$

$f(2) = 2$ 0 está entre $f(1)$ e $f(2)$ = $f(x_0) = 0$

9 - c) F é contínua (função polinomial)

$f\left(\frac{1}{2}\right) = 1 + \frac{\sqrt{2}}{4} > 0$ $\quad f\left(\frac{3}{2}\right) = 1 - \frac{3\sqrt{2}}{4} < 0$

0 está entre $f\left(\frac{1}{2}\right)$ e $f\left(\frac{3}{2}\right)$ = $f(x_0) = 0$

5 - c) $\lim_{x \to 0} \left(1 + \frac{x}{3}\right)^{1/x}$ $\quad \lim_{x \to 0} \left(1 + \frac{1}{z}\right)^{3z}$

$z = \frac{x}{3}$

$\lim_{x \to 0} \left[\left[1 + \frac{1}{z}\right]^{z}\right]^{3}$

$z \cdot 3 = x$

$\lim_{x \to 0} e^{1/3} \Rightarrow \lim_{x \to 0} \sqrt[3]{e}$

5 - d) $\lim_{x \to +\infty} \left(1 + \frac{2}{x}\right)^{x} \to \left(1 + \frac{1}{x/2}\right)^{x}$ $\quad \left(1 + \frac{1}{t}\right)^{2t}$

$t = \frac{x}{2}$

$2t = x$

$\left(\left(1 + \frac{1}{t}\right)^{t}\right)^{2} = e^{2}$

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