Lista 2 derivadas - CC

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1-(i) $f(x) = 4 - x^2$; $f'(-3)$. $f'(0)$. $f'(1)$

$f(x) = \frac{d}{dx}(4) - \frac{d}{dx}(x^2) \rightarrow \frac{d}{dx}(x^2) = 2x$

$0 - 2x =$

$-2x$

- $f'(-3) = -2 . -3 = 6$
- $f'(0) = -2 . 0 = 0$
- $f'(1) = -2 . 1 = -2$

(ii) $g(t) = \frac{1}{t^2}$; $g'(-1)$, $g'(2)$, $g'(\sqrt{3})$

$g(t) = \frac{d}{dt}\left(\frac{1}{t^2}\right) = \frac{2}{t^3}$

$\frac{d}{dt}.(t^{-2}) = -2t^{-2-1} = \frac{-2}{t^3}$

- $g'(-1) = \frac{-2}{-1^3} = 2$
- $g'(2) = \frac{-2}{2^3} = \frac{-2}{8} = \frac{-1}{4}$
- $g'(\sqrt{3}) = \frac{-2}{\sqrt{3}^3} = \frac{-2}{\sqrt{3}^3}$

3- a) Qual é o seu deslocamento depois dos primeiros 4 segundos?

$t = 4$

$x(t) = 3t^2 - t^3$

$x(t) = 3(4)^2 - (4)^3 =$

$-16\ m$

Deslocamento

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3- b) Qual é a velocidade da partícula ao terminar cada um dos 4 primeiros segundos?

$x(t) = 3t^2 - t^3$

$x'(t) = 6t - 3t^2$

$x'(0) = 6(0) - 3(0)^2 = 0 \ m.s^{-1}$

$x'(1) = 6(1) - 3(1)^2 = 3 \ m.s^{-1}$

$x'(2) = 6(2) - 3(2)^2 = 0 \ m.s^{-1}$

$x'(3) = 6(3) - 3(3)^2 = -9 \ m.s^{-1}$

$x'(4) = 6(4) - 3(4)^2 = -24 \ m.s^{-1}$

3-c) Qual é a aceleração da partícula em cada um dos 4 primeiros segundos?

$x'(t) = 6t - 3t^2$

$x''(t) = 6 - 6t$

$x''(0) = 6 - 6(0) = 6 \ m.s^{-2}$

$x''(1) = 6 - 6(1) = 0 \ m.s^{-2}$

$x''(2) = 6 - 6(2) = -6 \ m.s^{-2}$

$x''(3) = 6 - 6(3) = -12 \ m.s^{-2}$

$x''(4) = 6 - 6(4) = -18 \ m.s^{-2}$

4 - a) $f(x) = 10(3x^2 + 7x - 3)^{10}$

$f'(x) = 10 . 10(3x^2 + 7x - 3)^9 . (6x + 7)$

$f'(x) = 100 . (6x + 7) . (3x^2 + 7x - 3)^9$

$10 = f(z) = (3z^2 + 7z - 3)^{10}$

$f(z) = 10 . (3z^2 + 7z - 3) . (6z + 7)$

4 - b) $f(t) = (7t^2 + 6t)^7 . (3t - 1)^4$

$u' . v + v' . u$

$7(7t^2 + 6t)^6 . (14t + 6) . (3t - 1)^4 + 4(3t - 1)^3 . 3 . (7t^2 + 6t)^7$

4- c) $f(x) = \sqrt[3]{(3x^2 + 6x - 2)^2}$

$f(g(x))' = f'(g(x)) . g'(x)$

$f(x) = \sqrt[3]{(3x^2 - 6x - 2)^2}$

$f(x) = (3x^2 - 6x - 2)^{2/3}$

$f'(x) = \frac{2}{3}(3x^2 - 6x - 2)^{-\frac{1}{3}} . (6x - 6)$

$$f'(x) = \frac{2(6x - 6)}{3\sqrt[3]{3x^2 - 6x - 2}}$$

4- d) $\sqrt{\frac{2t+1}{t-1}} = \left(\frac{2t+1}{t-1}\right)^{1/2}$ $f'(t) = \left(\frac{2t+1}{t-1}\right)' . \frac{1}{2} . \left(\frac{2t+1}{t-1}\right)^{\frac{1}{2}-1}$

$$f'(t) = \frac{(2t+1)' . (t-1) - (2t+1) . (t-1)'}{(t-1)^2} . \frac{1}{2} . \left(\frac{2t+1}{t-1}\right)^{-\frac{1}{2}}$$

$$f'(t) = (2 . 1 . t^{1-1} + 0) . (t-1) - (2t-1) . (1 . t^{1-1} + 0) . \frac{1}{2} . \frac{1}{\sqrt{\frac{2t+1}{t-1}}}$$

$$f'(t) = \frac{2 . (t-1) - (2t+1)}{(t-1)^2} . \frac{1}{2 . \sqrt{\frac{2t+1}{t-1}}}$$

$$f'(t) = -\frac{3 . (t-1)^{1/2 - 2}}{2 . \sqrt{2t+1}} = f'(t) = -\frac{3 . (t-1)^{-3/2}}{2 . \sqrt{2t+1}}$$

$$f'(t) = -\frac{3}{2} . \sqrt{\frac{1}{(t-1)^3 . (2t+1)}}$$

4 - e) $f(x) = 2^{3x^2+6x}$ $= 2(3.2+6)\,2(3x.2+6)$

$2(6+6)\,2(6x+6)$

$12+12$

4 - f) $e^{t/2}(t^2+5t) = (a.b)' = a'.b + a.b'$

$f'(t) = (e^{(t/2)}).(\frac{1}{2}).(t^2+5t) + (e^{(t/2)}).(2t+5)$

$f'(t) = (e^{(t/2)}).((\frac{1}{2}).(t^2+5t) + (2t+5))$

$f'(t) = (e^{(t/2)}).\left(\frac{t^2}{2} + \frac{5t}{2} + 2t + 5\right)$

$f'(t) = \left(e^{(t/2)}).\left(\frac{t^2}{2}\right)\right).\left(\frac{t^2}{2} + \frac{9t}{2} + 5\right)$

4 - g) $\log_3 \sqrt{s+1}$ $= \frac{\log_3(\sqrt{s+1})}{\sqrt{s+1}}$ $\quad s \in \mathbb{R} \setminus \{-1\}$

$s+1$ $\quad s \geq -1$

$s \in \mathbb{R}$

$s \in \{-1, +\infty]$

$\therefore s > -1$

4 - h) $f(u) = \cos(\pi/2 - u)$ $=$ $f(u) = \cos(u)$ e $g(u) = pi/2 - u$. log. $f(g(u)) = \cos(pi/2-u)$

$f'(g(u)) = (\cos(pi/2-u))'.(pi/2-u)'$

$f'(g(u)) = -\text{sen}(pi/2-u).(-1) = \text{Sen}(pi/2-u) = \cos(u)$

4- i) $f(x) = sen^3(3x^2 + 6x)$

$f'(x) = 3 sen^2(3x^2 + 6x) . cos(3x^2 + 6x) . (6x + 6)$

4- j) $\frac{3 \sec^2 x}{x} = 0 = \frac{3 \sec(x)^2}{2} =$

$0 = \frac{3 \sec(x)^2}{x}, \quad x \neq \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}, \; x \neq 0$

$\frac{3 \sec(x)^2}{x} = 0$

$3 \sec(x)^2 = 0 \qquad x \in \emptyset$

$\sec(x)^2 = 0$

$\sec(x = 0$

4-k $f(\theta) = -\operatorname{cosec}^2 \theta^3 = f(\theta) = -\csc(\theta^3)^2$

$\theta \in \mathbb{R} \qquad \csc(\theta^3)^2$

$\theta \in \mathbb{R} \setminus \{\sqrt[3]{k\pi}\}, \; k \in \mathbb{Z} \qquad \csc(\theta^3)$

$\theta \in \mathbb{R} \qquad \theta$

$\theta \in \mathbb{R} \setminus \{\sqrt[3]{k\pi}\}, \; k \in \mathbb{Z}$

5- Calcular $f'(0)$, se $f(x) = e^{-x} \cos 3x$. $: f'(0)$

$f(x) = e^{-x} \cos(3x)$

$f'(x) = -x' \, e^{-x} \cos(3x) + \cos(3x)' \, e^{-x} =$

$-e^{-x} \cos(3x) + (-3x') \, sen(3x) \, e^{-x} =$

$-\frac{1}{e^{x}} \cos(3x) - 3 sen(3x) \, e^{-x} =$

$\frac{-\cos(3x) - 3 sen(3x)}{e^{x}}$

$f'(0) = \frac{-\cos(0) - 3 sen(0)}{e^0} = \frac{-1 - 0}{1} = -1$

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6- Mostrar que a função $y = x\,e^{-x}$ satisfaz a equação $x y' = (1-x)\,y$.

$y = x \cdot e^{(-x)}$

$y' = -x \cdot e^{(-x)}$

$x y' = (1-x)\,y$

$x y' = y - xy$

$-x^2 \cdot e^{(-x)} = x \cdot e^{(-x)} - x^2 \cdot e^{(-x)}$

$x \cdot e^{(-x)} = 0$

$x = 0$

7- $h'(t) = \lim_{\Delta t = 0} \dfrac{h(t+\Delta t) - h(t)}{\Delta t} = \lim_{\Delta t \to 0}$

$\dfrac{10^6 + 10^4(t+\Delta t) - 10^3(t+\Delta t)^2 - (10^6 + 10^4 - 10^3 t^2)}{\Delta t}$

$= \lim_{\Delta t \to 0} \dfrac{10^4 \Delta t - 10^3(t^2 + 2t\Delta t + (\Delta t)^2) + 10^3 t^2}{\Delta t} =$

$\lim_{\Delta t \to 0} \dfrac{10^4 \Delta t - 10^3 \cdot 2t\Delta t - 10^3 \cdot (\Delta t)^2}{\Delta t} =$

$\lim_{\Delta t \to 0} \dfrac{\cancel{\Delta t}(10^4 - 10^3 \cdot 2t - 10^3 \Delta t)}{\cancel{\Delta t}} = \lim_{\Delta t \to 10} = 10^4 - 10^3 \cdot 2t - 10^3 \Delta t$

$10^4 - 10^3 \cdot 2t =$

7- a) $h'(0) = 10^4 - 10^3 \cdot 2 \cdot (0) = 10^4$

7- b) $h'(5) = 10^4 - 10^3 \cdot 2 \cdot (5) = 10^4 - 10^4 = 0$

7- c) $h'(10) = 10^4 - 10^3 \cdot 2 \cdot (10) = 10^4 - 10^4 \cdot 2 = -10^4$

2- $f(x) = x + \frac{9}{x}$ Coeficiente angular:

$f'(x) = 1 + \frac{9}{x^2}$ $1 - \frac{9}{(-3)^2} = 1 - \frac{9}{9} = 1 - 1 = 0$

$\frac{9}{x^2} - \frac{1s}{x}$