

# Prova 1 - Calculo 1

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$$2-(i) \lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^3 - 4x^2 + x - 4} : \text{Simplificando: } \frac{x}{x^2 + 1}$$
$$\frac{4}{4^2 + 1} = \frac{4}{17}$$
$$\lim_{x \rightarrow 4} \left( \frac{x}{x^2 + 1} \right)$$

$$2-(ii) \lim_{x \rightarrow 2} \frac{\sqrt{4x+1} - 3}{x-2} ; = \lim_{x \rightarrow 2^+} \left( \frac{\sqrt{4x+1} - 3}{x-2} \right) = \infty$$
$$\lim_{x \rightarrow 2^-} \left( \frac{\sqrt{4x+1} - 3}{x-2} \right) = -\infty$$

se  $\lim_{x \rightarrow a} -f(x) \neq \lim_{x \rightarrow a} +f(x)$

Então o limite não existe

Não existe

$$2-(iii) \lim_{x \rightarrow \infty} \frac{3x^4 - 6x^2 + 1}{6x - x^3 - 2x^4} ; = \frac{3 - \frac{6}{m^2} + \frac{1}{m^4}}{\frac{6}{m^3} - \frac{1}{m} - 2}$$
$$\lim_{m \rightarrow -\infty} \left( 3 - \frac{6}{m^2} + \frac{1}{m^4} \right) = 3$$
$$\lim_{m \rightarrow -\infty} \left( \frac{6}{m^3} - \frac{1}{m} - 2 \right) = -2$$

$$\frac{3}{-2} = -\frac{3}{2}$$

$$2-(iv) \lim_{x \rightarrow +\infty} \left(1 + \frac{4}{x}\right)^{x+2} = \lim_{n \rightarrow \infty} \left(1 + \frac{4}{n}\right)^n \left(1 + \frac{4}{n}\right)^2$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{4}{n}\right)^n = e^4$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{4}{n}\right)^2 = 1 = e^4 \cdot 1 = \boxed{e^4} = 54.598$$

$$4 - x^2 - e^x + 2 = 0$$

usando 1,3 e 1,4

$$1,3^2 - 2,72^{1,3} + 2 = 0$$

$$1,69 - 3,67 + 2$$

$$0,02$$

$$1,4^2 - 2,72^{1,4} + 2 = 0$$

$$1,96 - 4,05 + 2 =$$

$$-2,09 + 2$$

$$-0,09$$

$$\{0,02; -0,09\}$$

$$6-(i) y = \frac{x^2 - 1}{x^3 - \sin(x)} = \frac{d}{dx} \left( \frac{x^2 - 1}{x^3 - \sin(x)} \right)$$

Regra da quociente

$$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2} = \frac{\frac{d}{dx}(x^2 - 1) \cdot (x^3 - \sin x) - \frac{d}{dx}(x^3 - \sin x) \cdot (x^2 - 1)}{(x^3 - \sin x)^2}$$

$$\frac{d}{dx}(x^2 - 1) = 2x \quad \frac{d}{dx}(x^3 - \sin x) = 3x^2 - (\sin 1) =$$

$$\frac{2x(x^3 - \sin x) - (3x^2 - (\sin 1))(x^2 - 1)}{(x^3 - \sin x)^2}$$



6-(ii)  $y = (x - \cos x^2) \ln(3x^4 - 2)$

Aplicando a regra do Produto:

$$f = x - \cos(x^2)$$

$$g = \ln(3x^4 - 2)$$

$$(x - \cos(x^2)) \ln(3x^4 - 2) + (\ln(3x^4 - 2))(x - \cos(x^2))$$

$$\frac{d}{dx} (\ln(3x^4 - 2)) = \frac{12x^3}{3x^4 - 2}$$

$$(1 + 2x \sin(x^2)) \ln(3x^4 - 2) + \frac{12x^3}{3x^4 - 2} (x - \cos(x^2))$$

Multiplicando frações:

$$\ln(3x^4 - 2) \cdot (1 + 2x \sin(x^2)) + \frac{12x^3(x - \cos(x^2))}{3x^4 - 2}$$

6-(iii)  $y = \lg(x^3 - 7x) - 3 + \cotg(5x)$

Aplicando a regra da soma e diferença

$$\frac{d}{dx} (\lg(x^3 - 7x)) = \frac{d}{dx} (3) + \frac{d}{dx} (\cotg(5x))$$

$$\frac{d}{dx} (\lg(x^3 - 7x)) = \lg^2(x^3 - 7x) \cdot (3x^2 - 7) = (3) = 0$$

$$\frac{d}{dx} (\cotg(5x)) = \operatorname{cosec}^2(5x) \cdot 5$$

$$\lg^2(x^3 - 7x) \cdot (3x^2 - 7) = 0 + \operatorname{cosec}^2(5x) \cdot 5$$

$$\lg^2(x^3 - 7x) \cdot (3x^2 - 7) + 5 \operatorname{cosec}^2(5x)$$



$$b-(iv) = y = 3(4x^3 - 5)^5 \cdot \frac{3}{x^6} + e^{x-x^2}$$

$$\text{Soma e diferença} = \frac{d}{dx} \left( \tan(3) \cdot (4x^3 - 5)^5 \right) - \left( \frac{3}{x^6} \right) + \frac{d}{dx} \left( e^{x-x^2} \right)$$

$$\frac{d}{dx} \left( \tan(3) (4x^3 - 5)^5 \right) = 60 \tan(3) x^2 (4x^3 - 5)^4$$

$$\frac{d}{dx} \left( \frac{3}{x^6} \right) = -\frac{18}{x^7} = \frac{d}{dx} \left( e^{x-x^2} \right) = e^{x-x^2} (1-2x)$$

$$60 \tan(3) x^2 (4x^3 - 5)^4 - \left( -\frac{18}{x^7} \right) + e^{x-x^2} (1-2x) =$$

$$60 \tan(3) x^2 (4x^3 - 5)^4 + \frac{18}{x^7} + e^{x-x^2} (1-2x)$$

$$b-(v) = y = \sec(3-2x^3) + x \operatorname{cosec}^3(2x+4) + \sqrt{1+x^2}$$

$$\text{Simplificando } \tan \left( \sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + \sqrt{1+x^2} \right);$$

$$\frac{d}{dx} \left( \tan(\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2) \right) =$$

Regra cadeia:

$$\sec^2(\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2)$$

$$\frac{d}{dx} (\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2) =$$

$$\sec^2(\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2) \frac{d}{dx} (\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2)$$

$$= \sec^2(\sec(3-2x^3) + \operatorname{cosec}^3(2x+4) + 1+x^2) (-6x^2 \sec(-2x^3+3) + \tan(-2x^3+3) + 2x - 6 \operatorname{cosec}^8(2x+4) \cotan(2x+4))$$

$$3- f(x) = \frac{2x^2}{x^2-4} \cdot 2^2-4=0 \quad f(x) = \frac{2x^2}{x^2-4} \quad | \quad |$$

$$\lim_{x \rightarrow -2} \frac{2x^2}{x^2-4} = +\infty \quad \lim_{x \rightarrow +2} \frac{2x^2}{x^2-4} = -\infty$$

$$\lim_{x \rightarrow 2} \frac{2x^2}{x^2-4} = -2 = -\infty$$

$$y = -2; x = 2$$

$$f(x) = \frac{2x^2}{x^2-4}; \quad \lim_{x \rightarrow +\infty} \frac{2x^2}{x^2-4} = \frac{+\infty}{+\infty} = \text{Indeterminado}$$

$$\frac{2x^2}{x^2-4} = \frac{2}{1-\frac{4}{x^2}} = \frac{2}{1-\frac{4}{x^2}} = \frac{2}{1} = 2$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{x^2-4} = \text{Indeter.} \quad \frac{2x^2}{x^2} \cdot \left(1 - \frac{4}{x^2}\right) = \frac{2}{1-\frac{4}{x^2}} = \frac{2}{1} = 2$$

(horizontal 2, vertical 2, x=2)