/ /

Igor Radtke - Geometria Analítica - CC

1- Se $\vec{u} = 3\vec{i} - \vec{j} - 2\vec{k}$, $\vec{v} = 2\vec{i} + 4\vec{j} - k$ e $\vec{w} = -\vec{i} + \vec{k}$, determine:

a) $|\vec{u} \times \vec{u}| = \begin{vmatrix} i & j & k \\ u_i & u_j & u_k \\ v_i & v_j & v_k \end{vmatrix} = i(-2.(-1)-(-2).4) + j(-2.2-3.(-1)) + k(3.4-(-2).2)$

$i:(2-(-8)) + j(-4-(-3)) + k(12-(-4))$

$i:(2-(-8)) + j(-4-(-3)) + k(12-(-4))$

$\vec{u} \times \vec{v} = 10i + (-1)j + 16k$

b) $(2\vec{v}) \times (3\vec{v}) = 2v = (4,8,-2)$ $3v = (6,12,-3)$

$\begin{vmatrix} i & j & k \\ 4 & 8 & -2 \\ 6 & 12 & -3 \end{vmatrix}$ $i(-24+24) - j(-12+12)j + k(48-48) =$

$0i + 0j + 0k = (0,0,0)$

d) $(\vec{u} \times \vec{v}).\vec{v}$

$u \times v = \begin{vmatrix} i & j & k \\ 3 & -1 & -2 \\ 2 & 4 & -1 \end{vmatrix}\begin{matrix} i & j \\ 3 & -1 \\ 2 & 4 \end{matrix}$

$u \times v = 9i - j + 14k$

$(9,-1,14).(2,4,-1) = 0$

$(u \times v).v = 0$

2 - Determine os valores simultaneamente ortogonal aos vetores $\vec{u} + 2\vec{v}$ e $\vec{v} - \vec{u}$, $\vec{u} = (-3, 2, 0)$ e $\vec{v} = (0, -1, -2)$

$a = u + 2v = (-3, 0, -4)$ → AB

$b = v - u = (3, -3, -2)$

Determinante

$AB = -12i - 1j + 9k$

$\boxed{AB = (-12, -1, 9)}$

3 - Determine um vetor de módulo 2 ortogonal a $\vec{u} = (3, 2, 2)$ e $\vec{v} = (0, 1, 1)$

$\|w\| = \sqrt{(a^2 + b^2 + c^2)} = 2$ $\quad a^2 + b^2 + c^2 = 4$ $\quad w.u = 0$

$(a, b, c) \cdot (3, 2, 2) = 0$ $\quad (a, b, c) \cdot (0, 1, 1) = 0$

$3a, 2b, 2c = 0$ $\quad w.v = 0$ $\quad b + c = 0$

$3a + 2b + 2c = 0$ $\quad c = -b$

$3a + 2b - 2b = 0$ $\quad \sqrt{(a^2 + b^2 + c^2)} = 2$

$3a = 0$ $\quad \sqrt{(0^2 + (-c)^2 + c^2)} = 2$ $\quad c1 = \sqrt{2}, c2 = -\sqrt{2}$

$a = 0$ $\quad \sqrt{(2c^2)} = 2$

$2c^2 = 4$ $\quad b1 = -c1$ $\quad b2 = -c2 =$

$c^2 = 2$ $\quad b1 = -\sqrt{2}$ $\quad \sqrt{2}$

$W1 = (a, b1, c1) = (0, -\sqrt{2}, \sqrt{2})$

$W2 = (a, b2, c2) = (0, \sqrt{2}, -\sqrt{2})$

5 - Determine $\vec{u} \cdot \vec{v}$, dado que $|\vec{u} \times \vec{v}| = 12$, $|\vec{u}| = 13$ e $\vec{v}$ é unitário.

$|u| = 12$ $\quad |u.v| = |u|.|v|. \operatorname{sen}(a)$

$|v| = 1$ $\quad 12 = 13 . 1 \operatorname{sen}(a)$

$\operatorname{sen}(a) = \frac{12}{13}$

$u.v = |u|.|v|.\cos(a)$

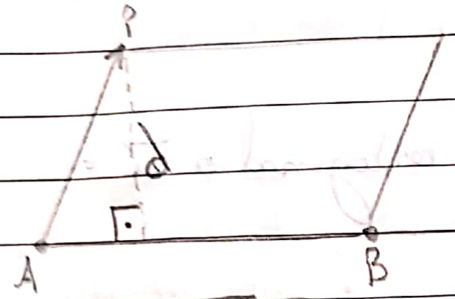
$\operatorname{sen} 2(a) + \cos 2(a) = 1$

$u.V = 13.1.\frac{5}{13} = 5$ $\quad \frac{144}{169} + \cos 2(a) = \frac{169}{169}$

$\cos(a) = \frac{25}{169} = \frac{5}{13}$

7- Calcular o ponto $P(4,3,3)$ á reta que passa por $A(1,2,-1)$ e $B(3,1,1)$ $\quad A=(\underset{x_1}{1}, \underset{y_1}{2}, \underset{z_1}{-1}) \quad B=(\underset{x_2}{3}, \underset{y_2}{1}, \underset{z_2}{1})$

$$d = \frac{|\vec{AB} \times \vec{AP}|}{|\vec{AB}|} \qquad \vec{AB} = B - A = (3,1,1)-(1,2,-1) = (2,-1,2)$$
$$\vec{AP} = P - A = (4,3,3) - (1,2,-1) = (3,1,4)$$



$$\vec{AB} \times \vec{AP} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 2 & -1 & 2 & 2 & -1 \\ 3 & 1 & 4 & 3 & 1 \end{array}\right|$$

$$-4\vec{i} + 6\vec{j} + 2\vec{k} - (-3\vec{k} + 2\vec{i} + 8\vec{j})$$
$$-4i + 6j + 2k + 3k - 2i - 8j$$
$$-6i - 2j + 5k$$
$$(-6,-2,5) \rightarrow \sqrt{(-6)^2 + (-2)^2 + 5^2} = \sqrt{36+4+25} = \sqrt{65}$$

$$\vec{AB} = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$d = \frac{\sqrt{65}}{3}$$

8- $A=(-4,1,1) \quad B=(1,0,1) \quad C=(0,-1,3)$

$AB;\ AC;\ BC \qquad AB = B - A$

$AB = (1-(-4));(0-1);(1-1)$

$AB = (5,-1,0)$

$$AB \times AC = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 5 & -1 & 0 & 5 & -1 \\ 4 & -2 & 2 & 4 & -2 \end{array}\right|$$

$AC = C - A$

$AC = (0,-1,3) - (-4,1,1)$

$AC = (4,-2,2)$

$i = -2 \quad j = -10 \quad k = -6$

$(-2,-10,-6)$

$BC = (0,-1,3) - (1,0,1)$

$BC = (-1,-1,2)$

$A^{\text{área}} = \sqrt{-2^2 + -10^2 + -6^2}$

$A = \sqrt{4+100+36}$

$A = \sqrt{140}$

$A = \frac{\sqrt{140}}{2}$

140	2
70	2
35	5
7	7
1	

$\sqrt{140} = 2\sqrt{5 \cdot 7} = 2\sqrt{35}$

$\text{Área} = \frac{2\sqrt{35}}{2} = \sqrt{35}$

$|BC| = \sqrt{1^2+1^2+2^2}$ (Base . Altura)/2

$|BC| = \sqrt{6}$ Base = $|BC|$ H: altura

área = $\sqrt{35}$ $\sqrt{35} = \frac{1}{2} \cdot \sqrt{6} \cdot$ altura

$\frac{2 \cdot \sqrt{35}}{\sqrt{6}} = H$

9- Calcular z, sabendo-se que A(2,0,0), B(0,2,0) e C(0,0,z) são vértices de um triângulo de área 6.

$u = B - A = (0,2,0) - (2,0,0) = (-2,2,0)$

$v = C - A = (0,0,z) - (2,0,0) = (-2,0,z)$

$$\left|\begin{array}{ccc|cc} i & j & k & i & j \\ -2 & 2 & 0 & -2 & 2 \\ -2 & 0 & z & -2 & 0 \end{array}\right|$$

$|u \times v| = \sqrt{((2z)^2 + (2z)^2 + (4)^2)} =$

$\sqrt{(4z^2 + 4z^2 + 16} =$

$\sqrt{(8z^2 + 16)}$

$A = |u \times v| / 2 = 6$

$\frac{\sqrt{(8z^2+16)}}{2} = 6$

$\sqrt{(8z^2+16)} = 12$

$(8z^2 + 16) = 144$

$8z^2 = 144 - 16 = 128$

$z^2 = \frac{128}{8} = 16$

$z_1 = 4, z^2 = -4$