Prova 02 — Igor Rodtke

6- Se $x^3 - xy + 4xz - 5 = 0$, calcular $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$ usando a regra de derivação de função implícita.

Queremos encontrar $\frac{\partial z}{\partial x}(x,y)$ e $\frac{\partial z}{\partial y}(x,y)$.

Por outro lado,

$$F(x,y) = 0 \Rightarrow \frac{\partial F}{\partial x}(x,y,z) \cdot \frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}(x,y,z) \cdot \frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}(x,y,z) \cdot \frac{\partial z}{\partial x}(x,y) = 0$$

Como $x$ e $y$ são variáveis e não funções, $\frac{\partial x}{\partial x} = 1$ e $\frac{\partial y}{\partial x} = 0$

Logo: $\frac{\partial F}{\partial x}(x,y,z) + \frac{\partial F}{\partial z}(x,y,z) \cdot \frac{\partial z}{\partial x}(x,y) = 0$

$$\frac{\partial z}{\partial x}(x,y) = -\left(\frac{\frac{\partial F}{\partial x}(x,y,z)}{\frac{\partial F}{\partial z}(x,y,z)}\right) \qquad \frac{\partial z}{\partial y}(x,y) = -\left(\frac{\frac{\partial F}{\partial y}(x,y,z)}{\frac{\partial F}{\partial z}(x,y,z)}\right)$$

Substituindo as equações:

$$\frac{\partial z}{\partial x}(x,y) = -\left(\frac{\frac{\partial F}{\partial x}(x,y,z)}{\frac{\partial F}{\partial z}(x,y,z)}\right) = \left(\frac{3x^2 - y + 4z}{4x}\right) = \frac{-3x^2 + y - 4z}{4x}$$

$$\frac{\partial z}{\partial y}(x,y) = -\left(\frac{\frac{\partial F}{\partial y}(x,y,z)}{\frac{\partial F}{\partial z}(x,y,z)}\right) = -\left(\frac{-x}{4x}\right) = \frac{1}{4}$$

1- Calcule as seguintes derivadas parciais $f_{xy}$ e $f_{yx}$ para $\ln\sqrt{x^2+y^2}$.

Derivada de x: $f(x,y) = \ln(x^2+y^2)^{1/2}$

$$f_x(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot [(x^2+y^2)^{1/2}]$$

$$f_x(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2} \cdot 2x \cdot (x^2+y^2)^{1/2 - 1}$$

$$f_x(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot x \cdot (x^2+y^2)^{-1/2}$$

$$f_x(x,y) = \frac{x}{(x^2+y^2)^{1/2}} \cdot \frac{1}{(x^2+y^2)^{1/2}} = f_x(x,y) \frac{x}{(x^2+y^2)^{1/2+1/2}}$$

$$f_x(x,y) = \frac{x}{(x^2+y^2)^1} = f_x(x,y) = \frac{x}{(x^2+y^2)}$$

Derivada de y = $f_y(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot [(x^2+y^2)^{1/2}]$

$$f_y(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{2} \cdot 2y \cdot (x^2+y)^{1/2-1}$$

$$f_y(x,y) \frac{1}{(x^2+y^2)^{1/2}} \cdot y(x^2+y^2)^{-1/2} = f_y(x,y) = \frac{y}{(x^2+y^2)^{1/2}} \cdot \frac{1}{(x^2+y^2)^{1/2}}$$

$$f_y(x,y) = \frac{y}{(x^2+y^2)^{1/2+1/2}}$$

$$f_y(x,y) = \frac{y}{(x^2+y^2)^1} = f_y(x,y) = \frac{y}{(x^2+y^2)}$$

2- Obtenha $\frac{df}{dt}$ sendo F a função:

a) $f(x,y) = x^2 y^3 + x^3 y^2 \quad x(t) = \frac{1}{t}$ e $y(t) = \frac{1}{t^2}$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} \quad \frac{\partial f}{\partial x} = 2x \cdot y^3 + 3x^2 \cdot y^2$$

$$\frac{1}{t} = t^{-1}$$

$$\frac{\partial f}{\partial y} = 3y^2 \cdot x^2 + 2y \cdot x^3$$

$$\frac{1}{t^2} = t^{-2}$$

$$\frac{dx}{dt} = -1 \cdot t^{-1-1} = -t^{-2} = \frac{-1}{t^2}$$

$$\frac{dy}{dt} = -2 \cdot t^{-2-1} = -2t^{-3} = \frac{-2}{t^3}$$

$$\frac{dz}{dt} = (2x \cdot y^3 + 3x^2 \cdot y^2) \cdot \left(\frac{-1}{t^2}\right) + (3y^2 \cdot x^2 + 2y \cdot x^3 \cdot \left(\frac{-2}{t^3}\right)$$

Após a substituição de x e y:

$$\frac{dz}{dt} = \frac{2}{1}\left(\frac{1}{t}\right) \cdot \left(\frac{1}{t^2}\right) + \frac{3}{1}\left(\frac{1}{t}\right)^2 \cdot \left(\frac{1}{t^2}\right) \cdot \left(\frac{-1}{t^2}\right) + \frac{3}{1} \cdot \left(\frac{1}{t^2}\right)^2 - \left(\frac{1}{t}\right) + \frac{2}{1}\left(\frac{1}{t^2}\right) \cdot \left(\frac{1}{t}\right)^3 \cdot \left(\frac{-2}{t^3}\right)$$

$$\frac{dz}{dt} = \left(\frac{2}{t^7}\right) + \left(\frac{-3}{t^3}\right) + \frac{3}{t^6} + \left(\frac{-4}{t^3}\right) + \left(\frac{-7}{t^3}\right) + \left(\frac{2}{t^7}\right) + \left(\frac{3}{t^6}\right)$$

$$\frac{dz}{dt} = \left(\frac{-7}{t^3}\right) + \left(\frac{2}{t^7}\right) + \left(\frac{3}{t^6}\right)$$

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b-- $f(x,y) = e^{x+y}$ $x(t) = t^2$ e $y(t) = 2t^3 - 1$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = e^{x+y} \cdot 2t + e^{x+y} \cdot 6t^2 \rightarrow \frac{dz}{dt} = e^{x+y} \cdot (6t^2 + 2t)$$

$$\frac{\partial f}{\partial x} = e^{x+y} \cdot 1$$

Usando a substituição:

$$\frac{\partial f}{\partial y} = e^{x+y} \cdot 1$$

$$\frac{dz}{dt} = e^{t^2 + 2t^3 - 1} \cdot (6t^2 + 2t)$$

$$\frac{\partial x}{dt} = 2t$$

$$\frac{dy}{dt} = 6t^2$$

3- Considere a função $f(x,y) = \ln(e^{xy} - x^2 y^3)$

a) Calcule $f_x(10;15)$

$$\frac{\partial f}{\partial x} = \frac{1}{e^{xy} - x \cdot y^3} \cdot e^{xy} \cdot y - 2x \cdot y^3$$

$$\frac{\partial f}{\partial x} = \frac{e^{xy} \cdot y - 2x \cdot y^3}{e^{xy} - x^2 \cdot y^3} = \frac{y - 2x}{-x^2}$$

$$\frac{\partial f}{\partial x}(10,15) = \frac{15 - 2.(10)}{(-10)^2} = \frac{15 - 20}{100} = \frac{-5}{100} = -0,05$$

b) Calcule $f(11;15)$ e $f(10;15)$ e compare com o resultado obtido em (a).

$$\ln(e^{11.15} - 11^2 . 15)^3 \to \ln(e^{165} - 121 . 3.375)$$

$$\ln(e^{165} - 408.375) = 165$$

c) Calcule $f_y(10;15)$

$$\ln(e^{xy} - x^2 y^3) \quad \frac{\partial f}{\partial y} = \frac{1}{e^{xy} - x^2 . y^3} . e^{xy} . x - 3 . x^2 . y^2$$

$$\frac{\partial f}{\partial y} = \frac{e^{xy} . x - 3 . x^2 . y^3}{e^{xy} - x^2 . y^3} \to \frac{x - 3}{-y}$$

$$\frac{\partial f}{\partial y}(10,15) = \frac{10 - 3}{-15} = \simeq 0,466$$

d) Calcule $f(10;16)$ e $f(10;15)$ e compare com o resultado obtido em (c).

$$\ln(e^{10.16} - 10^2 . 16^3) \to \ln(e^{160} - 100 . 4096)$$

$$\ln(e^{160} - 409.600)$$

$$f(10,15 = \ln(e^{10.15} - 10^2 . 15^3) \to \ln(e^{150} - 100 . 3375)$$

$$\ln(e^{150} - 337500)$$

150

4- Sabendo-se que $\frac{\partial f}{\partial x} = f_x$ e $\frac{\partial f}{\partial y} = f_y$, e $f(x,y) = \operatorname{sen}(x y z) + x^2 \cdot \frac{y^3}{3}$, calcule:

$u = y^3$
$u' = 3y^2$
$v = 3$
$v' = 0$
$f.q = \frac{u' \cdot v - v' \cdot u}{v^2}$

a) $f121 = \frac{\partial f}{\partial x} = y \cdot z \cdot \cos(x \cdot y \cdot z) + 2x \frac{y^3}{3}$

$\frac{3y^2 \cdot 3}{3^2} = \frac{3y^2}{3}$

$\frac{\partial f}{\partial y} = 1 \cdot z \cdot -x \cdot z \cdot \operatorname{sen}(x \cdot y \cdot z) + 2x \cdot \frac{3y^2}{3}$

$\frac{\partial f}{\partial x} = -z \cdot z \cdot y \cdot \cos(x, y, z) + 2 \cdot \frac{3y^2}{3} \rightarrow$

$f121 = -z^2 \cdot y \cos(x, y, z) + \frac{6y^2}{3} \rightarrow f121 = -z \cdot y \cdot \cos(xyz) + 6y^2$

b) $f221 = \frac{\partial f}{\partial y} = x \cdot z \cdot \cos(xyz) + x^2 \cdot \frac{3y^2}{3}$

$\frac{\partial f}{\partial y} = x \cdot z - x \cdot z \cdot \operatorname{sen}(x \cdot y \cdot z) + x^2 \cdot \frac{6y}{3}$

$\frac{\partial f}{\partial x} = -y^2 \cdot y \cdot z \cos(x \cdot y \cdot z) + 2x \cdot \frac{6y}{3}$

$f221 = -z^2 \cdot y \cdot \cos(x \cdot y \cdot z) + 2x \cdot 6y$

$u = y^3$
$u' = 0$
$v = 3$
$v' = 1$

$f(x \cdot y \cdot z) = \operatorname{sen}(xyz) + x^2 \cdot \frac{y^3}{3} - \frac{y^3}{3^2}$

c) $f_{zzz}$ $\frac{\partial f}{\partial z} = x \cdot y - x \cdot y \cdot sen(x \cdot y \cdot z) + x^2 \cdot \left(\frac{-y^3}{z^2}\right)$

$U = -y^3$
$U' = 0$
$V = z^2$
$V' = 2z$

$\frac{-2z \cdot -y^3}{z^4} = \frac{2y^3}{z^3}$

$\frac{\partial f}{\partial z} = x \cdot y \cdot -x \cdot y \cdot sen(x \cdot y \cdot z) + x^2 \cdot \frac{2y^3}{z^3}$

$\frac{\partial f}{\partial y} = -x^2 \cdot x \cdot z \cos(x \cdot y \cdot z) + x^2 \cdot \frac{6y^2}{z^3}$

$f_{zzz} = -x^3 \cdot z \cdot \cos(x \cdot y \cdot z) + x^2 \cdot \frac{6y^2}{z^3}$

5- $f(x,y) = x^3 e^x + 10y$

$\frac{\partial f}{\partial x} =$ $U = x^3$, $U' = 3x^2$, $V = e^x$, $V' = e^x$

$\underline{3x^2 \cdot e^x + e^x \cdot x^3} = e^x(x^3+3x^2)$

$u' \cdot V + V' \cdot U$

$\frac{\partial f}{\partial y} = \cancel{x^3 \cdot e^x}_{0} + 10y = 10$

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$f(x,y) = 2y^2 \ln(x)$

$\frac{\partial f}{\partial x} =$ $u = 2y^2$, $u' = 0$, $V = \ln(x$, $V' = \frac{1}{x}$

$\boxed{\frac{1}{x} \cdot 2y^2}$

$0 \cdot \ln(x) + \frac{1}{x} \cdot 2y^2 \rightarrow u' \cdot V + V' \cdot U$

$\frac{\partial f}{\partial x} =$ $U = 2y^2$, $U' = 4y$, $V = \ln(x)$, $V' = 0$

$\boxed{4y \cdot \ln(x)}$

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$f(x,y) = 3y^2 \cos(x)$

$\frac{\partial f}{\partial x} = \underline{3y^2 \cdot -\text{sen}(x)}$

$U: 3y^2$, $U': 0$, $V: \cos(x)$, $V': -\text{sen}(x)$

$U = 3y^2$, $U': 6y$, $V = \cos(x)$, $V': 0$

$\frac{\partial f}{\partial y} = \underline{6y \cdot \cos(x)}$

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$f(x,y)\ 4y^2 e^y + 6x^2 =$

$\frac{\partial f}{\partial x} = 12x$

$\frac{\partial f}{\partial y} = 8y \cdot e^y$

$f(x,y) = 20x^2 y^2 \operatorname{sen}(x) = \frac{\partial f}{\partial x} = \underline{40xy^2 . \cos(x)}$

$U'.V + V'.U$

$\frac{\partial f}{\partial y} = \underline{40yx^2 . \operatorname{sen}(x)}$

$U = 20x^2y^2$

$U' = 40yx^2$

$V = \operatorname{sen}(x)$

$V' = 0$

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$f(x,y) = \frac{x+y}{x-y} = \frac{\partial f}{\partial x} = \left.\begin{array}{l} U = x+y \\ U' = x \\ V = x-y \\ V' = x \end{array}\right\}$

$\frac{x.(x-y) - x.(x+y)}{(x-y)^2}$

$\frac{\cancel{yx} - y^2 + yx - y^2}{(x-y)^2}$

$\frac{2yx - 2.y^2}{(x-y)^2}$

$\frac{\partial f}{\partial y} = \left.\begin{array}{l} U = x+y \\ U' = x \\ V = x-y \\ V' = x \end{array}\right\}$

$\frac{x.(x-y) - x.(x+y)}{(x-y)^2}$

$\frac{\cancel{x^2} - xy - \cancel{x^2} - xy}{(x-y)^2}$

$\boxed{\frac{-2xy}{(x-y)^2}}$

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$f(x,y) = \frac{e^x}{2x+3y} = \frac{\partial f}{\partial x} = \left.\begin{array}{l} U = e^x \\ U' = e^x \\ V = 2x+3y \\ V' = 3y+2 \end{array}\right\}$

$\frac{e^x . 2x + 3y - 3y + 2 . e^x}{(2x+3y)^2}$

$\frac{e^x (2x + 3y - 3y + 2)}{(2x+3y)^2}$

$\frac{e^x . (2x+2)}{(2x+3y)^2}$

$\frac{\partial f}{\partial y} = \left.\begin{array}{l} U = e^x \\ U' = 0 \\ V = 2x+3y \\ V' = 2x+3 \end{array}\right\}$

$\frac{-2x + 3 . e^x}{(2x+3y)^2}$

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7- a) Determine os pontos de máximos e de mínimos da função $f(x,y) = 3xy^2 + x^3 - 3x$

Vetor gradiente = $\nabla f(x,y) = (f_x, f_y)$

$f_x = \frac{\partial}{\partial x}(3xy^2) + \frac{\partial}{\partial x}(x^3) - \frac{\partial}{\partial x}(3x) = 3y^2 + 3x^2 - 3$

$f_y = \frac{\partial}{\partial y}(3xy^2) + \frac{\partial}{\partial y}(x^3) - \frac{\partial}{\partial y}(3x) = 6xy$

$(3y^2 + 3x^2 - 3, 6xy) \longrightarrow$ Gradiente

$\begin{cases} 3y^2 + 3x^2 - 3 = 0 \\ 6xy = 0 \end{cases}$ | $3x^2 = -3y^2 + 3$ $\quad x = \sqrt{\frac{-3y^2+3}{3}}$

Se $y = 0$ $\quad x = \sqrt{\frac{-3.0+3}{3}}$ $\quad x = \pm 1$

$P_1(1,0)$ $\quad P_2(-1,0) \longrightarrow$ Ponto Crítico

Ponto Máximo $(x,y) = (-1,0)$

Ponto mínimo $(x,y) = (1,0)$