SPH approaches to river modelling

of varied complexity

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Outline

What do we model?

Landscape evolution by water erosion: water flow plus sediment.

We route water

- from the slopes using a rainfall-runoff model
- from upstream sources
- downstream
- to a sink

Sediment is:

- entrained through erosion
- suspended over varied time scales
- advected with the flow (entrained)
- deposited due to gravity

General advantages of SPH

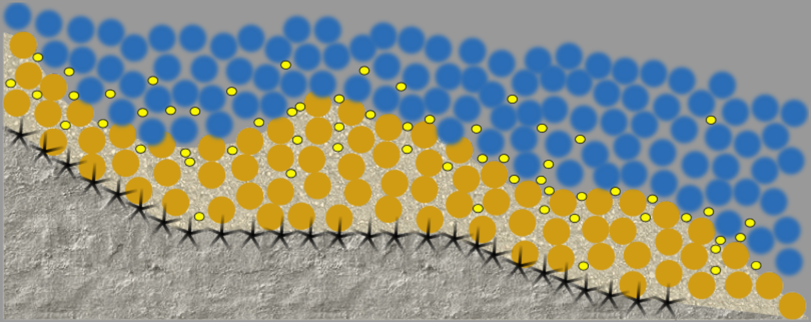
- Bridges the gap between the continuum and fragmentation in a natural way.
- Physically based approximation. Advection is treated exactly. Conservations laws are obeyed. Falls under the broader area of the statistical mechanics of particle systems. Complex physics may be incorporated into the formulation relatively easy.
- Accuracy. The motion of a continuum can be represented with very high accuracy by simulating the advection of an increasing large number of such particles. Resolution (h) can be made time and/or location dependent.
- Naturally allows complex deformable boundaries.
- A wide range of problems can be modelled: interaction of several fluid phases (different sets of particles, interfaces are trivial); free-surface flows, compressible or incompressible, viscous or inviscid, Newtonian or Non-Newtonian, turbulent or laminar; gravity, shear flow and other external fields; fractures and damaged solids.
- Computationally effective: only where the matter actually is.
- Parallel code implementations are available.
- One of the hottest current topic of research in numerous fields: engineering, environmental, and physical sciences.

Reduced complexity SPH for river modelling

Hierarchy of SPH reduced complexity approaches to the river flow:

1. "Full" 3d SPH multi-phase flow: water and sediment particles, "movable bed", grain size, suspended sediment
2. 3d SPH for water only, sediment is governed by a transport equation
3. 2d SPH particles dynamics constrained to the terrain (a Riemannian manifold); sediment transport is governed by a transport equation or empirical laws.

3D multi-phase multi-resolution SPH



3D multi-phase multi-resolution SPH: discussion

The model where everything is particles:

- Water and suspended sediment particles
- “Movable bed” made of of large sediment particles
- Fixed bed – the boundary

Problems:

- Very large number of particles, even when we lump water or sediment into large particles
- Even greater number of particles if we attempt to model suspended sediment as small particles
- In the SPH framework, suspended particles cannot be as small as to represent real individual sediment particles, they are still have to be “macroscopic”
- Great difference in density or particle sizes between the “phases”

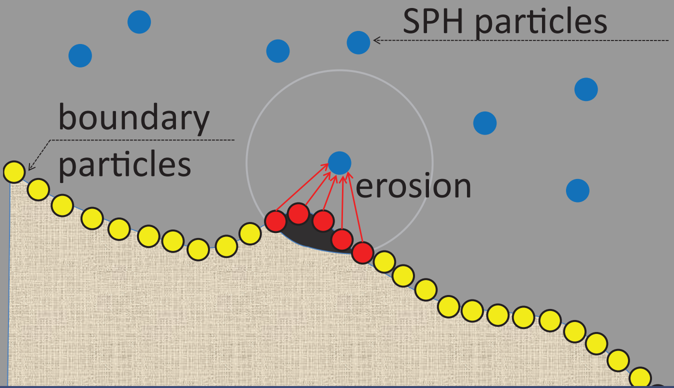
Conclusions:

- Due to the high computational cost could only be applied to small-scale problems
- Possibly less adequate method compared to other particle methods, such as DEM
- Instead, in the SPH approach suspended particles may be better viewed as “dissolved” sediment

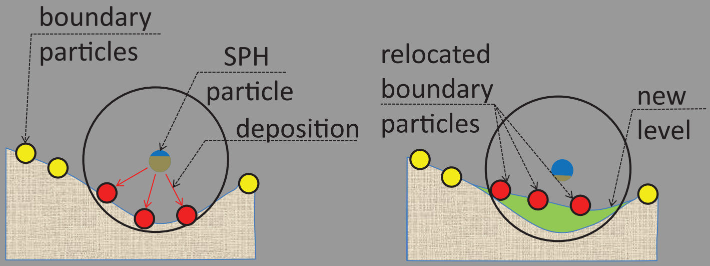
3D SPH of water and dissolved sediment

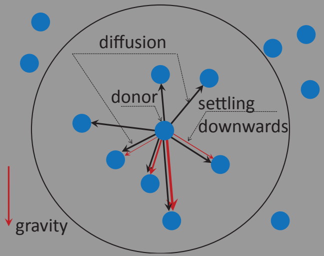
Making use of the big advantage of SPH: particles can be considered interpolation centers of other smooth fields, “carry” quantities at “sub-particle” scale.

- sample both liquid and solid phase at one type of particles (SPH particles)
- each particle carries one more attribute C – sediment volume fraction of the particle’s volume
- dissolved sediment will have a local relative settling velocity v_s ($u \to u + v_s$ for sediment dynamics)
- need to introduce an explicit advection into the solver



Sediment erosion, deposition, advection and diffusion





Erosion

- Shear stress:

$$\tau = K\theta^n$$

where $n = 1$ for a Newtonian fluid, and K is the shear stress constant

- Shear rate:

$$\theta = \frac{v_{rel}}{r_0}$$

where v_{rel} is the velocity of fluid close to the boundary and r_0 is the particle radius (smoothing length)

- Erosion rate:

$$J_\epsilon = K_\epsilon(\tau - \tau_c)^a$$

where K_ϵ is the erosion strength, τ_c is the critical shear stress, a measure of erosion resistance, a is a power coefficient (=1)

Sediment transport

- Advection-diffusion equation

$$\frac{dC}{dt} = -v_s \nabla C + \frac{1}{\rho} \nabla (D \nabla C) + J$$

where D is the molecular diffusivity, J stands for sources and sinks, primarily erosion and deposition rates, $-v_s \nabla C$ – advection the term

- SPH interpolants
  - the "donor-acceptor" scheme for the advection term

$$\frac{dC_i}{dt} = -\sum_j (v_s \cdot r'_{ij}) F(|r_{ij}|, h) \begin{cases} \frac{m_j C_j}{\rho_j} & , v_s \cdot r_{ij} \geq 0, \text{ i-acceptor} \\ \frac{m_i C_i}{\rho_i} & , v_s \cdot r_{ij} < 0, \text{ i-donor} \end{cases}$$

where $\nabla W = r'_{ij} F(|r_{ij}|, h)$ is the derivative of the smoothing kernel W, $r_{ij} = r_i - r_j$, $r'_{ij} = \frac{r_{ij}}{|r_{ij}|}$

  - the diffusion term (the flow from a higher concentration to a lower)

$$\frac{dC_i}{dt} = \sum_j \frac{m_j}{\rho_i \rho_j} D(C_i - C_j) F(|r_{ij}|, h)$$

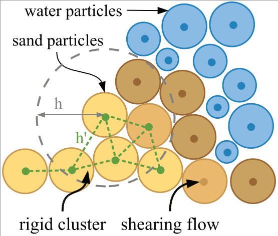
which is positive when $C_j > C_i$.

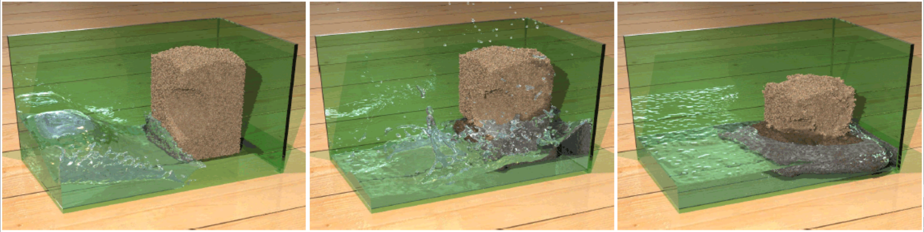
Deposition

- Same as the “donor-acceptor” advection scheme, with the SPH particles as donors and boundary particles as acceptors
- The exact expression will depend on the boundary implementation: triangular mesh over the height-field, dynamic boundary particles, etc.

3D SPH of water and granular matter

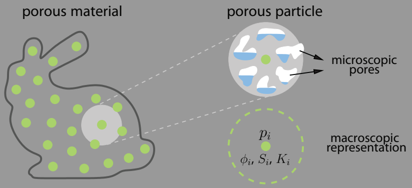
- decomposing the sand domain in regions moving rigidly and regions of shearing flow
- find an SPH expression for displacement gradient and strain rate tensor
- compare stress to material yielding using Mohr-Coulomb condition
- move particles marked rigid as a rigid body



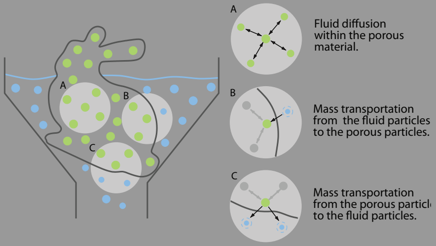


(Porous particles)

Same basic: idea intra- and inter-particle dynamics in addition to particle flow



Porous materials are represented at a macroscopic scale. Pores or cavities of a certain region in the volume are not modeled explicitly. Instead, the region is represented at a coarser level by the use of porous particles (right), which have porosity and permeability parameters that characterize the surrounding material.



Three different cases for fluid transportation in the presence of porous materials. A: Within the porous object fluid mass is diffused at a macroscale between the porous particles. B: Fluid particles near the porous material are treated as porous particles and mass is taken away from them if fluid is absorbed by the porous material. C: When the porous material emits fluid, mass is added to neighboring fluid particles, which are again treated as porous particles. If, after the diffusion process, more fluid remains to be emitted, new fluid particles will be created in the neighborhood of the porous particle.

3D SPH for dam breaking with debris

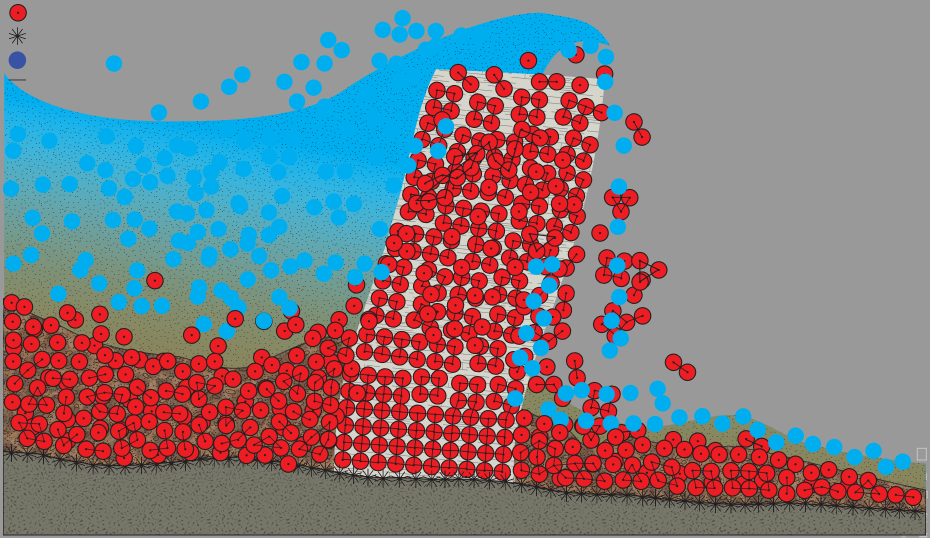
Supra-particle clusters

- Breaking. Each solid erodable particle can be initially a part of a larger local cluster to represent big solid constructs. The clusters are to be broken above critical shear stress.
- Coalescence. Cohesion or solidification can be modelled by allowing the liquid particles to coalesce into larger clusters when the sand concentration is within some cohesion interval (wetting when adding water to the pure sand? making clay by adding sand to water? Which fluids cannot be described by just increasing the viscosity and have to be modelled by clusterization?)

Sub-particle concetration

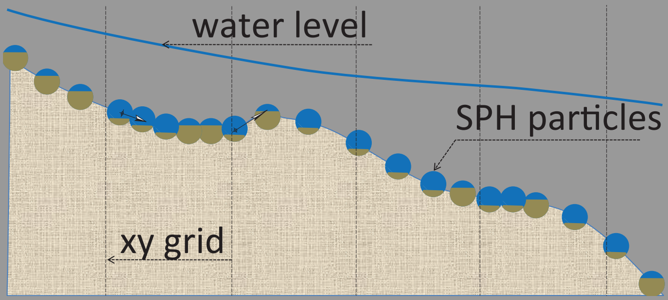
- Suspended, dissolved sediment
- Advection, diffusion, and erosion-deposition through SPH transport and donor-acceptor intercation

3D SPH for dam breaking with debris: model sketch



2D SPH over smooth terrain

- Taking one step further in reducing the complexity of the model.
- When do we really need the 3d SPH river flow?
  - flowing over or formation of vertical heterogeneous composition of the terrain: boulders, caves, vegetation, waterfalls, bridges, buildings
  - for smaller scale problems, when we may need to model turbulence
- Otherwise, a reduction from 3d to 2d can be a good idea



2D SPH over smooth terrain: formulation

Water flow:

- Columns of water undergoing a depth-averaged (shallow water) Navier-Stokes dynamics
- Surface roughness enters as a tangential resistance force
- What is the equation of state (how do we find the 2D pressure)?
- What is the expression for the local water depth, given the number density?

Sediment transport:

- A 2D diffusion-advection version of the 3D equation above
- Raster rules used in cellular automata modelling (we can call it a hybrid particle-raster method)

Boundaries:

- Smooth enough manifold, which
  - ia a surface of the type $z = z(x, y)$
  - is parameterized with the coordinates x, y, therefore requiring the tangential derivatives and other methods of differential geometry to write the dynamics equation correctly
  - represents the height-field of a various resolutions, obtainable from terrestrial laser scanning
  - reduces the multiresolution subgrid surface roughness to an effective friction parameter.

2D fluid equations on a manifold

On the manifold $z = h(x, y)$, metric induced from its embedding in $\mathbb{R}^3$,

$$g_{ij} = \begin{pmatrix} 1 + h_x^2 & h_x h_y \\ h_x h_y & 1 + h_y^2 \end{pmatrix}$$

Conservation of mass is still $\frac{\partial \rho}{\partial t} = \mathrm{div}(\rho \vec{v})$ if $\mathrm{div} X = \frac{1}{\sqrt{g}} \sum \frac{\partial}{\partial x^i} \left(\sqrt{g} X^i \right)$

Euler inviscid fluid linear momentum balance equation becomes $\frac{\partial v_i}{\partial t} + v^j \frac{\partial v_i}{\partial x^j} = -\frac{1}{\rho} \frac{\partial p}{\partial x^i} + f_i$ with the velocity covector $v = v_i dx^i$ and $v_i = g_{ij} v^j$. (Velocity covectors acting on a tangent gives the corresponding projection) The volume density of the forces in i-direction is given by f_i.

2D SPH over smooth terrain: advantages

Advantages:

- The least possible number of particles, therefore potentially the most effective for the large scale spatio-temporal modelling, such as landscape evolution
- Only tangential forces and dynamics, another big saving on the computational time
- Overcomes shortcomings of the *ad hoc* water routing tricks in the CA models
- At the same time, may take advantage of raster methods such as sediment erosion/deposition or rainfall-runoff models (per-cell birth of SPH particles off the slopes to the channel)