Algorithmic complexity and practical performance of Top-k compression for communication efficient distributed optimization

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Team

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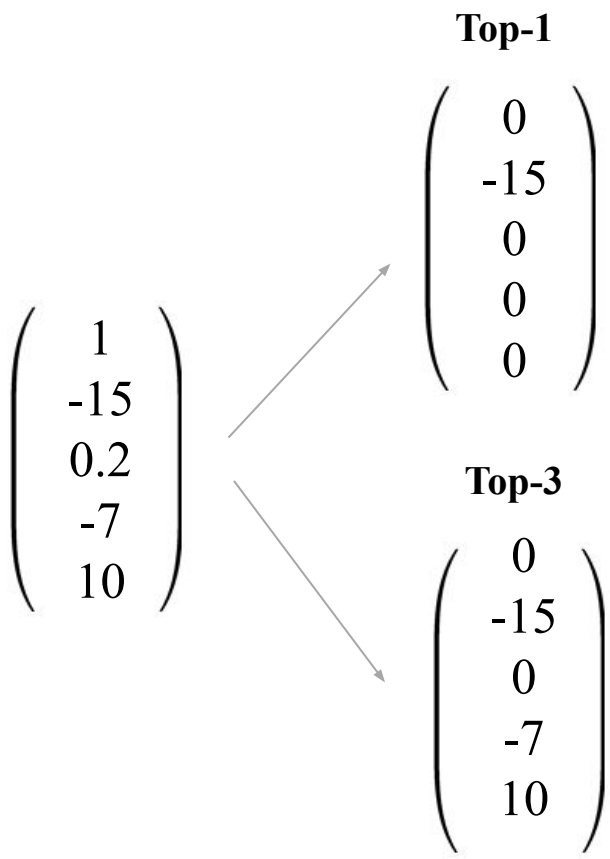
Goal of the project

Top-k as a Compressor

$$\begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix}$$

Top-k as a Compressor



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$$\begin{pmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_d \end{pmatrix}$$

Assuming that d is large ($d \approx 10^5$)

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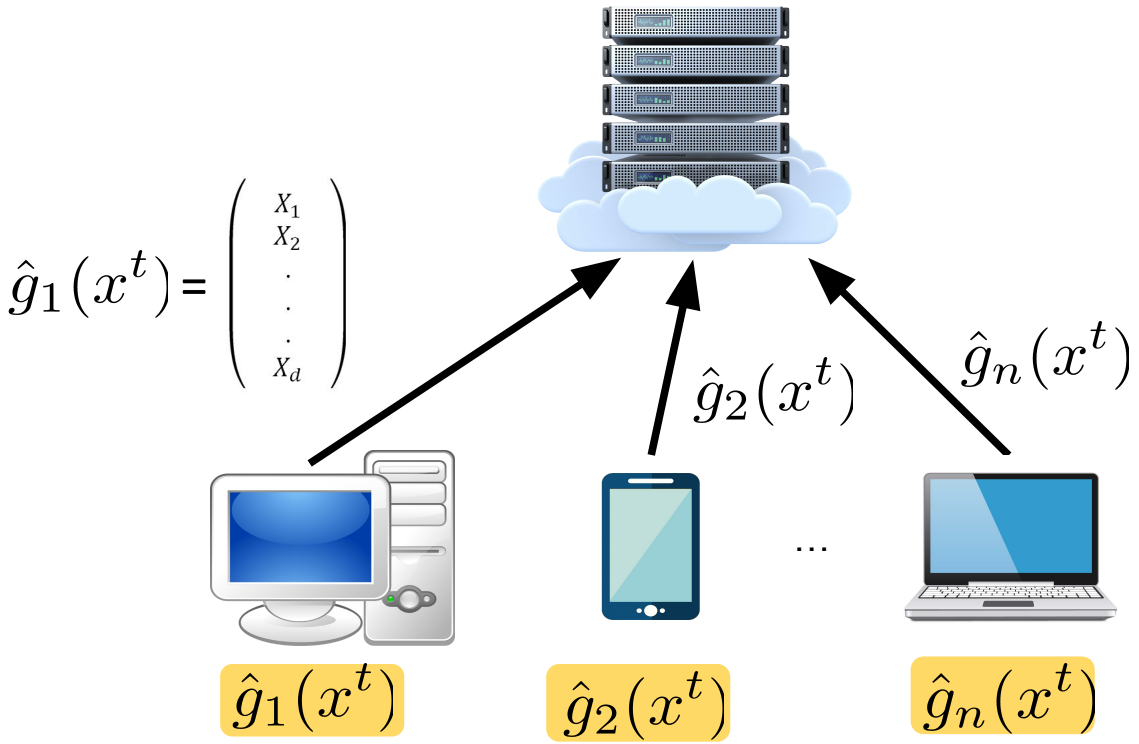
Goal:
Measure the time complexity of Top-k compression for large d

Why is Top-k important?

DSGD: Distributed Stochastic Gradient Descent



DSGD: Distributed Stochastic Gradient Descent

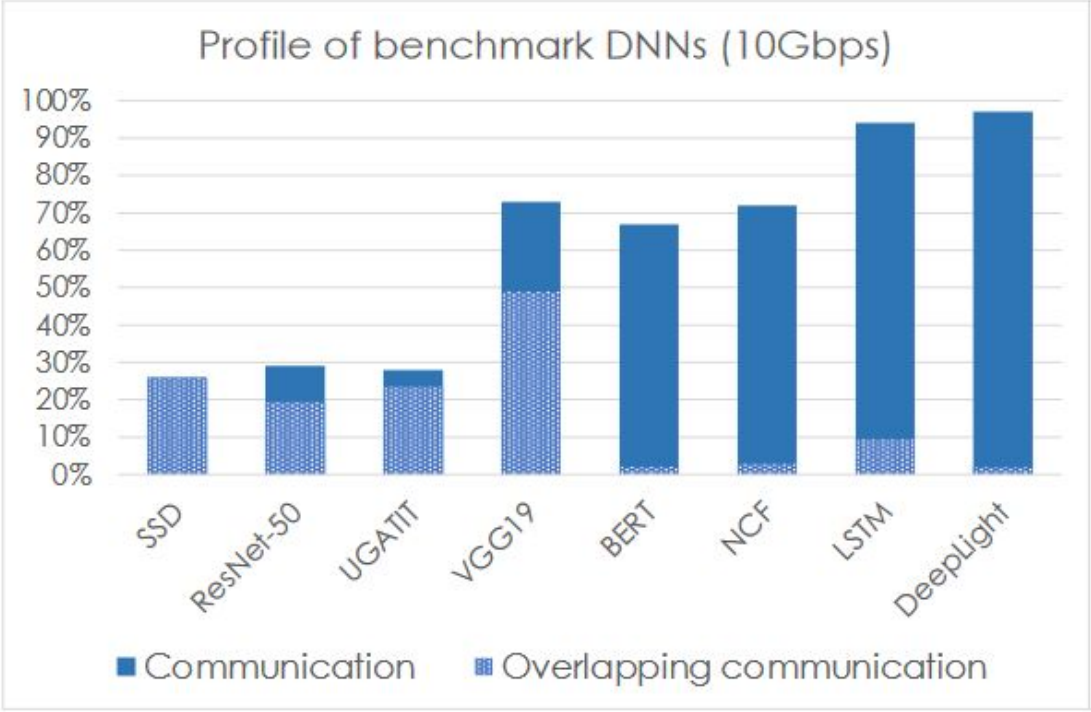
Machine Learning models are increasing in size:

Architecture	Year	Top-1 Accuracy	# Parameters
DenseNet-169	2017	76.2%	14M
Inception-v3	2016	78.8%	24M
Inception-resnet-v2	2017	80.1%	56M
PolyNet	2017	81.3%	92M
SENet	2018	82.7%	146M
GPipe	2018	84.3%	557M
ResNeXt-101 32x48d	2019	85.4%	829M

Source: Efficient inference in Deep Learning - where is the problem? Deci. (2021, November 22). Retrieved December 4, 2021, from https://deci.ai/resources/blog/problem-efficient-inference-deep-learning/.

DSGD: Distributed Stochastic Gradient Descent

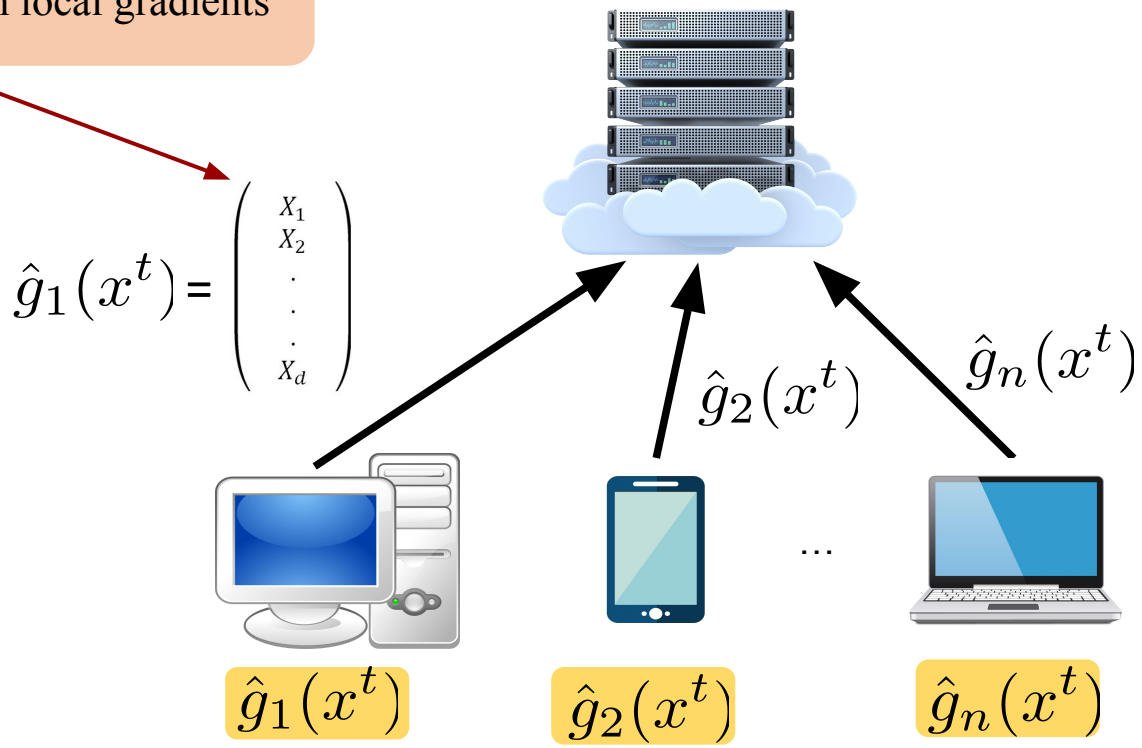
Benchmark on datacenter with 10 Gbps network speed



Source: DeepReduce A Sparse-tensor Communication Framework for Federated Deep Learning Deci. (2021, November 29). Retrieved December 4, 2021, from KAUST CS Graduate Seminar, Professor Panos Kalnis

DSGD: Distributed Stochastic Gradient Descent

Objective:
Apply Top-k compression on local gradients



Algorithms that performs Top-k compression

Algorithms

1. Tournament Method — Worst case: $O(d + k \log(d))$

2. Quickselect
   a. Deterministic Pivot — Worst case: $O(d^2)$ Average: $O(d)$
   b. Random Pivot — Worst case: $O(d^2)$ Average: $O(d)$
   c. Median of medians — Worst case: $O(d)$ Average: $O(d)$
      but worse than other pivots

Algorithms Modifications

We are interested in finding all elements larger than or equal to kth element.
Not only kth largest element!

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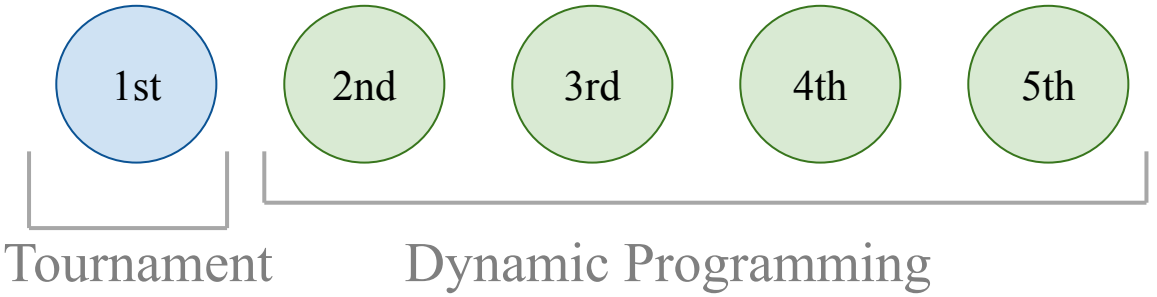
Find the Top-5 elements:

Tournament Method + DP



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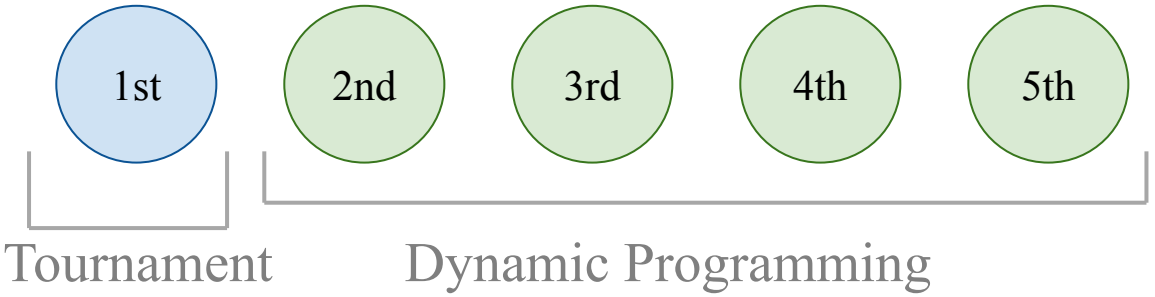
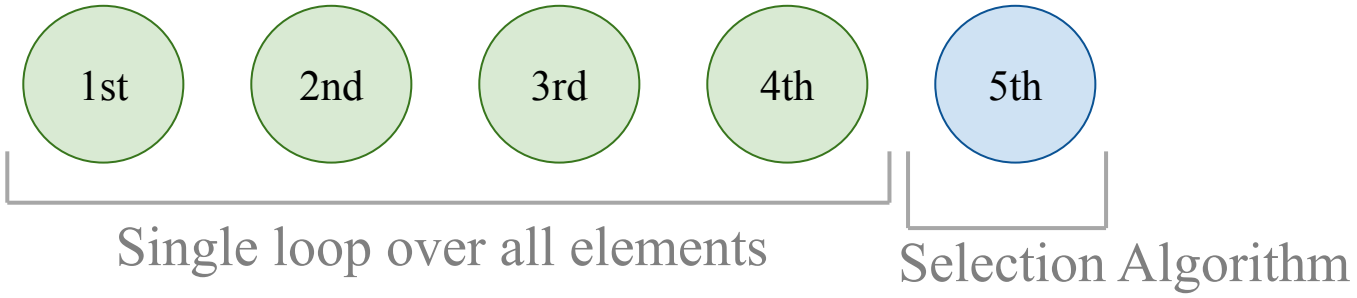


Better Approach



Implementation, Experiments

Experiments

1. **Synthetic experiments: inputs are sampled from known priors**

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2. **Real experiments: inputs are taken from ML model training procedure**

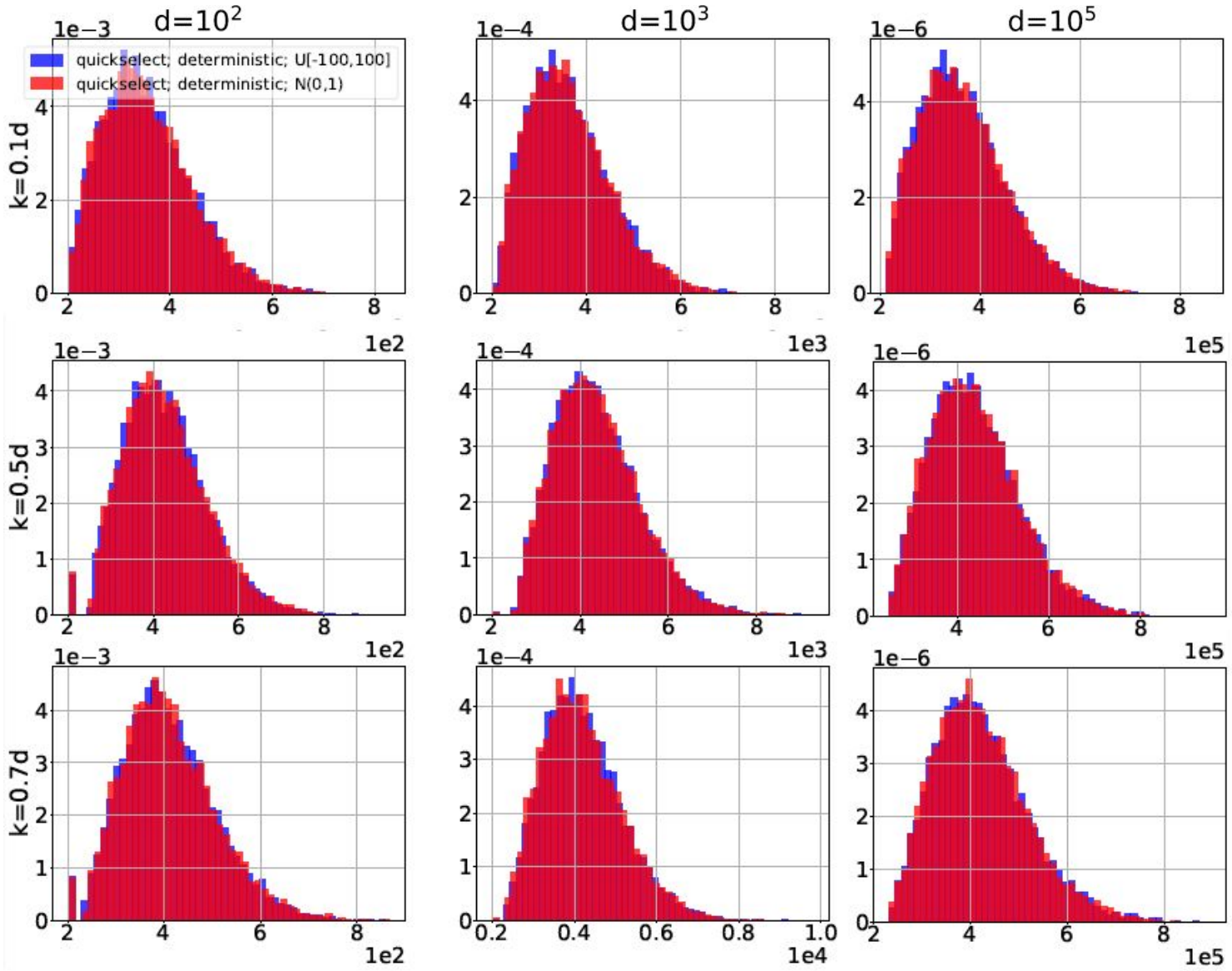
Synthetic Experiments

Synthetic experiments

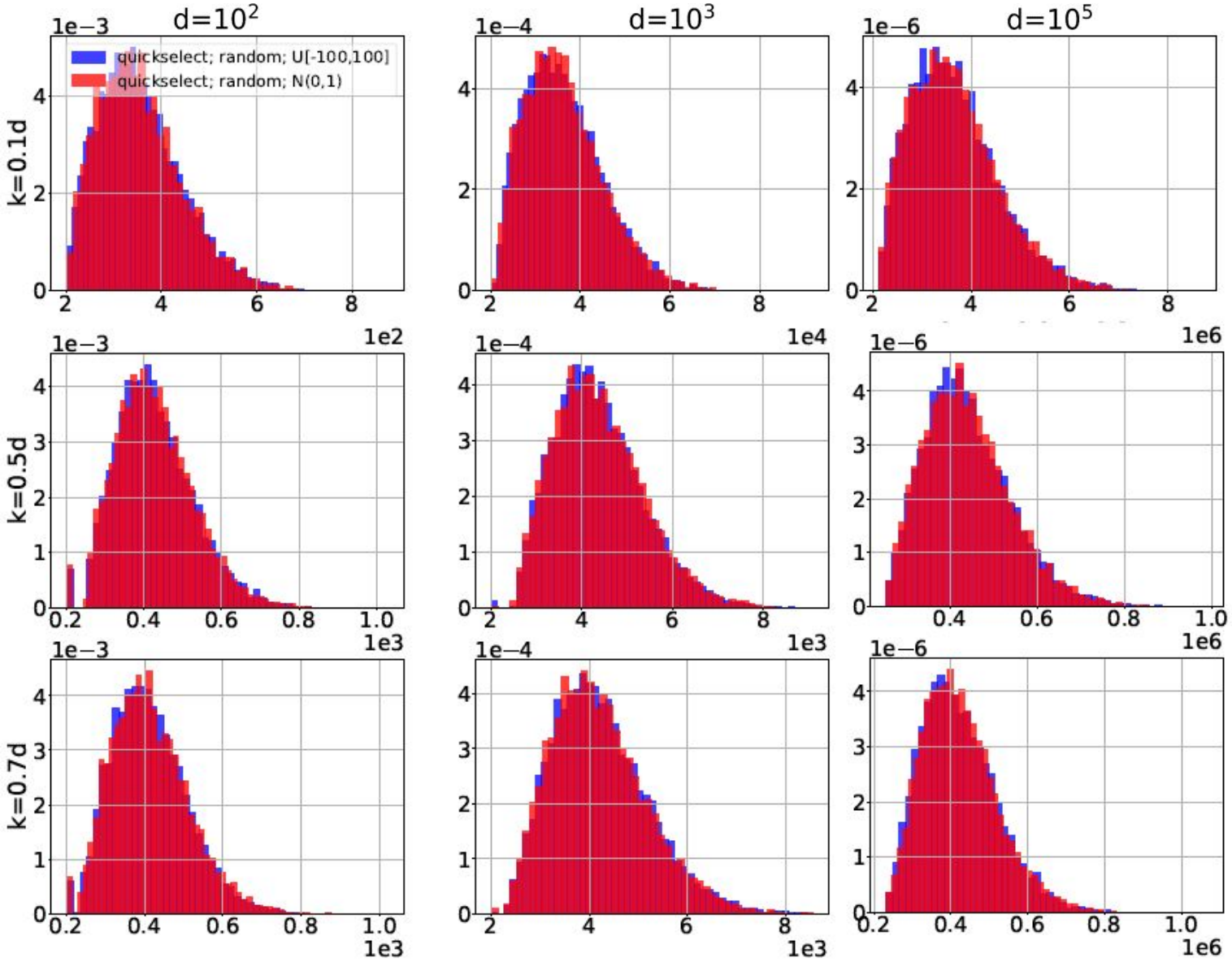
Synthetic data

- Fix some values of d from 100 to 100000 and for each d fix k from 0.01d to 0.9d
- Sample ~10000 vectors from known prior and compute time complexity for each vector
- Plot the empirical distribution of the time complexity
- Compare medians of the empirical distributions

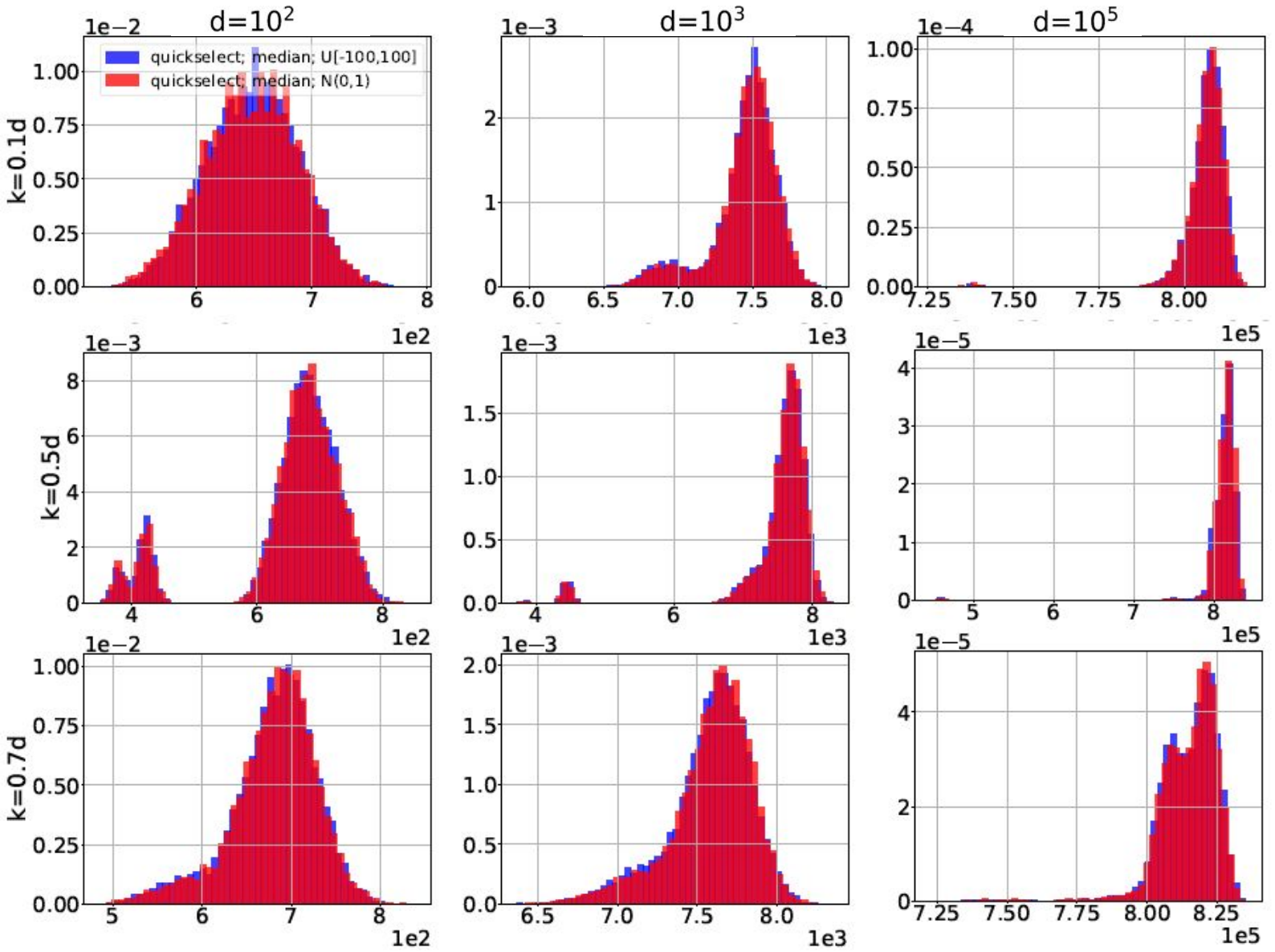
Prior Impact (Deterministic Quickselect)



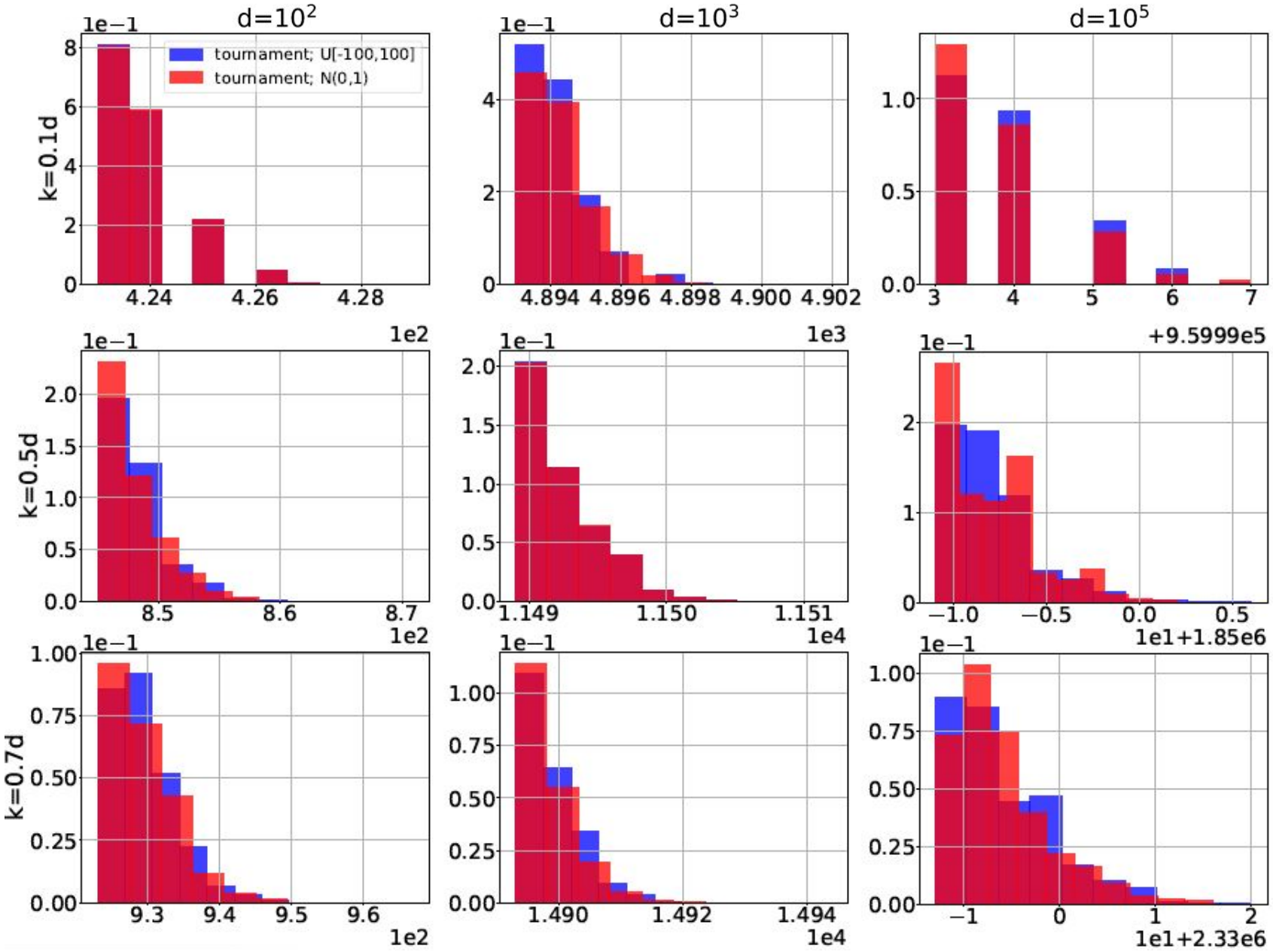
Prior Impact (Random Quickselect)



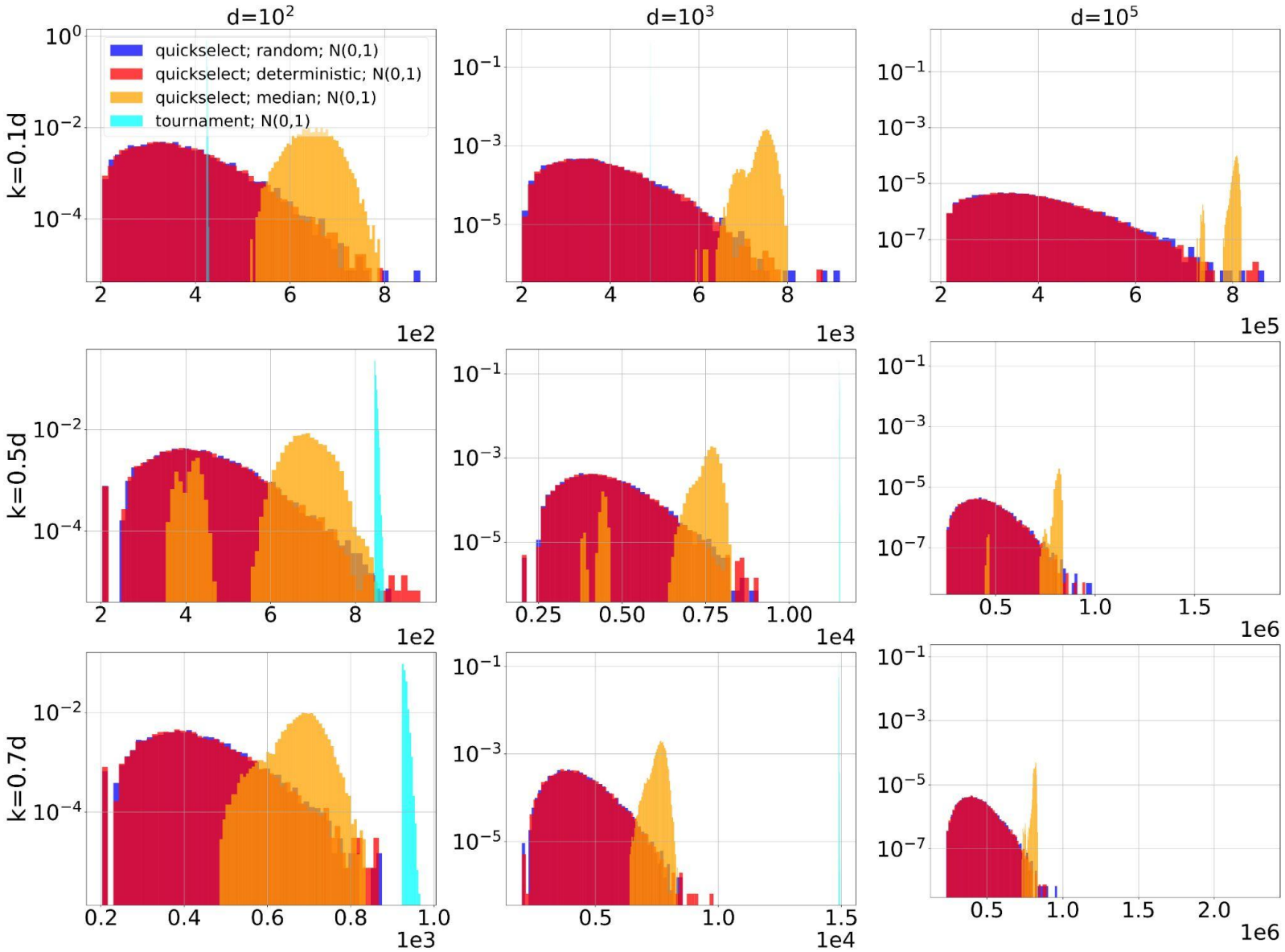
Prior Impact (Median Quickselect)



Prior Impact (Tournament)



Number of comparison histograms (log scale)



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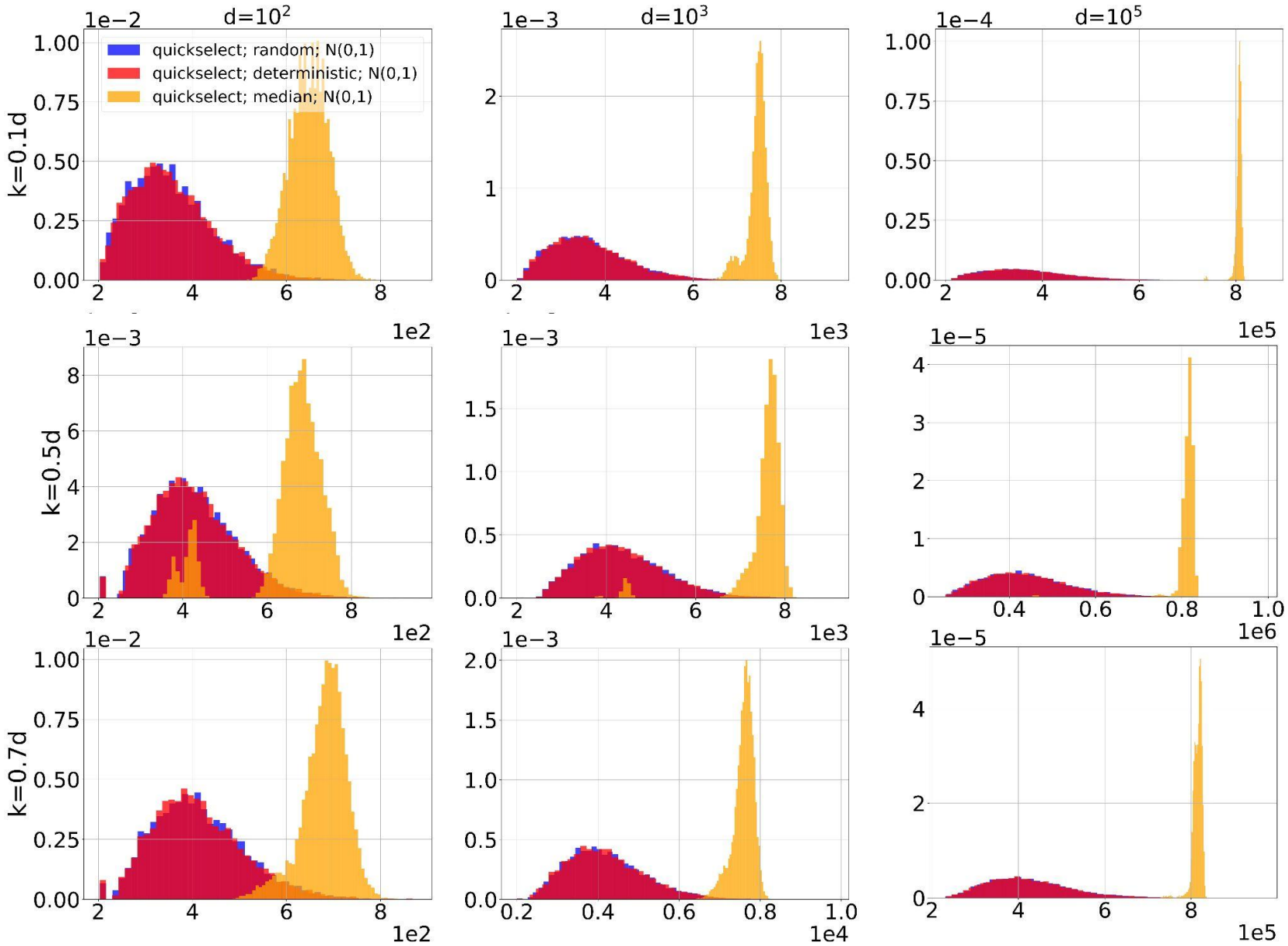
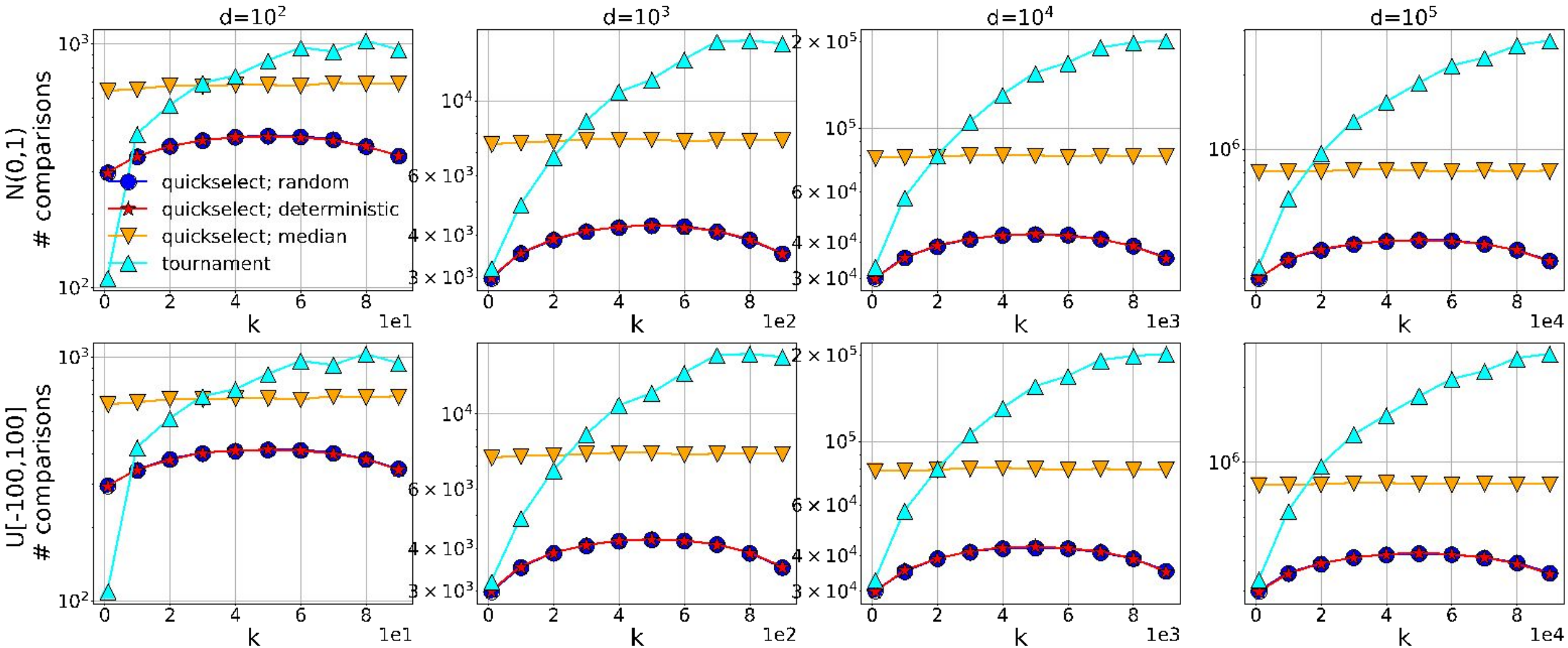
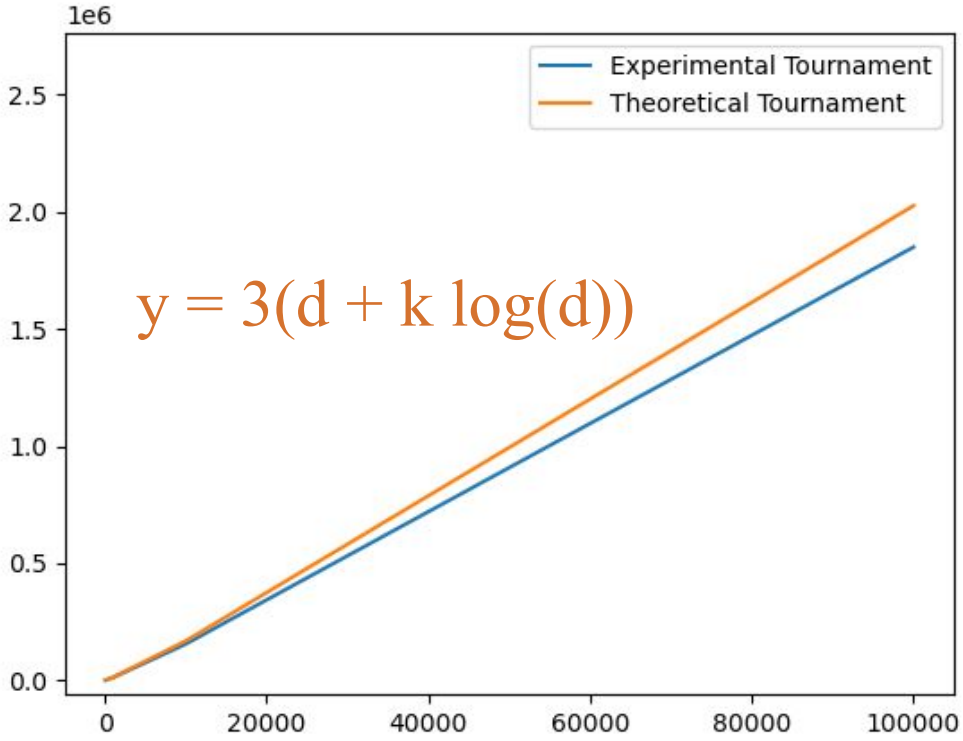
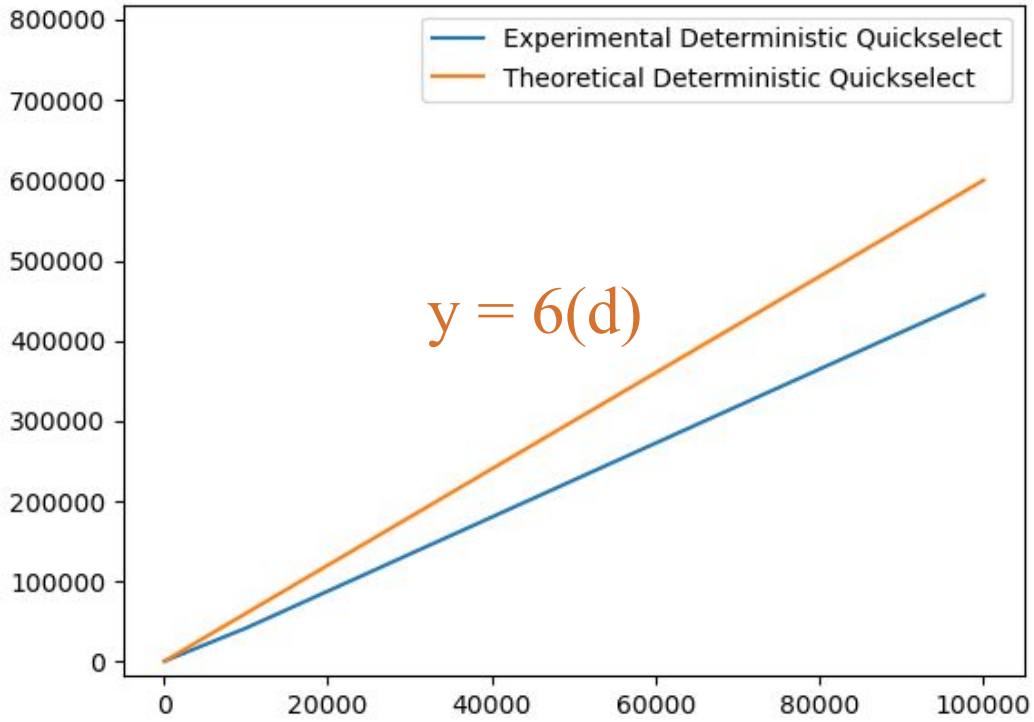


Number of comparisons over k values



Theoretical vs Experimental Time Complexity





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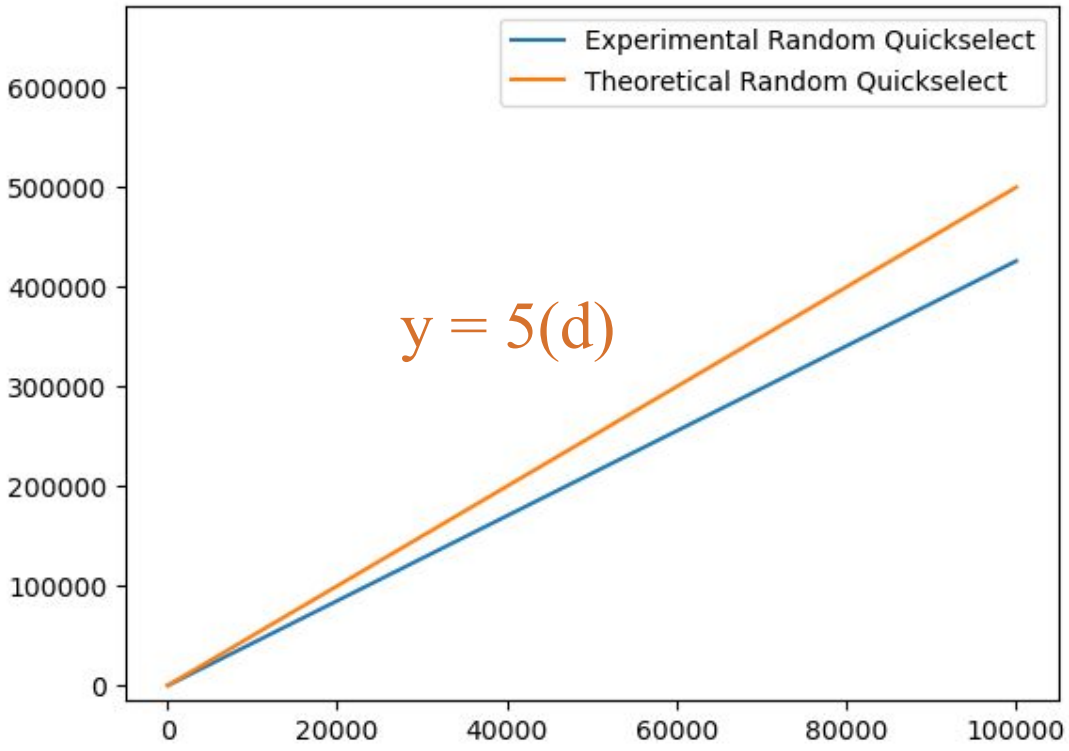

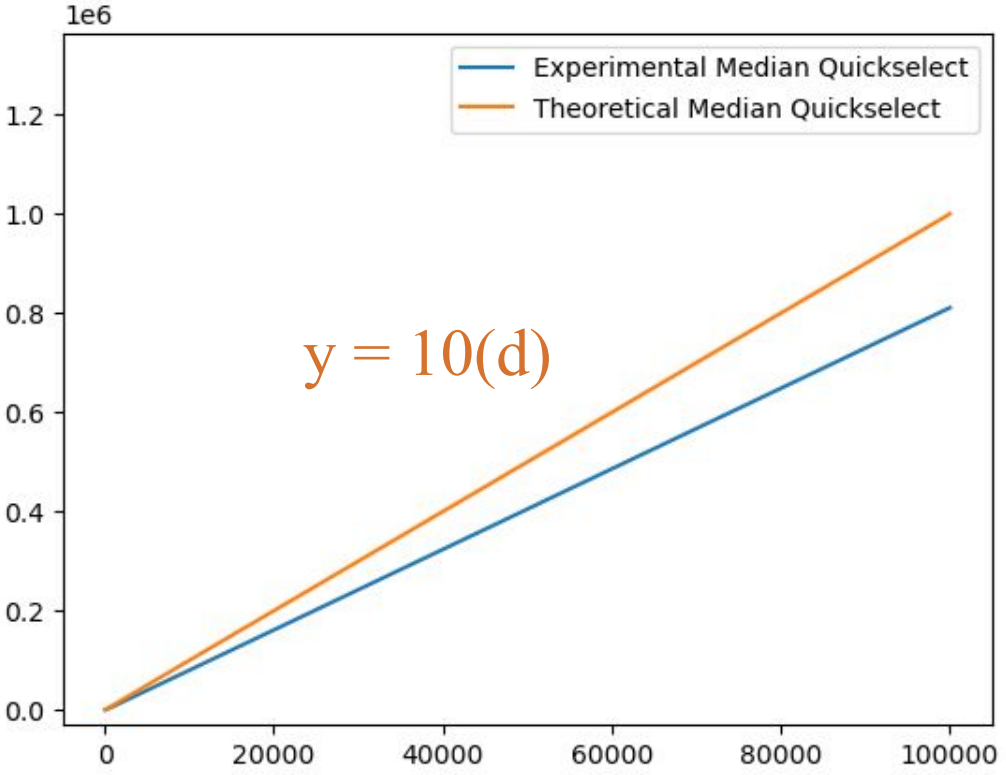

Real Experiments

Real experiments

$$\min_{x\in\mathbb{R}^d}\left\{f(x)=\frac{1}{N}\sum_{i=1}^{N}\log\left(1+\exp\left(-b_i a_i^\top x\right)\right)+\lambda\sum_{j=1}^{d}\frac{x_j^2}{1+x_j^2}\right\}$$

where $a_i\in\mathbb{R}^d, b_i\in\{-1,1\}$ are the training data provided by [1] and $\lambda>0$ is the regularization parameter;

[1] Chih-Chung Chang and Chih-Jen Lin. **Libsvm: a library for support vector machines.** ACM TIST, 2(3):1–27, 2011.

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Dataset	n	N (total # of datapoints)	d (# of features)	N_i
w8a	100	49,749	300	497
a9a	100	32,560	123	325

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Instead of the classical distributed sgd we will consider more stable method - **EF21** [2],[3]

[1] Chih-Chung Chang and Chih-Jen Lin. **Libsvm: a library for support vector machines.** ACM TIST, 2(3):1–27, 2011.
[2] Peter Richtárik, Igor Sokolov and Ilyas Fatkhullin **EF21: A new, simpler, theoretically better, and practically faster error feedback**, NeurIPS, 2021 (oral)
[3] Ilyas Fatkhullin, Igor Sokolov, Eduard Gorbunov, Zhize Li, Peter Richtárik **EF21 with Bells & Whistles: Practical Algorithmic Extensions of Modern Error Feedback**, NeurIPS, 2021 (OptML workshop)

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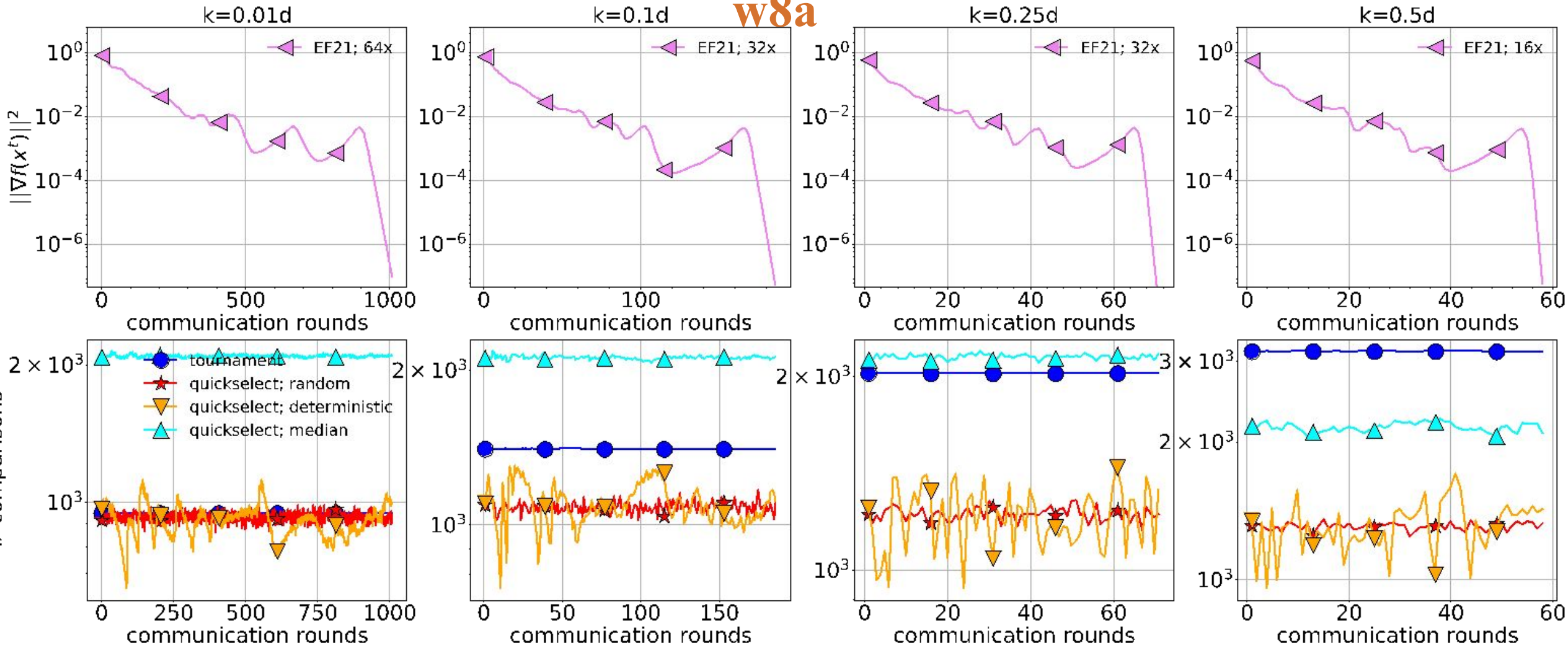
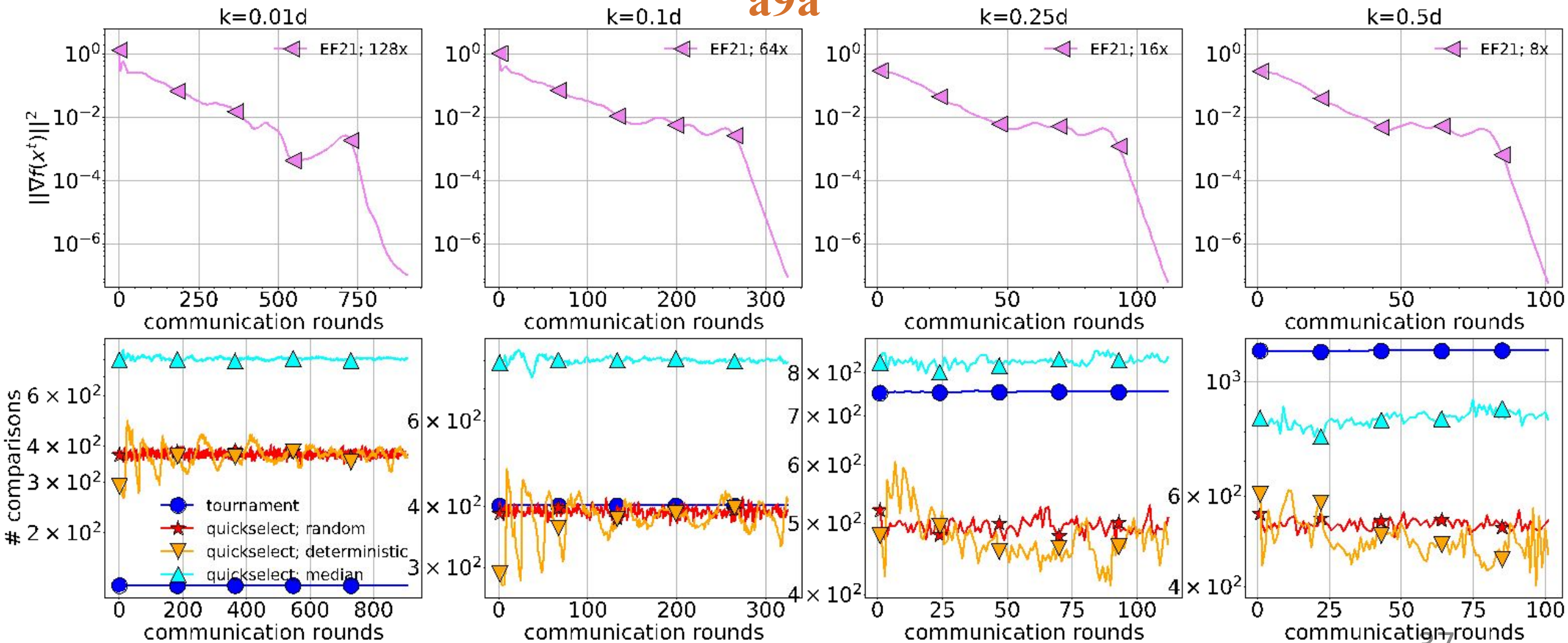


Real experiments

a9a



Conclusion

1. Synthetic experiments: inputs are sampled from known priors

- Choice of prior does not affect on the distribution of the time complexity
- For small d and $k < 0.1d$ Tournament method has the lowest time complexity, whereas for $k > 0.2d$ it is the slowest method
- Quick select with random and deterministic pivots show similar results, outperforming median one and Tournament method

Conclusion

2. Real experiments: inputs are taken from ML model training procedure

- For both datasets, for $k<0.1d$ Tournament methods shows the best time complexity and degrades with the increase of k

- Quick select with random and deterministic pivots shows similar results, and do not increase significantly with the increase of k

Thank you for your attention!

Questions?