Numerical Methods – Laboratory 5
Root - finding algorithms

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1 Introduction

The goal of the lab is to implement the two root-finding algorithms: bisection and secant method and use them to solve the following two problems:

1. Figure 1 shows a circuit with a resistor, an inductor, and a capacitor in parallel. Kirchhoff's rules can be used to express the impedance magnitude of the system as:

$$|Z| = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} \tag{1}$$

   where ω is the angular frequency. Find the ω that results in an impedance of 75 Ω. For the following parameters: $R = 725\Omega$, $C = 8 \times 10^{-5}F$, $L = 2H$, with initial interval $\omega \in \langle 0, 50 \rangle$ and $\epsilon_f = 10^{-12}$.

2. The upward velocity of a rocket can be computed by the following formula:

$$v = u\ln\left(\frac{m_0}{m_0 - qt}\right) - gt. \tag{2}$$

   where t denotes the time, v is the upward velocity, m_0 - the initial mass of the rocket at time $t = 0$, u - the velocity at which fuel is expelled relative to the rocket, q - the fuel consumption rate, $g = 9.81m/s^2$ - the downward acceleration of gravity.

   Compute the time, at which $v = 750m/s$, assuming:

   $m_0 = 150000kg$, $q = 2700kg/s$, $u = 2000m/s$. Set the initial interval to: $t \in \langle 0, 50 \rangle$ and the tolerance to: $\epsilon_f = 10^{-12}$.

2 Tasks

1. (4 points) Implement in Matlab the two root-finding methods: bisection and secant in a form of functions with the following input parameters:

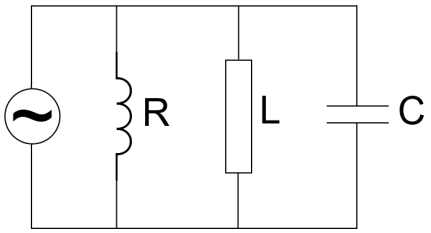

Figure 1: A circuit with a resistor, an inductor, and a capacitor in parallel.

- initial interval
- tolerance: (ϵ_f)
- handle to a proper function (use a Matlab handle : @, see the attached example files)

2. (3 points) Compare the two methods in terms of the number of iterations. To this end, you need to implement formulas expressing the differences between sought quantities (1) - (2) and their target values (75 *Omega* and 750m/s). Then the names of these functions can be used directly as the parameters for the bisection and secant procedures, using the @ handle. Generate the *.png plots:

   - approximated root value x_i in a function of the iteration i,
   - difference between x_i and x_{i-1} in a function of the iteration i. What can you say about the convergence rate?

3. (1 points) Use the embedded Matlab function: *fzero* to find the roots of the *tg(x)* function, for the guess root-value: 6.0 and 4.5. Set the proper parameters in *OPTIONS* structure to obtain the convergence data:

   - options = optimset('Display','iter')
   - fzero(@tan,6, options)

   Interpret and comment the obtained results in:
   NMlab4_index_task3.txt.

Remember to upload your plots, codes and data files to e-nauczanie website.