Fractal-Wave Algebra: A New Mathematical Framework Based on Collatz Dynamics

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Abstract

This paper introduces the concept of **Fractal-Wave Algebra**, a mathematical framework that generalizes linear algebra to describe branching, self-similar, and dynamically complex processes. Inspired by the Collatz conjecture and fractal geometry, this algebra is designed to model natural phenomena—such as lightning, river networks, plant growth, and turbulence—where classical linear methods fail. The core of this approach is the integration of fractal dimension, recursive branching, and wave dynamics into a unified set of equations and algorithms.

1. Introduction

Classical linear algebra is powerful for systems with linear relationships and integer dimensions. However, many real-world processes exhibit fractal, nonlinear, and dynamically branching behavior. **Fractal-Wave Algebra** addresses this gap by introducing new mathematical objects and operations that capture the essence of self-similarity, scale invariance, and recursive dynamics, using the Collatz process as a foundational branching rule

2. Key Definitions

- **Fractal-Wave Function:**
  ψ~(x,t,δ)ψ~(x,t,δ) — describes the system's state at position xx, time tt, and fractal scale δδ.
- **Fractal Dimension:**
  DfDf — a non-integer dimension characterizing the degree of branching and self-similarity in the structure.
- **Collatz Branching Operator:**

$$\mathcal{K}(N) = \begin{cases} N/2, & \text{if } N \text{ is even} \\ 3N+1, & \text{if } N \text{ is odd} \end{cases}$$

where N is the number of branches or elements at level k.

3. Fractal-Wave Equation

The evolution of a fractal-wave system is governed by:

$$\frac{\partial^2 \tilde{\psi}}{\partial t^2} + \alpha(\delta)\frac{\partial^2 \tilde{\psi}}{\partial \mu^2} = c^2 \nabla^2_{D_f} \tilde{\psi} + \mathcal{K}(\tilde{\psi}, k)$$

$\nabla^2_{D_f}$: Fractal Laplacian operator in dimension D_f.

$\alpha(\delta)$: Scale-dependent coefficient.

$\mathcal{K}(\tilde{\psi}, k)$: Collatz branching applied to the wave function at level k.

4. Collatz Dynamics in Fractal Space

The Collatz process is generalized to continuous and even complex domains, enabling fractal visualization and analysis

$$\text{For } z \in \mathbb{C}, \quad \text{Collatz}(z) = \frac{z}{2} \cdot \cos^2\left(\frac{\pi}{2} z\right) + (3z+1) \cdot \sin^2\left(\frac{\pi}{2} z\right)$$

This function reduces to the standard Collatz rule for integer arguments and generates intricate fractal structures when iterated in the complex plane.

5. Algorithmic Implementation (Pseudocode)

```
def collatz_branching(N):
    if N % 2 == 0:
        return N // 2
    else:
        return 3 * N + 1


def fractal_wave_process(state, levels):
    for k in range(levels):
        state = collatz_branching(state)
        # Update the fractal-wave function here
    return state
```

6. Applications

- Modeling the branching of lightning, rivers, and biological systems.
- Simulating the evolution of fractal waves in physics and engineering.
- Visualizing self-similar patterns and recursive energy transfer.

7. Conclusion

Fractal-Wave Algebra provides a new language for describing and simulating complex, branching, and self-similar processes. By uniting fractal geometry, wave dynamics, and Collatz-like recursion, it opens new directions for both theoretical research and practical modeling beyond the limits of linear algebra.

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8. References

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