IEL – protokol k projektu

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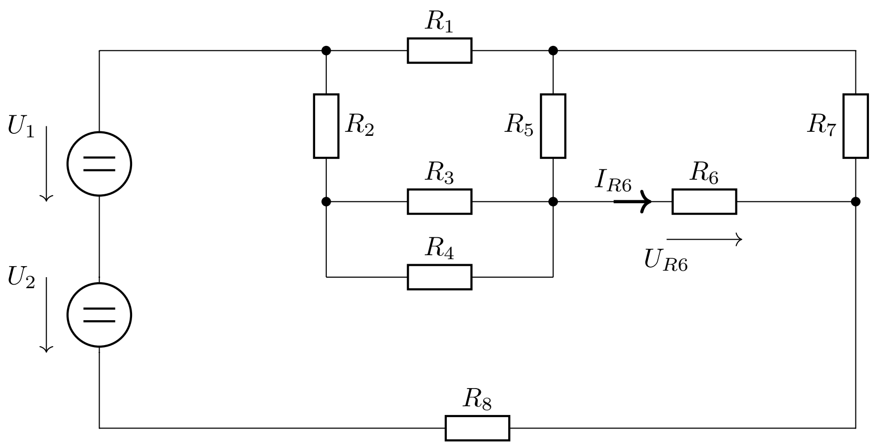
Obsah

Príklad 1H

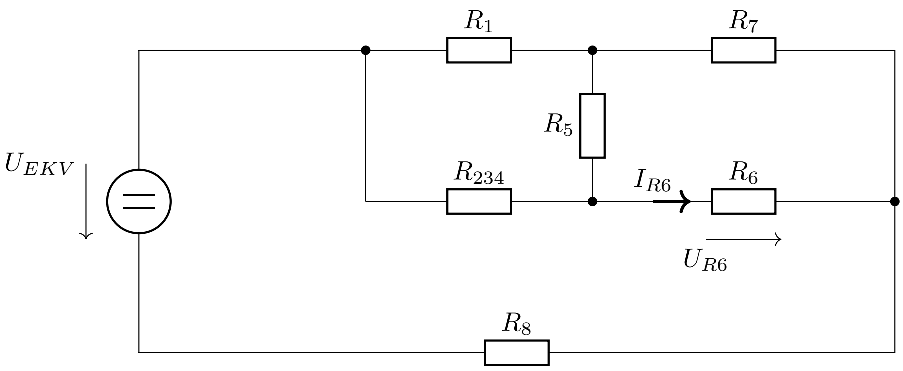
Zadané hodnoty:

$U_1 = 125\text{V},\ U_2 = 80\text{V}$

$R_1 = 680\Omega,\ R_2 = 600\Omega,\ R_3 = 260\Omega,\ R_4 = 310\Omega$
$R_5 = 575\Omega,\ R_6 = 870\Omega,\ R_7 = 355\Omega,\ R_8 = 265\Omega$



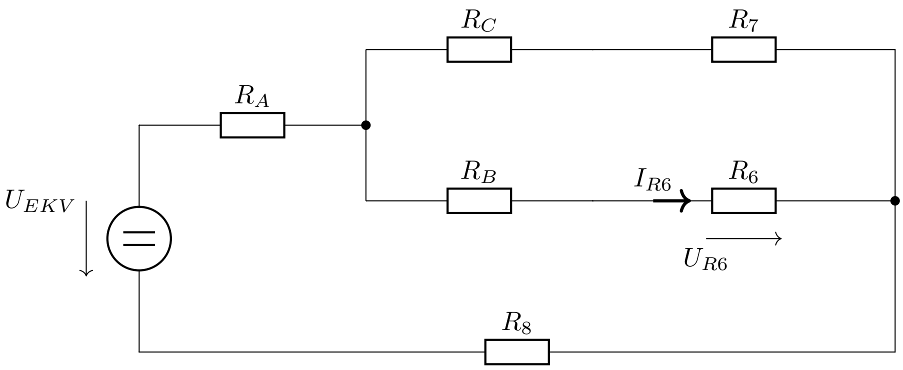
Obvod upravíme pomocou metódy postupného zjednodušovania:



$$R_{234} = \frac{R_3 R_4}{R_3 + R_4} + R_2 = \frac{260 \times 310}{570} + 600 = \frac{42260}{57}\Omega$$

$$U_{EKV} = U_1 + U_2 = 215V$$

Aplikujeme prevod hviezdy na trojuholník:

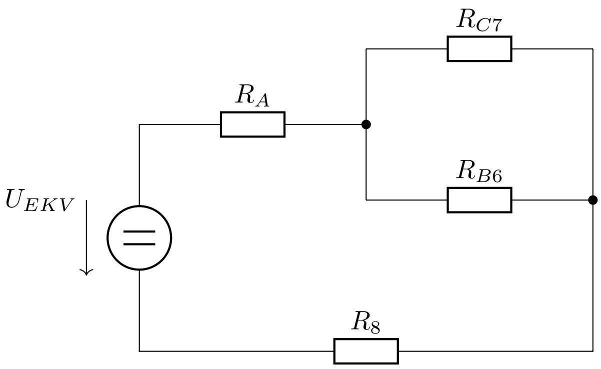


$$R_A = \frac{R_{234}R_1}{R_1 + R_5 + R_234} = \frac{574360}{22759}\Omega$$

$$R_B = \frac{R_{234}R_5}{R_1 + R_5 + R_234} = \frac{4859900}{22759}\Omega$$

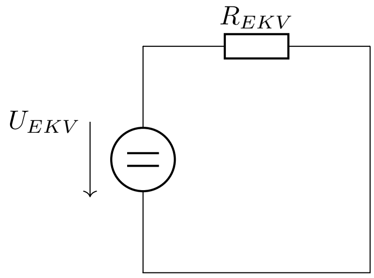
$$R_C = \frac{R_1 R_5}{R_1 + R_5 + R_234} = \frac{4457400}{22759}\Omega$$

Pokračujeme v zjednodušovaní obvodu:



$$R_{C7} = R_C + R_7 = \frac{12536845}{22759}\Omega$$

$$R_{B6} = R_B + R_6 = \frac{24660230}{22759}\Omega$$



$$R_{EKV} = R_A + \frac{R_{C7} \times R_{B6}}{R_{C7} + R_{B6}} + R_B =$$

$$= \frac{574360}{22759} + \frac{12366459246974}{33862729197} + \frac{24660230}{22759} = \frac{1313391701}{1487883}\Omega$$

Vypočítame I_{EKV}:

$$I_{EKV} = \frac{U_{EKV}}{R_{EKV}} = 0,243564A$$

Teraz môžeme dopočítať hladané hodnoty U_6 a I_6:

$$U_{R_{C7B6}} = I_{EKV} \times R_{C7B6} = \frac{125501011667915639}{1410947049875000}V$$

$$I_{R_6} = I_{R_{B6}} = \frac{\frac{R_{C7B6}}{}}{R_{B6}} = \frac{50892068593}{619951250000} \doteq 82,0904mA$$

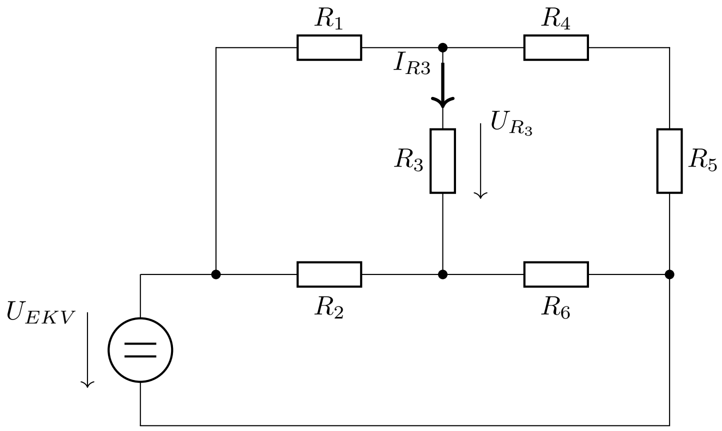
$$U_{R_6} = I_{R_6} \times R_6 = \frac{4427609967591}{61995125000} \doteq 71,4187V$$

Príklad 2F

Zadané hodnoty:

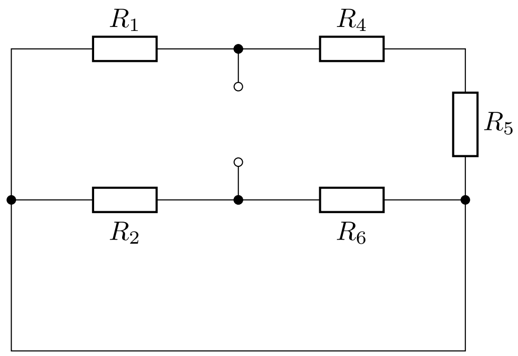
$U = 130\text{V}$

$R_1 = 180\Omega,\ R_2 = 350\Omega,\ R_3 = 600\Omega,$
$R_4 = 195\Omega,\ R_5 = 650\Omega,\ R_6 = 250\Omega$

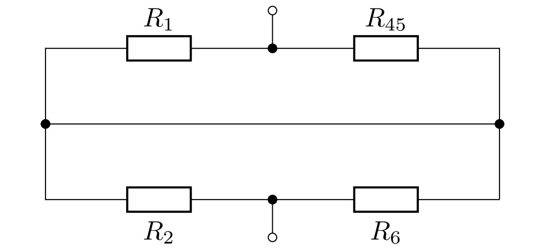


Odvodenie a výpočet R_{TH}

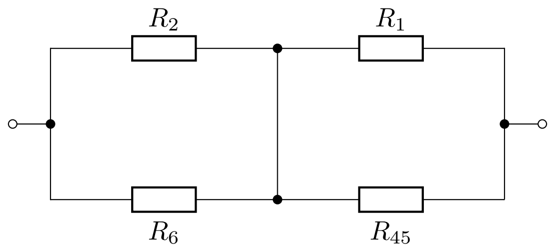
Odstránime hľadaný rezistor a skratujeme zdroj:



Následne obvod upravíme:



$$R_{45} = R_4 + R_5 = 845\Omega$$



$$R_{145} = \frac{R_{45} \times R_1}{R_{45} + R_1} = \frac{845 \times 180}{1025} = \frac{6084}{41}\Omega$$

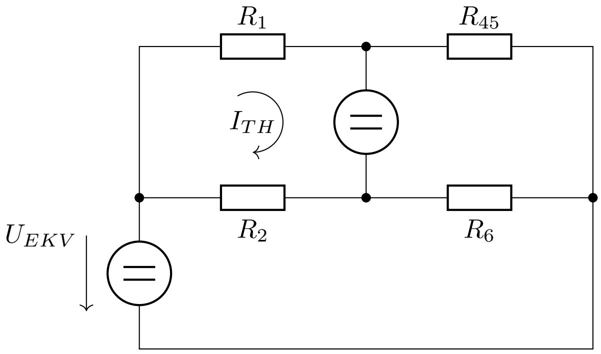
$$R_{26} = \frac{R_2 \times R_6}{R_2 + R_6} = \frac{350 \times 250}{600} = \frac{875}{6}\Omega$$

Vypočítame hodnotu R_{TH}:

$$R_{TH} = R_{145} + R_{26} = \frac{875}{6} + \frac{6084}{41} = \frac{72379}{246}\Omega$$

Odvodenie a výpočet U_{TH}

Namiesto R_3 dosadíme fiktívny napäťový zdroj a pomocou slučky I_{TH} vypočítame U_{TH}:



Najprv vypočítame U_{R_1} a U_{R_2}:

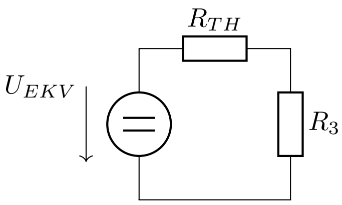
$$U_{R_1} = U_{EKV} \times \frac{R_1}{R_1 + R_{45}} = 130 \times \frac{180}{1025} = \frac{936}{41} V$$

$$U_{R_2} = U_{EKV} \times \frac{R_2}{R_2 + R_6} = 130 \times \frac{350}{600} = \frac{455}{6} V$$

Vypočítame U_{TH}:

$$U_{TH} = U_{R_2} - U_{R_1} = \frac{455}{6} - \frac{936}{41} = \frac{13039}{246} V$$

Vypočítané hodnoty dosadíme do náhradnej schémy:



Vypočítame I_{R_3} a U_{R_3}:

$$I_{R_3} = \frac{U_{TH}}{R_{TH} + R_3} = \frac{\frac{13039}{246}}{\frac{72379}{246} \times 600} \doteq 59,2738 mA$$

$$U_{R_3} = I_{R_3} \times R_3 = 0,0592738 \times 600 \doteq 35,5643 V$$

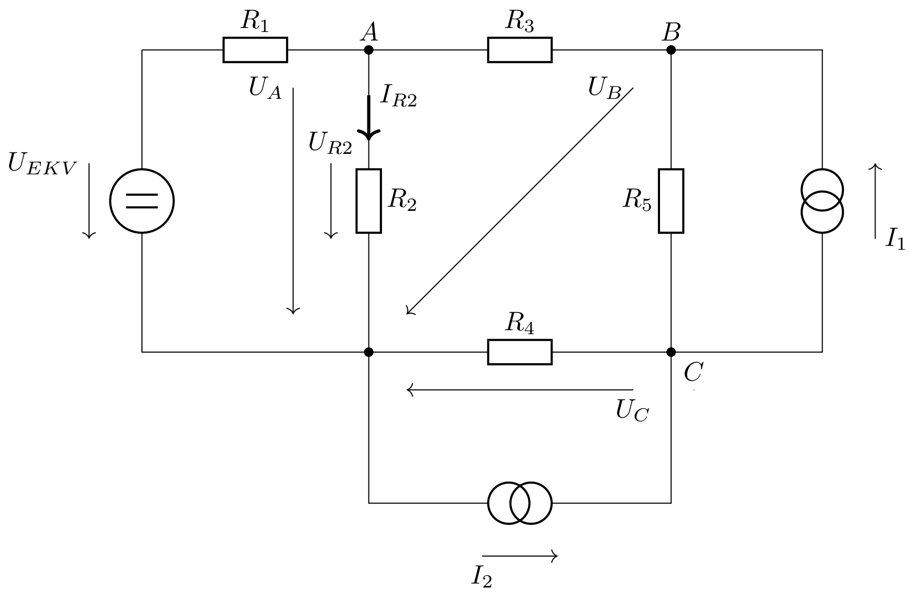
Príklad 3C

Zadané hodnoty:

$U = 110\text{V}$,

$I_1 = 0{,}85\text{A}$, $I_2 = 0{,}75\text{A}$

$R_1 = 44\Omega$, $R_2 = 31\Omega$, $R_3 = 56\Omega$,
$R_4 = 20\Omega$, $R_5 = 30\Omega$



Určíme jednotlivé uzly:

$$A \rightarrow I_{R_1} - I_{R_2} - I_{R_3} = 0$$

$$B \rightarrow I_{R_3} + I_1 - I_{R_5} = 0$$

$$C \rightarrow I_{R_5} - I_{R_4} + I_2 - I_1 = 0$$

Odvodíme jednotlivé napätia:

$$A \to \frac{U - U_A}{R_1} - \frac{U_A}{R_2} - \frac{U_A - U_B}{R_3} = 0$$

$$A \to \frac{110 - U_A}{44} - \frac{U_A}{31} - \frac{U_A - U_B}{56} = 0$$

$$A \to 190960 - 1736U_A - 2464U_A - 1364U_A + 1364U_B = 0$$

$$A \to U_A = \frac{190960 + 1364U_B}{5564}$$

$$B \to \frac{U_A - U_B}{R_3} + I_1 - \frac{U_B - U_C}{R_5} = 0$$

$$B \to \frac{U_A - U_B}{56} + 0,85 - \frac{U_B - U_C}{30} = 0$$

$$B \to 30U_A - 30U_B + 1428 - 56U_B + 56U_C = 0$$

$$B \to 30U_A - 86U_B + 56U_C = -1428$$

$$C \to \frac{U_B - U_C}{R_3} - \frac{U_C}{R_4} + I_2 - I_1 = 0$$

$$C \to \frac{U_B - U_C}{30} - \frac{U_C}{20} + 0,75 - 0,85 = 0$$

$$C \to 20U_B - 20U_C - 30U_C - 60 = 0$$

$$C \to U_C = \frac{20U_B - 60}{50}$$

Dosadíme U_A a U_C do U_B:

$$30U_A - 86U_B + 56U_C = -1428$$

$$30\left(\frac{190960 + 1364U_B}{5564}\right) - 86U_B + 56\left(\frac{20U_B - 60}{50}\right) = -1428$$
$$\frac{5728800 + 40920U_B}{5564} - 86U_B + \frac{1120U_B - 3360}{50} = -1428$$

$$50(5728800 + 40920U_B) - 23925200U_B + 5564(1120U_B - 3360) = -397269600$$

$$-15647520U_B + 267744960 = -397269600$$

$$U_B = \frac{665014560}{-15647520}$$

$$U_B \doteq 42,49968V$$

Vypočítame $U_{R_2}(U_A)$ a I_{R_2}:

$$U_{R_2} = U_A = \frac{190960 + 1364U_B}{5564}$$

$$U_{R_2} = \frac{190960 + 1364(42,49968)}{5564} = \frac{190960 + 57969,56352}{5564} = \frac{388952443}{8693750}$$

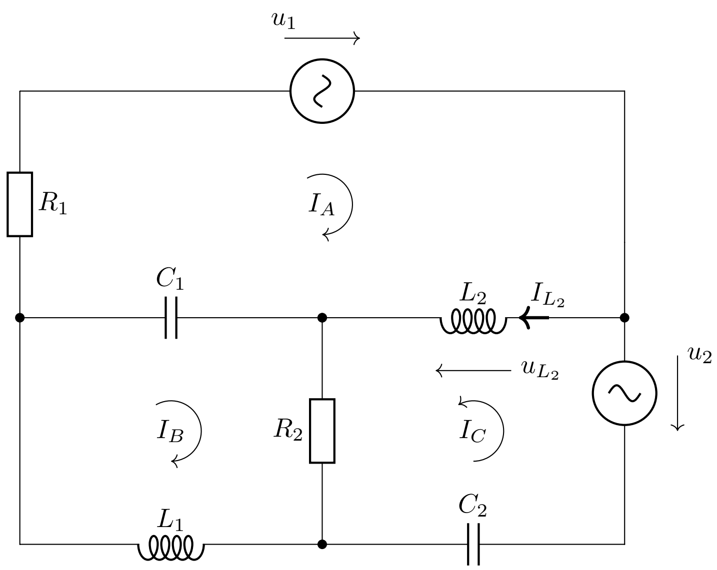
$$U_{R_2} \doteq 44,7393V$$

$$I_{R_2} = \frac{U_A}{R_2} = \frac{44,7393}{31} \doteq 1,4432A$$

Príklad 4H

Zadané hodnoty:

$U_1 = 65\text{V}$, $U_2 = 60\text{V}$
$R_1 = 10\Omega$, $R_2 = 10\Omega$
$L_1 = 160\text{mH}$, $L_2 = 75\text{mH}$,
$C_1 = 155\text{uF}$, $C_2 = 70\text{uF}$,



Určíme slučky:

$$I_A \rightarrow Z_{L_2}(I_A + I_C) + Z_{C_1}(I_A - I_B) + R_1 I_A + u_1 = 0$$

$$I_B \rightarrow R_2(I_B + I_C) + Z_{L_1} I_B + Z_{C_1}(I_B - I_A) = 0$$

$$I_B \rightarrow Z_{L_2}(I_C + I_A) + R_2(I_C + I_B) + Z_{C_2} I_C - u_2 = 0$$

Vytvoríme maticu na základe určených slučiek:

$$\begin{bmatrix} Z_{L_2} + Z_{C_1} + R_1 & -Z_{C_1} & Z_{L_2} \\ -Z_{C_1} & Z_{L_1} + Z_{C_1} + R_2 & R_2 \\ Z_{L_2} & R_2 & Z_{L_2} + Z_{C_2} + R_2 \end{bmatrix} \times \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} -u_1 \\ 0 \\ u_2 \end{bmatrix}$$

Dosadíme hodnoty do matice:

$$\begin{bmatrix} 10+33,95921i & 10,80849i & 44,7677i \\ 10,80849i & 10+84,69593i & 10 \\ 44,7677i & 10 & 10+20,84463i \end{bmatrix} \times \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} -65 \\ 0 \\ 60 \end{bmatrix}$$

Vypočítame determinanty pomocou Sarusovho pravidla:

$$D = \begin{bmatrix} 10+33,95921i & 10,80849i & 44,7677i \\ 10,80849i & 10+84,69593i & 10 \\ 44,7677i & 10 & 10+20,84463i \end{bmatrix}$$

$$D = -41963,01420089 + 122778,8205584i$$

$$D_A = \begin{bmatrix} -65 & 10,80849i & 44,7677i \\ 0 & 10+84,69593i & 10 \\ 60 & 10 & 10+20,84463i \end{bmatrix}$$

$$D_A = 342253,1151458 - 88976,89i$$

$$D_C = \begin{bmatrix} 10+33,95921i & 10,80849i & -65 \\ 10,80849i & 10+84,69593i & 0 \\ 44,7677i & 10 & 60 \end{bmatrix}$$

$$D_C = -406019,7340711 + 93266,5705i$$

Vypočítame i_A a i_C:

$$i_A = \frac{D_A}{D} = -1,501969045700 - 2,274219327747i$$

$$i_C = \frac{D_C}{D} = -1,501969045700 - 2,274219327747i$$

Vypočítame i_{L_2}:

$$i_{L_2} = i_A + i_C = 0,190223937874 + 0,454347491409i$$

Vypočítame u_{L_2}:

$$u_{L_2} = Z_{L_2} \times i_{L_2} = -20,3400921911506893 + 8,5158881835618698i$$

Vypočítame $|u_{L_2}|$ a φ_{L_2}:

$$|u_{L_2}| = \sqrt{Re(u_{L_2})^2 + Im(u_{L_2})^2}$$

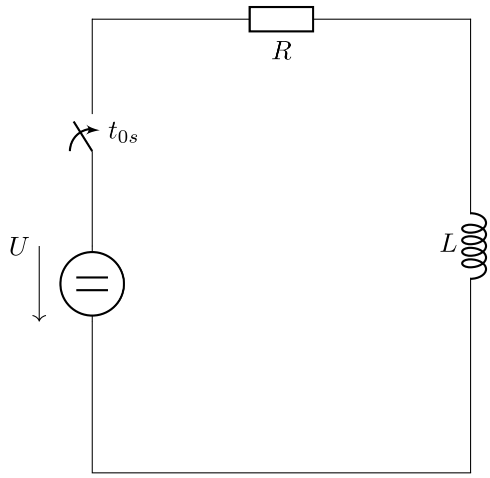
$$|u_{L_2}| = \sqrt{(-20,3400921911506893)^2 + (8,5158881835618698)^2} \doteq 22,0508V$$

$$\varphi_{L_2} = \arctan \frac{Re(u_{L_2})}{Im(u_{L_2})} = \frac{-20,3400921911506893}{8,5158881835618698} \doteq -1,1743rad = -67°16'55.85''$$

Príklad 5F

Zadané hodnoty:

$U = 22\text{V}$,
$L = 30\text{H}$
$R = 15\Omega$
$i_{L(0)} = 5A$



Zo zadaných hodnôt odvodíme rovnice:

$$i_L = \frac{u_R}{R} => u_R = u_L \times R$$

$$u_R + u_2 - U = 0 => u_L = U - u_R$$

Napíšeme a doplníme diferenciálnu rovnicu pre i_L:

$$i'_L = \frac{u_C}{L}$$

$$i'_L = \frac{U - u_R}{L}$$

$$i'_L = \frac{U - i_L \times R}{L}$$

$$Li'_L = U - i_L \times R => Li'_L + Ri_L = U$$

Dosadíme hodnoty do výslednej rovnice:

$$30i^{'}{}_L + 15i_L = 22$$

Definujeme charakteristickú rovnicu:

$$30\lambda + 15 = 0$$

$$\lambda = -\frac{1}{2}$$

Očakávané riešenie:

$$i_L(t) = K(t) \times e^{\lambda t}$$

$$i_L(t) = K(t) \times e^{-\frac{1}{2}t}$$

Zderivujeme i_L a dosadíme i_L a i'_L do diferenciálnej rovnice:

$$i'_L(t) = K'(t) \times e^{-\frac{1}{2}t} + K(t) \times \left(-\frac{1}{2}\right) e^{-\frac{1}{2}t}$$

$$30\left(K'(t) \times e^{-\frac{1}{2}t} + K(t) \times \left(-\frac{1}{2}\right) e^{-\frac{1}{2}t}\right) + 15\left(K(t) \times e^{-\frac{1}{2}t}\right) = 22$$

$$30\left(K'(t) \times e^{-\frac{1}{2}t}\right) - 15\left(K(t) \times e^{-\frac{1}{2}t}\right) + 15\left(K(t) \times e^{-\frac{1}{2}t}\right) = 22$$

$$30\left(K'(t) \times e^{-\frac{1}{2}t}\right) = 22$$

$$K'(t) \times e^{-\frac{1}{2}t} = \frac{22}{30}$$

Zintegrujeme rovnicu:

$$K'(t) = \frac{11}{15} e^{\frac{1}{2}t}$$

$$K(t) = \frac{11}{15} \int e^{\frac{1}{2}t} \times dt$$

$$K(t) = \frac{11}{15} \times 2e^{\frac{1}{2}t} + k$$

$$K(t) = \frac{22}{15} e^{\frac{1}{2}t} + k$$

Hodnotu dosadíme do očakávaného riešenia:

$$i_L(t) = K(t) \times e^{-\frac{1}{2}t}$$

$$i_L(t) = K\left(\frac{22}{15}e^{\frac{1}{2}t} + k\right) \times e^{-\frac{1}{2}t}$$

$$i_L(t) = \frac{22}{15} + k \times e^{-\frac{1}{2}t}$$

Dosadíme do počiatočnej podmienky:

$$i_L(0) = \frac{22}{15} + k \times e^{-\frac{1}{2}0}$$

$$10 - \frac{22}{15} = k$$

$$k = \frac{128}{15}$$

k dosadíme do očakávaného riešenia:

$$i_L(t) = \frac{22}{15} + \frac{128}{15}e^{-\frac{1}{2}t}$$

Odvodíme i'_L:

$$30i'_L + 15\left(K(t) \times e^{-\frac{1}{2}t}\right) = 22$$

$$i'_L + \frac{\frac{22}{15} + \frac{128}{15}e^{-\frac{1}{2}t}}{2} = \frac{22}{30}$$

$$i'_L + \frac{11}{15} + \frac{64}{15}e^{-\frac{1}{2}t} = \frac{11}{15}$$

$$i'_L = -\frac{64}{15}e^{-\frac{1}{2}t}$$

Pre kontrolu výpočtov dosadíme hodnoty do diferenciálnej rovnice:

$$Li'_L + Ri_L = U$$

$$i'_L + \frac{\frac{22}{15} + \frac{128}{15}e^{-\frac{1}{2}t}}{2} = \frac{22}{30}$$

$$-\frac{64}{15}e^{-\frac{1}{2}t} + \frac{64}{15}e^{-\frac{1}{2}t} + \frac{11}{15} = \frac{11}{15}$$

$$\frac{11}{15} = \frac{11}{15}$$

$$0 = 0$$

Tabuľka výsledkov:

1H	2F	3C	4H	5F
$I_{R_6} = 82{,}0904\text{mA}$ $U_{R_6} = 71{,}4187\text{V}$	$I_{R_3} = 59{,}2738\text{mA}$ $U_{R_3} = 35{,}5643\text{V}$	$U_{R_2} = 44{,}7393\text{V}$ $I_{R_2} = 1{,}4432\text{A}$	$\lvert u_{L_2}\rvert = 22{,}0508\text{V}$ $\varphi_{L_2} = -1{,}1743\text{rad}$	$i_L(t) = \frac{22}{15} + \frac{128}{15}e^{-\frac{1}{2}t}$