ADIC-DAG: A Higher-Dimensional p-Adic Ultrametric Tangle with Feeless Consensus and the ADIC Token

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Abstract

We introduce *ADIC-DAG*, a feeless, reputation-weighted distributed ledger that organizes messages as a higher-dimensional directed acyclic *hypergraph* (a Tangle of d-simplices). The core novelty is a *p-adic ultrametric* structure on message features, enforcing multi-axis diversity at attachment time via *d-approvals*: each new message forms a d-simplex by approving $d+1$ parents across distinct p-adic neighborhoods. This yields strong resistance to collusion and sybils without transaction fees. Finality is certified by (i) a k-core coverage test with axis-diversity thresholds and (ii) stabilization in persistent homology on the induced simplicial complex.

We define the **ADIC** token as a native asset for sponsorships, bounties, storage commitments, and governance. Consensus remains feeless and independent of token ownership; a refundable deposit (anti-spam) and a non-transferable *ADIC-Rep* reputation score regulate participation. We specify the *Genesis* procedure, token allocation schedule, diversity rules, tip selection by multi-axis random walks, conflict resolution by an energy descent functional, and the security controls that make the system robust. This white paper provides the mathematical foundations, protocol rules, and a practical deployment blueprint for an initial parameter set $(p, d) = (3, 3)$.

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1 Overview

1.1 Vision

ADIC-DAG is a minimal, mathematically principled ledger:

- *Feeless base layer:* no gas; participants escrow a refundable anti-spam deposit.
- *Ultrametric security:* p-adic radii enforce multi-axis diversity on each attachment.
- *Higher-dimensional Tangle:* every message creates a d-simplex of approvals.
- *Deterministic finality tests:* k-core coverage and persistent homology stabilization.

- *Two ledgers in one:* transferable **ADIC** token for utility and a non-transferable **ADIC-Rep** reputation for consensus weighting.

1.2 System parameters (v1 defaults)

Prime $p = 3$, dimension $d = 3$, axis radii $\rho = (2, 2, 1)$, diversity threshold $q = 3$, refundable deposit $D = 0.1$ ADIC, reputation exponents $(\alpha, \beta) = (1, 1)$, k-core threshold $k = 20$, depth $D^* = 12$, homology window $\Delta = 5$.

2 Mathematical Preliminaries

2.1 p-adic ultrametric

Fix a prime p. The p-adic valuation $v_p : \mathbb{Q} \to \mathbb{Z} \cup \{\infty\}$ and norm $|x|_p = p^{-v_p(x)}$ satisfy the ultrametric inequality

$$|x + z|_p \leq \max(|x|_p, |z|_p).$$

Balls $B^{(p)}(c, r) := \{x : v_p(x - c) \geq r\}$ are both open and closed and are naturally hierarchical. We use $\mathbb{Q}_p$-coordinates for message features to induce strongly structured neighborhoods.

2.2 Simplicial hypergraph

A d-simplex is a set of $d+1$ vertices. The ADIC-DAG is a directed acyclic *simplicial* hypergraph $\mathcal{T}_d = (V, \Sigma_d)$ where each new message x adds a d-simplex $\{x, a_0, \ldots, a_d\}$ to Σ_d via approvals $A(x) = \{a_0, \ldots, a_d\}$ respecting acyclicity. We leverage persistent homology of the induced complex for finality checks.

3 Data Model and Encodings

3.1 Messages, features, axes

Each message x carries:

$$\begin{aligned}
\text{payload} &: \text{application data (UTXO/account ops, notes, DA pointers)}, \\
\text{meta} &: \text{timestamp, axis tags, conflict set id}, \\
\text{approvals} &: A(x) = \{a_0, \ldots, a_d\}, \\
\text{signature} &: \sigma_x,\ \text{public key } P_x, \\
\text{features} &: \Phi(x) = (\phi_1(x), \ldots, \phi_d(x)) \in \mathbb{Q}_p^d, \\
\text{reputation} &: R(P_x) \in \mathbb{R}_{\geq 0}\ \text{(ADIC-Rep)}.
\end{aligned}$$

Axes (dimension d) encode distinct *orthogonal* aspects, e.g. Time-bucket, Topic-LSH, Stake-Tier, Region/ASN. Each ϕ_j is a canonical map into $\mathbb{Q}_p$ (e.g. p-adic digit embedding of a bucket index).

3.2 Admissibility & diversity

Given radii $\rho = (\rho_1, \ldots, \rho_d) \in \mathbb{Z}^d_{\geq 0}$ and diversity $q \in \{2, \ldots, d+1\}$, a candidate approval set $A = \{a_0, \ldots, a_d\}$ for x is *admissible* if:

$$v_p\big(\phi_j(x) - \phi_j(a_j)\big) \geq \rho_j \quad \forall j, \tag{C1}$$

$$\#\big\{B^{(p)}(\phi_j(a_k), \rho_j) : k = 0, \ldots, d\big\} \geq q \quad \forall j, \tag{C2}$$

$$\sum_{k=0}^{d} R(a_k) \;\geq\; R_{\min}, \quad \min_k R(a_k) \geq r_{\min}. \tag{C3}$$

3.3 Tip selection: multi-axis random walk (MRW)

Let T be current tips. Define proximity and trust for candidate parent y relative to proposer x:

$$\mathrm{prox}_p(x, y) := \prod_{j=1}^{d} \Big(1 + p^{\rho_j - v_p(\phi_j(x) - \phi_j(y))}\Big)^{-1}, \quad \mathrm{trust}(y) := R(y)^{\alpha}(1 + \mathrm{age}(y))^{-\beta}.$$

With a conflict penalty $\mathrm{conflict}(y) \in \{0, 1, \ldots\}$, the transition probability is

$$\Pr(t \to y) \;\propto\; \exp\big(\lambda\, \mathrm{prox}_p(x, y)\, \mathrm{trust}(y) - \mu\, \mathrm{conflict}(y)\big)\,,$$

with $\lambda, \mu > 0$. We run MRW per axis, intersect candidates, and sample $d+1$ distinct parents satisfying (C1)–(C3).

4 Consensus Invariants and Finality

4.1 Conflict sets and energy descent

For each conflict set C (e.g. UTXO double-spends), define the *support* of a node $z \in C$:

$$\mathrm{supp}(z; C) = \sum_{y \text{ descends to } z} \frac{R(y)}{1 + \mathrm{depth}(y)}.$$

Define total energy

$$\mathcal{E} = \sum_{C} \left| \sum_{z \in C} \mathrm{sgn}(z)\, \mathrm{supp}(z; C) \right|.$$

Under MRW + admissibility, $\mathcal{E}$ almost surely decreases with time (negative drift), ensuring a unique winner per conflict set. (Formal drift bounds are provided in Appendix B.)

4.2 Finality tests

We certify finality by either:

(F1) ***k*-core finality** Node x is final if the induced future cone contains a k-core exceeding thresholds on (i) distinct balls per axis ($\geq q$), (ii) total reputation $\geq R^*$, and (iii) depth $\geq D^*$.

(F2) Homology finality Consider the simplicial complex formed by approvals in the future cone of x. If the weighted persistent homology H_d stabilizes over a window Δ and the H_{d-1} bottleneck distance falls below ε, mark x final.

5 Economics: Feeless Base, Deposits, and ADIC Token

5.1 Feeless consensus; refundable deposits

Consensus is feeless. Each proposal escrows a deposit D (anti-spam), refunded at finality. Objective faults (invalid signatures, malformed approvals, provable sybil overlap) incur slashing of D.

5.2 ADIC token

ADIC is a *transferable* utility/governance token that is *not* used for consensus security. Uses include:

- Sponsoring compute/storage bounties (PoUW hooks) and data availability payments.
- Long-term storage commitments and archival incentives.
- Governance (parameter updates, grants), subject to quadratic voting caps.
- Optional staking for *service* guarantees (bandwidth, indexing), *not* for consensus weight.

5.3 ADIC-Rep (reputation)

ADIC-Rep is a non-transferable reputation (soul-bound) score $R(P)$ for public key P:

$$R_{t+1}(P) = \gamma R_t(P) + (1-\gamma)\left(\underbrace{\text{finalized approvals weight}}_{\text{good}} - \underbrace{\text{overlap penalty}}_{\text{bad}}\right), \quad 0 < \gamma < 1.$$

Consensus weighting only uses R, never ADIC.

6 Genesis and Supply

6.1 Genesis

The Genesis hyperedge G consists of $d+1$ system identities $\{g_0, \ldots, g_d\}$, each pre-loaded with:

- a fixed ADIC-Rep score $R(g_k) = R_{\text{seed}}$,
- a locked deposit pool for bootstrapping refunds,
- an initial set of axis anchors $\Phi(g_k)$ spanning distinct p-adic balls.

The Genesis manifest (hash, parameters, anchors, multiparty signatures) is published and permanently anchored (e.g. to multiple L1s for timestamping).

6.2 Supply & allocation

- **Max supply:** $S_{\max} = 10^9$ ADIC.
- **Genesis mint:** $S_0 = 3 \cdot 10^8$ ADIC: 20% Treasury (multisig), 30% Liquidity and Community R&D grants, 50% Genesis.
- **Emissions:** $S_{\text{PoUW}} \leq 1\%/yr$ (sponsor-funded or treasury-matched), decaying half-life 6 years.

Consensus remains independent of ADIC. No fees are required; deposits are refunded. ADIC is optional utility.

7 Protocol Algorithms

7.1 Admissibility scoring

For candidate parents $A = \{a_0, \dots, a_d\}$ define

$$S(x; A) = \sum_{j=1}^{d} \min_{a \in A} p^{-\max\{0,\ \rho_j - v_p(\phi_j(x) - \phi_j(a))\}}.$$

Require $S(x; A) \geq d$ and (C2)–(C3).

7.2 Selection (pseudocode)

```
SelectApprovals(x):
  C = {} # candidates per axis
  for j in 1..d:
     C_j = MRW_on_axis(j, x, tips, lambda, mu)
  C = IntersectionOrDiverseMerge(C_1,...,C_d)
  Sample subsets A of size d+1 from C:
     if Admissible(x, A): return A
  fallback: widen radii or extend MRW horizon
```

7.3 Validation and finality

```
Validate(x):
  check signature, format
  ensure acyclicity (vector clocks per axis)
  ensure Admissible(x, A(x))
  ensure conflict rules; escrow deposit D

Finalize():
  periodically test nodes by F1 and/or F2
  refund deposits for finalized nodes
  update ADIC-Rep; slash objective faults
```

8 Security Analysis

Sybil Defended by refundable deposits, non-transferable reputation, and mandatory axis diversity (q distinct balls per axis). Coordinated clusters cannot cheaply satisfy (C2) over time.

Censorship MRW across axes prevents capture by any single neighborhood. Finality requires multi-ball coverage that censorship coalitions struggle to block without broad presence.

Double-spend Conflict sets with energy descent ensure a unique winner; attackers must sustain approvals across many distinct balls and reputations—economically prohibitive.

Liveness Any honest node can attach via MRW and admissibility; anti-entropy checkpoints and archival peering guarantee catch-up.

9 Implementation Plan

9.1 Phase 0 (prototype)

Rust/TypeScript node with: message format, feature encoders, MRW tip selection, admissibility, k-core finality, deposits/refunds, explorer.

9.2 Phase 1 (beta)

Persistent homology-based finality (streaming lib), ADIC-Rep SBT, axis-aware gossip overlays, anti-entropy checkpoints, optional anchors.

9.3 Phase 2 (mainnet-candidate)

PoUW sponsor hooks (optional), storage markets, governance module (quadratic), parameter sweeps, adversarial testing.

10 ADIC Token: Utility and Governance

ADIC pays for optional services: storage duration, indexing, prioritized DA, PoUW bounties. Governance scopes: axis catalog updates, parameter tuning (ρ, q, k, D), treasury grants. Quadratic caps mitigate plutocracy; consensus remains reputation/deposit driven.

11 Conclusion

ADIC-DAG unifies p-adic ultrametrics, higher-dimensional tangles, and topological finality into a feeless, robust ledger. The ADIC token funds useful services and governance without entangling economic weight with consensus safety. A concrete $(p, d) = (3, 3)$ design is ready for implementation, audit, and open beta.

A Ultrametric Feature Encoders

Time axis Map blocktime t to bucket $b = \lfloor t/\tau \rfloor$; encode $\phi_1 = b$ as a p-adic integer by digit expansion base p.

Topic axis SimHash or LSH on payload; map to integer u; $\phi_2 = u$ in $\mathbb{Z}_p$.

Stake/Service tier Quantize stake or service score into tiers $\{0, \dots, M\}$; set $\phi_3 =$ tier.

B Energy Drift

Let X_t be the process of approvals and $\mathcal{E}(t)$ the energy. Under MRW with $\lambda > 0$, admissibility (C1)–(C3), and ergodicity of tip renewal, there exist $c > 0$ and $\delta > 0$ such that

$$\mathbb{E}[\mathcal{E}(t+1) - \mathcal{E}(t) \mid X_t] \;\leq\; -c\,\mathbb{1}_{\{\mathcal{E}(t)>\delta\}}.$$

Thus $\mathcal{E}(t)$ is a supermartingale with negative drift away from a small neighborhood, implying eventual absorption to a unique winner per conflict set with high probability.

C Persistent Homology for Finality

We view approvals as simplices with weights w (reputation/age). Using a filtration by cumulative weight, compute H_d and H_{d-1}; finality requires stabilization of H_d bars and small bottleneck distance for H_{d-1} over Δ rounds. Efficient streaming implementations provide $O(m \log m)$ amortized updates for m simplices.

D Genesis Manifest

- System: ADIC-DAG v1.0, $(p, d) = (3, 3)$.
- Radii $\rho = (2, 2, 1)$, diversity $q = 3$, $k = 20$, $D^* = 12$, $\Delta = 5$.
- Genesis identities $\{g_0, g_1, g_2, g_3\}$ with anchors $\Phi(g_k)$ in distinct balls.
- Deposit pool: X ADIC, Treasury: Y ADIC.
- Multisig: M-of-N keys (hashes listed).
- Anchors: SHA-256 of manifest posted to $\{\mathrm{L1}_1, \mathrm{L1}_2\}$.

E ADIC-Rep Update Rules

Let $\mathcal{N}_P(t)$ be approvals by P in $(t - \Delta, t]$, $\mathrm{div}(\mathcal{N}_P)$ the number of distinct balls per axis touched. Define

$$\mathrm{good}(P) = \sum_{x \in \mathcal{N}_P} \frac{\mathbb{1}_{\{x \text{ final}\}} \cdot \mathrm{div}(\mathcal{N}_P)}{1 + \mathrm{depth}(x)}, \quad \mathrm{bad}(P) = \eta \cdot \mathrm{overlap\_score}(P).$$

Then $R_{t+1}(P) = \gamma R_t(P) + (1 - \gamma)(\mathrm{good} - \mathrm{bad})$ with caps to avoid runaway.

F Genesis

The Genesis phase begins when BTC and ETH and SOL sent to the following addresses as ADIC seed until October 20, 2025 at 23:59 UTC are taken for core legacy L1 coordination of ADIC:

BTC
bc1qnykv3t8fqpar7aguaas3sxtlsqyndxrpa0g7h8

ETH
0x7EB0c7ea79D85d2A3Ac45aF6A8CB0F7AC9A125bE

SOL
GrUy83AAsibyrcUtpAVA8VgpnQSgyCAb1d8Je8MXNGLJ

G Conclusion & Genesis Contribution

ADIC in one paragraph. ADIC-DAG delivers a feeless, reputation-weighted base layer that organizes messages as a higher-dimensional Tangle and enforces multi-axis diversity via a p-adic ultrametric. Finality is certified deterministically through k-core coverage and stabilization of persistent homology—yielding robust liveness, Sybil resistance, and conflict resolution without tying safety to token wealth. The ADIC token funds useful services (storage, DA, PoUW bounties) and governance; consensus weight comes only from refundable deposits and non-transferable reputation.

How contributions are used. Funds support engineering and research aligned to the protocol invariants: admissibility & diversity enforcement, MRW, conflict energy descent, k-core and homology finality, archive/anti-entropy, and explorer/SDKs. Phase 0 implements message format, encoders, MRW, admissibility, k-core, deposits/refunds; Phase 1 adds streaming homology finality, ADIC-Rep SBT, axis-aware gossip, checkpoints; Phase 2 hardens PoUW sponsor hooks, storage markets, quadratic governance, parameter sweeps, and adversarial testing.

Alignment with token economics. The protocol is feeless at its core; deposits are escrowed and refunded at finality, with slashing only for objective faults. ADIC's maximum supply and genesis allocation schedule are fixed; token utility covers optional services and governance with quadratic caps, while consensus weight remains independent of token balance.

Genesis Contribution — Start Signal and Addresses

Start signal. The Genesis Contribution phase *begins* the moment inbound transactions start accumulating to the following addresses, which we designate for core legacy L1 coordination of ADIC and for anchoring the Genesis manifest across chains.[1]

Contribution addresses

- **BTC**: `bc1qnykv3t8fqpar7aguaas3sxtlsqyndxrpa0g7h8`
- **ETH**: `0x7EB0c7ea79D85d2A3Ac45aF6A8CB0F7AC9A125bE`
- **SOL**: `GrUy83AAsibyrcUtpAVA8VgpnQSgyCAb1d8Je8MXNGLJ`

What ADIC delivers

- **Feeless base layer at scale:** Diversity-enforced attachments and refundable deposits replace gas fees, making everyday participation accessible.
- **Deterministic finality:** Dual tests—k-core coverage and persistent homology—tighten safety without sacrificing liveness.
- **Token utility without plutocracy:** ADIC funds storage, DA, PoUW bounties, and governance with quadratic caps; consensus remains reputation/deposit driven.

In closing. ADIC marries p-adic ultrametrics, higher-dimensional tangles, and topological finality to deliver a safer, fairer, feeless ledger. If this architecture should exist, the fastest and most durable way to make distributed ledger technology beyond blockchain.

[1] For transparency and timestamping, the Genesis manifest (parameters, anchors, multisig) is permanently anchored to multiple L1s; contribution receipts will be mirrored alongside those anchors. No investment promise.