NBER WORKING PAPER SERIES

HOW CREDIBLE IS HONG KONG'S CURRENCY PEG?

Urban Jermann
Bin Wei
Vivian Yue

Working Paper 34300
http://www.nber.org/papers/w34300

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2025

We would like to thank Saleem Bahaj, Xiang Fang, Yi Huang, Dmitry Mukhin, Ali Ozdagli, Ricardo Reis (discussant), Andrea Vedolin, Russell Wong, and participants at seminars and conferences. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve System. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

How Credible is Hong Kong's Currency Peg?
Urban Jermann, Bin Wei, and Vivian Yue
NBER Working Paper No. 34300
September 2025
JEL No. F3, F31, G13, G15

ABSTRACT

This paper develops an asset-pricing model to evaluate the credibility of Hong Kong's Linked Exchange Rate System (LERS). Allowing for imperfect peg credibility, we derive closed-form solutions for exchange rates and option prices under potential regime shifts. Using HKD option data, we estimate market-implied probabilities of peg survival and the fundamental value of the HKD. Our results show that credibility fluctuates with U.S. interest rate hikes, local liquidity conditions, and Chinese currency dynamics. Compared with more standard Black-Scholes-based models, our approach provides more realistic assessments of peg sustainability.

Urban Jermann
University of Pennsylvania
Finance Department
and NBER
jermann@wharton.upenn.edu

Vivian Yue
Emory University
Economics Department
and NBER
vivianyue1@gmail.com

Bin Wei
Federal Reserve Bank of Atlanta
bin.wei@atl.frb.org

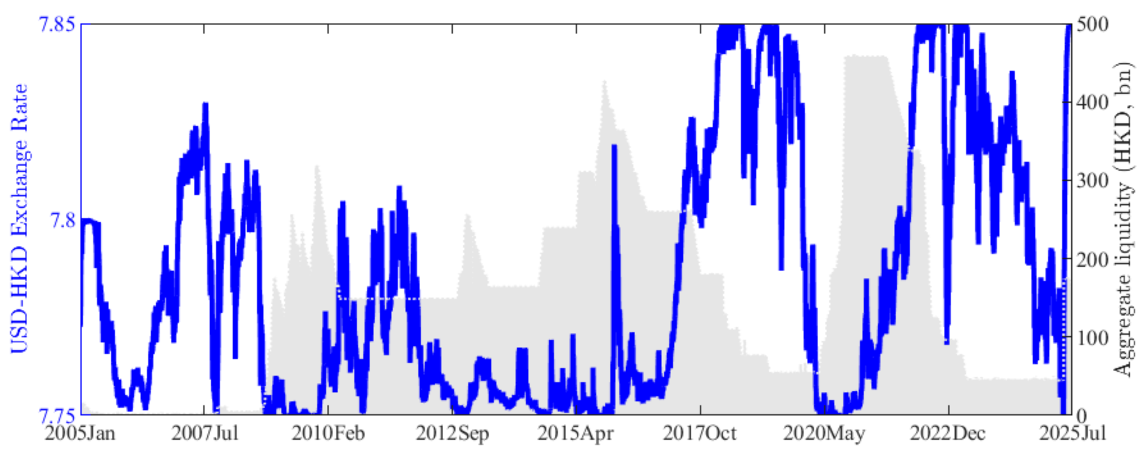
"Confidence in the LERS has been tested countless times through difficult events ... The LERS sailed through them all. [W]e have no intention and we see no need to change the LERS."

— *Eddie Yue, HKMA Chief Executive*[1]

1 Introduction

The Hong Kong Dollar (HKD) has been pegged to the U.S. Dollar (USD) at 7.8 HKD per USD since 1983. Under the current currency board arrangement—known as the *Linked Exchange Rate System* or "LERS"—the Hong Kong Monetary Authority (HKMA) has a mandate to keep the spot exchange rate between 7.75 to 7.85 HKD per USD, according to a band set in 2005. This arrangement has long been considered a guarantee of financial stability and economic prosperity in Hong Kong. Figure 1 plots the historical USD-HKD exchange rate since 2005.

Figure 1: **USD-HKD Exchange Rate and Aggregate Liquidity**



NOTE: This figure plots the USD-HKD exchange rate (solid line, left axis) and aggregate banking liquidity in Hong Kong in billion HKD (shaded area, right axis) between January 2005 and July 2025.

When the HKD reaches either end of the band, the authority has intervened to defend the

[1]See Eddie Yue's remarks in "The Linked Exchange Rate System – 40 Years On" on October 17, 2023, at URL: https://www.hkma.gov.hk/eng/news-and-media/insight/2023/10/20231017/.

peg, through buying or selling HKD in the foreign exchange market. For instance, by selling US dollars to buy Hong Kong dollars, the HKMA withdraws cash from the banking system, reducing its aggregate balance and causing interest rates to rise, which would strengthen the Hong Kong dollar and keeps it within the band. If a currency board depletes its foreign reserves, loses its credibility, or faces severe economic shocks, maintaining the fixed exchange rate becomes unsustainable.[2]

Recently, Hong Kong's currency peg has been under increasing pressure. As shown in Figure 1, in early 2022 the Hong Kong dollar has traded on the weak-side of the trading band for much of the time since the U.S. started to raise interest rates at that time. As a result, the HKMA had to intervene by buying a large amount of Hong Kong dollars to maintain the peg. Consequently, the aggregate balance or liquidity in the Hong Kong banking system, depicted by the grey shaded area in Figure 1, has declined by more than 90% from its peak in 2021. Most recently between May and July 2025, the Hong Kong dollar swung from strong-side pressure to weak-side stress. In early May, capital inflows pushed the HKD toward 7.75, prompting the HKMA to sell HKD and inject liquidity. This lowered local interest rates and fueled carry trades. By late June 2025, capital outflows reversed the trend. The HKD weakened toward 7.85, triggering multiple HKMA interventions to buy HKD 86.93 billion in order to defend the peg.

An important question thus arises: Is Hong Kong's currency peg still credible? As quoted above, Eddie Yue, chief executive of the HKMA, reiterated the institution's steadfast commitment to the LERS in his speech on October 17, 2023. As another example, he said in May 2023 that local banks have sufficient liquidity to manage with.[3] However, not all market participants share this view. For example, some hedge funds, such as Pershing Square Capital Management, have reportedly taken large positions against the HKD. Bill Ackman, the owner of Pershing Square Capital Management, tweeted in November 2023 that "it is a matter of time" before the peg breaks.

In this paper, we develop an asset pricing model of the HKD and measure financial markets' views about the likelihood that the peg would be abandoned based on derivatives data. In the model, the HKD-USD peg is not perfectly credible. The peg will persist into the next period with probability p and will break with a probability $1 - p$. If the peg continues, the HKD-USD exchange rate is determined by a no-arbitrage equilibrium condition. If the peg breaks, the exchange rate equals its fundamental value, reflecting the levels that would

[2]For example, Argentina's quasi-currency board collapsed in 2002 due to its failure to maintain full foreign reserves and large persistent trade deficits (Na et al., 2018). Under speculative attacks, Estonia's currency board nearly failed during the Russian financial crisis in 1998.

[3]See Eddie Yue's remarks in "On the operation of the Linked Exchange Rate System" on May 3, 2023, at URL: https://www.hkma.gov.hk/eng/news-and-media/insight/2023/05/20230503.

prevail under a free-floating regime, V.

Under a continuous-time formulation, we show that the equilibrium exchange rate can be expressed as a function of the fundamental value in closed form (Proposition 1). The function is non-linear and S-shaped. This is reminiscent of the so-called "honeymoon effect" in the target-zone literature (see Krugman, 1991) that represents the expectation of potential interventions even when the exchange rate lies strictly inside the zone.

We also derive an analytical formula for option prices implied by the model (Proposition 3). Due to the possibility of a regime shift in the exchange rate system, the option pricing formula differs significantly from the standard Black-Scholes formula. Specifically, our model implies a U-shaped distribution for the HKD-USD exchange rate, closely matching the empirical distribution observed in the data—a stark contrast to the bell-shaped distribution under standard Black-Scholes models. This result implies distinct differences in assessing the credibility of Hong Kong dollar peg. Because it ignores the stabilizing features of the currency board, the Black-Scholes model implies probability of 67% (17%) that the USD-HKD exchange rate would break out of the upper (lower) bound at the 90 day horizon. We estimate the credibility of the Hong Kong dollar peg based on our model using HKD options data. Our model offers more realistic estimates, implying an average probability of 3% of breaking out of either bound. These results illustrate the importance of incorporating possible regime shifts in our framework. Models that neglect potential regime shifts can dramatically overstate the fragility of the peg.

Our estimation reveals several episodes in which market confidence in the peg was significantly weakened. Most notably, in late 2022, the estimated probability of peg survival (p) fell sharply to around 50%, coinciding with aggressive U.S. Federal Reserve rate hikes, capital outflows from HKD assets, and a steep decline in the Aggregate Balance from over HKD 300 billion to below HKD 100 billion (Figure 1). Other stress events include the 2008–09 global financial crisis, which triggered volatility spikes; August 2019, when market tensions rose after the Chinese currency weakened to more than 7 yuan per dollar amid U.S.–China trade frictions; and 2025, when the peg faced two-sided pressure due to rapid capital flow swings, which destabilized local interest rates and liquidity conditions and further tested the credibility of the exchange rate regime.

Our analysis shows that higher Hong Kong interest rates strengthen both the fundamental value of the HKD (V) and the peg continuation probability (p), reinforcing confidence in the LERS. Conversely, rising U.S. interest rates have the opposite effect. Liquidity in the banking system supports peg credibility by increasing p, but its decline weakens V, reflecting capital outflows that put stress on the currency board system. Lastly, China's onshore and offshore exchange rates, CNY and CNH, play a crucial role. A stronger yuan supports the

HKD's fundamental value, while depreciation in the Chinese currency raises concerns over the peg's sustainability. These findings underscore that the HKD peg's credibility hinges not only on local monetary conditions but also on external pressures from U.S. rate hikes and China's financial stability.

This paper is related to the literature on exchange rate target zones. The canonical target zone model presented in Krugman (1991) has been extended in various ways.[4] All these traditional target zone models rely on the assumption of uncovered interest parity (UIP), which does not hold in the data (see, e.g., Engel, 1996; Lustig et al., 2011). In contrast, we relax UIP and derive the exchange rate through a no-arbitrage condition following Jermann (2017) which uses this approach to estimate the implied survival probability of the Swiss franc cap in 2015. We extend the framework in Jermann (2017) to a continuous-time setting and derive both the equilibrium exchange rate and option prices in closed form. This has the added benefit of making the estimation of the model much more efficient.[5] The analytical option pricing formula derived in our framework with imperfect credibility contributes to the literature on option pricing in a target zone (e.g. Ball and Roma, 1993; Dumas et al., 1995; Hertrich and Zimmermann, 2017).

Beyond the target zone literature, our paper also relates to the broader exchange rate literature that studies deviations from uncovered and covered interest parity, as well as the profitability of carry trades. A large body of work has documented the failure of UIP in the data, giving rise to systematic excess returns from currency speculation strategies (see, e.g., Engel, 1996; Lustig et al., 2011).[6] This literature provides important context for our analysis, since interest rate differentials and CIP deviations shape capital flows, carry trades, and thus the credibility of exchange rate pegs. Our framework builds on these insights by incorporating no-arbitrage conditions into the modeling of exchange rate dynamics, while allowing for imperfect credibility of the peg.

Our paper is closely related to Genberg and Hui (2011).[7] They extract the risk-neutral probability density function of exchange rate expectations from option prices based on Malz (1997) and study how market expectations respond to reforms of the LERS in September

[4]For example, Bertola and Svensson (1993) incorporates time-varying realignment risk; Froot and Obstfeld (1991a,b) and Delgado and Dumas (1991) introduce intra-marginal mean-reverting interventions; Svensson (1991a,b) derives and tests implications for the term structure of interest rate differentials. Lindberg and Söderlind (1994) contains a structural estimation on Swedish data of an extended target zone model that incorporates both intra-marginal interventions and time-varying realignment risk.

[5]Other papers studying on Swiss franc are Amador et al. (2019); Mirkov et al. (2019); Auer et al. (2021); Breedon et al. (2023).

[6]More recently, research has emphasized deviations from covered interest parity (CIP), particularly following the global financial crisis. Du et al. (2018) show that CIP violations are persistent, reflecting limits to arbitrage and balance sheet constraints faced by financial intermediaries.

[7]For other studies on the Hong Kong dollar, see, for example, Kwan and Lui (1999) and Latter (2007).

1998 and May 2005. The methodology in Malz (1997), which is based on the standard Black-Scholes model, usually leads to a bell-shaped probability density function. In contrast, the exchange rate distribution in a target zone like Hong Kong's is U-shaped, indicating that the probability of the exchange rate being near the boundaries is significantly higher than the likelihood of it being near the midpoint of the band. In our paper, we introduce a probability of regime shifts and derive option prices in closed form for target-zone economies. Not only does our model reproduce the U-shaped exchange rate distribution, but it also leads to a more realistic estimate of the probability of the exchange rate breaking out of the band. Moreover, our options-based framework differentiates itself from studies employing regime-switching DSGE or econometric models to assess currency board credibility (e.g., Jeanne and Masson, 2000; Blagov and Funke, 2019; Feng et al., 2023).

In other related work, Hanke et al. (2018) examines how probabilities from betting markets can be combined with risk-neutral exchange rate densities derived from currency options to forecast exchange rate movements around major political events. Our paper complements broader discussions on currency boards and their credibility, as reviewed in Ghosh et al. (2020).

The rest of the paper is organized as follows. In Section 2, we provide a brief overview of Hong Kong's linked exchange rate system. In Section 3, we present our model for the Hong Kong dollar. In Section 4, we report estimation results and discuss key implications. Section 5 concludes.

2 Hong Kong's Linked Exchange Rate System: An Overview

The monetary institution in Hong Kong has evolved from the silver standard (1863–Nov 1935) and the Sterling standard (Dec 1935–Jun 1972), to a brief fixed exchange rate regime against the US dollar (Jul 1972–Nov 1974), then to a free-floating regime (Nov 1974–Oct 1983), and finally to the current currency board with a US dollar link (Oct 1983–present).[8]

The current currency board arrangement is implemented through the Linked Exchange Rate System (LERS), introduced on October 17, 1983. Under the LERS, the Hong Kong dollar monetary base is fully backed by US dollar assets held in the Exchange Fund at the fixed exchange rate of HKD 7.80 to one US dollar (and all changes in the monetary base are also fully matched by corresponding changes in US dollar assets).

[8]See, e.g., Greenwood (2007) and Latter (2007) for a more detailed description of the evolution of the monetary system in Hong Kong.

The monetary base in Hong Kong comprises the following components: *(i)* Certificates of Indebtedness (CIs), which provide full backing to the banknotes issued by note-issuing banks; *(ii)* Government-issued notes and coins in circulation; *(iii)* Aggregate Balance, which is the sum of clearing account balances kept with the HKMA; and *(iv)* Exchange Fund Bills and Notes issued by the HKMA on behalf of the government for the account of the Exchange Fund.[9]

The government, through the HKMA, has given authorization to three commercial banks to issue banknotes in Hong Kong.[10] Holding CIs give the note issuing banks the right to issue HKD bills with a fixed exchange rate at 7.8 between a CI and the USD. The fixed value of the CI thus effectively translates into a fixed exchange rate between the HKD and the USD.

With the implementation of real-time gross settlement on December 9, 1996, all banks became obliged to maintain settlement accounts directly with the Exchange Fund. The combined total of clearing balances is known as the Aggregate Balance, which enables the authorities to control the volume of funds in the system directly through open market operations.

Following the Asian Financial Crisis in 1997, the HKMA announced on September 5, 1998 a package of measures—subsequently known as the "Seven Technical Measures"—to strengthen the mechanism.[11] A central elemtn was the introduction of a weak-side Convertibility Undertaking (CU) on the Aggregate Balance, under which the HKMA stood ready to purchase unlimited amounts of HKDs against USDs at a fixed rate of HKD 7.75 per USD, thereby preventing the exchnage rate from depreciating beyond that level. The weak-side CU was gradually moved from 7.75 to 7.8 between April 1999 and July 2000. In the early 2000s, large capital inflows generated persistent appreciation pressure on the currency. To achieve symmetry and strengthen credibility, the HKMA on May 18, 2005 introduced a strong-side CU at 7.75—committing to sell Hong Kong dollars against U.S. dollars to licensed banks at that rate—and simultaneously announced a gradual adjustment of the weak-side CU from 7.80 to 7.85 over a five-week period. Since then, the Linked Exchange Rate system has operated within a transparent symmetric convertibility zone, with the HKMA committed to sell (buy) Hong Kong dollars at the strong-side (weak-side) CU.

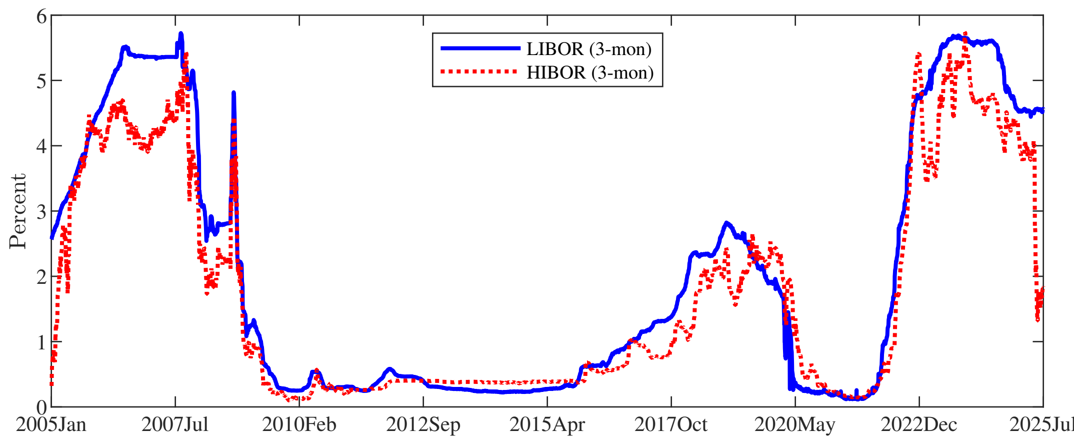
[9] To facilitate the development of Hong Kong's debt markets, the Exchange Fund Bills program was created in March 1990, and Notes were introduced in March 1993. The Exchange Fund Bills and Notes market enables the authorities to conduct open market operations.

[10] Namely, the Hongkong and Shanghai Banking Corporation Limited, the Bank of China (Hong Kong) Limited and the Standard Chartered Bank (Hong Kong) Limited. See, for example, https://www.hkma.gov.hk/eng/key-functions/money/hong-kong-currency/notes/.

[11] See HKMA press release at URL: https://www.hkma.gov.hk/eng/news-and-media/press-releases/1998/09/980905/?utm_source=chatgpt.com.

The stability of the Hong Kong dollar exchange rate is maintained through an automatic interest rate adjustment mechanism and the firm commitment by the HKMA to honour the CUs. When the demand for Hong Kong dollars is greater (less) than the supply and the market exchange rate strengthens (weakens) to the strong-side (weak-side) CU of HKD 7.75 (7.85) to one US dollar, the HKMA stands ready to sell (buy) Hong Kong dollars to banks for US dollars. The Aggregate Balance will then expand (contract) to push down (up) Hong Kong dollar interest rates, creating monetary conditions that move the Hong Kong dollar away from the strong-side (weak-side) limit to within the Convertibility Zone of 7.75 to 7.85.

Figure 2: **Interest Rates in the US and Hong Kong**



Note: This figure plots 3-month interbank interest rates in the US (solid blue line) and in Hong Kong (dotted red line) between January 2005 and July 2025.

Several factors are typically thought to contribute to pressure on the HKD-USD peg. First, when there is a significant interest rate gap between the U.S. dollar and the Hong Kong dollar, investors engage in carry trades to exploit the differential. The U.S. Federal Reserve began aggressive rate hikes in March 2022, as reflected in the sharp rise of the 3-month LIBOR shown in Figure 2.[12] In search of higher yields, investors borrowed in Hong Kong dollars—where interest rates remained low—and sold them to purchase higher-yielding U.S. dollar assets. This selling pressure further weakened the HKD. Throughout 2022, the Hong Kong Monetary Authority (HKMA) was slower to raise rates compared to its U.S. counterpart, as Hong Kong was still recovering from the pandemic-driven economic

[12] USD LIBOR based on panel bank submissions was discontinued on June 30, 2023. To facilitate the transition of outstanding contracts, "synthetic" USD LIBOR for selected tenors (1-, 3-, and 6-month) continued to be published until September 30, 2024. In this paper, we impute the 3-month LIBOR after October 1, 2024 using the historical relationship between the 3-month LIBOR and the 3-month Secured Overnight Financing Rate (SOFR) over the period January 2019 to September 2024.

slowdown and lingering political uncertainty following the 2020 national security law. In fact, the 3-month HIBOR declined during this period, as shown by the dotted red line in Figure 2. The widening rate differential further amplified depreciation pressure on the HKD. This dynamic reemerged in June 2025, when the interest rate gap between the two regions reached a new peak. In APril 2025, the sharp depreciation of the USD alongside record high net capital inflows from mainland China led to HKD appreciation. The HKD hit the strong side of the 7.75 HKD per USD in May As a result, the HKD experienced sharp volatility, swinging from the strong side to the weak side of the Convertibility Undertaking (CU). Since then, the exchange rate has hovered at 7.85, the weak end of the Convertibility Zone.

Second, as one of the world's major international financial centers, Hong Kong maintains an open market environment that allows capital to flow freely to and from mainland China. While this openness attracts foreign investors and institutions, it can also amplify capital flow volatility, including episodes of capital flight. For instance, Hong Kong experienced severe capital outflows during the Asian Financial Crisis in 1997. More recently, massive capital inflows entered the city, driven by foreign participation in blockbuster share offerings and record investment by Chinese investors in Hong Kong-listed equities. This wave of inflows created appreciation pressure, pushing the HKD to the strong-side limit of 7.75. In response, the HKMA intervened by selling Hong Kong dollars to defend the peg and contain further currency strength.

As HKMA Chief Executive Eddie Yue noted in his statement on July 11, 2025, "Future fluctuations of the Hong Kong dollar will largely depend on evolving market conditions and various global factors, such as U.S. monetary policies, the trajectory of interest rates, stock market sentiment, global financial markets, and fund flows." These same factors also contribute to the pressures faced by Hong Kong's currency peg.

3 Model

In this section we build an asset pricing model for the Hong Kong dollar (HKD) that accounts for the imperfect credibility of its peg. We derive both the equilibrium exchange rate and option prices in closed form. This provides intuition about the determinants and allows for efficient estimation of the model.

3.1 Setup

We assume that there is a probability p that the HKD-USD peg continues, and a probability $(1-p)$ that it ends tomorrow. If it ends, the exchange rate equals the fundamental exchange rate V_t that satisfies

$$\frac{1+r^{\$}}{1+\widetilde{r}^{H}} E_t^{Q}\left[V_{t+1}\right] = V_t, \tag{1}$$

where $\widetilde{r}^{H}$ denotes the interest rate in Hong Kong in the free-floating regime and $r^{\$}$ denotes the US interest rate.[13]

In the current peg regime, the HKD-USD exchange rate is constrained to be between 7.75 and 7.85. Denote the HKD-USD peg exchange rate $S^P \equiv 7.8$ and $b = 0.05$. The equilibrium exchange rate $\widetilde{S}_t$ is given by

$$\begin{aligned} \tilde{S}_t(V_t) &= \frac{1+r^{\$}}{1+r^{H}} \left[p E_t^{Q} H\left(\tilde{S}_{t+1}; S^P, b\right) + (1-p)\, E_t^{Q} V_{t+1} \right] \\ &= \frac{1+r^{\$}}{1+r^{H}} p E_t^{Q} H\left(\tilde{S}_{t+1}; S^P, b\right) + \frac{1+\widetilde{r}^{H}}{1+r^{H}} (1-p)\, V_t, \end{aligned} \tag{2}$$

where $E_t^{Q}[\cdot]$ refers to expectations under the HKD risk-neutral measure, r^H denotes the interest rate in Hong Kong in the current peg regime, and we denote

$$H\left(x; S^P, b\right) \equiv \max\left(\min\left(x, S^P + b\right), S^P - b\right).$$

Intuitively, the current spot exchange rate is the risk-adjusted expected value of the exchange rate in the two regimes, appropriately adjusted for the interest rates. In particular, the second line in in equation (2) is derived under the assumption in equation (1).

If the equilibrium exchange rate $\widetilde{S}_t$ falls within the band around the peg exchange rate, the observed spot exchange rate at the close is equal to $\widetilde{S}_t$. Otherwise, the spot exchange rate is equal to $\widetilde{S}_t$ truncated at the (lower or upper) boundary of the band. Therefore, the model-implied spot exchange rate at the close then equals

$$S_t^{CL} = H\left(\tilde{S}_t; S^P, b\right). \tag{3}$$

Normalizing key exchange rates by S^P, we define $\widehat{S}_t \equiv \tilde{S}_t / S^P$ and $\widehat{V}_t \equiv V_t / S^P$. As a result, the determination of equilibrium exchange rate $\tilde{S}_t$ boils down to the determination

[13] Jermann (2017) makes a similar assumption when studying the Swiss franc-Euro exchange rate.

of the univariate function $\widehat{S}(\widehat{V}_t)$. Based on this, we can simplify equation (2) to

$$\begin{aligned}
\widehat{S}(\widehat{V}_t) &= \frac{\tilde{S}\left(V_t; S^P\right)}{S^P} \\
&= \frac{1+r^{\$}}{1+r^H} p E_t^Q \left[H\left(\frac{\tilde{S}\left(V_{t+1}; S^P\right)}{S^P}; 1, \frac{b}{S^P} \right) \right] + \frac{1+\widetilde{r}^H}{1+r^H} (1-p) \widehat{V}_t \\
&= \frac{1+r^{\$}}{1+r^H} p E_t^Q \left[H\left(\widehat{S}\left(\widehat{V}_{t+1}\right); 1, \widehat{b} \right) \right] + \frac{1+\widetilde{r}^H}{1+r^H} (1-p) \widehat{V}_t, \qquad (4)
\end{aligned}$$

where we denote $\widehat{b} \equiv \frac{b}{S^P} = \frac{0.05}{7.8}$=0.0064. For ease of notation from now on we simply write $H(\widehat{S}(\widehat{V}_{t+1}); 1, \widehat{b})$ as $H(\widehat{S}(\widehat{V}_{t+1}))$. That is,

$$\widehat{S}(\widehat{V}_t) = \frac{1+r^{\$}}{1+r^H} p E^Q \left[H\left(\widehat{S}(\widehat{V}_{t+1}) \right) \right] + \frac{1+\widetilde{r}^H}{1+r^H} (1-p) \widehat{V}_t. \qquad (5)$$

We can derive the equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$ in closed form when we cast the model in continuous time. Specifically, under the HKD risk-neutral measure Q, the dynamics of the fundamental exchange rate V_t in continuous time is given below:

$$\begin{aligned}
\frac{dV_t}{V_t} &= \left(\widetilde{r}_{HKD} - r_{USD}\right) dt + \sigma_V dW_{V,t} \qquad (6) \\
&\equiv \mu_V dt + \sigma_V dW_{V,t},
\end{aligned}$$

where $W_{V,t}$ is independent Brownian motions under the measure Q, and $\mu_V \equiv \widetilde{r}_{HKD} - r_{USD}$ with $\widetilde{r}_{HKD}$ and r_{USD} being instantaneous interest rates for the HKD and the USD, respectively. Let Δt denote the length of a period in discrete time. Then the per-period interest rates in the preceding discrete-time setup satisfy: $1+r^{\$} = \exp\left(r_{USD}\Delta t\right)$ and $1+\widetilde{r}^H = \exp\left(\widetilde{r}_{HKD}\Delta t\right)$. The drift in the above equation is specified so as to exclude any arbitrage opportunities. Similarly, the per-period probability $p = \exp(-\lambda \Delta t)$ and we assume that the current managed floating regime will be abandoned upon arrival of a Poisson process with intensity λ.

To ensure the existence of a solution to equation (5), we assume that $\frac{1+r^{\$}}{1+r^H} p < 1$, which can be rewritten as equation (7) in Assumption 1. Note that the right-hand side of equation (5) can be considered as an operator on the equilibrium exchange rate. The condition (7) in Assumption 1 guarantees Blackwell's sufficient conditions for a contraction mapping, which implies the existence of a unique fixed point.

Assumption 1 *We assume*

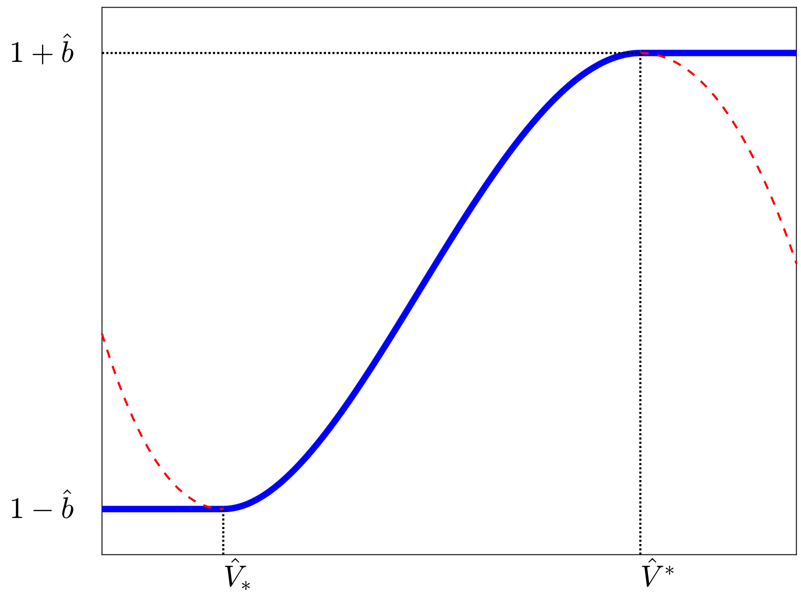
$$\lambda + r_{HKD} - r_{USD} > 0. \qquad (7)$$

We also assume that the Hong Kong interest rate in the current peg regime equals the counterpart in the free-floating regime: $r_{HKD} = \widetilde{r}_{HKD}$, which allows us to pin down the drift of the fundamental exchange rate μ_V using observable rates.

Assumption 2 *We assume*

$$\mu_V = r_{HKD} - r_{USD}. \tag{8}$$

Figure 3: **Equilibrium Exchange Rate**



NOTE: This figure plots the equilibrium exchange rate as a function of the (scaled) fundamental value.

In the proposition below, we derive the equilibrium exchange rate in closed form.

Proposition 1 *In the continuous-time model, the scaled equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$ is determined as follows:*

$$\widehat{S}(\widehat{V}) = \begin{cases} 1-\widehat{b}, & \text{if } \widehat{V} \leq \widehat{V}_*; \\ C_0\widehat{V} + C_1\widehat{V}^{\eta_1} + C_2\widehat{V}^{\eta_2}, & \text{if } \widehat{V}_* < \widehat{V} < \widehat{V}^*; \\ 1+\widehat{b}, & \text{if } \widehat{V} \geq \widehat{V}^*, \end{cases} \tag{9}$$

where the thresholds $\widehat{V}_*$ and $\widehat{V}^*$ are endogenously determined and the expressions of η_1, η_2, and C_0 through C_2 are given in the proof in the appendix. Under Assumption 2, $C_0 = 1$.

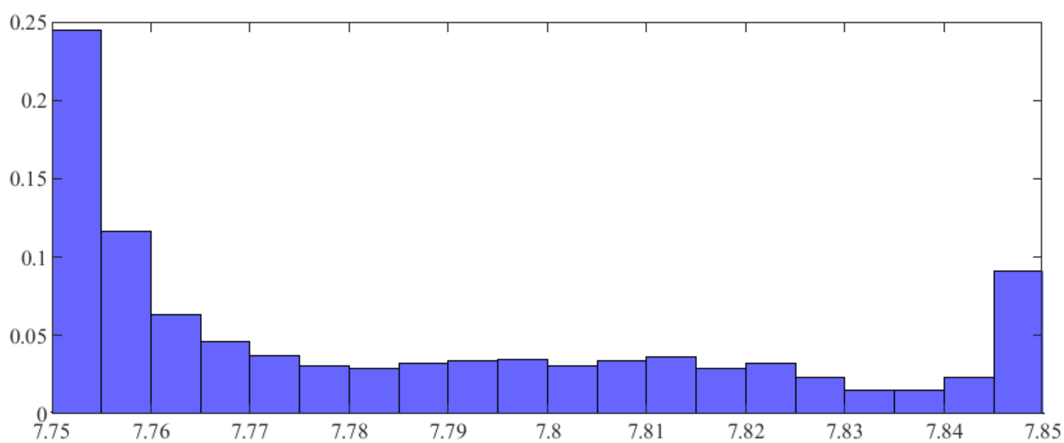
Proof. See Appendix A. ■

Based on our estimation results, we find that $\widehat{S}(\widehat{V}_t)$ is S-shaped in the middle range when $\widehat{V}$ falls within the interval between $\widehat{V}_*$ and $\widehat{V}^*$. Figure 3 plots the equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$. Mathematically, under Assumption 1 the exponents η_1 and η_2 in equation (9) have opposite signs. Without loss of generality, suppose $\eta_1 > 0 > \eta_2$ and thus $C_1 < 0 < C_2$. Intuitively, as the fundamental rate deviates from the central parity in the positive direction, $\widehat{V}_t$ increases, so does the term $\widehat{V}^{\eta_1}$. When $\widehat{V}$ gets closer to $\widehat{V}^*$, the shape of $\widehat{S}$ gets flatter because of expected interventions in the near future. This is reminiscent of the exchange rate behavior in a target-zone model (see Krugman, 1991) that the expectation of possible interventions affects exchange rate behavior even when the exchange rate lies inside the zone.

3.2 Exchange Rate Distribution

The empirical distribution of the USD-HKD exchange rate is U-shaped in the data, as shown in Figure 4. The U-shaped distribution is at odds with the bell-shaped distribution under standard Black-Scholes models. In sharp contrast, our model implies a distribution that closely resembles the empirical one in the data as we show below.

Figure 4: **Histogram of Historical USDHKD Exchange Rate**



NOTE: This figure plots the empirical distribution of the daily USD-HKD exchange rate between January 2007 and July 2025.

In our model, with probability $e^{-\lambda\tau}$, the current regime remains in place at $t+\tau$. Depending on whether the (scaled) exchange rate, $\widehat{S}_{t+\tau}$, hits one of the boundaries or remains strictly within the band, there are three possibilities: (i) with probability $1-\Phi(-d^*)=\Phi(d^*)$, $\widehat{S}_{t+\tau}$ is equal to its upper bound $1+\widehat{b}$; (ii) with probability $\Phi(-d_*)$, it is equal to the lower bound $1-\widehat{b}$; and (iii) with probability $\Phi(-d^*)-\Phi(-d_*)$, it is equal to $C_0\widehat{V}_{t+\tau}+C_1\widehat{V}_{t+\tau}^{\eta_1}+C_2\widehat{V}_{t+\tau}^{\eta_2}$, where

$$d^* \equiv \frac{\log\left(\widehat{V}_t/\widehat{V}^*\right)+\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}, \tag{10}$$

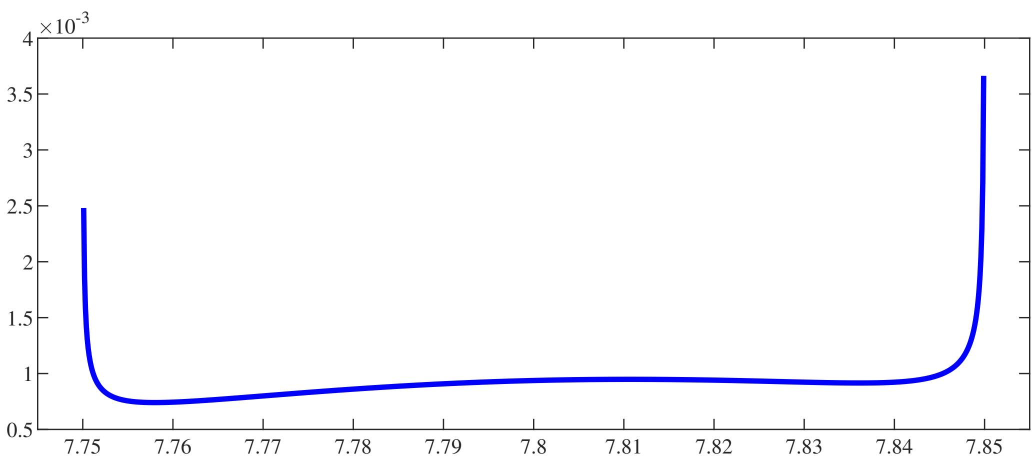
$$d_* \equiv \frac{\log\left(\widehat{V}_t/\widehat{V}_*\right)+\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}. \tag{11}$$

and $\widehat{V}_*$ and $\widehat{V}^*$ are defined in Proposition 1. In the last possibility, when the (scaled) fundamental value is between $\widehat{V}_*$ and $\widehat{V}^*$, the probability density function of the exchange rate within the band can be derived by a change of variable

$$f^{TZ}\left(\widehat{S}_{t+\tau}\right)=\frac{LN_{\left(\widehat{V}_*,\widehat{V}^*\right)}\left(\widehat{V}_{t+\tau}\right)}{\widehat{S}'\left(\widehat{V}_{t+\tau}\right)}=\frac{\exp\left(-\frac{\left(\ln\left(\widehat{V}_{t+\tau}/\widehat{V}_t\right)-\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau\right)^2}{2\sigma_V^2\tau}\right)}{\sigma_V\sqrt{2\pi\tau}\widehat{V}_{t+\tau}\widehat{S}'\left(\widehat{V}_{t+\tau}\right)}1_{\left\{\widehat{V}_{t+\tau}\in\left(\widehat{V}_*,\widehat{V}^*\right)\right\}}, \tag{12}$$

where $LN_{(\widehat{V}_*,\widehat{V}^*)}(\widehat{V}_{t+\tau})$ denotes a truncated log-normal distribution between $\widehat{V}_*$ and $\widehat{V}^*$, and $\widehat{V}_{t+\tau}=\widehat{S}^{-1}(\widehat{S}_{t+\tau})$. It is straightforward to see that the integral of $f^{TZ}(\widehat{S}_{t+\tau})$ from $1-\widehat{b}$ to $1+\widehat{b}$ equals $\Phi(-d^*)-\Phi(-d_*)$. Figure 5 plots $f^{TZ}(\widehat{S}_{t+\tau})$ within the band for $\widehat{S}_{t+\tau}\in(1-\widehat{b},1+\widehat{b})$.

Figure 5: **Model-implied Distribution within the Band**



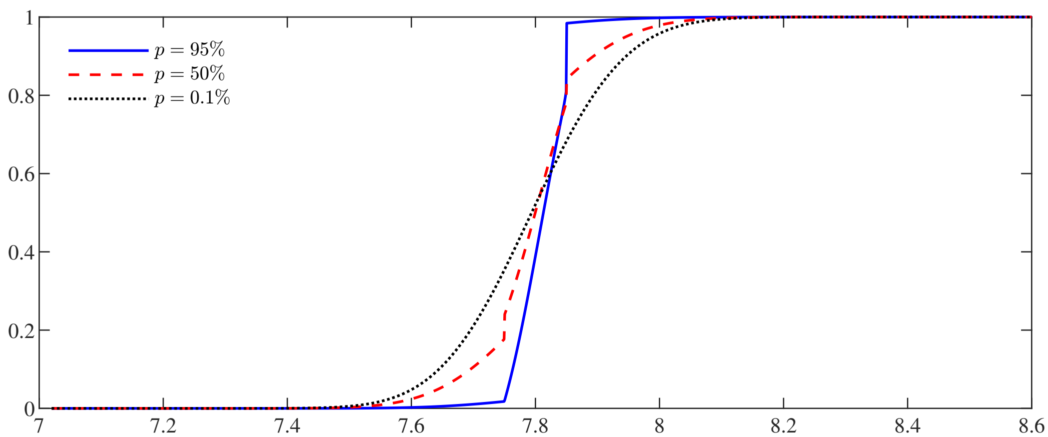
NOTE: This figure plots the model-implied density distribution of the equilibrium exchange rate within the band in the current regime (i.e., $f^{TZ}(\widehat{S}_{t+\tau})$ in equation (12)).

With probability $1-e^{-\lambda\tau}$, the economy is in the free-floating regime at $t+\tau$ in which the (scaled) exchange rate at $t+\tau$ is equal to the fundamental value: $\widehat{S}_{t+\tau}=\widehat{V}_{t+\tau}$, which follows a log-normal distribution with mean $\ln\widehat{V}_t+\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau$ and variance $\sigma_V^2\tau$, since $\widehat{V}_{t+\tau}=\widehat{V}_t\exp\left(\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau+\sigma_V\sqrt{\tau}z_V\right)$ where z_V follows a standard normal distribution. That is, conditional on the regime being the free-floating one, the probability density function of $\widehat{S}_{t+\tau}$, which equals $\widehat{V}_{t+\tau}$, is given by

$$f^{FF}\left(\widehat{S}_{t+\tau}\right)=LN\left(\widehat{V}_{t+\tau}\right)=\frac{\exp\left(-\frac{\left(\ln\left(\widehat{V}_{t+\tau}/\widehat{V}_t\right)-\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau\right)^2}{2\sigma_V^2\tau}\right)}{\sigma_V\sqrt{2\pi\tau}\widehat{V}_{t+\tau}}, \tag{13}$$

where $LN(\widehat{V}_{t+\tau})$ denotes the above log-normal distribution.

Figure 6: **Model-implied Full Cumulative Distribution Function**



NOTE: This figure plots the full cumulative distribution function of the model-implied equilibrium exchange rate.

Taking into account possible regime shifts, Figure 6 plots the cumulative distribution function for three cases: 1) $\lambda=27.6$ (or $p=0.1\%$); 2) $\lambda=2.8$ (or $p=50\%$); and 3) $\lambda=0.2$ (or $p=95.1\%$). The distribution function for $p=0.1\%$ closely mirrors the log-normal distribution assumed in standard Black-Scholes models. This resemblance arises from the high likelihood in the first case that the existing regime will transition to a free-floating one. Consequently, this scenario tends to overstate the probability of the exchange rate breaking out of the band compared with the other two more realistic cases.

In contrast, when $p=95\%$, which represents the model-implied density for an average day in the data, the possibility of the regime switching to the free-floating one is taken into account. However, this case assesses a substantial likelihood for the current regime to persist.

Therefore, the exchange rate predominantly fluctuates within the band, though there exists a non-trivial probability of it breaking out. In the next section, we will use our estimation results to further investigate this important implication.

3.3 Forward and Option Pricing

We estimate the model to match the spot rate and four options. In this subsection, we price HKD forwards and options in closed form.

First, let $F(\tau) \equiv E_t^Q\left[S_{t+\tau}^{CL}\right]$ denote the forward rate, which is defined as the date-t conditional expectation of the HKD's spot rate at date $t+\tau$. Suppose there are N periods until $t+\tau$; that is, $N=\tau/\Delta t$. In the proposition below, we provide the closed-form formula for the HKD forward rate.

Proposition 2 *Under the model, the HKD forward rate has the following closed-form formula:*

$$
\begin{aligned}
F(\tau) &= p^N E_t^Q\left[H\left(\widetilde{S}_{t+\tau}; S^P, b\right)\right] + \left(1-p^N\right) E_t^Q\left[V_{t+\tau}\right] \\
&\equiv p^N F_S(\tau) + \left(1-p^N\right) F_V(\tau),
\end{aligned}
$$

where

$$
\begin{aligned}
F_S(\tau) &\equiv E_t^Q\left[H\left(\widetilde{S}_{t+\tau}; S^P, b\right)\right] \\
&= \left(1-\Phi\left(-d^*\right)\right)\left(S^P+b\right) + \Phi\left(-d_*\right)\left(S^P-b\right) \\
&\quad + \left(\Phi\left(-d^*\right)-\Phi\left(-d_*\right)\right) S^P \sum\nolimits_{i=0}^{2} C_i \Gamma\left(\widehat{V}_t; \eta_i\right),
\end{aligned} \tag{14}
$$

and

$$
F_V(\tau) \equiv E_t^Q\left[V_{t+\tau}\right] = V_t \exp\left(\mu_V \tau\right), \tag{15}
$$

and

$$
\Gamma(\widehat{V}_t; \eta) = \widehat{V}_t^{\eta} e^{\eta\mu_V\tau + \frac{1}{2}\eta(\eta-1)\sigma_V^2\tau} \frac{\Phi\left(-d^* - \eta\sigma_V\sqrt{\tau}\right) - \Phi\left(-d_* - \eta\sigma_V\sqrt{\tau}\right)}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)}. \tag{16}
$$

Proof. See Appendix A. ■

Next, we turn to pricing options. Consider a call option with strike K and maturity τ.

Its price is given by

$$
\begin{aligned}
& C(K,\tau) \\
= \; & p^N e^{-r_{HKD}\tau} E^Q \left[\max\left(H\left(\widetilde{S}_{t+\tau}; S^P, b\right) - K, 0\right)\right] \\
& + \sum_{n=1}^{N} p^{n-1}(1-p)\, e^{-r_{HKD} n\Delta t - \widetilde{r}_{HKD}(N-n)\Delta t} E^Q \left[\max\left(V_{t+\tau} - K, 0\right)\right] \\
\equiv \; & p^N C_S(K,\tau) + \left(1-p^N\right)\chi C_V(K,\tau), \qquad (17)
\end{aligned}
$$

where $\chi \equiv \frac{1}{1-p^N}\sum_{n=1}^{N} p^{n-1}(1-p)\, e^{-r_{HKD} n\Delta t + \widetilde{r}_{HKD} n\Delta t} = 1$ under Assumption 2, and

$$
\begin{aligned}
C_S(K;\tau) & \equiv e^{-r_{HKD}\tau} E_t^Q \left[\max\left(H\left(\widetilde{S}_{t+\tau}; S^P, b\right) - K, 0\right)\right], \\
C_V(K;\tau) & \equiv e^{-\widetilde{r}_{HKD}\tau} E_t^Q \left[\max\left(V_{t+\tau} - K, 0\right)\right].
\end{aligned}
$$

In the following proposition, we derive closed-form formula for the above option price.

Proposition 3 *The price of a call option with strike K and maturity τ is a weighted average of two components $C_S(K,\tau)$ and $C_V(K,\tau)$ as shown in equation (17).*

(i) The component $C_V(K,\tau)$ has the following Black-Scholes formula:

$$
C_V(K,\tau) \equiv e^{-\widetilde{r}_{HKD}\tau} E_t^Q \max\left[V_{t+\tau} - K, 0\right] = e^{-r_{USD}\tau} V_t \Phi\left(d_{1,V}\right) - e^{-\widetilde{r}_{HKD}\tau} K \Phi\left(d_{2,V}\right), \qquad (18)
$$

where

$$
\begin{aligned}
d_{1,V} & = d_{2,V} + \sigma_V\sqrt{\tau} = \frac{\log\left(V_t/K\right) + \left(\mu_V + \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}, \qquad (19) \\
d_{2,V} & = \frac{\log\left(V_t/K\right) + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}. \qquad (20)
\end{aligned}
$$

(ii) The component $C_S(K,\tau)$ is given by:

$$
C_S(K,\tau) = \begin{cases}
0, & \text{if } K \geq S^P + b \\
e^{-r_{HKD}\tau}\left(F_S(\tau) - K\right), & \text{if } K \leq S^P - b \\
\left[\begin{array}{c}
\left(1-\Phi\left(-d^*\right)\right) e^{-r_{HKD}\tau}\left[S^P + b - K\right] \\
+\left(\Phi\left(-d^*\right) - \Phi\left(-d_*\right)\right) e^{-r_{HKD}\tau} \\
\times\left[S^P \sum_{i=0}^{2} C_i \Gamma^*\left(\widehat{V}_t;\eta_i\right) - K\Gamma^*\left(\widehat{V}_t;0\right)\right]
\end{array}\right], & \text{otherwise}
\end{cases} \qquad (21)
$$

where

$$\Gamma^*(\widehat{V}_t;\eta) \equiv \widehat{V}_t^{\eta} e^{\eta\mu_V\tau+\frac{1}{2}(\eta^2-\eta)\sigma_V^2\tau} \frac{\Phi\left(-d^*-\eta\sigma_V\sqrt{\tau}\right)-\Phi\left(-d_K-\eta\sigma_V\sqrt{\tau}\right)}{\Phi\left(-d^*\right)-\Phi\left(-d_*\right)},$$

$$d_K \equiv \frac{\log\left(\widehat{V}_t/\widehat{V}_K\right)+\left(\mu_V-\frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}},$$

and $\widehat{V}_K$ *is the cutoff value satisfying* $\widehat{S}(\widehat{V}_K)=\widehat{K}\left(\equiv K/S^P\right)$, *which has a unique solution when* $\widehat{K}\in[1-\widehat{b},1+\widehat{b}]$.

Proof. See Appendix A. ■

We can similarly derive the closed-form formula for the price of a put option with the same strike and maturity.

$$P\left(K,\tau\right)\equiv p^N P_S\left(K,\tau\right)+\left(1-p^N\right)\chi P_V\left(K,\tau\right), \tag{22}$$

where

$$P_V\left(K,\tau\right)=e^{-\widetilde{r}_{HKD}\tau}E_t^Q\max\left[K-V_{t+\tau},0\right]=-V_te^{-r_{USD}\tau}N\left(-d_{1,V}\right)+Ke^{-\widetilde{r}_{HKD}\tau}N\left(-d_{2,V}\right),$$

and

$$P_S\left(K,\tau\right)=\begin{cases} e^{-r_{HKD}\tau}\left(K-F_S\left(\tau\right)\right), & \text{if } K\geq S^P+b;\\ 0, & \text{if } K\leq S^P-b;\\ \left[\begin{array}{c}\Phi\left(-d_*\right)e^{-r_{HKD}\tau}\left[K-\left(S^P-b\right)\right]\\ +\left(\Phi\left(-d^*\right)-\Phi\left(-d_*\right)\right)e^{-r_{HKD}\tau}\\ \times\left[K\Gamma_*\left(\widehat{V}_t;0\right)-S^P\sum_{i=0}^2 C_i\Gamma_*\left(\widehat{V}_t;\eta_i\right)\right]\end{array}\right], & \text{otherwise,}\end{cases} \tag{23}$$

and

$$\Gamma_*(\widehat{V}_t;\eta)\equiv\widehat{V}_t^{\eta}e^{\eta\mu_V\tau+\frac{1}{2}(\eta^2-\eta)\sigma_V^2\tau}\frac{\Phi\left(-d_K-\eta\sigma_V\sqrt{\tau}\right)-\Phi\left(-d_*-\eta\sigma_V\sqrt{\tau}\right)}{\Phi\left(-d^*\right)-\Phi\left(-d_*\right)}.$$

4 Model Estimation and Results

In this section, we describe estimation methodology and results.

4.1 Data

Our sample period is from 2007 to 2025. We obtain most data at a daily frequency from Bloomberg, such as, the historical USD-HKD exchange rate, the 3-month Hong Kong interbank offer rate (HIBOR) for the HKD and London interbank offer rate (LIBOR) for the USD, and derivatives data. The derivatives data include forwards/futures and options written on the HKD.

The HKD option data consist of implied volatility quotes for at-the-money options ("ATM"), risk reversals ("RR"), and butterfly spreads ("BY"), with a maturity of 3 months.[14] The RR and BY quotes are available for strike prices corresponding to 25% and 10% delta (labeled as "25Δ" and "10Δ" respectively). These quotes can then be used to infer implied volatilities of 25Δ or 10Δ options by the standard approach (e.g., see Bisesti et al., 2005; Jermann, 2017, for more details). For 25Δ calls and puts, for instance, implied volatilities are computed as

$$
\begin{aligned}
\sigma_{25C} &= \sigma_{ATM} + \sigma_{25BY} + \frac{1}{2}\sigma_{25RR}, \\
\sigma_{25P} &= \sigma_{ATM} + \sigma_{25BY} - \frac{1}{2}\sigma_{25RR}.
\end{aligned}
$$

Following the quoting convention of the RMB options markets, the Black-Scholes formula is then used to compute the prices and strike prices corresponding to 25Δ and 10Δ options. In the end, we obtain four option price series: two for puts and two for calls.

Table 1: Summary Statistics of HKD Option Prices

	strike				price	IV
	mean	25%	50%	75%	mean	mean
Put (10Δ)	7.68	7.64	7.68	7.72	0.38	2.05
Call (10Δ)	7.85	7.82	7.85	7.88	0.27	1.46
Put (25Δ)	7.75	7.73	7.74	7.77	0.71	1.22
Call (25Δ)	7.80	7.78	7.80	7.83	0.54	0.93

NOTE: This table presents summary statistics for strike prices, option prices, and implied volatility (expressed as percentages) for four HKD options spanning the sample period. We provide the average strike price along with the 25th, 50th, and 75th percentiles for strike prices. Additionally, we report the average option prices and implied volatilities.

Table 1 reports summary statistics for HKD options for these four options during the sample period. As shown in Table 1, the 10Δ put options have an average strike of 7.68.

[14] The price of a risk reversal equals the difference in implied volatility between an out-of-the-money call and an out-of-the-money put with the same delta. The price of a butterfly equals the difference between the average implied volatility of those out-of-the-money options and that of the at-the-money option.

Interestingly, these strike prices are lower than the strong-side CU of 7.75 for more than 75% of the time. Similarly, the strike prices for the 10Δ call options are greater than the weak-side CU of 7.85 for about half of the time. Given that that our main interest is in the implied probabilities of exchange rate realizations breaking out the convertibility zone, it is useful that our data include options with strike prices that are typically outside the convertibility zone.

The model is used to estimate the values for (V, p, σ_V) that best fit the spot rate and the prices of the four three-month HKD options. Interest rates are taken as given from the data. Model fit is evaluated by the sum of squared deviations between model-implied and observed prices (for both the options and the exchange rate), with equal weights assigned in the benchmark specification. We repeat this procedure for every day in the sample to generate time series for $(V_t, p_t, \sigma_{V,t})$. Because the model becomes degenerate and numerically unstable under perfect credibility ($p = 100\%$), we impose an upper bound of 97.5% on the credibility parameter in estimation.

4.2 Results

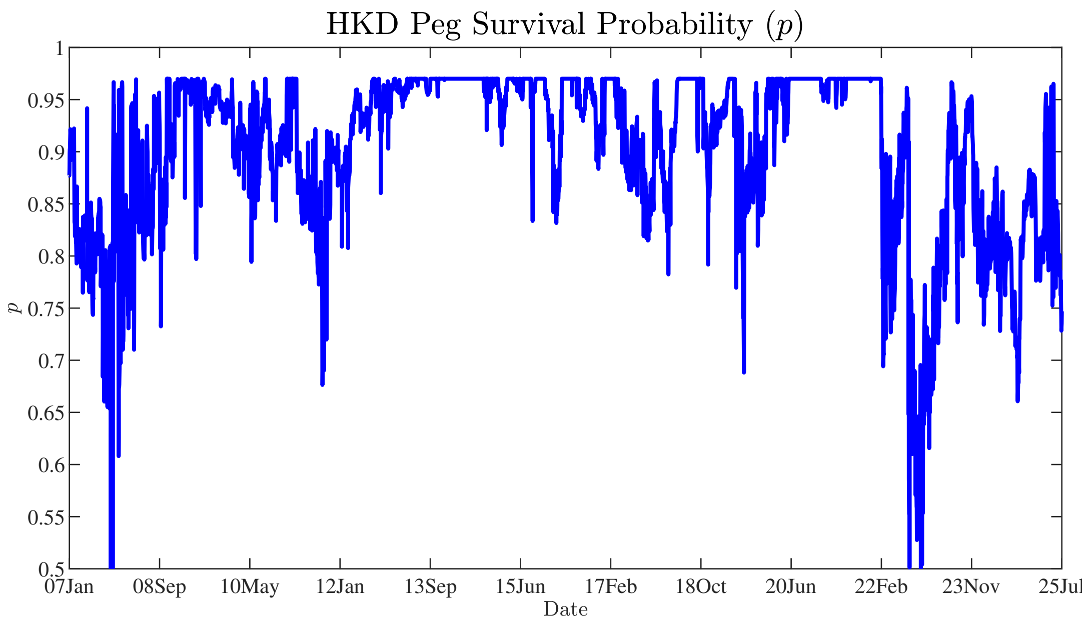
We present the empirical results in this subsection. Figure 7 illustrates the main estimation results. The first panel (Figure 7a) displays the estimated probability, p, that the HKD peg remains intact. At several key stress points, this probability experiences significant declines. Notably, it dropped to 69% in August 2019 when China's currency renminbi (RMB) weakened past 7 per USD amid the US-China trade war. It further declined to 50% in late 2022, coinciding with rising U.S. interest rates and declining interbank liquidity.[15] Such an elevated risk of the peg breaking, as viewed by financial markets, had not been observed since the global financial crisis.

The second panel (Figure 7b) plots the estimated fundamental exchange rate value, V, over time. The fluctuations in V are considerably wider than the band, particularly during major financial and geopolitical events. For example, the estimated value of V increased during periods of economic uncertainty, including the US-China trade war and the COVID-19 pandemic, indicating increased depreciatory pressure on the HKD. Additionally, the estimated volatility of the fundamental exchange rate (not reported here but available upon request) closely tracks the option-implied volatility.

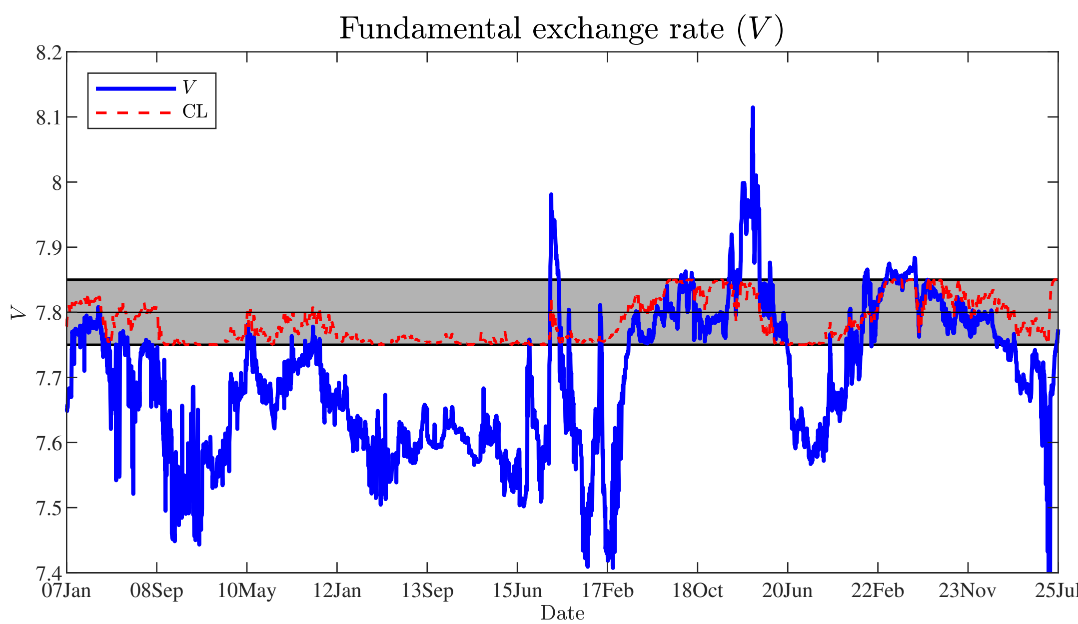
As discussed earlier, the Black-Scholes model tend to overstate the probability of breaking out the band. This finding is confirmed by Figure 8 by the large gap between the estimates based on the Black-Scholes model and our model. As shown by the dotted black line, ignoring

[15] The interbank liquidity dropped from more than 300 billion HKD in March 2022 to about 50 billion HKD in April 2023.

Figure 7: **Estimation Results**



(a)

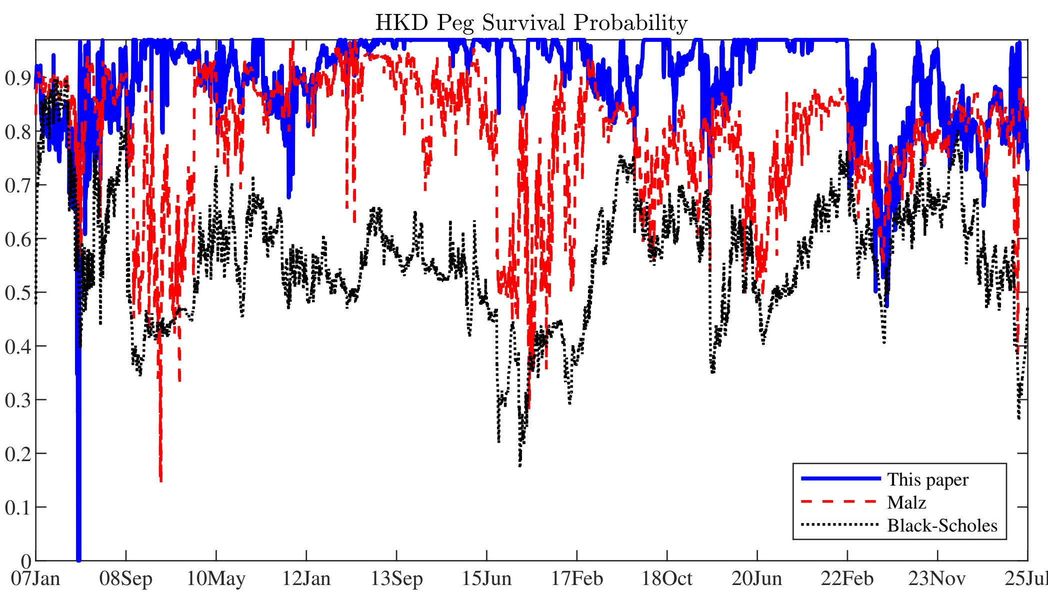


(b)

NOTE: Panel (a) plots the 3-month probability p that the HKD peg remains credible in 90 days. Blue line in Panel (b) plots the estimated fundamental exchange rate value V. The red line is the spot USD-HKD exchange rate. The shaded area is the band set for the USD-HKD exchange rate.

possible regime shifts, the Black-Scholes model assigns an average probability of 67% (17%) that the USD-HKD exchange rate would break out of the upper (lower) bound. Malz (1997) extracts the risk-neutral probability density function of exchange rate expectations using the second derivative of the price of a European call option (or put option) with respect to its strike price. Genberg and Hui (2011) applies this methodology to study HKD. The dashed red line shows the estimated continuation probability of the peg in our sample, which is higher than the estimates based on the Black-Scholes model. However, it is still substantially lower than our estimates. In contrast, our model offers more realistic estimates, implying an average probability of 3% of breaking out of either bound. These results illustrate the importance of incorporating possible regime shifts in our framework, compared with the benchmark Black-Scholes model (see, e.g., Genberg and Hui, 2011).

Figure 8: **Peg Survival Probability**



NOTE: This figure plots our estimated probability that the USD-HKD exchange rate will remain within the band based on our model (solid blue lines) and the method in Malz (1997) (dashed red lines), and the standard Black-Scholes model (dotted black lines).

Our estimation captures financial market participants' expectations regarding the Hong Kong dollar, particularly as reflected in the derivatives market. Figure 9 plots the prices of risk reversals and butterflies. In the HKD options market, risk reversals indicate directional expectations: a positive value suggests anticipated U.S. dollar strength, while a negative value points to expected Hong Kong dollar appreciation. A near-zero value reflects a neutral outlook. Butterflies, by contrast, provide insights into the market's expectations of price stability or potential volatility, helping to identify periods of uncertainty or events likely to trigger large exchange rate movements. These option-based indicators show heightened pressure for significant exchange rate shifts during 2016–17, 2019–2023, and early 2025, aligning with the periods of lower peg survival probability p estimated by our model. Furthermore, the estimated value of the underlying exchange rate fundamentals, V, tracks closely with the behavior of risk reversals. Notably, the spikes in 2016 and 2019 reflect depreciation pressure, while the sharp decline in May 2025 coincides with volatile HKD movements, including a brief appreciation to the strong-side limit of 7.75 before reversing course.

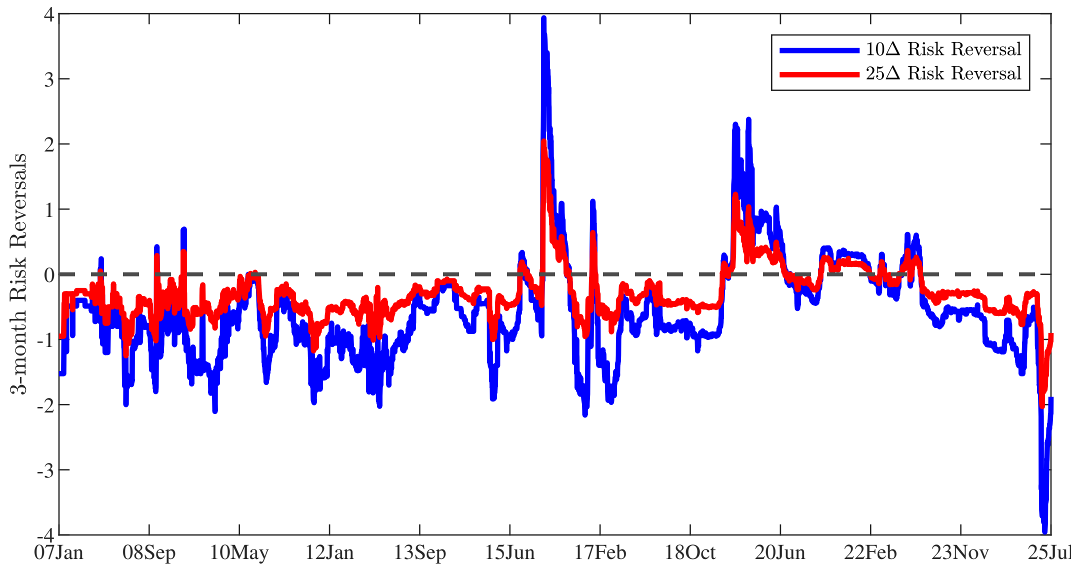
Our estimates depend on factors such as the spot exchange rate, option-implied volatilities, and interest rate differentials between Hong Kong and the U.S. To examine the relative importance of these input variables in the estimation, we run regressions of the estimated fundamental value (V_t) on these input variables. The regression results are reported in Table 2 Panel A. The regression results for the estimated continuation probability (p_t) are reported in Table 2 Panel B. The results in these two tables indicate that all input variables play a significant role in our estimation. Collectively, they account for about 88.5% of the variation in the estimated fundamental value and 53.3% of the variation in the estimated continuation probability (Columns (1) in Table 2).

The most important input variables for the fundamental exchange rate are the spot exchange rate as well as the implied volatility of the two risk-reversals. Dropping the splot rate or the two risk reversals reduces R^2 from 88.5% to 80% (column (3) and (5) in Table 2) Panel B) The implied-volatility for butterfly spreads are most important for the estimation fo continuation probability. Dropping the two butterflies reduces R^2 from 53.3% to 33.6% (Column (2)) in Table 2) Panel A.

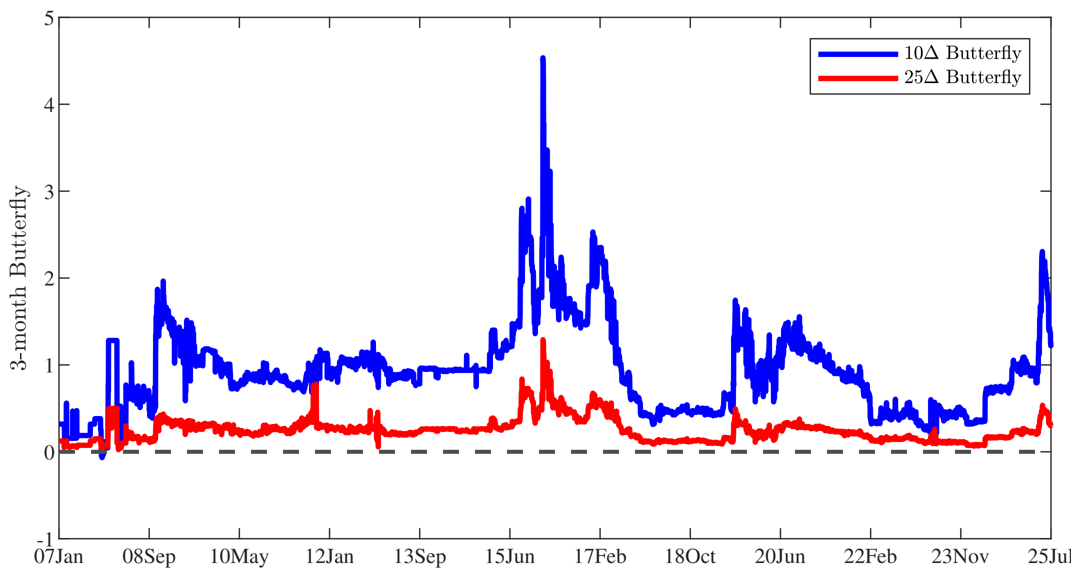
4.3 Economic fundamentals and sustainability of the Hong Kong Dollar currency board

We next examine the importance of liquidity conditions, U.S. monetary policy, and exchange rate expectations in shaping the credibility of Hong Kong's currency board arrangement. Figure 10 plots the smoothed continuation probability based on our estimation results. As shown by the solid blue line, the estimated probability declines to around 60% in March 2022, coinciding with the start of the Federal Reserve's tightening cycle. At the same time, interbank liquidity (represented by the dashed red line) drops sharply—from over 300 billion HKD in March 2022 to approximately 50 billion HKD by April 2023. The model also estimates a decline in the continuation probability in May 2025, when the HKD briefly appreciated to the strong-side limit of the Convertibility Undertaking (CU). In response, the

Figure 9: **HKD-USD Risk Reversals and Butterflies**



(a) Risk Reversals



(b) Butterflies

Note: Panel A plots 10Δ (blue) and 25Δ (red) risk-reversal quotes, defined as the difference in implied volatility between a call and a put of the same delta. Panel B shows butterfly quotes, calculated as the average implied volatility of the matching call and put minus the at-the-money implied volatility.

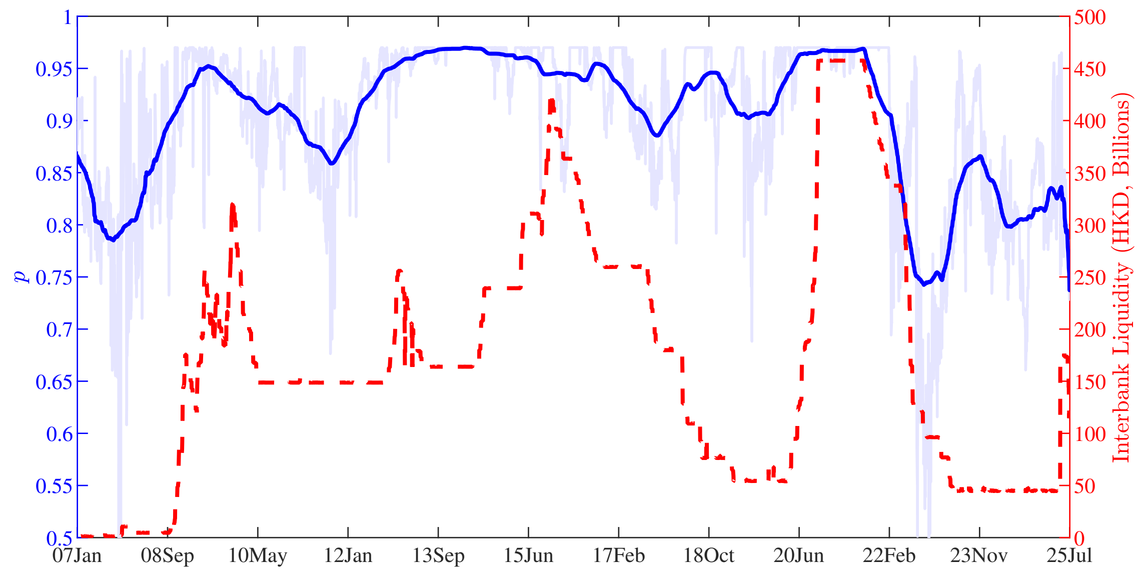
Table 2: Sensitivity Analysis

	(1)	(2)	(3)	(4)	(5)	(6)
A. Fundamental Exchange Rate (V)						
$i^{HKD} - i^{USD}$	0.02***	0.03***	0.08***	0.01***	−0.04***	
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
S^{CL}	1.86***	2.59***	2.52***	2.31***		1.65***
	(0.03)	(0.03)	(0.04)	(0.03)		(0.03)
σ_{ATM}	0.09***	0.04***	0.13***		0.16***	0.09***
	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)
σ_{25RR}	0.02*	−0.15***		−0.04***	0.15***	0.03***
	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)
σ_{10RR}	0.04***	0.12***		0.08***	0.00	0.04***
	(0.00)	(0.00)		(0.00)	(0.01)	(0.00)
σ_{25BY}	0.17***		0.19***	0.24***	0.16***	0.18***
	(0.01)		(0.02)	(0.01)	(0.02)	(0.01)
σ_{10BY}	−0.11***		−0.10***	−0.09***	−0.18***	−0.11***
	(0.00)		(0.00)	(0.00)	(0.00)	(0.00)
R^2(%)	88.5	84.5	80.3	85.3	79.1	88.1
Obs.	4833	4833	4833	4833	4833	4833
B. Continuation Probability (p)						
$i^{HKD} - i^{USD}$	0.05***	0.03***	0.04***	0.07***	0.02***	
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	
S^{CL}	0.75***	−0.46***	0.64***	−0.05		0.30***
	(0.05)	(0.04)	(0.04)	(0.05)		(0.04)
σ_{ATM}	−0.17***	−0.09***	−0.17***		−0.14***	−0.18***
	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)
σ_{25RR}	−0.09***	0.19***		0.01	−0.03**	−0.05***
	(0.01)	(0.02)		(0.02)	(0.01)	(0.01)
σ_{10RR}	0.04***	−0.09***		−0.02***	0.02***	0.04***
	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)
σ_{25BY}	−0.32***		−0.31***	−0.45***	−0.32***	−0.31***
	(0.02)		(0.02)	(0.02)	(0.02)	(0.02)
σ_{10BY}	0.19***		0.18***	0.16***	0.16***	0.18***
	(0.01)		(0.01)	(0.01)	(0.01)	(0.01)
R^2(%)	53.3	33.6	52.9	35.4	50.6	50.2
Obs.	4833	4833	4833	4833	4833	4833

NOTE: This table reports regression results for the fundamental exchange rate (V_t) in Panel A and the continuation probability (p_t). Independent variables include the difference between Hong Kong and US interest rates ($i^{HKD} - i^{USD}$), the USD-HKD exchange rate (S^{CL}), implied volatility quotes for at-the-money options (σ_{ATM}), risk-reversals (σ_{25RR} or σ_{10RR}), and butterfly spreads (σ_{25BY} or σ_{10BY}). Column (1) reports the results from the regression with all the above regressors, while Columns (2)-(6) report the results with one or two regressors dropped at a time. Significance levels: * $p < .10$; ** $p < .05$; and *** $p < .01$.

HKMA intervened by selling HKD, which rapidly increased interbank liquidity. However, as the exchange rate quickly reversed toward the weak-side limit of 7.85, the HKMA resumed intervention—this time by selling USD—leading to a renewed decline in liquidity.

Figure 10: **Market-based Measure of the Continuation Probability**



NOTE: Solid blue line shows the smoothed 3-month probability as the moving average over 250 days. The light blue line is the estimated 3-month probability as in Figure 7. Dashed red line shows the interbank liquidity.

Table 3 presents regression results examining the relationship between key economic variables and two estimated measures: the fundamental exchange rate value (V) and the probability of the HKD peg continuing (p). In particular, we focus on the economic fundamentals related to Hong Kong's banking sector liquidity, foreign reserve, and its money base, the onshore and offshore exchange rate for China's renminbi, as well as the interest rate differentials between the US and Hong Kong, Panel A focuses on V, showing that a decrease in Hong Kong's interbank liquidity (LIQ.) is significantly associated with a lower fundamental value of the HKD. This suggests that liquidity constraints in the financial system exert downward pressure on the HKD's fundamental value. Additionally, the regression reveals a strong positive correlation between V and the CNY and CNH exchange rates, with the coefficients of the latter variables being 0.10 and 0.14, respectively. This indicates that financial markets perceive the HKD's fundamental value as closely linked to the Chinese currency, suggesting that in the event of a peg break, the HKD might move toward the value of the yuan as an alternative to a free float. Lastly, a higher interest rate in Hong

Kong relative to the US counterparts strengthens the Hong Kong dollar, as indicated by the negative coefficient on V. When all the variables are included in the regression as in column (8), the R^2 is around 50%.

Panel B analyzes the how the economic variables are related to the probability, p, that the HKD peg remains intact. Liquidity has a strong positive relationship with p, with the coefficients rangin from 0.08 to 0.11, indicating that higher liquidity supports peg stability. Meanwhile, the foreign reserve and monetary base have an insignificant effect. The CNY and CNH exchange rates exhibit strong negative correlations with p, suggesting that as the Chinese renminbi depreciates, market participants perceive a higher risk of the HKD peg breaking. Lastly, a higher Hong Kong interest rate relative to the US interest rate strengthens the HKD currency board. With all the variables in the regression, the R^2 in column (8) is also high at 42%.

Overall, the regression results highlight the HKD's sensitivity to both local liquidity conditions and broader developments in China's currency markets. They reinforce the view that external pressures—particularly those from U.S. monetary policy and China's financial stability—play a crucial role in shaping market expectations about the peg's future. Our estimates also shed light on the market's view about the alternative of LERS. If the LERS were to be abandoned, what could be alternative exchange rate regimes to replace the LERS? In the market commentary, several alternatives have been proposed besides the free float: (1) a link to another currency, say, CNY or CNH; (2) a link to a basket of currencies. Our estimates of the fundamental value V can be used to assess which alternative is considered to be more realistic by financial market participants.

5 Conclusion

This paper develops a structural asset-pricing model to assess the credibility of Hong Kong's Linked Exchange Rate System (LERS). By incorporating regime shifts and estimating with HKD options data, we extract market-implied probabilities of peg survival and the HKD's fundamental value. While overall credibility remains high in normal conditions, the peg has faced notable pressures, especially during U.S. rate hikes, liquidity squeezes, and episodes of geopolitical or economic stress in 2019, 2022, and 2025.

Our framework offers a practical tool for monitoring and quantifiable assessing regime credibility in market perceptions in real time. It can also be extended to evaluate alternative exchange rate arrangements or applied to other economies with fixed or semi-fixed exchange rates. As discussions about Hong Kong's long-term monetary anchor continue, this model provides a useful lens for interpreting how markets perceive and price such risks. Regular updates to our estimates can serve as an early warning system, enabling regulators and investors to anticipate and respond to changing market conditions.

Table 3: Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A. Fundamental Exchange Rate (V)								
LIQ	-0.26^{***}							-0.20^{**}
	(0.06)							(0.09)
RES		0.40^{***}						1.93^{***}
		(0.06)						(0.31)
MB			0.06^{***}					-0.25^{***}
			(0.01)					(0.09)
CNY				0.10^{***}				-1.09^{***}
				(0.02)				(0.41)
CNH					0.14^{***}			1.11^{***}
					(0.02)			(0.41)
CFETS						0.00^{***}		-0.00
						(0.00)		(0.00)
IR							-0.08^{***}	-0.05^{***}
							(0.02)	(0.02)
$R^2(\%)$	8.5	14.8	9.6	13.3	21.4	3.4	10.9	49.6
Obs.	220	220	220	220	177	220	220	177
B. Continuation Probability (p)								
LIQ	0.29^{***}							0.26^{***}
	(0.04)							(0.07)
RES		0.06						1.17^{***}
		(0.05)						(0.23)
MB			0.01					-0.31^{***}
			(0.01)					(0.07)
CNY				-0.11^{***}				0.42
				(0.01)				(0.30)
CNH					-0.11^{***}			-0.43
					(0.01)			(0.30)
CFETS						0.00		$0.00*$
						(0.00)		(0.00)
IR							0.09^{***}	0.03^{**}
							(0.01)	(0.01)
$R^2(\%)$	21.7	0.1	-0.3	31.7	26.2	0.0	25.5	42.1
Obs.	220	220	220	220	177	220	220	177

NOTE: This table reports regression results for the fundamental value (Vt) in Panel A and the continuation probability (p_t) in Panel B. Columns (1)-(7) report the results from univariate regressions using interbank liquidity ("LIQ"), reserves ("RES"), monetary base ("MB"), exchange rates of USD versus CNY and CNH (("CNY" and "CNH"), the CFETS index ("CFETS"), as well as the interest rate differential ("IR"), respectively. Column (8) reports the multi-variate regression results using all variables, except the USD-CNH exchange rate to avoid collinearity. The sample period is between January 2007 and May December 2025 (and between August 2010 and May 2025 for the regressions using the USD-CNH exchange rate). Monthly observations represent the average of daily data for each month. Significance levels: * $p < .10$; ** $p < .05$; and *** $p < .01$.

References

Amador, M., J. Bianchi, L. Bocola, and F. Perri (2019): "Exchange Rate Policies at the Zero Lower Bound," *The Review of Economic Studies*, 87, 1605–1645.

Auer, R., A. Burstein, and S. M. Lein (2021): "Exchange Rates and Prices: Evidence from the 2015 Swiss Franc Appreciation," *American Economic Review*, 111, 652–86.

Ball, C. A. and A. Roma (1993): "A jump diffusion model for the European monetary system," *Journal of International Money and Finance*, 12, 475–492.

Bertola, G. and L. E. O. Svensson (1993): "Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models," *The Review of Economic Studies*, 60, 689–712.

Bisesti, L., A. Castagna, and F. Mercurio (2005): "Consistent Pricing and Hedging of an FX Options Book," *Kyoto Economic Review*, 74, 65–83.

Blagov, B. and M. Funke (2019): "The regime-dependent evolution of credibility: A fresh look at Hong Kong's linked exchange rate system," *Macroeconomic Dynamics*, 23, 2434–2468.

Breedon, F., L. Chen, A. Ranaldo, and N. Vause (2023): "Judgment day: Algorithmic trading around the Swiss franc cap removal," *Journal of International Economics*, 140, 103713.

Delgado, F. and B. Dumas (1991): "Target zones Big and Small," NBER Working Paper No. 3601.

Du, W., A. Tepper, and A. Verdelhan (2018): "Deviations from Covered Interest Rate Parity," *The Quarterly Journal of Economics*, 133, 2005–2057.

Dumas, B., L. P. Jennergren, and B. Naslund (1995): "Realignment Risk and Currency Option Pricing in Target Zones," *European Economic Review*, 39, 1523–1544.

Engel, C. (1996): "The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence," *Journal of Empirical Finance*, 3, 123–192.

Feng, S., L. Fu, C.-Y. Ho, and W.-Y. Alex Ho (2023): "Political stability and credibility of currency board," *Journal of International Money and Finance*, 137, 102911.

Froot, K. A. and M. Obstfeld (1991a): "Exchange-rate dynamics under stochastic regime shifts: A unified approach," *Journal of International Economics*, 31, 203–229.

——— (1991b): "Stochastic Process Switching: Some Simple Solutions," *Econometrica*, 59, 241–250.

Genberg, H. and C.-H. Hui (2011): "The Credibility of Hong Kong's Link from the Perspective of Modern Financial Theory," *Journal of Money, Credit and Banking*, 43, 185–206.

Ghosh, A., A.-M. Gulde, and H. Wolf (2020): "Currency boards," in *Handbook of the History of Money and Currency*, ed. by S. Battilossi, Y. Cassis, and K. Yago, Singapore: Springer, 687–715.

Greenwood, J. (2007): *Hong Kong's Link to the US Dollar: Origins and Evolution*, Hong Kong University Press.

Hanke, M., R. Poulsen, and A. Weissensteiner (2018): "Event-Related Exchange-Rate Forecasts: Combining Information from Betting Quotes and Option Prices," *Journal of Financial and Quantitative Analysis*, 53, 2663–2683.

Hertrich, M. and H. Zimmermann (2017): "On the Credibility of the Euro/Swiss Franc Floor: A Financial Market Perspective," *Journal of Money, Credit and Banking*, 49, 567–578.

Jeanne, O. and P. Masson (2000): "Currency crises, sunspots and Markov-switching regimes," *Journal of International Economics*, 50, 327–350.

Jermann, U. J. (2017): "Financial Markets' Views about the Euro–Swiss Franc Floor," *Journal of Money, Credit and Banking*, 49, 553–565.

Krugman, P. R. (1991): "Target Zones and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 106, 669–682.

Kwan, Y. K. and F. Lui (1999): "Hong Kong's currency board and changing monetary regimes," in *Changes in exchange rates in rapidly developing countries: theory, practice, and policy issues*, University of Chicago Press, 403–436.

Latter, T. (2007): "Rules versus discretion in managing the Hong Kong dollar, 1983-2006," Tech. rep., Hong Kong Institute for Monetary Research.

Lindberg, H. and P. Söderlind (1994): "Intervention Policy and Mean Reversion in Exchange Rate Target Zones: The Swedish Case," *The Scandinavian Journal of Economics*, 96, 499–513.

Lustig, H., N. Roussanov, and A. Verdelhan (2011): "Common Risk Factors in Currency Markets," *Review of Financial Studies*, 24, 3731–3777.

Malz, A. M. (1997): "Estimating the probability distribution of the future exchange rate from option prices," *Journal of Derivatives*, 5, 18–36.

Mirkov, N., I. Pozdeev, and P. Söderlind (2019): "Verbal interventions and exchange rate policies: The case of Swiss franc cap," *Journal of International Money and Finance*, 93, 42–54.

Na, S., S. Schmitt-Grohé, M. Uribe, and V. Yue (2018): "The Twin Ds: Optimal Default and Devaluation," *American Economic Review*, 108, 1773–1819.

Svensson, L. E. (1991a): "Target zones and interest rate variability," *Journal of International Economics*, 31, 27–54.

——— (1991b): "The term structure of interest rate differentials in a target zone: Theory and Swedish data," *Journal of Monetary Economics*, 28, 87–116.

A Proofs

Proof of Proposition 1. Below we use lowercase variables to denote the logarithm of the corresponding uppercase variables. For example, $v_t \equiv \log V_t$, $s^P \equiv \log S^P$, $\widehat{v}_t \equiv \log \widehat{V}_t$, etc.

Similarly, we derive the dynamics of $\widehat{V}_t$ as follows:

$$\frac{d\widehat{V}_t}{\widehat{V}_t} = \frac{dV_t}{V_t} = \mu_V dt + \sigma_V dW_{V,t}. \tag{24}$$

We are now ready to solve the (scaled) equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$. It is straightforward to prove that $\widehat{S}(\widehat{V}_t)$ is monotonically increasing. Define $\widehat{V}_*$ and $\widehat{V}^*$ such that $\widehat{S}(\widehat{V}_*) = 1 - \widehat{b}$ and $\widehat{S}(\widehat{V}^*) = 1 + \widehat{b}$. As the length of the period Δt converges to zero, with probability one $\widehat{V}_{t+\Delta t} > \widehat{V}^*$ (or $\widehat{V}_{t+\Delta t} < \widehat{V}_*$) if $\widehat{V}_t > \widehat{V}^*$ (or $\widehat{V}_t < \widehat{V}_*$). Therefore, from equation (5), it must be true that: $\widehat{S}(\widehat{V}_t) = 1 - \hat{b}$ if $\widehat{V} < \widehat{V}_*$, and $1 + \hat{b}$ if $\widehat{V} > \widehat{V}^*$.

If $\widehat{V} \in (\widehat{V}_*, \widehat{V}^*)$, it is straightforward to show that $\widehat{S}(\widehat{V}_t)$ must satisfy the following equation based on equation (5):

$$\widehat{S}'\left(\widehat{V}_t\right)\widehat{V}_t\mu_V + \frac{1}{2}\widehat{S}''\left(\widehat{V}_t\right)\widehat{V}_t^2\sigma_V^2 + \left(r_{USD} - r_{HKD} - \lambda\right)\widehat{S}\left(\widehat{V}_t\right) + \lambda\widehat{V}_t = 0. \tag{25}$$

The solution to this ordinary differential equation is: $\widehat{S}(\widehat{V}_t) = C_0\widehat{V} + C_1\widehat{V}^{\eta_1} + C_2\widehat{V}^{\eta_2}$, where η_1 and η_2 are the two roots of the quadratic equation:

$$\frac{1}{2}\sigma_V^2\eta^2 + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)\eta + \left(r_{USD} - r_{HKD} - \lambda\right) = 0, \tag{26}$$

and the coefficient C_0 is given by

$$C_0 = \frac{\lambda}{\lambda + r_{HKD} - r_{USD} - \mu_V}, \tag{27}$$

and the coefficients C_1, C_2 and the thresholds $\widehat{V}_*$, $\widehat{V}^*$ are determined from the value-matching and smooth-pasting conditions:

$$\begin{aligned}
\widehat{V}_* + C_1(\widehat{V}_*)^{\eta_1} + C_2(\widehat{V}_*)^{\eta_2} &= 1 - \widehat{b}, && (28)\\
\widehat{V}^* + C_1(\widehat{V}^*)^{\eta_1} + C_2(\widehat{V}^*)^{\eta_2} &= 1 + \widehat{b}, && (29)\\
C_0 + \eta_1 C_1(\widehat{V}_*)^{\eta_1 - 1} + \eta_2 C_2(\widehat{V}_*)^{\eta_2 - 1} &= 0, && (30)\\
C_0 + \eta_1 C_1(\widehat{V}^*)^{\eta_1 - 1} + \eta_2 C_2(\widehat{V}^*)^{\eta_2 - 1} &= 0. && (31)
\end{aligned}$$

■

Proof of Proposition 2. Because $S_t^P \equiv S^P (= 7.8)$ is a constant, $\widehat{V}_t$ follows the same process in equation (6), implying the following expression for $\widehat{V}_{t+\tau}$

$$\widehat{V}_{t+\tau} = \widehat{V}_t \exp\left\{\left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau + \sigma_V\sqrt{\tau}z_V\right\},$$

where z_V denotes a random variable with a standard normal distribution. Therefore, $\widehat{V}_{t+\tau} \leq \widehat{V}^*$ if and only if $z_V \leq -d^*$ and $\widehat{V}_{t+\tau} \geq \widehat{V}_*$ if and only if $z_V \geq -d_*$, where the expressions of d^* and d_* are given in equations (10) and (11). Therefore, $H\left(\widetilde{S}_{t+\tau}; S^P, b\right)$ equals $S^P + b$ with probability $1 - \Phi(-d^*) = \Phi(d^*)$; $S^P - b$ with probability $\Phi(-d_*)$; and $S^P\left(C_0\widehat{V}_{t+\tau} + C_1\widehat{V}_{t+\tau}^{\eta_1} + C_2\widehat{V}_{t+\tau}^{\eta_2}\right)$ with probability $\Phi(-d^*) - \Phi(-d_*)$.

Second, the date-t conditional expectation of $\widehat{V}_{t+\tau}^{\eta}$ conditional on $\widehat{V}_* \leq \widehat{V}_{t+\tau} \leq \widehat{V}^*$ is given below

$$\begin{aligned}
& E_t^Q\left[\widehat{V}_{t+\tau}^{\eta}\middle| \widehat{V}_* \leq \widehat{V}_{t+\tau} \leq \widehat{V}^*\right] \\
= \; & \frac{\widehat{V}_t^{\eta} e^{\eta\left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau}}{\Phi(-d^*) - \Phi(-d_*)} \int_{-d_*}^{-d^*} \frac{1}{\sqrt{2\pi}} e^{\eta\sigma_V\sqrt{\tau}z_V - \frac{1}{2}z_V^2} dz_V \\
= \; & \widehat{V}_t^{\eta} e^{\eta\mu_V\tau + \frac{1}{2}\eta(\eta-1)\sigma_V^2\tau} \frac{\Phi\left(-d^* - \eta\sigma_V\sqrt{\tau}\right) - \Phi\left(-d_* - \eta\sigma_V\sqrt{\tau}\right)}{\Phi(-d^*) - \Phi(-d_*)} \\
\equiv \; & \Gamma\left(\widehat{V}_t; \eta\right).
\end{aligned}$$

Substituting the above results into $E_t^Q\left[H\left(\widetilde{S}_{t+\tau}; S^P, b\right)\right]$ yields the expression for $F_S(\tau)$ in equation (14) where we denote $\eta_0 = 1$. ■

Proof of Proposition 3. (i) Note that

$$\begin{aligned}
C_V(K,\tau) &\equiv e^{-\widetilde{r}_{HKD}\tau} E^Q\left[\max\left(V_{t+\tau} - K, 0\right)\right] \\
&= e^{-\widetilde{r}_{HKD}\tau} \int_{-d_{2,V}}^{\infty} \left[V_t \exp\left\{\left(\mu_V - \sigma_V^2/2\right)\tau + \sigma_V\sqrt{\tau}z_V\right\} - K\right] \phi(z_V)\, dz_V \\
&= e^{-\widetilde{r}_{HKD}\tau} V_t e^{\left(\mu_V - \sigma_V^2/2\right)\tau} \int_{-d_{2,V}}^{\infty} e^{\sigma_V\sqrt{\tau}z_V} \phi(z_V)\, dz_V - e^{-\widetilde{r}_{HKD}\tau} K\Phi(d_{2,V}) \\
&= V_t e^{(\mu_V - \widetilde{r}_{HKD})\tau}\left(1 - \Phi\left(-d_{2,V} - \sigma_V\sqrt{\tau}\right)\right) - e^{-\widetilde{r}_{HKD}\tau} K\Phi(d_{2,V}) \\
&= V_t e^{-r_{USD}\tau}\Phi(d_{1,V}) - e^{-\widetilde{r}_{HKD}\tau} K\Phi(d_{2,V}),
\end{aligned}$$

where

(ii) There are three possibilities.

(a) If $K \geq S^P + b$, then $H\left(\widetilde{S}_{t+\tau}; S^P, b\right) \leq S^P + b \leq K$, implying $C_S(K,\tau) = 0$.

(b) If $K \leq S^P - b$, then $H\left(\widetilde{S}_{t+\tau}; S^P, b\right) \geq S^P + b \geq K$, implying $C_S(K,\tau) = e^{-r_{HKD}\tau} E^Q\left[H\left(\widetilde{S}_{t+\tau}; S^P, b\right) - K\right] = e^{-r_{HKD}\tau}\left(F_S(\tau) - K\right)$, where $F_S(\tau)$ is given in equation (14).

(c) If $S^P - b < K < S^P + b$, then there exists a cutoff value, denoted by $\widehat{V}_K \in \left(\widehat{V}_*, \widehat{V}^*\right)$, such that $\widehat{S}(\widehat{V}_K) = \frac{K}{S^P} \equiv \widehat{K}$. Consider three cases. In the first case where $\widehat{V}_{t+\tau} \geq \widehat{V}^*$, we have $H\left(\widetilde{S}_{t+\tau}; S^P, b\right) = S^P + b > K$, implying that the present value of the expected option payoff is $e^{-r_{HKD}\tau}\left(S^P + b - K\right)$. In the second case where $\widehat{V}_{t+\tau} \leq \widehat{V}_*$, we have $H\left(\widetilde{S}_{t+\tau}; S^P, b\right) = S^P - b < K$, implying that the present value of the expected option payoff is zero. In the third case where $\widehat{V}_* \leq \widehat{V}_{t+\tau} \leq \widehat{V}^*$, in this case $H\left(\widetilde{S}_{t+\tau}; S^P, b\right) \geq K$ is equivalent to $\widehat{V}_{t+\tau} \geq \widehat{V}_K$, or equivalently $z_V \geq -d_K$, where the expression of d_K is given in equation (22). Therefore,

$$
\begin{aligned}
& e^{-r_{HKD}\tau} E^Q\left[\max\left(H\left(\widetilde{S}_{t+\tau}; S^P, b\right) - K, 0\right) \middle| \widehat{V}_* \leq \widehat{V}_{t+\tau} \leq \widehat{V}^*\right] \\
= & \frac{e^{-r_{HKD}\tau}}{\Phi(-d^*) - \Phi(-d_*)} \int_{-d_K}^{-d^*} \left(\widetilde{S}_{t+\tau} - K\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= & \frac{e^{-r_{HKD}\tau} S^P}{\Phi(-d^*) - \Phi(-d_*)} \int_{-d_K}^{-d^*} \left[C_0 \widehat{V}_{t+\tau} + C_1 \widehat{V}_{t+\tau}^{\eta_1} + C_2 \widehat{V}_{t+\tau}^{\eta_2} - \widehat{K}\right] \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= & e^{-r_{HKD}\tau}\left[S^P \sum\nolimits_{i=0}^{2} C_i \Gamma^*\left(\widehat{V}_t; \eta_i\right) - K\Gamma^*\left(\widehat{V}_t; 0\right)\right],
\end{aligned}
$$

where in deriving the second-to-last equality we have used the following result

$$
\begin{aligned}
& \frac{1}{\Phi(-d^*) - \Phi(-d_*)} \int_{-d_K}^{-d^*} \widehat{V}_{t+\tau}^{\eta} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= & \frac{\widehat{V}_t^{\eta} e^{\eta\left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau}}{\Phi(-d^*) - \Phi(-d_*)} \int_{-d_K}^{-d^*} \frac{1}{\sqrt{2\pi}} e^{\eta\sigma_V\sqrt{\tau} z_V - \frac{1}{2} z_V^2} dz_V \\
= & \widehat{V}_t^{\eta} e^{\eta\mu_V\tau + \frac{1}{2}\left(\eta^2 - \eta\right)\sigma_V^2\tau} \frac{\Phi\left(-d^* - \eta\sigma_V\sqrt{\tau}\right) - \Phi\left(-d_K - \eta\sigma_V\sqrt{\tau}\right)}{\Phi(-d^*) - \Phi(-d_*)} \\
\equiv & \Gamma^*\left(\widehat{V}_t; \eta\right).
\end{aligned}
$$

Therefore, in summary, the option price is given below when $S^P - b < K < S^P + b$,

$$
\begin{aligned}
& C_S(K,\tau) \\
= & \left(1 - \Phi(-d^*)\right) e^{-r_{HKD}\tau}\left[S^P + b - K\right] \\
& + \left(\Phi(-d^*) - \Phi(-d_*)\right) e^{-r_{HKD}\tau}\left[S^P \sum\nolimits_{i=0}^{2} C_i \Gamma^*\left(\widehat{V}_t; \eta_i\right) - K\Gamma^*\left(\widehat{V}_t; 0\right)\right].
\end{aligned}
$$

The derivation for put options is similar. For example,

$$
\begin{aligned}
& e^{-r_{HKD}\tau} E^{Q}\left[\max\left(K - H\left(\widetilde{S}_{t+\tau}; S^P, b\right), 0\right) \middle| \widehat{V}_* \leq \widehat{V}_{t+\tau} \leq \widehat{V}^*\right] \\
= \; & \frac{e^{-r_{HKD}\tau}}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)} \int_{-d_*}^{-d_K} \left(K - \widetilde{S}_{t+\tau}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= \; & \frac{e^{-r_{HKD}\tau} S^P}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)} \int_{-d_*}^{-d_K} \left[\widehat{K} - \left(C_0 \widehat{V}_{t+\tau} + C_1 \widehat{V}_{t+\tau}^{\eta_1} + C_2 \widehat{V}_{t+\tau}^{\eta_2}\right)\right] \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= \; & e^{-r_{HKD}\tau}\left[K \Gamma_*\left(\widehat{V}_t; 0\right) - S^P \sum_{i=0}^{2} C_i \Gamma_*\left(\widehat{V}_t; \eta_i\right)\right],
\end{aligned}
$$

where

$$
\begin{aligned}
& \frac{1}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)} \int_{-d_*}^{-d_K} \widehat{V}_{t+\tau}^{\eta} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z_V^2}{2}\right) dz_V \\
= \; & \frac{\widehat{V}_t^{\eta} e^{\eta\left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau}}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)} \int_{-d_*}^{-d_K} \frac{1}{\sqrt{2\pi}} e^{\eta\sigma_V\sqrt{\tau} z_V - \frac{1}{2} z_V^2} dz_V \\
= \; & \widehat{V}_t^{\eta} e^{\eta\mu_V\tau + \frac{1}{2}\left(\eta^2 - \eta\right)\sigma_V^2\tau} \frac{\Phi\left(-d_K - \eta\sigma_V\sqrt{\tau}\right) - \Phi\left(-d_* - \eta\sigma_V\sqrt{\tau}\right)}{\Phi\left(-d^*\right) - \Phi\left(-d_*\right)} \\
\equiv \; & \Gamma_*\left(\widehat{V}_t; \eta\right).
\end{aligned}
$$

■