Contents

Part I

Chapter 1

1 Definition of Irreducible

For equivalent definitions, see Wiki HERE; Also see "dense" on Wiki HERE. See Atiyah's [?] Exercise 19 from Chapter 1 for more information...

2 Example 1.1.3.

See Atiyah's [?] Exercise 19 from Chapter 1, which proves it must be dense.

For the sake of contradiction, assume a non-empty subset $A \subset X$ is reducible. Hence there exist two proper closed subset $A_1, A_2 \subset A$ such that $A = A_1 \cup A_2$. Then we have

$$X = (A^c \cup A_1) \cup (A^c \cup A_2),$$

which implies X is reducible. ✕!

2.1

See a post HERE ✔

First approach applied some density argument. While the second approach, similarly, gave a decomposition of the whole space given the open set is reducible.

3 Example 1.1.4.

See Atiyah's [?] Exercise 20 from Chapter 1.

4 Prop 1.2 (d)

According to Hilbert's Nullstellensatz, I agree we'll get $I(Z(\mathfrak{a})) \subset \sqrt{\mathfrak{a}}$. For the reverse inclusion, pick any $f \in A$ such that $f^r \in \mathfrak{a}$ where $r \in \mathbb{Z}_{>0}$. We wish to show that $f(P) = 0$ for any $P \in Z(\mathfrak{a})$. By definition, $f^r(P) = 0$ given $f^r \in \mathfrak{a}$. And this implies

$$f^r(P) = (f(P))^r = 0 \ \Rightarrow \ f(P) = 0$$

given the polynomial ring A is an integral domain. Therefore we get the inclusion

$$I(Z(\mathfrak{a})) \supset \sqrt{\mathfrak{a}}.$$

See Theorem 6 on Page 183, Strong Nullstellensatz, [1]. ✔

5 Theorem 1.8A.

For transcendence degree, see HERE and a NOTE by Milne James.

6 References

[1] David Cox, John Little, and Donal OShea. *Ideals, varieties, and algorithms: an introduction to computational algebraic geometry and commutative algebra.* Springer Science & Business Media, 2013.