Analysis of Psychological Data

Lab 3. Hooked on a Distribution: Central Tendency and Variability

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Some announcements

Exam 1 on February 15 (next Tuesday)

Big and red scantron forms

Pencil and eraser to fill in scantrons

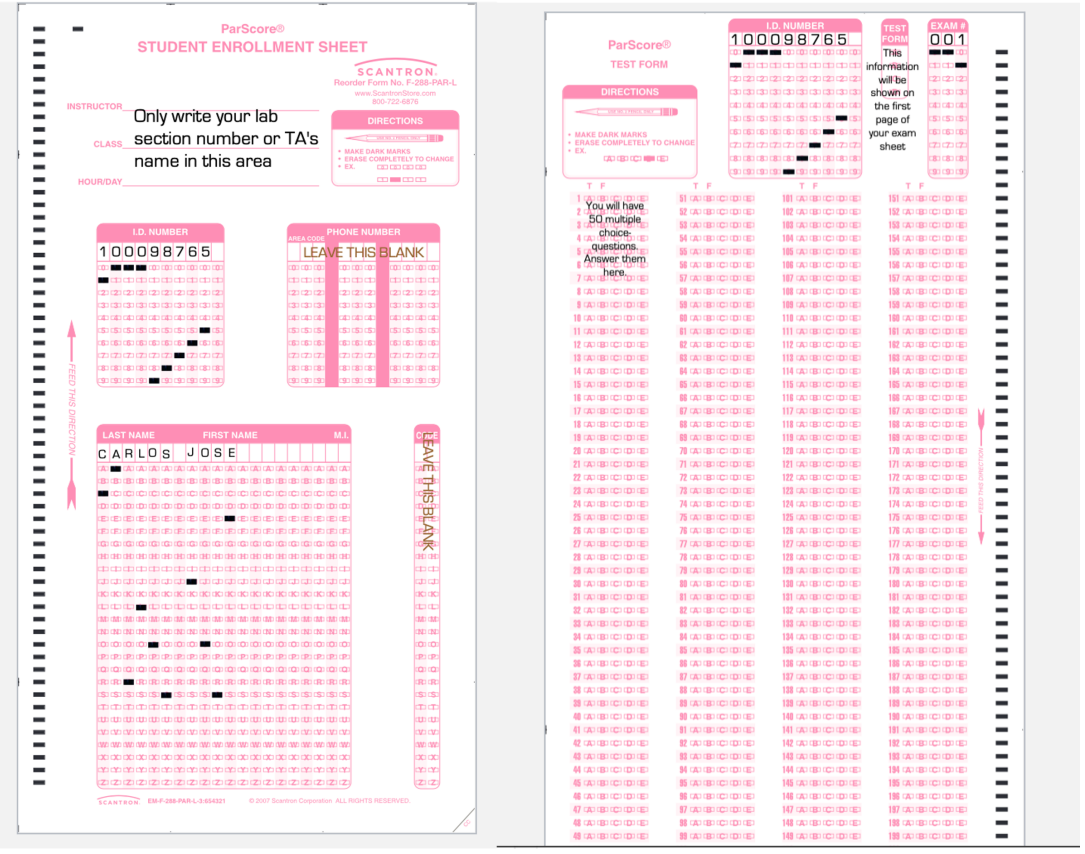
Blank scratch paper

Class-approved calculator

One 8.5 x11 crib sheet (i.e., a concise set of notes, could be double-sided)

Some announcements

Big and red scantron forms



What are we going to do?

Recap to give you a big picture

Central tendency & variability

(Primers on the formula included)

Group activity

Q&A session

Reflecting upon three weeks

Fundamentals (statistics, variables)

Some basic stat terms (population, parameter, sample, statistic)

Focus on variables (type, measurement scale)

Fiddling with variables (visualization, distribution)

We are playing the game with the distribution

Distribution

An intuitive way to understand how the values of a variable are spread over

We are interested in describing the distribution

What would be efficient and informative ways?

How to describe any distributions

Measures of central tendency

Reflect where 'values of a variable (i.e., distribution)' are centered

Mean, median, and mode

Measures of variability

Reflect how much 'values of a variable (i.e., distribution)' are dispersed

Range, deviation score, sum of squares, **variance**, **standard deviation**

Primers on formula

Notation

- Uppercase Roman letters that are usually near the end of the alphabet denote variables (e.g., X and Y)
  - A numerical subscript represents an individual value of that variable (e.g., X_3 is the third value of the variable X)
- Greek letters are used represent population parameters (e.g., μ, σ^2, σ)
- Roman letters are used to represent sample statistics (e.g., $\bar{X}$, s^2, s)
  - An alphabetical subcript represents the corresponding variables (e.g., σ_X^2, s_X)
- Lowercase n denotes the sample size whereas the uppercase N indicates the population size

Primers on formula

Notation

- Σ (the uppercase Greek letter sigma) indicates the operation of summation (i.e., the addition of scores)
  - $\Sigma_{i=1}^{5} X_i = X_1 + X_2 + X_3 + X_4 + X_5$
- Numerical exponents indicate multiplying that number itself multiple times
  - $X^5 = X \times X \times X \times X \times X$
- (Positive) square root
  - $\sqrt{X^2} = (X^2)^{\frac{1}{2}} = X$
- Summation and exponents combined
  - $\Sigma_{i=1}^{5} (X_i - \bar{X})^2 = (X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 + (X_4 - \bar{X})^2 + (X_5 - \bar{X})^2$
    $= (X_1 - \bar{X}) \times (X_1 - \bar{X}) + (X_2 - \bar{X}) \times (X_2 - \bar{X}) + \ldots + (X_5 - \bar{X}) \times (X_5 - \bar{X})$

Central tendency

Summary

	Sample	**Population**
Mean	$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$	$\mu_X = \frac{\sum_{i=1}^{N} X_i}{N}$
Median	(When aranged) The $\frac{n+1}{2}$th value	(When aranged) The $\frac{N+1}{2}$th value
Mode	The most frequent value	The most frequent value

Key ideas

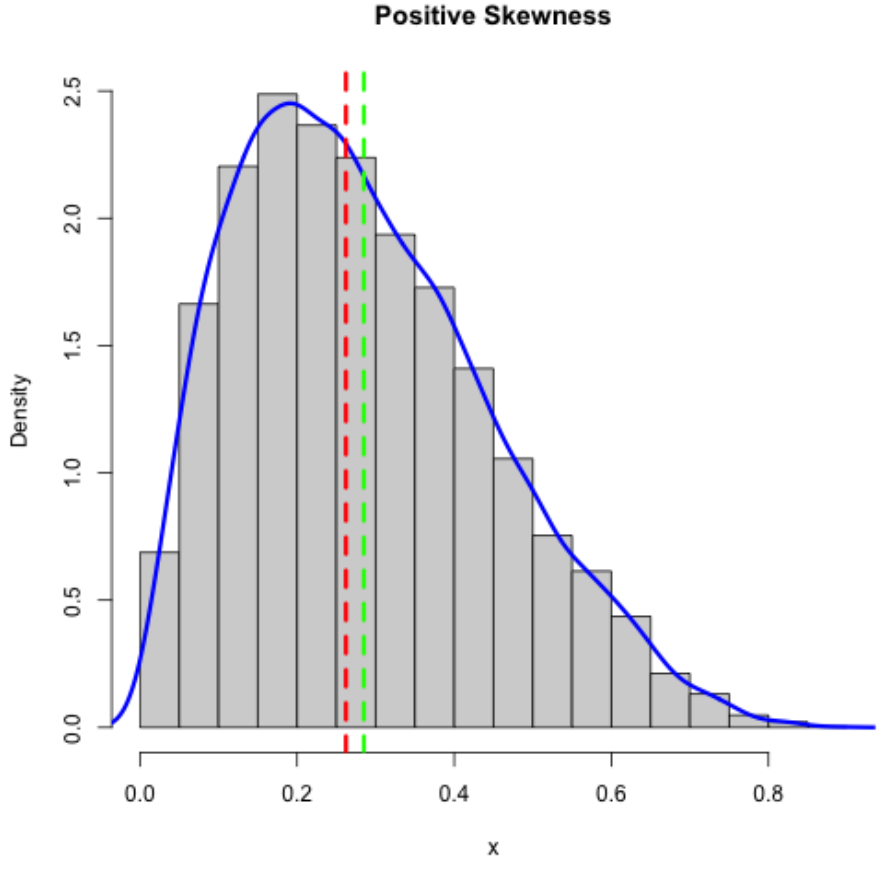
- How these measures are related to distributions?
- Which measures are useful and when?
- Calculation

Central tendency

Recall Positive Skewness

```
set.seed(322)
x = rbeta(10000,2,5)
hist(x, main="Positive Skewness", freq=FALSE)
lines(density(x), col='blue', lwd=3)
abline(v = c(mean(x),median(x)),
       col=c("green", "red"),
       lty=c(2,2), lwd=c(3, 3))
```

- Trailing off toward the right end
- Mode < Median < Mean
- Score distribution of difficult exams



Central tendency

Recall Negative Skewness

```
set.seed(322)
x = rbeta(10000,5,2)
hist(x, main="Negative Skewness", freq=FALSE)
lines(density(x), col='blue', lwd=3)
abline(v = c(mean(x),median(x)),
       col=c("green", "red"),
       lty=c(2,2), lwd=c(3, 3))
```

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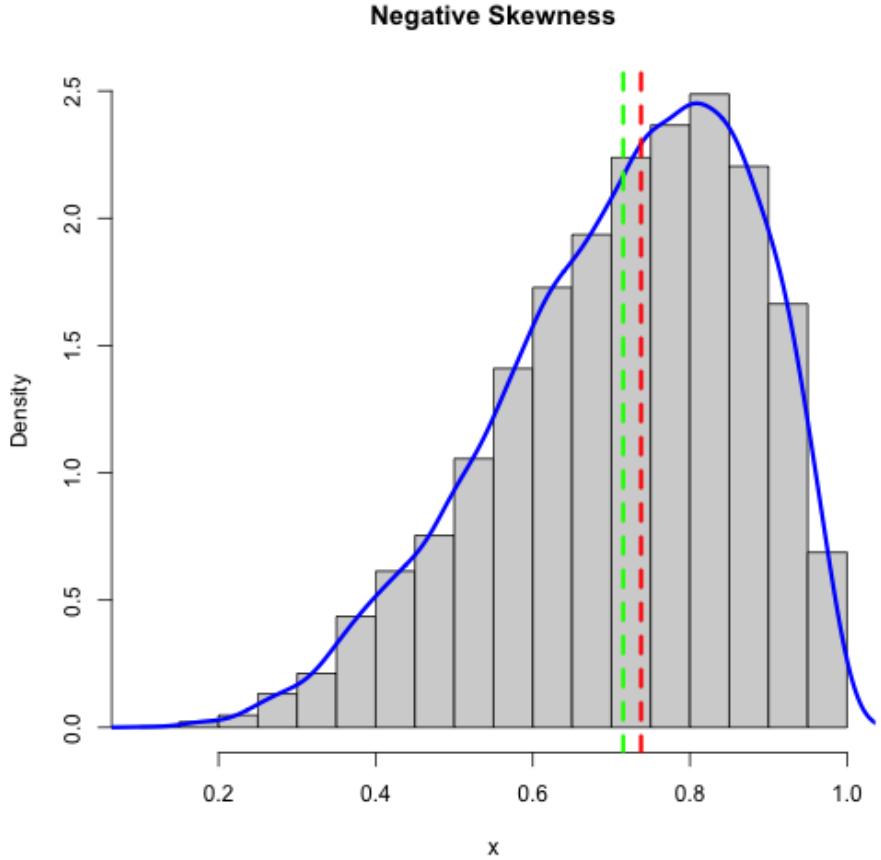


Central tendency

Mean vs. median

When **outliers** (i.e., extreme values) exist, consider using the median as a central tendency measure

Imagine Elon Musk enters this room when we calculate our monthly income...

Mode

Useful for variables measured on nominal and ordinal scales

Do you remember the distributional forms such as unimodal and bimodal?

Central tendency

Medians are resistant to the outliers... and hey means, don't be mean...

Variability

Range

$max - min$

Deviation score

$X_i - \bar{X}$

Sum of squares

$\Sigma_{i=1}^{n}(deviation\ score)^2 = \Sigma_{i=1}^{n}(X_i - \bar{X})^2$

Variability

Variance

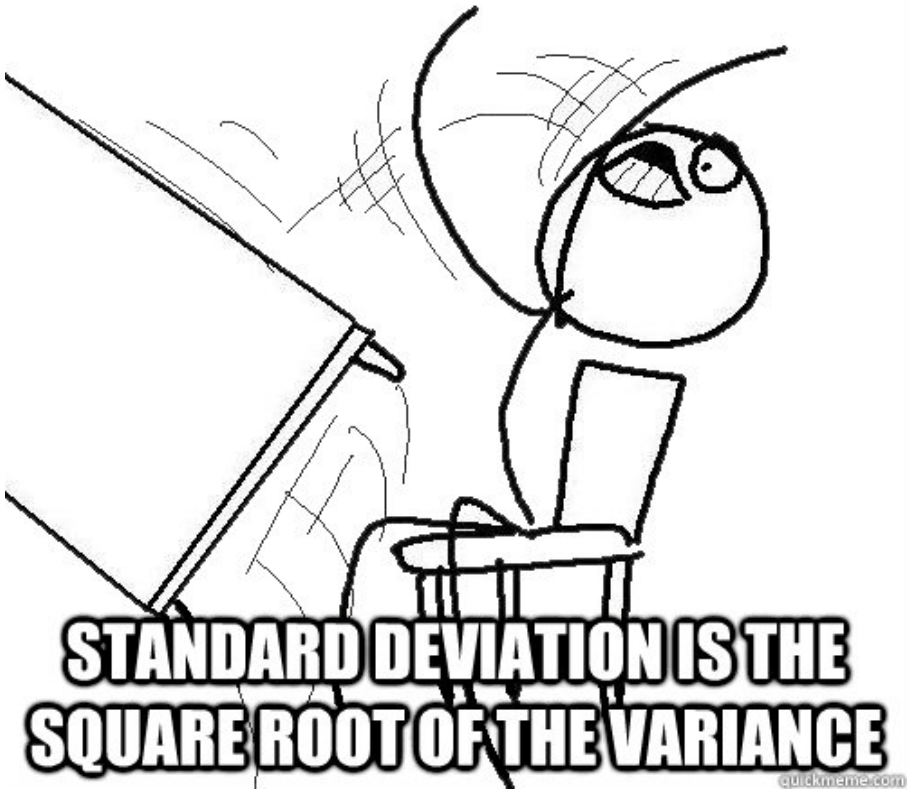
- Sample variance = $s^2 = \frac{\Sigma_{i=1}^{n}(deviation\ score)^2}{n-1} = \frac{sum\ of\ squares}{n-1} = \frac{\Sigma_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$
- Population variance = $\sigma^2 = \frac{\Sigma_{i=1}^{N}(deviation\ score)^2}{N} = \frac{sum\ of\ squares}{N} = \frac{\Sigma_{i=1}^{N}(X_i-\mu_X)^2}{N}$

Standard deviation

- Sample standard deviation = $\sqrt{sample\ variance} = \sqrt{s^2} = s = \sqrt{\frac{\Sigma_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$
- Population standard deviation = $\sqrt{population\ variance} = \sqrt{\sigma^2} = \sigma = \sqrt{\frac{\Sigma_{i=1}^{N}(X_i-\mu)^2}{N}}$

Variability

Remember!



Variability

Some tips on interpreting the standard deviation

Average distance from the mean

$\longrightarrow$ Say, the average height of males in the Netherlands is 6 ft, and the standard deviation is 2.5 inches. What do these central tendency and variability measures mean?

What if the standard deviation is zero?

Central tendency and variability

Summary

	Sample	**Population**
Mean	$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$	$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$
Median	(When aranged) The $\frac{n+1}{2}$th value	(When aranged) The $\frac{N+1}{2}$th value
Mode	The most frequent value	The most frequent value
Variance	$s^2 = \frac{\Sigma_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$	$\sigma^2 = \frac{\Sigma_{i=1}^{N}(X_i - \mu_X)^2}{N}$
Standard deviation	$s = \sqrt{\frac{\Sigma_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$	$\sigma = \sqrt{\frac{\Sigma_{i=1}^{N}(X_i - \mu_X)^2}{N}}$

Group activity

Case study

David, a statistician at the United States Air Force, has collected the data on pilots' satisfaction with aviation training. With the data collected, his research team would like to evaluate the current training system and know how to improve it in the future. See the collected data following:

25, 35, 18, 20, 15, 5, 25, 10, 22, 25

Can you answer below?

- What are the mean, the median, and the mode?
- What are the variance, and the standard deviation?
- What is the relative frequency of the score 25?
- What is the cumulative frequency of the score equal to 18 or lower?

Q&A session

Before you go home...

Lab materials are available at

https://github.com/lhnwhiHeo/PSY010

Any questions or comments?

Office hours or my email

Thanks! Good luck with your exam!