Case 1.

Stephen, a statistician at the United States Air Force, has collected the data on pilots' satisfaction with aviation training. With the data collected, his research team would like to evaluate the current training system and know how to improve it in the future. See the collected data following:

25, 35, 18, 20, 15, 5, 25, 10, 22, 25

I should have been more clear, but please regard those data as sample data! Sorry!

- What is $\Sigma_{i=1}^{10} X_i$ (Assuming X refers to the score variable)?

$$\Sigma_{i=1}^{10} X_i = X_1 + X_2 + X_3 + \ldots + X_9 + X_{10} = 25 + 35 + 18 + \ldots + 22 + 25 = 200$$

- What are the mean, the median, and the mode?

Mean = $\bar{X} = \dfrac{\Sigma_{i=1}^{10}}{10} = \dfrac{200}{10} = 20$

To get the median, first order values in an ascending order, which becomes 5, 10, 15, 18, 20, 22, 25, 25, 25, 35.

Median = $\dfrac{10+1}{2}$th value=5.5th value=The mean between the 5th and the 6th values=$\dfrac{20+22}{2} = 21$

Mode = The most frequent value = 25

- What are the range, the sum of squares, the variance, and the standard deviation?

Range = max - min = 35 - 5 = 30

To calculate the sum of squares, the variance, and the standard deviations, I recommend you to make an organized table:

X_i	25	35	18	20	15	5	25	10	22	25
$\bar{X}$	20	20	20	20	20	20	20	20	20	20
$X_i - \bar{X}$	5	15	-2	0	-5	-15	5	-10	2	5
$(X_i - \bar{X})^2$	25	225	4	0	25	225	25	100	4	25

Therefore, the sum of squares = $\Sigma_{i=1}^{10}(X_i - \bar{X})^2$ = 25 + 225 + 4 + 0 + $\cdots$ + 25 + 100 + 4 + 25 = 658

The variance = $s_X^2 = \dfrac{\Sigma_{i=1}^{10}(X_i - \bar{X})^2}{10 - 1} = \dfrac{658}{9} = 73.11$

The standard deviation = $s_X = \sqrt{s_X^2} = \sqrt{73.11} = 8.55$

- What is the relative frequency of the score 25?

For this question and the next one, I recommend you to make a frequency table:

Scores	Frequency
5	1
10	1
15	1
18	1
20	1
22	1
25	3
35	1

The relative frequency fo the score 25 = $\dfrac{3}{1+1+1+1+1+1+3+1} = \dfrac{3}{10} = 0.3 = 30\,\%$

- What is the cumulative frequency of the score equal to 18 or lower?

The cumulative frequency of the score equal to 18 or lower = 1 + 1 + 1 + 1 = 4

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Case 2.
Makram is interested in programming, so he took two programming courses in R and Python. His achievement was excellent, but he is curious to compare his performances. In the R programming course, he scored 125 where the mean and the standard deviation are 80 and 18. In the Python programming course, he scored 92 where the mean and the standard deviation are 72 and 10. Assume that all scores are normally distributed.

- What are the z-score in each programming course? Can you interpret them?

Z-score in the R programming course = $\dfrac{X_i - \mu_X}{\sigma_X} = \dfrac{125 - 80}{18} = \dfrac{45}{18} = 2.5$

Makram's score is 2.5 standard deviation unit above the mean in the R programming course.

Z-score in the Python programming course = $\dfrac{X_i - \mu_X}{\sigma_X} = \dfrac{92 - 72}{10} = \dfrac{20}{10} = 2$

Makram's score is 2 standard deviation unit above the mean in the Python programming course.

- What is the percentage of students who did better than Makram in the Python programming course?
1 - 0.9772 = 0.0228. There are 2.28% of students who did better than Makram in the Python programming course.

- What course did Makram showed a better achievement relative to his classmates?
R programming course because he has a higher z-score.

- Say Ihnwhi also took the same R programming course, and his z-score is -0.25.
  - What would be Ihnwhi's raw score?

Let's denote Ihnwhi's raw score as x_{ihnwhi}

$$-0.25 = \frac{x_{ihnwhi} - 80}{18}$$

$$-0.25 \times 18 = x_{ihnwhi} - 80$$

$$x_{ihnwhi} = -0.25 \times 18 + 80 = 75.5$$

Therefore, Ihnwhi's raw score is 75.5.

- What is the percentage of students who did worse than Ihnwhi?

40.13% (Can you find 0.4013 from the z-table?)

- What is the percentage of students who scored between Makram and Ihnwhi?

0.9938 - 0.4013 = 0.5925. Therefore, there are 59.25% of students who scored between Makram and Ihnwhi.