Analysis of Psychological Data

Lab 7. Hey Means, Don't Be Mean To Us: Z-Test and T-Test

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Reminders

Homework 3 is **on now** and due October 29 (50 points)

Homework 4 is available on October 21 and due October 29 (7 points)

Difficulties in homework? Join office hours!

What are we going to do?

Recapto give you a big picture

z-test

One-sample t-test

Independent-sample t-test

Do it together

Statistical inference

Idea 1

Let's make the best guess about the population parameter and test if that guess is true
$\longrightarrow$ Estimation and hypothesis testing

Idea 2

Let's assume we are interested in one sample statistic (e.g., a sample mean) and have the distribution of all the possible sample statistics
$\longrightarrow$ Sampling distribution

Usually, we are interested in the mean level of something in the population! Let me show you one example

Statistical inference

Idea 1

Estimation $\longrightarrow$ Our best guess about the population mean is the sample mean

Hypothesis testing $\longrightarrow$ How likely our sample mean (or more extreme) is to be observed if the null hypothesis is true? If the p-value is lower than α-level, reject the null hypothesis!

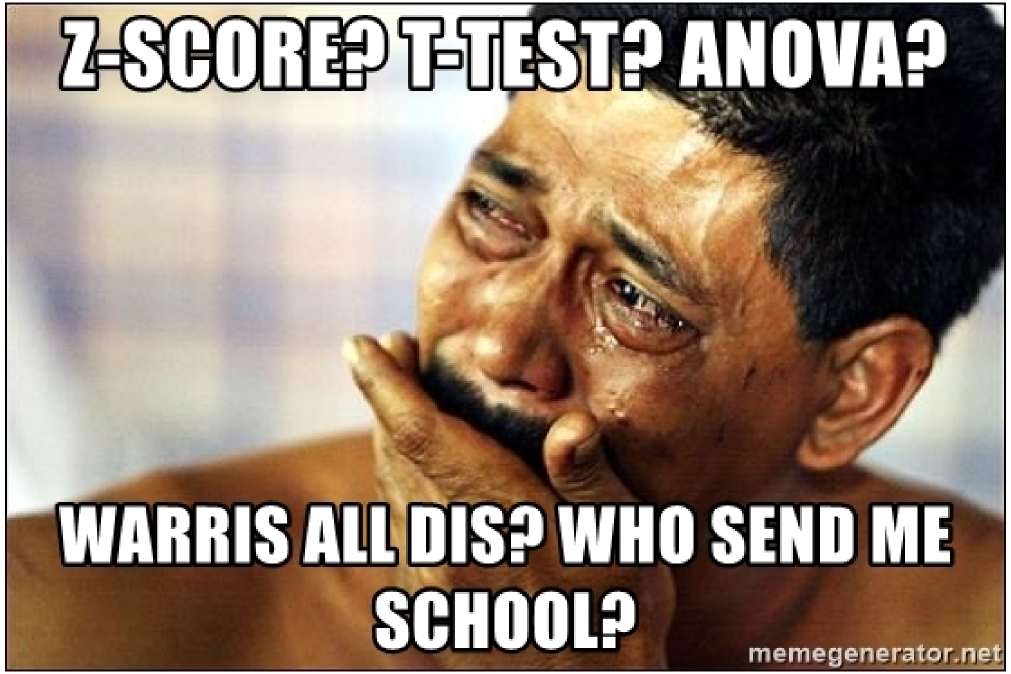
Idea 2

Sampling distribution $\longrightarrow$ Collect a sample and calculate the sample mean, repeat this process infinite times, and get the sampling distribution of the mean. Then, quantify how close/far our sample mean is (i.e., **test-statistic**)

Are you ready?

z-test, t-test, ANOVA, ANCOVA, MANOVA, MANCOVA, RM-ANCOVA... WHAT?!

Playing the game of statistical inference about population 'means' (as I showed before)



Z-test

What does it test?

If a sample mean is different from a specified value μ_0

Answers how many SEs away from the null distribution is our sample mean?

But, when?

We know the population mean and population standard deviation (VERY IDEAL!)

Z-test

Idea

We have a sample mean (estimation) and want to test the likelihood of observing this sample (or more extreme) if the null hypothesis were true (hypothesis testing)

In doing estimation and hypothesis testing, we have made a sampling distribution of the mean. Now, let's quantify how close/far our sample mean is from our expectation using **test-statistic**

Test-statistic

$$\frac{\text{Observed Data - What We Expect if the Null is True}}{\text{Average Variation}}$$

Z-test

Idea

We have a sample mean (estimation) and want to test the likelihood of observing this sample (or more extreme) if the null hypothesis were true (hypothesis testing)

In doing estimation and hypothesis testing, we have made a sampling distribution of the mean. Now, let's quantify how close/far our sample mean is from our expectation using

Test-statistic (**z-statistic**) when we know μ_0 and $\sigma_{\overline{X}}$

$$z = \frac{\overline{X} - \mu_0}{\sigma_{\overline{X}}} \quad \text{where} \quad \sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}}$$

Z-test

Effect size

Remember the practical significance! Let's quantify the size of effect (if significant)

Means how many standard deviations (SDs) the sample mean lies from the μ_0 in the population

Rule of thumb: 0.2 (small), 0.5 (medium), 0.8 (large)

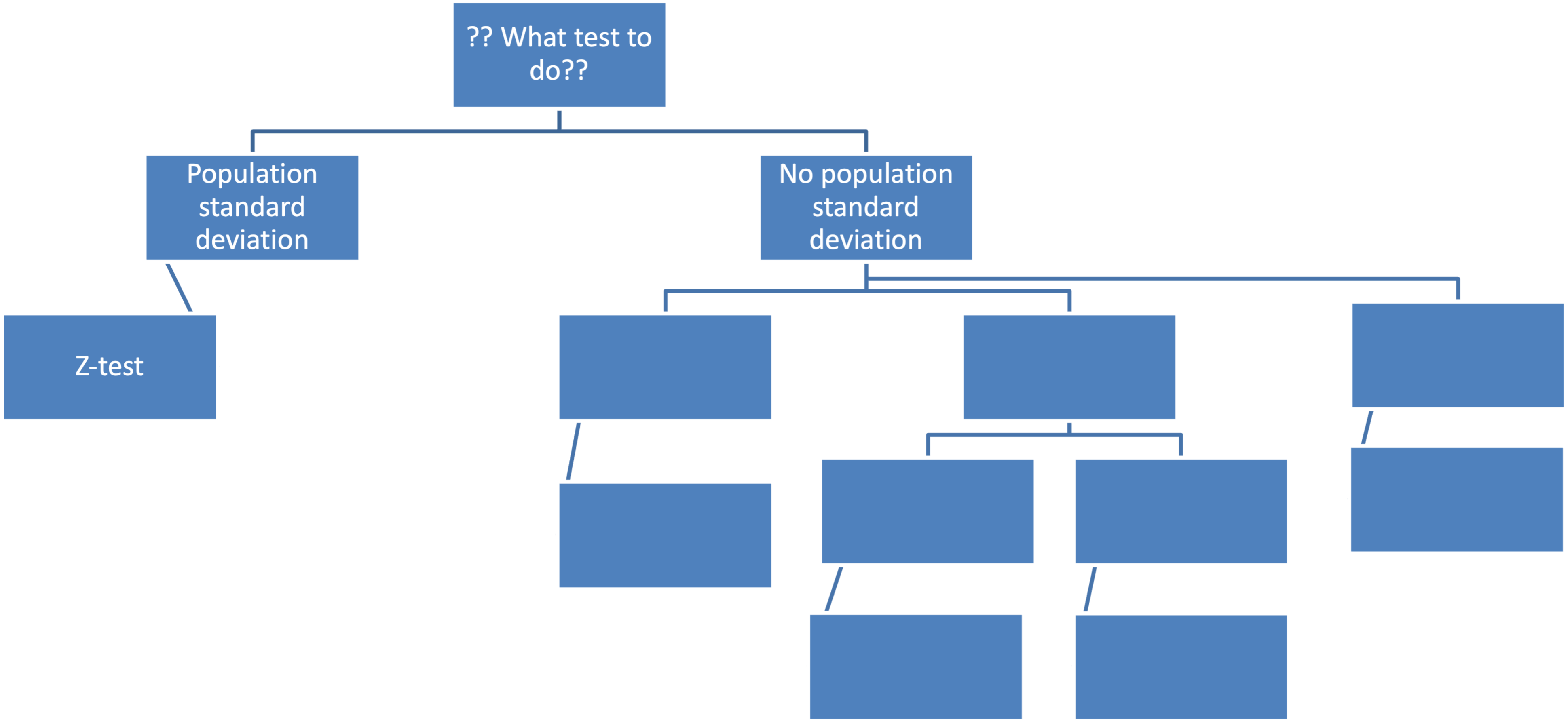
In z-test, the effect size is calculated as

$$d = \frac{\overline{X} - \mu_0}{\sigma_X} \quad \text{where} \quad \sigma_X = \sqrt{\frac{SS}{n-1}}$$

Do it together

Say the average person spends 120 minutes (with the standard deviation of 30 minutes) on social media per day. Does the average time on social media in **this class** differ from the general population? Use the α-level of .05.

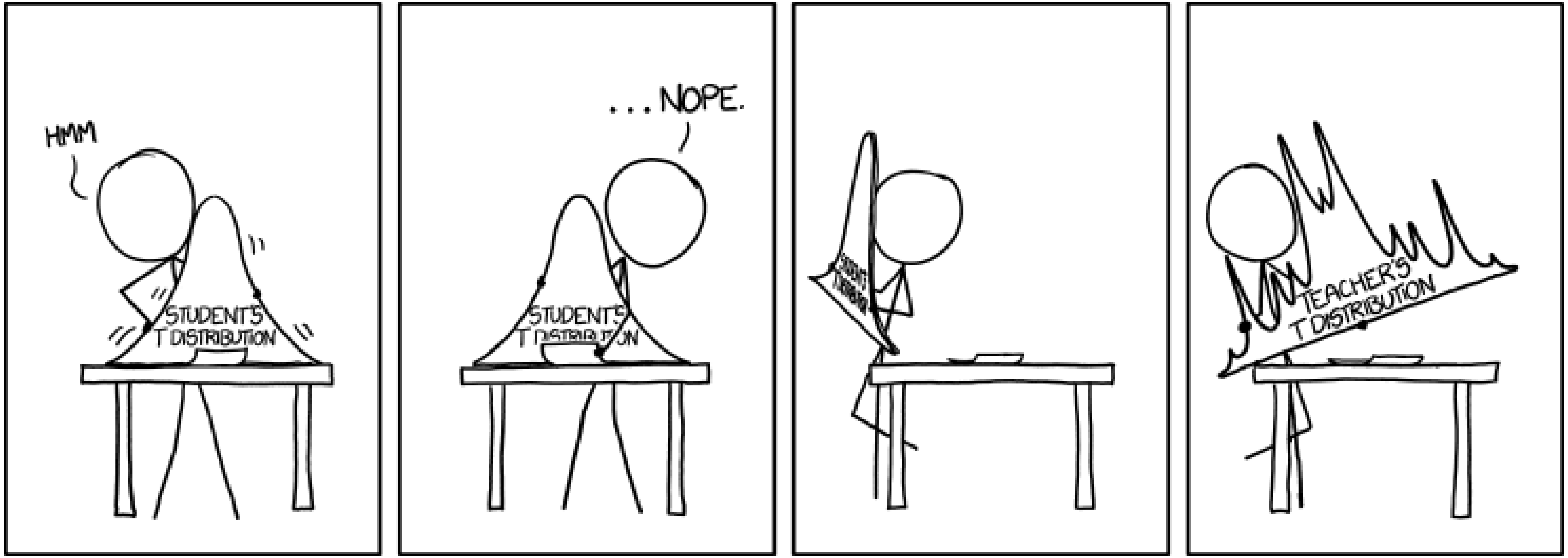
Big picture



?? What test to do??
Population standard deviation
No population standard deviation
Z-test

T-test

Student's t-distribution vs. teacher's t-distribution



T-test

Idea

Still interested in population means from either one sample or two samples (independent vs. related)

But, when?

We know the population mean but not the population standard deviation (MORE REALISTIC!)

T-test

Degrees of freedom (df)

of independent pieces of information $\longrightarrow$ # of values that are free to vary in a data set

Determines the shape of the t-distributions (i.e., df is the parameter of the t-distributions)

One-sample t-test = $n - 1$

Independent-sample t-test = $n_1 + n_2 - 2$

One-sample t-test

What does it test?

If a sample mean is different from a specified value μ_0

Note

We use not the normal distribution but the t-distribution based on df of $n-1$

But, when?

We know the population mean but not the population standard deviation (MORE REALISTIC!)

One-sample t-test

Idea

We have a sample mean (estimation) and want to test the likelihood of observing this sample (or more extreme) if the null hypothesis were true (hypothesis testing)

In doing estimation and hypothesis testing, we have made a sampling distribution of the mean. Now, let's quantify how close/far our sample mean is from our expectation

Test-statistic (**t-statistic**) when we know μ_0 but do not know $\sigma_{\overline{X}}$

$$t = \frac{\overline{X} - \mu_0}{s_{\overline{X}}} \quad \text{where} \quad s_{\overline{X}} = \frac{s_X}{\sqrt{n}}$$

One-sample t-test

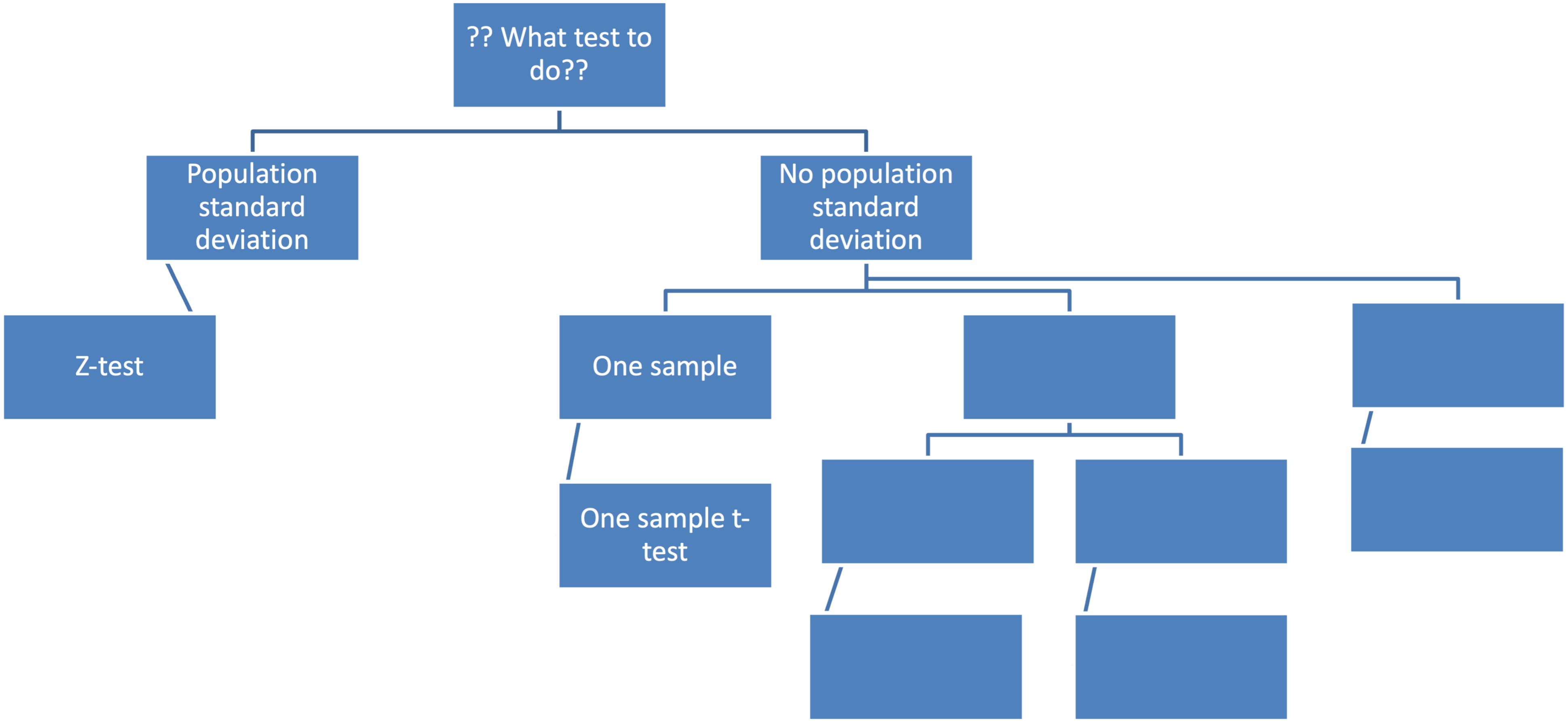
Effect size

$$d = \frac{\overline{X} - \mu_0}{s_X} \quad \text{where} \quad s_X = \sqrt{\frac{SS}{n-1}}$$

Do it together

Say we are interested in knowing whether this class's ratings of Pavilion food differs from the rest of the UC population. The average rating of Pavilion food is 50, and we do not have any information about the standard deviation in the population. Use the α-level of .05.

Big picture



?? What test to do??
Population standard deviation
No population standard deviation
Z-test
One sample
One sample t-test

Independent-sample t-test

What does it test?

To compare two samples whether they are different from each other

Note

We use the t-distribution based on df of $n_1 + n_2 - 2$

But, when?

We know the population mean but not the population standard deviation (MORE REALISTIC!)

Independent-sample t-test

Idea

We have a difference between the two sample means (estimation) and want to test the likelihood of observing this sample mean difference (or more extreme) if the null hypothesis were true (hypothesis testing)

In doing estimation and hypothesis testing, we have made a sampling distribution of the difference of the two means. Now, let's quantify how close/far our sample mean difference is from our expectation

Independent-sample t-test

Test-statistic (**t-statistic**) when we know μ_0 but do not know $\sigma_{\overline{X}}$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}} \quad \text{where} \quad s_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}$$

$$s_p^2 = \frac{s_1^2 + s_2^2}{2} \quad \text{when} \quad n_1 = n_2$$

$$s_p^2 = \frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2} \quad \text{when} \quad n_1 \neq n_2$$

Independent-sample t-test

Effect size

$$d = \frac{\overline{X}_1 - \overline{X}_2}{s_p} \quad \text{where}$$

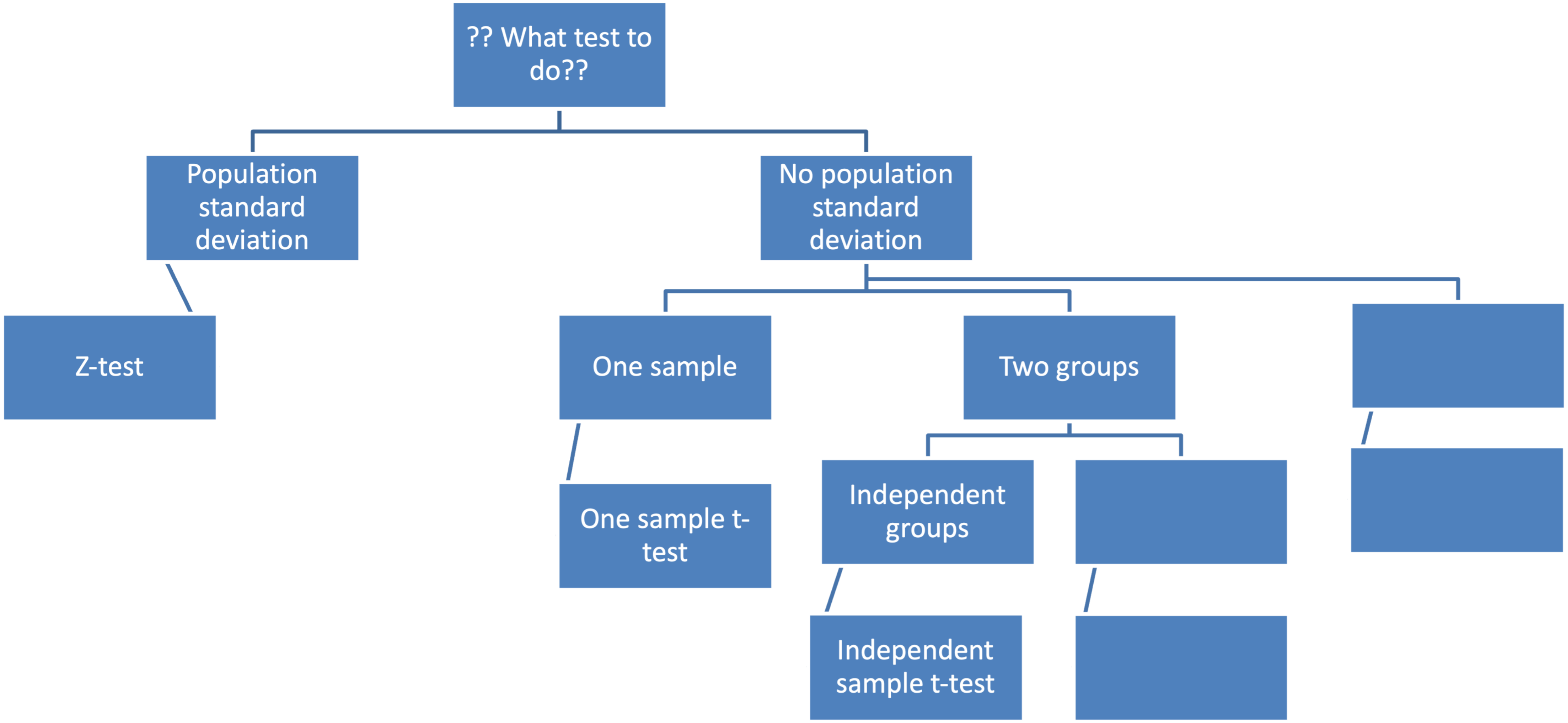
$$s_p = \sqrt{\frac{s_1^2 + s_2^2}{2}} \quad \text{when} \quad n_1 = n_2$$

$$s_p = \sqrt{\frac{s_1^2(n_1 - 1) + s_2^2(n_2 - 1)}{n_1 + n_2 - 2}} \quad \text{when} \quad n_1 \neq n_2$$

Do it together

Does the flight distance of a paper plane differ by plane size (small vs. large)? Use the α-level of .05.

Big picture



?? What test to do??
Population standard deviation
No population standard deviation
Z-test
One sample
Two groups
One sample t-test
Independent groups
Independent sample t-test

Summary

Z-test

When to compute (one sample mean; know population SD) & how to compute?

One-sample T-test

When to compute (one sample mean; don't know population SD) & how to compute?

Independent-sample T-test

When to compute (difference of two sample means; don't know population SD) & how to compute?

Before you go home...

Any questions or comments?

Thanks! Have a nice weekend!