Analysis of Psychological Data

Lab 13. We Go Together: Correlation and Regression

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Announcement

Assignment 5 is due December 7 (next Tuesday)

Exam 3 is on December 9 (next Thursday)

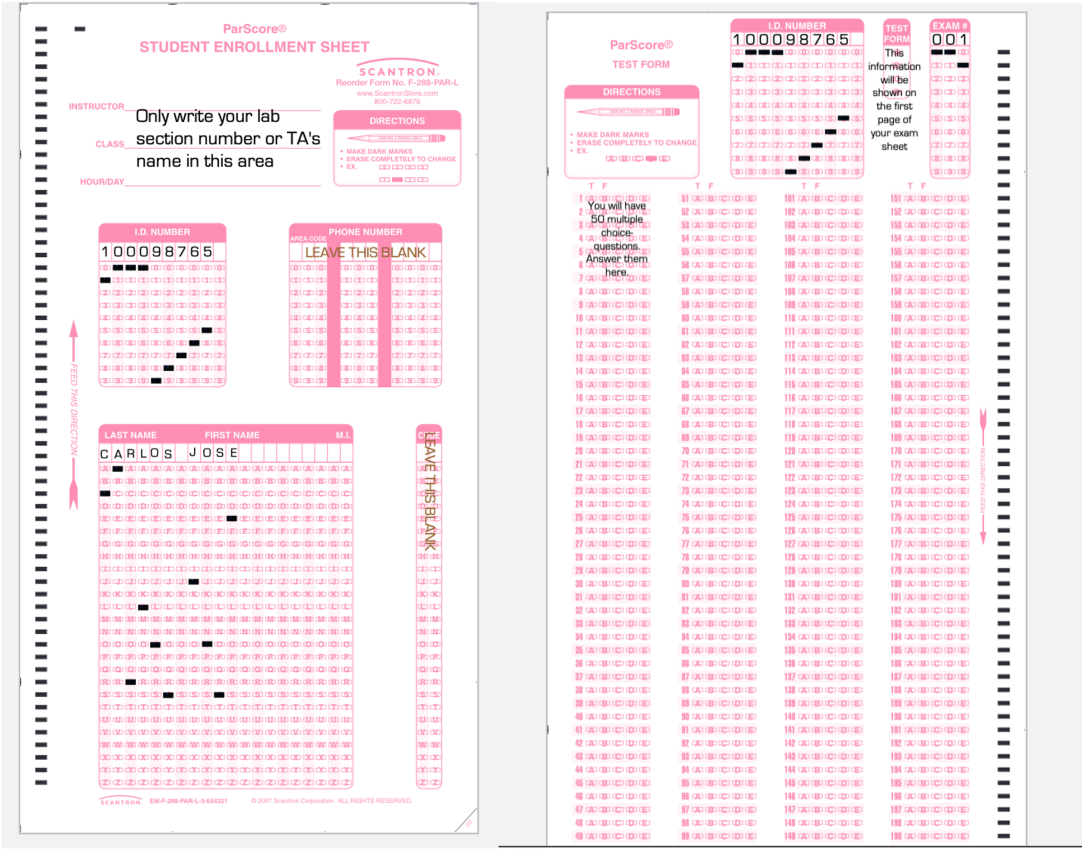
Prepare for the correct scantron form (see next slide) $\longrightarrow$ Otherwise, you get a score of 0

Work individually or in pairs

Bring pencil, eraser, pen, calculator, notes, F-table, correlation table

Announcement

Scantron form (red and large one)



What are we going to do?

Recap to give you a big picture

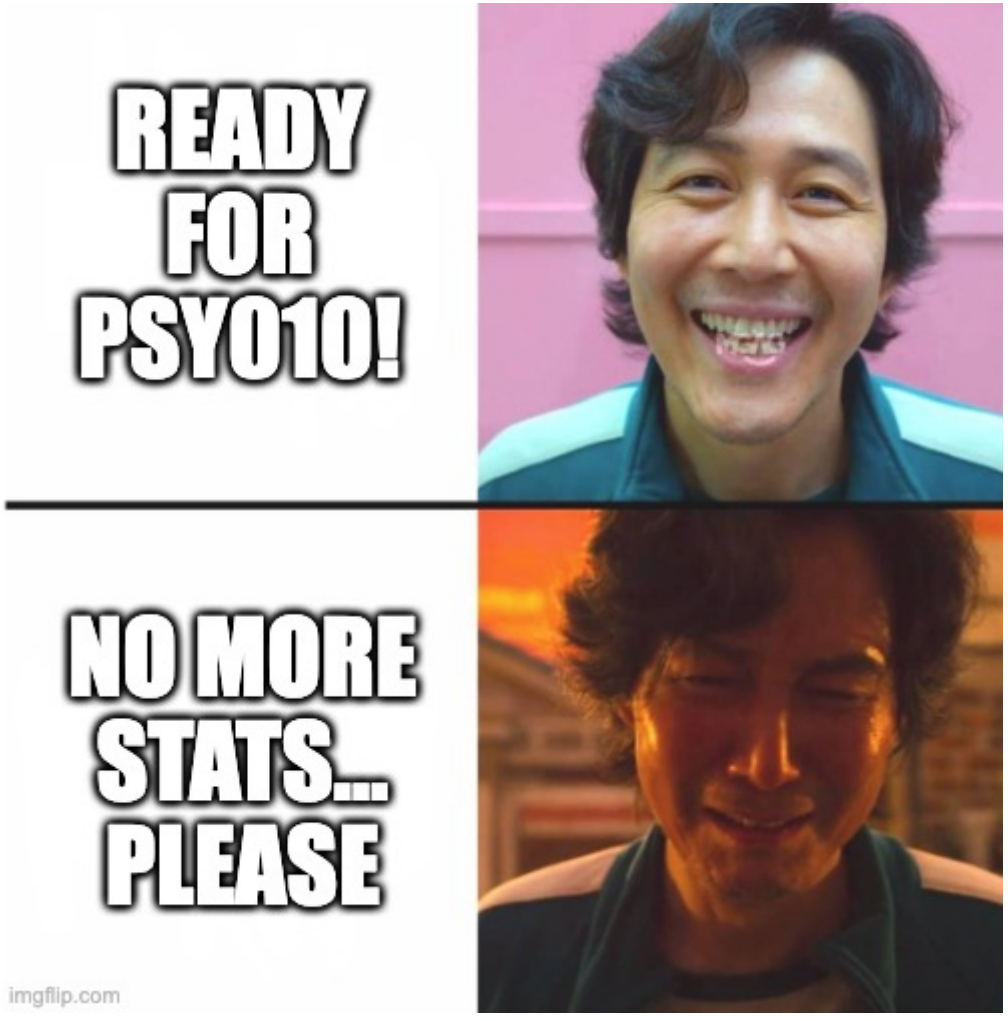
Correlation

Regression

... and Metascience

Do it together

It's been a long, long semester



It's been a long, long semester

What you've learned...

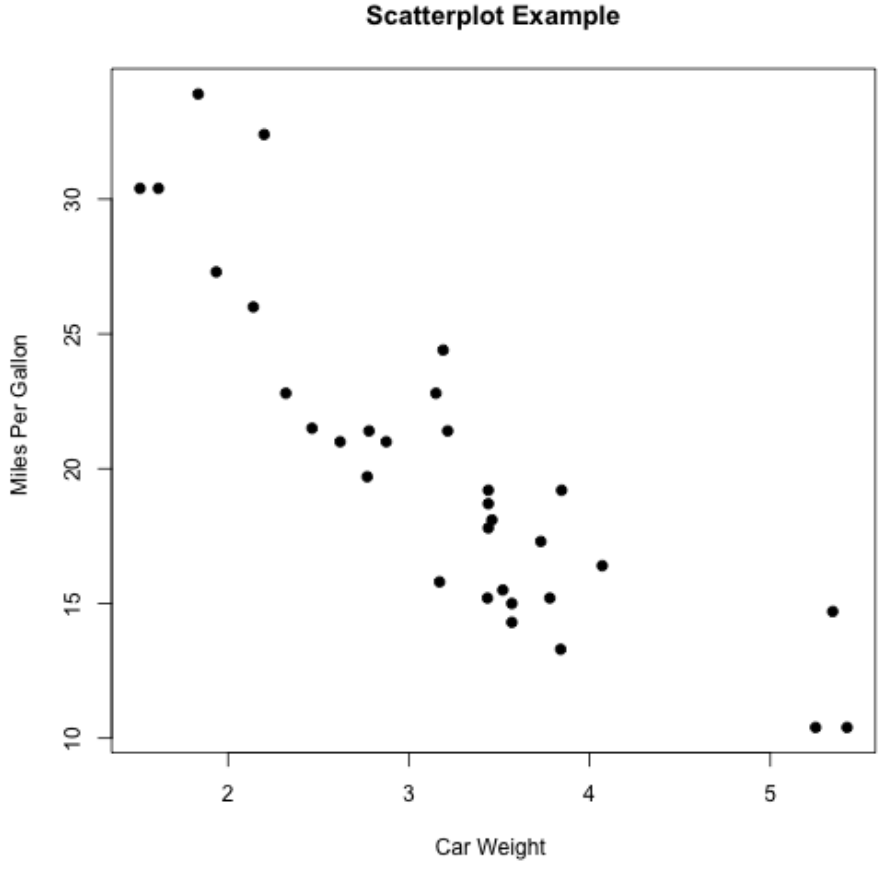
Fundamentals (design, measurement, analysis), different research designs (experimental, quasi-experimental, correlational), descriptive vs. inferential statistics, measurement scale (nominal, ordinal, interval, ratio), data visualization (bar chart, pie chart, histogram, scatterplot), central tendency (mean, median, mode), variability (range, variance, standard deviation), population parameters vs. sample statistics, normal distribution, z-score, central limit theorem, standard error, (frequentist) null hypothesis significance testing, significance level, p-value, z-test, test-statistic, effect size (Cohen's d, eta-squared, partial eta-squared), z-table, t-test (one-sample, independent-sample, related-sample), t-table, type I and type II error, power, one-way ANOVA (sum of squares, mean square, F-value), F-table, factorial ANOVA, repeated measures ANOVA, correlation, correlation table, regression

Correlation

Do you remember scatterplot?

Shows how two variables are related

```
plot(mtcars$wt, mtcars$mpg,
   main="Scatterplot Example",
   xlab="Car Weight ",
   ylab="Miles Per Gallon ", pch=19)
```



Correlation

Goal

To describe a relationship between two variables (i.e., how two variables are related)

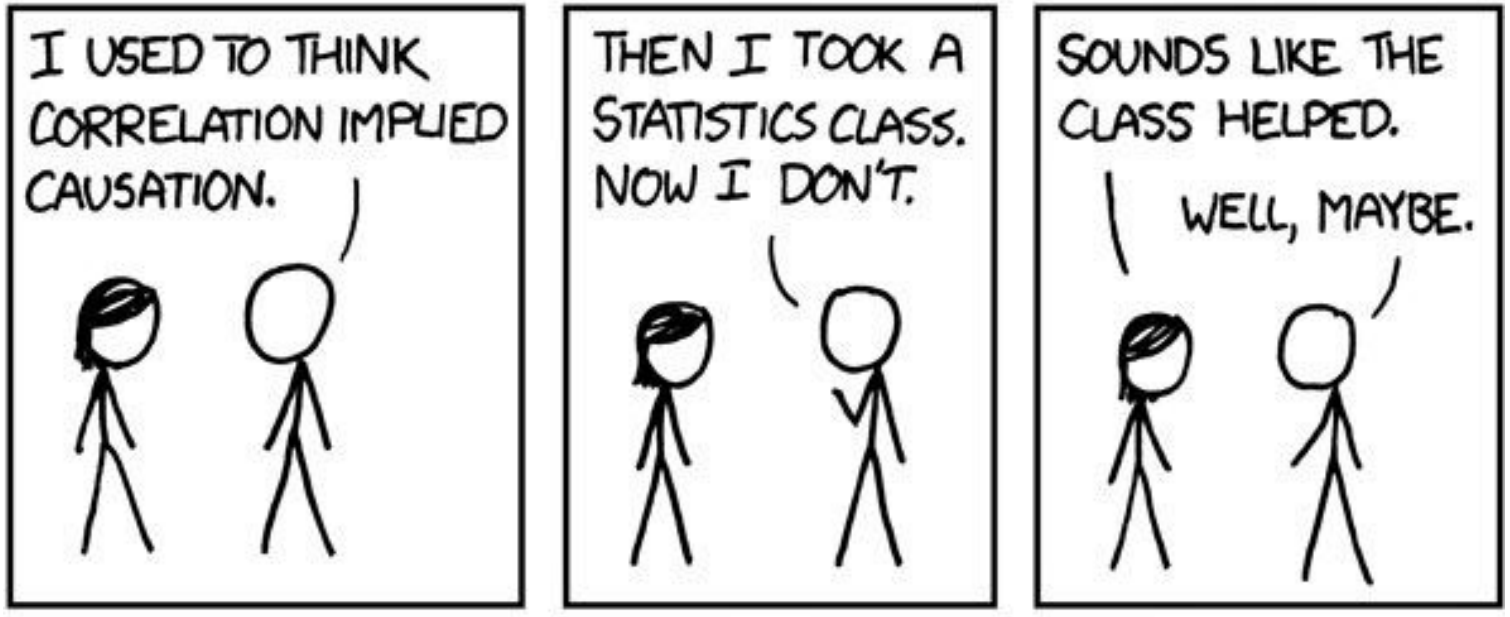
Real-life examples

The longer time you drive, the more gasoline your will need

The taller the person is, the heavier the person is

Correlation

Does not mean causation



Correlation

Two key ideas

Direction $\longrightarrow$ positive vs. negative

Strength $\longrightarrow$ perfect vs. strong vs. weak vs. no

Need to know how to interpret

Scatterplot

Correlation coefficient

Correlation

Direction

Positive $\longrightarrow$ As one variable increases, the other increases

Negative $\longrightarrow$ As one variable increases, the other decreases

Strength

How much two variables covary $\longrightarrow$ Degree to which the data fall on a straight line

Correlation

Scatterplot

One variable is on the x-axis whereas the other variable is on the y-axis

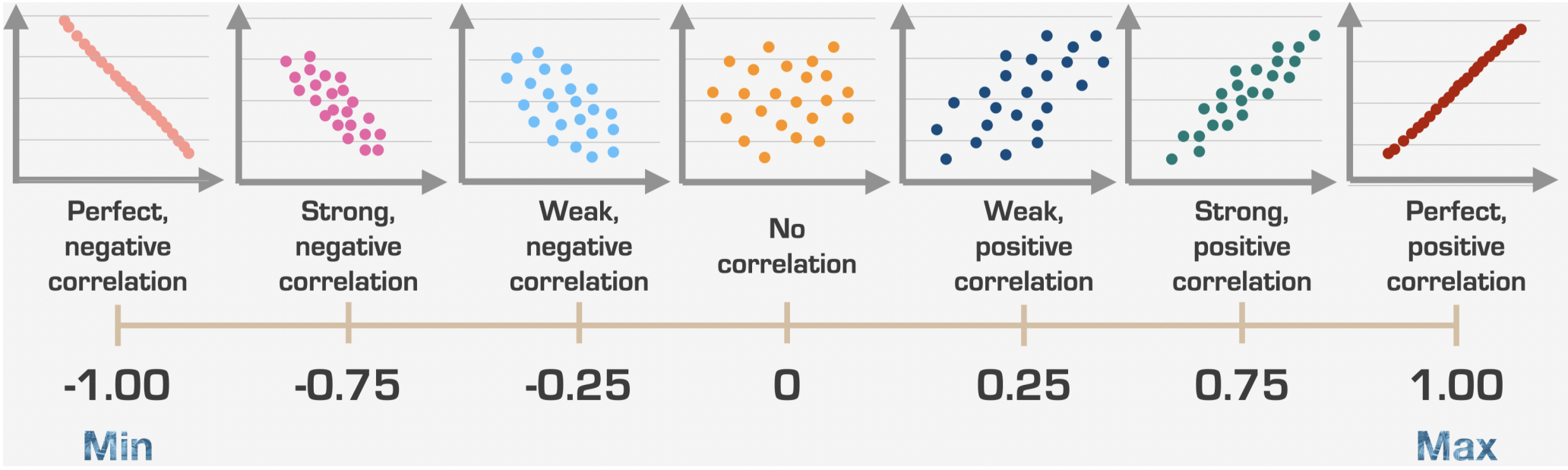
Correlation coefficient

Correlation coefficient ragnes from -1 (perfect negative) to 1 (perfect positive)

Mathematically, correlation is a standardized covariance $\longrightarrow$ We'll do it together

Correlation

Direction and strength



Correlation

Pearson's correlation coefficient ← standardized covariance

A statistic that describes the direction and the strength of the linear relationship

$$r_{XY} = \frac{Cov_{XY}}{s_X s_Y} = \frac{SS_{XY}}{\sqrt{SS_X SS_Y}}$$

SS_{XY} is the sum of products of deviations for two variables X and Y

SS_X is the sum of squares for one variable X

SS_Y is the sum of squares for the other variable Y

Do it together - Correlation

Adrian, Alejandro, Amy, and Roberto are data scientists at Instagram. They want to investigate whether the length of time spent on Instagram is correlated with satisfaction with the app. They collect a sample of 5 Instagram users. Each user reported length of time on the app per week and satisfaction with the app on a 1-5 scale (1 = not satisfied, 5 = very satisfied).

Test whether Pearson's correlation coefficient is significant for this dataset.

Do it together - Correlation

1. State the null and alternative hypothesis

2. Choose your α-level of significance

3. Determine the degrees of freedom

4. Locate the critical r value

5. Calculate Pearson's r

6. Compare observed and critical r value $\longrightarrow$ Reject the null hypothesis if the absolute value of the observed r value is greater than the absolute value of the critical r value

Do it together - Correlation

State hypotheses

$H_0 : r = 0$

The population correlation coefficient between time and satisfaction is zero.

$H_1 : r \neq 0$

The population correlation coefficient between time and satisfaction is different from zero.

Choose your α-level of significance

Let's use two-tailed test with the α-level of 0.05

Do it together - Correlation

Determine the degrees of freedom

$df = n - 2$ where n refers to the sample size

In our example...

$$df = n - 2 = 5 - 2 = 3$$

Do it together - Correlation

Locate the critical r value, which is 0.878

Table of Critical Values: Pearson Correlation

	1-tailed		
	0.05	0.025	0.005
	2-tailed *Degrees of Freedom = N-2*		
df	**0.1**	**0.05**	**0.01**
1	0.988	0.997	0.999
2	0.900	0.950	0.990
3	0.805	0.878	0.959
4	0.729	0.811	0.917
5	0.669	0.754	0.875
6	0.621	0.707	0.834
7	0.584	0.666	0.798
8	0.549	0.632	0.765

Do it together - Correlation

Pearson's correlation coefficient ← standardized covariance

Formula is as follows:

$$r_{XY} = \frac{Cov_{XY}}{s_X s_Y} = \frac{SS_{XY}}{\sqrt{SS_X SS_Y}}$$

SS_{XY} is the sum of products of deviations for two variables X and Y

SS_X is the sum of squares for one variable X

SS_Y is the sum of squares for the other variable Y

Do it together - Correlation

Data in a table

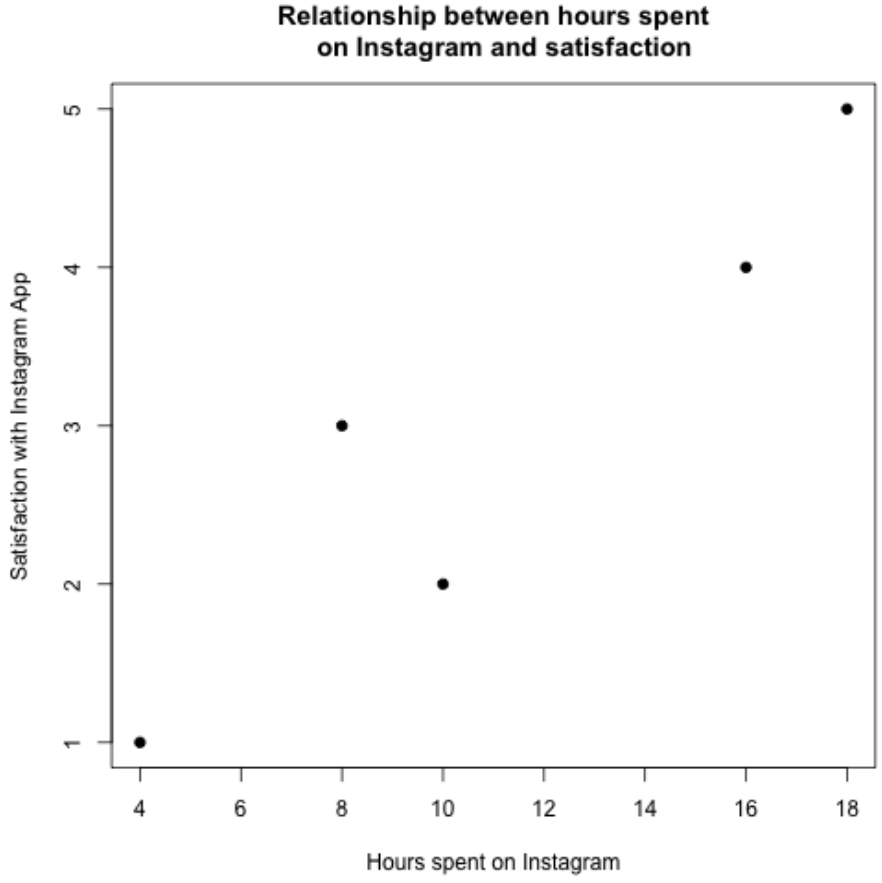
Time (X)	Satisfaction (Y)	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})(Y - \bar{Y})$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$
18	5					
8	3					
4	1					
16	4					
10	2					
$\bar{X} = ?$	$\bar{Y} = ?$			$SS_{XY} = ?$	$SS_X = ?$	$SS_Y = ?$

Do it together - Correlation

Scatterplot of the Instagram example

```
insta.time <- c(18,8,4,16,10)
insta.satis <- c(5,3,1,4,2)
insta.dat <- data.frame(insta.time, insta.satis)

plot(insta.dat$insta.time, insta.dat$insta.satis,
   main="Relationship between hours spent
   on Instagram and satisfaction",
   xlab="Hours spent on Instagram",
   ylab="Satisfaction with Instagram App", pch=19)
```



Do it together - Correlation

Calculation

Time (X)	Satisfaction (Y)	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})(Y - \bar{Y})$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$
18	5	6.8	2	13.6	46.24	4
8	3	-3.2	0	0	10.24	0
4	1	-7.2	-2	14.4	51.84	4
16	4	4.8	1	4.8	23.04	1
10	2	-1.2	-1	1.2	1.44	1
$\bar{X} = 11.2$	$\bar{Y} = 3$			$SS_{XY} = 34$	$SS_X = 132.8$	$SS_Y = 10$

Do it together - Correlation

Correlation coefficient

$$r_{XY} = \frac{Cov_{XY}}{s_X s_Y} = \frac{SS_{XY}}{\sqrt{SS_X SS_Y}} = \frac{34}{\sqrt{132.8 \times 10}} = 0.933$$

What does this correlation coefficient signal to us?

Direction: Positive

Strength: Strong

Do it together - Correlation

Make a decision

$r_{obt} = 0.933$

$r_{crit} = 0.878$

Reject H_0 if p-value < α

$\iff |r_{obt}| > |r_{crit}| \iff |0.933| > |0.878| \iff 0.933 > 0.878$

In our example, the null hypothesis is rejected. The conclusion is that there is a significant positive correlation between time spent on the Instagram app and satisfaction with the app (i.e., correlation is not 0, there is an effect).

Do it together - Correlation

Effect size $\longrightarrow$ Coefficient of determination r^2

$$r^2 = 0.933^2 = 0.87049$$

Interpretation

The length of time spent on the Instagram app accounted for 87.05% of the variance in the user's satisfaction with the Instagram app

You can also say in the other way around $\longrightarrow$ User's satisfaction with the Instagram app accounts for 87.05% of the variance in the length of time spent on the Instagram app

Regression

Goal

To predict values of the dependent variable (Y) from values of the independent variable (X)

Real-life examples

If you drive 5 hours, how many gallons of gasoline you will need

If the person is 6 feet, how heavy the person would be

Regression

Regression equation

$$\hat{Y}_i = b + mX_i$$

$\hat{Y}_i$ is a value of Y on the regression line for individual i $\longrightarrow$ predicted value of Y
X_i is an individual i's value on the independent variable

b is an intercept $\longrightarrow$ a value of Y when X is 0
m is a slope $\longrightarrow$ rate of change in Y when X changes by one-unit

Regression equation about $\hat{Y}_i$ is also referred to as a prediction line

Regression

Regression equation

$$Y_i = b + mX_i + e_i$$

Y_i is an individual i's value on the dependent variable $\longrightarrow$ observed value of Y
X_i is an individual i's value on the independent variable

b is an intercept $\longrightarrow$ a value of Y when X is 0
m is a slope $\longrightarrow$ rate of change in Y when X changes by one-unit

e_i an error (a residual) for individual i $\Longleftrightarrow e_i = Y_i - \hat{Y}_i$

Regression

Slope $\longrightarrow m$

$$\hat{Y}_i = b + mX_i$$

Rate of change in Y when X changes by one-unit

$$m = \frac{SS_{XY}}{SS_X} = \frac{\Sigma_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma_{i=1}^{N}(X_i - \bar{X})^2}$$

SS_{XY} is the sum of products of deviations for two variables X and Y
SS_X is the sum of squares for one variable X

Regression

Intercept $\longrightarrow b$

$$\hat{Y}_i = b + mX_i$$

A value of Y when X is 0

After you calculate the slope, input $\bar{X}$ to X_i and $\bar{Y}$ to $\hat{Y}_i$ to obtain the intercept

Do it together - Regression

Blanca, Carly, Karla, Bianca, and Anthoni, another team of data scientists at Instagram, wants to investigate if hours spent on Instagram predicts satisfaction with the Instagram app. They use the same data collected for correlation analysis.

In this case...

Dependent variable (Y): satisfaction

Independent variable (X): Hours

Do it together - Regression

Several values we obtained before

Time (X)	Satisfaction (Y)	$X - \bar{X}$	$Y - \bar{Y}$	$(X - \bar{X})(Y - \bar{Y})$	$(X - \bar{X})^2$	$(Y - \bar{Y})^2$
18	5	6.8	2	13.6	46.24	4
8	3	-3.2	0	0	10.24	0
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$\bar{X} = 11.2$	$\bar{Y} = 3$			$SS_{XY} = 34$	$SS_X = 132.8$	$SS_Y = 10$

Do it together - Regression

Regression equation

$$\hat{Y}_i = b + mX_i$$

Slope

$$m = \frac{SS_{XY}}{SS_X} = \frac{\Sigma_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma_{i=1}^{N}(X_i - \bar{X})^2} = \frac{34}{132.8} = 0.26 \rightarrow \hat{Y}_i = b + 0.256X_i$$

Intercept

Input $\bar{X}$ to X_i and $\bar{Y}$ to $\hat{Y}_i \rightarrow 3 = b + 0.256 \times 11.2 \rightarrow b = 3 - 0.256 \times 11.2 = 0.133$

Do it together - Regression

Regression equation

$$\hat{Y}_i = 0.133 + 0.256 X_i$$

Interpretation

Slope of 0.256 $\longrightarrow$ 1-hour increase in using the Instagram app leads to 0.256 point increase in satisfaction

Intercept of 0.133 $\longrightarrow$ When the independent variable (i.e., hours) is 0, the dependent variable (i.e., satisfaction) is 0.133

Do it together - Regression

Regression equation

$$\hat{Y}_i = 0.133 + 0.256 X_i$$

Let's do prediction

If the user uses 10 hours, what would be a satisfaction score? Input 10 to X

$$\hat{Y}_i = 0.133 + 0.256 X_i = 0.133 + 0.256 \times 10 = 0.133 + 2.56 = 2.693$$

Satisfaction score would be 2.693

Regression

Other measures you can compute (FYI; details are on lecture slides)

Correlation coefficient between the dependent variable and the independent variable

$$r_{XY} = \frac{SS_{XY}}{\sqrt{SS_X SS_Y}}$$

Coefficient of determination $\longrightarrow$ Proportion of variance in Y explained by X

$$r_{XY}^2 = (r_{XY})^2 = r_{XY} \times r_{XY}$$

Regression

Other measures you can compute (FYI; details are on lecture slides)

Sum of squares in the DV explained by IV

$$r^2 SS_Y = r^2 \times SS_Y$$

Sum of squares in the DV NOT explained by IV

$$(1 - r^2) SS_Y = (1 - r^2) \times SS_Y$$

Regression

Other measures you can compute (FYI; details are on lecture slides)

We can use ANOVA to examine if IV predicts DV well

F-value $\longrightarrow$ test statistics

$$F = \frac{MS_{regression}}{MS_{residual}} = \frac{\frac{SS_{regression}}{df_{regression}}}{\frac{SS_{residual}}{df_{residual}}} = \frac{\frac{r^2 SS_Y}{1}}{\frac{(1-r^2)SS_Y}{N-2}}$$

where N is the sample size

Regression

Other measures you can compute (FYI; details are on lecture slides)

Standard error

Standard deviation or distance that data points in a sample fall from the regression line

$$S_e = \sqrt{MS_{residual}} = \frac{SS_{residual}}{df_{residual}} = \frac{(1-r^2)SS_Y}{N-2}$$

where N is the sample size

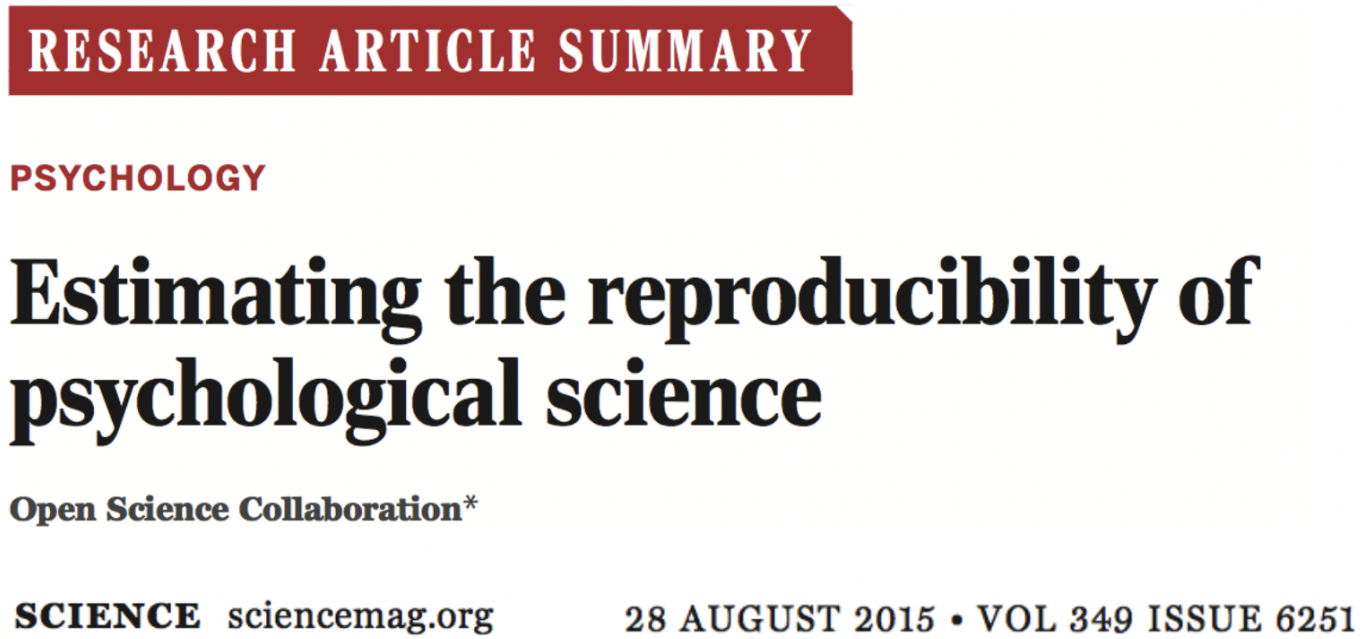
Metascience

Recall that I kept emphasizing the problems of p-values... but why?

Metascience

The replication crisis in psychology

Only 36% of the significant findings (p-value below 0.05) was replicable



RESEARCH ARTICLE SUMMARY

PSYCHOLOGY

Estimating the reproducibility of psychological science

Open Science Collaboration*

SCIENCE sciencemag.org 28 AUGUST 2015 • VOL 349 ISSUE 6251

Metascience

One of the problems pointed out was the p-value

The Earth Is Round ($p < .05$)

Jacob Cohen

After 4 decades of severe criticism, the ritual of null hypothesis significance testing—mechanical dichotomous decisions around a sacred .05 criterion—still persists. This

sure how to test H_0, chi-square with Yates's (1951) correction or the Fisher exact test, and wonders whether he has enough power. Would you believe it? And would you

Metascience

A movement to redefine or ban the p-value

nature human behaviour

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Comment | Published: 01 September 2017

Redefine statistical significance

Daniel J. Benjamin ✉, James O. Berger, ... Valen E. Johnson ✉ + Show authors

Nature Human Behaviour **2**, 6–10 (2018) | Cite this article

139k Accesses | **964** Citations | **854** Altmetric | Metrics

We propose to change the default *P*-value threshold for statistical significance from 0.05 to 0.005 for claims of new discoveries.

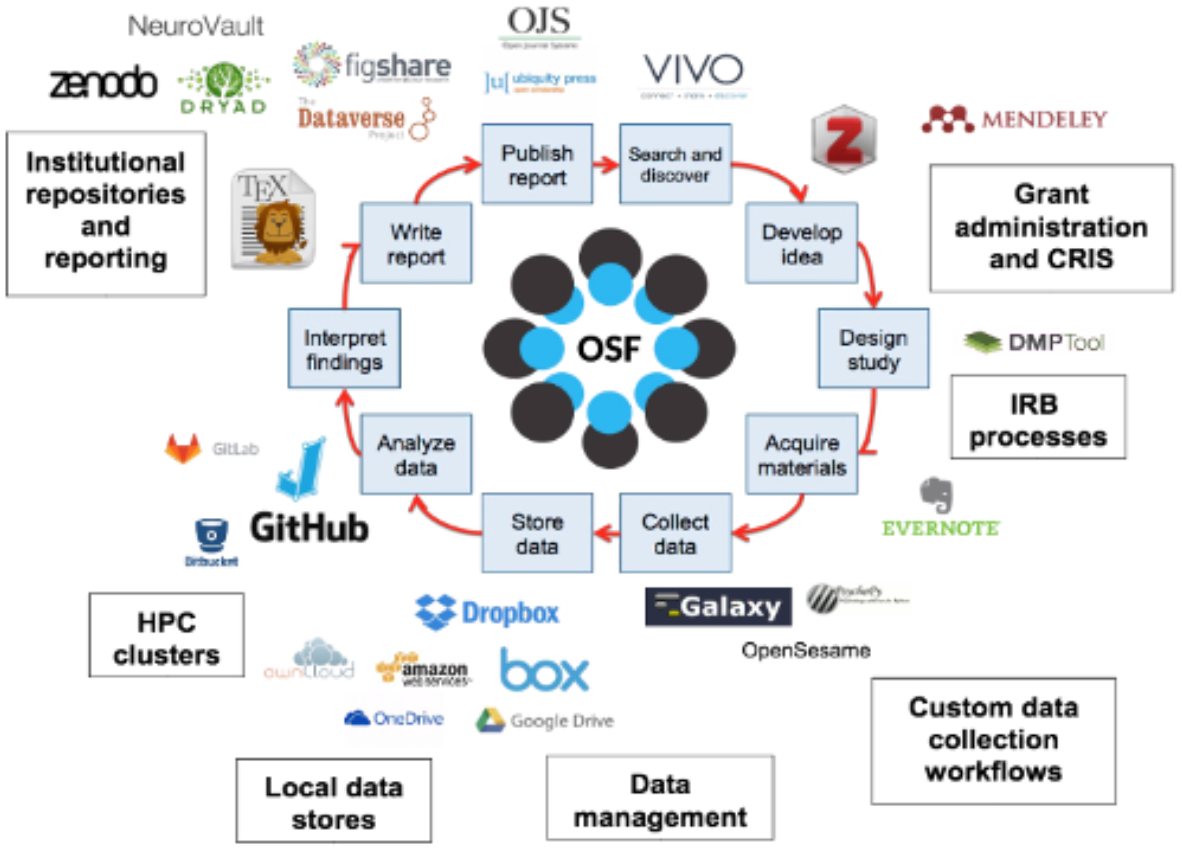
Metascience

Stop using p-values!

Prof. Dr. Rens van de Schoot (https://www.youtube.com/watch?v=owb5A8lVS0o)

Metascience

Open Science Movement



Concluding remark

What I want to convey to you...

Be critical in doing everything in your life

Be transparent in doing science

Enjoy your undergraduate life (jealous of you!)

Before we finish our semester...

Any **FINAL** questions or comments?

Thank you for the great semester! Cheers!