Measuring and Predicting Running Time

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CSC220 Programming II – Spring 2016

Outline

Running times of different implementations

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- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.

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- Each has implementations of find, add, and removeEntry.

Running times of different implementations

- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.
- Each has implementations of find, add, and removeEntry.
- Can we compare their speeds?

find

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- ArrayBasedPD.find

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- SortedPD.find
  - Only really helpful when n (size) is large.

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  - Look for Vic?
  - Have to compare Vic with n entries, where $n =$ size, which is 6.
- SortedPD.find
  - Only really helpful when n (size) is large.
  - Requires $\log_2 n$ comparisons

add

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  - Abe, Ann, Bob, Eve, Ian, Jay, Zoe
  - n array accesses. Actually $n - 1$ reads and n writes, where n is 7. So $2n - 1$.
  - Total time is $\log_2 n$ comparisons plus $2n - 1$ array accesses.

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  - Call to find takes n comparisons.

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  - The program still uses 2 array accesses to “remove” Eve (but it could be smarter).

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  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.

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  - find takes 1 comparison to find Jay.
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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
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  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?

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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - The program still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.

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  - Who takes longest to remove? Jay?
  - removeEntry calls find.
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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
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  - Time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - So Eve is worst case, requiring time for n comparisons and 2 array accesses.
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  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - Bob, Eve, Ian, Jay, Zoe
  - n array reads and writes to move everyone else back.

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  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - Bob, Eve, Ian, Jay, Zoe
  - n array reads and writes to move everyone else back.
  - Total is $\log_2 n$ comparisons and $2n$ array accesses. Actually the first n should be $n - 1$.

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- Accurate, up to a constant factor, for large n.

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  - SortedPD add is slower.

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  - SortedPD removeEntry is the same.

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- We can measure the running time for one value of n and use that to extrapolate the running time for another value of n. Here is how to do it.
- Suppose ArrayBasedPD.find takes 10 microseconds for $n = 100$.
- Since the running time t is in $\mathrm{O}(n)$, we have
  - $t = c \cdot n$

How to predict running time.

- So what use is $\mathrm{O}(1)$ or $\mathrm{O}(\log n)$, $\mathrm{O}(n)$, or $\mathrm{O}(n \log n)$ if we don't know that constant, especially if it is a different constant in each case?
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- Suppose ArrayBasedPD.find takes 10 microseconds for $n = 100$.
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  - $t = c \cdot n$
  - $10 = c \cdot 100$

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  - $t = 1/10 \cdot 1000$
  - $t = 100$
- So the answer is 100 microseconds.

Another extrapolation

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  - $50 = c \cdot 2$

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  - $50 = c \cdot 2$
  - $c = 25$

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  - $t = 25 \cdot 3$

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- Even though the original analysis of binary search was for $\log_2 n$, I can use any base I want to because all logs differ by a constant factor.
- But you must use the *same* base for *every* log in the calculation.
- For $n = 1000$,
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  - $t = 75$

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  - $t = 25 \cdot 3$
  - $t = 75$
- So 75 microseconds.

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  - $t = 25 \cdot 3$
  - $t = 75$
- So 75 microseconds.
- Notice that I used the same log base 10. You can't switch log bases in the middle, or you will get a different (and wrong) answer.

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  - $t = c \cdot \ln n$
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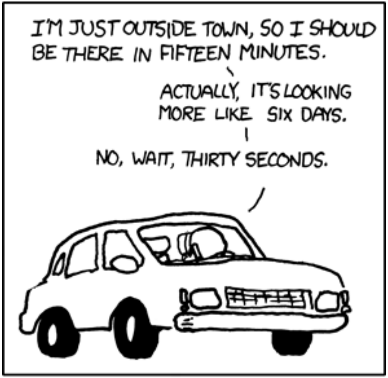
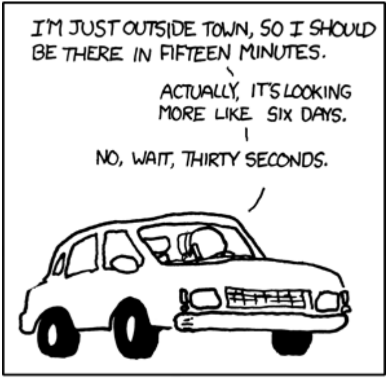
  - $t = 10.857 \cdot 6.9077$
  - $t = 74.997$
- Different log. Same answer!



THE AUTHOR OF THE WINDOWS FILE COPY DIALOG VISITS SOME FRIENDS.



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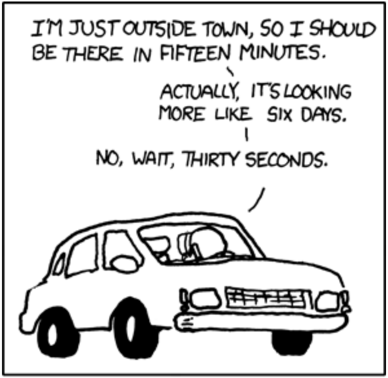
- Let's discuss this joke. In general the estimate is terrible and then gets better and better. Why?



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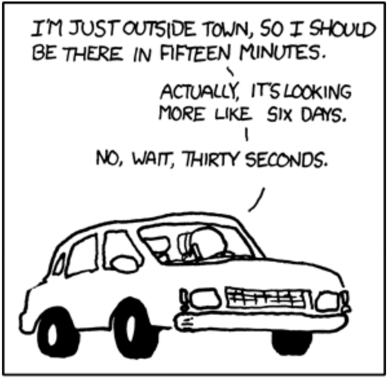

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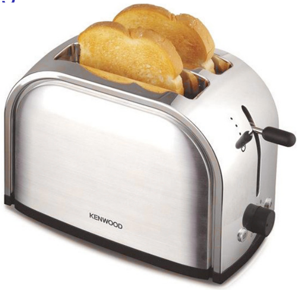
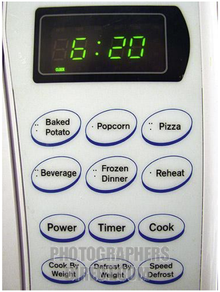
- Let's say you do an experiment and it generates a number. It could be a time or a mass or anything. Just something you can measure. Unfortunately, the result is not very accurate. How can we increase the accuracy?
- Answer: repeat the experiment many times and take the average.

Timing the Toaster





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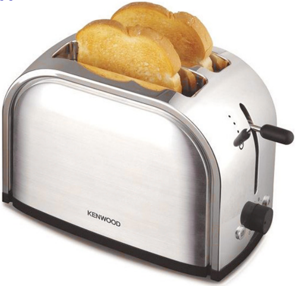
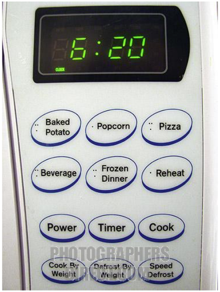
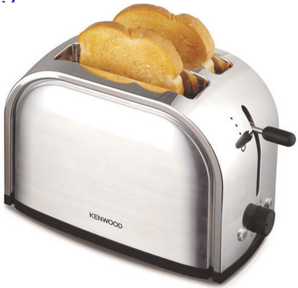
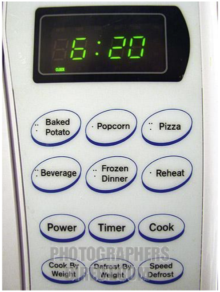




- I want to know how long my toaster takes (on setting 4–lightly browned),

Timing the Toaster





- I want to know how long my toaster takes (on setting 4–lightly browned),
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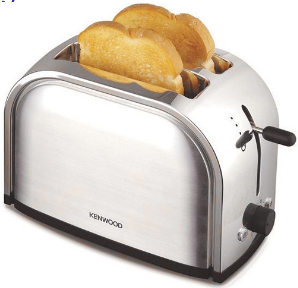
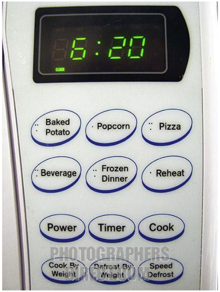
Timing the Toaster





- I want to know how long my toaster takes (on setting 4–lightly browned),
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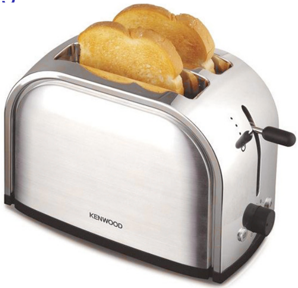
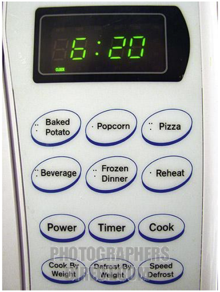
Timing the Toaster





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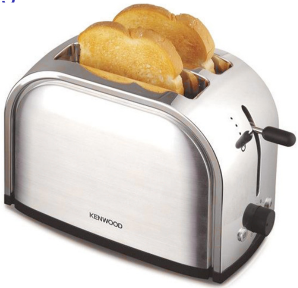
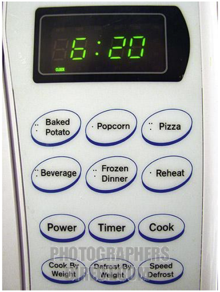
- I start the toaster when the clock says 6:20 (I could not find an image with 5:30 on it). What time will it say when the toaster pops?

Timing the Toaster





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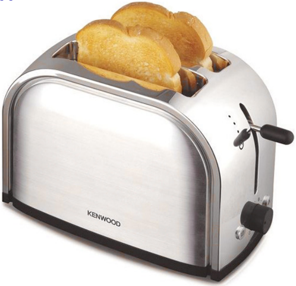
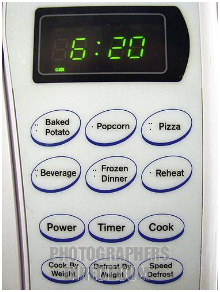
  a. 6:22

Timing the Toaster





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  a. 6:22
  b. 6:23

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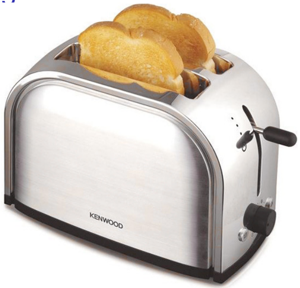

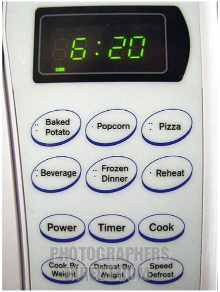

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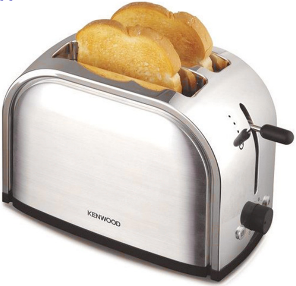

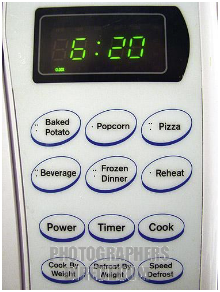

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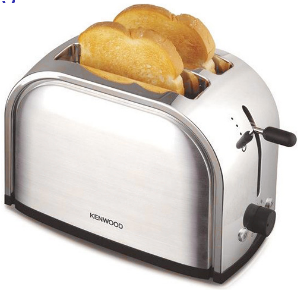

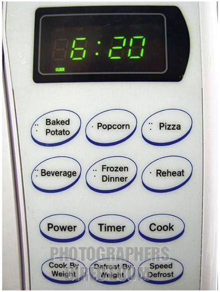

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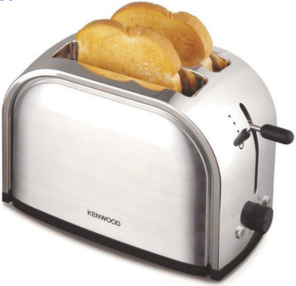

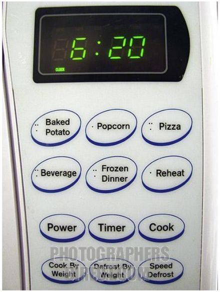

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- Answer: c.
- It will say 6:22 with probability $2/3$ and 6:23 with probability $1/3$.

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1000	2.328
10000	2.3438
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- The Central Limit Theorem implies that averaging over n trials increases the accuracy by a factor of $\sqrt{n}$.

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  - $n \cdot t = 10^8 \cdot 0.1 = 10^7$ microseconds, which is 10 seconds.

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- Accurate predictions can make or break a business and save millions of dollars.
- To improve the accuracy of a measurement, repeat it many times and take an average.
- According to the Central Limit Theorem, if the time is t, we need to repeat it $n = 10^6/t^2$ times to get three significant figures of accuracy.